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DESIGN OF REINFORCED CONCRETE STRUCTURES

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DESIGN OF REINFORCED CONCRTE STRUCTURES

PART I—THEORY

Introduction

§1. Cement concrete consists of hard inorganic materials called aggregates such as gravel, sand, crushed stone slag *etc.* cemented together with Portland Cement and water. When the above materials are mixed together so as to form a workable mixture it can be moulded or cast into beams and slabs. A few hours after mixing, the materials undergo a chemical combination and the mixture solidifies and hardens. Concrete attains greater strength with age.

Concrete possesses a high compressive strength and is usually more economical than steel and is not subjected to corrosive weathering and such effects. Hence concrete is used in all present day constructions. But, concrete has a poor tensile strength and is liable to be cracked when subjected to tension. It also develops shrinkage stresses. Hence, by providing steel reinforcement within the concrete mass at the time of pouring, the resulting member will have both the properties of concrete and steel. By reinforcing the concrete with steel, the defects of concrete are made good.

§2. Requirements of Good Concrete

In making sound and durable concrete the prime requirements are the following :

(*i*) The aggregates should be hard and durable.

(*ii*) The aggregates shall be properly graded in size from fine to coarse.

(*iii*) Cement should be of sufficient quantity to produce the required water-tightness and strength.

(*iv*) The water used while mixing shall be free from organic material or any deleterious minerals.

(*v*) The quantity of water should be such as to produce the needed consistency.

(*vi*) Mixing should be done thoroughly so as to produce homogeneity.

(*vii*) Concrete should fill every part in the forms. This is done by *ramming* or *puddling*.

(*viii*) Until the concrete is thoroughly hard it is necessary to ensure that the temperature of concrete is maintained above the freezing point. This is done to avoid retarded hardening.

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As mentioned earlier, concrete is a product obtained by mixing cement, aggregates and water in a planned definite proportion so that the resulting concrete is satisfactory in meeting the need for the job for which it is meant, especially regarding its workability, strength, economy and durability. The strength of cement concrete is governed by the properties of its constituents and their relative quantities and the process of mixing them. It is therefore desirable to study in some detail the properties of the various components of concrete.

§3. Ordinary Portland Cement (*I S. 269*)

This is the cement used for normal concrete construction. It has adhesive and cohesive properties so as to render it to form a good bond with other materials. It solidifies when mixed with water. It is the most active binding medium and is perhaps the most scientifically controlled component in concrete. Cement is obtained by burning a mixture of the following materials in a definite proportion :

(*i*) Silicious materials (containing silica),

(*ii*) Argillaceous materials (containing alumina).

(*iii*) Calcareous materials (containing lime).

The temperature to which the mixture must be burnt is about 1400°C. The clinker so obtained is cooled and powdered to the required fineness. The product so obtained is called cement. Cements of different properties are obtained by mixing the above components in different proportions along with small percentages of other chemicals. Portland Cement is the most important type of cement and is widely used.

Portland Cement consists of the following components :

(*i*) Lime	... 60% to 67%
(*ii*) Silica	... 17% to 25%
(*iii*) Alumina	... 3% to 8%
(*iv*) Iron oxide	...0·5% to 6%
(*v*) Magnesia	...0·1% to 4%
(*vi*) Soda and Potash	...0·2% to 1%
(*vii*) Sulphur trioxide	1% to 2·75%
(*viii*) Free Lime	0% to 1%

Cement is a complicated mixture of chemical compounds. Ordinary Portland Cement contains the following compounds :

(*i*) Tricalcium Silicate $(CaO)_3\ SiO_2$ denoted by C_3S.

(*ii*) Dicalcium Silicate $(CaO)_2\ SiO_2$ denoted by C_2S.

(*iii*) Tricalcium Aluminate $(CaO)_3\ Al_2O_3$ denoted by C_3A.

(*iv*) Tetracalcium Aluminoferrite $(CaO)_4\ Al_2O_3Fe_2O_3$ denoted by C_4AF.

About 70% to 80% of cement is contributed by C_3S and C_2S which are responsible for the strength of cement. These compounds

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provide to cement the property to resist the attacks of acids and alkalies and make the cement durable.

Tricalcium Silicate has the property of hydrating rapidly and is responsible to provide not only early strength but also the ultimate strength.

Dicalcium Silicate has the property of hydrating less rapidly and provides the strength after a duration of 7 days.

Tricalcium Aluminate gets hydrated rapidly and is also responsible to provide early strength but is found to slightly retard the ultimate strength. This compound is susceptable to be attacked by alkalies and salts.

Tetracalcium Aluminoferrite appears to be a redundent compound not known to provide any strength and is at times considered an undesirable compound which gets into cement.

§4. Rapid Hardening Portland Cement (*I S. 8041*)

This type of cement is manufactured in a manner similar to that of Ordinary Portland Cement; but it differs in respect of chemical composition, degree of burning and grinding. This contains a greater lime content than Ordinary Portland Cement. This requires a better control of burning the ingredients over a longer time. The clinker is ground to a much finer extent than the Ordinary Portland Cement. The quick rate of gain of strength is due to a higher Content of Tricalcium Silicate and finer grinding of clinker. For the same water cement ratio, it gains a strength in 3 days what the ordinary cement gains in 7 days; and it gains a strength in 7 days what the ordinary cement gains in 28 days The main advantage in using rapid hardening Portland Cement is that the shuttering may be struck considerably earlier, thus saving considerable time and expenses This cement is particularly satisfactory under conditions of frost.

This type of cement can also be used for road work, so that the road can be open to traffic with least delay.

§5. Low Heat Portland Cement (*I S. 269*)

During the process of setting and hardening of cement, an appreciable rise of temperature occurs. The heat generated in ordinary cement at the end of 3 days may be at the order of 80 *calories per gram* of cement. In massive constructions like abutments, retaining walls, dams *etc.* the rate at which the heat can be lost at the surface is lower than at which the heat is initially generated. The temperature rise is an important factor because the shrinkage which may take place during the cooling process introduces tensile stresses in concrete. This will therefore affect the water tightness of the structure and may lead to deterioration. In the low-heat cement the heat evolved is less. As per the code the heat of hydration for low-heat cement shall not exceed 65 *cal /gm.* at 7 days, and 75 *cal /gm.* at 28 days. The strength of this cement is a little lower than the ordinary cement during the first two or three months but its strength

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after this period is practically the same as that of ordinary cement. This cement offers a better resistance to chemical deterioration than Ordinary Portland Cement.

Low heat cement is obtained by increasing Dicalcium Silicate content, decreasing Tricalcium Silicate content and considerably bringing down Tricalcium Aluminate. The lesser rate of gain of strength in the initial period is due to the reduction of Tricalcium Silicate and Tricalcium Aluminate. However in order the rate of gain of strength may be ensured sufficiently the code recommends a specific surface requirement of 3200 $cm.^2/gm.$

§6. Portland Blast Furnace Slag Cement *(I.S. 455)*

This is the cement obtained by intergrinding Portland Cement clinker with blast furnace slag. The blast furnace slag content shall not exceed 65%. The property of cement is not detracted by the above blending On the contrary the blending provides some advantages. The granulated slag possesses latent hydraulic properties which are activated to a very great extent when the slag is catalysed and interground with Portland Cement clinker. From practical considerations, Portland blast furnace slag cement may be used in all situations for which ordinary Portland Cement is used. In addition, due to low heat evolution, it can be used in huge mass concrete structures like dams, retaining walls, foundations and abutments of bridg s. This cement has a good sulphate resistance and is very suitable for use in constructions in sea water.

§7. Portland–Pozzolana Cement *(I.S. 1489)*

This cement is made either by intergrinding Portland Cement clinker and Pozzolana or by uniformly blending Portland Cement and fine pozzolana. Intergrinding does not present any difficulty, whereas, blending tends to produce a non-uniform product. The I.S. code has suggested that the latter method may be confined to factories where intimate blending can be guaranteed through mechanical means.

The pozzolana content varies from 10% to 25% by weight of cement. Pozzolanas do not possess cementing value themselves but have the property of combining with lime to produce a stable lime-pozzolana compound which possesses cementing property. Since the free lime (which is readily attacked chemically) is removed, the pozzolana concretes have a greater resistance to chemical agencies. They can also resist attack by sea water better than ordinary Portland cement concrete, since the pozzolana cement has a lower heat evolution and is popularly used in the construction of dams (Example : *Bhakra Dam*).

The pozzolana used in the manufacture of cement in India consists of burnt clay or shale or flyash. The pozzolana must satisfy very strictly certain requirements in order to use it successfully. The properties of the Portland Cement will obviously depend upon the quantities of pozzolana used. Thus Portland pozzolana cement is

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TABLE 1. Physical Requirements for Cements (*IS : 269 ; 455 ; 1489*)

Type of Cement	*Ordinary Portland Cement*	*Rapid-hardening Portland Cement*	*Low Heat Portland Cement*	*Portland Blast Furnace Slag Cement*	*Portland Pozzolana Cement*
Fineness #Residue by weight on 90-micron IS sieve	⩾ 10%	⩾ 5%	—	⩾ 10%	⩾ 5%
Minimum specific surface $cm.^2/gm.$	2250	3250	3200	2250	3000
Soundness Expansion (*Le Chatelier*)	⩾ 10 *mm*	⩾ 10 *mm.*	⩾ 10 *mm.*	⩾ 10 *mm.*	⩾ 10 *mm.*
Setting time Initial	⩽ 30 *min.*	⩽ 30 *min.*	⩽ 60 *min.*	⩽ 30 *min.*	⩽ 30 *min.*
Final	⩾ 600 *min.*	⩾ 600 *min.*	⩾ 600 *min.*	⩾ 600 *min.*	⩾ 600 *min.*
Minimum compressive strength 4 *hours*	—	160 *kg./cm.*^{2} (16 *N/mm.*^{2})	—	—	—
72 *hours*	160 *kg./cm.*^{2} (16 *N/mm.*^{2})	275 *kg./cm.*^{2} (28 *N/mm.*^{2})	100 *kg./cm.*^{2} (10 *N/mm.*^{2})	160 *kg./cm.*^{2} (16 *N/mm.*^{2})	—
168 *hours*	200 *kg./cm.*^{2} (22 *N/mm.*^{2})	—	160 *kg./cm.*^{2} (16 *N/mm.*^{2})	220 *kg./cm.*^{2} (22 *N/mm.*^{2})	175 *kg./cm.*^{2} (18 *N/mm.*^{2})
336 *hours*	—	—	—	—	250 *kg./cm.*^{2} (25 *N/mm.*^{2})
672 *hours*	—	—	350 *kg./cm.*^{2} (35 *N/mm.*^{2})	—	—
			Heat of hydration not to exceed 65 *cal./gm.* at 7 days and 75 *cal./gm.* at 28 days		**Average drying shrinkage of mortar not to exceed 0·15%**

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TABLE 2

Chemical Requirements for Cements (*I.S 269 ; 455 ; 1489*)

Type of Cement	*Ordinary and rapid hardening Portland Cement*	*Low heat Portland Cement*	*Portland blast furnace slag cement*	*Portland Pozzolana Cement*
	←— — — *Maximum*	*Permissible values* — — —→		
Insoluble Residue	1·5%	1·5%	1·5%	$x+\frac{1\cdot5(100-x)}{100}$ x = declared percentage of pozzolana in the cement
Magnesia (MgO)	6%	6%	8%	6%
Sulphur as SO_3	2·75%	2·75%	3%	2·75%
Sulphur as Sulphide	—	—	1·5%	—
Oxide of Manganese (Mn_2O_3)	—	—	2%	—
Ignition Loss	4%	4%	4%	5%

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generally stated to possess a lower rate of development of strength, compared to ordinary Portland cement. The following table shows the compressive strengths reached by ordinary Portland cement and Portland pozzolana cement. When pozzolana is selected carefully and is calcined and well ground with Portland cement clinker under controlled conditions, the 7 day strength reached is practically as that reached by Ordinary Portland Cement. We also conclude that using Portland-pozzolana cement will not involve any delay in the removal of shuttering and may therefore be used in reinforced concrete constructions without any difficulty.

TABLE 3

Compressive Strength of Portland Pozzolana Cement and Ordinary Portland Cement

Age in Days	*Compression Strength*	
	Ordinary Portland Cement	*Portland Pozzolana Cement*
1	77 *kg /cm.*^{2} 8 *N/mm*^{2}	77 *kg /cm.*^{2} 8 *N/mm.*^{2}
3	192 *kg./cm.*^{2} 19 *N/mm*^{2}	165 *kg./cm.*^{2} 16 *N/mm.*^{2}
7	256 *kg /cm*^{2} 26 *N/mm.*^{2}	247 *kg./cm.*^{2} 25 *N/mm.*^{2}
14	310 *kg /cm.*^{2} 31 *N/mm.*^{2}	301 *kg./cm.*^{2} 30 *N/mm.*^{2}
15	375 *kg /cm.*^{2} 38 *N/mm*^{2}	375 *kg./cm.*^{2} 38 *N/mm.*^{2}

§8. White Cement

White cement has practically the same composition and has the same strength as Ordinary Portland cement. To obtain the white colour the iron oxide content is considerably reduced (which shall be limited to 1%). The raw materials of this cement are :

(*i*) Limestone (or chalk).

(*ii*) China clay.

Both the above must have low iron content. Sometimes *cryolite* (Sodium aluminium fluoride) is added as a flux to aid in burning. The cement is burnt in a rotary kiln. Fuel firing is adopted in place of coal firing. The trade name for this cement is *Snowcrete*. This cement is generally used as a base for coloured cements. This cement, due to its white colour is mainly used for interior and exterior decorative work like external renderings of buildings, facing slabs, floorings, ornamental concrete products, paths of gardens, swimming pools *etc.*

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§9. Coloured Cement

This is obtained by adding 5% to 10% of ground pigments to ordinary or white cement. The following table indicates the pigments generally used to obtain the designed colours.

TABLE 4

Pigments used to produce Coloured Cement

Colour desired	*Name of pigment used*	*Quantity required in kg. per bag of cement*	
		Light shade	*Medium shade*
Green	*Chromium oxide or greenish blue ultra marine*	2·7	—
Brown to reddish brown	*Metallic brown (oxide)*	2·3	4·1
Bright red to vermilion	*Mineral turkey red*	2·3	4·1
Brownish red to dull brick red	*Red oxide of iron*	2·3	4·1
Blue	*Ultramarine blue*	2·3	4·1
Blue-black	*Carbon black*	0·25	0·5
Black	*Black oxide of manganese*	0·50	1·0
Grey	*German town lamp black*	0·25	0·5

§10. Sulphate Resisting Cement

This cement is similar to ordinary Portland cement, but in this case the cement contains a greater proportion of Tricalcium Silicate and Dicalcium Silicate and a low content of Tricalcium Aluminate and Tetra Calcium Aluminoferrite. The chemical combination of Tricalcium Aluminate and Tetracalcium Aluminoferrite renders the cement, resistant to sulphate attack. This type of cement is particularly useful in the arid western regions of India where the ground water deposits a good amount of sulphates in the soil. Hence in these regions sulphate resistant cement can be used with advantage in culverts, siphons, canal-linings *etc.*

§11. Hydrophobic Cement (*I.S. 8043*)

This is a special type of cement manufactured by the Associated Cement Companies Limited, which is claimed to have the property

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of repelling water. Hence the cement stored does not get spoiled even during the monsoons. This cement is claimed to remain uneffected when transported during rains also. This cement is made by adding water repellent chemicals to ordinary Portland cement clinker in the process of grinding. The chemicals added form a monomolecular coating over every grain of cement and thus prevents any absorption of moisture from air by the cement In the concreting process, in the mixer, the coating gets removed by abrasion. After this the process of hydration will take place in exactly the same manner as in the case of ordinary Portland cement.

§12. Oil-Well Cement

After the oil wells are dug it is necessary to fill the clearance between the steel lining and the wall of the well. It is necessary to grout the porous strata so that the wall of the well is prevented from collapse. It is also necessary to keep the ground waters out of the shaft of the well Oil wells are dug to great depths (1000 *m* to 3000 *m*) and at high temperatures Thus in such conditions the cement adopted for sealing the oil wells must be in such a state, the slurry if pumped into the oil well casing will remain in the fluid state for a few hours before it can harden. For such conditions the Associated Cement Companies Limited have manufactured the *oil-well cement* which is drained to be best suited for sealing the deep oil wells at temperatures not exceeding 100°C. They also manufacture cement suited for still greater depth involving higher temperatures also In such cases, the cement is retarded at the site by adding certain retarders to facilitate satisfactory sealing.

§13. Storage of Portland Cement

Portland cement is in the very finely powdered state and is highly *hygroscopic*, *i e* it absorbs moisture readily from the surroundings, not only as free water but also as moisture from the air. Thus one can realize the need to take measures to protect cement from dampness to maintain it in a satisfactory condition. In this connection the following points may be noted :

1. Constructional Features of the Storing Shed

(*a*) Water proof masonry walls should be provided for the storing shed.

(*b*) A leak proof roof say, of RCC with a cover of a water proof course should be provided.

(*c*) There should be few windows only. These windows shall be small and shall be kept tightly shut so that the outside moisture is prevented from entering the shed.

(*d*) A 15 *cm* thick concrete slab on a dry course of soling should be provided.

(*e*) The ground should be drained away from the shed so as to prevent accumulation of rain water in the region. This will ensure an absolutely dry floor.

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2. *Mode of Storing Cement Bags*

(*a*) If the storing shed is recently built, sufficient time may be allowed for the interior of the shed to dry thoroughly.

(*b*) The bags should not be piled against the wall. A clearance of atleast 30 *cm.* should be left all-round between the piles and the exterior walls.

(*c*) Bags may not be allowed to rest directly on the floor ; they should preferably be piled on wooden planks. Of course, at the discretion of the Engineer-in-charge, if the flooring is absolutely a dry concrete flooring, the piles may be placed directly over the floors.

(*d*) The clearance between the piles of bags may be very small to minimise the circulation of air.

(*e*) Not more than 15 bags be piled one over the other ; otherwise it may become very tedious to stack or remove them The width of the piles may not be more than 3 *metres*.

(*f*) If the number of bags in a pile is more than 8 it is advisible to arrange the bags in header and stretcher form to provide stability.

(*g*) If the cement is meant to be stored during monsoon, or if it is to be stored for long periods, it is advisible to fully cover the piles by 700 gauge polythene sheet.

§14. Testing of Portland Cement

For quality control of Portland cement the following specifications and tests have been suggested by the I S. code.

1. **Sampling**. Like other manufactured goods, cement may also be subjected to some variations in quality. Hence, the sampling of cement should be made, so that the sample shall sufficiently represent the whole bulk in the stock. If there is no judicious sampling, the tests on the sample may not have any value. A sample meant for testing shall be drawn from at least 12 different bags or barrels or containers or from 12 different positions in a heap if the cement is loose.

The final sample shall weigh at least 5 *kg* The sample may be placed in a clean, dry metal can closed with a tight cover to exclude external air and moisture. It may be further advisable to even put a strip of adhesive tape round the seam of the cover of the can to make a good seal.

2. **Chemical Composition**. The I.S. specification has stipulated the following requirements :

(*i*) *Ignition Loss*. One *gram* of the sample is weighed and heated in a porcelain crucible continuously for a period of 1 *hour* at 900°C to 1000°C. If a platinum crucible is used, it is enough to heat the sample for 15 *minutes*. After so heating the sample, it is cooled and again weighed. The percentage loss of weight on ignition should not exceed 4%.

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(*ii*) *Insoluble Residue.* One *gram* of cement is weighed and put in 40 *cc* of water and stirred. Then 10 *cc* of hydrochloric acid of specific gravity 1·18 should be added and stirred. Any lumps present shall be carefully broken and the mixture is boiled for 10 minutes. The mixture, now, shall be filtered. Using hot water the container should be rinsed five times and the filter should be washed ten times. Now the residue left on the filter should be washed in a beaker containing 30 *cc* of hot distilled water. This is boiled for a duration of 10 minutes with 30 *cc* of sodium carbonate solution of twice the normal strength. The solution obtained is again filtered through the same filter paper. The filter paper is now washed five times with water. Now it is washed with hydrochloric acid solution of twice the normal strength and then with water so that it is free from chlorides. The filter paper is dried, then ignited and weighed. The insoluble residue shall not exceed 1·5 %.

(*iii*) *Lime and Alumina.* The ratio of percentage of alumina to the percentage of iron oxide shall not be less than 0·66. The percentage of lime to silica alumina and iron oxide when determined by the formula

$$\frac{CaO - 0{\cdot}7\ SO_3}{2{\cdot}8\ SiO_2 + 1{\cdot}2\ Al_2O_3 + 0{\cdot}65\ Fe_2O_3},$$

in percentage should not exceed 1·02 nor should it be less than 0·65.

(*iv*) *Magnesia.* Free magnesia shall not be present at more than 5 percent.

(*v*) *Sulphur.* This shall not exceed 2·75% (For details see I S. 269 specification).

In this connection the BS specification for chemical composition of cement has stipulated the following requirements :

TABLE 5

B.S. Specification for Chemical Composition of Cement

Test	*Ordinary and rapid hardening portland cement B S. 12* %	*Portland slast furnace cement B.S. 146* %	*Low heat portland cement B.S. 1310* %
Insoluble residue	1	1	1
Magnesia	4	5	4
Sulphur as (SO_3)	—	2	2·75
as Sulphite	2·75	1·2	—
Loss on ignition temperate climates	3	3	3
Hot climates	4	4	4

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3. **Fineness.** *Sieve Test* 100 *gms.* of cement is weighed accurately and placed on I.S. Sieve No. 9. Any air set lumps are broken using fingers. Now gentle sieving is done continuously for fifteen minutes. The residue left is weighed. This shall not exceed 10% by weight of the sample of cement.

The sieve test is not fully purposeful, since it assists only in measuring the quantity of grit present in the cement and it gives no adequate comparative measure of fineness. Hence, special methods have been devised in which it is possible to estimate the particle size distribution This is estimated in terms of the *specific surface, i.e*, the surface area per unit weight. This is expressed in *sq. cm. per gram.* The requirements are as follows :

Type of Cement	*Specific Surface Requirement*
Ordinary Portland cement and Portand blast furnace slag cement	2250 $cm.^2/gm.$
Rapid hardening Portland cement	3250 $cm.^2/gm.$
Low heat Portland cement	3200 $cm.^2/gm.$
Portland Pozzolana cement	3000 $cm.^2/gm.$

4. **Consistency.** This is a test conducted to estimate the quantity of water to be mixed in cement to form a paste of normal consistency for use in other tests.

300 *gms.* of cement should be weighed. To this about 25% of water should be mixed The paste obtained should be filled in the mould of the Vicat apparatus. The interval of time between the instant of adding water to the dry cement and the instant of commencement of filling the mould is called the *time of gauging.* The time of gauging must be $4 \pm 0{\cdot}25$ minutes.

The plunger, the biggest of the three needles (Fig. 1) 10 *mm.* diameter, is lowered gently on to the paste in the mould. Now the settlement of the plunger is noted. If the settlement is between 5 and 7 *mm.* from the bottom of the mould the amount of water added is correct. If this condition is not satisfied, the test must be again repeated changing the percentage of water until the stipulated extent of penetration of the plunger is reacted.

Let W_1 = Weight of cement taken for the test.

W_2 = Weight of water added corresponding to the condition of the stipulated extent of penetration of the plunger.

Percentage of water,

$$= P_a = \frac{W_2}{W_1} \times 100\%.$$

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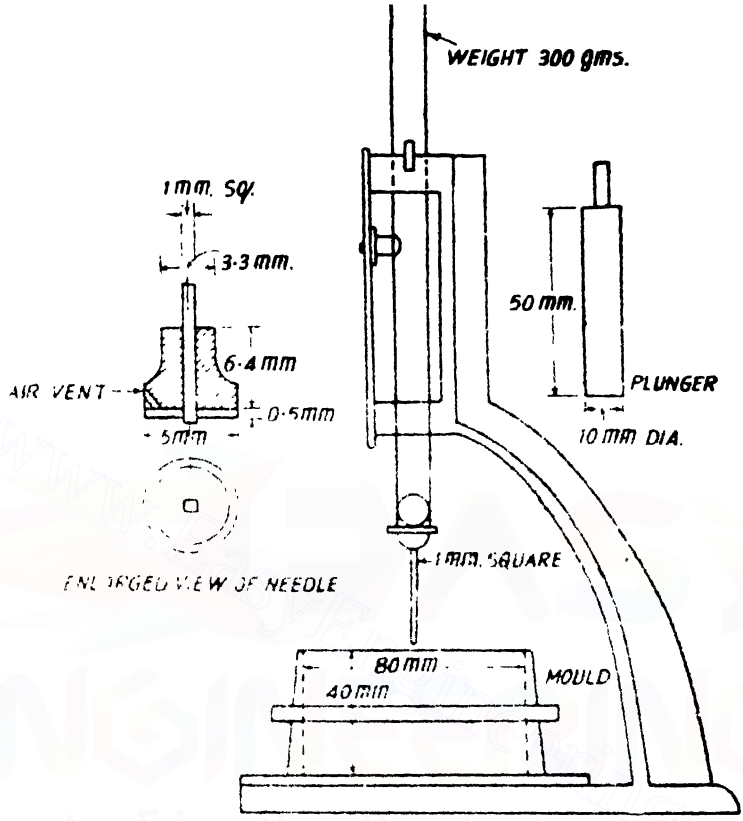

Fig. 1

5. Test for Setting Time. The object of this test is to make a distinction between normal setting and quick setting types of cement and also to detect deterioration due to storage.

Weigh 300 *gms.* of cement. Add P_a percentage of water (which is the percentage of water required to have normal consistency. See test for consistency). The paste obtained after mixing is filled in the mould. In the Vicat's apparatus, attach the needle of square section, *i e.* 1 *mm.* × 1 *mm.* to the Vicat rod. Lower the system so that the needle just touches the surface of the paste and gently release. Find out if the needle pierces into the paste fully. If it does, again release the needle on the paste after a few seconds and again observe whether the needle pierces fully into the paste. This is repeated till the needle does not pierce into the paste completely. Now the interval of time between this instant and the instant at which water was added to the cement is called the *initial setting time.*

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Now change the needle to the third one which has a projecting sharp point in the centre with an annular attachment Now release this needle as before on the same paste. The needle as well as the attachment will make their impressions on the paste. Repeat this process till only the needle makes the impression but not the attachment. The interval of time between this instant and the instant at which the water was added is called the *final setting time.*

6. Test for Soundness. If excess lime is present during the manufacture of cement, it is liable to remain uncombined and overburnt in the kilns. Such over burnt lime particles slake at a very slow rate and expand. If after setting of cement is completed the overburnt lime expands, it will lead to the disruption of the cement mass The object of the soundness test is to detect the presence of overburnt uncombined lime. The cement is tested by using *Le Chatelier apparatus.* See Fig 2. This apparatus

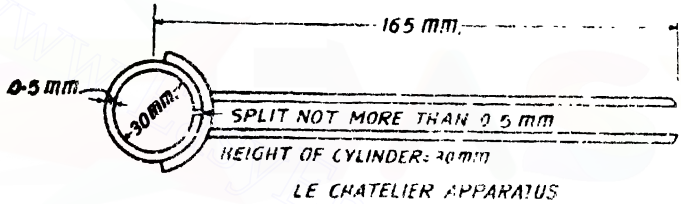

Fig. 2

consists of a small split cylinder of brass 0·5 *mm.* thick. This forms a mould 30 *mm* internal diameter and 30 *mm* high On each side of the split is attached an indicator having pointed end The distance between the centre of the cylinder and the pointed end of each indicator is 165 *mm.* The mould is placed on a glass plate and is then filled with cement paste. The cement paste should be formed by gauging with 0·78 of the water required to give a paste of normal consistency. After filling the mould another glass plate is placed on the top and is immediately submerged in water at a temperature of 27°C to 32°C for 24 hours After this interval of time, the distance between the ends of the pointers is measured Now the mould is again submerged and brought to boiling point in 25 to 30 minutes and maintained at boiling point for a duration of one hour Now the mould is taken out of water and the distance between the ends of the pointers is again measured. The difference between this measurement and the original measurement should not exceed 10 *mm.*

7. Tensile Strength. The tensile strength of cement is estimated by determining the tensile strength of cement-sand mortar. For conducting the test briquettes of standard size are prepared. The briquettes are 2·54 *cm.* thick and will have a minimum sectional area of 2·54 *cm.* × 2·54 *cm.* The cement mortar is made by mixing one part of cement to three parts of Leighton Buzzard sand. The

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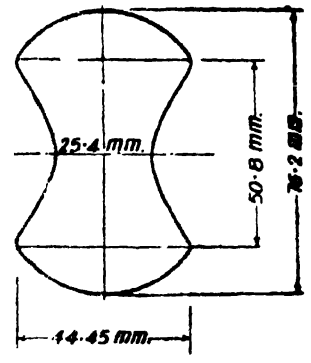
percentage of water to be used for making the mix is taken at 0·2 P_a+2·5 where, P_a is the percentage of water for normal consistency of cement At-least 12 briquettes are to be made. After the mould is made a small heap of paste is placed on the top and is lightly beaten with the standard spatula until on the surface water appears. Now the same is repeated on the other face of the briquette. The briquettes are now kept in a damp tank, the relative humidity in the tank being 90%. Now six briquettes are tested after 3 days, (and the other after 7 days) in a testing machine to destruction. The load must be increased at the rate of 35 *kg /cm.*^{2} (3·5 *N/mm.*^{2}) of section for each minute.



BRIQUETTE FOR TENSION TEST

Fig 3

$$\text{Stress at failure} = \frac{\text{Load at failure}}{\text{Minimum sectional area of briquette}}$$

The following are the requirements for good cement.

(*a*) *Portland cement.* After 3 days, the stress at failure shall not be less than 20 *kg /cm.*^{2} (2 *N/mm*^{2}). After 7 days, the stress at failure shall not be less than 25 *kg./cm.*^{2} (2·5 *N/mm*^{2}).

(*b*) *Rapid hardening cement* After one day stress at failure shall not be less than 20 *kg /cm.*^{2} (2 *N/mm.*^{2}) After 3 days the stress at failure shall not be less than 30 *kg /cm.*^{2} (3 *N/mm.*^{2}).

8. **Compressive Strength Test** This is a test to study the quality of cement about its compressive strength The compressive strength is studied by determining the compressive strength of cement-sand mortar

185 *gm.* of cement should be mixed with 3 times its weight, of standard Leighton Buzzard sand, for one minute and then mixed with water whose percentage is given by,

$$P = \frac{P_a}{4} + 3{\cdot}5$$

where P_a is the percentage of water for normal consistency. The mixing should be done for 3 minutes and in this duration the mixture must reach a uniform colour. The mould shall be a cube of 7·06 *cm.* side. The mortar is filled in the mould and the compaction is done by using a standard vibration machine for 2 minutes. After this the cubes are kept at a temperature for about 27°C in a chamber for 24 hours, in which the relative humidity is 90% After this the cubes are removed from the moulds and are kept under fresh water till they are tested

Testing the cubes shall be done on their sides. 3 cubes may be tested at the end of 3 days, and 3 more may be tested at the end of

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7 days, the loading intensity may be gradually increased. The strength requirements are given below :

After 3 days : 115 $kg./cm^2$ (11·5 $N/mm.^2$) for ordinary cement.
210 kg/cm^2 (21 N/mm^2) for rapid hardening cement

After 7 days : 175 $kg./cm.^2$ (17·5 N/mm^2) for ordinary cement.

§15. Aggregates

Aggregates are inert materials which when bound together by cement, will form concrete. Aggregates shall comply with the requirements of IS : 383. Aggregates used in India are the naturally occurring aggregates like sand, crushed rock and gravel. In some cases artificial and processed aggregates are also used to make concrete Broken brick, crushed air-cooled blast furnace slag are also used as aggregates. Sometimes, light weight aggregates like pumice, furnace clinker, expanded clays, expanded slates, *etc* are also adopted to produce low density concrete. In some cases of unimportant works, aggregate as it comes from the pit is used, such aggregate is called *all in aggregate*.

Classification of aggregates Aggregates may be classified into (*i*) fine aggregates and (*ii*) coarse aggregates depending on their sizes. The size of a fine aggregate shall not exceed 4·75 *mm* gauge. Aggregates of bigger sizes are called coarse aggregates.

Quality of aggregates. As mentioned earlier the aggregates shall conform to the requirements of IS : 383. Aggregates shall be of chemically inert mineral particles They shall be strong, hard and durable. They should be non-absorbent. They shall be free from adherent coatings, coal residues, clay lumps, *etc*. They shall not contain organic or other admixtures which are likely to retard the setting of cement and may reduce the strength of concrete and may chemically act with the reinforcement.

Generally, aggregates used in concrete may be of sand, gravel, crushed stone, broken brick, furnace slag, *etc*. Brick and furnace slag may be used for plain concrete. For reinforced concrete only sand, gravel, crushed rock should be used as aggregates. Aggregate pieces should be preferably approaching cubical or spherical shapes instead of being elongated thin and angular. Surface of aggregate shall be rough. Aggregates used for fire resisting concrete should have low thermal conductivity and low coefficient of expansion. Lime stone aggregate should not be used for fire resisting concrete. Overburnt broken brick, blast furnace slag are the proper materials to be used for this work.

Sand or gravel obtained from the beds of deep rivers with slow motion are likely to consist of clay and silt coating Hence, these should not be used without properly washing them, for reinforced concrete work.

Stone metal of rocks like granites, gneiss, trap, *etc*. should be a satisfactory aggregate.

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Size of Aggregates. The nominal maximum size of aggregate should be as large as possible within the limits specified earlier, but in no case greater than one-fourth of the minimum thickness of the member, provided that the concrete can be placed without difficulty so as to surround the reinforcement thoroughly and fill the corners of the form. For reinforced concrete work, aggregates having a nominal size of 20 *mm.* are generally considered satisfactory.

For heavily reinforced concrete members, such as in the case of ribs of main beams, the nominal maximum size of the aggregate should usually be restricted to 5 *mm.* less than the minimum clear distance between the main bars or 5 *mm.* less than the minimum cover to the reinforcement whichever is smaller. Where the reinforcement is widely spaced as in solid slabs, limitations of the size of the aggregate may not be so important and the nominal maximum size may sometimes be as great as or greater than the minimum cover.

Grading of Aggregates. The next in the order of importance about the property of aggregates is its *gradation.* Gradation means the distribution of the various sizes of particles. This is studied by sieve analysis.

Sieve Analysis and Fineness Modulus. The object of this test is to study the grading of sand. The sample taken should be not less than 1·5 *kg.* Find the actual weight of the sample. The sample is separated into different sizes by sieving *i.e.* passing it through standard sieves. The percentage by weight of the residue on each sieve is noted. The standard sieves used are, 80 *mm.*, 63 *mm.*, 40 *mm.*, 20 *mm.*, 10 *mm.*, No. 480 (480 *micron*), No. 240 (240 *micron*), No. 120 (120 *micron*, No. 60 (60 *micron*) No. 30 (30 *micron*), and No. 15 (15 *micron*).

The sum of the percentage amount of fine aggregate retained on each sieve divided by 100 gives the fineness modulus of the aggregate.

Based on the fineness modulus sand can be classified into (*i*) fine sand (FM 2·20 to 2·60), (*ii*) medium sand (FM 2·60 to 2·90,, and (*iii*) coarse sand (FM 2·90 to 3·20).

The following results relate to a test for fineness modulus of sand : (see table)

$$\text{Fineness modulus of fine aggregates} = \frac{294{\cdot}5}{100} = 2{\cdot}945$$

$$\text{Fineness modulus of coarse aggregate} = \frac{668{\cdot}5}{100} = 6{\cdot}685.$$

Test for Unit Weight of Fine Aggregate. A measuring cylinder 15 *cm* diameter and 17 *cm.* high is first weighed empty. Now it is filled with water and weighed. From these the weight of water in the cylinder is known. Now the cylinder is dried and filled with dry sand without tamping and the top level is struck flush with the

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IS sieve number	*Percentage amount of fine aggregate retained on each sieve*	*Percentage amount of coarse aggregate retained on each sieve*
80 *mm.*	—	—
63 *mm.*	—	—
40 *mm.*	—	3·5
20 *mm.*	—	15 0
10 *mm.*	—	59·5
No. 480	3·5	90·5
No. 240	19·5	100
No. 120	29·5	100
No. 60	53 5	100
No. 30	88 5	100
No. 15	100	100
Total	294·5	668·5

top end of the cylinder. The cylinder with its contents is weighed. Thus the weight of sand can be computed. From these observations the unit weight of loose dry sand is determined. This experiment is repeated a number of times and the results are averaged.

To determine the unit weight of compacted sand, the sand is filled in the cylinder in three layers. Each layer should be compacted with a 16 *mm.* rod 60 *cm.* long whose lower end is bullet pointed. After filling, the level of sand is struck to the level of the upper end of the cylinder. The cylinder with its contents is now weighed, and the unit weight of compacted sand is determined. The experiment is repeated several times and the results are averaged.

§16. Bulking Factor of Sand

Take about 6 *litres* of dry compacted sand, and weigh it and dump it into a mixing pan. Add a certain known percentage of water by weight of dry sand. Mix rapidly and thoroughly till a uniform colour is obtained and fill the container with the wet sand without any tamping. Now strike off the top surface and weigh and thus find the weight of wet sand. Repeat the experiment a number of times increasing the percentage of water from 0 to 20%.

Let W_1 = weight of 1 *cu.m.* of compacted dry sand,

W_2 = weight of dry sand contained in 1 *cu.m.* of wet loose sand.

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W_3 = weight of 1 *cu.m.* of loose wet sand,

x = percentage of water added,

W_2 = weight of dry sand + weight of water.

$$\therefore \quad W_3 = W_2 + \frac{x}{100} \times W_2$$

$$\therefore \quad W_3 = W_2\left(1 + \frac{x}{100}\right)$$

$$\therefore \quad W_2 = \frac{W_3}{\left[1 + \frac{x}{100}\right]}$$

$$\% \text{ of bulking} = \frac{W_1 - W_2}{W_2} \times 100$$

$$\text{Bulking factor} = \frac{W_1}{W_2}.$$

§17. Water

Water is an essential ingredient of concrete since it takes part in chemical reactions with cement to form a binding paste that fills the innumerable minute surface irregularities of sand and aggregates, and thus bring them into closer adhesion. Water is needed not only to bring about hydration but also to render it to a workable state. It is from the latter consideration additional water is added in order to make the mix workable and to render its easy placing inside the forms and around the reinforcement. Such additional water should of course be present to the minimum requirement. Excessive water brings down the strength of cement paste. Excessive water is found to produce other defects in concrete. As the water evaporates it will leave blank spaces or voids. This will bring down the density, strength and durability. Too much water will make the mixture of the flowy part of the cement and water to the surface resulting in the formation of a scum or a thin layer of material known as *laitance*. This scum layer will prevent the proper bonding of the next layer of concrete resulting in the formation of a weakness plane. This should be seriously taken note of in concreting for dams and tanks in which water tightness is an essential requirement besides strength. Water if provided excessively is liable to leak out of shuttering and such leaking water may carry away some of the cement resulting in honeycomb formation in concrete.

Quality of Water for Concrete. Water used for concreting should not contain chemicals which can cause harmful effects. Water fit for drinking is obviously fit for concreting too. In fact water need not be as pure as drinking water. Unfiltered water is generally used for concreting. We will now briefly discuss the effects of certain impurities present commonly in water, on concrete.

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(*i*) **Presence of carbonates and bicarbonates of sodium and potassium**. These affect the setting times of cements. Sodium carbonate may cause very rapid setting. Bicarbonates may accelerate or retard setting When present in large extent these salts reduce the strength of concrete. In cases where these salts are present to an extent exceeding 1000 *ppm* (0·1 percent) then it is necessary to conduct tests for setting time and 28-day cube strength.

(*ii*) **Presence of sodium chloride and sulphate**. These salts are not found to produce any damaging effects. However, the maximum limits to which these may be present in water are the following .

Sodium chloride concentration : 20,000 *ppm*.

Sodium sulphate concentration : 10,000 *ppm*.

(*iii*) **Presence of carbonates and bicarbonates of calcium and magnesium**. Since carbonates are mostly insoluble in water, they are not therefore present to any appreciable extent. However the bicarbonates are soluble in water, and are generally present in the water supplied by municipality. The bicarbonate content in water for concreting shall not exceed 4·0 *ppm*.

(*iv*) **Presence of magnesium sulphate and magnesium chloride**. These have not been found to produce any harmful effects on concrete. Even with 40000 *ppm* concentration, the strength of concrete is not found to be affected.

(*v*) **Presence of calcium chloride**. Water containing this salt can be used for ordinary concrete up to 2 percent by weight of cement and is found to produce an accelerating effect on setting as well as hardening However, this may not be allowed in prestressed concrete.

(*vi*) **Presence of iron salts** It is observed that acid mine waters contain large proportions of iron salts. Mortar strengths are unaffected when these salts are present up to 40,000 *ppm*.

(*vii*) **Presence of miscellaneous inorganic salts**. The strength of concrete is reduced due to the presence of salts of manganese, zinc, copper and lead. Chlorides of zinc and copper produce a retarding effect on setting to such an extent that no strength tests are practically possible for two to three days. Lead nitrate is very harmful and should be avoided completely. Salts like Sodium iodate, Sodium phosphate, Sodium arsenate, Sodium borate bring down the initial strength of concrete to a very great extent Of particular mention is the salt Sodium sulphide which is most harmful to concrete. If present to an extent of even 100 *ppm*. the concrete needs testing.

I.S. Recommendations. In this connection I.S. 456 has specified the following requirements :

Water used for mixing and curing shall be clear and free from injurious amounts of oils, acids, alkalis, salts, sugar, organic materials or other substances that may be deleterious to concrete or steel. Potable water is generally considered satisfactory for mixing

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concrete. As a guide the following concentrations represent the maximum permissible values :

(*a*) To neutralize 200 *ml.* sample of water, using phenolphthalein as an indicator, it should not require more than 2 *ml.* of 0·1 normal NaOH. The details of test shall be as given in I.S. : 3025-1964.

(*b*) To neutralize 200 *ml* sample of water, using methyl orange as an indicator, it should not require more than 10 *ml.* of 0·1 normal HCl. The details of test shall be as given in I.S. : 3025-1964.

(*c*) Permissible limits for solids when tested in accordance with I S. : 3025-1964 shall be as given in the table below :

Permissible Limit for Solids

	Permissible limit (*Max*)
Organic	...200 *mg./l.*
Inorganic	...3000 *mg./l.*
Sulphates (as SO_4)	...500 *mg./l.*
Chloride (as Cl)	...2000 *mg./l.* for plain concrete work and 1000 *mg./l.* for reinforced concrete work
Suspended matter	...2000 *mg./l.*

In case of doubt regarding development of strength, the suitability of water for making concrete shall be ascertained by the compressive strength and initial setting time tests.

The sample of water taken for testing shall represent the water proposed to be used for concreting, due account being paid to seasonal variation. The sample shall not receive any treatment before testing other than that envisaged in the regular supply of water proposed for use in concrete The sample shall be stored in a clear container previously rinsed out with similar water.

Average 28 days compressive strength of at least 15 *cm.* concrete cubes prepared with water proposed to be used shall not be less than 90 percent of the average strength of three similar concrete cubes prepared with distilled water. The cubes shall be prepared, cured and tested in accordance with the requirements of I.S. 516.

The initial setting time of test block made with the appropriate cement and the water proposed to be used shall not be less than 30 minutes and shall not differ by ± 30 minutes from the initial setting time of control test block prepared with the same cement and distilled water and tested in accordance with I.S. 4031.

The *pH* value of water shall generally be not less than 6.

Sea water. Mixing or curing of concrete with sea water is not recommended because of presence of harmful salts in sea water. Under unavoidable circumstances, sea water may be used for mixing

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or curing in plain concrete or such reinforced concrete which are permanently under sea water.

Water found satisfactory for mixing is also suitable for curing concrete. However, water used for curing should not produce any objectionable strain or unsightly deposit on the concrete surface. The presence of tannic acid or iron compounds is objectionable.

§18. Grades of Concrete

In constructions certain standard mixes alone are used. A set of mix for concrete should be well defined either in terms of the proportion of cement, fine and coarse aggregates or in terms of the 28 day compressive strength requirement.

Concrete mix in terms of proportions of the components. The mixes usually used are 1 : 3 : 6, 1 : 2 : 4, 1 : 1½ : 3, 1 : 1 : 2.

The 1 : 3 : 6 mix is used for mass concreting and the rear sides of dams.

The 1 : 2 : 4 mix is used for general reinforced concrete work.

The 1 : 1½ : 3 mix is used for front faces of dams, water tanks, columns *etc.*

The 1 : 1 : 2 mix is used for piles.

Concrete mix based on the 28 day cube strength. In the *MKS* units the following grades of concrete are used :

Nomenclature of Grade of concrete	*Approximate proportion of components*	*28 day characteristic compressive (f_{ck}) strength*
M 100	1 : 3 : 6	100 *kg./cm*^{2}
M 150	2 : 4	150 *kg./cm.*^{2}
M 200	1½ : 3	200 *kg./cm.*^{2}
M 250	1 : 2	250 *kg./cm.*^{2}
M 300		300 *kg./cm.*^{2}
M 350		350 *kg./cm.*^{2}
M 400		400 *kg./cm.*^{2}

In the *SI* units the following grades of concrete are used as per I.S. 456 code :

Namenclature of Grade of concrete	*Approximate proportion of components*	*28 day characteristic compressive (f_{ck}) strength*
M 10	1 : 3 : 6	10 *N/mm.*^{2}
M 15	1 : 2 : 4	15 *N/mm.*^{2}
M 20	1 : 1½ : 3	20 *N/mm.*^{2}
M 25	1 : 1 : 2	25 *N/mm.*^{2}

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M 30	30 $N/mm.^2$
M 35	35 $N/mm.^2$
M 40	40 $N/mm.^2$

Note : (*i*) M 5 and M 7·5 Grades may be used for lean concrete bases and simple foundations for masonry walls. These mixes need not be designed.

(*ii*) Grades of concrete lower than M 15 shall not be used in reinforced concrete.

In the above table characteristic strength means the strength of material below which not more than 5 per cent of the test results are expected to fall.

§19. Properties of Concrete (*I.S. recommendations*)

Increase in strength with Age. Where it can be shown that a member will not receive its full design load/stress within a period of 28 days after the casting of the member (for example, in foundations and lower columns in multistorey buildings), the characteristic compressive strength given in the table above may be increased by multiplying by the factors given below :

Minimum age of member when full design load/stress is expected (Months)	*Age factor*
1	...1·0
3	...1·10
6	...1·15
12	...1·20

Note 1. No increase in respect of age at loading should be allowed where high alumina cement concrete is used.

Note 2. Where members are subjected to lower direct load during construction, they should be checked for stresses resulting from combination of direct load and bending during construction.

Note 3. The permissible stresses or design strengths shall be based on the increased value of the compressive strength.

Tensile strength of concrete. The flexural and split tensile strengths shall be obtained as described in I.S. 516 and I.S. 5816 respectively. When the designer wishes to use an estimate of the tensile strength from the compressive strength, the following formula may be used':

Flexural strength $f_{cr}=0{\cdot}7\sqrt{f_{ck}}$ $N/mm.^2$

where f_{ck} = characteristic compressive strength of concrete in $N/mm.^2$

Elastic deformation. The modulus of elasticity is primarily influenced by the elastic properties of the aggregate and to a lesser extent by the conditions of curing and age of the concrete. the mix

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proportions and the type of cement. The modulus of elasticity is normally related to the compressive strength of concrete.

In the absence of test data, the modulus of elasticity for structural concrete may be assumed as follows :

$$E_c = 5700\sqrt{f_{ck}}$$

where E_c = The short term static modulus of elasticity in N/mm^2

and f_{ck} = the characteristic cube strength of concrete in N/mm^2

§20. Shrinkage of Concrete

The total shrinkage of concrete depends upon the constituents of concrete, size of the member and environmental conditions. For a given environment, the total shrinkage of concrete is most influenced by the total amount of water present in the concrete at the time of mixing and, to a lesses extent, by the cement content.

In the absence of test data, the approximate value of the total shrinkage strain for design may be taken as 0·0003. (For additional information see I S. 1343).

Creep of concrete. Creep of concrete depends, in addition to the factors mentioned for shrinkage, on the stress in concrete, age at loading and the duration of loading. As long as the stress in concrete does not exceed one-third of its characteristic compressive strength, creep may be assumed to be proportional to stress.

Creep coefficient. This is the ratio of the ultimate creep strain to the elastic strain at the age of loading. In the absence of experimental data the creep coefficient may be taken as per the table given below :

Age at loading	*Creep coefficient*
7 days	2·2
28 days	1·6
1 year	1·1

§21. Thermal Expansion

The coefficient of thermal expansion for concrete may be taken at the values given below :

Type of Aggregate	*Coefficient of expansion for concrete per degrees C*
Quartzite	...1·2 to $1·3 \times 10^{-5}$
Sandstone	...0·9 to $1·2 \times 10^{-5}$
Granite	...0·7 to $0·95 \times 10^{-5}$
Basalt	...0·8 to $0·95 \times 10^{-5}$
Limestone	...0·6 to $0·9 \times 10^{-5}$

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§22. Nominal Mix Concrete

Nominal mix concrete may be used for concrete of grades M 5, M 7·5, M 10, M 15 and M 20. The proportions of materials for nominal mix concrete shall be in accordance with the table below :

Proportions for Nominal Mix Concrete

Grade of concrete	*Total quantity of dry aggregates by Mass per 50 kg. of cement to be taken as the sum of the individual masses of fine and coarse aggregates (Max.) Kilogram*	*Proportion of fine aggregate to coarse aggregate (by Mass)*	*Quantity of water per 50 kg. of cement Max. Litres*
M 5	800	Generally 1 : 2 but subject to an upper limit of 1 : 1½ and a lower limit of 1 : 2½	60
M 7·5	625		45
M 10	480		34
M 15			32
M 20			30

NOTE – The proportion of the fine to coarse aggregates should be adjusted from upper limit to lower limit progressively as the grading of the fine aggregates becomes finer and the maximum size of coarse aggregate becomes larger. Graded coarse aggregate shall be used.

§23. Stripping Time

Forms can be struck only after the concrete has attained a certain amount of strength. In this connection the I.S. code has made the following recommendations.

Forms shall not be struck until the concrete has reached a strength at least twice the stress to which the concrete may be subjected at the time of removal of formwork. The strength referred to shall be that of concrete using the same cement and aggregates, with the same proportions and cured under conditions of temperature and moisture similar to those existing on the work. Where possible the formwork shall be left longer as it would assist the curing.

NOTE 1 – In normal circumstances and where ordinary portland cement is used, forms may generally be removed after the expiry of the following periods.

(*a*) Walls, columns and vertical faces of all structural members ...24 to 48 hours as may be decided by the engineer-in-charge.

(*b*) Slabs (props left under) ...3 days

(*c*) Beam soffits (props left under) ...7 days

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(*d*) Removal of props under slabs :

(1) Spanning up to 4.5 *metres* ...7 days

(2) Spanning over 4·5 *metres* ...14 days

(*e*) Removal of props under beams and arches :

(1) Spanning up to 6 *metres* ...14 days

(2) Spanning over 6 *metres* ...21 days

NOTE 2—The number of props left under, their sizes and disposition shall be such as to be able to safely carry the full dead load of the slab, beam or arch as the case may be together with any live load likely to occur during curing or further construction.

§24. Compaction Test

This is a test developed to estimate more judiciously the workability of concrete.

The apparatus consists of two hopper vessels *A* and *B* provided with hinged doors at their bottom.

A cylindrical vessel *C* is placed below the hopper *B* The vessel *A* is filled with the sample of concrete. The hinged door at its bottom is opened so that the concrete falls into the vessel *B*. After this, the hinged door of the vessel *B* is opened so that the concrete will fall into the cylinder *C*. The surplus concrete from this cylinder is struck off with steel floats. The contents of the cylinder are now weighed. Let the weight of the contents be W_1. Now after removing the contents the cylinder is again filled with the sample in 5 *cm*. layers ; the concrete being compacted by ramming and vibrating and then weighed. Let the weight of the compacted concrete be W_2 The compacting factor $= \frac{W_1}{W_2}$.

The following conclusions are made based on the value of the compacting factor.

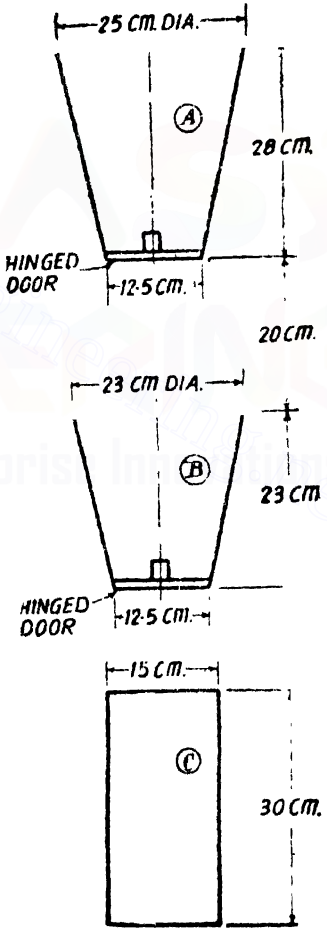

Fig. 4

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Value of compacting factor	*Standard of workability*
0·95	Good
0·92	Medium
0·85	Low

§25. Test for Consistency of Concrete

The object of this test is to study the workability of the concrete mix. Workability means the *ease* with which mixing, handling and placing of concrete can be done. Consistency of concrete is a measure of the fluidity of concrete.

Slump test. Concrete is filled in the standard slump cone which consists of a vessel whose shape is a frustum of a cone. The dimensions of the vessel are given in Fig. 5.

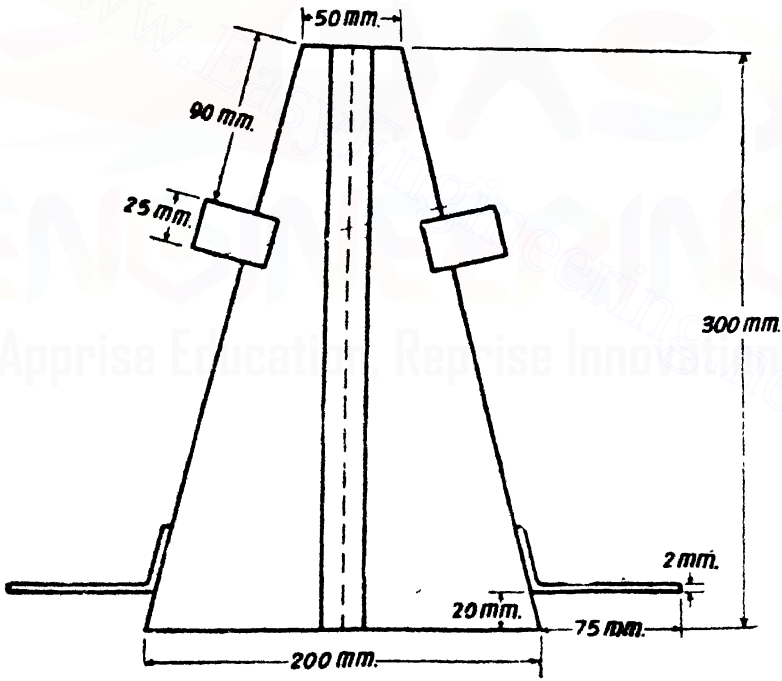

Fig. 5

Filling the slump cone with concrete is done in layers of thickness 8 *cm*. After laying each layer, it is compacted with 25 strokes using a rod of 16 *mm*. diameter 60 *cm*. long whose striking end is bullet pointed. After filling the vessel with concrete, the vessel is carefully removed. *i.e.* lifted. The vertical settlement of the body of concrete is called the *slump* of concrete.

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The following table shows the slump requirement for various situations :

Concrete for road work	...20 *mm.* to 30 *mm.*
Concrete for beams and slabs	...50 *mm.* to 100 *mm.*
Concrete for walls, thin vertical members, columns	...75 *mm.* to 150 *mm.*
Vibrated concrete	...15 *mm.* to 25 *mm.*
Mass concrete	...25 *mm.* to 50 *mm*

The workability is said to be good when the concrete can be easily handled, placed and tamped. When the concrete is set it should not contain voids. Good workability can be reached by

(*i*) Using aggregates of bigger size,
(*ii*) Varying the shape of the aggregates,
(*iii*) Varying the grading of aggregates,
(*iv*) Increasing the quantity of cement,
(*v*) Increasing the quantity of water and
(*vi*) Adding resins or pozzolans.

§26. Frequency of Sampling

Sampling procedure. A random sampling procedure shall be adopted to ensure that each concrete batch shall have a reasonable chance of being tested ; that is, the sampling should be spread over the entire period of concreting and cover all mixing units.

Frequency. The minimum frequency of sampling of concrete of each grade shall be in accordance with the following :

Quantity of concrete in the work (metre³)	*Number of samples*
1—5	1
6—15	2
16—30	3
31—50	4
51 and above	4 plus one additional sample for each additional 50 *metre³* or part thereof

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§27. Test Strength of Sample

The test strength of the sample shall be the average of the strength of 3 specimens. The individual variation should not be more than ±15 percent of the average.

§28. Sampling and Strength Test of Concrete (*IS 456*)

Samples from fresh concrete shall be taken and cubes shall be made and tested at 28 days.

NOTE 4—For relatively small and unimportant buildings and works in which quantity of concrete is less than 15 *metre*^{3}, the strength tests may be waived by the engineer-in-charge at his discretion.

In order to get a relatively quicker idea of the quality of concrete, optional tests on beams for modulus of rupture at 72±2 hours or at 7 days, or compressive strength tests at 7 days may be carried out in addition to 28 days compressive strength tests. For this purpose the values given in the table below may be taken for general guidance in the case of concrete made with ordinary Portland cement. In all cases, the 28 days compressive strength specified shall alone be the criterion for acceptance or rejection of the concrete.

Test Requirements of Concrete (S I. System)

Grade of concrete	*Compressive strength on 15 cm. cubes at 28 days N/mm.*^{2}	*Compressive strength on 15 cm. cubes at 7 days N/mm*^{2}	*Modulus of rupture by beam tests*	
			at 72 ± 2 hours N/mm.^{2}	*at 7 days N/mm.*^{2}
M 10	10	7·0	1·2	1·7
M 15	15	10·0	1·5	2·1
M 20	20	13·5	1·7	2·4
M 25	25	17·0	1·9	2·7
M 30	30	20·0	2·1	3·0
M 35	35	23·5	2·3	3·2
M 40	40	27·0	2·5	3·4

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Test Requirements of Concrete (M K.S. System)

Grade of concrete	*Compressive strength on 15 cm. cubes at 28 days kg/cm.²*	*Compressive strength on 15 cm. cubes at 7 days kg/cm.²*	*Modulus of rupture by beam tests*	
			at 72 ± 2 hours kg /cm.²	*at 7 days kg./cm.²*
M 100	100	70	12	17
M 150	150	100	15	21
M 200	200	135	17	24
M 250	250	170	19	27
M 300	300	200	21	30
M 350	350	235	23	32
M 400	400	270	25	34

§29. Permissible Stresses in Concrete

The I.S. code has specified the permissible stresses in concrete for various grades. They are given in the tables below :

Permissible Stresses in Concrete in Compression and Average Bond

SI UNITS				*MKS UNITS*			
Grade of concrete	*Bending stress in compression σ_{cbc} N/mm²*	*Direct stress in compression σ_{cc} N/mm.²*	*Average bond stress for plain tension bars N/mm².*	*Grade of concrete*	*Bending stress in compression σ_{cbc} kg./cm.²*	*Direct stress in compression σ_{cc} kg./cm.²*	*Average bond stress for plain tension bars kg./cm.²*
M 10	3·0	2·5	—	M 100	30	25	—
M 15	5·0	4·0	0·6	M 150	50	40	6
M 20	7·0	5·0	0·8	M 200	70	50	8
M 25	8·5	6·0	0·9	M 250	85	60	9
M 30	10·0	8·0	1·0	M 300	100	80	10
M 35	11·5	9·0	1·1	M 350	115	90	11
M 40	13·0	10·0	1·2	M 400	130	100	12

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Note 1. The bond stress given above shall be increased by 25 percent for bars in compression.

Note 2. The bond stress given above shall be increased by 40 percent for deformed bars.

Permissible Direct Stress in Tension in Concrete on the Equivalent Concrete Area

S.I. UNITS		*M.K.S. UNITS*	
Grade of concrete	*Permissible tensile stress N/mm.²*	*Grade of concrete*	*Permissible tensile stress kg./cm.²*
M 10	1·2	M 100	12
M 15	2·0	M 150	20
M 20	2·8	M 200	28
M 25	3·2	M 250	32
M 30	3·6	M 300	36
M 35	4·0	M 350	40
M 40	4·4	M 400	44

Tensile stress shall be calculated as

$$\frac{\text{Tension}}{(\text{Actual concrete area}) + (m \times \text{steel area})}$$

§30. **Modular Ratio** m

This is the ratio of Modulus of Elasticity for steel to the Modulus of Elasticity for concrete.

As per the code, the modular ratio shall be taken as follows :

$$m = \frac{\text{Constant}}{3 \times \sigma_{cbc}}$$

where σ_{cbc} = Permissible compressive stress in concrete in bending.

(*i*) *When S.I. units are adopted for* σ_{cbc}

$$m = \frac{280}{3\sigma_{cbc}}$$

(*ii*) *When M.K.S. units are adopted for* σ_{cbc}

$$m = \frac{2800}{3\sigma_{cbc}}$$

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The following table shows the values of the modular ratio for different grades of concrete.

When S.I. units are adopted			*When M.K.S. units are adopted*		
Grade of concrete	σ_{cbc} N/mm²	$m=\frac{280}{3\sigma_{cb}}$	*Grade of concrete*	σ_{cbc} kg./cm.²	$m=\frac{2800}{3\sigma_{cb_0}}$
10	3·0	31	100	30	31
15	5·0	18	150	50	18
20	7·0	13	200	70	13
25	8·5	11	250	85	11
30	10·0	9	300	100	9
35	11·5	8	350	115	8
40	13·0	7	400	130	7

§31. Permissible Shear Stress in Concrete

The I S. code has introduced the concept of *nominal shear stress* defined by the following relation :

Nominal shear stress in beams or slabs of uniform depth

$$=q_v=\frac{V}{bd}$$

where

V = Shear force due to design loads.

b = Breadth of the member (or breadth of web).

d = Effective depth.

In the case of beams of varying depth

$$q_v=\frac{V\pm\frac{M}{d}\tan\beta}{bd}$$

where

M = Bending moment at the section.

β = Angle between the top and bottom edges of the beam.

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The negative sign in the formula should be used when the bending moment increases numerically in the same direction as the effective depth increases ; and the positive sign in the formula should be used when the bending moment decreases numerically in the direction.

The permissible shear stress in concrete in beams without shear reinforcement is given in the table below :

Permissible Shear Stress in Concrete in Beams

S.I. Units

$\frac{100\ A_t}{bd}$	*Permissible shear stress in concrete q_c in N/mm² for various Grades of concrete*					
	M 15	*M 20*	*M 25*	*M 30*	*M 35*	*M 40*
0·25	0·22	0·22	0·23	0·23	0·23	0·23
0·50	0·29	0·30	0·31	0·31	0·31	0·32
0·75	0·34	0·35	0·36	0·37	0·37	0·38
1·00	0·37	0·39	0·40	0·41	0·42	0·42
1·25	0·40	0·42	0·44	0·45	0·45	0·46
1·50	0·42	0 45	0·46	0·48	0·49	0·49
1·75	0·44	0·47	0·49	0·50	0·52	0·52
2·00	0 44	0·49	0·51	0·53	0·54	0·55
2·25	0 44	0 51	0·53	0·55	0·56	0·57
2 50	0·44	0·51	0·55	0 57	0·58	0·60
2·75	0·44	0·51	0·56	0 58	0 60	0·62
3·00 and above	0·44	0·51	0·57	0·60	0·62	0·63

Note – A_t is that area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at supports where the full area of tension reinforcement may be used.

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Permissible Shear Stress in Concrete in Beams M.K.S Units

$\frac{100\ A_t}{bd}$	*Permissible shear stress in concrete τ_c in kg./cm.² for various grades of concrete*					
	M 150	*M 200*	*M 250*	*M 300*	*M 350*	*M 400*
0·25	2·2	2·2	2·3	2·3	2 3	2·3
0·50	2 9	3·0	3·1	3 1	3·1	3·2
0·75	3·4	3·5	3·6	3·7	3·7	3·8
1 00	3·7	3·9	4·0	4·1	4·2	4 2
1·25	4·0	4·2	4·4	4·5	4·5	4·6
1·50	4·2	4·5	4·6	4·8	4 9	4·9
1·75	4·4	4·7	4·9	5·0	5 2	5·2
2·00	4·4	4·9	5·1	5·3	5·4	5·5
2·25	4 4	5·1	5·3	5·5	5·6	5·7
2·50	4·4	5·1	5·5	5·7	5·8	6·0
2·75	4·4	5·1	5·6	5·8	6·0	6·2
3·00 and above	4·4	5·1	5·7	6·0	6·2	6·3

NOTE—A_t is that area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at supports where the full area of tension reinforcement may be used.

§32. Types and Grades of Reinforcement Bars

The following table shows the various types of steel permitted for use as reinforcement bars and their characteristics yield strength f_y.

Type of steel	*I.S. code*	*Bar diameter*	*Charocteristic yield strength f_y*	
			MKS	*SI*
Mild steel (plain bars)	I.S. : 432-(part I)-1966	upto 20 *mm.* over 20 *mm.*	2600 *kg./cm.*² 2400 *kg./cm.*²	250 *N/mm*² 240 *N/mm.*²
Mild steel (hot rolled deformed bars)	I.S. : 1139-1966	upto 20 *mm.* over 20 *mm.*	2600 *kg./cm.*² 2400 *kg./cm.*²	250 *N/mm.*² 235 *N/mm.*²
Medium tensile steel (plain bars)	I.S. : 432-(part I)-1966	upto 20 *mm.* 20*mm.*-40 *mm.* over 40 *mm.*	3600 *kg./cm.*² 3450 *kg./cm.*² 3300 *kg./cm2.*	350 *N/mm.*² 340 *N/mm.*² 320 *N/mm.*²
Medium tensile steel (hot rolled deformed bars)	I.S. : 1139-1966	upto 20 *mm.* 20*mm.*-40 *mm.* over 40 *mm.*	3600 *kg./cm.*² 3450 *kg./cm.*² 3300 *kg /cm.*²	350 *N/mm.*² 340 *N/mm.*² 320 *N/mm.*²
High yield strength steel (hot rolled deformed bars)	I.S. : 1139-1966	All sizes	4250 *kg./cm.*²	415 *N/mm.*²
High yield strength steel (cold twisted deformed bars)	I.S. : 1786-1979	All sizes	5100 *kg./cm*²	500 *N/mm.*²
Hard drawn steel wire fabric	I S. : 1566-1967 and I.S : 432 (part II)-1966		4900 *kg./cm.*²	480 *N/mm.*²

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[S.I. Units] Permissible Stresses in Steel Reinforcement—(*I.S. 456*)

Type of stress in steel reinforcement	*Permissible stresses in Newtons/mm.*^{2}		
	Mild steel bars conforming to grade I of I.S. 432 (Part I) or deformed mild steel bars conforming to I.S. : 1139	*Medium tensile steel conforming to I.S. 432 (Part I) or Deformed medium tensile steel bars conforming to I.S. : 1139*	*High yield strength deformed bars conforming to I.S. : 1139 or I.S. : 1786 grade (Fe 415) (Ribbed Torsteel)*
Tension (σ_{st} or σ_{sv})			
(*a*) Up to and including 20 *mm.*	140	Half the guaranteed yield stress subject to a maximum of 190	230
(*b* Over 20 *mm.*	130		230
Compression in column bars (σ_{sc})	130	130	190
Compression in bars in a beam or slab when the compressive resistance of the concrete is taken into account	The calculated compressive stress in the surrounding concrete multiplied by 1·5 times the modular ratio or σ_{sc} whichever is lower		
Compression in bars in a beam or slab when the compressive resistance of the concrete is not taken into account :			
(*a*) Up to and including 20 *mm.*	140	Half the guaranteed yield stress subject to a maximum of 190	190
(*b*) Over 20 *mm.*	130		190

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[M.K.S. Units] Permissible Stresses in Steel Reinforcement (*I.S. 456*)

Type of stress in Steel Reinforcement	*Permissible stresses in kg./cm.²*		
	Mild steel bars conforming to grade I of I.S. 432 (Part I) or deformed mild steel bars conforming to I.S. : 1139	*Medium tensile steel conforming to I S. 432 (Part I) or deformed medium tensile steel bars comforming to I S. : 1139*	*High yield strength deformed bars conforming to I.S. : 1139 or I.S. : 1786 grade (Fe 415) (Ribbed-Torsteel)*
Tension σ_{st} or σ_{sn}			
(*a*) Up to and including 20 *mm.*	1400	Half the guaranteed yield stress subject to a maximum of 1900	2300
(*b*) Over 20 *mm.*	1300		2300
Compression in column bars (σ_{sc})	1300	1300	1900
Compression in bars in a beam or slab when the compressive resistance of the concrete is taken into account	The calculated compressive stress in the surrounding concrete multiplied by 1.5 times the modular ratio or σ_{sc} whichever is lower		
Compression in bars in a beam or slab where the compressive resistance of the concrete is not taken into account			
(*a*) Up to and including 20 *mm*	1400	Half the guaranteed yield stress subject to a maximum of 1900	1900
(*b*) Over 20 *mm*	1300		1900

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§33. Loads

Dead load. This consists of the weight of walls, partitions, floors, roofs including the weights of all other permanent constructions in the building. A schedule of unit weights of building materials is given in I.S. : 1911 code.

The I.S. recommendations for dead load are given below :

Provision for loads from partition walls. Loads due to partitions shall be assessed on the basis of the actual constructional details of the proposed partitions and their positioning in accordance with plans, and the loads thus assessed shall be included in the dead load for the design of floors and the supporting structures. Where, however, the actual loads of the partitions cannot be assessed beforehand owing to lack of knowledge of the final positioning of the partitions, the floors and the supporting structures shall be designed to carry, in addition to other loads, a uniformly distributed load per *square metre* of not less than 33⅓ percent of the weight per *metre* run of finished partitions over the entire floor area subject to a minimum uniformly distributed load of 100 $kg./metre^2$ in the case of floors used for office purposes.

No partition shall be allowed to be erected which may, in effect, result in stresses greater than those allowed for in the design.

Live Load

Live Loads on Floors

Live loads on floors of various types of buildings. Live loads on floors shall comprise all loads other than dead loads. The minimum live loads on different floors for different uses shall be as given in the Table (See pages 38-39). The loads specified in this table are uniformly distributed static loads $kg./metre^2$ on the plan area and provide for normal effects of impact and acceleration, but do not take into consideration special concentrated loads, snow loads and other loads.

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Live Loads on Floors—*I S. 875*

Loading Class No.	*Types of floors*	*Minimum live loads in kg./m² of floor area*	*Alternative minimum live load*
200	Floors in dwelling houses, tenements, hospital wards, bed rooms and private sitting rooms in hostels, and dormitories	200	Subject to a minimum total loads of 2·5 times the given values for any given slab panel and 6 times the given values for any given beam. This total load shall be assumed uniformly distributed on the entire area of the slab panel of the entire length of the beam
250	Office floors other than entrance halls, floors of light work rooms.	*250 400	
300	Floors of banking halls, office entrance halls, and reading rooms	300	
400	Shop floors used for the display and sale of merchandise ; floors of work rooms generally ; floors of class rooms in schools, floors or places of assembly with fixed seating, restaurants ; circulation space in machinery halls, power stations, *etc.*, where not occupied by plant or equipment.	400	
500	Floors of ware-houses, workshops, factories and other buildings or parts of buildings of similar category for light weight loads ; office floors for storage and filing purposes ; floors of places of assembly without fixed seating, public rooms in hotels, dance halls, waiting halls *etc.*	500	
7 0	Floors of ware-houses, workshops, factories and other buildings or parts of buildings of similar category for medium-weight loads	750	
1000	Floors of ware-houses, workshops, factories and other buildings or parts of buildings of similar category for heavy-weight loads, floors of book stores and libraries, roofs and pavement lights over basements projecting under the public foot-path	1000	
Garage Light	Floors used for garages for vehicles not exceeding 2·5 tonnes gross weight : Slabs	400	or the worst combination of actual wheel loads, whichever is greater

(Contd.)

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(*Contd.*)

	Beams	250	or the worst combination of actual wheel loads, whichever is greater
Garage Heavy	Floors used for garages for vehicles not exceeding 4 tonnes gross weight	750	Subject to a minimum of 1·5 times the maximum wheel load but not less than 900 *kg.* considered to be distributed over 75 *cm.* square
Stairs	Stairs, landings and corridors for class 200 loading but not liable to overcrowding	300	Subject to a minimum of 130 *kg.* concentrated load at the unsupported end of each step for stairs constructed out of structurally independent cantilever steps
	Stairs, landings and corridors for class 200 loading but liable to overcrowding, and for other classes	500	
Balcony	Balconies not liable to overcrowding :		
	For class 200 loading	300	
	For all other classes	500	
	Balconies liable to overcrowding	500	

* The lower value of 250 *kg./m*^{2} should be taken where separate facilities are provided and the higher value of 400 *kg./m*^{2} should be taken where such provisions are lacking.

§34. Reduction in Floor Live Loads

The following reductions in assumed total live loads on floors may be made in designing column, walls, piers, their supports and foundations :

Number of floors carried by member under consideration	*Percent reduction of total live load on all floors above the member under consideration*
1	0
2	10
3	20
4	30
5 or more	40

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No reduction shall be made in the case of warehouses, garages and other buildings used for storage purposes and for factories and workshops designed for 500 *kg./m²*. However, for buildings, such as factories and workshops, designed for a live load of more than 500 *kg./m²*. the reductions given in the table may be made provided that the loading assumed for any column, *etc.*, is not less than it would have been if all the floors had been designed for 500 *kg./m.²* with no reduction.

Where, a single span of a beam or girder supports not less than 50 *metre²* of floor at one general level, the live load may be reduced in the design of the beam or girder by 5 percent, for each 50 *metre²* supported, subject to a maximum reduction of 25 percent. This reduction or that given in the table above, whichever is greater, may be taken into account in the design of columns, supporting such a beam, but, shall not be made where the floors are used for storage purposes nor in the weight of any plant or machinery which is specifically allowed for.

Live Loads on Roofs

Live loads on various types of roofs. On flat roofs, sloping roofs and curved roofs allowance for live load shall be as per table on page 41.

Roofs of buildings used for promenade or incidental assembly purposes shall be designed for a minimum load of 400 *kg /metre²* or heavier, if required.

Snow load. If a roof is subjected to snow load, it should be designed for the actual load due to snow or for the live loads spcified in table on page 41 whichever is more severe. Actual load due to snow will depend upon the shape of the roof and its capacity to retain the snow and each case shall be treated on its own merits. In the absence of any specific information, the loading due to the collection of snow may be assumed to be 2·5 *kg./metre²* per *cm.* depth of snow. The possibility of total or partial snow load should be considered, that is one-half of the roof fully loaded with the design snow load and the other half loaded with half the design snow load. In the case of roofs with slopes greater than 50°, snow load may be disregarded ; where, however, there are possibilities of formation of snow pockets, these should be taken into account.

Loads due to rain. On surfaces whose positioning, shape and drainage system are such as to make accumulation of rain-water possible, load due to such accumulation of water and the live loads for the roofs as given in the table shall be considered separately and the more critical of the two shall be adopted in the design.

Horizontal loads on parapets and balustrades. Where it is desired to design parapets and balustrades against horizontal pressures, the following loads expressed as horizontal loads acting at handrail or coping level shall be provided for. The values given are for guidance

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Live Loads on Roofs (*I S. 875*)

Sl. No. (*1*)	*Type of roof* (*2*)	*Live load measured on plan* (*3*)	*Minimum live load measured on plan* (*4*)
(*i*)	Flat, sloping or curved roof with slopes up to and including 10 degrees		
	(*a*) Access provided	150 *kg./m.*^{2}	375 *kg.* uniformly distributed over any span of 1 *metre* width of the roof slab and 900 *kg* uniformly distributed over the span in the case of all beams
	(*b*) Access not provided, except for maintenance	75 *kg /m.*^{2}	190 *kg.* uniformly distributed over any span of 1 *metre* width of the roof slab and 450 *kg.* uniformly distributed over the span in the case of beams
(*ii*)	Sloping roof with slope greater than 10 degrees	(*a*) For roof membrane, sheets or purlins —75 *kg /m*^{2} less 2 *kg./m*^{2} for every degree increase in slope over 10 degrees	Subject to a minimum of 40 *kg./m*^{2}.
		(*b*) For members supporting the roof memorane and roof purlins, such as trusses, beams, girders, *etc.* —$\frac{3}{4}$ of the load in (*a*)	
		(*c*) Loads in (*a*) and (*b*) do not include loads due to snow, rain, dust collection, *etc.*, and the effects of such load shall be appropriately considered	
(*iii*)	Curved roofs with slope at springing greater than 10 degrees	(75—345) r^2) *kg /m*^{2} where, $r = \frac{h}{l}$ *h* = The height of the highest point of the structure measured from its springing ; and *l* = Chord width of the roof if singly curved and shorter of the two sides, if doubly curved	Subject to a minimum of 40 *kg./m*^{2}.

NOTE—**For special types of roofs with highly permeable and absorbent material, the contingency of roof material increasing in weight due to absorption of moisture shall be provided for.**

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only and where values for actual loadings are available, they shall be used instead.

Description	*Horizontal load kg./metre run*
(*a*) Stairways, landings and balconies (private and domestic).	
Internal	35
External	75
(*b*) all other stairways, parapets and handrails to roofs.	75
(*c*) Parapets and balustrades in places of assembly likely to be over-crowded.	225

§35. Impact and Vibrations

For structures carrying live loads which induce impact or vibration, as far as possible, calculations shall be made for the increase in the live load due to impact or vibration. In the absence of sufficient data for such calculation, the increase in the live load shall be as follows :

Structure	*Impact factor*
(*a*) For frames supporting lifts and hoist	100 percent
(*b*) For foundations, footings and piers supporting lifts and hoisting apparatus	40 percent
(*c*) For light machinery, shaft or motor units	20 percent, *Min.*
(*d*) For reciprocating machinery or power units	50 percent, *Min.*

Concentrated live loads with impact and vibrations. Concentrated live loads with impact and vibration which may be due to installed machinery shall be considered and provided for in the design. The impact factor shall not be less than 20 percent which is the amount allowable for light machinery.

Provision shall also be made for carrying any concentrated equipment loads while the equipment is being installed or moved for servicing and repairing.

For crane gantry girders and supporting columns, the following allowances shall be deemed to cover all forces set up by vibration, shock from slipping of slings, kinetic action of acceleration and retardation, and impact of wheel loads.

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Type of load	*Additional load*
(*a*) Vertical loads for electric overhead cranes	25 percent of maximum static wheel loads
(*b*) Vertical loads for hand-operated cranes	10 percent of maximum wheel loads
(*c*) Horizontal forces transverse to the rails :	
(*i*) For electric over-head cranes	10 percent of weight of the crab and the weight lifted on the crane
(*ii*) For hand-operated cranes	5 percent of weight of the crab and the weight lifted on the crane
(*d* Horizontal forces along the rails	5 percent of the static wheel loads

The horizontal forces specified in (*c*) and (*d*) shall be considered as acting at the rail level and being appropriately transmitted to the supporting system for both electric and hand operated-cranes.

Gantry girders and their vertical supports shall be designed on the assumption that either of the horizontal loads (*c*) and (*d*) may act at the same time as the vertical load.

§36. Wind Load on Buildings

The IS-875 specification has stated that for structures of various plan shapes other than rectangular plan shape, the external pressures acting on the projected area in the plane perpendicular to the wind shall be the product of basic pressure stipulated by the code and the shape factors given in the following table.

Shape (in Plan) Factors

Plan shape of the structure	*Ratio of height to base width 0—4*	*Ratio of height to base width 4—8*	*Ratio of height to base width 8 or over*
Circular	0·7	0·7	0·7
Octagonal	0 8	0·9	1·0
Square (wind perpendicular to diagonal	0·8	0·9	1·0
Square (wind perpendicular to face)	1·0	1·15	1·3

NOTE 1 – In the case of projections above the general roof level, the base width shall be taken as the width of the construction where it rises from the surface of the roof; and the height governing the ratio of height to base width shall be that from the roof surface to the top of the construction.

NOTE 2 – The division of wind load into that acting on windward and leeward sides shall be the same as that for rectangular buildings as affected by the openings.

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1

Analysis of Composite Sections by Elastic Theory

A reinforced concrete member shall be designed for all conditions of stresses that may occur and in accordance with principles of mechanics. This chapter deals with the analysis of reinforced concrete members.

The characteristic property of a reinforced concrete member is that its components namely concrete and steel act together as a single unit as long as they remain in the elastic condition *i.e.* the two components are bound together so that there can be no relative displacement between them.

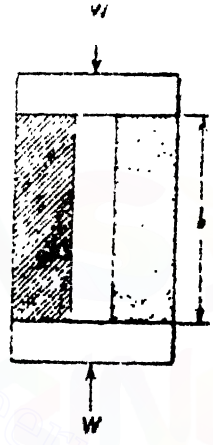

Fig. 6

§37 Reinforced Concrete Members Subjected to Direct Load

Let us consider a reinforced concrete member of length l.

Let A_s = area of steel and

A_c = area of concrete.

For the sake of analysis the two components have been shown separately in Fig. 6. Let the member be subjected to an axial load W. Let W_s and W_c be the load components transmitted to steel and concrete respectively

Assuming both the components undergo the same change in length, Strain in steel = Strain in concrete.

$$\therefore \quad \frac{W_s}{A_s E_s} = \frac{W_c}{A_c E_c} = \frac{W_s + W_c}{A_s E_s + A_c E_c} = \frac{W}{A_s E_s + A_c E_c} \qquad \ldots(i)$$

where, E_s and E_c are the elastic moduli for steel and concrete respectively.

From equation (i) $\quad \frac{W_s}{A_s} = \frac{F_s}{E_c} \cdot \frac{W_c}{E_c}$

But $\quad \frac{W_s}{A_s}$ = stress in steel

and $\quad \frac{W_c}{A_c}$ = stress in concrete

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Let $\frac{E_s}{E_c} = m$ = modular ratio between steel and concrete.

$\therefore$ *Stress in steel* $= m \times$ *stress in concrete*

Again, from equation (*i*) $W_s = \left[\frac{A_s E_s}{A_s E_s + A_c E_c}\right] W$

and $W_c = \left[\frac{A_c E_c}{A_s E + A_c E_c}\right] W$

Putting $E_s = mE_c$ the above relations can be reduced to

$$W_s = \left[\frac{mA_s}{mA_s + A_c}\right] W$$

and $$W_c = \left[\frac{A_c}{mA_s + A_c}\right] W$$

Suppose $A_s = A_c = A$

Then, $$W_s = \left(\frac{m}{m+1}\right) W$$

and $$W_c = \left(\frac{1}{m+1}\right) W$$

i e, if $m = 18$ and the two components have the same area

$$W_s = \frac{18}{19} W$$

and $$W_c = \frac{1}{19} W.$$

Hence we find that the steel member is subjected to a greater load than concrete. This explains, that when steel is provided in combination with concrete, it will be very useful in sharing a considerable part of the load on the composite member.

Equivalent or transformed concrete area. Let f_s and f_c be the stresses in steel and concrete.

Since strains are equal in steel and concrete

$$\frac{f_s}{E_s} = \frac{f_c}{E_c} \qquad \therefore \quad f_s = \frac{E_s}{E_c} f_c$$

$\therefore$ $f_s = m f_c$

Load on steel + load on concrete = total load

$$f_s A_s + f_c A_c = W$$

$$m f_c A_s + f_c A_c = W$$

$\therefore$ $$f_c = \left[\frac{W}{A_c + mA_s}\right]$$

$$f_s = m \left[\frac{W}{A_c + mA_s}\right]$$

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The quantity $(A_c + mA_s)$ is called the equivalent concrete area.

Equivalent concrete area

$= A_e =$ Actual concrete area $+ m \times$ steel area.

§38. Composite Member Subjected to Bending

Consider an R.C. beam simply supported on a span l. Let us consider the mid section of the beam. Let b be the width of the beam and d the depth of the beam from the top compression edge to the centre of steel.

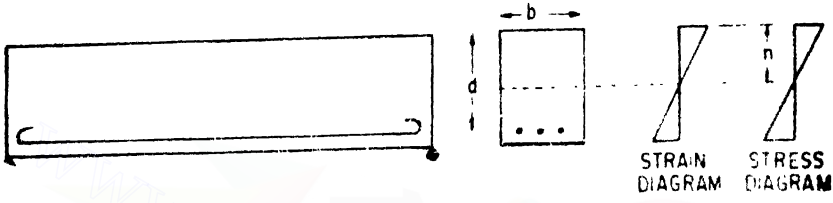

Fig. 7

Fig. 7 shows the strain and stress diagrams for concrete at the section. We assume that the strain varies linearly consistent with the assumption that transverse sections which were plane before bending remain plane after bending. With the assumption of no slip between steel and the surrounding concrete, it means, that the strains in steel and the surrounding concrete are equal. Suppose the strain in concrete at the level of the steel is e, then the strain in steel is also equal to e. The corresponding stresses in concrete and steel at the same level will be eE_c and eE_s respectively. Thus we find the stress in steel is again modular ratio times the stress in concrete at the level of steel.

Neutral axis of the beam section. We know in the case of beams of homogeneous material the neutral axis is a centroidal axis. In the case of composite section the neutral axis can be determined by considering the equivalent or transformed section of one material only. The neutral axis therefore is the centroidal axis of this equivalent section. This treatment is satisfactory when the components of the beam remain as one unit in the elastic condition. For instance in the case of a composite beam consisting of a timber beam strengthened by steel plates, the analysis can be made by replacing steel by its equivalent timber section and taking the neutral axis at the centroid of the transformed section. But concrete is a material strong in compression and weak in tension. After the concrete is strained beyond a limit it develops cracks in the tension zones. After this stage the concrete in the tension zone becomes ineffective to offer tensile resistance. This condition shifts the position of the neutral axis. Hence the neutral axis has to be determined ignoring the concrete in the tension zone. This aspect is discussed in detail in chapter 2 on *Singly Reinforced Beams.*

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Problem 1. (S.I.). *A 30 cm. × 30 cm. R.C. member reinforced with 12·57 cm.² of steel supports an axial compressive load of 440 kilonewtons. Calculate the stresses in concrete and steel. Take m = 18.*

Solution. $A = 30 \times 30 = 900$ *cm.*² ; $A_s = 12{\cdot}57$ *cm.*² ;

$A_c = 900 - 12{\cdot}57 = 887{\cdot}43$ *cm.*²

Equivalent concrete area

$$= A_e = A_c + mA_s$$

$$= 887{\cdot}43 + 18 \times 12{\cdot}57 = 1113{\cdot}69 \text{ cm.}^2$$

Compressive load $= W = 440$ *kilonewtons*

$= 440000$ *newtons*

$$\therefore \quad \text{Stress in concrete} = f_c = \frac{W}{A_e} = \frac{440000}{1113{\cdot}69} = 395{\cdot}1 \text{ N/cm.}^2$$

$$= 3{\cdot}95 \text{ N/mm.}^2$$

$$\therefore \quad \text{Stress in steel} \quad = f_s = mf_c = 11 \times 395{\cdot}1 = 7111{\cdot}8 \text{ N/cm.}^2$$

$$= 71{\cdot}11 \text{ N/mm.}^2$$

Problem 2. (M.K.S.). *A 30 cm. × 30 cm. R.C. member reinforced with 12·57 cm.² of steel supports an axial compressive load of 44000 kg. Calculate the stresses in concrete and steel. Take m = 18.*

Solution. $A = 30 \times 30 = 900$ *cm.*² ; $A_s = 12{\cdot}57$ *cm.*² ;

$A_c = 900 - 12{\cdot}57 = 887{\cdot}43$ *cm.*²

Equivalent concrete area

$$= A_e = A_c + mA_s$$

$$= 887{\cdot}43 + 18 \times 12{\cdot}57 = 1113{\cdot}69 \text{ cm.}^2$$

$\therefore$ *Stress in concrete*

$$= f_c = \frac{W}{A_e} = \frac{44000}{1113{\cdot}69} = 39{\cdot}51 \text{ kg./cm.}^2$$

$\therefore$ *Stress in steel* $= f_s = m f_c = 18 \times 39{\cdot}51 = 711{\cdot}18$ *kg./cm.*²

Problem 3. (S.I.). *A 25 cm. × 25 cm. reinforced concrete member has to support an axial compressive load of 300 kilonewtons. If the stress in concrete is not to exceed 4 N/mm.², calculate the area of steel required. Take m = 18.*

Solution. $A = 25 \times 25 = 625$ *cm.*² ; A_s = area of steel = ?

$A_c = (625 - A_s)$

Equivalent concrete area

$$= A_e = A_c + mA_s = (625 - A_s) + 18\, A_s$$

$$= (625 + 17\, A_s) \text{ cm.}^2$$

Limiting the stress in concrete to 4 N/mm^2 *i e.* 400 $N/cm.^2$,

$$\frac{W}{A_e} = \frac{300{,}000}{625 + 17\, A_s} = 400$$

$\therefore 625 + 17\, A_s = 750$

$\therefore \quad A_s = 7{\cdot}35$ *cm.*²

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Problem 4. (M.K.S.). *A 25 cm. × 25 cm. reinforced concrete member has to support an axial compressive load of 30,000 kg. If the stress in concrete is not to exceed 40 kg./cm.², calculate the area of steel required Take m = 18.*

Solution. $A = 25 \times 25 = 625\ cm.^2$; A_s = area of steel = ?
$A_c = (625 - A_s)$

Equivalent concrete area

$$= A_e = A_c + mA_s = (625 - A_s) + 18\ A_s$$
$$= (625 + 17\ A_s)\ cm.^2$$

Limiting the stress in concrete to 40 *kg./cm.*²,

$$\frac{W}{A_e} = \frac{30{,}000}{625 + 17\ A_s} = 40$$

$\therefore 625 + 17\ A_s = 750$

$\therefore \quad A_s = 7{\cdot}35\ cm.^2$

EXERCISE 1

1. Write a note on requirements of good concrete.
2. What are the various types of cement ?
3. What precautions should be taken while storing cement for a long time ?
4. What do you mean by *specific surface* of a sample of cement ?
5. What do you mean by consistency of a cement paste ?
6. Describe the Vicat apparatus.
7. How do you determine the initial and final setting times of cement ?
8. How are aggregates classified ?
9. What do you mean by fineness modulus ?
10. What do you mean by grades of concrete ? What are the grades recommended by the I S. code ?
11. Describe the slump test for concrete. What are the slump requirements for various situations ?
12. What do you mean by equivalent concrete area of a reinforced concrete member ?
13. What are the basic assumptions in the analysis of reinforced concrete members ?
14. A 40 *cm* × 40 *cm* R.C. member reinforced with 20·32 *cm.*² of steel supports an axial compressive load of 750 *kilonewtons*. Calculate the stresses in concrete and steel. Take $m = 18$.
(*3·85 N/mm.²* ; *69·39 N/mm.²*)
15. A 30 *cm* × 30 *cm.* reinforced concrete member has to support an axial compressive load of 425 *kilonewtons*. If the stress in concrete is not to exceed 4 *N/mm.²*, calculate the area of steel required. Take $m = 18$. (*9·56 cm.²*)
16. A 30 *cm.* × 30 *cm* reinforced concrete member is provided with 4 bars of 18 *mm.* diameter. Calculate the maximum axial compressive load on the member, if the stress in concrete is limited to 4 *N/mm.²*
(*429080 Newtons*)

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2

Singly Reinforced Beams

§39. Assumptions

A singly reinforced beam is a beam provided with longitudinal reinforcement in the tension zone only.

The analysis and design of a reinforced concrete member subjected to bending are based on the following assumptions :

(*a*) Plane sections transverse to the centre line of a member before bending remain plane sections after bending.

(*b*) Elastic modulus of concrete has the same value within the limits of deformation of the member.

(*c*) Elastic modulus for steel has the same value within the limits of deformation of the member.

(*d*) The reinforcement does not slip from concrete surrounding it.

(*e*) Tension is borne entirely by steel.

(*f*) The steel is free from initial stresses when embedded in concrete.

(*g*) There is no resultant thrust on any transverse section of the member.

Of the above assumptions, the assumption that plane sections transverse to the centre line of a member, before bending remain plane after bending is further explained below.

Fig. 8 (*a*) shows a beam subjected to an external loading. Consider sections 1-1, 2-2, 3-3, 4-4, 5-5, *etc.*, which are at right angles to the centre line of the member. After the beam bends, the various fibres are subjected to deformations of such amounts that these planes respectively occupy the new positions shown in Fig. 8 (*b*).

Fig. 9 (*a*) shows a simply supported singly reinforced beam. Consider two sections 1-1 and 2-2 unit distance apart. Let the beam be subjected to an external loading.

Fig. 9 (*b*) shows the deflected form of the beam. The uppermost fibre of concrete AC deforms to A_1C_1. A fibre BD of concrete at the level of the reinforcement deforms to B_1D_1.

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We have strain in concrete in the top fibre

$$= AC - A_1C_1 = e_c$$

Similarly, strain in concrete just surrounding the steel

$$= B_1D_1 - BD = e_t$$

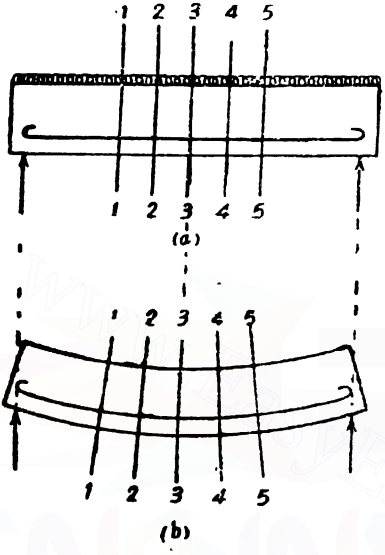

Fig. 8

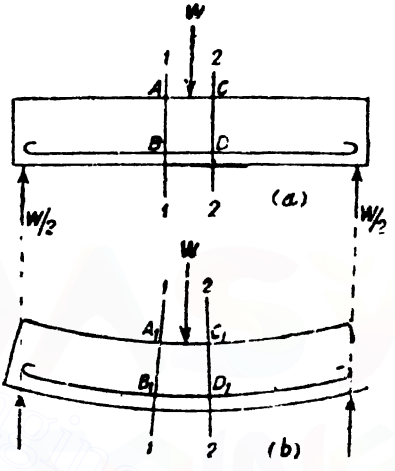

Fig. 9

Since there is no slip between steel and the concrete surrounding it, the strain in steel is also equal to e_t

∴ Stress in steel

$$= t = E_s e_t = m E_c e_t \quad N$$

∴ Stress in concrete surrounding steel

$$= C_t = E_c\ e_t = E_c \frac{t}{E_s}$$

$$\therefore \quad C_t = \frac{t}{m}$$

$C_t = \frac{t}{m}$

Fig. 10

Fig. 10 shows the stress distribution in concrete across the section of the beam.

§40. Neutral Axis

The neutral axis for a beam section is the line of intersection of the neutral layer with the beam section. This is a straight line dividing the cross-section into a tension and a compression zone

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One of the basic assumptions made in the analysis of reinforced concrete beams is that the tension is borne completely by steel. Hence, it is important to note that in determining the neutral axis, the concrete in the tension zone should not be taken into account. The tension should be considered as resisted by the steel. If the area of the reinforcement is A_t and the tensile stress in the reinforcement is t, the total tension resisted

$$= T = A_t t$$
$$= A_t m C_t$$
$$= (mA_t)\, C_t$$

Hence, a reinforcement of area A_t can be regarded as equivalent to an area (mA_t) of concrete. Let b and d be the breadth and effective depth of the beam section. Effective depth means the depth from the compression edge to the centre of the tensile reinforcement. Let n be the depth of neutral axis.

One of the assumptions in the analysis is that there is no resultant thrust on the section.

i.e., Total compression = total tension.

$\therefore$ Compression area × average compressive stress

= area of tensile reinforcement × stress in steel.

$$\therefore \quad bn \frac{c}{2} = A_t t \qquad \ldots(i)$$

Further, by the geometry of the stress diagram (Fig. 11) we have,

$$\frac{c}{t/m} = \frac{n}{d-n} \qquad \ldots(ii)$$

Equating the moments of area of the compression and tension zones about the neutral axis, we have,

$$\frac{bn^2}{2} = mA_t (d-n) \qquad \ldots(iii)$$

Solving the above quadratic equation in n we get,

$$= -mA_t + \sqrt{(mA_t)^2 + 2bdmA_t} \qquad \ldots(iv)$$

Besides the above results, the following result will be found interesting.

We know, $\quad \frac{c}{t/m} = \frac{n}{d-n}$

$\therefore \quad \frac{mc}{t} = \frac{n}{d-n}$

Putting $\quad \frac{t}{c} = r, \quad \frac{m}{r} = \frac{n}{d-n}$

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$$\therefore \quad md - mn = nr$$

$$n(m+r) = md$$

$$\therefore \quad n = \left[\frac{m}{m+r}\right] d \qquad \ldots(v)$$

Putting $n = n_1 d$, we have, $n_1 = \dfrac{m}{m+r}$ $\qquad \ldots(vi)$

With $m = 18$, we have,

$$n_1 = \frac{18}{18+r}$$

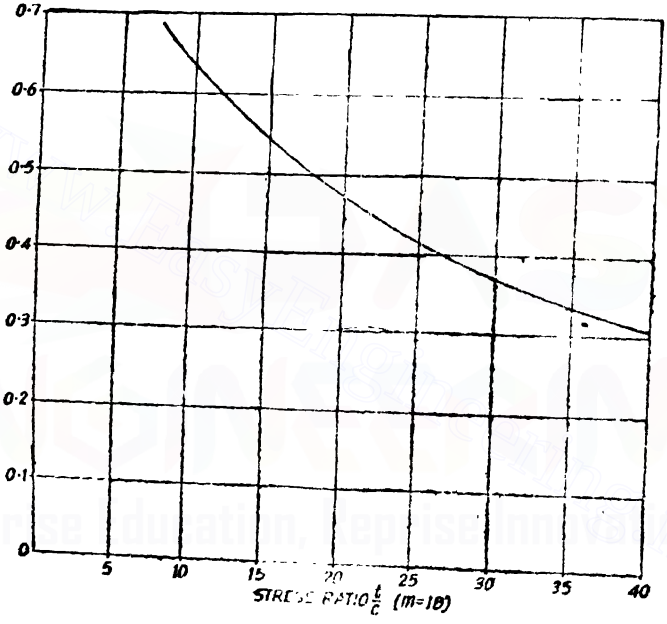

Fig. 11

Fig. 11 shows the values of n_1 for various values of r

Alternative expression for n

$$n_1 = \frac{m}{m+r} = \frac{m}{m+\dfrac{t}{c}}$$

$$\therefore \quad n_1 = \frac{mc}{mc+t}$$

Position of neutral axis for a given percentage of steel.

Rearranging equation (*iii*), we get,

$$\frac{n^3}{2} + A_s \frac{mn}{b} - A_s \frac{md}{b} = 0$$

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Let the percentage of steel be $p\%$

$$\therefore \qquad A_t = \frac{p}{100}(bd)$$

Substituting in the above equation,

$$\frac{n^2}{2} + \frac{mnpd}{100} - \frac{mpd^2}{100} = 0$$

$$\therefore \qquad \frac{n^2}{2} + \frac{mpd}{100}(n-d) = 0$$

$$\therefore \qquad p = \frac{50}{md}\,\frac{n^2}{d-n}$$

Putting $n = n_1 d$,

$$p = \frac{50 n_1^2}{m(1-n_1)} \qquad \ldots(vii)$$

Fig. 12 shows the relation between the percentage of steel and the position of neutral axis for $m = 18$.

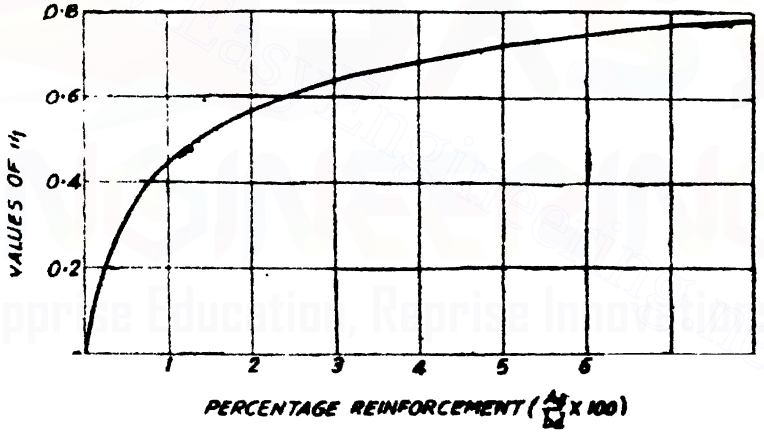

Fig. 12

§41. Lever Arm

This is the distance between the line of action of the resultant compression and the line of action of the resultant tension. The line of action of the resultant compression is at the level of the centroid of the compressive stress diagram, *i.e.*, at a depth of $\frac{n}{3}$ from the compression edge. But the resultant tension is at the level of the reinforcement since the tensile resistance of concrete is ignored.

$$\text{Lever arm} = a = d - \frac{n}{3} \qquad \ldots(viii)$$

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Sometimes the lever arm is expressed as the product of a coefficient a_l and the effective depth.

$$\therefore \quad a = a_l d$$

But $$a = d - \frac{n}{3}$$

$$\therefore \quad a_l d = d - \frac{n}{3}$$

$$\therefore \quad a_l = 1 - \frac{1}{3}\,\frac{n}{d}$$

$$\therefore \quad a_l = 1 - \frac{n_1}{3} \qquad \ldots(ix)$$

§42. Moment of Resistance

This is the resisting moment offered by a beam section to resist the bending moment at the section.

Moment of resistance

= Total compression or total tension × lever arm

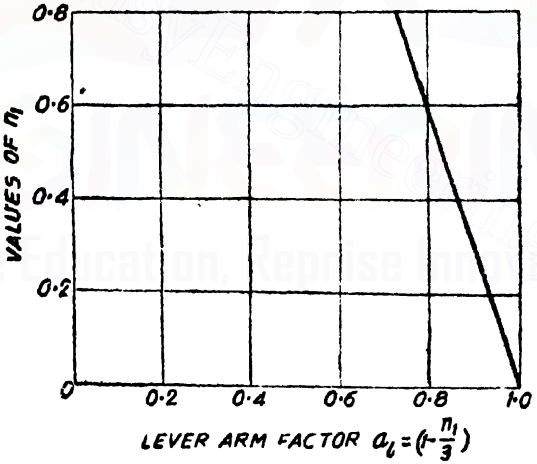

Fig. 13

$$\therefore \quad \text{M.R.} = bn\frac{c}{2}\left(d - \frac{n}{3}\right) \qquad \ldots(x)$$

and $$\text{M.R.} = A_s t\left(d - \frac{n}{3}\right) \qquad \ldots(xi)$$

The moment of resistance can also be expressed as follows :

$$\text{M.R.} = M = bn\,\frac{c}{2}\left(d - \frac{n}{3}\right)$$

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Put $n = n_1 d$

$$\therefore \quad M = b n_1 d \left(d - \frac{n_1 d}{3} \right)$$

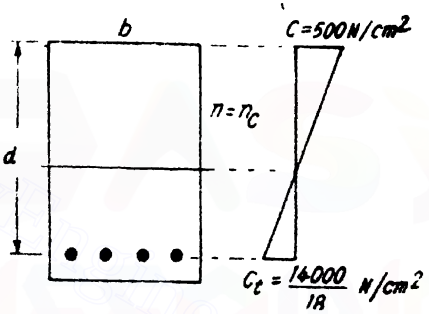
$$\therefore \quad M = \tfrac{1}{2} n_1 \left(1 - \frac{n_1}{3} \right) c\, bd^2$$

$$\therefore \quad M = Q\, bd^2$$

where $$Q = \tfrac{1}{2} n_1 \left(1 - \frac{n_1}{3} \right) c$$

§43 Balanced or Economic or Critical Section

This is a section, in which the quantity of steel provided is such that, when the most distant concrete fibre in the compression zone reaches the allowable stress in compression the tensile stress in the reinforcement reaches its allowable stress. For example let the allowable stresses in concrete and steel be 5 $N/mm.^2$ and 140 $N/mm.^2$ and let $m = 18$. Hence for the balanced section, when the extreme stress in concrete reaches 5 $N/mm.^2$ the stress in steel will be 140 N/mm^2. The neutral axis corresponding to this condition is called the *critical neutral axis* Let n_c be the depth of critical neutral axis.



Balanced section
Fig. 1.6

Analysis of the Balanced-Section Adopting S I. Units

Position of neutral axis

$$c = 5\ N/mm.^2 = 500\ N/cm^2.$$

$$t = 140\ N/mm.^2 = 14000\ N/cm^2.$$

By the geometry of the stress diagram

$$\frac{mc}{t} = \frac{n_c}{d - n_c}$$

$$\frac{18 \times 500}{14000} = \frac{n_c}{d - n_c}$$

$$\frac{n_c}{d - n_c} = \frac{9}{14} \qquad \therefore \quad n_c = \frac{9}{23} d = 0{\cdot}39\, d$$

Neutral axis factor

$$= n_1 = \frac{9}{23} = 0{\cdot}39.$$

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Lever arm $= a = d - \frac{n_c}{3}$

$= d - \frac{3}{23}d = \frac{20}{23}\ d = 0{\cdot}87\ d$

$\therefore$ *Lever arms factor* $= a_1 = \frac{20}{23} = 0{\cdot}87.$

Moment of resistance $=$ M.R. $= bn_c \frac{c}{2}\left(d - \frac{n_c}{3}\right)$

$= b \times \frac{9}{23}\ d\ \frac{500}{2} \times \frac{20}{23} d$

$= \left[\frac{9}{23} \times \frac{500}{2} \times \frac{20}{23}\right] bd^2$

$\therefore$ $M.R. = 85\ bd^2\ N\ cm.$

Total compression = Total tension

$$bn_c \frac{c}{2} = A_t\ t \quad \therefore \quad b \times \frac{9}{23}\ d \times \frac{500}{2} = A_t \times 14000$$

$$\therefore \quad \frac{A_t}{bd} = 0{\cdot}00699 = 0{\cdot}699\%$$

or $A_t = 0{\cdot}699\%$ *of* $bd.$

Analysis of the Balanced Section Adopting M.K.S. Units

Position of neutral axis

$c = 50\ kg/cm.^2$, $t = 1400\ kg./cm.^2$ and $m = 18$

By the geometry of the stress diagram

$$\frac{mc}{t} = \frac{n_c}{d - n_c}$$

$$\frac{18 \times 50}{1400} = \frac{n_c}{d - n_c}$$

$$\therefore \quad \frac{n_c}{d - n_c} = \frac{9}{14}$$

$$\therefore \quad n_c = \frac{9}{23} d$$

$$= 0{\cdot}39\ d$$

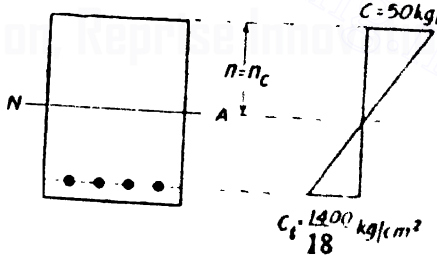

Fig. 17

$\therefore$ Neutral axis factor

$$= n_1 = \frac{9}{23} = 0{\cdot}39.$$

Lever arm

$$= a = d - \frac{n_c}{3} = d - \frac{3}{23} d$$

$$= \frac{20}{23}\ d = 0{\cdot}87\ d$$

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$$\therefore \quad \textit{Lever arm factor} = a_1 = \frac{20}{23} = 0{\cdot}87.$$

$$\textit{Moment of resistance} = \text{M.R.} = bn_c \frac{c}{2}\left(d - \frac{n_c}{3}\right)$$

$$= b \times \frac{9}{23} d \times \frac{50}{2} \times \frac{20}{23} d$$

$$= \left[\frac{9}{23} \times \frac{50}{2} \times \frac{20}{23}\right] bd^2$$

$$\therefore \quad M.R. = 8{\cdot}5\, bd^2 \text{ kg. cm.}$$

Total compression = Total tension

$$bn_c \frac{c}{2} = A_t t$$

$$b \times \frac{9}{23} d \times \frac{50}{2} = A_t \times 1400$$

$$\therefore \quad \frac{A_t}{bd} = 0{\cdot}00699 = 0{\cdot}699\%$$

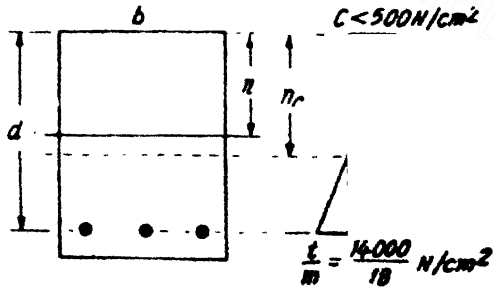
or $A_t = 0{\cdot}699\%$ of bd.

§44. Unbalanced Section

This is a section in which the quantity of steel provided is different from what is required for the balanced section. Unbalanced sections may be classified into *under-reinforced* and *over-reinforced* sections.

§45. Under-Reinforced Section

This is a section in which the quantity of steel provided is less than what is required for a balanced section. In this case when the stress in steel reaches its permissible value, the corresponding extreme compressive stress reached in concrete will be less than its permissible value. For example, taking the permissible stresses in concrete and steel as 5 $N/mm.^2$ and 140 $N/mm.^2$ and $m = 18$, we find that when the stress in steel reaches 140 $N/mm.^2$, the extreme compressive stress in concrete will be less than 5 $N/mm.^2$ (Fig. 16). The depth n of the actual neutral axis will be less than the depth n_c of the critical neutral axis. The moment of resistance of the section will be less than that of the balanced section.



Under-reinforced section
Fig. 16

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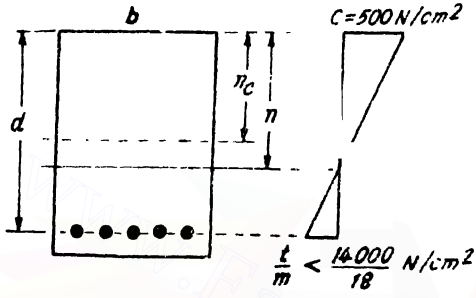
∴ Moment of resistance of the section is given by,

$$M.R. = A_s t \left(d - \frac{n}{3} \right)$$

The stress t being taken at the allowable stress for steel.

§46. Over-Reinforced Section

This is a section in which the quantity of steel provided is more than what is required for a balanced section. In this case when the extreme compressive stress in concrete reaches its permissible value, the corresponding tensile stress in steel will be less than its permissible value. For example, taking the permissible stresses in concrete and steel as 5 $N/mm.^2$ and 140 N/mm^2 and $m = 18$, we find that, when the extreme compressive stress in concrete reaches 5 $N/mm.^2$ the corresponding tensile stress in steel will be less than 140 N/mm^2. (See Fig. 17). The depth n of the actual neutral axis will be greater than the depth n_c of the critical neutral axis. The moment of resistance of the section will be greater than that of the balanced section.



Over-reinforced section.
Fig. 17

Moment of resistance of the section is given by

$$M.R. = bn \frac{c}{2} \left(d - \frac{n}{3} \right)$$

The stress c being taken at the allowable compressive stress for concrete.

Summary of Results

S.I. Units	*M.K.S. Units*
Grade of concrete : M 15	Grade of concrete : M 150
Permissible stresses	Permissible stresses
$C = 5 N/mm.^2 = 500 N/cm.^2$	$c = 50 kg./cm.^2$
$t = 140 N/mm.^2 = 14000 N/cm.^2$	$t = 1400 kg./cm.^2$
$m = 18$	$m = 18$
Properties of balanced section	*Properties of balanced section*
$n = n_c = \frac{9}{23} d = 0{\cdot}39\, d$	$n = n_c = \frac{9}{23} d = 0{\cdot}39\, d$
$= 0{\cdot}87\, d$	$a = 0{\cdot}87\, d$

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M.R. $= 85\ bd^2\ N\ cm.$	M.R. $= 8{\cdot}5\ bd^2\ kg.\ cm.$
$At = 0{\cdot}699\%$ of bd	$At = 0{\cdot}699\%$ of bd
Properties of under-reinforced section	*Properties of under-reinforced section*
$At < 0{\cdot}699\%$ of bd	$At < 0{\cdot}699\%$ of bd
$n < n_c$	$n < n_c$
Steel reaches its permissible stress earlier to concrete	Steel reaches its permissible stress earlier to concrete
M.R. $< 85\ bd^2\ N\ cm.$	M.R. $< 8{\cdot}5\ bd^2\ kg.\ cm.$
$M.R. = At\ t\left(d - \frac{n}{3}\right)$	$M.R. = At\ t\left(d - \frac{n}{3}\right)$
where $t = 140{\cdot}00\ N/cm.^2$	where $t = 1400\ kg./cm.^2$
Properties of over-reinforced section	*Properties of over-reinforced section*
$At > 0{\cdot}699\%$ of bd	$At > 0{\cdot}699\%$ of bd
$n > n_c$	$n > n_c$
Concrete reaches its permissible stress earlier to steel	Concrete reaches its permissible stress earlier to steel
M.R. $> 85\ bd^2\ N\ cm.$	M.R. $> 8{\cdot}5\ bd^2\ kg\ cm.$
$M.R. = bn\frac{c}{2}\left(d - \frac{n}{3}\right)$	$M.R. = bn\frac{c}{2}\left(d - \frac{n}{3}\right)$
where $c = 500\ N/cm.^2$	where $c = 50\ kg./cm.^2$

§47. Types of Problems

In singly-reinforced beams we come across the following types of problems :

Type A *Data* : Dimensions of the section, permissible stresses in concrete and steel and modular ratio.

Required : Moment of resistance of the section.

This type of problem may be solved as follows :

(*i*) First determine the position of the actual neutral axis by equating the moment of the concrete area in compression about the neutral axis to the moment of equivalent tension area about the neutral axis,

i.e., use the relation,

$$\frac{bn^2}{2} = mA_t(d-n) \text{ and find } n.$$

(*ii*) Find the position of critical neutral axis corresponding to the given safe stresses in concrete and steel.

(*iii*) Ascertain whether the section is under-reinforced or over-reinforced. If the actual neutral axis lies above the critical neutral axis, the section is under-reinforced. But, if the actual neutral axis is below the critical neutral axis, the section is over-reinforced.

(*iv*) If the section is over-reinforced concrete attains the maximum stress earlier than steel, and the moment of resistance is given by

$$M.R. = bn\frac{c}{2}\left(d - \frac{n}{3}\right),$$

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taking c = permissible stress in concrete

and n = depth of actual neutral axis.

If the section is under-reinforced, steel attains the maximum stress earlier than concrete. and the moment of resistance is given by

$$\text{M.R.} = A_s t \left(d - \frac{n}{3} \right),$$

taking t = permissible stress in steel

and n = depth of actual neutral axis

Type B *Data* : Dimensions of the section, Area of reinforcement, Bending moment M and modular ratio.

Required : Stresses in concrete and steel.

This type of problem may be solved as follows :

(*i*) Determine the position of the actual neutral axis.

(*ii*) Find the stress in concrete by equating the moment of resistance to the given bending moment.

i.e. use the relation,

$$bn\frac{c}{2}\left(d - \frac{n}{3} \right) = M \text{ and find } c.$$

(*iii*) Find the stress in steel from the relation,

$$\frac{mc}{t} = \frac{n}{d-n}$$

Type C *Daia* : Permissible stresses in concrete and steel, Bending moment M and modular ratio.

Required : To design the section.

This type of problem may be solved as follows :

(*i*) Determine the depth of critical neutral axis in terms of the effective depth d

i.e. use the relation,

$$\frac{c}{t/m} = \frac{n_c}{d-n_c} \text{ and find } n_c \text{ in terms of } d.$$

(*ii*) Choose a convenient width b. By equating the moment of resistance to the given bending moment find the effective depth

i.e. use the relation, $bn_c \frac{c}{2}\left(d - \frac{n_c}{3} \right) = M$ and find d.

(*iii*) Find the area of steel by equating the total compression on the beam section to the total tension on the beam section.

i.e. use the relation,

$$bn_c\frac{c}{2} = A_s t \text{ and find } A_s.$$

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The following problems illustrate the above types of problems.

Problem 5. (M.K.S). *The cross-section of a singly reinforced concrete beam is 30 cm. wide and 40 cm. deep to the centre of the reinforcement which consists of four bars of 14 mm. dia. If the stresses in concrete and steel are not to exceed 50 kg./cm.² and 1400 kg./cm.² determine the moment of resistance of the section. Take m = 18.*

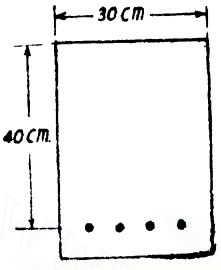

Fig. 18

Solution. See Fig. 18.

$$A_s = 4 \times 1{\cdot}54 = 6{\cdot}16 \text{ cm.}^2$$

Position of actual neutral axis. Equating moments of compression and equivalent tension zones about the actual neutral axis, we have

$$\frac{30n^2}{2} = 18 \times 6{\cdot}16\,(40-n)$$

$$\therefore \quad n^2 + 7{\cdot}392\,n - 295{\cdot}7 = 0$$

$$\therefore \quad n = 13{\cdot}89 \text{ cm.}$$

For a balanced section,

$$\text{Steel ratio} = r = \frac{t}{c} = \frac{1400}{50} = 28$$

Depth of critical neutral axis

$$= n_c = \frac{m}{m+r}\,d$$

$$= \frac{18}{18+28} \times 40 \text{ cm.}$$

$$= 15{\cdot}65 \text{ cm.}$$

But $n = 13.89$ cm.

Hence $n < n_c$.

$\therefore$ The section is *under-reinforced* and steel will attain the maximum stress earlier to concrete

$$\therefore \quad \text{M.R.} = A_{st}\left(d - \frac{n}{3}\right)$$

$$= 6{\cdot}16 \times 1400\left(40 - \frac{13{\cdot}89}{3}\right) \text{kg. cm.}$$

$$= 305030 \text{ kg. cm.}$$

Problem 6. (S.I.). *The cross-section of a singly reinforced concrete beam is 30 cm. wide and 40 cm. deep to the centre of the reinforcement which consists of four bars of 14 mm. diameter. If the stresses in concrete and steel are not to exceed 5 N/mm.² and 140 N/mm.² determine the moment of resistance of the section. Take m = 18.*

Solution. $A_s = 4 \times 1{\cdot}54 = 6{\cdot}16 \text{ cm}^2$.

Position of actual neutral axis. Equating moments of com-

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pression and equivalent tension zones about the actual neutral axis, we have

$$\frac{30n^2}{2} = 18 \times 6{\cdot}16\ (40-n)$$

$$n^2 + 7{\cdot}392\ n - 295\ 7 = 0$$

$$\therefore \qquad n = 13{\cdot}89\ cm.$$

For a balanced section,

$$\text{Steel ratio} = r = \frac{t}{c} = \frac{140}{5} = 28$$

Depth of critical neutral axis,

$$n_c = \left(\frac{m}{m+r}\right)d$$

$$= \left(\frac{18}{18+28}\right)40 \quad = 15{\cdot}65\ cm.$$

Hence $n < n_c$

$\therefore$ The section is *under-reinforced* and steel will attain the maximum stress earlier to concrete.

$$\text{M.R.} = A_s t \left(d - \frac{n}{3}\right)$$

$$= 6{\cdot}16 \times 14000 \left(40 - \frac{13{\cdot}89}{3}\right) N\ cm.$$

$$= 3050310\ N\ cm.$$

Problem 7. (M.K.S.) *A singly reinforced concrete beam 25 cm. wide and 40 cm deep to the centre of tensile reinforcement has a span of 5 metres and carries a total uniformly distributed load of 1250 kg. per metre including its weight. The stresses in concrete and steel are not to exceed 50 kg./cm.² and 1400 kg./cm.² respectively. Find the steel reinforcement necessary. Take m = 18.*

Solution. Total load per *metre*

$$= 1250\ kg./metre$$

Span $= 5\ metres$

Maximum bending moment

$$M = \frac{1250 \times 5^2}{8} \times 100\ kg.\ cm.$$

$$= 390625\ kg.\ cm.$$

$$\text{M.R.} = Qbd^2 = Q \times 25 \times 40^2 = 390625\ kg.\ cm.$$

$$\therefore \qquad Q = \frac{390625}{25 \times 1600} = 9{\cdot}77$$

$M = 9{\cdot}77\ bd^2$

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If the section had been a balanced section,

$$n_c = \frac{m}{m+r} = \frac{18}{18+\frac{1400}{50}} = 0{\cdot}39$$

$$\text{Balanced M.R.} = b \times 0{\cdot}39\, d \times \frac{50}{2}\left(d - \frac{0{\cdot}39\, d}{3} \right)$$

$$= 8{\cdot}50\, bd^2.$$

Since the moment of resistance of the beam has to be greater than that of the balanced section, the beam is to be *over-reinforced.* Concrete attains its maximum stress earlier to steel.

$\therefore$ taking $c = 50$ *kg./cm.*^{2}

$$\text{M.R. of the beam} = b\, n \frac{c}{2}\left(d - \frac{n}{3} \right)$$

$$= 25\, n \times \frac{50}{2}\left(40 - \frac{n}{3} \right) = 390625 \text{ kg. cm.}$$

$$\therefore \quad n\left(40 - \frac{n}{3} \right) = \frac{390625}{625} = 625$$

$$40\, n - \frac{n^2}{3} = 625$$

$$n^2 - 120\, n + 1875 = 0$$

Solving,

we get $n = 18{\cdot}46$ *cm.*

Stress in steel is given by

$$\frac{mc}{t} = \frac{n}{d-n}$$

$$\frac{18 \times 50}{t} = \frac{18{\cdot}46}{40 - 18{\cdot}46}$$

$$\therefore \quad t = \frac{18 \times 50 \times 21{\cdot}54}{18{\cdot}46} \text{ kg./cm.}^2$$

$$= 1050 \text{ kg./cm.}^2$$

Total compression = Total tension

$$bn\frac{c}{2} = A_s.t$$

$$25 \times 18{\cdot}46 \times \frac{50}{2} = A_s \times 1050$$

$$A_s = \frac{25 \times 18{\cdot}46 \times 50}{2 \times 1050} \text{ cm.}^2 = 10{\cdot}98 \text{ cm.}^2$$

Problem 8. (S.I.) *A singly reinforced concrete beam 25 cm. wide and 40 cm. deep to the centre of the tensile reinforcement has a span of 5 metres and carries a total uniformly distributed load of*

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12500 Newtons per metre including its weight. The stresses in concrete and steel are not to exceed 5 N/mm.² and 140 N/mm.² respectively. Find the steel reinforcement necessary. Take m = 18.

Solution Total load per *metre*

$$= 12500 \text{ Newtons/metre}$$

$$\text{Span} = 5 \text{ metres}$$

$\therefore$ Maximum bending moment

$$= M = \frac{12500 \times 5^2}{8} = 39062{\cdot}5 \text{ N m.}$$

$$= 3906250 \text{ N cm.}$$

$$M.R = Qbd^2 = Q \times 25 \times 40^2 = 3906250 \text{ N cm.}$$

$$\therefore \quad Q = \frac{3906250}{25 \times 1600} = 97{\cdot}7$$

$$\therefore \quad M.R. = 97{\cdot}7\, bd^2$$

If the section had been a balanced section,

$$n_c = \frac{m}{m + r} = \frac{18}{18 + \frac{140}{5}} = 0{\cdot}39$$

$$\text{Balanced M.R.} = b \times 0{\cdot}39\, d \times \frac{500}{2}\left(d - \frac{0{\cdot}39\, d}{3} \right)$$

$$= 85\, bd^2.$$

Since the moment of resistance of the beam section has to be greater than that of the balanced section, the beam is to be *over-reinforced.* Concrete attains its maximum stress earlier to steel taking $c = 5 \text{ N/mm.}^2 = 500 \text{ N/cm.}^2$

$$\text{M.R. of the beam} = b\, n \frac{c}{2}\left(d - \frac{n}{3} \right)$$

$$= 25\, n \times \frac{500}{2}\left(40 - \frac{n}{3} \right) = 3906250 \text{ N cm.}$$

$$n\left(40 - \frac{n}{3} \right) = \frac{3906250}{6250} = 625$$

$$40\, n - \frac{n^2}{3} = 625$$

$\therefore \; n^2 - 120\, n + 1875 = 0$

Solving, we get $n = 18{\cdot}46$ *cm.*

Stress in steel is given by,

$$\frac{mc}{t} = \frac{n}{d - n}$$

$$\frac{18 \times 500}{t} = \frac{18{\cdot}46}{40 - 18{\cdot}46}$$

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$$\therefore \quad t = \frac{18 \times 500 \times 21{\cdot}54}{18{\cdot}46} \ N/cm.^2$$

$$= 10500 \ N/cm.^2$$

Total compression = Total tension

$$bn\frac{c}{2} = A_t \, t$$

$$25 \times 18{\cdot}46 \times \frac{500}{2} = A_t \times 10500$$

$$\therefore \quad A_t = \frac{25 \times 18{\cdot}46 \times 500}{2 \times 10500} = 10{\cdot}98 \ cm.^2$$

Problem 9. *A reinforced concrete beam 30 cm. wide by 60 cm. total depth has a span of 6 metres. Find the necessary tension reinforcement at the mid span to enable the beam to carry a load of 800 kg /m. in addition to its own weight.*

Concrete cover below the steel centre $= 3{\cdot}5$ *cm.*

Weight of R.C.C. $= 2500$ *kg./m.*^{3}

Allowable stress in steel $= 1400$ *kg./cm.*^{2}

Allowable stress in concrete $= 50$ *kg./cm.*^{2}

Modular ratio $m = 18$

Solution. Dead load per *metre* length

$$= 0{\cdot}3 \times 0{\cdot}6 \times 1 \times 2500 = 450 \ kg./m.$$

Superimposed load on the beam $= 800$ *kg./m.*

Total load per *metre* $= 1250$ *kg./m.*

$$\text{Maximum bending moment} = M = \frac{1250 \times 6^2}{8} \times 100 \ kg.\ cm.$$

$$= 562500 \ kg.\ cm.$$

Let the M.R. of the beam be Qbd^2

$$\therefore \quad Qbd^2 = Q \times 30 \times 56{\cdot}5^2 = 562500$$

$$\therefore \quad Q = \frac{56{\cdot}500}{30 \times 56{\cdot}5^2}$$

$$Q = 5{\cdot}87$$

$$\text{M.R.} = 5{\cdot}87 \ bd^2$$

For a balanced section,

with $c = 50 \ kg./cm^2$

$t = 1400 \ kg./cm.^2$

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$$\therefore \quad c = \frac{f}{m} \cdot \frac{n}{d-n}$$

$$\therefore \quad c = \frac{14000}{18} \times \frac{n}{d-n}$$

Effective depth $= 56{\cdot}5$ *cm.*

$$\therefore \quad c = \frac{14000}{18} \times \frac{n}{56{\cdot}5-n}$$

$$\text{M.R.} = b\, n \frac{c}{2}\left(d - \frac{n}{3}\right)$$

$$= 30\, n \times \frac{14000}{2 \times 18} \times \frac{n}{56{\cdot}5-n}\left(56{\cdot}5 - \frac{n}{3}\right)$$

$$= 5625000$$

$$\therefore \frac{n^2\left(56{\cdot}5 - \frac{n}{3}\right)}{56{\cdot}5-n} = \frac{5625000 \times 36}{30 \times 14000}$$

$$\therefore \frac{n^2\left(56{\cdot}5 - \frac{n}{3}\right)}{56{\cdot}5-n} = 482{\cdot}14$$

Solving the above equation by trial and error,

we get $n = 18{\cdot}99$ *cm.*

$$\therefore \quad c = \frac{14000}{18} \times \frac{18{\cdot}99}{(56{\cdot}5-18{\cdot}99)} = 393{\cdot}8 \ N/cm.^2$$

Total compression = Total tension

$$bn\frac{c}{2} = A_s f$$

$$30 \times 18{\cdot}99 \times \frac{393{\cdot}8}{2} = A_s \times 14000$$

$$\therefore \quad A_s = \frac{30 \times 18{\cdot}99}{14000} \times \frac{393{\cdot}8}{2} \ cm^2.$$

$$= 8{\cdot}01 \ cm.^2$$

Problem 11. (M.K.S). *The moment of resistance of a rectangular singly reinforced beam of width b cm. and effective depth d cm. is 10 bd^2 kg. cm. If the stresses in the outside fibre of the concrete and in steel do not exceed 50 kg./cm.^{2} and 1400 kg./cm.^{2} respectively and the modular ratio equals 18, determine the ratio of the depth of neutral axis from the extreme compression fibres to the effective depth of the beam and the ratio of the area of the tensile steel to the effective area of the beam.*

Solution. For a balanced section, with $c = 50$ *kg./cm.*^{2}, $t = 1400$ *kg./cm.*^{2} and $m = 18$, the M.R. of the section $= M = 8{\cdot}50\ bd^2$.

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But the M.R. of the given beam $= 10\ bd^2$. Since the M.R. of the given beam is greater than the M. R. of the balanced section, the beam should be *over-reinforced.* Hence concrete attains its maximum stress earlier to steel.

Let n = actual depth of neutral axis

$$\frac{n}{d} = n_1$$

$$\text{M.R.} = bn\ \frac{c}{2}\left(d - \frac{n}{3}\right) = 10\ bd^2$$

$$\therefore \quad bn_1 d \times \frac{50}{2}\left(d - \frac{n_1 d}{3}\right) = 10\ bd^2$$

$$\therefore \quad n_1\left(1 - \frac{n_1}{3}\right) = \frac{10}{25} = 0{\cdot}4$$

$$n_1 - \frac{n_1^2}{3} = 0{\cdot}4$$

$$n_1^2 - 3n_1 + 1{\cdot}2 = 0$$

Solving, we get $n_1 = 0{\cdot}476$

Stress in steel is given by,

$$\frac{mc}{t} = \frac{n}{d-n} = \frac{n_1 d}{d - n_1 d} = \frac{n_1}{1 - n_1}$$

$$\therefore \quad t = mc\ \frac{1-n_1}{n_1} = 18 \times 50 \times \frac{(1 - 0{\cdot}476)}{0{\cdot}476}\ kg./cm.^2$$

$$= 990{\cdot}7\ kg./cm.^2$$

Total compression = Total tension

$$bn\frac{c}{2} = A_t . t$$

$$b \times 0{\cdot}476d \times \frac{50}{2} = A_t \times 990{\cdot}7$$

$$\therefore \quad \frac{A_t}{bd} = \frac{0{\cdot}476 \times 25}{990{\cdot}7} = 0{\cdot}012.$$

Problem 12. (S.I.). *The moment of resistance of a rectangular singly reinforced beam of width b cm. and effective depth d cm. is 100 bd^2 Newton centimetre. If the stresses in the outside fibre of the concrete and steel do not exceed 5 N/mm.^{2} and 140 N/mm.^{2} respectivley and the modular ratio equals 18, determine the ratio of the depth of neutral axis from the extreme compression fibres to the effective depth of the beam, and the ratio of the area of the tensile steel to the effective area of the beam.*

Solution. For a balanced section with $c = 5$ *N/mm.*^{2}, $= 500$ *N/cm.*^{2}, $t = 140$ *N/mm*^{2} $= 14000$ *N/cm.*^{2} and $m = 18$, the M.R. of the section $= M = 85\ bd^2$ *N cm.*

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But, the M. R. of the given beam section is 100 bd^2. Since the M.R. of the given beam section is greater than the M.R. of the balanced section, the beam should be *over-reinforced.* Hence, concrete attains its maximum stress eartier to steel.

Let n = actual depth of neutral axis

Let $\frac{n}{d} = n_1$

$$\text{M. R.} = bn\,\frac{c}{2}\left(d - \frac{n}{3}\right) = 100\ bd^2$$

$$\therefore \quad bn_1 d \times \frac{500}{2}\left(d - \frac{n_1 d}{3}\right) = 100\ bd^2$$

$$\therefore \quad n_1\left(1 - \frac{n_1}{3}\right) = 0{\cdot}4$$

$$\therefore \quad n_1^2 - 3n_1 + 1{\cdot}2 = 0$$

Solving, we get $n_1 = 0{\cdot}476$

Stress in steel is given by,

$$\frac{mc}{t} = \frac{n}{d-n} = \frac{n_1 d}{d - n_1 d} = \frac{n_1}{1-n_1}$$

$$\therefore \quad t = mc\,\frac{1-n_1}{n_1} = 18 \times 500 \times \left[\frac{1-0{\cdot}476}{0{\cdot}476}\right] N/cm.^2$$

$$= 9907\ N/cm^2.$$

Total compression = Total tension

$$bn\frac{c}{2} = A_t t$$

$$b \times 0{\cdot}476\ d \times \frac{500}{2} = A_t \times 9907$$

$$\therefore \quad \frac{A_t}{bd} = \frac{0{\cdot}476 \times 250}{9907} = 0{\cdot}012.$$

Problem 13. (M.K.S.). *The moment of resistance of a singly reinforced rectangular reinforced concrete beam of breadth b cm. and effective depth d cm. is 6 bd^2 kg. cm. If the stresses in the extreme fibre of concrete and in steel do not exceed 50 kg./cm.^{2} and 1400 kg./cm.^{2} respectively and the modular ratio equals 18, determine the ratio of the depth of neutral axis from the outside compression fibres to the effective depth of the beam and the ratio of the area of the tensile steel to the effective area of the beam.*

Solution. For a balanced section, with $c = 50$ *kg./cm.*^{2}, $t = 1400$ *kg./cm.*^{2} and $m = 18$, the moment of resistance of the balanced section. $= 8{\cdot}5\ bd^2$ *kg. cm.*

But the M.R. of the given beam is only 6 bd^2. *kg. cm.*

Since the M.R. of the given beam is less than the M.R. of the balanced section, the beam section should be *under-reinforced.* Hence steel attains the maximum stress earlier to concrete

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Hence taking $t = 1400\ kg./cm.^2$,
corresponding maximum stress in concrete is given by,

$$\frac{18c}{1400} = \frac{n}{d-n}$$

$$\therefore \quad c = \frac{1400}{18} \cdot \frac{n}{d-n}$$

Let $n = n_1 d$

$$\therefore \quad c = \frac{1400}{18} \cdot \frac{n_1 d}{d - n_1 d}$$

$$\therefore \quad c = \frac{1400}{18} \cdot \left(\frac{n_1}{1-n_1} \right)$$

$$\text{M.R.} = bn \frac{c}{2} \left(d - \frac{n}{3} \right) = 6\ bd^2$$

$$\therefore \quad b\ n_1 d \frac{1400}{2 \times 18} \left(\frac{n_1}{1-n_1} \right) \left(d - \frac{n_1 d}{3} \right) = 6\ bd^2$$

$$\frac{n_1^2 \left(1 - \frac{n_1}{3} \right)}{1-n_1} = \frac{6 \times 2 \times 18}{1400}$$

$$\therefore \quad \frac{n_1^2 \left(1 - \frac{n_1}{3} \right)}{1-n_1} = 0{\cdot}154$$

Solving the above equation by trial and error, we get

$$n_1 = 0{\cdot}34$$

$$\therefore \quad n = 0{\cdot}34\ d$$

$$\therefore \quad c = \frac{1400}{18} \cdot \frac{0{\cdot}34}{1-0{\cdot}34}\ kg./cm.^2$$

$$= 41{\cdot}3\ kg./cm.^2$$

Total compression = Total tension

$$\therefore \quad bn \frac{c}{2} = A_t\ t$$

$$\therefore \quad b \times 0{\cdot}34\ d \times \frac{41{\cdot}3}{2} = A_t \times 1400$$

$$\therefore \quad \frac{A_t}{bd} = \frac{0{\cdot}34 \times 41{\cdot}3}{2 \times 1400} = 0{\cdot}005$$

Problem 14. (S I.). *The moment of resistance of a singly reinforced rectangular reinforced concrete beam of breadth b cm. and effective depth d cm is 60 bd^2 Newton cm. If the stresses in the extreme fibre of concrete and steel do not exceed 5 N/mm.² and 140 N/mm.² respectively and the modular ratio equals 18, determine the ratio*

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of the depth of neutral axis from the outside compression fibres to the effective depth of the beam and the ratio of the area of the tensile steel to the effective area of the beam.

Solution. For a balanced section, with $c = 5\ N/mm.^2 = 500\ N/cm.^2$, $t = 140\ N/mm.^2 = 1400\ N/cm.^2$ and $m = 18$, the moment of resistance of the balanced section $= 85\ bd^2\ N\ cm.$ But the M.R. of the given beam is only $60\ bd^2\ N\ cm.$

Since the M.R. of the given beam is less than the M.R. of the balanced section, the beam section should be *under-reinforced.* Hence steel attains the maximum stress earlier to concrete.

Hence, taking $t = 14000\ N/cm.^2$

corresponding maximum stress in concrete is given by,

$$\frac{18c}{14000} = \frac{n}{d-n}$$

$$\therefore \quad c = \frac{14000}{18} \cdot \frac{n}{d-n}$$

Let $n = n_1 d$

$$\therefore \quad c = \frac{14000}{18} \cdot \frac{n_1 d}{d - n_1 d}$$

$$\therefore \quad c = \frac{14000}{18}\left(\frac{n_1}{1-n_1}\right)$$

$$\text{M.R.} = bn\,\frac{c}{2}\left(d - \frac{n}{3}\right) = 60\ bd^2$$

$$\therefore \quad bn_1 d \times \frac{14000}{2 \times 18}\left(\frac{n_1}{1-n_1}\right)\left(d - \frac{n_1 d}{3}\right) = 60\ bd^2$$

$$\frac{n_1^2\left(1 - \frac{n_1}{3}\right)}{1-n_1} = \frac{60 \times 2 \times 18}{14000}$$

$$\therefore \quad \frac{n_1^2\left(1 - \frac{n_1}{3}\right)}{1-n_1} = 0{\cdot}154$$

Solving by trial and error, we get,

$$n_1 = 0{\cdot}34$$

$$\therefore \quad c = \frac{14000}{18} \times \frac{0{\cdot}34}{1 - 0{\cdot}34}\ N/cm.^2$$

$$= 413\ N/cm^2.$$

Total compression = Total tension

$$\therefore \quad bn\,\frac{c}{2} = A_t\ t$$

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$$\therefore \quad b \times 0{\cdot}34\, d \times \frac{413}{2} = A_t \times 14000$$

$$\therefore \quad \frac{A_t}{bd} = \frac{0{\cdot}34 \times 413}{2 \times 14000} = 0{\cdot}005.$$

Problem 15. (M.K.S.). *A reinforced concrete slab has an overall depth of 10 cm., the effective cover to the centre of the reinforcement being 2 cm. If the stresses in concrete and steel are not to exceed 50 kg./cm.^{2} and 1400 kg./cm.^{2}, find the safe uniformly distributed load which can be placed on the slab. The slab is supported on beams spaced at 3 metres centres. Find also the spacing of 10 mm. diameter bars to resist the maximum bending moment. The maximum bending moment for a one metre wide strip of the slab may be taken as* $\frac{wl^2}{12}$ *kg. m. where w is the load on the slab in kg. per metre^2, and l is the spacing of the beams in metres. Take* $m = 18$.

Solution. For a balanced section, with $c = 50$ kg./cm.^{2}, $t = 1400$ kg./cm.^{2} and $m = 18$, the moment of resistance of the balanced section $= 8{\cdot}5\, bd^2$ kg. cm.

and lever arm $a = 0{\cdot}87\, d$

Hence, the moment of resistance offered by the balanced section per *metre* width

$$= 8{\cdot}5 \times 100 \times (10-2)^2 \text{ kg. cm.}$$
$$= 54400 \text{ kg. cm.}$$

Let the safe distributed load on the slab be w kg./metre^2.
Maximum bending moment for a 1 *metre* wide strip

$$\frac{wl^2}{12} \quad \frac{w \times 3^2}{12} \times 100 \text{ kg. cm.}$$

Equating the maximum bending moment to the moment of resistance, we get,

$$w \times \frac{3^2}{12} \times 100 = 54{,}400$$

$$w = \frac{54400 \times 12}{9 \times 100} \text{ kg./metre}^2$$

$$= 725{\cdot}3 \text{ kg /metre}^2$$

The above load is the total load on the slab including the weight of the slab also.

Dead load of the slab per *square metre*

$$= 1 \times \frac{10}{100} \times 2500 = 250 \text{ kg./metre}^2$$

$\therefore$ *Net external load which the slab can support*

$$= 725{\cdot}3 - 250 = 475{\cdot}3 \text{ kg./metre}^2$$

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Area of steel $A_t = \frac{M}{ta}$

$$= \frac{54400}{1400 \times 0{\cdot}87 \times 8} \text{ cm.}^2$$

$$= 5{\cdot}58 \text{ cm.}^2$$

∴ Spacing of 10 *mm.* diameter bars

$$= \frac{\text{area of 1 bar} \times 100}{\text{Total area of steel per } \textit{metre} \text{ width}}$$

$$= \frac{0{\cdot}79 \times 100}{5{\cdot}58} \text{ cm.}$$

$$= 14{\cdot}2 \text{ cm.}$$

Hence, let us provide *10 mm. diameter bars at 14 cm. centres.*

Problem 16. (S.I.). *A reinforced concrete slab has an overall depth of 10 cm. the effective cover to the centre of the reinforcement being 2 cm. If the stresses in concrete and steel are not to exceed 5 N/mm.² and 140 N/mm² find the safe uniformly distributed load which can be placed on the slab The slab is supported on beams spaced at 3 metres centres Find also the spacing of 10 mm. diameter bars to resist the maximum bending moment. The maximum bending moment for a one metre wide strip of the slab may be taken as* $\frac{wl^2}{12}$ *Newton metre where w is the load on the slab in Newton/metre², and l is the spacing of the beams in metres. Take m = 18.*

Solution. For a balanced section, with $c = 5$ *N/mm.²* $= 500$ *N/cm²*, $t = 140$ *N/mm.²* $= 14000$ *N/cm²*, and $m = 18$. the moment of resistance of the balanced section $= 85\ bd^2$ *N cm* and the lever arm $= a = 0{\cdot}87\ d$.

Hence, the moment of resistance offered by the balanced section per *metre* width

$$= 85 \times 100\ (10-2)^2 \text{ N cm.}$$

$$= 544{,}000 \text{ N cm.}$$

Let the safe distributed load on the slab be *w Newton/metre²*.

Maximum bending moment for a 1 *metre* wide strip

$$= \frac{wl^2}{12} = \frac{w \times 3^2}{12} \times 100 \text{ N cm.}$$

Equating the maximum bending moment to the moment of resistance, we get,

$$w \times \frac{3^2}{12} \times 100 = 544000$$

$$\therefore \quad w = \frac{544000 \times 12}{9 \times 100} = 7253 \text{ N/metre}^2$$

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The above load is the total load on the slab including the weight of the slab also.

Dead load of the slab per *square metre*

$$= 1 \times \frac{10}{100} \times 25000 = 2500 \ N/metre^2$$

Net external load which the slab can support

$$= (7253 - 2500) \ N/metre^2$$

$$= 4753 \ N/metre^2$$

Area of steel $= A_t = \dfrac{M}{ta}$

$$= \frac{544000}{14000 \times 0{\cdot}87 \times 8} cm.^2$$

$$= 5{\cdot}58 \ cm.^2$$

$\therefore$ Spacing of 10 *mm.* diameter bars

$$= \frac{\text{area of 1 bar} \times 100}{\text{total area of steel per } metre \text{ width}}$$

$$= \frac{0{\cdot}79 \times 100}{5{\cdot}58} \ cm.$$

$$= 14{\cdot}2 \ cm.$$

Hence, let us provide *10 mm. diameter bars at 14 cm. centres.*

§48. Effect of Varying the Steel Ratio on the Depth of Neutral Axis and the Moment of Resistance

For this discussion let us consider *M* 15 grade concrete (S.I. units).

Permissible compressive stress in concrete

$$= 5 \ N/mm.^2 = 500 \ N/cm.^2$$

Permissible tensile stress in steel

$$= 140 \ N/mm.^2 = 14000 \ N/cm.^2$$

Modular Ratio $= m = 18.$

For a balanced section, the depth of the neutral axis is given by,

$$\frac{mc}{t} = \frac{n}{d-n}$$

$$\frac{18 \times 500}{14000} = \frac{n}{d-n}$$

$$n = 0{\cdot}39 \ d$$

Lever arm $= a = d - \dfrac{n}{3}$

$$= d - \frac{0{\cdot}39 \ d}{3}$$

$$= 0{\cdot}87 \ d$$

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Moment of resistance

$$= \text{M.R.} = bn\frac{c}{2}\left(d - \frac{n}{3}\right)$$

$$= b \times 0{\cdot}39\, d \times \frac{500}{2} \times 0{\cdot}87\, d$$

$$= 85\, bd^2$$

$$= Q\, bd^2 \qquad \text{where} \quad Q = 85$$

Total tension = Total compression

$$A_t\, t = bn\, c$$

$$A_t \times 14000 = b \times 0\,39\, d \times 500$$

$$\therefore \text{ Steel ratio } = p = \frac{A_t}{bd} = \frac{0{\cdot}39 \times 250}{14000} = 0{\cdot}007$$

$\therefore$ percentage of steel

$$= 100\, p = 0{\cdot}7\%$$

Now for all values of n less than $0{\cdot}39\ d$ the corresponding values of the lever arm a, $100\ p$ and the $M.R.$ are tabulated below :

Properties of Under-Reinforced sections

$m = 18$ *Stress in steel* $= 14000\ N/cm.^2$ or $1400\ kg./cm.^2$

Concrete : M 15 in S.I. units and M 150 in M.K.S. units

Depth of neutral axis n (cm.)	*Lever arm a cm.)*	*Percentage of steel 100 p*	*Safe Moment of Resistance*		*M.R. Factor Q*	
			M = p b d t a N cm.	*M = p b d t a kg. cm.*	*S.I. Units*	*M K.S. Units*
$0{\cdot}10\ d$	$0{\cdot}967\ d$	0 03	$4{\cdot}1\ bd^2$	$0{\cdot}41\ bd^2$	4·1	0·41
$0{\cdot}15\ d$	$0{\cdot}950d$	0·07	$9{\cdot}3\ bd^2$	$0{\cdot}93\ bd^2$	9·3	0·93
$0{\cdot}20\ d$	$0{\cdot}933\ d$	0·14	$18\ 3\ bd^2$	$1{\cdot}83\ bd^2$	18·3	1·83
$0{\cdot}25\ d$	$0{\cdot}9\ 7\ d$	0·23	$29{\cdot}6\ bd^2$	$2\ 96\ bd^2$	29·6	2 95
$0{\cdot}30\ d$	$0{\cdot}900\ d$	0·36	$45{\cdot}4\ bd^2$	$4\ 54\ bd^2$	45·4	4·54
$0{\cdot}35\ d$	$0{\cdot}883\ d$	0·52	$64{\cdot}3\ bd^2$	$6\ 43\ bd^2$	64·3	6·43
$0{\cdot}39\ d$	$0{\cdot}870\ d$	0·70	$85\ bd^2$	$8{\cdot}50\ bd^2$	85	8·50

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For this range in the values of n the section will be an *under-reinforced* section. The moment of resistance should therefore be estimated corresponding to the maximum tensile stress in steel.

Now let us consider a further range in the values of n from $0.39\,d$ to $0.7\,d$. These sections are *over-reinforced sections* and hence for these sections the moment of resistance should be computed corresponding to the condition of maximum stress in concrete. The values of a, $100\,p$ and the moment of resistance for various values of n are tabulated below :

Properties of Over-Reinforced Sections

$m = 18$ *Stress in concrete* $= 500\ N/cm.^2$ or $50\ kg./cm.^2$

Depth of neutral axis n cm.	*Lever arm a (cm.)*	*Percentage of steel 100 p*	*Safe Moment of Resistance*		*M.R. Factor Q*	
			$M = bn\frac{c}{2}a$ *N cm*	$M = bn\frac{c}{2}a$ *kg. cm.*	*S.I. Units*	*M.K.S. Units*
$0.39\,d$	$0.870\,d$	0.70	$85\ bd^2$	$8.5\ bd^2$	85	8.50
$0.45\,d$	$0.850\,d$	1.02	$95\ bd^2$	$9.56\ bd^2$	95.6	9.56
$0.50\,d$	$0.833\,d$	1.39	$104.1\ bd^2$	$10.41\ bd^2$	104.1	10.41
$0.55\,d$	$0.817\,d$	1.87	$112.3\ bd^2$	$11.23\ bd^2$	112.3	11.23
$0.60\,d$	$0.800\,d$	2.50	$120\ bd^2$	$12.00\ bd^2$	120	12.00
$0.65\,d$	$0.783\,d$	3.35	$127.2\ bd^2$	$12.72\ bd^2$	127.2	12.72
$0.70\,d$	$0.767\,d$	4.54	$134.2\ bd^2$	$13.42\ bd^2$	130.2	13.42

It is worthy to note :

(*i*) As the percentage of steel increases the depth of neutral axis n also increases.

(*ii*) As the percentage of steel increases the lever arm decreases.

(*iii*) The safe moment of resistance increases rapidly and somewhat uniformly as $100\,p$ (percentage of steel) varies from 0 to 0.69. For further increase in $100\,p$ the safe moment of resistance increases at an *appreciably slower rate*. The critical point at which the change in rate of increase of moment of resistance occurs represents the condition for a balanced section. Hence it is important to note that by providing more steel than the requirement of the balanced section though it is possible to increase the moment of resistance it is uneconomical due to appreciable slower rate of increase of the moment of resistance.

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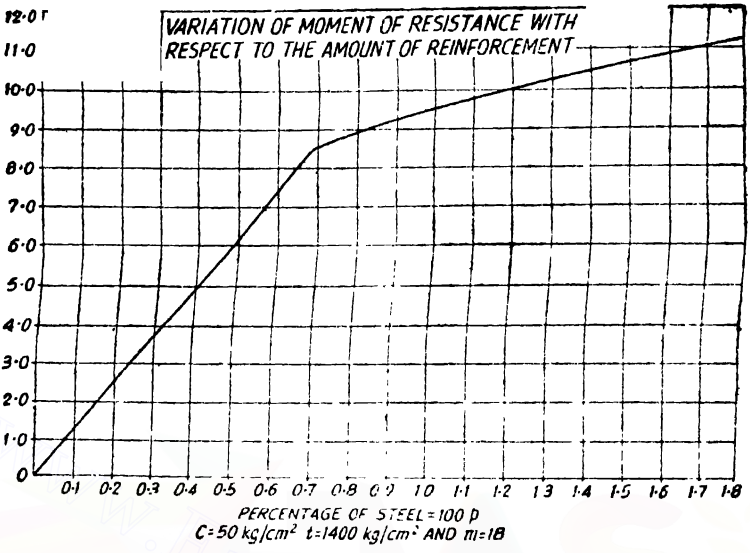

Fig. 19

Problem 17. (S.I.). *Fig. 20 shows a trapezoidal reinforced concrete beam section. Determine the depth of neutral axis and the moment of resistance of the beam section. Permissible stresses in concrete and steel are 5 N/mm.² and 140 N/mm.² respectively. Take m = 18.*

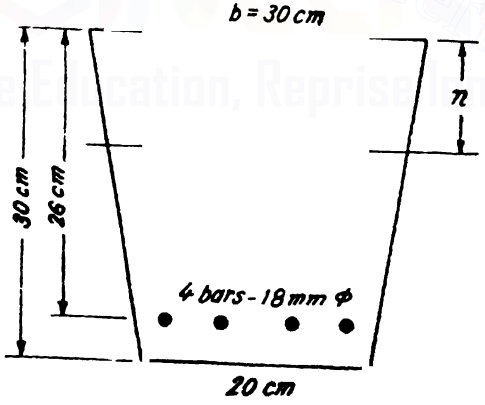

Fig. 20

Solution. Let n be the depth of neutral axis

Let b' = width of the beam at neutral axis

$$b' = 30 - \left[\frac{30-20}{30}\right] n$$

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$$b' = 30 - \frac{n}{3}$$

Area of compression zone

$$= \frac{30 + \left(30 - \frac{n}{3}\right)}{2} \times n = 30\ n - \frac{n^2}{6}$$

Moment of the trapezoidal compression zone about the neutral axis

$$= \left[\frac{b' + b}{2} \times n\right]\left[\frac{b' + 2b}{b' + b} \times \frac{n}{3}\right] = (b' + 2b)\ \frac{n^2}{6}$$

$$= \left[30 - \frac{n}{3} + (2 \times 30)\right]\frac{n^2}{6} = \left[90 - \frac{n}{3}\right]\frac{n^2}{6} = 15\ n^2 - \frac{n^3}{18}$$

Area of steel $A_t = 4 \times 2{\cdot}54 = 10{\cdot}16\ cm^2$

Taking moments about the neutral axis,

$$15\ n^2 - \frac{n^3}{18} = 18 \times 10{\cdot}16\ (26 - n)$$

$$\therefore \quad n^3 - 270\ n^2 - 3291{\cdot}84\ n + 85587{\cdot}84 = 0$$

Solving, by trial and error, we get, $n = 12{\cdot}93\ cm$.

Depth of critical neutral axis

$$= n_c = \frac{d}{23} = \frac{}{23} \times 26 = 10{\cdot}17\ cm.$$

Since $n > n_c$ the section is over-reinforced. Hence concrete attains its maximum stress earlier to steel.

Width of beam at N.A.

$$= 30 - \frac{12\ 93}{3} = 25{\cdot}69\ cm.$$

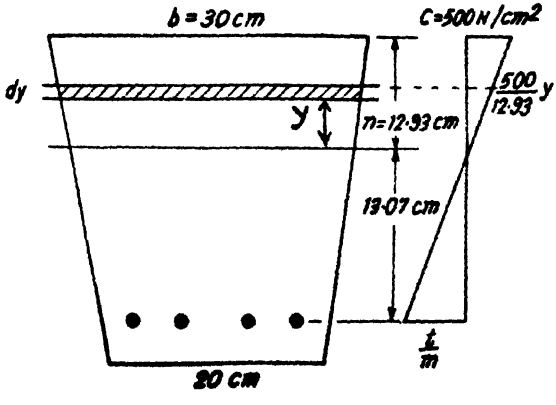

Fig. 21

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At a distance y from N.A. in the compression zone, width of beam

$$= 25{\cdot}69 + \left[\frac{30 - 25{\cdot}69}{12{\cdot}93}\right] y = 25{\cdot}69 + \frac{y}{3}$$

Stress in concrete at a distance y from N.A.

$$= \frac{y}{12{\cdot}93} \times 500 \ N/cm.^2$$

$$= \left[\frac{500}{12{\cdot}93}\right] y \ N/cm.^2$$

Consider an elemental strip of height dy at a distance y from N.A.

Force on the elemental strip

$$= \left(25{\cdot}69 + \frac{y}{3}\right) dy \times \frac{500}{12{\cdot}93} y$$

$$= \left[\frac{500}{12{\cdot}93}\right] y \left(25{\cdot}69 + \frac{y}{3}\right) dy$$

Moment of the above force about the line of action of the tension in the tension zone

$$= \left[\frac{500}{12{\cdot}93}\right] y \left(25{\cdot}69 + \frac{y}{3}\right) dy \ (13{\cdot}07 + y)$$

$$= \left[\frac{500}{12{\cdot}93 \times 3}\right] [y^3 + 90{\cdot}14y^2 + 1007{\cdot}30y] dy$$

$\therefore$ Moment of resistance of the section

$$= \left[\frac{500}{12{\cdot}93 \times 3}\right] \int_0^{12{\cdot}93} (y^3 + 90{\cdot}14y^2 \times 1007{\cdot}3y) dy$$

$$= 12{\cdot}89 \left[\frac{12{\cdot}93^4}{4} + 90{\cdot}14 \times \frac{12{\cdot}93^3}{3} + 1007{\cdot}3 \times \frac{12{\cdot}93^2}{2}\right]$$

$$= 2282890 \ N \ cm.$$

§49. Modular Ratio m

The I.S. code has specified that the modular ratio for any grade of concrete is given by the relation

$$m = \frac{280}{3c}$$

where, c = permissible stress in concrete in compression in bending in $N/mm.^2$

When M.K.S. units are adopted, the modular ratio is given by

$$m = \frac{2800}{3c}$$

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where, c = permissible stress in concrete in compression in bending in $kg./cm.^2$

Constancy of position of critical neutral axis for all grades of concrete

We know for a balanced section, the depth of critical neutral axis is given by the relation

$$\frac{mc}{t} = \frac{n_c}{d - n_c}$$

Putting $n_c = n_1 d$

$$\therefore \quad \frac{mc}{t} = \frac{n_1}{1 - n_1}$$

$$\therefore \quad n_1 = \frac{1}{1 + \frac{t}{mc}}$$

Since the product mc is constant as per the code n_1 becomes a constant for all grades of concrete, when the quality of steel used is the same.

For all grades of concrete,

$mc = \frac{280}{3}$ $N/mm.^2$ in S.I. units ; and $\frac{2800}{3}$ $kg./cm.^2$ in M K.S. units.

Taking $t = 140$ $N/mm.^2$ or 1400 $kg./cm.^2$

$$\frac{t}{mc} = \frac{140}{280} = 1{\cdot}5 \text{ for all grades of concrete}$$

$$\therefore \quad n_1 = \frac{1}{1 + 1{\cdot}5} = 0{\cdot}4 \text{ for all grades of concrete}$$

$$\therefore \text{ Lever arm } \quad a = d - \frac{n_1 d}{3} = d - \frac{0{\cdot}4d}{3} = \frac{13}{15}\ d = 0{\cdot}87\ d$$

for all grades of concrete.

Considering M 15 concrete, in *S.I. units*,

M.R. of the balanced section

$$= \text{M R.} = b \times 0{\cdot}4d \times \frac{500}{2} \times \frac{13}{15}\ d = \frac{260}{3}\ bd^2$$

$$= 86{\cdot}67\ bd^2\ N\ cm.$$

Considering M 150 concrete, *in M.K.S. units*,

M.R. of the balanced section

$$= \text{M.R.}' = b \times 0{\cdot}4d \times \frac{50}{2} \times \frac{13}{15}\ d = \frac{26}{3}\ bd^2$$

$$= 8{\cdot}67\ bd^2\ kg.\ cm.$$

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Total compression = Total tension

$$bn_1\frac{c}{2} = A_s t$$

$$bn_1 d\,\frac{c}{2} = A_s t$$

$$\frac{A_s}{bd} = \frac{1}{2}\,n_1\frac{c}{t} = \frac{1}{2}\,(0{\cdot}4)\,\frac{500}{14000}$$

$$= 0{\cdot}0071 \qquad \text{or} \qquad 0{\cdot}71\%.$$

The values given above hold good for the case when the modular ratio is taken equal to the exact value of the quantity $\left(\frac{280}{3c}\right)$

But in practice, the quantity $\left(\frac{280}{3c}\right)$ is rounded off to the nearest lesser integer and this integral value is adopted as the modular ratio.

Accordingly, for M 15 grade of concrete, $c = 5\ N/mm.^2$ and the exact value of modular ratio

$$= m = \frac{280}{3c} = \frac{280}{3\times 5} = \frac{56}{3} = 18\ 67$$

But suppose the modular ratio be rounded off to the value 18, the coefficients obtained above will be slightly modified.

With $c = 5\ N/mm.^2$, $t = 140\ N/mm^2$ and $m = 18$,

we get $n_1 = \frac{9}{23}$ $\quad a = \frac{20}{23}\ d = 0{\cdot}87\ d$

and M.R. $= 85\ bd^2\ N\ cm.$

Similarly with $c = 50\ kg./cm.^2$, $t = 1400\ kg./cm.^2$ and $m = 18$,

we get $n_1 = \frac{9}{23}$ $\quad a = \frac{20}{23}\ d = 0{\cdot}87\ d$

and M.R. $= 8{\cdot}5\ bd^2$

§50. Types of Bars used in R.C.C.

Usually the following types of bars are used in R.C.C. works :

(*i*) Mild steel conforming to Grade I of I.S. : 432,

(*ii*) Medium tensile steel conforming to I.S. : 432,

(*iii*) Deformed medium tensile steel bars conforming to I.S. : 1139, and

(*iv*) High yield strength deformed bars conforming to I.S. : 1139.

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Properties of Balanced Sections

Safe stress in steel f_s		Grade of concrete		Safe stress in concrete c		m	n_1	a_1	$100\,p$	Q	
S.I. N/mm.²	M.K.S. kg./cm.²	S.I.	M.K.S.	S.I. N/mm.²	M.K.S. kg./cm.²					S.I.	M.K.S
140	1400	M 15	M 150	5	50	18	0·39	0·87	0·70	85	8·5
140	1400	M 20	M 200	7	70	13	0·39	0·87	0·98	119·8	11·98
140	1400	M 25	M 250	8·5	85	11	0·40	0·87	1·21	147·3	14·73
140	1400	M 30	M 300	10	100	9	0·39	0·87	1·40	170	17
190	1900	M 15	M 150	5	50	18	0·32	0·89	0·42	71·7	7·17
190	1900	M 20	M 200	7	70	13	0·32	0·89	0·60	101	10·10
190	1900	M 25	M 250	8·5	85	11	0·33	0·89	0·74	124·7	12·47
190	1900	M 30	M 300	10	100	9	0·32	0·89	0·85	144	14·4
230	2300	M 15	M 150	5	50	18	0·28	0·91	0·31	64	6·4
230	2300	M 20	M 200	7	70	13	0·28	0·91	0·43	90	9
230	2300	M 25	M 250	8·5	85	11	0·29	0·91	0·53	111	11·1
230	2300	M 30	M 300	10	100	9	0·28	0·91	0·61	127	12·7

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The permissible tensile stresses for these bars are given below :

Type of steel bars	*Permissible tensile stress*	
	S I. Units $N/mm.^2$	*M K.S. Units* kg/cm^2
Mild steel conforming to Grade I of I.S. : 432		
Upto 20 *mm.*	140	1400
Over 20 *mm.*	130	1300
Medium tensile steel conforming to I.S. : 432 *Or* Deformed medium tensile steel bars conforming to I.S. : 1139	Half the guaranteed yield stress subject to a maximum of 190	Half the guaranteed yield stress subject to a maximum of 1900
High yield strength deformed bars conforming to I.S. : 1139 (Ribbed-Torsteel)	230	2300

§51. Properties of Balanced Sections for Various Grades of Concrete with Various Types of Steel

The table on page 83 shows the properties of balanced sections for various grades of concrete using various types of steel mentioned above.

EXERCISE 2

1. The cross-section of a singly reinforced concrete beam is 30 *cm.* wide and 50 *cm.* deep to the centre of the tension reinforcement which consists of four bars of 16 *mm.* diameter. If the stresses in concrete and steel are not to exceed 5 $N/mm.^2$ and 140 $N/mm.^2$ respectively, determine the moment of resistance of the beam. Take $m = 18$ (*4,972,000 N cm.*)

2. The cross-section of a singly reinforced concrete beam is 30 *cm.* wide and 50 *cm.* deep to the centre of the tension reinforcement which consists of four bars of 16 *mm* diameter. If the stresses in concrete and steel are not to exceed 50 $kg/cm.^2$ and 1400 $kg./cm.^2$ respectively, determine the moment of resistance of the beam. Take $m = 18$. (*497200 kg. cm.*)

3. A singly reinforced beam is 20 *cm.* wide and 40 *cm.* deep to the centre of the tension reinforcement which consists of four bars of 16 *mm.* diameter. If the stresses in concrete and steel are not to ex-

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ceed 5 *N/mm.*^{2} and 140 *N/mm.*^{2}, find the moment of resistance of the beam. Take $m=18$. (*3256000 N cm.*)

4. The moment of resistance of a rectangular singly reinforced beam of width *b cm* and effective depth *d cm.* is 90 bd^2 *Newton cm.* If the stresses in the outside fibres of concrete and in steel do not exceed 5 *N/mm* ^{2} and 140 *N/mm.*^{2} respectively and the modular ratio equals 18, determine the ratio of the depth of neutral axis from the extreme compression fibres to the effective depth of the beam and the ratio of the area of the tensile steel to the effective area of the beam. (*0·4255 ; 0 00875*)

5. Find the percentage of tension reinforcement necessary for a singly reinforced balanced rectangular section if the permissible stresses in concrete and steel are *c* and *t Newton/cm* ^{2} respectively and the modular ratio is *m*. $\left\{ 50\frac{mc^2}{t(mc+t)}\% \right\}$

6. A reinforced concrete beam 30 *cm.* ..ide by 60 *cm.* total depth has a span of 8 *metres*. Find the necessary tension reinforcement at the centre of the span to enable the beam to carry a load of 6000 *Newton/metre* in addition to its own weight.

Concrete cover below the steel centre = 4 *cm.*

Weight of beam	= 25000 *N/metre*^{3}
Permissible stress in concrete	= 5 *N/mm.*^{2}
Permissible stress in steel	= 140 *N/mm.*^{2}
Modular ratio	= 18

(*13 69 cm.*^{2})

7. The section of a singly reinforced concrete beam is subjected to a sagging bending moment of 200,000 *N metre*. If the stresses in concrete and steel are not to exceed 5 *N/mm.*^{2} and 140 *N/mm.*^{2} respectively, find the dimensions of the beam. The width of the beam may be made $\frac{3}{4}$ the effective depth. Take $m = 18$.

(*51 cm.* × *68 cm. effective*)

8. A simply supported slab is subjected to a bending moment of 8500 *Newton metre* per *metre* width. If the stresses in concrete and steel are not to exceed 5 *N/mm.*^{2} and 140 *N/mm.*^{2}, find the effective depth required and the area of steel. Take $m=18$.

(*10 cm ; 6 98 cm.*^{2} *per metre width*)

9. A simply supported rectangular concrete beam is reinforced for tension only. The beam is subjected to a bending moment of 91800 *N metre*. If the beam is 30 *cm.* wide and the stresses in concrete and steel are not to exceed 5 *N/mm.*^{2} and 140 *N/mm.*^{2} respectively, find the effective depth and the area of steel. Take $m=18$.

(*60 cm. ; 12·53 cm.*^{2})

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3

Doubly Reinforced Beams

Introduction. Beams reinforced with steel in compression and tension zones are called doubly reinforced beams. This type of beam will be found necessary when due to head room consideration or architectural consideration the depth of the beam is restricted. The beam with its limited depth, if reinforced on the tension side only, may not have enough moment of resistance, to resist the bending moment. By increasing the quantity of steel in the tension zone, the moment of resistance cannot be increased indefinitely. Usually, the moment of resistance can be increased by not more than 25 percent over the balanced moment of resistance, by making the beam over reinforced on the tension side. Hence, in order to further increase the moment of resistance of a beam section of limited dimensions, a doubly reinforced beam is provided. Besides this a doubly reinforced beam is also used in the following circumstances :

(*i*) The external live loads may alternate *i..e.* may occur on either face of the member.

Ex. : A pile may be lifted in such a manner that the tension and compression zones may alternate.

(*ii*) The loading may be eccentric and the eccentricity of the load may change from one side of the axis to another side.

(*iii*) The member may be subjected to a shock or impact on accidental lateral thrust

§52. Analysis of a Doubly Reinforced Section*—General Elastic Theory

Fig. 22 shows a doubly reinforced beam of width *b cm.* and depth *d cm.* to the centre of the tensile reinforcement.

Let A_c and A_t be the areas of reinforcement in the compression and tension zones.

Let *n* be the depth of neutral axis.

Equating moments of areas on either side of the neutral axis, about the neutral axis,

$$\frac{bn^2}{2} + mA_c(n-d_c) - A_c(n-d_c)$$

$$m\ A_t(d-n)$$

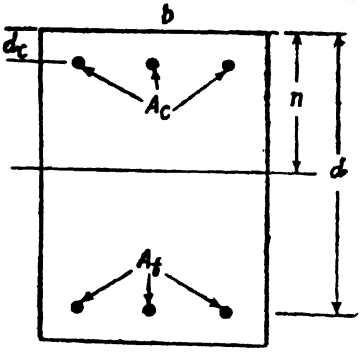

Fig. 22

*See also analysis of doubly reinforced section by revised elastic theory needed by the I.S. code on page 95.

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$$\frac{bn^2}{2}+(m-1)\,A_c(n-d_c)=mA_t(d-n) \qquad ...(1)$$

If the stresses c and t, *i.e.*, the stresses in concrete and tension reinforcement are known, we have

$$\frac{mc}{t}=\frac{n}{d-n} \qquad ...(2)$$

It is very important to note that in the case of a singly reinforced beam of chosen dimensions, in order that the actual neutral axis may coincide with the critical neutral axis, there is a certain definite amount of steel required.

But, in a doubly reinforced beam of chosen dimensions, the reinforcement A_c and A_t may be adjusted in an infinite number of ways so that the actual neutral axis and the critical neutral axis may coincide.

Since total compression = Total tension, we have,

$$C=T$$

$$bn\frac{c}{2}+(m-1)\,A_c\,c'=A_t\,t \qquad ...(3)$$

Stress in compression steel. If c' is the stress in concrete at the level of the compression steel, the stress in compression steel $=mc'$

But $c'=\frac{n-d_c}{n}c$

$\therefore$ Stress in compression steel $=m.\frac{n-d_c}{n}c$

§53. Moment of Resistance

This is computed by taking moments of the compressive forces about the centre of gravity of the tension reinforcement.

$$\text{M.R.}=bn\frac{c}{2}\left(d-\frac{n}{3}\right)+(m-1)A_c\,c'\,(d-d_c) \qquad ...(4)$$

§54 Types of Problems

The following are the types of problems we come across in the analysis of doubly reinforced sections.

Type I. *Data* : Overall section, A_c, A_t and the working stresses c and t.

Required : Moment of resistance.

Solution. The position of the actual neutral axis is first determined (*see* eq. 1). The position of the critical neutral axis is found from eq. (2) with the given values of the working stresses c and t. If the depth of actual neutral axis is greater than the depth of critical neutral axis, concrete will attain its maximum stress earlier. Hence the $M\,R.$ is worked out from eq. (4) taking c at the given working stress, c' should be taken as $\frac{n-d_c}{n}c$.

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If the depth of neutral axis is less than the depth of critical axis, the steel in the tension zone reaches its maximum stress earlier. The stress in concrete corresponding to the working stress of t in the tension reinforcement is found from eq. (2). With this value of c the $M.R.$ is found from eq. (4).

The following example illustrates the above type.

Problem 18. (M.K.S.) *A beam of reinforced concrete is 25 cm. wide and 40 cm deep to the centre of tensile steel. It is reinforced with four bars of 16 mm. diameter as compressive steel at an effective cover of 5 cm. and with four bars of 20 mm. diameter as tensile steel. If the stresses in concrete and steel are not to exceed 50 kg./cm.² and 1400 kg./cm.² respectively, determine the moment of resistance of the section. Take $m = 18$.*

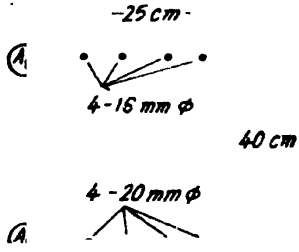

Fig. 23

Solution.

Area of compression steel
$= A_c = 4 \times 2{\cdot}01 = 8{\cdot}04\ cm.^2$

Area of tension steel $= A_t = 4 \times 3{\cdot}14 \quad 12{\cdot}56\ cm.^2$

Depth of neutral axis

Taking moments about the neutral axis, we have,

$$\frac{bn^2}{2} + (m-1)A_c\,(n-d_c) = mA_t\,(d-n)$$

$$\frac{25n^2}{2} + (18-n)8{\cdot}04(n-5) = 18 \times 12{\cdot}56(40-n)$$

$$\therefore \quad n^2 + 29{\cdot}02\,n - 778{\cdot}128 = 0$$

Solving, we get $n = 16{\cdot}93\ cm.$

Depth of critical neutral axis is given by

$$\frac{18 \times 50}{1400} = \frac{n_c}{40-n_c}$$

$$\therefore \quad n_c = 15{\cdot}65\ cm.$$

Since, the depth of actual neutral axis is greater than the depth of critical neutral axis, concrete in compression attains the maximum stress earlier to steel in tension.

$\therefore$ Stress in concrete will be allowed to reach $50\ kg./cm.^2$

$$\therefore \quad c = 50\ kg./cm.^2$$

$$\therefore \quad c' = \frac{n-d_c}{n}\,c = \frac{(16{\cdot}93-5)}{16{\cdot}93} \times 50 = 35{\cdot}23\ kg./cm.^2$$

$$M.R. = bn\,\frac{c}{2}\left(d - \frac{n}{3}\right) + (m-1)A_c\,c'(d-d_c)$$

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$$= 25 \times 16{\cdot}9 \times \frac{50}{2}\left(40 - \frac{16{\cdot}93}{3}\right) + (18-1)\,8{\cdot}04 \times 35\,23(40-5)$$

$$= 363536 + 168533 = 532069 \text{ kg. cm.}$$

Problem 19. (S.I.). *A beam of reinforced concrete is 25 cm. wide and 40 cm. deep to the centre of tensile steel. It is reinforced with four bars of 16 mm. diameter as compressive steel at an effective cover of 5 cm. and with 4 bars of 20 mm. diameter as tensile steel. If the stresses in concrete and steel are not to exceed 5 N/mm.^{2} and 140 N/mm.^{2} respectively, determine the moment of resistance of the section. Take $m = 18$.*

Solution.

Area of compression steel $= A_c = 4 \times 2\,01 = 8{\cdot}04$ *cm.*^{2}

Area of tension steel $= A_t = 4 \times 3{\cdot}14 = 12{\cdot}56$ *cm.*^{2}

Depth of neutral axis

Taking moments about the neutral axis,

$$\frac{bn^2}{2} + (m-1)A_c\,(n-d_c) = mA_t(d-n)$$

$$\frac{25n^2}{2} + (18-1)8\,04(n-5) = 18 \times 12{\cdot}56(40-n)$$

$$n^2 + 29{\cdot}02\,n - 778{\cdot}128 = 0$$

Solving, we get $n = 16{\cdot}93$ *cm.*

Depth of critical neutral axis is given by,

$$\frac{18 \times 500}{14000} = \frac{n_c}{40 - n_c}$$

$$\therefore \quad n_c = 15{\cdot}65 \text{ cm.}$$

Since the depth of actual neutral axis is greater than the depth of critical neutral axis, concrete in compression attains the maximum stress earlier to steel in tension.

$$\therefore \quad c = 5 \text{ N/mm.}^2 = 500 \text{ N/cm.}^2$$

$$\therefore \quad c' = \left[\frac{n - d_c}{n}\right] c = \left[\frac{16{\cdot}93 - 5}{16{\cdot}93}\right] \times 500 = 352{\cdot}3 \text{ N/cm.}^2$$

$$M.R. = bn\,\frac{c}{2}\left(d - \frac{n}{3}\right) + (m-1)A_c\,c'(d-d_c)$$

$$= 25 \times 16{\cdot}93 \times \frac{500}{2}\left(40 - \frac{16{\cdot}93}{3}\right) + (18-1) \times 8\,04 \times 352{\cdot}3 \times (40-5)$$

$$= 3635365 + 1685333 = 5320698 \text{ N cm.}$$

Type 2. *Data* : Overall section, A_c and A_t, Max. $B.M.$

Required : Stresses c and t in concrete and tensile steel and also in compression steel.

Solution. The position of the actual neutral axis is first determined from Eq. (1). Express the moment of resistance in terms of

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the stress c in concrete and equate to the given bending moment. From this equation the stress c in concrete is determined.

The stress in compression steel $= mc' = \frac{(n-d_c)}{n} c.$

The stress in tension steel is given by the relation

$$\frac{mc}{n} = \frac{n}{d-n}$$

The following example illustrates the above type.

Problem 20. (M K.S). *A doubly reinforced concrete beam is 25 cm. wide and 50 cm. deep to the centre of tension reinforcement. The centre of compression steel is 5 cm. from the compression edge. The areas of the compression and tension steel are respectively 10·16 cm² and 12·56 cm². If $m = 18$ and the bending moment at the section is 700,000 kg. cm , calculate the stresses in concrete and steel.*

Solution.

Position of neutral axis

Taking moments about the neutral axis,

$$\frac{bn^2}{2} + (m-1)A_c(n-d_c) = m\,A_t(d-n)$$

$$\therefore \frac{25\,n^2}{2} + (18-1)10{\cdot}16(n-5) = 18 \times 12{\cdot}56\,(50-n)$$

$$\therefore \quad n^2 + 31{\cdot}90\,n - 973{\cdot}41 = 0$$

Solving, we get $n = 19{\cdot}09$ *cm.*

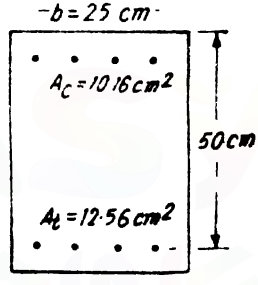

Fig. 24

Let the maximum compressive stress in concrete be c *kg /cm²*

∴ Stress in concrete at the level of the compression steel

$$= c' = \left[\frac{n-d_c}{n}\right] c$$

$$= \left[\frac{19{\cdot}09-5}{19{\cdot}09}\right] c$$

$$= 0{\cdot}738\ c$$

$$\text{M.R.} = bn\,\frac{c}{2}\left(d - \frac{n}{3}\right) + (m-1)\,A_c\,c'\,(d-d_c)$$

$$= 25 \times 19{\cdot}09 \times \frac{c}{2}\left(50 - \frac{19{\cdot}09}{3}\right) + (18-1) \times 10{\cdot}16 \times 0{\cdot}738\ c \times (50-5)$$

$$= 10412{\cdot}80\ c + 5736{\cdot}03\ c$$

$$= 16148{\cdot}83\ c$$

Equating the M.R. to the bending moment

$$16148{\cdot}83\ c = 700{,}000$$

$$\therefore \quad c = \frac{700{,}000}{16148{\cdot}83} = 43{\cdot}35\ kg./cm^2$$

$$c' = 0{\cdot}738 \times 43{\cdot}35 = 31{\cdot}99\ kg./cm.^2$$

∴ *Stress in compression steel*

$$= 18 \times 31{\cdot}99 = 575{\cdot}82\ kg\ /cm.^2$$

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Stress in tension steel

$$= \left[\frac{d-n}{n}\right] mc$$

$$= \left[\frac{50-19{\cdot}09}{19{\cdot}09}\right] \times 18 \times 43{\cdot}35$$

$$= 1263{\cdot}44 \text{ kg./cm.}^2$$

Problem 21 (S.I.). *A doubly reinforced concrete beam is 25 cm. wide and 50 cm. deep to the centre of tension reinforcement. The centre of compression steel is 5 cm from the compression edge. The areas of the compression and tension steel are respectively 10·16 cm.² and 12·56 cm². If m = 18 and the bending moment at the section is 7,000,000 N cm., calculate the stress in concrete and steel.*

Solution. $b = 25$ cm. ; $d = 50$ cm. ; $d_c = 5$ cm.

$A_c = 10{\cdot}16$ cm.² ; $A_t = 12{\cdot}56$ cm.²

Position of neutral axis

Taking moments about the neutral axis,

$$\frac{bn^2}{2} + (m-1)A_c(n-d_c) = m\,A_t(d-n)$$

$$\frac{25}{2}\,n^2 + (18-1)\,10{\cdot}16\,(n-5) = 18 \times 12{\cdot}56\,(50-n)$$

$$\therefore \quad n^2 + 31{\cdot}90\,n - 973{\cdot}41 = 0$$

Solving, we get $n = 19{\cdot}09$ cm.

Let the maximum compressive stress in concrete be c N/cm.²

∴ Stress in concrete at the level of the compression steel

$$= c' = \left[\frac{n-d_c}{n}\right] c$$

$$= \left[\frac{19{\cdot}09-5}{19{\cdot}09}\right] c = 0{\cdot}738\,c$$

$$M.R. = bn\,\frac{c}{2}\left(d - \frac{n}{3}\right) + (m-1)\,A_c\,c'\,(d-d_c)$$

$$= 25 \times 19{\cdot}09 \times \frac{c}{2}\left(50 - \frac{19{\cdot}09}{3}\right) + (18-1)\,10{\cdot}16 \times 0{\cdot}738\,c\,(50-5)$$

$$= 10412{\cdot}8\,c + 5736{\cdot}03\,c$$

$$= 16148{\cdot}83\,c \text{ N cm.}$$

Equating the M.R. to the bending moment,

$$16148{\cdot}83\,c = 7{,}000{,}000$$

$$\therefore \quad c = \frac{7{,}000{,}000}{16148{\cdot}83} = 433{\cdot}5 \text{ N/cm.}^2$$

$$= 4{\cdot}335 \text{ N/mm.}^2$$

$$c' = 0{\cdot}738 \times 433{\cdot}5 = 319{\cdot}92 \text{ N/cm.}^2$$

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$\therefore$ *Stress in compression steel*

$$= 18 \times 319{\cdot}92 = 5758{\cdot}6 \ N/cm.^2$$
$$= 57{\cdot}586 \ N/mm.^2$$

$\therefore$ *Stress in tension steel*

$$= \left[\frac{d-n}{n}\right] mc$$
$$= \left[\frac{50 - 19{\cdot}09}{19{\cdot}09}\right] 18 \times 433{\cdot}5$$
$$= 12634{\cdot}4 \ N/cm.^2$$
$$= 126{\cdot}344 \ N/mm.^2$$

Type 3. *Data* Overall section, the maximum bending moment, safe stresses c and t in concrete and steel.

Required A_c and A_t.

Solution With working stresses c and t determine the depth of critical neutral axis. Find the M R. in terms of the stress in concrete and equate to the given bending moment. This equation will involve A_c as the unknown which can be determined.

By equating total compression to total tension find A_t.

i e., in this case the beam section is designed as a balanced section.

The following example illustrates this case.

Problem 22 (M.K.S.). *A rectangular beam reinforced on both sides is 30 cm. wide and 55 cm deep. The centres of steel are 5 cm. from the respective edges If the limiting stresses in concrete and steel are 50 kg /cm.² and 1400 kg /cm ² respectively, determine the steel areas for a bending moment of 800,000 kg. cm. Take m = 18.*

Solution. The section will be designed as a balanced section.

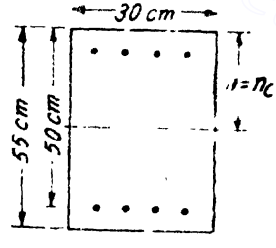

Fig. 25

$\therefore$ $n = n_c$

$\therefore$ $c = 50 \ kg./cm.^2$

and $t = 1400 \ kg./cm.^2$

The depth of critical neutral axis is given by,

$$\frac{18 \times 50}{1400} = \frac{50 - n_c}{n_c}$$

$\therefore$ $n_c = 19{\cdot}56 \ cm.$

Stress in concrete $c = 50 \ kg./cm.^2$

$\therefore$ Stress in concrete at the level of compression steel

$$= c' = \left(\frac{n - d_c}{n}\right) c = \frac{(19{\cdot}56 - 5)}{19{\cdot}56} \times 50$$
$$= 37{\cdot}22 \ kg./cm.^2$$

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$$M.R. = bn\ \frac{c}{2}\left(d - \frac{n}{3}\right) + (m-1)\ A_c\ c'\ (d - d_c)$$

$$= 8,00,000\ kg.\ cm.$$

$$\therefore\ 30 \times 19.56 \times \frac{50}{2}\left(50 - \frac{19.56}{3}\right) + (18-1)\ A_c \times 37.22\ (50-5) = 800,000$$

$$637852 + 28473.3\ A_c = 800,000$$

$$\therefore\ A_c = 5.69\ cm.^2$$

Total compression = Total tension

$$bn\frac{c}{2} + (m-1)\ A_c\ c' = A_t\ t$$

$$30 \times 19.56 \times \frac{50}{2} + (18-1)\ 5.69 \times 37.22 = A_t \times 1400$$

$$14670 + 3600 = 1400\ A_t$$

$$\therefore\ A_t = 13.05\ cm^2.$$

Problem 23 (S.I.). *A rectangular beam reinforced on both sides is 30 cm. wide and 55 cm deep. The centres of steel are 5 cm. from the respective edges. If the limiting stresses in concrete and steel are 5 N/mm.² and 140 N/mm.² respectively, determine the steel areas for a bending moment of 8,000,000 N cm. Take m = 18.*

Solution. The section will be designed as a balanced section.

$$\therefore\ n = n_c$$

$$\therefore\ c = 5\ N/mm.^2 = 500\ N/cm.^2$$

and $t = 140\ N/mm.^2 = 14000\ N/cm.^2$

The depth of critical neutral axis is given by,

$$\frac{18 \times 500}{14000} = \frac{n_c}{50 - n_c}$$

$$\therefore\ n_c = 19.56\ cm.$$

Stress in concrete = $c = 500\ N/cm.^2$

∴ Stress in concrete at the level of the compression steel

$$= c' = \left(\frac{n - d_c}{n}\right) c$$

$$= \left[\frac{19.56 - 5}{19.56}\right] \times 500 = 372.2\ N/cm.^2$$

$$M.R. = bn\ \frac{c}{2}\left(d - \frac{n}{3}\right) + (m-1)\ A_c\ c'\ (d - d_c)$$

$$= 8,000,000\ N\ cm.$$

$$\therefore\ 30 \times 19.56 \times \frac{500}{2}\left(50 - \frac{19.56}{3}\right) + (18-1)\ A_c \times 372.2\ (50-5) = 8,000,000$$

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$$6378520+284733\ A_c=8000{,}000$$

$$\therefore \quad A_c=5{\cdot}69\ cm.^2$$

Total compression = Total tension

$$bn\frac{c}{2}+(m-1)\ A_c\ c'=A_t\ t$$

$$30\times 19{\cdot}56\times\frac{500}{2}+(18-1)\ 5{\cdot}69\times 372{\cdot}2=A_t\times 14000$$

$$146700+36000=14000\ A_t$$

$$\therefore \quad A_t=13{\cdot}05\ cm.^2$$

§55. The Steel Beam Theory

This is a method of designing doubly reinforced beams. This theory assumes the following :

(*i*) compression is resisted only by compression steel ;

(*ii*) tension is resisted only by tension steel ;

(*iii*) stress in compression steel = stress in tension steel ;

(*iv*) concrete serves only as a web of an I beam whose flanges are represented by the compression and tension reinforcement.

In this method areas of compression and tension steel are equal, *i.e.*, $A_c=A_t$.

The *M.R.* of the beam is, therefore, given by,

$$A_c\ t(d-d_c) \text{ or } A_t\ t(d-d_c)$$

$$\therefore \quad A_c=A_t=\frac{M}{t(d-d_c)} \qquad ...(5)$$

A stress of $t=1400\ kg./cm.^2$ or $140\ N/mm.^2$ may be assumed in determining the reinforcement.

Problem 24 (M.K.S.). *A rectangular beam reinforced on both sides is 30 cm. wide and 75 cm. deep. The centres of steel are 5 cm. from the respective edges. Find the reinforcement required on the steel beam theory, for a bending moment of 1,400,000 kg. cm.*

Solution. Distance between compression and tension reinforcement

$$=75-5-5=65\ cm.$$

Allowing $t=1400\ kg./cm.^2$

$$A_c=A_t=\frac{1{,}400{,}000}{1{,}400\times 65}\ cm^2$$

$$=15{\cdot}4\ cm^2.$$

Problem 25 (S.I.). *A rectangular beam reinforced on both sides is 30 cm. wide and 75 cm. deep. The centres of steel are 5 cm from the respective edges. Find the reinforcement required on the steel beam theory, for a bending moment of 14,000,000 N cm.*

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Solution. Distance between compression and tension reinforcement

$$= 75 - 5 - 5 = 65 \text{ cm.}$$

Allowing $t = 140 \text{ N/mm.}^2 = 14000 \text{ N/cm.}^2$

$$\therefore \quad A_c = A_t = \frac{14{,}000{,}000}{14000 \times 65} = 15{\cdot}4 \text{ cm.}^2$$

§56. Analysis of doubly Reinforced Sections by Revised Elastic Theory (*I.S. 456*)

We know concrete possesses the property of creep by virtue of which, when maintained in a stressed condition, it undergoes additional strains with the passage of time, besides the elastic strains. Moreover due to the fact that concrete is not exactly elastic, it is seen that the strains in concrete increase more rapidly. Besides creep effect there is also the shrinkage effect due to which also concrete gets shortened. These additional strains in concrete introduce corresponding additional strains in steel provided in the compression zones of beams and slabs. Thus it can be realized that the actual strain in steel in the compression zone of a beam or slab is considerably greater than the elastic strain calculated surrounding the steel. Thus the steel in the compression zone is subjected to greater stress than what is calculated by the simple elastic theory. To take this into account the I.S. 456 code has made the following recommendations.

The compressive stress in steel in the compression zone of a beam or slab shall be taken as 1·5 m times the calculated compressive stress in the surrounding concrete or σ_{sc} whichever is lesser,

where σ_{sc} = permissible compressive stress in steel.

According to the revised theory, in the compression zone corresponding to a steel area A_c, the equivalent concrete area will be taken as 1·5 mA_c instead of mA_c. Accordingly the various equations obtained earlier by the simple elastic theory are suitably modified. The modified equations are given below.

1. *Position of neutral axis*

$$\frac{bn^2}{2} + (1{\cdot}5\, m - 1)\, A_c\, (n - d_c) = m\, A_t\, (d - n)$$

2. Total compression = Total tension

$$bn\frac{c}{2} + (1{\cdot}5\, m - 1)\, A_c\, c' = A_t\, t$$

3. Moment of resistance

$$= bn\, \frac{c}{2}\left(d - \frac{n}{3} \right) + (1{\cdot}5\, m - 1)\, A_c\, c'\, (d - d_c)$$

Problem 26 (S.I.). *A beam of reinforced concrete is 25 cm. wide and 40 cm. deep to the centre of tensile steel. It is reinforced*

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with four bars of 16 mm. diameter as compressive steel and four bars of 20 mm. diameter as tensile steel. If the stresses in concrete and steel are not to exceed 5 N/mm.² and 140 N/mm.² respectively, determine the moment of resistance of the section according to revised elastic theory. Take m = 18

Solution. $b = 25$ *cm.* ; $d = 40$ *cm.* ; $d_c = 5$ *cm.*

Area of compression steel $= A_c = 4 \times 2{\cdot}01 = 8{\cdot}04$ *cm.*²

Area of tension steel $= A_t = 4 \times 3{\cdot}14 = 12{\cdot}56$ *cm.*²

Depth of neutral axis

Taking moments about the neutral axis, we have,

$$\frac{bn^2}{2} + (1{\cdot}5\ m - 1)\ A_c\ (n - d_c) = m\ A_t\ (d - n)$$

$$\frac{25\ n^2}{2} + (1{\cdot}5 \times 18 - 1)\ 8{\cdot}04\ (n - 5) = 18 \times 12{\cdot}56\ (40 - n)$$

$$\therefore \quad n^2 + 34{\cdot}81\ n - 807{\cdot}07 = 0$$

Solving, we get $n = 15{\cdot}91$ *cm.*

Depth of critical neutral axis is given by

$$\frac{18 \times 500}{14000} = \frac{n_c}{40 - n_c}$$

$$\therefore \quad n_c = 15{\cdot}65 \text{ cm.}$$

Since the depth of neutral axis is greater than the depth of critical neutral axis, concrete in compression attains the maximum stress earlier to steel in tension.

∴ Stress in concrete will be allowed to reach 500 *N/cm.*²

$$\therefore \quad c = 500\ N/cm.^2$$

$$\therefore \quad c' = \left(\frac{n - d_c}{n}\right) c = \left[\frac{15{\cdot}91 - 5}{15}\right] \times 500$$

$$= 342{\cdot}9\ N/cm.^2$$

$$\text{M.R.} = bn\ \frac{c}{2}\left(d - \frac{n}{3}\right) + (1{\cdot}5\ m - 1) A_c\ c'\ (d - d_c)$$

$$= 25 \times 15{\cdot}91 \times \frac{500}{2}\left(40 - \frac{15{\cdot}91}{3}\right) + (1{\cdot}5 \times 18 - 1) 8{\cdot}04 \times 342{\cdot}9\ (40 - 5)$$

$$= 3450150 + 2508790$$

$$= 5958940\ N\ cm.$$

Problem 27. (S.I.). *A doubly reinforced concrete beam is 25 cm. wide and 50 cm. deep to the centre of tension reinforcement The centre of compression steel is 5 cm. from the compression edge. The areas of the compression and tension steel are respectively 10·16 cm.² and 12·56 cm.² If m = 18 and the bending moment at the section is 7,000,000 N cm., calculate the stresses in concrete and steel by the revised elastic theory.*

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Solution. $b=25$ *cm.* ; $d=50$ *cm.* ; $d_c=5$ *cm.*

$A_c=10{\cdot}16$ *cm.*^{2} ; $A_t=12{\cdot}56$ *cm.*^{2}

Position of neutral axis

Taking moments about the neutral axis,

$$\frac{bn^2}{2}+(1{\cdot}5\,m-1)A_c(n-d_c)=mA_t(d-n)$$

$$25\,\frac{n^2}{2}+(1{\cdot}5\times 18-1)10{\cdot}16\,(n-5)=18\times 12{\cdot}56(50-n)$$

$$\therefore \qquad n^2+39{\cdot}22\,n-1009{\cdot}98=0$$

Solving, we get $n=17{\cdot}73$ *cm.*

Let the maximum compressive stress in concrete be c *N/cm.*^{2}

$\therefore$ Stress in concrete at the level of the compression steel

$$=c'=\left[\frac{n-d_c}{n}\right]c=\left[\frac{17{\cdot}73-5}{17{\cdot}73}\right]c=0{\cdot}708\,c.$$

$$\text{M.R.}=bn\,\frac{c}{2}\left(d-\frac{n}{3}\right)+(1{\cdot}5\,m-1)A_c\,c'(d-d_c)$$

$$=25\times 17{\cdot}73\times\frac{c}{2}\left(50-\frac{17{\cdot}73}{3}\right)+(1{\cdot}5\times 18-1)10{\cdot}16\times 0{\cdot}708\,c\,(50-5)$$

$$=9771{\cdot}4\,c+8416{\cdot}1\,c=18187{\cdot}5\,c$$

Equating the M.R. to the bending moment

$$18187{\cdot}5\,c=7{,}000{,}000$$

$\therefore$ $c=384{\cdot}9$ *N/cm.*$^2=3{\cdot}85$ *N/mm.*^{2}

$\therefore$ $c'=0{\cdot}708\times 384{\cdot}0=272{\cdot}5$ *N/cm.*^{2}

$\therefore$ *Stress in compression steel*

$=1{\cdot}5\,mc'=1{\cdot}5\times 18\times 272{\cdot}5=7357{\cdot}5$ *N/cm.*^{2}

$=73{\cdot}58$ *N/mm.*^{2}

$\therefore$ Stress in tension steel

$$=\left[\frac{d-n}{n}\right]mc=\left[\frac{50-17{\cdot}73}{17{\cdot}73}\right]\times 18\times 384{\cdot}9=12610\ \text{N/cm.}^2$$

$=126{\cdot}1$ *N/mm.*^{2}

Problem 28. (S.I.). *A rectangular beam reinforced on both sides is 30 cm. wide and 55 cm. deep. The centres of steel are 5 cm. from the respective edges. If the limiting stresses in concrete and steel are 5 N/mm.*^{2} *and 140 N/mm.*^{2} *respectively, determine the steel areas for a bending moment of 8,000,000 N cm. based on the revised elastic theory. Take* $m=18$.

Solution. The section will be designed as a balanced section.

$\therefore$ $n=n_0$ $\therefore$ $c=5$ *N/mm.*$^2=500$ *N/cm.*^{2}

and $t=140$ *N/mm.*$^2=14000$ *N/cm.*^{2}

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The depth of critical neutral axis is given by,

$$\frac{18\times 500}{14000}=\frac{n_c}{50-n_c} \qquad \therefore \quad n_c=19\cdot 56 \text{ cm.}$$

Stress in concrete $=c=500 \text{ N/cm.}^2$

Stress in concrete at the level of the compression steel

$$=c'=\left(\frac{n-d_c}{n}\right)c=\left(\frac{19\cdot 56-5}{19\cdot 56}\right)\times 500=372\cdot 2 \text{ N/cm.}^2$$

$$\text{M R.}=bn\,\frac{c}{2}\left(d-\frac{n}{3}\right)+(1\cdot 5\,m-1)A_c\,c'(d-d_c)$$

$$=8{,}000{,}000 \text{ N cm.}$$

$$\therefore\ 30\times 19\cdot 56\times\frac{500}{2}\left(50-\frac{19\cdot 56}{3}\right)+(1\cdot 5\times 18-1)A_c\times 372\cdot 2(50-5)$$

$$=8{,}000{,}000$$

$$\therefore\ 6378520+435474\,A_c=8{,}000{,}000$$

$$\therefore\quad A_c=3\cdot 72 \text{ cm.}^2$$

Total compression = Total tension

$$bn\,\frac{c}{2}+(1\cdot 5\,m-1)A_c\,c'=A_t t$$

$$30\times 19\cdot 56\times\frac{500}{2}+(1\cdot 5\times 18-1)3\cdot 72+372\cdot 2=A_t\times 14000$$

$$146700+35999\cdot 14000\,A_t$$

$$\therefore \qquad A_t=12\cdot 05 \text{ cm.}^2$$

EXERCISE 3

1. A doubly reinforced rectangular beam is 30 *cm.* wide and 50 *cm.* deep to the centre of tension steel. It is reinforced with four bars of 18 *mm.* diameter as compressive steel at an effective cover of 4 *cm.* and with four bars of 20 *mm.* diameter as tensile steel. If the stresses in concrete and steel are not to exceed 5 *N/mm.*^{2} and 140 *N/mm.*^{2} respectively, find the moment of resistance of the section. Take $m=18$. *Adopt revised elastic theory.* *(78530 Nm)*

2. A doubly reinforced rectangular beam is 30 *cm.* wide and 50 *cm.* deep to the centre of tensile steel. It is reinforced with four bars of 20 *mm.* diameter as compressive steel at an effective cover of 4 *cm.* and with four bars of 24 *mm* diameter as tensile steel. If the stresses in concrete and steel shall not exceed 50 *kg./cm.*^{2} and 1400 *kg./cm.*^{2} respectively find the moment of resistance of the section. *Adopt simple elastic theory.* Take $m=18$.

(1,053,200 kg. cm.)

3. A doubly reinforced concrete beam is 25 *cm.* wide and 45 *cm.* deep to the centre of tensile reinforcement. The areas of the compression and tension steel are 12·57 *cm.*^{2} each. The centre of the compression steel is 5 *cm.* from the compression edge. If $m=18$

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and the section is subjected to a bending moment of 6,500,000 *N cm.* find the stresses in concrete and steel as per *revised elastic theory.*

($c = 3{\cdot}88$ *N/mm.*^{2} ; $t = 129{\cdot}8$ *N/mm.*^{2} ; $t_c = 71{\cdot}5$ *N/mm.*^{2})

4. A doubly reinforced concrete beam is 25 *cm.* wide and 55 *cm.* deep to the centre of tensile reinforcement. The areas of the compression and tension steel are 15 *cm.*^{2} each. The centre of the compression steel is 5 *cm.* from the compression edge. If $m = 18$ and the section is subjected to a maximum bending moment of 750,000 *kg. cm.* find the stresses in concrete and steel. *Adopt simple elastic theory.* ($c = 34$ *kg./cm.*^{2} ; $t = 1020$ *kg./cm.*^{2} ; $t_c = 463{\cdot}5$ *kg./cm.*^{2})

5. A rectangular beam reinforced on both sides is 30 *cm.* wide and 70 *cm.* deep. The centres of steel are 4 *cm.* from the respective edges. If the limiting stresses in concrete and steel are 5 *N/mm.*^{2} and 140 *N/mm.*^{2} respectively, find the steel areas for a bending moment of 15,000,000 *N cm., according to revised elastic theory.* Take $m = 18$. ($A_c = 5{\cdot}70$ *cm.*^{2} ; $A_t = 18{\cdot}31$ *cm.*^{2})

6. A pile 30 *cm.* × 30 *cm.* in section is reinforced with four bars of 24 *mm.* diameter, the effective cover to the centre of reinforcement being 5 *cm.* The pile section is subjected to a bending moment of 1600 *kg. m.* while hoisting it. If the modular ratio is 18, find the stresses induced in concrete and steel according to *simple elastic theory.*

($c = 47{\cdot}56$ *kg./cm.*^{2} ; $t = 1226$ *kg /cm* ^{2} ; $t_c = 439{\cdot}6$ *kg./cm.*^{2})

7. A doubly reinforced rectangular beam is 25 *cm.* by 40 *cm.* and is subjected to a bending moment of 40,000 *Nm.* the effective cover to the reinforcement being 4 *cm.* Using *M 150* concrete calculate the steel areas required in the compression and the tension zone *as per revised elastic theory.* Take $m = 18$.

($A_c = 4{\cdot}17$ *cm.*^{2} ; $A_t = 9{\cdot}06$ *cm.*^{2})

8. A doubly reinforced beam 30 *cm.* wide and 65 *cm.* deep is reinforced with four bars of 20 *mm.* diameter as compression and an equal amount as tensile steel. The respective reinforcements are provided at an effective cover of 4 *cm.* If the stresses in concrete and steel are not be exceed 5 *N/mm.*^{2} and 140 *N/mm* ^{2} and the modular ratio is 18, find the moment of resistance of the section based on : (*i*) revised elastic theory, and (*ii*) steel beam theory.

(*148,261 Nm.* ; *100,229 Nm.*)

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4

Shear in Beams

§57. Shear Stresses in Homogeneous Sections

Fig. 26 shows a simply supported beam subjected to concentrated load W at the centre.

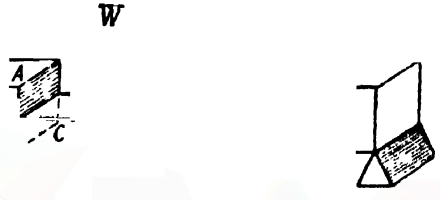

Fig. 26

We find that the shear force at the section $ABCD = \frac{W}{2}$

Hence, the cross-section $ABCD$ has to offer an equal and opposite resistance. If really the intensity of the shear resistance is uniform, the shear stress at the section would have been $\frac{W/2}{\text{area } ABCD}$. But, actually, the intensity of shear resistance is not uniform.

Now consider the beam shown in Fig. 27 subjected to a load system. Consider two sections 1-1 and 2-2, dx apart. Let the

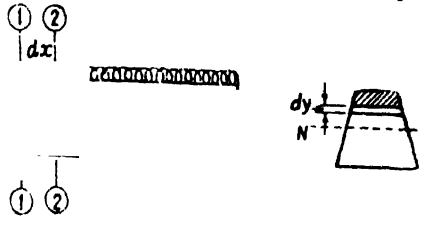

Fig. 27

bending moments at sections 1-1 and 2-2 be M and $(M+dM)$ respectively. Now consider an elemental part of the beam, of width b and thickness dy and length dx, situated at a height y from the neutral layer. This part is separately shown in Fig. 28.

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Let I be the moment of inertia of the cross-section of the beam about the neutral axis.

Bending stress on the elemental part at section (1-1)

$$=f=\frac{M}{I}y$$

Bending stress on the elemental part at section (2-2)

$$=f+df=\frac{M+dM}{I}y$$

∴ Net force at the elemental part

$=df \times$ area of cross-section of the elemental part.

$$=df\, bdy=\frac{dM}{I}y\,.\,bdy$$

∴ Total force on the part of the beam of length dx and of area as shown shaded (Fig. 27).

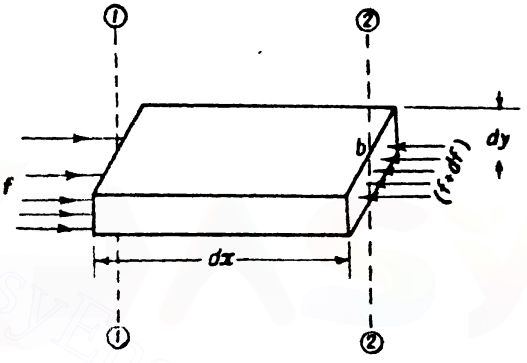

Fig. 28

$$=\int_{y_c}^{y_e}\frac{dM}{I}\,b\,.\,y\,dy=\frac{dM}{I}\int_{y_c}^{y_e}bdy\,.\,y$$

$$=\frac{dM}{I}\times \text{moment of shaded area about the neutral axis}$$

$$=\frac{dM}{I}\cdot A\bar{y}$$

where A = area shaded

$\bar{y}$ = distance of C.G. of shaded area from the N.A.

But this net force should be balanced by horizontal shear.

Let q be the intensity of horizontal shear stress.

∴ Horizontal shear resistance = force on the part of the beam of length dx and of area shown shaded.

$$\therefore \quad b\,dx\,.\,q=\frac{dM}{I}A\bar{y}$$

$$\therefore \quad q=\frac{dM}{dx}\cdot\frac{A\bar{y}}{Ib}$$

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But $\frac{dM}{dx}$ = S.F at the section = S

$$q = \frac{SA\bar{y}}{Ib} \qquad ...(i)$$

This is the intensity of shear stress in a horizontal direction. But, this also represents the intensity of shear stress in a vertical direction (by the principle of complementary shear).

§58. Shear Distribution in a Beam of Rectangular Section

Consider a beam b *cm.* by d *cm.* Let the shear force at the section be S. The shear stress q at a point y *cm.* above the neutral axis is given by

$$q = \frac{S}{Ib} \times \text{moment of area above}$$

the point considered about the N.A.

$$= \frac{S}{Ib} \cdot b\left(\frac{d}{2} - y\right)\left(\frac{\frac{d}{2} + y}{2}\right)$$

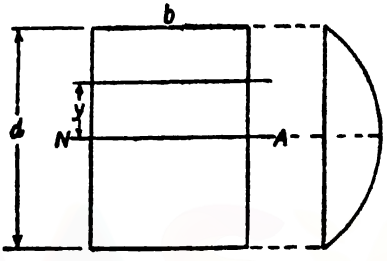

Fig. 29

$$q = \frac{SI}{2I}\left[\frac{d^2}{4} \quad\right] \qquad ...(ii)$$

Hence, the distribution of shear intensity follows a parabolic law. This is shown in Fig 29.

We find that the shear stress

$$= 0 \text{ at } y = \frac{d}{2}$$

and the shear stress is maximum at

$$y = 0$$

$$\therefore \quad q_{max} = \frac{S}{2I} \cdot \frac{d^2}{4} = \frac{Sd^2}{8I}$$

Since $$I = \frac{bd^3}{12}$$

$$q_{max} = \frac{Sd^2}{8} \frac{12}{bd^3} = \frac{3}{2} \cdot \frac{S}{bd}$$

But $\frac{S}{bd}$ = average shear stress.

$$\therefore \quad q_{max} = \frac{3}{2} q_{average}.$$

59 Shear Stresses in an R.C. Beam

In the case of reinforced concrete beam in which it is assumed

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that tensile stresses are not resisted by concrete, the distribution of shear stress will not exactly follow the law of equations (*i*) and (*ii*). However, for the compression zone of the section. the shear distribution follows the law of equations (*i*) and (*ii*). The shear stress in concrete in the tension zone is constant, as shown in Fig. 32.

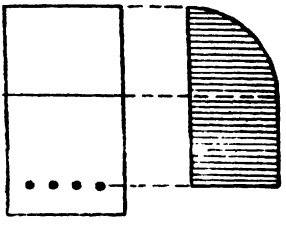

Fig. 30

The shear stress in concrete in the tension zones therefore is the maximum shear stress. This can be determined as follows :

Consider the concrete beam subjected to a loading. Consider two sections 1-1 and 2-2 *dx* apart. Let the cross-section of the beam be *b cm.* wide and *d cm.* effective depth.

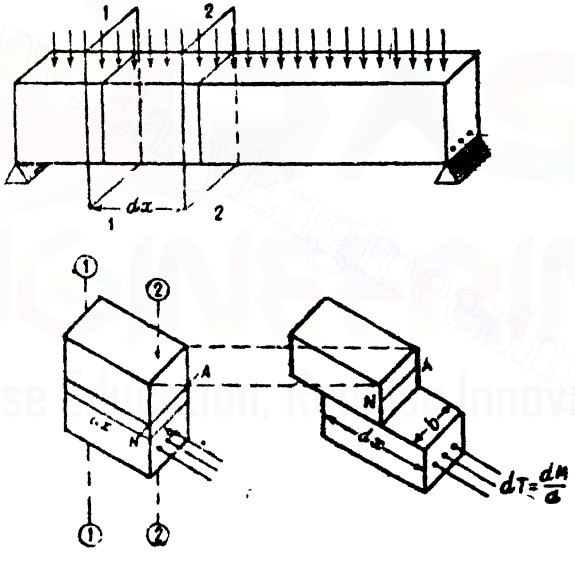

Fig. 31

Let the bending moments at sections 1-1 and 2-2 be M and $(M+dM)$ respectively. Let the lever arm

$$=a=d-\frac{n}{3}$$

Tension in the reinforcement at section 1-1

$$=T=\frac{M}{a}$$

Tension in the reinforcement at section 2-2

$$=T+dT$$

$$M+dM$$

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∴ **Net force on the reinforcements tending them to move**

$$= dT = \frac{dM}{a}$$

If the reinforcement be firmly bonded with concrete so that the reinforcement will not slip out of concrete, then the net force in the reinforcement will induce shear stress equal to

$$\frac{dT}{\text{Horizontal area of beam between sections 1-1 and 2-2}}$$

$$\therefore \quad q = \frac{dT}{bdx} = \frac{dM}{a} \cdot \frac{1}{bdx} = \frac{1}{ab} \cdot \frac{dM}{dx}$$

$$= \frac{S}{ab} \text{ since the S.F.} = S = \frac{dM}{dx}$$

$$q = \frac{S}{ab} \qquad \ldots(iii)$$

This is the horizontal shear stress in concrete which will also be equal to vertical shear stress intensity.

§60. Effect of Shear Stresses

Consider a rectangular block $ABCD$ acted upon by shear stresses q as shown in Fig. 32 The effect of the shear stresses is to deform the block to the shape shown in Fig. 32 (*c*). We find that compressive stresses are developed on the diagonal plane AC and tensile stresses are developed on the diagonal plane BD. It can be easily shown that the intensity of diagonal compression or diagonal tension is also equal to the shear stress q applied on this block. If the material of the block is weak in tension, the failure will occur along the diagonal BD and the block will be split up into two along this plane. If the material is weak in compression, then failure can occur by crushing along the plane AC.

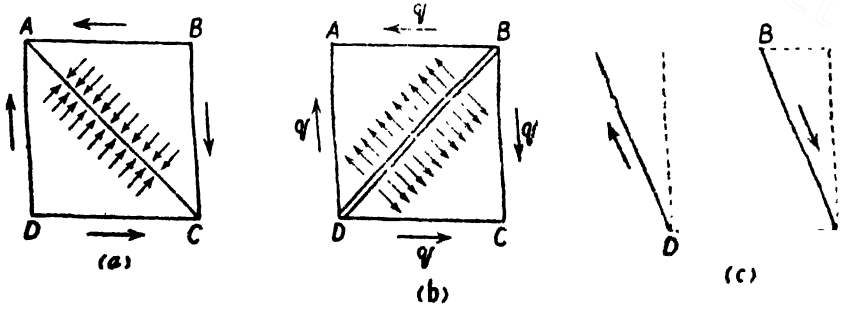

Fig. 32

Concrete is a material weak in tension and strong in compression. It is at least ten times as strong in compression as in tension. Hence if a concrete block is subjected to shear stresses failure may result by diagonal tension.

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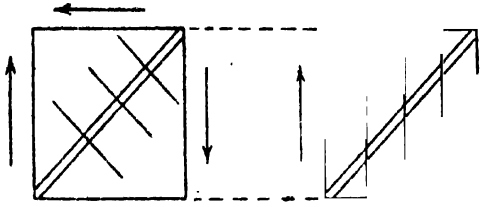

(a) (b)

Fig. 33

Fig. 33 shows a concrete block which has developed a diagonal tensile crack as a consequence of shear stresses. Hence to maintain the block in the compact form, the two parts into which the block is liable to be split should be connected either by inclined reinforcement or by vertical reinforcement These reinforcements (which are provided at the time of casting the block) which are provided to prevent the failure of concrete by excessive diagonal tension are called shear reinforcements.

Figs. 34 (*a*) and (*b*) show how such diagonal tensile stresses are developed in a beam Suppose a failure by diagonal tension occurs so that the part *ABCD* is split up into the parts *ABC* and *DCB*. In order to safeguard the structure against such failure by diagonal tension, reinforcements connectiong the two parts *ACB* and *DCB* are required. These reinforcements are called

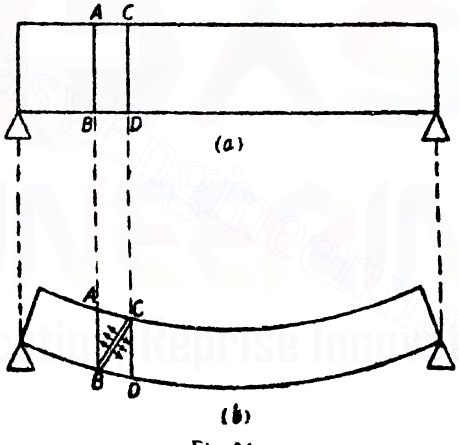

Fig. 34

NOMINAL BARS FOR ANCHORING THE VERTICAL STIRRUPS

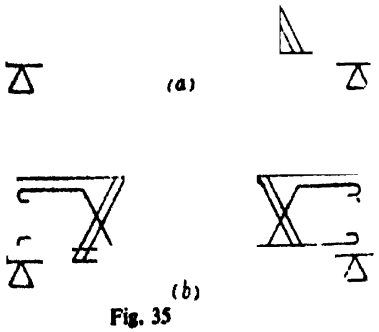

Fig. 35

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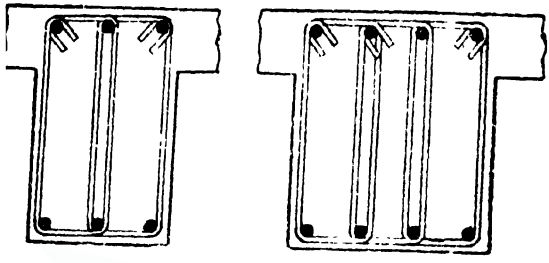
shear reinforcements. As already suggested, [in Figs. 35 (*a*) and (*b*)] the reinforcement may be provided vertically or diagonally.

When the shear reinforcements are provided vertically they are called stirrups. These consist of bars of 6 *mm*. to 10 *mm*. diameter bent round the tensile reinforcement, as shown in Fig. 35 (*a*).



(*a*) (*b*)

Fig. 36

It is also necessary to provide small diameter bars of 10 *mm*. to 12 *mm*. in the compression zone of the beam in order to properly anchor the stirrups. In Fig. 35 (*a*) each stirrup consists of two legs as shown in the cross-section. In such a case we say the stirrups are two-legged. In some case in order to resist greater shear stresses it becomes necessary to provide several legged stirrups (four-legged, six-legged, *etc.*). These are shown in Figs. 36 (*a*) and (*b*)

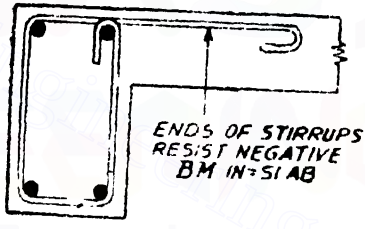


Fig. 37. Two-Legged Surrups in *L* Beam

§61. Design of Vertical Stirrups* — Conventional Method

Let us assume that the concrete has failed by diagonal tension so that the stirrups which are all crossing the crack are subjected to a load equal to the maximum shear force *S*, which is the same as the end reaction. See Fig. 38. Let us also assume that the cracked plane is inclined at 45° to the centre line of the beam and that it extends over a horizontal distance equal to the lever arm *a* of the beam.

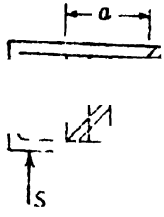


Fig. 38

*See also method suggested by the I.S. code, on page 111

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Let A_w = area of one stirrup

t_w = allowable stress in tension in one stirrup
= 140 $N/mm.^2$ or 14000 $N/cm.^2$ (14000 $kg./cm.^2$)

p = pitch of stirrups

Number of stirrups in the horizontal distance of $a = \frac{a}{p}$

$\therefore$ Total load on the stirrups $= \frac{a}{p} \cdot A_w \cdot t_w = S$

$\therefore$ $$p = \frac{A_w t_w a}{S} \qquad ...(iv)$$

A_w should be determined taking into account the number of legs provided in each stirrup.

Since the shear force decreases from the ends towards the centre, the spacing of these stirrups will be close at the ends and can be increased towards the centre. As per conventional practice the spacing of these stirrups shall not exceed the lever arm distance of the beam.

Strictly at sections where the shear stress $q = \frac{S}{ab}$ is less than the allowable limit of 0·5 $N/mm.^2$ or 5 $kg/cm.^2$, shear reinforcement is not theoretically required But it is a practice to always provide stirrups even if the induced shear stress is less than the above limit.

Fig. 39 shows simply supported beam carrying a distributed load of w *kg./metre*. Let the span of the beam be l *metres*.

We have $q_{max} = \frac{S}{ab} = \frac{\left[\frac{wl}{2}\right]}{ab}$ N/cm^2. This occurs at the end.

Let the section where the shear stress is 0·5 N/mm^2 or 50 N/cm^2 be at a distance of x *metres* from the centre

$\therefore$ $\frac{x}{l/2} = \frac{50}{q_{max}}$

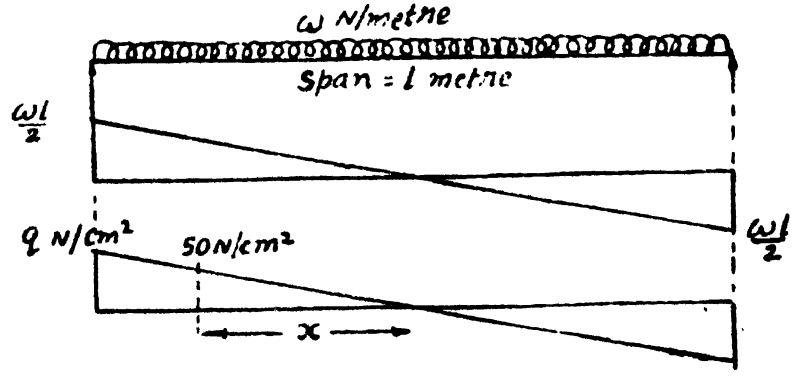

Fig. 39

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From the above relation, x can be determined.

Hence for a distance of x on each side of the middle point, shear reinforcement is not theoretically required, but practically provided to a nominal extent at a spacing not exceeding the lever arm distance of the beam.

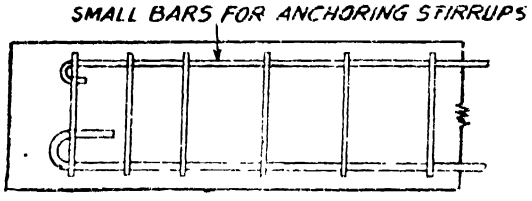

Fig. 40

For the distance of $\left(\frac{l}{2}-x\right)$ from each end stirrups are provided in accordance with the requirement of eq (*iv*). The distance $\left(\frac{l}{2}-x\right)$ from each end may be divided into a number of sections and the shear forces at these sections can be determined. The pitch of stirrups corresponding to these sections can now be calculated. This will assist in deciding about changing the pitch at suitable sections.

Problem 29. (S.I.). *A beam 25 cm. wide, 50 cm. effective depth and 6 metres span supports a total load of 190,500 Newtons including its weight. Find the maximum shear stress and determine the spacing of 10 mm. stirrups by conventional method*

Solution. Assume $c=500\ N/cm.^2$ and $t=14000\ N/cm^2$, $m=18$

$$a=0{\cdot}87\ d$$

$$S=\frac{190500}{2}=95250\ N$$

$$q=\frac{S}{ab}=\frac{95250}{(0{\cdot}87\times 50\times 25)}=87{\cdot}5\ N/cm.^2$$

This is greater than 50 $N/cm.^2$

$\therefore$ Shear reinforcement is necessary.

Suppose two-legged 10 *mm.* bars are suggested for stirrups, their spacing will be

$$p=\frac{A_w t_w a}{S}$$

$$\frac{2\times 0{\cdot}79\times 14000\times 0{\cdot}87\times 50}{95250}=10{\cdot}3\ cm.$$

Let us, therefore, provide 10 *mm.* diameter two-legged stirrups at 10 *cm.* centres.

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Let the point where the shear stress equals the allowable or safe shear stress of 50 $N/cm.^2$ be x *metres* from the centre.

$$\frac{x}{3} = \frac{50}{87{\cdot}5}$$

$$x = \frac{150}{87{\cdot}5} = 1{\cdot}71 \text{ metre.}$$

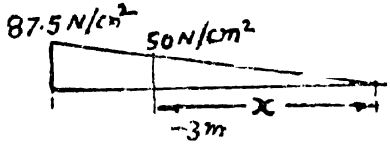

Fig. 41

Hence shear reinforcement is needed for $3 - 1{\cdot}71 = 1{\cdot}29$ *metres* say 1·30 *metres* from the support.

For the middle 3·4 *metres* nominal stirrups at 25 *cm.* centres may be provided.

§62. Inclined or Diagonal Reinforcement* — Conventional Method

When inclined reinforcements are provided, they consist of main tension reinforcements bent up at a certain angle, as shown in Fig. 42 Bars are usually bent at 45° with the horizontal. Bars can be bent up only if the remaining bars are sufficient to resist the prevailing bending moments.

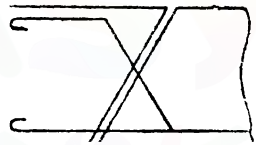

Fig. 42

Suppose A_w is the area of reinforcement bent up. If t_w is the stress in these bars the vertical component of the tension in these bars $= A_w t_w \sin 45° = 0{\cdot}707\ A_w t_w$.

Usually t_w is taken at 1400 $kg./cm^2$. or 140 $N/mm.^2$

If this quantity exceeds the maximum shear force S, then the shear is safely resisted by the bent up bars. Sometimes with the bars which can be spared to be bent up the shear value of $0{\cdot}707\ A_w t_w$ may be less than the maximum shear force. In such cases for the balance shear force of $S - (0{\cdot}707\ A_w t_w)$ vertical stirrups should be provided. However, vertical stirrups are always provided with a spacing not exceeding the lever arm distance of the beam.

§63 Lattice Girder Effect

(*a*) **Single system.** It is not enough if bars are bent up just near the ends to resist the shear. *In order that the beam is safe against shear failure, shear reinforcement is provided throughout the length* whether with diagonal steel or with vertical steel.

When a number of bars are available to be bent up to resist shear, it is usual to assume that the beam is equivalent to a truss consisting of concrete compression members and steel tension members as shown in Fig. 43.

* See also method suggested by the I.S. code.

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The usual arrangement is to assume the first imaginary compression member AB at $67\frac{1}{2}°$ with the horizontal. The inclined steel tension members are at 45° with the horizontal.

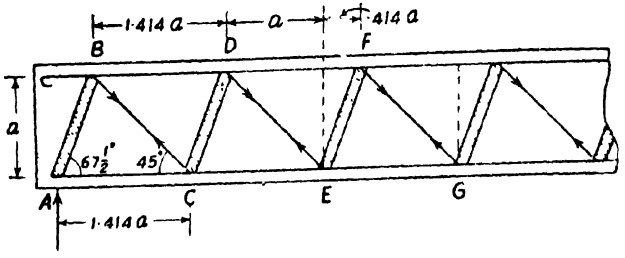

Fig. 43

With such a system, the shear resistance at any section $= A_w t_w \sin 45° = 0{\cdot}707\ A_w t_w$.

The inclined tension members of this imaginary truss are provided by the bent up bars. Bending up bars are possible only when a good number of bars are present at the bottom and a number of them are no longer required to resist the bending moment. The arrangement shown in Fig. 3 is called a single system of bent up bars. The height of the imaginary truss is equal to the lever arm distance a.

From the geometry of the triangle ABC of the truss, we have

$$AC = CB = a\sqrt{2} = 1{\cdot}414\ a$$

i.e., the bars bent up at C are bent from a point at a distance of $1{\cdot}414\ a$ from the support.

If the inclination of the bars be at any angle θ with the horizontal, the arrangement would be as shown in Fig. 44.

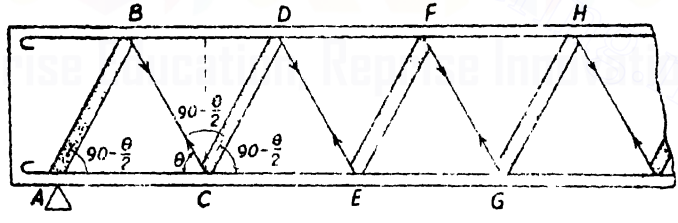

Fig. 44

It is worth noting that if the bent up bar say CB is at θ with the horizontal, the compression member CD should be bisecting the angle $B\hat{C}E$ so that $B\hat{C}D = D\hat{C}E = \dfrac{180-\theta}{\theta} = 90 - \dfrac{\theta}{2}$. If this arrangement is not done, *i.e.*, if the compression members are not taken at $90 - \dfrac{\theta}{2}$ with the horizontal, we find the design of shear reinforcement becomes uneconomical.

Sometimes the imaginary compression members are also taken at 45° with the horizontal while the bars are also bent up at 45° as shown in Fig. 45.

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In such cases while calculating the shear resistance at any section by the expression 0·707 $A_w t_w$, care should be taken to assume a value of t_w such that excessive tensile stresses are not developed in the horizontal part of the bent up bars.

A value of $t_w = 0{\cdot}707 \times$ allowable tensile stress in steel, should be used while calculating the shear resistance. Suppose the allowable

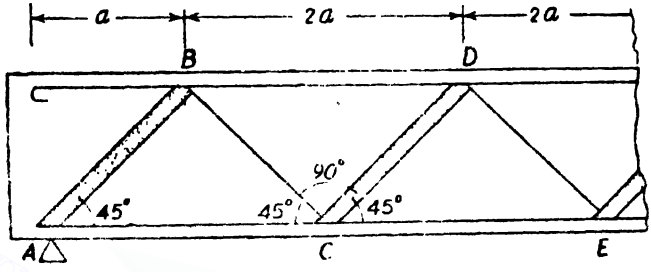

Fig. 45

tensile stress is 14000 $N/cm.^2$ The value of t_w in this case should be taken at 0·707 × 14000 = 9900 N/cm^2.

(*b*) **Double system.** If in addition to the bars bent up as shown in Fig. 46, additional bars are also bent up as shown in Fig. 46, the arrangement is called a double system arrangement.

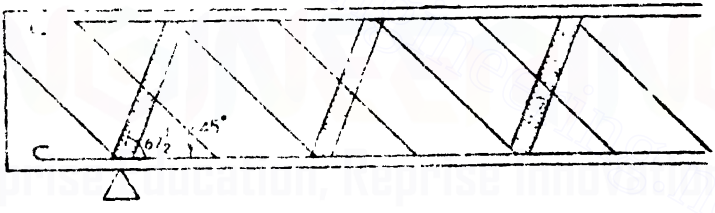

Fig. 46

In this case the shear resistance at any section $= 2 \times 0{\cdot}707\ A_w t_w$.

64. Revised Theory Recommended by the I.S. Code

The I.S. 456-1978 code has proposed a revised theory for analysis and design for shear in the case of beams and slabs. The basic changes proposed by the code are the following :

(*i*) **Nominal shear stress.** The code introduces the concept of *nominal shear stress*. The nominal shear stress q_v in beams or slabs of uniform depth shall be calculated by the following equation :

$$q_v = \frac{S}{bd}$$

where, S = shear force due to design loads

b = breadth of the member, which for flanged sections shall be taken as the breadth of the web b_w and

d = effective depth.

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In the case of beams of varying depth, the above equation shall be modified as :

$$q_v = \frac{S \pm \frac{M}{d} \tan \beta}{bd}$$

where, q_v, S, b and d are the same as before

β = angle between the top and the bottom edges of the beam.

The negative sign in the formula applies when the bending moment M increases numerically in the same direction as the effective depth d increases, and the positive sign when the moment decreases numerically in this direction.

(*ii*) **Permissible nominal shear stress** q_c. The permissible nominal shear stress in concrete in beams without shear reinforcement is given in the table below.

Permissible Nominal Shear Stress in Concrete in Beams [*S.I. Units*]
I.S. 456-1978

$100\frac{A_t}{bd}$	*Permissible Nominal Shear Stress in Concrete* q_c *in N/mm*^{2}					
	M 15	*M 20*	*M 25*	*M 30*	*M 35*	*M 40*
0·25	0·22	0·22	0·23	0·23	0·23	0·23
0·50	0·29	0·30	0·31	0·31	0·31	0·32
0·75	0·34	0·35	0·36	0·37	0·37	0·38
1·00	0·37	0·39	0·40	0·41	0·42	0·42
1·25	0·40	0·42	0·44	0·45	0·45	0·46
1·50	0·42	0·45	0·46	0·48	0·49	0·49
1·75	0·44	0·47	0·49	0·50	0·52	0·52
2·00	0·44	0·49	0·51	0·53	0·54	0·55
2·25	0·44	0·51	0·53	0·55	0·56	0·57
2·50	0·44	0·51	0·55	0·57	0·58	0·60
2·75	0·44	0·51	0·56	0·58	0·60	0·62
3·00 and above	0·44	0·51	0·57	0·60	0·62	0·63

NOTE 1 ... at least one effective depth beyond the section being considered except at supports where the full area of tension reinforcement may be used.

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Permissible Nominal Shear Stress in Concrete in (*M.K.S. Units*)

$100\frac{A_t}{bd}$	*Permissible Nominal Shear Stress in Concrete q_c in kg./cm.²*					
	M 150	*M 200*	*M 250*	*M 300*	*M 350*	*M 400*
0·25	2·2	2·2	2·3	2·3	2·3	2·3
0·50	2·9	3·0	3·1	3·1	3·1	3·2
0·75	3·4	3·5	3·6	3·7	3·7	3·8
1·00	3·7	3·9	4·0	4·1	4·2	4·2
1·25	4·0	4·2	4·4	4·5	4·5	4·6
1·50	4·2	4·5	4·6	4·8	4·9	4·9
1·75	4·4	4·7	4·9	5·0	5·2	5·2
2·00	4·4	4·9	5·1	5·3	5·4	5·5
2·25	4·4	5·1	5·3	5·5	5·6	5·7
2·50	4·4	5·1	5·5	5·7	5·8	6·0
2·75	4·4	5·1	5·6	5·8	6·0	6·2
3·00 and above	4·4	5·1	5·7	6·0	6·2	6·3

NOTE—A_t is that area of the longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at supports where the full area of tension reinforcement may be used.

For solid slabs the permissible nominal shear stress in concrete shall be kq_c where k has the value given below :

Overall depth of slab (mm.)	*300 or more*	*275*	*250*	*225*	*200*	*175*	*150 or less*
k	1·00	1·05	1·10	1·15	1·20	1·25	1·30

NOTE—This does not apply to flat slabs.

(*iii*) **Maximum nominal shear stress with shear reinforcement.** When shear reinforcement is provided the nominal shear stress q_v in beams shall not exceed $q_{c\,max}$ given in the table below. For slabs q_v shall not exceed half the value of $q_{c\,max}$ given in this table.

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Maximum Nominal Shear Stress $q_{c\ max}$ (N/mm^2) (*S.I. Units*)

Concrete grade	*M 15*	*M 20*	*M 25*	*M 30*	*M 35*	*M 40*
$q_{c\ max}$ (N/mm^2)	1·6	1·8	1·9	2·2	2·3	2·5

Maximum Nominal Shear Stress $q_{c\ max}$ (*kg./cm.*^{2}) (*M.K.S. Units*)

Concrete grade	*M 150*	*M 200*	*M 250*	*M 300*	*M 350*	*M 400*
$q_{c\ max}$ (*kg/cm.*^{2})	16	18	19	22	23	25

(*iv*) **Minimum Shear Reinforcement in Beams.** When the nominal shear stress q_v is less than the permissible nominal shear stress q_c, minimum shear reinforcement shall be provided in the form of stirrups such that

$$\frac{A_w}{bp} \geq \frac{0{\cdot}4}{f_y}$$

where, A_w = total cross-sectional area of stirrup legs effective in shear,

p = spacing of stirrups along the length of the member,

b = breadth of the beam or the breadth of the web of flanged beam, and

f_y = characteristic strength of the stirrup reinforcement in $N/mm.^2$ which shall not be taken greater than 415 $N/mm.^2$

$$\therefore \quad p \leq \frac{f_y}{0{\cdot}4} \frac{A_w}{p}$$

If $f_y = 250\ N/mm.^2$, $p \leq 625 \frac{A_w}{b}$

If $f_y = 415\ N/mm.^2$, $p \leq 1037{\cdot}5 \frac{A_w}{b}$.

However, in members of minor structural importance such as lintels or where the maximum nominal shear stress calculated is less than half the permissible value, this provision need not be complied with.

(*v*) **Design of shear reinforcement.** When the nominal shear stress q_v exceeds the permissible shear stress q_c, shear reinforcement shall be provided in any of the following forms :

(*a*) Vertical stirrups,

(*b*) Bent-up bars along with stirrups, and

(*c*) Inclined stirrups.

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Where bent-up bars are provided, their contribution towards shear resistance shall not be more than half that of the total shear reinforcement.

Shear reinforcement shall be provided to carry a shear equal to $(S - q_c\, bd)$. The strength of shear reinforcement S_w shall be calculated as below :

(*a*) For vertical stirrups

$$S_w = A_w\, t_w\, d$$

(*b*) For inclined stirrups or a series of bars, bent-up at different cross-sections

$$S_w = \frac{A_w\, t_w\, d}{p}\,(\sin\alpha + \cos\alpha)$$

(*c*) For single bar or single group of parallel bars, all bent-up at the same cross-section,

$$S_w = A_w\, t_w \sin\alpha$$

where, A_w = total cross-sectional area of stirrup legs or bent-up bars within a distance p,

p = spacing of stirrups or bent-up bars along the length of the member,

q_c = design nominal shear strength of concrete,

b = breadth of the member, which for flanged beams, shall be taken as the breadth of the web b_w,

t_w = permissible tensile stress in shear reinforcement which shall not be taken greater than 230 $N/mm.^2$,

α = angle between the inclined stirrup or bent-up bar and the axis of the member, not less than 45°, and

d = effective depth.

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5
Bond

One of the main assumptions in developing the theory of reinforced concrete is that the reinforcements do not slip from the concrete surrounding it. When concrete sets and thus hardens, it will firmly grip round the reinforcements. It is because of this grip between concrete and steel, the two materials share the applied loads. Once this grip is absent the reinforcement provided would serve no purpose.

Suppose the reinforcement of the beam shown in Fig. 47 has lost the grip with concrete.

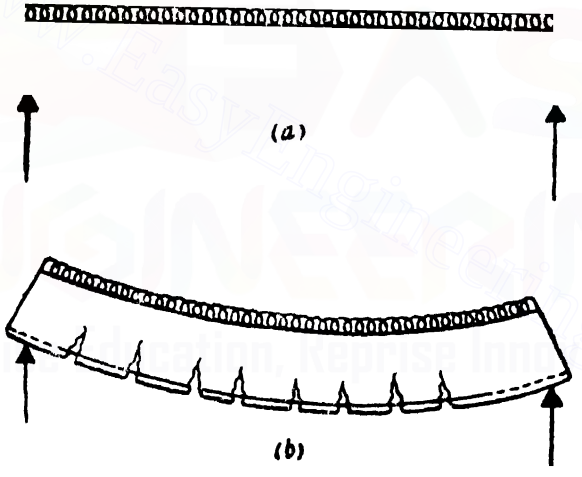

Fig. 47

Then it is just as good as loosely providing a reinforcement in holes present earlier. When the beam is loaded the steel rod slips and the beam fails since the reinforcement has not really shared the loading. Hence this grip between the concrete and steel is very important and the gripping stress, hereafter called the bond stress, should therefore be within a limit.

It may be realised that the function of this bond in reinforced concrete members is exactly the same as the function of rivets in built up plate girders which consist of web plates and flange angles

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and cover plates. If for instance the flange angles are not properly riveted to the web plate the flange angles will not function. If the number of rivets connecting the flange angles and the web is insufficient, these rivets will fail and immediately the flange becomes separated from the web. In almost the same manner when the induced bond stresses are very large the bars get separated from the concrete or let us say the bond between the concrete and the steel is broken and this will result in the load of the beam to be resisted only by the concrete. The beam will thus fail by tension even though sufficient amount of steel from bending moment considerations has been provided. Bond stresses often are not receiving that much good attention which they deserve. While designing particularly, beams subjected to heavy loads and footings, bond stresses shall always be determined and shall be compared with the allowable bond stress.

§65. Direct Bond

In Fig. 48 (*a*) is shown a bar embedded in concrete only for a short length ; while in Fig. 48 (*b*) is shown a bar embedded for a

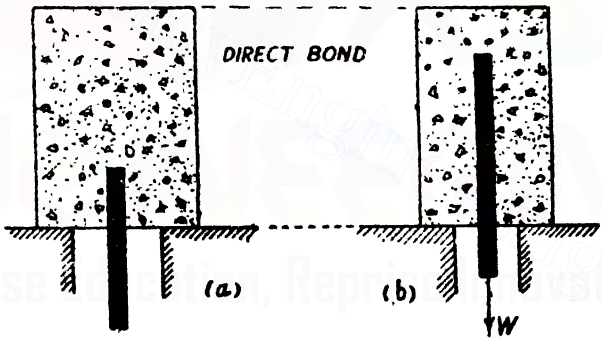

Fig. 48

greater length. It is obvious that assuming the bars not to fail, it requires a greater load to pull out the bar in the arrangement of Fig. 48 (*b*) than what is required to pull out the bar in the arrangement of Fig. 48 (*a*).

The intensity of bond stress

$$= \frac{\text{Load in the bar}}{\text{Contact area of the bar with concrete}}$$

Let l be the length of embedment.

Bond stress $= S_b = \dfrac{W}{\pi d l}$

where d = diameter of bar

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S_b is called the *average bond stress.* Let the bar be subjected to its maximum stress t.

$$\therefore \quad W = \frac{\pi d^2}{4} t.$$

$$\therefore \quad S_b = \frac{\frac{\pi d^2}{4} t}{\pi d l} = \frac{dt}{4l}$$

$$\therefore \quad \text{Length of embedment} = l = \frac{dt}{4S_b} \qquad ...(i)$$

M.K.S. Units	S.I. Units
Taking $t = 1400\ kg./cm.^2$ and $S_b = 6\ kg./cm.^2$,	Taking $t = 140\ N/mm.^2 = 14000\ N/cm.^2$ and $S_b = 0{\cdot}6\ N/mm.^2 = 60\ N/cm.^2$
Length to embedment $= l = \frac{d \times 1400}{4 \times 6} = 58{\cdot}33\ d$ *say* $58\ d$	Length to embedment $= l = \frac{d \times 14000}{4 \times 60} = 58{\cdot}33\ d$ *say* $58\ d$

The average bond stress permissible for different grades of concrete are given below :

M.K.S. Units		S.I. Units	
Grade	*Average bond stress in kg./cm.²*	*Grade*	*Average bond stress in N/mm.²*
M 150	6	M 15	0·6
M 200	8	M 20	0·8
M 250	9	M 25	0·8
M 300	10	M 30	1·0
M 350	11	M 35	1·1
M 400	12	M 40	1.2

NOTE—The bond stress given above shall be increased by 25 percent for bars in compression.

The following are the factors that give the property of a good bond between concrete and reinforcement :

(*a*) Sufficient cover for reinforcement.

(*b*) Richness of concrete.

(*c*) Using twisted bars, welding the stirrup bars with the main bars.

(*d*) Roughness of steel.

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§66. Local Bond

Consider an R.C. beam subjected to a certain loading. Consider two sections 1-1 and 2-2 distant dx apart. See Fig. 49. Let the bending moments at sections 1-1 and 2-2 be M and $(M+dM)$ respectively. Let the lever arm distance be a.

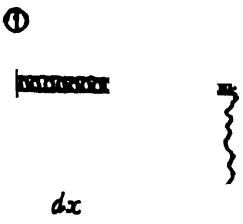

Fig. 49

$\therefore$ Tension in steel reinforcement at section 1-1 $= \frac{M}{a}$ and Tension in steel reinforcement at section 2-2 $= \frac{M+dM}{a}$.

$\therefore$ Net force on the tension reinforcement in a length $dx = \frac{dM}{a}$

For, this length of reinforcement, the bond force between the steel and the concrete $= (\Sigma o)\, dx\, S_b$ where (Σo) is the total perimeter of reinforcement. Equating the force of bond to the net force on the tensile reinforcement, we have, $(\Sigma o)\, dx . S_b = \frac{dM}{a}$

$$S_b = \frac{dM}{dx} \cdot \frac{1}{a(\Sigma o)}$$

But $$\frac{dM}{dx} = \text{S.F.} = S.$$

$$S_b = \frac{S}{a\Sigma o} \qquad \ldots(2)$$

This bond stress induced due to the rate of change of bending moment is called the *local bond stress.*

A local bond stress of 10 $kg./cm.^2$ is permissible for M 150 concrete.

In designs the Eq. (2) is put in the form $\Sigma o = \frac{S}{aS_b}$. $\qquad \ldots(3)$

From this, the perimeter of bars required to limit the bond stress to 10 $kg./cm.^2$ can be computed.

Hence reinforcement required from bond considerations must be provided though from bending moment considerations no reinforcement may be required.

Eq. (3) shows that the required perimeter of reinforcement is proportional to the shear force. For a simply supported beam, for instance, the bending moment at the support is zero and the shear force is maximum.

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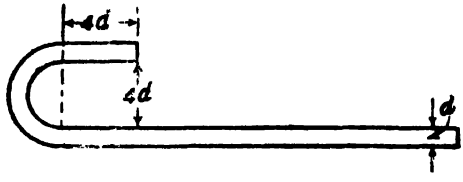
Hence some bars are to be maintained in the bottom such that the perimeter of bars provided is at least equal to $\frac{S}{aS_b}$. The remaining bars may be curtailed at such places where they are no longer required for bending moment. These bars may also be bent up at suitable places to serve as shear reinforcement.

§67. End anchorage

Bars embedded in concrete are sometimes hooked so as to have proper anchorage with concrete. If bars are provided with hooks, the necessary grip or bond length can be reduced. A hook shall always confirm to the specifications shown in Fig. 50. The anchorage value of the hook alone is considered as 16 d where d is the diameter of the bar.



A standard hook.
Fig. 50

§68. Anchorage Value of Bends

A bend in a reinforcing bar may be assumed to have an anchorage value equivalent to that of the length of the bar equal to four times the diameter of the round bar for each 45° the bar is bent, provided that :

(*a*) The radius of the bend be not less than twice the diameter of the round bar.

(*b*) The length of the straight part of the bar beyond the end of the curve be at least four times the diameter of the round bar.

(*c*) Whatever be the angle through which the bar is bent, the assumed anchorage value should not be taken as more than equivalent to a length of bar equal to sixteen times the diameters of the round bar.

§69. Bars in Tension

In the tensile reinforcement the length measured from any section to the end of the bar plus the equivalent anchorage value of hook shall be such that the average bond stress induced to develop the actual stress at the section shall not exceed the permissible average bond stress. In the case of the tensile reinforcement of circular section, the length measured from such section up to the beginning of the hook shall at least be equal to n times the diameter of the bar minus the anchorage value of the hook.

where $$n = \frac{\text{Actual tensile stress in the bar}}{\text{Four times the permissible average bond stress}}$$

In no case shall such value of n be less than 12.

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§70. Bars in Compression

In this case, the length measured from any section to the end of the bar shall be such that the average bond stress induced to develop the actual stress at the section shall not exceed 1·25 times the permissible average bond stress. In the case of bars of circular section the length measured from such section shall be at least equal to n times the diameter of the bar where

$$n = \frac{\text{Actual compressive stress in the bar}}{\text{Five times the permissible average bond stress}}$$

In no case the value of n be less than 12. Hooks are unnecessary. But when a hook is provided it shall not be accounted for anchorage purposes.

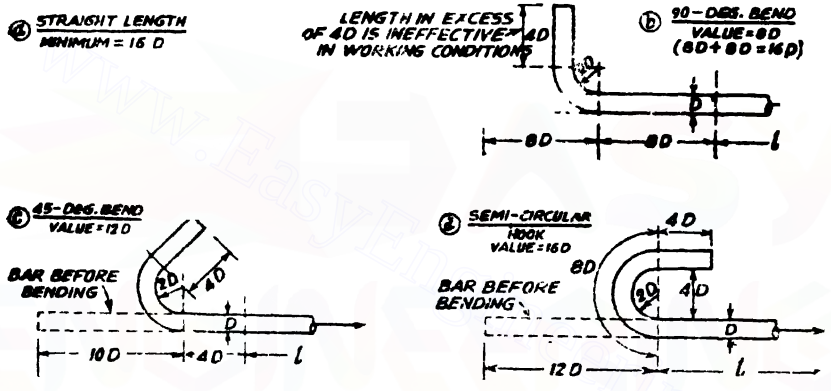

Fig. 51. Dimensions and Equivalent Bond-lengths of Bends and Hooks.

§71. I.S. Recommendations—Development Length

The revised I.S. code has deleted the concept of local bond stress and has made special recommendations, as given below :

1. Development of stress in reinforcement. The calculated tension or compression in any bar at any section shall be developed on each side of the section by an appropriate development length or end anchorage or by a combination thereof.

2. Development length of bars. The development length L_d is given by

$$L_d = \frac{d\sigma_s}{4S_b}$$

where d = nominal diameter of the bar

σ_s = stress in the bar at the section considered at design load

and S_b = Design bond stress.

Note—1. The development length includes anchorage value of hooks in tension reinforcement.

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2. For bars of sections other than circular, the development length should be sufficient to develop the stress in the bar by bond.

3. Bars bundled in contact. The development length of each bar of bundled bars shall be that for the individual bar, increased by 10 percent for two bars in contact, 20 percent for three bars in contact and 33 percent for four bars in contact.

4. Anchoring Reinforcing Bars

(*a*) *Anchoring bars in tension.* Deformed bars may be used without end anchorages provided development length requirement is satisfied. Hooks should normally be provided for plain bars in tension.

(*b*) *Bends and hooks.* The anchorage value of bend shall be taken as 4 times the diameter of the bar for each 45° bend subject to a maximum of sixteen times the diameter of the bar.

The anchorage value of a standard U-type hook shall be equal to sixteen times the diameter of the bar.

Anchoring bars in compression. For bars in compression the value of the permissible average bond stress given above, shall be increased by 25 percent Accordingly the development length of bars in compression

$$L_d = \frac{\text{dia. of bar} \times \text{stress in steel}}{4 \times (1.25 \times \text{usual average permissible bond stress})}$$

$$= \frac{\text{dia. of bar} \times \text{stress in steel}}{5 \times \text{usual average permissible bond stress}}$$

The projected length of hooks, bends and straight lengths beyond bends if provided for a bar in compression, shall be considered for development length.

Anchorage shear reinforcement

(*a*) *Inclined bars.* The development length shall be as for bars in tension ; this length shall be measured as under :

(*i*) In tension zone, from the end of the sloping or inclined portion of the bar, and

(*ii*) In the compression zone, from the mid depth of the beam.

(*b*) *Stirrups.* For stirrups and ties, the complete development lengths and anchorage shall be deemed to have been provided when the bar is bent through an angle of at least 90° round a bar of atleast its own diameter and is continued beyond the end of the curve for a length of atleast eight diameters, or when the bar is bent through an angle of 135° and is continued beyond the end of the curve for a length of atleast six bar diameters or when the bar is bent through an angle of 180° and is continued beyond the end of the curve for a length of atleast four bar diameters.

5. Curtailment of tension reinforcement in flexural members. For curtailment, reinforcement shall extend beyond the point at

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which it is no longer required to resist flexure for a distance equal to the effective depth of the member or 12 times the bar diameter, whichever is greater except at simple support or end of cantilever.

Positive moment reinforcement

(*a*) At least one-third the positive moment reinforcement in simple members and one-fourth the positive reinforcement in continuous members shall extend along the same face of the member into the support to a length equal to $L_d/3$.

(*b*) When a flexural member is part of the primary lateral load resisting system, the positive reinforcement required to be extended into the support as described in (*a*) shall be anchored to develop its design stress in tension at the face of the support.

(*c*) At simple supports and at points of inflexion, positive moment tension reinforcement shall be limited to a diameter such that L_d computed for f_d does not exceed

$$\frac{M_1}{V} + L_0$$

where, M_1 = moment of resistance of the section assuming all reinforcement at the section to be stressed to f_d,

f_d = permissible stress in tension in steel (σ_{st}),

V = shear force at the section due to design loads,

L_0 = sum of the anchorage beyond the centre of the support and the equivalent anchorage of any hook or mechanical anchorage at simple support; and at a point of inflexion, L_0 is limited to the effective depth of the members or 12 diameters whichever is greater.

The value of M_1/V in the above expression may be increased by 30 percent when the ends of the reinforcement are confined by a compressive reaction.

Negative moment reinforcement. At least one-third of the total reinforcement provided for negative moment at the support shall extend beyond the point of inflexion for a distance not less than the effective depth of the member or 12 times the bar diameter or one-sixteenth of the clear span whichever is greater.

Curtailment of bundled bars. Bars in a bundle shall terminate at different points spaced apart by not less than 40 times the bar diameter except for bundles stopping at a support.

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6
Reinforcement

Reinforcement used shall conform to the requirements of *I.S. 432 specification for Mild steel and high tensile steel bars and hard drawn steel wire for concrete reinforcement (Revised)*. The reinforcement shall be free from loose mill scale, loose rust, oil and grease or any such harmful matter, immediately before placing the concrete. The reinforcement shall be placed and positioned strictly following the requirements shown in the structural drawings.

§72. Bending Bars

Bending bars shall be done with great caution. Often this job does not receive that much attention which it deserves Bending bars are expected to fulfil certain definite functions and hence bars must be bent so that there is good advantage of the worked out design. There are instances of failures of structures for want of correct bending, even though the designs worked out are not faulty.

§73. The Necessity of Bending Reinforcement

Bars are bent under different circumstances. They may be bent to form hooks so as to develop proper anchorage. Sometimes bars have to be bent so as to form loops as in the case of stirrups as shear reinforcement. Bars may also be bent to resist diagonal tension. They may also be bent up to form necessary reinforcement for hogging bending moments. The following are the types of bend we normally come across :

(*a*) Hooks at the end of bars in beams.

(*b*) Bars bent up at ends and hooked in beams for resisting diagonal tension.

(*c*) Bars which serve for positive bending moment which are bent up to resist negative bending moment.

(*d*) Bars bent to form loops to serve as shear reinforcement.

These are shown in Fig. 52.

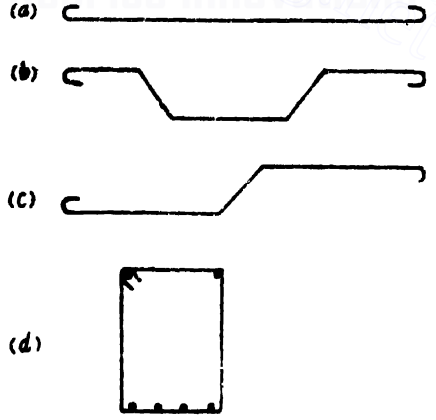

Fig. 52

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[S.I. Units] Permissible Stresses in Steel Reinforcement—(*I.S. 456*)

Type of stress in steel reinforcement	*Permissible stresses in Newtons/mm.*[2]		
	Mild steel bars conforming to grade I of I.S. 432 (Part I) or deformed mild steel bars conforming to I.S. : 1139	*Medium tensile steel conforming to I.S. 432 (Part I) or Deformed medium tensile steel bars conforming to I.S. : 1139*	*High yield strength deformed bars conforming to I.S. : 1139 or I.S. : 1786 grade (Fe 415) (Ribbed-Torsteel)*
Tension (σ_{st} **or** σ_{sc})			
(*a*) Up to and including 20 *mm.*	140	Half the guaranteed yield stress subject to a maximum of 190	230
(*b* Over 20 *mm.*	130		230
Compression in column bars (σ_{sc})	130	130	190
Compression in bars in a beam or slab when the compressive resistance of the concrete is taken into account	The calculated compressive stress in the surrounding concrete multiplied by 1·5 times the modular ratio or σ_{sc} whichever is lower		
Compression in bars in a beam or slab when the compressive resistance of the concrete is not taken into account :			
(*a*) Up to and including 20 *mm.*	140	Half the guaranteed yield stress subject to a maximum of 190	190
(*b*) Over 20 *mm.*	130		190

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[M.K S. Units] Permissible Stresses in Steel Reinforcement (*I.S. 456*)

Type of stress in Steel Reinforcement	*Permissible stresses in kg./cm.²*		
	Mild steel bars conforming to grade I of I.S. 432 (Part I) or deformed mild steel bars conforming to I.S. : 1139	*Medium tensile steel conforming to I S. 432 (Part I) or deformed medium tensile steel bars comforming to I S. : 1139*	*High yield strength deformed bars conforming to I S : 1139 or I.S : 1786 grade (Fe 415 (Ribbed Torsteel)*
Tension σ_{st} or σ_{sv}			
(*a*) Up to and including 20 *mm.*	1400	Half the guaranteed yield stress subject to a maximum of 1900	2300
(*b*) Over 20 *mm*	1300		2300
Compression in column bars (σ_{sc})	1300	1300	1900
Compression in bars in a beam or slab when the compressive resistance of the concrete is taken into account	The calculated compressive stress in the surrounding concrete multiplied by 1 5 times the modular ratio or σ_{sc} whichever is lower		
Compression in bars in a beam or slab where the compressive resistance of the concrete is not taken into account			
(*a*) Up to and including 20 *mm*	1400	Half the guaranteed yield stress subject to a maximum of 1900	1900
(*b*) Over 20 *mm*	1300		1900

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The I.S. code has further recommended the following :

Splices in tensile reinforcement. Splices at point of maximum tensile stress shall be avoided wherever possible ; splices where used shall be welded, lapped or otherwise fully developed. In any case the splice shall transfer the entire computed stress from bar to bar. Lapped splices in tension shall not be used for bars of sizes larger than 36 *mm.* diameter, and such splices shall preferably be welded.

For contact splices, spaced laterally closer than 12-bar diameters or located closer than 15 *cm.* or 6-bar diameters from the outside edge, the lap shall be increased by 20 per cent or stirrups or closely spaced spirals shall enclose the splice for its full length.

Where more than one half of the bars are spliced within a length of 40-bar diameters or where splices are made at points of maximum stress special precaution shall be taken such as increasing the length of the lap, and/or using spirals of closely spaced stirrups around and for the length of the splice.

Splices in compression reinforcement. Where lapped splices are used, the lap lengths shall conform to the requirements mentioned earlier. Welded splices may be used instead of lapped splices. Where bar size exceeds 36 *mm.* diameter welded splices shall preferably be used. In bars required for compression only the compressive stress may be transmitted by bearing of square cut ends held in concrete contact by a suitably welded sieve or mechanical device.

In columns where longitudinal bars are offset at a splice, the slope of the inclined portion of the bar with the axis of the column shall not exceed 1 in 6 and the portions of the bar above and below the offset shall be parallel to the axis of the column. Adequate horizontal support at the offset bands shall be treated as a matter of design, and shall be provided by metal ties, spirals or parts of the floor construction Metal ties or spirals so designed shall be placed near (not more than eight-bar diameters from) the point of bend. The horizontal thrust to be resisted shall be assumed as $1\frac{1}{2}$ times the horizontal component of the nominal stress in the inclined portion of the bar. Offset bars shall be bent before they are placed in forms.

Joining or Lapping. The length of lap in reinforcement shall not be less than :

(*a*) For bars in tension :

$$\text{bar diameter} \times \frac{\text{actual tensile stress}}{\text{four times the permissible average bond stress}}$$

or 30-bar diameters whichever is greater.

(*b*) For bars in compression :

$$\text{bar diameter} \times \frac{\text{actual compressive stress}}{\text{five times the permissible average bond stress}}$$

or 24-bar diameter whichever is greater.

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Minimum spacing of reinforcement. The minimum horizontal distance between parallel reinforcements shall not be less than the following :

(*i*) Diameter of bar when bars are of the same diameter and diameter of the thickest bar when bars of more than one size are used.

(*ii*) Maximum size of coarse aggregate plus 5 *mm*. *A greater distance should be provided when convenient.*

The vertical distance between two horizontal main bars shall be not less than 15 *mm*., two-thirds the nominal maximum size of aggregate or the maximum size of the bar whichever is greatest.

Cover. All reinforcements shall have a cover of concrete and the thickness of such a cover exclusive of plaster or other decorative finish shall be as follows :

(*a*) At each end of a reinforcing bar—not less than 25 *mm*. nor less than twice the diameter of such bar.

(*b*) For longitudinal reinforcement in a column—not less than 40 *mm*. nor less than the diameter of bar. In the case of columns of minimum dimensions of 20 *cm*. or less, whose bars do not exceed 12 *mm*. dia—25 *mm* cover may be used.

(*c*) For longitudinal reinforcement in beams—not less than 25 *mm*. nor less than diameter of such bar.

(*d*) For tensile, compressive, shear and other reinforcement in a slab—not less than 15 *mm*. nor dia. of reinforcement.

(*e*) For any other reinforcement—not less than 15 *mm*. nor less than the diameter of reinforcement.

For reinforced concrete members totally immersed in sea water, the cover shall be 50 *mm*. more than that specified above. For concrete of grade over M 250 (M K.S.) or over M 25 (S.I.) the additional thickness of cover may be reduced to half the value stipulated above. In all cases the cover should not exceed 75 *mm*.

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7
T-Beams

Strictly rectangular beams are uncommon in reinforced concrete since the beam carries in almost all cases a slab with which it is monolithic Hence the structure becomes a slab which is stiffened by concrete ribs. The slab and the rib due to their monolithic nature form a T-beam. The flange of the T-beam provides the necessary resistance to compression while the vertical rib provides the depth and hence the necessary lever arm. The width of rib must be such as to accommodate the tensile reinforcement.

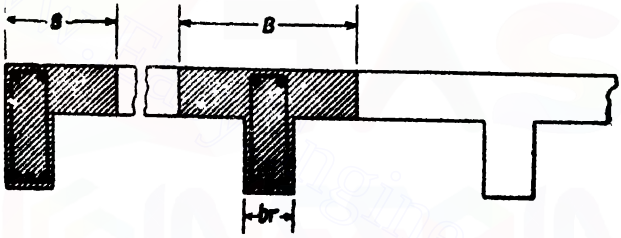

Fig. 53

§74. Effective Width of Flange of a T-beam

A certain portion of the slab on either side of the beam can be considered as forming the compression flange. The width B (*see* Fig. 53) of the flange which can be considered as acting effectively with the rib depends upon the span of the T-beam, the breadth of the rib, the overall thickness of the flange.

If the supporting beam happens to be an end beam, the flange of the beam is present only on one side of the beam and the beam in such a case is called an L-beam.

The width of flange may be taken as the following, but in no case greater than the breadth of the web plus half the sum of the clear distances to the adjacent beams on either side :

(*a*) For T-beams $B = \frac{l_o}{6} + b_r + 6d_s$

(*b*) For L beams, $B = \frac{l_o}{12} + b_r + 3d_s$

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(*c*) For isolated beams, the effective flange width shall be obtained as below, but in no case greater than the actual width

T-beam, $$B = \frac{l_o}{\frac{l_o}{b} + 4} + b_r$$

L-beam, $$B = \frac{0{\cdot}5\, l_o}{\frac{l_o}{b} + 4} + b_r$$

where, B = Effective width of flange.

l_o = Distance between points of zero moments in the beam.

b_r = Breadth of the rib (or also called web).

d_s = Thickness of flange.

b = Actual width of the flange.

NOTE—For continuous beams and frames, l_o may be assumed as 0·7 times the effective span.

For a T-beam or L-beam action the following conditions shall be satisfied.

(*a*) The slab shall be cast integrally with web, or the web and

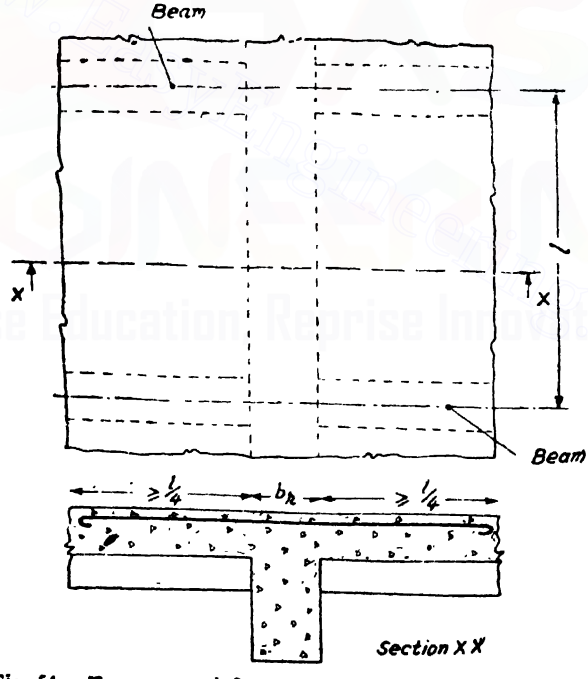

Fig. 54. Transverse reinforcement in flange of T-beam when main reinforcement of slab is parallel to the beam.

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the slab shall be effectively bonded together in any other manner ; and

(*b*) If the main reinforcement of the slab is parallel to the beam transverse reinforcement shall be provided as in Fig. 54; such reinforcement shall not be less than 60 percent of the main reinforcement at mid span of the slab.

§75. Control of Deflection—Span Depth Ratio

The deflection of a beam shall not adversely affect the appearance or efficiency of the beam or finishes or partitions. The deflection shall generally be limited to the following values as per the I.S. code.

(*a*) The final deflection due to all loads including the effects of temperature, creep and shrinkage and measured from the as-cast level of the supports of floors, roofs and all other horizontal members, should not normally exceed span/250.

(*b*) The deflection including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes should not normally exceed span/350 or 20 *mm.* whichever is less.

For beams and slabs, the vertical deflection limits may generally be assumed to be satisfied provided that the span to depth ratios are not greater than the values obtained as below :

(*a*) Basic values of span to effective depth ratios for spans up to 10 *metres* :

Cantilever	7
Simply supported	20
Continuous	26

(*b*) For spans above 10 metres, the values in (*a*) may be multiplied by $\left[\frac{10}{\text{span in } metres}\right]$, except for cantilever in which case deflection calculations should be made.

§76. Width of Rib

This shall be such as to accommodate the necessary tensile reinforcement. The width shall also be such as to prevent lateral instability. This width is generally between $\frac{1}{3}$ and $\frac{2}{3}$ of the depth of the rib. More often architectural requirements fix the width which shall be the same as the width of the supporting column.

§77. Neutral Axis of a T-Beam

The depth of the neutral axis can be determined by equating moments of areas on either side of the neutral axis. Three cases arise, *viz.*

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(*i*) The neutral axis may be situated within the flange.

(*ii*) The neutral axis may be just at the bottom edge of the slab.

(*iii*) The neutral axis may be below the slab.

Case (*i*). **Neutral axis within the flange.** (Fig. 55).

Such a beam will behave as a rectangular beam of width B.

Taking moments about the N.A.

$$\frac{Bn^2}{2} = mA_s(d-n) \quad ...(1)$$

Fig. 55

Case (*ii*). In this case $n = d_s$ = thickness of the slab.

Hence eq. (1) still holds good.

Case (*iii*). **Neutral axis below the slab** (Fig. 56).

This is the usual case. In this case, taking moments about N.A. we have,

$$Bd_s\left(n-\frac{d}{2}\right)+\frac{b_r(n-d_s)^2}{2} = mA_s(d-n) \quad ...(2)$$

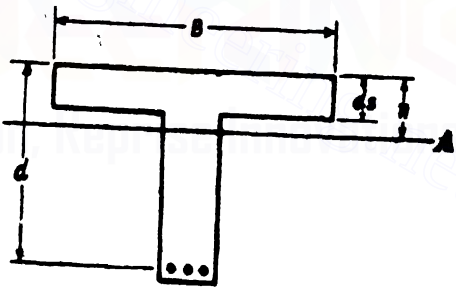

Fig. 56

In these computations it is usual to ignore the compressive force in the rib of the beam.

Hence, it is usual to discard the term b_r in eq. (2) and hence practically, we have

$$Bd_s\left(n-\frac{d_s}{2}\right) = m\,A_s(d-n) \quad ...(3)$$

It is to be noted that the above amendment on equation (2) is made only to simplify the calculations. The compression area of the T-beam has thus been taken as Bd_s.

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§78. Lever Arm of the T-Beam

This is the distance between the line of action of the resultant compression to the line of action of the resultant tension.

Let the maximum compressive stress in concrete be c *kg./cm*^{2}. Let the compressive stress in concrete at the bottom edge of the flange be c' *kg./cm.*^{2}

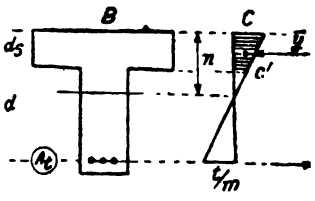

Fig. 57

The line of action of the resultant compression is situated at a depth y from the compression edge. y is the depth of centroid of the pressure trapezium corresponding to the flange (Fig. 57).

$$\bar{y} = \frac{c+2c'}{c+c'}\frac{d_s}{3} \qquad \ldots(4)$$

$\therefore$ Lever arm $= a = d - y$

But $c' = \frac{n-d_s}{n}c$

Substituting in eq. (4) we have,

$$y = \frac{c+\frac{2(n-d_s)}{n}c}{c+\frac{n-d_s}{n}c}\frac{d_s}{3}$$

$$y = \frac{3n-2d_s}{2n-d_s}\cdot\frac{d_s}{3} \qquad \ldots(5)$$

Eq. (5) may be found to be more convenient than eq. (4).

§79. Moment of Resistance of the Beam

M.R. = Total compression or total tension × lever arm

$\therefore$ $\quad M.R. = Bd_s \frac{c+c'}{2}(d-y) \qquad \ldots(6)$

Substituting for c'

$$M.R. = Bd_s \frac{c+\frac{n-d_s}{n}c}{2} \times (d-y)$$

$$M.R. = \frac{\left(n-\frac{d_s}{2}\right)}{2} \times c\, B\, d_s (d-y) \qquad \ldots(7)$$

M.R. is also equal to $A_t \cdot t\,(d-y)$. $\qquad \ldots(8)$

Problem 30 (S.I.). *An R.C. T-beam with a flange width of 102 cm. has a tension steel area of 22·8 cm*^{2}*. Taking the modular*

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ratio as 18 and the permissible stresses in concrete and steel as 5 N/mm.² and 140 N/mm.², determine the moment of resistance of the beam. The thickness of the flange is 13 cm. and the effective depth of the beam is 51 cm.

Solution.

$B = 102$ *cm.*,
$d = 51$ *cm.*,
$A_t = 22{\cdot}8$ *cm.*²,
$m = 18$

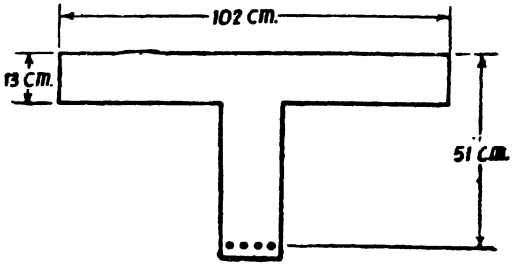

Fig. 58

Assuming the neutral axis to lie below the flange and taking moments about the neutral axis, we have

$$102 \times 13(n - 6{\cdot}5) = 18 \times 22{\cdot}8(51 - n)$$

$\therefore$ $n = 17$ *cm.* But thickness of slab $= 13$ *cm.*

Since the depth of neutral axis calculated is greater than the thickness of the flange, the assumption made above about the position of neutral axis is correct.

Depth of critical neutral axis is given by

$$\frac{18 \times 500}{14000} = \frac{n_c}{51 - n_c} \qquad \therefore \quad n_c = 19{\cdot}9 \text{ cm.}$$

Since $n < n_c$ steel attains its maximum stress earlier to concrete.

Distance of the line of action of the resultant compression in concrete from the top edge $= y = \frac{3n - 2d_c}{2n - d_c} \cdot \frac{d_c}{3}$

$$\frac{3 \times 17 - 2 \times 13}{2 \times 17 - 13} \times \frac{13}{3} = 5{\cdot}16 \text{ cm.}$$

Alternatively $y = \frac{c + 2c'}{c + c'} \cdot \frac{d_c}{3}$

But $c' = n - d_c$ $\quad c = \left[\frac{17 - 13}{17}\right] c = \frac{4}{17} c = 0{\cdot}235\, c$

$\therefore$ $y = \frac{c + 2(0{\cdot}235c)}{c + (0{\cdot}235c)} \cdot \frac{13}{3}$ *cm.* $= 5{\cdot}16$ *cm.*

$\therefore$ Lever arm $= a = d - y$

$= 51 - 5{\cdot}16 = 45{\cdot}84$ *cm.*

$\therefore$ $M.R. = A_t\, t(d - y)$

$= 22{\cdot}8 \times 14000 \times 45{\cdot}84$ *N cm.*

$= 14632000$ *N cm.*

$= 146320{\cdot}00$ *N m.*

$= 146{\cdot}30$ *KNm.*

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Problem 31. (M.K.S.). *An R.C. T-beam with a flange width of 102 cm. has a tension steel of area 22·8 cm². Taking the modular ratio as 18 and the permissible stresses in concrete and steel as 50 kg./cm.² and 1400 kg./cm.², determine the moment of resistance of the beam. The thickness of the flange is 13 cm. and the effective depth of the beam is 51 cm.*

Solution.

$B = 102$ *cm.* ; $d = 51$ *cm.* ; $d_s = 13$ *cm.* ; $A_t = 22{\cdot}8$ *cm.*² ; $m = 18$.

Taking moments about the neutral axis and assuming it to be situated below the slab, we have,

$$102 \times 13(n - 6{\cdot}5) = 18 \times 22\,8(51 - n)$$

$$\therefore \qquad n = 17 \text{ cm.}$$

Since this value of n is greater than the thickness of the slab our assumption that the neutral axis is below the slab is correct.

Depth of critical neutral axis is given by

$$\frac{18 \times 50}{1400} = \frac{n_c}{51 - n_c} \quad \therefore \quad n_c = 19{\cdot}9 \text{ cm.}$$

Since $n < n_c$ steel attains the maximum stress earlier.

Depth of line of action of resultant compression in concrete from the top edge

$$= y = \frac{3n - 2d_s}{2n - d_s} \cdot \frac{d_s}{3}$$

$$\frac{3 \times 17 - 2 \times 13}{2 \times 17 - 13} \cdot \frac{13}{3} = 5{\cdot}16 \text{ cm.}$$

or alternately,

$$y = \frac{c + 2c'}{c + c'} \cdot \frac{d_s}{3}$$

But $$c' = \frac{n - d_s}{n} c = \frac{17 - 13}{17} c =$$

$$= 0{\cdot}235c$$

$$\therefore \qquad y = \frac{c + 2 \times 0{\cdot}235c}{c + 0{\cdot}235c} \cdot \frac{13}{3} \text{ cm.}$$

$$= \frac{1{\cdot}47}{1{\cdot}235} \times \frac{13}{3} \text{ cm.}$$

$$= 5{\cdot}16 \text{ cm.}$$

$$\therefore \text{ Lever arm} = a = d - y$$

$$= 51 - 5{\cdot}16 \text{ cm.}$$

$$= 45{\cdot}84 \text{ cm.}$$

$$\therefore \qquad M.R. = A_t\, t(d - y)$$

$$= 22{\cdot}8 \times 1400 \times 45{\cdot}84 \text{ kg. cm.}$$

$$= 1{,}463{,}200 \text{ kg. cm.}$$

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Problem 32. (S I.). *A T-beam has a flange width of 120 cm, the flange thickness being 10 cm. The reinforcement consists of 5 bars of 24 mm. diameter placed at an effective depth of 40 cm. If the stresses in concrete and steel shall not exceed 5 N/mm.² and 140 N/mm.² respectively, find the moment of resistance of the beam section. Modular ratio = 18.*

Solution. $B = 120$ cm., $d_s = 10$ cm.

$A_t = 22{\cdot}62$ cm.², $d = 40$ cm.

Assuming the neutral axis to lie below the flange, taking moments about the neutral axis, we have,

$$120 \times 10(n - 5) = 18 \times 22{\cdot}62(40 - n)$$

$\therefore$ $n = 13{\cdot}87$ cm.

But $d_s = 10$ cm.

$\therefore$ $n > d_s$

$\therefore$ Our assumption about the position of neutral axis is correct.

Depth of critical neutral axis

$c = 5$ N/mm.$^2 = 50$ N/cm.^{2}

$t = 140$ N/mm.$^2 = 14000$ N/cm.^{2}

$$\therefore \quad \frac{18 \times 500}{14000} = \frac{n_c}{40 - n_c} \qquad \therefore \quad n_c = 15{\cdot}65 \text{ cm.}$$

Since $n < n_c$, steel attains its maximum stress earlier to concrete

$$y = \frac{3n - 2d_s}{2n - d_s} \cdot \frac{d_s}{3}$$

$$= \frac{3 \times 13{\cdot}87 - 2 \times 10}{2 \times 13{\cdot}87 - 10} \times \frac{10}{3} = 4{\cdot}06 \text{ cm.}$$

$\therefore$ Lever arm $= a = d - y$

$= 40 - 4{\cdot}06 = 35{\cdot}94$ cm.

$\therefore$ $M.R. = A_t f(d - y)$

$= 22{\cdot}62 \times 14{,}000 \times 35{\cdot}94$ N cm.

$= 11381500$ N cm.

$= 113815$ N m.

$= 113{\cdot}82$ KNm.

Problem 33. (M.K.S.). *The breadth and thickness of the flange of a T-beam are 170 cm. and 15 cm respectively. Steel reinforcement of area 45·24 cm.² is provided at an effective depth of 70 cm. If the modular ratio equals 18, find the stresses in concrete and steel if the beam section is subjected to a bending moment of 32000 kg m.*

Solution. Assuming the neutral axis to be below the flange, and taking moments about the neutral axis, we have,

$$170 \times 15(n - 7{\cdot}5) = 18 \times 45{\cdot}24(70 - n)$$

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$$= 22{\cdot}63 \text{ cm.}$$

$$= \frac{3n - 2d_s}{2n - d_s} \cdot \frac{d_s}{3}$$

$$\frac{3 \times 22{\cdot}63 - 2 \times 15}{2 \times 22{\cdot}63 - 15} \times \frac{15}{3} \text{ cm.} = 6{\cdot}26 \text{ cm.}$$

Lever arm $= a = d - y = 70 - 6{\cdot}26 = 63{\cdot}74$ cm.

$$M.R. = A_s t(d - y) = 32000 \times 100$$

$$45{\cdot}24 \times t \times 63{\cdot}74 = 32000 \times 100$$

$$t = \frac{32000 \times 100}{45{\cdot}24 \times 63{\cdot}74} \text{ kg./cm.}^2$$

$$= 1{,}110 \text{ kg./cm}^2$$

Stress in concrete is given by

$$\frac{18 \times c}{1110} = \frac{22{\cdot}63}{70 - 22{\cdot}63}$$

$$\therefore \quad c = 29{\cdot}45 \text{ kg./cm.}^2$$

Problem 34. (S.I.). *The breadth and thickness of the flange of a T-beam are 170 cm. and 15 cm. respectively. Steel reinforcement of area 45·24 cm.² is provided at an effective depth of 70 cm. If the modular ratio equals 18 find the stresses in concrete and steel if the moment of resistance of the beam section is 320 K Nm.*

Solution. Assuming the *N.A.* to be below the flange, taking moments about the *N.A.* we have,

$$170 \times 15(n - 7{\cdot}5) = 18 \times 45{\cdot}24\,(70 - n)$$

$$\therefore \quad n = 22{\cdot}63 \text{ cm.}$$

$$y = \frac{3n - 2d_s}{2n - d_s} \cdot \frac{d_s}{3}$$

$$= \frac{3 \times 22{\cdot}63 - 2 \times 15}{2 \times 22{\cdot}63 - 15} \cdot \frac{15}{3} \text{ cm.}$$

$$= 6{\cdot}26 \text{ cm.}$$

Lever arm $= (d - y) = 70 - 6{\cdot}26 = 63{\cdot}74$ cm.

$$M.R. = A_s t(d - y) = 320 \times 1000 \times 100 \text{ N cm.}$$

$$\therefore \quad = 45{\cdot}24 \times t \times 63\,74$$

$$= 320 \times 1000 \times 100 \text{ N cm.}$$

$$\therefore \quad t = \frac{320 \times 1000 \times 100}{45{\cdot}24 \times 63{\cdot}74} \text{ N/cm.}^2$$

$$= 11{,}100 \text{ N/cm.}^2 = 111 \text{ N/mm.}^2$$

Stress in concrete is given by

$$\frac{18 \times c}{11100} = \frac{22{\cdot}63}{70 - 22{\cdot}63}$$

$$c = 294{\cdot}5 \text{ N/cm.}^2 = 2{\cdot}945$$

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Permissible Shear Stress in Concrete q_c (S.I. Units)

$\frac{100\ A_t}{bd}$	Permissible nominal shear stress in concrete q_c in N/mm.^{2} for various grades of concrete					
	M 15	M 20	M 25	M 30	M 35	M 40
0·20	0·20	0·20	0·21	0·21	0·21	0·21
0·30	0·24	0·24	0·25	0·25	0·25	0·25
0·40	0 27	0·27	0·28	0·28	0·29	0·2
0·50	0·29	0·30	0·31	0·31	0·31	0·32
0 60	0 31	0·32	0·33	0·33	0·34	0·34
0.70	0·33	0·34	0·35	0·36	0·36	0·37
0·80	0·34	0·36	0·37	0·38	0·38	0·39
0 90	0 36	0·37	0·39	0·39	0·40	0·41
1.00	0·37	0·39	0·40	0·41	0·42	0·42
1·10	0·38	0·40	0·42	0·43	0·43	0·44
1·20	0·40	0·41	0·43	0·44	0·45	0·45
1·30	0 41	0·43	0·44	0 45	0·46	0·47
1·40	0·42	0· 4	0·45	0·46	0·47	0·48
1·50	0·42	0·45	0·46	0·48	0·49	0·49
1·60	0·43	0·46	0·47	0·49	0·50	0·51
1·70	0·44	0·47	0·48	0·50	0·51	0·52
1·80	0 44	0·47	0·49	0·51	0·52	0·53
1·90	0·44	0 48	0·50	0·52	0·53	0·54
2 00	0·44	0·49	0·51	0·53	0·54	0·55
2·10	0·44	0·50	0·52	0·54	0·55	0·56
2·20	0·44	0·51	0·53	0·54	0·56	0·57
2 30	0·44	0·51	0·53	0·55	0·57	0·58
2·40	0·44	0 51	0·54	0·56	0·57	0·59
2·50	0·44	0·51	0·55	0·57	0·58	0·60
2·60	0·44	0·51	0·56	0·57	0·59	0·60
2·70	0·44	0·51	0·56	0·58	0·60	0·61
2·80	0·44	0·51	0·57	0·59	0·60	0·62
2·90	0·44	0·51	0·57	0·59	0·61	0·63
3·00 and above	0·44	0·51	0·57	0·60	0·62	0·63

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Shear Stress in Concrete q_c (M.K.S. Units)

$\frac{100\ A_t}{bd}$	Permissible nominal shear stress in concrete q_c in kg./cm.^{2} for various grades of concrete					
	M 150	M 200	M 250	M 300	M 350	M 400
0·20	2·0	2·0	2·1	2·1	2·1	2·1
0·30	2·4	2·4	2·5	2·5	2·5	2·5
0·40	2·7	2·7	2·8	2·8	2·9	2·9
0·50	2·9	3·0	3·1	3·1	3·1	3·2
0·60	3·1	3·2	3·3	3·3	3·4	3·4
0·70	3·3	3·4	3·5	3·6	3·6	3·7
0·80	3·4	3·6	3·7	3·8	3·8	3·9
0·90	3·6	3·7	3·9	3·9	4·0	4·1
1·00	3·7	3·9	4·0	4·1	4·2	4·2
1·10	3·8	4·0	4·2	4·3	4·3	4·4
1·20	4·0	4·1	4·3	4·4	4·5	4·5
1·30	4·1	4·3	4·4	4·5	4·6	4·7
1·40	4·2	4·4	4·5	4·6	4·7	4·8
1·50	4·2	4·5	4·6	4·8	4·9	4·9
1·60	4·3	4·6	4·7	4·9	5·0	5·1
1·70	4·4	4·7	4·8	5·0	5·1	5·2
1·80	4·4	4·7	4·9	5·1	5·2	5·3
1·90	4·4	4·8	5·0	5·2	5·3	5·4
2·00	4·4	4·9	5·1	5·3	5·4	5·5
2·10	4·4	5·0	5·2	5·4	5·5	5·6
2·20	4·4	5·1	5·3	5·4	5·6	5·7
2·30	4·4	5·1	5·3	5·5	5·7	5·8
2·40	4·4	5·1	5·4	5·6	5·7	5·9
2·50	4·4	5·1	5·5	5·7	5·8	6·0
2·60	4·4	5·1	5·6	5·7	5·9	6·0
2·70	4·4	5·1	5·6	5·8	6·0	6·1
2·80	4·4	5·1	5·7	5·9	6·0	6·2
2·90	4·4	5·1	5·7	5·9	6·1	6·3
3·00 and above	4·4	5·1	5·7	6·0	6·2	6·3

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§80. Shear Stress in T-beams

The shear stress in concrete for a T-beam is given by

$$q = \frac{S}{ab_r}$$

where, S = maximum shear force at the section

b_r = width of the rib

The I.S. code has introduced the concept of nominal shear stress q_v given by

$$q_v = \frac{S}{b_r d}$$

where, d = effective depth of the beam.

The permissible shear stress in concrete in beams without shear reinforcement is given in thetables on pages 138-139 In these tables, A_t is the area of longitudinal tension reinforcement which continues one effective depth beyond the section being considered except at supports where the full area of tension reinforcement may be used provided the reinforcement is adequately anchored.

Provision of shear reinforcement.

(*i*) When the nominal shear stress q_v is less than q_c given in the table a minimum shear reinforcement shall be provided such that,

$$\frac{A_w}{bp} \geq \frac{0.4}{f_v}$$

where f_v = Characteristic strength of the stirrup reinforcement in $N/mm.^2$ which shall not be taken greater than 415 $N/mm.^2$

In M.K.S. designs

$$\frac{A_w}{bp} \geq \frac{4}{f_v}$$

where f_v = Characteristic strength of the stirrup reinforcement in $kg./cm^2$ which shall not be taken greater than 4150 kg/cm^2.

(*ii*) When the nominal shear stress q_v is greater than q_c and less than $q_{c\,max}$ given below, shear reinforcement should be provided for resisting a shear equal to $V_s (S - q_c bd)$. The shear reinforcement is provided either by stirrups or by bent-up bars.

(*iii*) When the nominal shear stress q_v exceeds $q_{c\,max}$ given below the section should be redesigned.

$$\text{Spacing of stirrups} = \frac{A_w f_v d}{V_s}$$

Spacing of stirrups shall not exceed (*i*) 0·75 d and (*ii*) 45 cm.

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Maximum Shear Stress ($q_{c\,max}$) (S I. Units)

Concrete grade	*M 15*	*M 20*	*M 25*	*M 30*	*M 35*	*M 40*
$q_{c\,max}$ *(kg./mm.²)*	1·6	1·8	1·9	2·2	2 3	2·5

Maximum Shear Stress ($q_{c\,max}$) (M.K.S. Units)

Concrete grade	*M 150*	*M 200*	*M 250*	*M 300*	*M 350*	*M 400*
$q_{c\,max}$ *(kg./cm.²)*	16	18	19	22	23	25

Problem 35. (M.K.S.) *A T-beam of overall depth 55 cm. has a rib width of 30 cm. Find the shear reinforcement required for the beam section, for a maximum shear force of 12000 kg. Concrete used is M 150. Permissible tensile stress for the stirrups is 1400 kg./cm². Percentage of tensile steel provided is 0·8%.*

Solution. Assuming 20 *mm.* diameter bars at a clear cover of 25 *mm.*

Effective cover $= 25 + 10 = 35$ *mm.* $= 3·5$ *cm.*

Effective depth $= d = 55 - 3·5 = 51·5$ *cm.*

Nominal shear stress in concrete

$$q_v = \frac{S}{bd} = \frac{12000}{30 \times 51·5} = 7·77 \text{ kg./cm.}^2$$

But, for M 150 concrete, $q_{c\,max} = 16$ *kg./cm.*²

$\therefore \quad q_v < q_{c\,max}$.

Corresponding to 0·8% steel, for M 150 concrete,

$q_c = 3·4$ *kg./cm.*²

$\therefore$ Shear capacity of concrete $= q_c bd$

$= 3·4 \times 30 \times 51·5 = 5253$ *kg.*

$\therefore$ Shear force to be resisted by the stirrups

$= V_s = 12000 - 5253 = 6747$ *kg.*

$t_w = 1400$ *kg./cm.*²

Spacing of two-legged 8 *mm.* diameter stirrups

$$= p = \frac{A_w t_w d}{V_s} = \frac{(2 \times 0·5) 1400 \times 51·5}{6747} = 10·7 \text{ cm.}$$

Provide 8 mm. ϕ two-legged stirrups at 10 cm. c/c.

Problem 36. (S.I.). *A T-beam of overall depth 55 cm. has a rib width of 30 cm. Find the shear reinforcement required for the beam section for a maximum shear force of 120 KN. Concrete used is M 15.*

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Permissible tensile stress for the stirrups is 140 N/mm.² *Percentage of tensile steel provided = 0·8%.*

Solution. Assuming 20 *mm.* diameter bars at a clear cover of 25 *mm*

Effective cover = 25 + 10 = 35 *mm.* = 3·5 *cm.*

Effective depth = $d = 55 - 3{\cdot}5 = 51{\cdot}5$ *cm.*

Nominal shear stress in concrete

$$= q_v = \frac{S}{bd} = \frac{120{,}000}{30 \times 51{\cdot}5} = 77{\cdot}7 \ N/cm.^2 = 0{\cdot}78 \ N/mm.^2$$

But, for M 15 concrete, $q_{c\ max} = 1{\cdot}6 \ N/mm.^2$

$\therefore \quad q_v < q_{c\ max}.$

Corresponding to 0·8% steel, for M 15 concrete,

$$q_c = 0{\cdot}34 \ N/mm.^2 = 34 \ N/cm.^2$$

$\therefore$ Shear capacity of concrete

$$= q_c b \cdot d = 34 \times 30 \times 51{\cdot}5 = 52530 \ N$$

$\therefore$ Shear force to be resisted by the stirrups

$$= V_s = 120{,}000 - 52{,}530 = 67{,}470 \ N$$

$$t_w = 140 \ N/mm^2 = 14000 \ N/cm.^2$$

Spacing of two-legged 8 *mm.* diameter stirrups

$$= p = \frac{A_w t_w d}{V_s} = \frac{(2 \times 0{\cdot}5) 14000 \times 51{\cdot}5}{67470} = 10{\cdot}7 \ cm.$$

Provide 8 mm. ϕ two-legged stirrups at 10 cm. c/c.

§81. Design of T-beams

The following steps may be followed in the design of T-beams :

1. Fix the overall dimensions of T-beam. The depth may be taken as $\frac{1}{12}$ to $\frac{1}{20}$ of the span. Width of the beam may be taken as $\frac{1}{3}$ to $\frac{2}{3}$ of the depth of rib.

2. Estimate the maximum bending moment due to the given loading and self-weight.

3. Approximately take the lever arm as $d - \frac{d_s}{2}$ where d = effective depth and d_s = overall thickness of slab.

4. Calculate the approximate steel requirement from the relation

$$A_t = \frac{M}{t \times a}$$

t = safe stress in steel say 14000 $N/cm.^2$ or 1400 $kg./cm.^2$

5. Provide now sufficient number of bars of a chosen diameter so that the area of reinforcement provided is nearly equal to what has been estimated in the previous step.

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6. The section being fully known determine the exact position of the neutral axis and the correct lever arm.

7. Now determine the stresses in concrete and steel with the known value of the actual lever arm.

Stress in steel $= \frac{M}{a \cdot A_t}$ where a = actual lever arm

and A_t = actual area of reinforcement provided.

Stress in concrete $\frac{t}{m} \cdot \frac{n}{d-n}$

If the stresses calculated in step 7 are within the allowable limits, the design is safe, otherwise the depth or the area of reinforcement can be increased and the steps 4 to 7 are repeated.

At least one-third of the positive reinforcement should be extended into the support, for a length equal to $\frac{L_d}{3}$, where L_d is the development length.

8. The design is now checked for shear stress.

Nominal shear stress at the support

$= q_v = \frac{S}{b_r d}$ where S = Maximum shear force based on clear span.

This is compared with the permissible shear stress q_c. If $q_v > q_c$ the shear reinforcement is calculated as mentioned earlier.

q_v shall not exceed $q_{c\,max}$ specified in the tables given earlier. If q_v exceeds $q_{c\,max}$ the section should be redesigned.

Problem 37. (S.I.). *The floor of a hall 12 metres by 6 metres to the centres of the supporting walls, consists of 3 beams spaced at 3 metres apart, the thickness of the slab being 13 cm. Design an intermediate beam. Allow a live load of 3500 N/metre². The dead load of the floor finish may be taken as 500 N/metre². Use M 15 concrete.*

Solution. Depth of the beam $= \frac{1}{15}$ span $= \frac{1}{15} \times 6 \times 100 = 40$ *cm.*

Depth of the rib $= 40 - 13 = 27$ *cm.*

Breadth of rib $= \frac{1}{2}$ to $\frac{2}{3}$ of the depth of the rib.

It is also convenient to choose a width from the point of view of accommodating a certain amount of reinforcement in one layer. Suppose we make a provision for 4 bars of 25 *mm.* diameter in one layer, width of rib required

$=$ 4 diameters + 3 spacings + 2 side covers

$= (4 \times 2{\cdot}5) + (3 \times 2{\cdot}5) + (2 \times 2{\cdot}5)$ *cm.* $= 10 + 7{\cdot}5 + 5 = 22{\cdot}5$ *cm.*

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Let us provide a width of 25 *cm.*

Suppose we provide the bars in two layers.

Clear space between the two layers = 15 *mm.*

∴ Effective cover to the centre of the two layers of bars

$= 25 + 25 + 7{\cdot}5 = 57{\cdot}5$ *mm.* $= 5{\cdot}75$ *cm.*

∴ Actual effective depth $= 40 - 5{\cdot}75 = 34{\cdot}25$ *cm.*

Assuming a bearing of 30 *cm.* at each end,

Clear span $= 6 - 0{\cdot}30 = 5{\cdot}70$ *m.*

Effective span. This is taken as the lesser of the following :

(*i*) Centre to centre of bearings = 6 *m*

(*ii*) Clear span + effective depth $= 5{\cdot}70 + 0{\cdot}3425 = 6{\cdot}0425$ *m.*

∴ Effective span = 6 *m.*

Flange width. This is taken equal to

$$\frac{l}{6} + b_r + 6\,d_s = \frac{600}{6} + 25 + 6 \times 13 = 203 \text{ cm.}$$

Approximate lever arm $= a = d - \frac{d_s}{2} = 34{\cdot}25 - 6{\cdot}50 = 27{\cdot}75$ *cm.*

Loads.

D.L. of the slab $= 250 \times 13$	3250 *N/metre*^{2}
Floor finish	500 *N/metre*^{2}
Live load	3500 *N/metre*^{2}
Total	7250 *N/metre*^{2}

Load transferred to the beam from the slab :

$7250 \times 3 = 21750$ *N/m*

D.L. of the rib $= 0{\cdot}25 \times 0{\cdot}27 \times 25000 =$ 1690 *N/m*

Total load on the beam $= 23440$ *N/m*

Maximum bending moment $= 23440 \times \frac{6^2}{8} \times 100$ *N cm.*

$= 10548000$ *N cm.*

Approximate steel required $= \frac{10548000}{14000 \times 27{\cdot}75} = 27{\cdot}15$ *cm.*^{2}

Provide 6 bars of 25 mm. diameter (29·45 *cm.*^{2})

These 6 bars can be arranged in two layers with three bars per layer.

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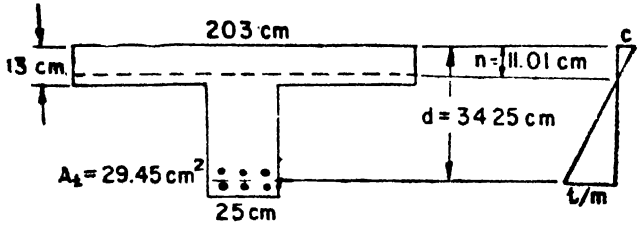

Fig. 59

Actual depth of neutral axis. Assuming the neutral axis to lie below the slab, and taking moments about the neutral axis, we have,

$$203 \times 13(n-6{\cdot}5) = 18 \times 29{\cdot}45(34{\cdot}25-n)$$

$$\therefore \qquad n = 11{\cdot}12 \text{ cm., this is less than 13 cm.}$$

Hence, the neutral axis lies within the slab.

Taking moments about the neutral axis,

$$203\,\frac{n^2}{2} = 18 \times 29{\cdot}45(34{\cdot}25-n)$$

$$\therefore \quad n^2 + 5{\cdot}22\,n - 178{\cdot}88 = 0$$

Solving we get $n = 11{\cdot}01$ *cm.*

$$\therefore \text{ Correct lever arm} = a = d - \frac{n}{3} = 34{\cdot}25 - \frac{11{\cdot}01}{3} = 30{\cdot}58 \text{ cm.}$$

$$\text{Actual stress in steel} = t = \frac{10548000}{29{\cdot}45 \times 30{\cdot}58} = 11710 \text{ N/cm.}^2$$

$$= 117 \text{ N/mm.}^2 \text{ (less than 140 N/mm.}^2\text{)}$$

Corresponding maximum stress in concrete

$$= c = \frac{11710}{18} \cdot \frac{11{\cdot}01}{34{\cdot}25-11{\cdot}01} = 308 \text{ N/cm.}^2$$

$$= 3{\cdot}08 \text{ N/mm.}^2 \text{ (less than 5 N/mm.}^2\text{)}$$

Design for shear.

Clear span $= 6 - 0{\cdot}30 = 5{\cdot}70$ *metre*

$$\text{Maximum S.F.} = S = \frac{23440 \times 5{\cdot}7}{2} = 66804 \text{ N}$$

Nominal shear stress at the support

$$= q_v = \frac{66804}{b\,d} = \frac{66804}{25 \times 34{\cdot}25} = 78 \text{ N/cm.}^2 = 0{\cdot}78 \text{ N/mm.}^2$$

For M 15 concrete, $q_{c\,max} = 1{\cdot}60$ *N/cm.*^{2}

$$\therefore \qquad q_v < q_{c\,max}$$

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Assuming that 50% of the longitudinal bars are curtailed, percentage of tensile steel at support

$$=\left(\frac{29{\cdot}45}{2}\right)\frac{1}{25\times 34{\cdot}25}\times 100=1{\cdot}7\%$$

Corresponding to 1·7% steel, for M 15 concrete,

$q_c=0{\cdot}44\ N/mm.^2=44\ N/cm.^2$

$\therefore$ Shear capacity of concrete

$=q_c bd=44\times 25\times 34{\cdot}25\ N=37675\ N$

$\therefore$ Shear force to be resisted by shear reinforcement

$=V_s=66804-37675=29129\ N$

Shear reinforcement in the form of stirrups

Spacing of 8 *mm* diameter stirrups

$$=p=\frac{A_{sv}\sigma_{st}d}{V_s}=\frac{(2\times 0{\cdot}5)14000\times 34{\cdot}25}{29129}=16{\cdot}5\ cm.$$

Provide two-legged 8 mm ϕ stirrups @ 16 cm. c/c.

In the region where all the 6 bars are present,

Percentage of steel

$=2\times 1{\cdot}7=3{\cdot}4\%$ and $q_c=0{\cdot}44\ N/mm.^2=44\ N/cm.^2$

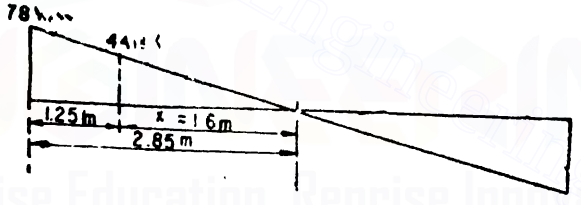

Fig. 60

Let the nominal shear stress be 44 $N/cm.^2$ at a distance x *metres* from the centre

$$\therefore\ \frac{x}{2{\cdot}85}=\frac{44}{78}\qquad\therefore\ x=1{\cdot}60\ m.$$

Hence shear reinforcement is needed for a distance of 2·85 − 1·60 = 1·25 *m* from each support. For the middle 3·20 *metres* length nominal two-legged 8 *mm.* diameter stirrups shall be used. Spacing of nominal stirrups shall be not greater than 0·75 *d*

that is, $\geq 0{\cdot}75\times 34{\cdot}25\ cm.$

that is, $\geq 25{\cdot}7\ cm.$

Provide 2-legged 8 mm. nominal stirrups @ 25 cm centres.

Curtailment of tensile reinforcement. B.M. at $\frac{1}{7}$ of the span is 50% of the maximum bending momemt. Hence half the number of

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bars can be curtailed at $\frac{1}{7}$ of the span *i.e.* at a distance of $\frac{6}{7}$ =0·86 *m.* or 86 *cm.* from the centre of bearing.

The I.S. code recommends that the bars shall extend beyond the point at which it is no longer required to resist flexure, for a distance equal to the effective depth of the beam or 12 times the bar diameter whichever is greater.

12 × dia. of bars = 12 × 2·5 = 30 *cm.*

Effective depth = 34·25 *cm.*

Let the 3 bars of the upper layer be curtailed at a distance of 86 − 35 = 51 *cm.* from the centre of bearing.

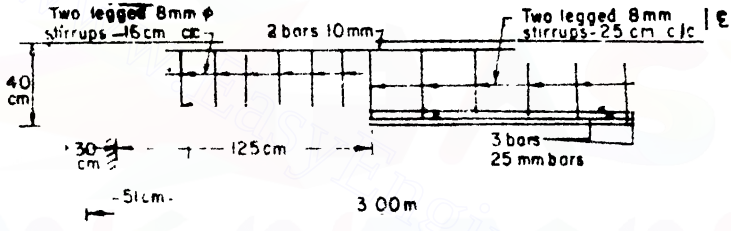

Fig. 61

Fig. 61 shows the details of reinforcement.

Shear reinforcement in the form of bent-up bars. Suppose the three bars of the upper layer be bent up at a distance 1·414*a* = 1·414 × 30·58 = 43 *cm.* from the edge of the support.

Shear value of the bent up bars = 0·707 $A_w f_w$

= 0·707 × (3 × 4·91) 14000

= 145800 *N*

But, the maximum S.F. is only 66804 *kg.*

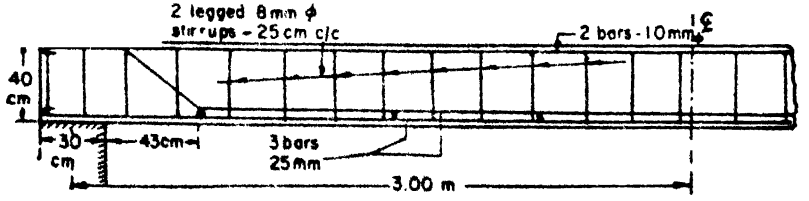

Fig. 62

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Hence the design is safe.

But by bending these bars at $1{\cdot}414\,a$ from the support, the beam becomes safe from shear consideration for a distance $1{\cdot}828\,a = 1{\cdot}828 \times 30{\cdot}58 = 55{\cdot}9$ *cm.* from the support.

S.F. at 55·9 *cm.* from the support

$$= \frac{2{\cdot}85 - 0{\cdot}559}{2{\cdot}85} \times 66804\ N = 53700\ N$$

Corresponding to 3·4% tensile steel,

$$q_c = 0{\cdot}44\ N/mm.^2 = 44\ N/cm.^2$$

$\therefore$ Shear capacity of concrete

$$= 44 \times 25 \times 34{\cdot}25\ N = 37675\ N$$

$\therefore$ Shear force to be resisted by stirrups

$$= V_s' = 53700 - 37675 = 16025\ N$$

$\therefore$ Spacing of two-legged 8 *mm.* ϕ stirrups

$$= \frac{A_w'\, t_w'\, d}{V_s'} = \frac{(2 \times 0{\cdot}5)1{\cdot}000 \times 34{\cdot}25}{16025}\ cm.$$

$$= 29\ cm.$$

Provide 2-legged 8 mm. ϕ stirrups @ 25 cm. c/c

Problem 38. *The floor of a hall 12 metres by 6 metres to the centres of the supporting walls, consists of 3 beams spaced at 3 metres apart, the thickness of the slab being 13 cm. Design an intermediate beam. Allow a live load of 350 kg./m². The dead load of the floor finish may be taken as 50 kg./metre². Wt. of RCC = 2500 kg./metre². Use M 150 concrete.*

Solution.

Depth of the beam = $\frac{1}{15}$ of span = $\frac{1}{15} \times 6 \times 100 = 40$ *cm.*

Depth of rib $= 40 - 13 = 27$ *cm.*

Breadth of rib : $\frac{1}{3}$ to $\frac{2}{3}$ of the depth of rib.

It is also convenient to choose a width from the point of view of accommodating a certain amount of reinforcement in one layer.

Suppose we make a provision for 4 bars of 25 *mm.* in one layer, width of rib required

$= 4$ diameters $+ 3$ spacings $+ 2$ side covers

$= (4 \times 2{\cdot}5) + (3 \times 2{\cdot}5) + (2 \times 2{\cdot}5) = 10 + 7{\cdot}5 + 5 = 22{\cdot}5$ *cm.*

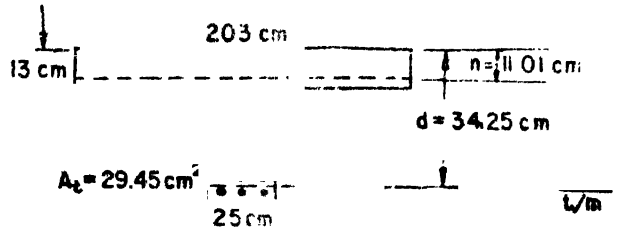

Fig. 63

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Let us provide a width of 25 *cm.*

Suppose we provide the bars in two layers.

Clear space between the two layers = 15 *mm.*
Clear cover to the bottom layer of bars = 25 *mm.*

∴ Effective cover to the centre of the two layers of bars
$= 25 + 25 + 7{\cdot}5 = 57{\cdot}5$ *mm.* $= 5{\cdot}75$ *cm.*

Actual effective depth
$= 40 - 5{\cdot}75 = 34{\cdot}25$ *cm.*

Assuming a bearing of 30 *cm.* at each end,
Clear span $= 6 - 0{\cdot}30 = 5{\cdot}70$ *m.*

Effective span : This is taken as the lesser of the following :

(*i*) Centre to centre of bearings
$= 6$ *m.*

(*ii*) Clear span + effective depth
$= 5{\cdot}70 + 0{\cdot}3425 = 6{\cdot}0425$ *m.*

∴ Effective span $= 6$ *m.*

Flange width : This is taken equal to $\frac{l}{6} + b_w + 6d_s$

$$= \frac{600}{6} + 25 + 6 \times 13 = 203 \text{ cm.}$$

Approximate lever arm

$$= a = d - \frac{d_s}{2} = 34{\cdot}25 - 6{\cdot}50 = 27{\cdot}75 \text{ cm.}$$

Loads

D.L. of the slab	$= 25 \times 13$	$= 325$ *kg./m.*^{2}
Floor finish		$= 50$ *kg./m.*^{2}
Live load		$= 350$ *kg./m.*^{2}
	Total	$= 725$ *kg./m.*^{2}

Load transferred to the beam from the slab,

	$= 725 \times 3$	$= 2{,}75$ *kg./m.*
D.L. of the rib	$= 0{\cdot}25 \times 0{\cdot}27 \times 2500$	$= 169$ *kg./m.*
	Total load on the beam	$= 2344$ *kg./m.*

Maximum bending moment

$$= 2344 \times \frac{6^2}{8} \times 100 \text{ kg. cm.}$$

$$= 1054800 \text{ kg. cm.}$$

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Approximate steel required

$$= \frac{1054800}{1400 \times 27{\cdot}75} = 27{\cdot}15 \text{ cm.}^2$$

Provide 6 bars of 25 mm. diameter (29 45 cm.²)

These 6 bars can be arranged in two layers with three bars per layer.

Actual depth of neutral axis. Taking moments about the neutral axis which is first assumed to be situated below the slab, we have,

$$203 \times 13\ (n - 6{\cdot}5) = 18 \times 29{\cdot}45\ (34\ 25 - n)$$

$\therefore$ $n = 11{\cdot}12$ *cm.* This is less than 13 *cm.*

Hence the neutral axis lies within the slab.

Taking moments about the neutral axis,

$$203\frac{n^2}{2} = 18 \times 29{\cdot}45(34{\cdot}25 - n)$$

$\therefore$ $n^2 + 5{\cdot}22\,n - 178\ 88 = 0$

Solving, we get $n = 11{\cdot}01$ *cm.*

$\therefore$ Correct lever arm $= a = d - \frac{n}{3} = 34{\cdot}25 - \frac{11.01}{3} = 30{\cdot}58$ *cm.*

$\therefore$ Actual stress in steel

$$= t = \frac{1054800}{29{\cdot}45 \times 30{\cdot}58} = 1171 \text{ kg /cm.}^2$$

Corresponding maximum stress in concrete

$$= c = \frac{1171}{18} \cdot \frac{11{\cdot}01}{34{\cdot}25 - 11{\cdot}01} = 30{\cdot}8 \text{ kg./cm.}^2$$

Design for shear

Clear span $= 6 - 0{\cdot}30 = 5{\cdot}70$ *m.*

Maximum S F. $= S = \frac{2344 \times 5{\cdot}7}{2} = 6680$ *kg.*

Nominal shear stress at the support

$$q_v = \frac{6680}{34{\cdot}25 \times 25} = 7{\cdot}80 \text{ kg./cm.}^2$$

For M 150 concrete

$$q_{c\ max} = 16 \text{ kg /cm.}^2$$

$\therefore$ $q_v < q_{c\ max}$

Assuming that 50% of the longitudinal bars are curtailed,

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Percentage of tensile steel at support

$$= \frac{\left(\frac{29{\cdot}45}{2}\right)}{25 \times 34{\cdot}25} \times 100 = 1{\cdot}7\%$$

Corresponding to 1·7%, steel for M 150 concrete,

$$q_c = 4{\cdot}4 \text{ kg./cm.}^2$$

Shear capacity of concrete

$$= q_c\, b_r\, d = 4{\cdot}4 \times 25 \times 34{\cdot}25 = 3767{\cdot}5 \text{ kg.}$$

$\therefore$ Shear force to be resisted by the stirrups

$$V_s = 6680 - 3767{\cdot}5 = 2912{\cdot}5 \text{ kg.}$$

Spacing of two-legged 8 *mm.* diameter stirrups

$$= p = \frac{A_w\, t_w\, d}{V_s} = \frac{(2 \times 0{\cdot}5)1400 \times 34{\cdot}25}{2912{\cdot}5}$$

$$= 16{\cdot}45 \text{ cm.}$$

Provide two-legged 8 mm. ϕ stirrups @ 16 cm. c/c.

In the region where all the 6 bars are present,

Percentage of steel $= 3{\cdot}4\%$

and $q_c = 4{\cdot}4 \text{ kg./cm.}^2$

Let the nominal shear stress be 4·4 *kg./cm.*^{2} at a distance x *metres* from the centre

$$\frac{x}{2{\cdot}85} = \frac{4{\cdot}4}{7{\cdot}8}$$

$$x = 1{\cdot}6 \text{ m.}$$

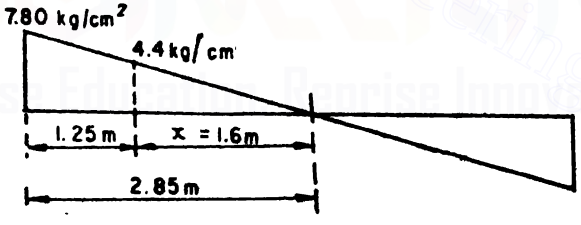

Fig. 64

Hence shear reinforcement is needed for a distance of 2·85 − 1·60 = 1·25 *m.* from each support. For the middle 3·20 *metres* length nominal two-legged 8 *mm.* diameter stirrups shall be used. Spacing of the nominal stirrups shall be not greater than 0·75 *d*

that is, $> 0{\cdot}75 \times 34{\cdot}25$ *cm.*

that is, $> 25{\cdot}7$ *cm.*

Provide the nominal stirrups at 25 cm. c/c.

Curtailment of tensile reinforcements

B.M. at $\frac{1}{7}$ of the span is 50% of the maximum bending moment.

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Hence half the number of bars can be curtailed at $\frac{1}{7}$ of the span *i.e.* at a distance of $\frac{6}{7} = 0{\cdot}86$ *m.* or 86 *cm.* from the centre of bearing.

The I.S. code recommends that the bars shall extend beyond the point at which it is no longer required to resist flexure, for a distance equal to the effective depth of the member or 12 times the bar diameter whichever is greater.

$12 \times$ dia of bars $= 12 \times 2{\cdot}5 = 30$ *cm.*

Effective depth $= 34{\cdot}25$ *cm.*

Let the 3 bars of the upper layer be curtailed at a distance of $86 - 35 = 51$ *cm.* from the centre of bearing.

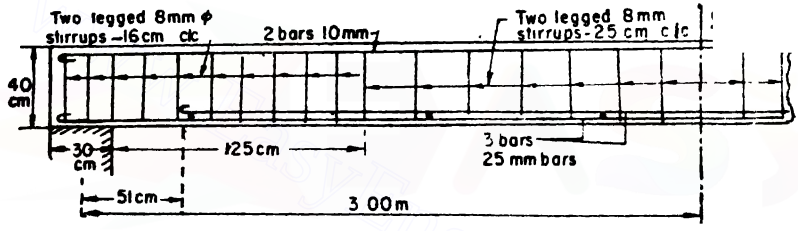

Fig. 65

Fig. 65 shows the details of reinforcement.

Shear reinforcement may also be provided by bent up bars.

Suppose the three bars of the upper layer be bent up at $1{\cdot}414\,a$

$= 1{\cdot}414 \times 30{\cdot}58 = 43$ *cm.* from the edge of support.

Shear value of bent-up bars

$= 0{\cdot}707\, A_w f_w = 0{\cdot}707 \times (3 \times 4{\cdot}91)\ 1400$

$= 14580$ *kg.*

But the maximum S.F. is only 6680 *kg.* Hence the design is safe.

But by bending these bars at $1{\cdot}414\,a$, the beam becomes safe from shear consideration for a distance of

$1{\cdot}828\,a = 1{\cdot}828 \times 30{\cdot}58 = 55{\cdot}9$ *cm.* from the support.

S.F. at 55·9 *cm.* from the support

$$= \frac{2{\cdot}85 - 0{\cdot}559}{2{\cdot}85} \times 6680 = 5370 \text{ kg.}$$

Corresponding to 3·4% of tensile steel,

$q_c = 4{\cdot}4$ *kg./cm.*^{2}

Shear capacity of concrete

$= 4{\cdot}4 \times 25 \times 34{\cdot}25 = 3767{\cdot}5$ *kg.*

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$\therefore$ Shear force to be resisted by stirrups

$$= V_s' = 5370 - 3767{\cdot}5 = 1602{\cdot}5 \text{ kg.}$$

Spacing of two-legged 8 *mm.* diameter stirrups

$$= \frac{(2 \times 0{\cdot}5) 1400 \times 34{\cdot}25}{1602{\cdot}5} = 29 \text{ cm.}$$

Provide 2-legged 8 mm. ϕ stirrups @ 25 cm. c/c

Fig. 66 shows the details of reinforcement.

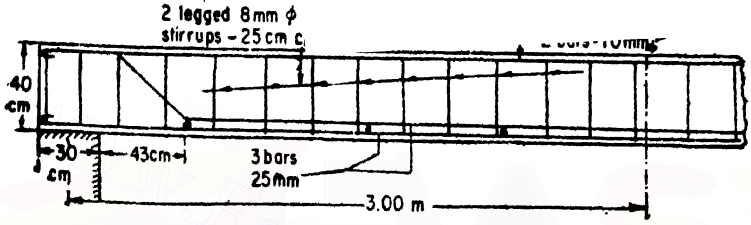

Fig. 66

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8

Axially Loaded Columns

A column forms a very important component of a structure. Columns support beams which in turn support walls and slabs. It should be realized that the failure of a column results in the collapse of the structure. The design of a column should therefore receive importance.

A column is defined as a compression member, the effective length of which exceeds three times the least lateral dimension compression members whose lengths do not exceed three times the least lateral dimension, may be made of plain concrete.

§82. R.C. Columns

Columns may be cast to any of the following shapes—square, circular, hexagonal, octagonal, etc Longitudinal reinforcement (or main steel) is provided to resist compressive loads along with concrete. As per I.S. 456 a reinforced concrete column shall have longitudinal steel reinforcement and the cross-sectional area of such reinforcement shall be not less than 0·8% nor more than 6% of the cross-sectional area of the column required to transmit all the loading. The object of stipulating a minimum percentage of steel is to make provision to prevent buckling of the column due to an accidental eccentricity of the load on it. The object of stipulating a maximum percentage of steel is to provide reinforcement within such a limit to avoid congestion of reinforcements which would make it very difficult to place the concrete and consolidate it. This may be best realized from the following two examples. Consider two columns 45 *cm.* × 45 *cm.* Reinforcement required at 0·8% of gross area.

$$= \frac{0{\cdot}8}{100} \times 45^2 = 16{\cdot}2 \ cm.^2$$

This may be provided by four bars of 24 *mm.* diameter with an area of 18·10 cm^2 [Fig. 67(*a*)]

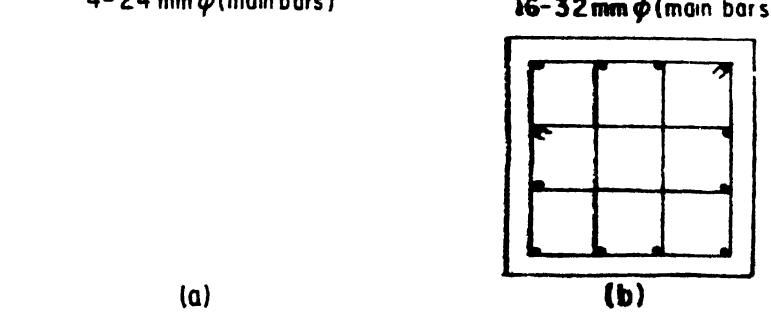

Fig. 67

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Reinforcement required at 6% of the gross area $= \frac{6}{100} \times 45^2$

$= 121{\cdot}5\ cm.^2$

Even if the bigger diameter bars be selected, say 32 *mm.* diameter bars, we will require 16 bars of 32 *mm.* diameter providing a total area of $8{\cdot}04 \times 16 = 128{\cdot}64\ cm.^2$ [Fig. 67]. The difficulty of placing concrete between the 16 bars of 32 *mm.* diameter with the overall size of 45 *cm.* × 45 *cm.* may be quite apparent. Practically the maximum percentage of steel may be limited to 4 per cent of the gross area so as to ensure a good and sound concrete.

§83. I S. Recommendations Regarding Longitudinal Reinforcements

The I.S. 456 code has stipulated the following :

(*a*) The cross-sectional area of longitudinal reinforcement shall be not less than 6 percent of the gross-cross sectional area of the column.

(*b*) In any column that has a larger cross-sectional area than that requ red to support the load, the minimum percenatge of steel shall be based upon the area of concrete required to resist the direct stress and not upon the actual area.

(*c*) The minimum number of longitudinal bars provided in a column shall be four in rectangular columns and six in circular columns.

(*d*) The bars shall not be less than 12 *mm.* in diameter.

(*e*) A reinforced concrete column having helical reinforcement shall have atleast six bars of longitudinal reinforcement within the helical reinforcement.

(*f*) In a helically reinforced column, the longitudinal bars shall be in contact with the helical reinforcement and equidistant around its inner circumference.

(*g*) Spacing of longitudinal bars measured along the periphery of the column shall not exceed 300 *mm.*

(*h*) In case of pedestals in which the longitudinal reinforcement is not taken into account in strength calculations, nominal longitudinal reinforcement not less than 0·15 percent of the cross-sectional area shall be provided.

Note—Pedestal is a compression member, the effective length of which does not exceed three times the least lateral dimension.

§84. Transverse Reinforcement

The longitudinal reinforcement should be laterally tied by transverse links to provide a restraint against outward buckling of each of the longitudinal bars. I S. 456 code stipulates that the diameter of longitudinal bars shall not be less than 12 *mm.* and that the diameter of the transverse reinforcement shall be not less than one-fourth of the diameter of the main rods and in no case less than

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5 *mm.* in diameter. The ends of the transverse links should be properly anchored.

Figs. 68 and 69 show how transverse reinforcements are provided in R-C-columns.

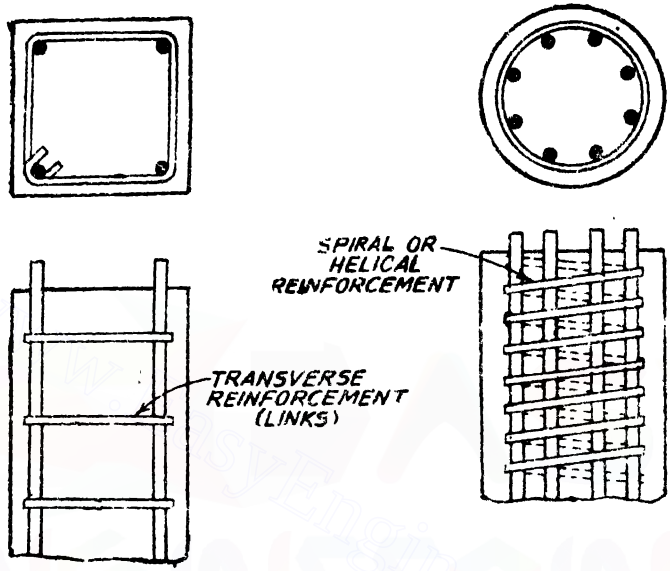

Fig. 68 R-columns.

Arrangement of transverse reinforcement—I.S. recommendations.

1. If the longitudinal bars are not spaced more than 75 *mm.* on either side, transverse reinforcement need only to go round corner and alternate bars for the purpose of providing effective lateral supports. See Fig. 70 (*a*).

2. If the longitudinal bars spaced at a distance of not exceeding 48 times the diameter of the tie are effectively tied in two directions, additional longitudinal bars in between these bars need to be tied in one direction by open ties [See Fig 70 (*b*)].

3. Where the longitudinal reinforcing bars in a compression member are placed in more than one row, effective lateral support to the longitudinal bars in the inner rows may be assumed to have been provided if :

(*i*) Transverse reinforcement is provided for the outermost row.

(*ii*) No bar of the inner row is closer to the nearest compression face than three times the diameter of the largest bar in the inner row [See Fig. 70 (*c*).]

4. Where the longitudinal bars in a compression member are grouped (not in contact) and each group adequately tied with

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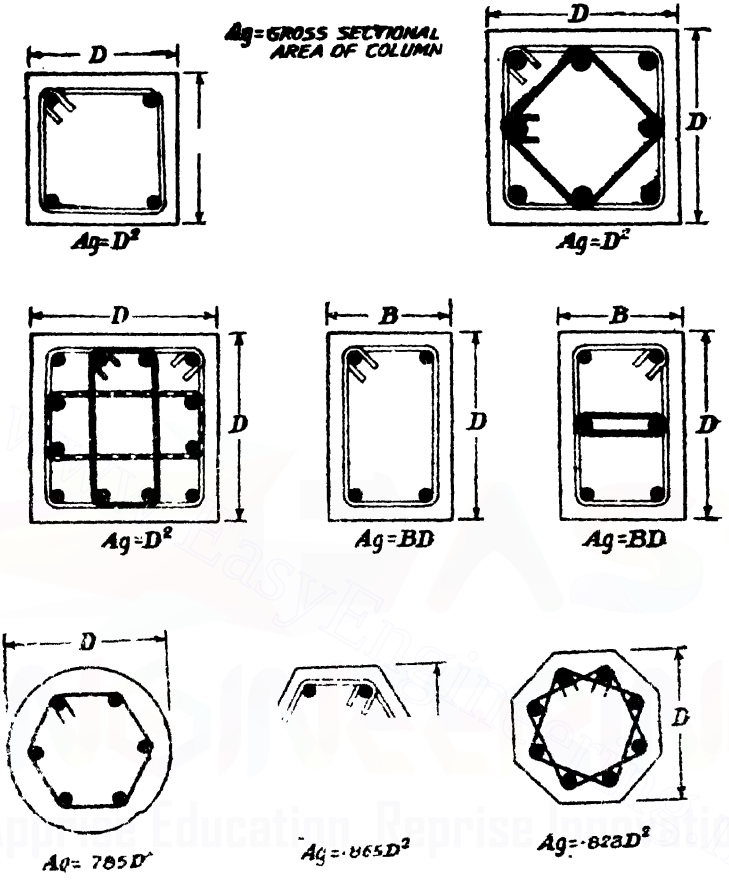

Fig. 69. Transverse reinforcement.

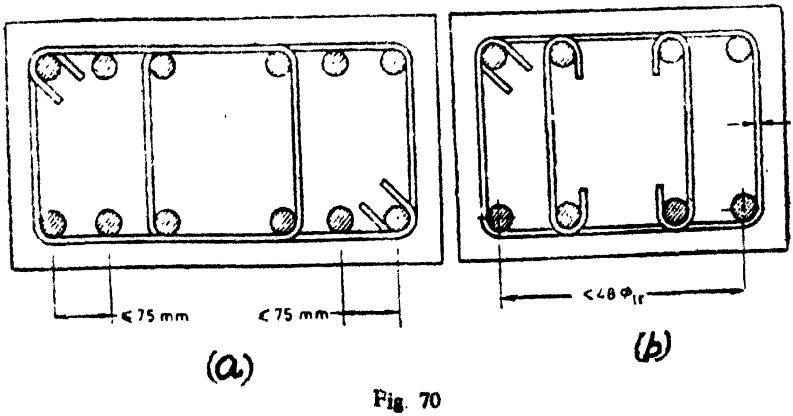

Fig. 70

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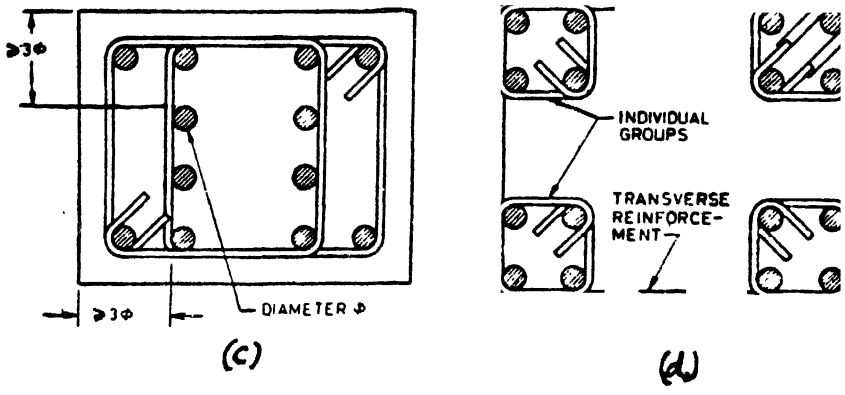

Fig. 70

transverse reinforcement then the transverse reinforcement for the compression member as a whole may be provided on the assumption that each group is a single longitudinal bar for the purpose of determining the pitch and the diameter of the transverse reinforcement. The diameter of such transverse reinforcement need not, however, exceed 20 *mm*. [See Fig. 70 (*d*).]

§85. Spacing of Transverse Links

This shall not exceed the least of the following :

(*a*) The least lateral dimension of the column.

(*b*) Sixteen times the diameter of the smallest longitudinal reinforcing rod in the column.

(*c*) Forty-eight times the diameter of the transverse reinforcement.

§86. Diameter of Transverse Links

The diameter of the transverse links shall not be less than

(*i*) One-fourth the diameter of the largest longitudinal bar,

(*ii*) 5 *mm*.

§87. Cover

The minimum cover to a column reinforcement equals 40 *mm*. or diameter of bar whichever is greater.

§88. Effective Length of a Column

The effective length of a column is not necessarily its actual length. It depends on the degree of fixity of the ends of the columns. The following table gives the effective length corresponding to the unsupported length *l* of the column from floor to floor or between properly restrained supports.

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Effective Length of Compression Members [*I.S. 456*]

Degree of end restraint of compression member	*Theoretical value of effective length*	*Recommended value of effective length*
Effectively held in position and restrained against rotation at both ends (*i.e*, both ends are fixed)	0·5 *l*	0·65 *l*
Effectively held in position at both ends, restrained against rotation at one end (*i.e.*, fixed at one end and hinged at the other end)	0·7 *l*	0·80 *l*
Effectively held in position at both ends but not restrained against rotation (*i.e.*, both ends are hinged)	1·00 *l*	1·00 *l*
Effectively held in position and restrained against rotation at one end, and the other restrained against rotation but not held in position	1·00 *l*	1·20 *l*
Effectively held in position and restrained against rotation at one end, and at the other partially restrained against rotation but not held in position		1·50 *l*
Effectively held in position at one end, but not restrained against rotation, and at the other end restrained against rotation but not held in position	2·00 *l*	2·00 *l*
Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end (*i.e*, fixed at one end and free at the other end	2·00 *l*	2·00 *l*

§89. Short and Long Columns

A column will be considered as short when the ratio of the effective length to its least lateral dimension is less than or equal to 12. When this ratio exceeds the columnn will be considered as a long column

§90. Slenderness Limits for Columns

The unsupported length between end restrains shall not exceed 60 times the least lateral dimension of the column.

If in any given plane, one end of a column is unrestrained, its unsupported length l, shall not exceed $\left(\frac{100\,b^2}{D}\right)$

where, b = width of that cross-section, and

D = depth of the cross-section measured in the plane under consideration.

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Safe Compressive Stress in Concrete Columns

S.I. Units		*M.K.S. Units*	
Grade of concrete	*Safe compressive stress in N/mm.²*	*Grade of concrete*	*Safe compressive stress in kg./cm.²*
M 10	2·5	M 100	25
M 15	4·0	M 150	40
M 20	5·0	M 200	50
M 25	6·0	M 250	60
M 30	8·0	M 300	80
M 35	9·0	M 350	90
M 40	10·0	M 400	100

Safe Compressive Stress in Steel Bars Used in RCC Columns

Type of bar	*Safe compressive stress in N./mm²*	*Safe compressive stress in kg./cm.²*
Mild steel bars conforming to I.S. 432 (Part I) or Deformed mild steel bars conforming to I.S. 1129	130	1100
Medium steel bars conforming to I.S. 432 (part I) or Deformed medium steel bars conforming to I.S. 1139	130	1300
High yield strength Deformed bars conforming to I.S. : 1139 or I.S. :1786 Grade (Fe 415) (Ribbed-T or steel)	190	1900

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§91. Minimum Eccentricity

All columns shall be designed for minimum eccentricity equal to $\left[\frac{l}{500}+\frac{b}{30}\right]$ subject to a minimum of 20 *mm*,

where l = unsupported length of the column, and
b = least lateral dimension of the column.

§92. Permissible Load on a Short Column

A short column may be analysed by the following methods:
(*i*) *Simple elastic theory.* (*ii*) *I.S. code method.*

(*i*) Simple elastic theory

Let A = area of the column section.
A_c = area of the compression steel.
Actual concrete area $=(A-A_c)$
$\therefore$ Equivalent concrete area
$= A_e$ = Actual concrete area + modular ratio × steel area
$=(A-A_c)+mA_c=[A+(m-1)A_c]$
Safe load on the column
= [Safe stress in concrete] × [Equivalent concrete area]
$W=\sigma_c[A+(m-1)A_c]$

(*ii*) I.S. code method [*Working stress method*]

The safe load on a short column is given by

$$W=\begin{bmatrix}\text{area of}\\ \text{concrete}\end{bmatrix}\times\begin{bmatrix}\text{safe stress}\\ \text{in concrete}\end{bmatrix}+\begin{bmatrix}\text{area of}\\ \text{steel}\end{bmatrix}\times\begin{bmatrix}\text{safe stress}\\ \text{in steel}\end{bmatrix}$$

NOTE—The minimum eccentricity mentioned earlier may be deemed to be incorporated in the above equation.

Safe compressive stress in concrete shall be taken as on page 160.

Problem 39. (S.I.). *A short column 30 cm. × 30 cm. in section is reinforced with 8 bars of 22 mm diameter. Find the safe load on the column by simple elastic theory. Take m = 18. Use M 15 mix.*

Solution. Area of column
$= A = 30\times 30 = 900\ cm.^2$
Area of steel $= A_c = 8\times 3{\cdot}8 = 30{\cdot}4\ cm.^2$
Equivalent concrete area
$= A_e = A+(m-1)A_c$
$= 900+(18-1)\ 30{\cdot}4$
$= 1416{\cdot}8\ cm.^2$

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Safe stress in concrete

$$= 4 N/mm.^2 = 400\ N/cm.^2$$

$$\therefore \quad \textit{Safe load} = \left[\begin{array}{c}\text{Safe stress in}\\ \text{concrete}\end{array}\right] \times \left[\begin{array}{c}\text{Equivalent}\\ \text{concrete area}\end{array}\right]$$

$$= 400 \times 1416{\cdot}8 \ \textit{Newtons}$$

$$= 566720 \ \textit{Newtons}$$

$$= 566{\cdot}72 \ \textit{Kilonewtons.}$$

Problem 40. (M.K.S). *An R.C. column 30 cm. × 30 cm in section is reinforced with 8 bars of 20 mm. diameter. If the permissible stress in concrete is 40 kg/cm² find the safe compressive load for the column, by simple elastic theory Take m = 18.*

Solution. Area of column $= A = 30 \times 30 = 900\ cm.^2$

Area of steel $= A_c = 8 \times 3{\cdot}14 = 25{\cdot}12\ cm.^2$

Equivalent concrete area $A_e = A + (m - 1)\ A_c$

$$= 900 + (18 - 1)\ 25{\cdot}12$$

$$= 1327{\cdot}04\ cm^2.$$

$$\textit{Safe load} = \left[\begin{array}{c}\text{Safe stress in}\\ \text{concrete}\end{array}\right] \times \left[\begin{array}{c}\text{Equivalent}\\ \text{concrete area}\end{array}\right]$$

$$= 40 \times 1327{\cdot}04 = 53082\ kg.$$

Problem 41. (S.I.). *A short column 30 cm. × 30 cm. in section is reinforced with 8 bars of 22 mm. diameter. Find the safe load on the column as per the I.S. code permissible stresses in concrete and steel are 4N/mm.² and 130 N/mm² respectively.*

Solution. Area of the column

$$= 30 \times 30 = 900\ cm.^2$$

Area of steel $A_c = 8 \times 3{\cdot}8 = 30{\cdot}40\ cm.^2$

$\therefore$ Actual area of concrete

$$= 900 - 30{\cdot}40 = 869{\cdot}6\ cm.^2$$

Safe stress in concrete

$$= 4\ N/mm.^2 = 400\ N/cm.^2$$

Safe stress in steel $= 130\ N/mm.^2 = 13000\ N/cm.^2$

$\therefore$ Safe load

$$= \left[\begin{array}{c}\text{area of}\\ \text{concrete}\end{array}\right] \times \left[\begin{array}{c}\text{Safe stress}\\ \text{in concrete}\end{array}\right] + \left[\begin{array}{c}\text{area of}\\ \text{steel}\end{array}\right] \times \left[\begin{array}{c}\text{Safe stress}\\ \text{in steel}\end{array}\right]$$

$$= [869{\cdot}6 \times 400] + [30{\cdot}4 \times 13000]$$

$$= 347840 + 395200$$

$$= 743040 \ \textit{Newtons.}$$

$$= 743{\cdot}04 \ \textit{Kilonewtons.}$$

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Problem 42. (M.K S.). *A short column 30 cm. × 30 cm. in section is reinforced with 8 bars of 22 mm. diameter. Find the safe load on the column as per the I.S. code. Permissible stresses in concrete and steel are 40 kg./cm.² and 1300 kg /cm.² respectively.*

Solution. Area of the column

$$= 30 \times 30 = 900 \text{ cm.}^2$$

Area of steel $= A_s = 8 \times 3{\cdot}8 = 30{\cdot}40 \text{ cm.}^2$

$\therefore$ Area of concrete $= 900 - 30{\cdot}4 = 869{\cdot}6 \text{ cm.}^2$

Safe load on the column

= concrete area × safe stress in concrete
+ steel area × safe stress in steel

$= 869{\cdot}60 \times 40 + 30{\cdot}4 \times 1300 \text{ kg.}$

$= 74304 \text{ kg.}$

Problem 43. (M.K.S.). *A short column of square section is to be designed to carry an axial load of 1,02,300 kg. Design the column, as per I.S. code.*

Solution. Let us assume that M 150 grade concrete is used.

Safe stress in concrete $= 40 \text{ kg./cm.}^2$

Safe stress in steel $= 1300 \text{ kg./cm.}^2$

Let the sectional area of the column be $A \text{ cm.}^2$

Let us assume 2% of area of column as steel reinforcement

$\therefore$ Area of steel $= 0{\cdot}02\ A \text{ cm.}^2$

$\therefore$ Area of concrete $= A - 0{\cdot}02\ A \text{ cm.}^2$

$= 0{\cdot}98\ A \text{ cm.}^2$

$\therefore$ Safe load = concrete area × safe stress in concrete
+ steel area × safe stress in steel

$\therefore \quad 0{\cdot}98A \times 40 + 0{\cdot}02A \times 1300 = 1{,}02{,}300 \text{ kg.}$

$\therefore \quad 65{\cdot}2A = 1{,}02{,}300$

$A = 1570 \text{ cm.}^2$

$\therefore$ Size of column $= \sqrt{1570} = 39{\cdot}62 \text{ cm.}$

Hence provide *40 cm. × 40 cm.*

Area of the column section $= 1600 \text{ cm.}^2$

$\therefore \quad 40(1600 - A_s) + A_c \times 1300$

$= 1{,}02{,}300$

$1260\ A_c = 38300$

$\therefore \quad A_s = 30{\cdot}4 \text{ cm.}^2$

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Provide 8 bars of 22 mm. diameter (30·41 cm.²)

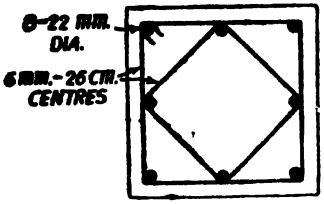

Fig. 71

Lateral ties. Diameter to be not less than

(*i*) 5 *mm.*

(*ii*) One-fourth of diameter of main steel $=\frac{1}{4}\times 22 = 5{\cdot}5$ *mm.*

Let us, therefore, provide 6 *mm.* diameter ties.

Spacing. This shall not exceed the following :

(*i*) Least lateral dimension $= 40$ *cm.*

(*ii*) 16 times the diameter of main bar $= 16\times 2{\cdot}2 = 35{\cdot}2$ *cm.*

(*iii*) 48 times the diameter of ties $= 48\times 0{\cdot}6 = 28{\cdot}8$ *cm.*

Provide 6 mm ties at 25 cm. centres

Problem. 44. (S.I.). *A short column of square section is to be designed to carry an axial load of 1023 kilonewtons. Design the column as per I S. code. Permissible stresses in concrete and steel are 4N/mm.² and 130 N/mm.²*

Solution. Safe stress in concrete

$= 4$ *N/mm.²* $= 400$ *N/cm.²*

Safe stress in steel $= 130$ *N/mm.²* $= 13000$ *N/cm.²*

Let us assume 2% of area of column as steel reinforcement.

Area of steel $= \frac{2}{100}\times A = 0{\cdot}02\,A$ *cm.²*

$\therefore$ Area of concrete $= 0{\cdot}98\,A$ *cm.²*

Safe load = concrete area × safe stress in concrete + steel area × safe stress in steel

$\therefore\ 0{\cdot}98A\times 400 + 0{\cdot}02A\times 13000 = 1023000$ *Newtons.*

$\therefore\ 652\,A = 1023000 \quad \therefore\ A = 1570$ *cm.²*

$\therefore$ Size of column $= \sqrt{1570} = 39{\cdot}62$ *cm.²*

Hence provide 40 *cm.* × 40 *cm.*

$\therefore$ Area of column section $= 1600$ *cm.²*

$\therefore\ 400\,(1600 - A_c) + A_c\times 13000 = 1023000$ *Newtons*

$\therefore\ 12600\,A_c = 383000 \quad \therefore\ A_c = (30{\cdot}4$ *cm.²*)

Provide 8 bars of 22 mm diameter (30·4 cm.²)

Lateral ties : Diameter to be not less than

(*i*) 5 *mm.*

(*ii*) one-fourth of diameter of main steel $= \frac{22}{4} = 5{\cdot}5$ *mm.*

Let us provide 6 *mm* diameter ties.

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Spacing : This shall not exceed the following :

(*i*) Least lateral dimension = 40 *cm*.

(*ii*) 16 times the diameter of main bar = $16 \times 2{\cdot}2 = 35{\cdot}2$ *cm*.

(*iii*) 48 times the diameter of ties = $48 \times 0{\cdot}6 = 28{\cdot}8$ *cm*.

Provide 6 mm ties at 25 cm centres.

§92. Long columns

When the ratio of the effective length of a column to its least lateral dimension exceeds 12 the column will be regarded as a long column. Such columns are liable to be buckled and to include this factor in the design the working stresses in concrete and steel are taken at a lower value, by multiplying the usual working stresses by a coefficient C_r called a *reduction coefficient*.

Hence for a long column

Safe stress in concrete = C_r × Corresponding safe stress for short column.

and safe stress in steel = C_r × Corresponding safe stress for short column

The coefficient C_r is to be determined from the following relation

$$C_r = 1{\cdot}25 - \frac{l_{ef}}{48b}$$

For more exact computations

$$C_r = 1{\cdot}25 - \frac{l_{ef}}{160\,K_m}$$

where C_r = reduction coefficient

l_{ef} = effective length of the column

b = least lateral dimension

K_m = least radius of gyration.

Values of C_r for various values of $\frac{l_{ef}}{b}$ are given in the table on page 166.

Problem 45. (S.I.). *A column 40 cm. × 40 cm × 6 metres long has to support a load of 875 kilonewtons. Find the necessary reinforcement for the column. Use M 15 concrete. For a short column the safe stresses in concrete and steel are 4 N/mm² and 130 N/mm². respectively.*

Solution.

b = least lateral dimension = 40 *cm*.

l_{ef} = Effective length of the column = 600 *cm*.

$$\frac{l_{ef}}{b} = \frac{600}{40} = 15$$

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Reduction Coefficient

$\frac{l_{ef}}{b}$	C_r	$\frac{l_{ef}}{b}$	C_r	$\frac{l_{ef}}{b}$	C_r	$\frac{l_{ef}}{b}$	C_r
12	1·000	25	0·729	38	0·458	51	0·188
13	0·979	26	0·708	39	0·438	52	0·167
14	0 958	27	0 688	40	0·417	53	0·146
15	0·937	28	0·667	41	0·396	54	0·125
16	0·917	29	0 646	42	0·375	55	0·104
17	0·896	30	0·625	43	0·354	56	0·083
18	0 875	31	0·604	44	0 333	57	0·063
19	0 854	32	0·583	45	0·313	58	0·042
20	0·833	33	0·563	46	0·292	59	0·021
21	0·813	34	0·542	47	0 271	60	0
22	0·792	35	0·521	48	0 250		
23	0·771	36	0·500	49	0·229		
24	0·750	37	0·479	50	0·208		

This is greater than 12. Therefore, the column is a long column.

Reduction coefficient $C_r = 1{\cdot}25 - \frac{l_{ef}}{48\,b}$

$= 1{\cdot}25 - \frac{15}{48} = 0{\cdot}937$

Safe load on long column = [Reduction coefficient] × [Safe load on short column]

Let the area of reinforcement be A_c $cm.^2$

$\therefore$ Area of concrete $= (40 \times 40) - A_c = (1600 - A_c)$ $cm.^2$

$\therefore$ Safe load $= 0{\cdot}937[400(1600 - A_c) + 13000A_c]$

$= 875000$ *Newtons*

$\therefore$ $A_c = 23{\cdot}32$ $cm.^2$

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Provide 8 bars of 20 mm. diameter (25·12 cm.²).

Lateral ties

Diameter = greater of : (*i*) $\frac{1}{4}$ diameter of main bars

$$= \frac{20}{4} = 5 \text{ mm.}$$

(*ii*) 5 *mm.*

Provide 5 *mm.* ties.

Spacing of ties = least of the following :

(*i*) least lateral dimension = 40 *cm.*

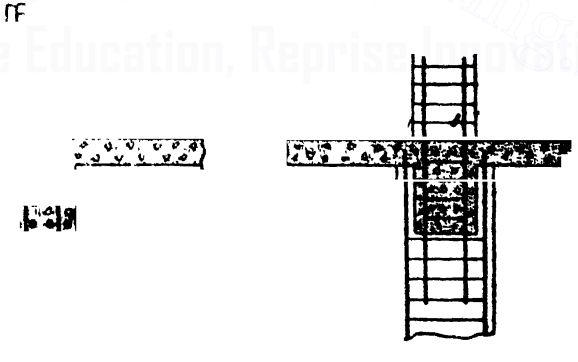
(*ii*) 16 times diameter of main bars = $16 \times 2{\cdot}0 = 32$ *cm.*

(*iii*) 48 times the diameter of ties = $48 \times 0{\cdot}5 = 24$ *cm.*

Provide 5 mm. ties at 24 cm. centres.

§94. Continuous Columns

Often in multistoried structures, a column continues up through a floor from one storey to another. In such cases the main bars of the column must be first continued up either within or outside the reinforcement of the floor beam which frames into this column. When the main bars continue up outside the reinforcement of the beam, it is necessary that the width of the column should be at least 8 *cm.* more than the width of the beam. Sometimes the column size in plan may be smaller above the floor than below it. In such cases the main bars of the column will have to be bent inwards at the



(*a*) Column reinforcement taken within the main bars of the beam.

Fig. 72

(*b*) Column reinforcement taken outside the main bars of the beam.

floor level, or alternatively these main bars may be stopped just below the floor level and separate lap bars may be provided for connecting the part of the column above and below the floor.

Figs. 72 (*a*) and (*b*) show two alternative arrangements.

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§95. Spirally Reinforced Circular Columns

These are circular columns, which are reinforced with closely and uniformly spaced spiral reinforcement in addition to longitudinal steel. Columns of circular section are usually spirally reinforced. Sometimes separate loops may also be provided in place of the spiral. The continuous spiral is adopted in preference to separate loops. A column with helical reinforcement shall have atleast six bars as longitudinal reinforcement. The strength of a column with helical reinforcement satisfying the requirement given below shall be taken as 1·05 times the strength of similar member with lateral ties.

The ratio of the volume of helical reinforcement to the volume of the core shall not be less than $0{\cdot}36\left[\frac{A_g}{A_k}-1\right]\frac{f_{ck}}{f_y}$,

where, A_g = Gross area of the section,

A_k = Area of the core of the helically reinforced column measured to the outside diameter of the helix,

f_{ck} = Characteristic compressive strength of concrete, (28 day strength of concrete),

f_y = Characteristic strength of the helical reinforcement but not exceeding 415 $N/mm.^2$ (4150 $kg./cm.^2$).

Pitch of helical reinforcement (*I.S. 456*). Helical reinforcement shall be of regular formation with the turns of the helix spaced evenly and its ends shall be anchored properly by providing one and half extra turns of the spiral bar. The pitch of the helical turns shall be not more than 75 *mm.*, nor more than one-sixth of the core diameter of the column, nor less than 25 *mm.*, nor less than three times the diamcter of the steel bar forming the helix.

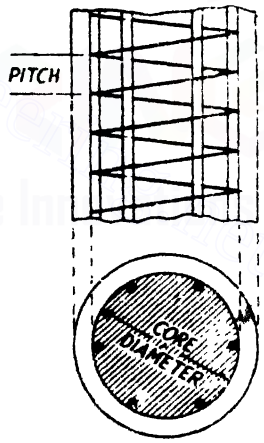

Fig 73.

Diameter of helical reinforcement. The diameter of the helical reinforcement shall be not less than one-fourth the diameter of the largest longitudinal bars and in no case less than 5 *mm.*

Long circular column A circular column will be considered as long if the ratio of the effective length to the core diameter exceeds 12. The reduction coefficient C_r for such a column is given by $C_r = 1{\cdot}25 - \frac{l_{ef}}{48b}$

where l_{ef} = Effective length of the column.

b = Diameter of the core.

Problem 46 (S.I.). ***Design a circular short column 40 cm. in diameter to support an axial load of 800 kilonewtons. Use M 15 concrete.***

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Solution.

Diameter of column $= 40$ *cm.*

Clear cover to longitudinal bars $= 4$ *cm.*

Let us provide 6 *mm.* diameter helical bars.

Clear cover to helical bar $= 4 - 0{\cdot}6 = 3{\cdot}4$ *cm.*

$\therefore$ Core diameter $= 40 - (2 \times 3{\cdot}4) = 33{\cdot}2$ *cm.*

Diameter of column corresponding to centre of helical bars

$= 33{\cdot}2 - 0{\cdot}6 = 32{\cdot}6$ *cm.*

Gross area of column $= A_g = \frac{\pi}{4} [40]^2 = 1256{\cdot}64$ *cm.*^{2}

Let the area of longitudinal steel $= A_c$ *cm.*^{2}

Safe stress in concrete $= 4$ *N/mm.*$^2 = 400$ *N/cm.*^{2}

Safe stress in steel $= 130$ *N/mm.*$^2 = 13000$ *N/cm.*^{2}

$$\text{Safe load} = 1{\cdot}05 \left[\begin{array}{l} \text{concrete area} \times \text{safe stress in concrete} \\ + \text{steel area} \times \text{safe stress in steel} \end{array} \right]$$

$\therefore \quad 1{\cdot}05[400(1256{\cdot}64 - A_c) + 13000\ A_c] = 800{,}000$ *Newtons*

$\therefore \quad A_c = 20{\cdot}58$ *cm.*^{2}

Provide 6 bars of 22 mm. diameter (22·8 cm.^{2}*)*

Provide 6 *mm.* helical bars at a pitch of 5 *cm.*

Length of helix per pitch length $= \sqrt{(\pi \times 32{\cdot}6)^2 + 5^2} = 102{\cdot}54$ *cm.*

Volume of helical steel per pitch length

$= 0{\cdot}28 \times 102{\cdot}54 = 28{\cdot}71$ *cm.*^{3}

Volume of core per pitch length $= 865{\cdot}70 \times 5 = 4328{\cdot}50$ *cm.*^{3}

$\therefore$ Ratio of volume of helical steel to volume of core

$$= \frac{28{\cdot}71}{4328{\cdot}50} = 0{\cdot}0076$$

This should not be less than $0{\cdot}36 \left[\frac{A_g}{A_k} - 1 \right] \frac{f_{ck}}{f_y}$

$$= 0{\cdot}36 \left[\frac{1256{\cdot}64}{865{\cdot}70} - 1 \right] \frac{15}{415} = 0{\cdot}005876$$

Hence provision of helical steel is satisfactory.

Problem 47. (M.K.S.). *Design a short column 40 cm. in diameter to support an axial load of 80,000 kg. Use M 150 concrete.*

Solution. Diameter of column $= 40$ *cm.*

Clear cover to longitudinal bars $= 4$ *cm.*

Let us provide 6 *mm.* helical bars.

$\therefore$ Clear cover to helical bar $= 4 - 0{\cdot}6 = 3{\cdot}4$ *cm.*

$\therefore$ Core diameter $= 40 - (2 \times 3{\cdot}4) = 33{\cdot}2$ *cm.*

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Diameter of column to centre of helical bar
$= 33{\cdot}2 - 0{\cdot}6 = 32{\cdot}6$ *cm.*

Gross area of column $= A_g = \frac{\pi}{4}(40)^2 = 1256{\cdot}64$ *cm.*^{2}

Let the area of longitudinal steel $= A_c$ *cm.*^{2}
Safe stress in concrete $= 40$ *kg./cm.*^{2}
Safe stress in steel $= 1300$ *kg./cm.*^{2}

$\therefore$ Safe load $= 1{\cdot}05\left[\begin{matrix}\text{concrete area} \times \text{safe stress in concrete} \\ + \text{steel area} \times \text{safe stress in steel}\end{matrix}\right]$

$\therefore \quad 1{\cdot}05[40(1256{\cdot}64 - A_c) + 1300\ A_c] = 80{,}000$ *kg.*

$\therefore \quad A_c = 20{\cdot}58$ *cm.*^{2}

Provide 6 bars of 22 mm. ϕ *(22·8 cm.*^{2}*)*

Provide 6 *mm.* helical reinforcement at a pitch of 5 *cm.*

Length of helix per pitch length
$= \sqrt{(\pi \times 32{\cdot}6)^2 + 5^2} = 102{\cdot}54$ *cm.*

Volume of helical steel per pitch length
$= 0{\cdot}28 \times 102{\cdot}54 = 28{\cdot}71$ *cm.*^{3}

Volume of core per pitch length $= 865{\cdot}70 \times 5 = 4328{\cdot}5$ *cm*^{3}

Ratio of volume of helical steel to volume of core

$$= \frac{28{\cdot}71}{4328{\cdot}5} = 0{\cdot}0076.$$

This should not be less than,

$$0{\cdot}36\left[\frac{A_g}{A_k} - 1\right]\frac{f_{ck}}{f_y} = 0{\cdot}36\left[\frac{1256{\cdot}64}{865{\cdot}70} - 1\right]\frac{150}{4150} = 0{\cdot}005876$$

Hence provision of helical reinforcement is satisfactory.

§96. Composite Columns

A composite column is a column which consists of a structural steel or cast iron column enclosed with concrete which is reinforced with longitudinal as well as spiral reinforcement.

As per I.S. code, the safe axial load P on a composite column is given by,

$$P = \left[\begin{matrix}\text{Safe stress} \\ \text{for} \\ \text{concrete}\end{matrix} \times \begin{matrix}\text{Area} \\ \text{of} \\ \text{concrete}\end{matrix}\right] + \left[\begin{matrix}\text{Safe stress} \\ \text{for} \\ \text{longitudinal} \\ \text{steel bars}\end{matrix} \times \begin{matrix}\text{Area} \\ \text{of} \\ \text{longitudinal} \\ \text{steel bars}\end{matrix}\right]$$
$$+ \left[\begin{matrix}\text{Safe stress for} \\ \text{structural steel} \\ \text{column or} \\ \text{cast iron column}\end{matrix}\right] \left[\begin{matrix}\text{Area} \\ \text{of} \\ \text{structural steel} \\ \text{or cast iron column}\end{matrix}\right]$$

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The safe stresses shall be taken as follows :

Component	*Safe stress*
Concrete	As in RCC columns
Longitudinal steel bars	As in RCC columns
Structural steel	125 $N/mm.^2$ or 1250 $kg./cm.^2$
Cast iron	70 $N/mm.^2$ or 700 $kg./cm.^2$

Placing of the metal core and reinforcement in the composite column. The cross-sectional area of the metal core shall not exceed 20 percent of the gross area of the column. If a hollow metal core is used, it shall be filled with concrete. The longitudinal steel shall be at least 0·8 percent and shall not exceed 6 percent of the gross sectional area of the column. The spacing of longitudinal bars shall not exceed 300 *mm*. The diameter of the helical bars shall be not less than one-fourth the diameter of the longitudinal bars nor less than 5 *mm*. A clearance of at least 75 *mm*. shall be maintained between the spiral and the metal core at all points, except that when the core consists of a structural steel *H*-column, the minimum clearance may be reduced to 50 *mm*.

Problem 48 *Fig. 74 shows the section of a composite column. Determine the safe load for the column.*

Solution. Area of longitudinal bars $= 8 \times 2{\cdot}01 = 16{\cdot}08\ cm.^2$

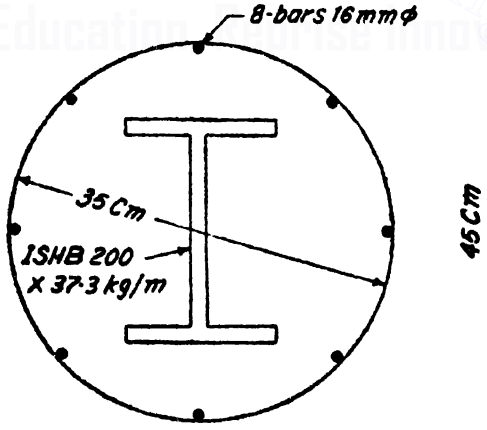

Fig. 74

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Area of *I*-section = 47·54 *cm.*²

Area of concrete = 45² − 16·08 − 47·54 = 1961·38 *cm.*²

Safe stress in concrete = 4 *N/mm.*² = 400 *N/cm.*²

Safe stress in steel bars = 130 *N/mm.*² = 13000 *N/cm.*²

Safe stress in structural steel = 125 *N/mm.*² = 12500 *N/cm.*²

∴ *Safe load on the column*

= [1961·38 × 400] + [16·08 × 13000] + [47·54 × 12500] *N*

= 784552 + 209040 + 594250

= *1,587,842 N say 1587 8 kilonewtons.*

§97. Concrete Filled Pipe Columns

A steel pipe filled with concrete can be used as a column. The safe load for such a column is given by

$$\left\{\begin{matrix}\text{area}\\ \text{of}\\ \text{concrete}\end{matrix} \times \begin{matrix}\text{Safe}\\ \text{stress}\\ \text{for}\\ \text{concrete}\end{matrix}\right\}\left[1 - 0{\cdot}000025\left(\frac{l}{K_c}\right)^2\right] + \left\{\begin{matrix}\text{area}\\ \text{of}\\ \text{steel}\\ \text{pipe section}\end{matrix} \times \begin{matrix}\text{safe}\\ \text{stress for}\\ \text{steel pipe}\\ \text{section}\end{matrix}\right\}$$

Safe stress for steel pipe $= 12000 - 0{\cdot}34\left(\frac{l}{K_s}\right)^2$ *N/cm.*²

or $1200 - 0{\cdot}034\left(\frac{l}{K_s}\right)$ *kg./cm.*²

where, K_c = radius of gyration of the concrete component.

K_s = radius of gyration of the steel pipe section.

Problem 49. (S I.). *Find the safe load for a concrete filled column whose details are given below :*

External diameter of the steel pipe = 35 cm.

Internal diameter of the steel pipe = 30 cm.

Length of the column = 6 metres

Solution. Area of steel pipe $= \frac{\pi}{4}(35^2 - 30^2) = 255{\cdot}25$ *cm.*²

Area of concrete $= \frac{\pi \times 30^2}{4} = 706{\cdot}86$ *cm.*²

Length of the column $= l = 600$ *cm.*

K_c = Radius of gyration of concrete component $\sqrt{\frac{\pi D^4/64}{\pi D^2/4}} = \frac{D}{4}$

$= \frac{30}{4} = 7{\cdot}50$ *cm.*

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K_s = Radius of gyration of steel component

$$K_s^2 = \frac{\frac{\pi}{64}[35^4 - 30^4]}{\frac{\pi}{4}[35^2 - 30^2]} = 132{\cdot}81$$

$$\therefore \quad K_s = \sqrt{132{\cdot}81} = 11\ 52\ cm.$$

$$\frac{l}{K_c} = \frac{600}{7{\cdot}50} = 80$$

$$\frac{l}{K_s} = \frac{600}{11{\cdot}52} = 52{\cdot}08$$

Safe stress for steel pipe $= 12000 - 0{\cdot}34\left(\frac{l}{K_s}\right)^2\ N/cm.^2$

$$= 12000 - 0\ 34 \times 52{\cdot}08^2 = 11080\ N/cm.^2$$

$\therefore$ *Safe load on the column*

$= [706\ 86 \times 400][1 - 0{\cdot}000025 \times 80^2]$
$+ [255{\cdot}25 \times 11080]$ *Newtons*
$= 282744 + 2828170 = 3110914$ *Newtons*
say 3110·9 kilonewtons.

§98 Design of a Column by Limit State Method

The I S code besides providing the specifications for working stress method, has also provided specifications for limit state method.

Minimum eccentricity. All columns shall be designed for minimum eccentricity equal to the unsupported length of column/500 plus lateral dimension/30 subject to a minimum of 20 *mm.*

i e., Minimum eccentricity

$$= e = \frac{l}{500} + \frac{D}{30} \text{ subject to a minimum of 20 } mm.$$

where l = upsupported length of the column,
D = lateral dimension of the column in the direction under consideration.

After calculating the eccentricity as mentioned above, the column section should be designed for the combined effect of the axial load and the bending moment due to the minimum specified eccentricity. But, as a simplification, when the minimum eccentricity as per above specification is less than or equal to 0·05 D. the column section can be designed as per the equation,

$$P_u = 0{\cdot}4\, f_{ck} A_c + 0{\cdot}67\, f_y A_t$$

where, P_u = ultimate axial load,
f_{ck} = characteristic compressive strength of concrete,
A_c = area of concrete,
A_t = area of the reinforcement, and
f_y = characteristic strength of the steel reinforcement (yield stress).

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Problem 50. *An R.C. column of unsupported length 2·75 metre has to be designed for an ultimate axial load of 3250 KN. Find the cross-sectional dimensions of the column and the reinforcement required. Use M 20 grade of concrete. Characteristic strength of reinforcement =415 N/mm.²*

Solution Let cross-sectional area of the column $= A$ *cm.*²

Let the area of steel $= A_t = 1\%$ of $A = 0{\cdot}01\ A$ *cm.*²

$\therefore$ Area of concrete $A_c = A - A_t = 0{\cdot}99\ A$ *cm.*²

$f_{ck} = 20$ *N/mm.*$^2 = 2000$ *N/cm.*^{2} ; $f_y = 415$ *N/mm.*^{2}

$= 41500$ *N/cm.*^{2}

Ultimate load $= 0{\cdot}4\, f_{ck} A_c + 0{\cdot}67\, f_y A_t = P_u$

$0{\cdot}4 \times 2000 \times 0{\cdot}99\ A + 0{\cdot}67 \times 41500 \times 0{\cdot}01\ A = 3250000$ *N.*

$792\ A + 278{\cdot}05\ A = 3250000$

$1070{\cdot}05\ A = 3250000$

$\therefore \quad A = 3037{\cdot}2$ *cm.*²

Provide 56 *cm.* × 56 *cm.*

Area of steel $= A_t = \frac{1}{100} \times 3037{\cdot}2 = 30{\cdot}37$ *cm.*²

Provide 8 bars of 22 mm. ϕ.

Check for minimum eccentricity

$$e_{min} = \frac{l}{500} + \frac{D}{30} = \frac{275}{500} + \frac{56}{30} = 0{\cdot}55 + 1{\cdot}87 = 2{\cdot}42 \text{ cm.}$$

$$\frac{e_{min}}{D} = \frac{2{\cdot}42}{56} = 0{\cdot}043 \text{ this is less than } 0{\cdot}05.$$

Hence the design of the section by the simplified method is valid.

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9

Combined Bending and Direct Stresses

Often we come across members subjected to direct stresses accompanied by bending stresses. A very common example is that of a column subjected to an eccentric load or a column of a storeyed building. Other instances are in arches, tank walls, chimneys, silos, bins *etc.* In some cases the direct stresses may be predominant and the bending stresses may be small. While in some cases the bending stresses may be predominant as compared with the direct stresses. There will also be instances in which both types of stresses may be predominant. Let us discuss the cases that we usually come across.

When the section is subjected to an axial load and a moment, the ratio of the moment to the load is called the eccentricity and such a section can be analysed as though the load has been applied at the above eccentricity.

Case 1. *Rectangular section subjected to compression and bending. Eccentricity is less than* $\frac{D}{6}$ *(See Fig. 75)*

In this case the resultant stresses in concrete will remain compressive and the resultant stresses are given by

$$c = -\frac{P}{A_e} \pm \frac{M}{Z}$$

where P = Eccentric load

M = B.M. due to eccentric load = $P.e$.

Z = Section modulus about the axis with respect to which the load is eccentric.

A_e = Equivalent concrete area.

For the section shown in the figure with symmetrical reinforcement we have the equivalent area of the section

$$= A_e = bD + (m-1)\, A_t$$

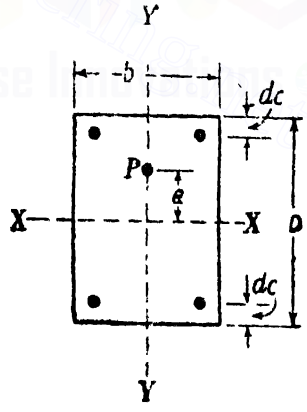

Fig 75

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where A_t = Total area of reinforcement

Equivalent moment of inertia about the axis XX

$$I_e = \frac{bD^3}{12} + (m-1)A_t\left[\frac{D}{2} - d_c\right]^2$$

$$Z = \frac{I_e}{\frac{D}{2}}$$

$\therefore$ The resultant stresses are

$$c = \frac{P}{A_e} \pm \frac{P.e}{I_e}\left(\frac{D}{2}\right)$$

The following examples illustrate the above case.

Problem 51 (S.I.). *A rectangular reinforced concrete section 40 cm. wide and 60 cm. deep is reinforced with 6 bars of 20 mm. diameter placed at a cover of 4 cm. from the top edge and 6 similar bars at the same cover from the bottom edge. Determine the maximum thrust on the section, which can be applied at a distance of 8 cm. from the centre line, if the compressive stress in concrete is not to exceed 5 N/mm². Take m = 18.*

Solution. $e = 8$ *cm.*

Effective cover to steel

$= 5$ *cm.*

$$\frac{D}{6} = \frac{60}{6} = 10 \text{ cm.}$$

$$e < \frac{D}{6}$$

Area of steel $= 12 \times 3{\cdot}14$

$= 37{\cdot}68$ *cm.*²

Equivalent area of concrete

$= A_e = bD + (m-1)A_t$

$= (40 \times 60) + (18-1)37{\cdot}68$

$= 2400 + 640{\cdot}56$

$= 3040{\cdot}56$ *cm.*²

Equivalent moment of inertia about the centroidal axis XX.

Fig. 76

$$= I_e \quad \frac{40 \times 60^3}{12} + 17 \times 37{\cdot}68 \times 25^2 \text{ cm.}^4$$

$$= 720000 + 400350 = 1120350 \text{ cm.}^4$$

Maximum compressive stress in concrete

$$= 5 \text{ N/mm.}^2 = 500 \text{ N/cm.}^2$$

$$\frac{P}{A_e} + \frac{Pe}{I_e} y_e = 500$$

$$\frac{P}{3040{\cdot}56} + \frac{P \times 8 \times 30}{1120350} = 500$$

$$P = 920630 \text{ N} = 920{\cdot}63 \text{ kN.}$$

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Problem 52. *A rectangular reinforced concrete section 70 cm. deep and 45 cm. wide is reinforced with 7 bars of 28 mm. diameter placed at an effective cover of 5 cm from the top edge and seven similar bars at the same effective cover from the bottom edge. Determine the maximum thrust on the section, which can be applied at a distance of 10 cm. from the centre line if the compressive stress in concrete is not to exceed 50 kg./cm². Take m = 18.*

Solution.

$e = 10$ *cm.*

$\frac{D}{6} = \frac{70}{6} = 11{\cdot}67$ *cm.*

$e < \frac{D}{6}$

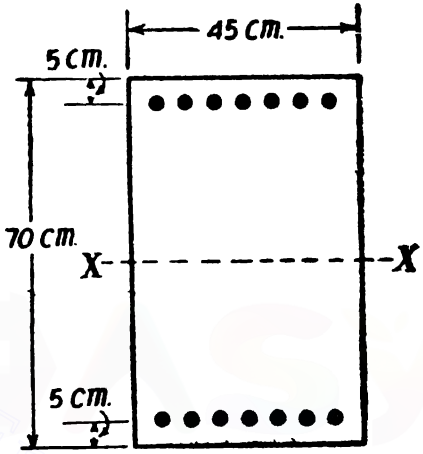

Fig. 77

Total area of steel provided $= 2 \times 43{\cdot}1 = 86{\cdot}2$ *cm².*

Equivalent area of concrete for the given section.

$= A_e = bD + (m-1)A_s$

$= 45 \times 70 + (18-1)86{\cdot}2$ *cm.²*

$= 4615{\cdot}4$ *cm.²*

Equivalent moment of Inertia about the centroidal axis *XX*

$$I_e = \frac{bD^3}{12} + (m-1)\ A_s \left(\frac{D}{2} - d_s\right)^2$$

$$\frac{45 \times 70^3}{12} + 17 \times 86{\cdot}2\ (35-5)^2\ cm.^4$$

$$= 2605{,}110\ cm.^4$$

Maximum compressive stress in concrete

$$= c = \frac{P}{A_e} + \frac{M}{I_e}\ y_e$$

$$50 = \frac{P}{4615{\cdot}4} + \frac{P \times 10 \times 35}{2605110}$$

$\therefore$ $P = 142400$ *kg.*

Problem 53. *The horizontal thrust for a two-hinged arch is found to be 110,000 N. The crown section of the arch is a square of 30 cm. side. It is reinforced with 6·03 cm.² of steel above the centroidal axis and an equal amount of steel below the axis. The centres of steel are respectively 5 cm. from the upper and lower edges of the crown section. The section is also subjected to a bending moment of 10,000 Nm. Assuming that the concrete does not fail in tension, calculate the maximum tensile and compressive stresses in concrete at the crown section. Find also the tensile and compressive stresses in steel. Take m = 18.*

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Solution. Equivalent concrete area of the arch section
$A_e = 30 \times 30 + (18-1) \times 2 \times 6{\cdot}03$ *cm.*^{2}
$= 1105$ *cm.*^{2}

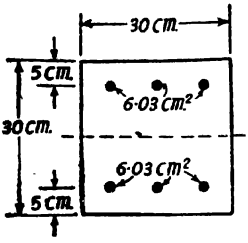

Fig. 78

Equivalent moment ef Inertia

$$= I_e = \frac{30 \times 30^3}{12} + (18-1)2 + 6{\cdot}03 \text{ cm.}^4$$

$= 88000$ *cm.*^{4}

Extreme stresses in concrete

$$= \frac{110000}{1105} \pm \frac{100,0000}{88000} \times 15 \text{ N/cm.}^2$$

$= 99{\cdot}5 \pm 170{\cdot}5$ *N/cm.*^{2}

$\therefore$ Maximum *compressive stress* in concrete
$= 99{\cdot}5 + 170{\cdot}5 = 270$ *N/cm.*$^2 = 2{\cdot}70$ *N/mm.*^{2}

Maximum *tensile stress* in concrete
$= 170{\cdot}5 - 99{\cdot}5 = 71$ *N/cm.*$^2 = 0{\cdot}71$ *N/mm.*^{2}

Compressive stress in steel $= 18\left[99{\cdot}5 + \frac{10}{15} \times 170{\cdot}5\right]$ *N/cm.*^{2}
$= 3837{\cdot}6$ *N/cm.*$^2 = 38{\cdot}376$ *N/mm.*^{2}

Tensile stress in steel $= 18\left[\frac{10}{15} \times 170{\cdot}5 - 99{\cdot}5\right]$ *N/cm.*^{2}
$= 256$ *N/cm*^{2}. $= 2{\cdot}56$ *N/mm.*^{2}

Problem 54 *The horizontal thrust for a two-hinged arch is found to be 11,000 kg. The crown section of the arch is a square of 30 cm. side. It is reinforced with 6·03 cm.*^{2} *of steel above the centroidal axis and an equal amount of steel below the axis. The centres of steel are respectively 5 cm. from the upper and lower edges of the crown section. The section is also subjected to a bending moment of 100,000 kg. cm. Assuming that the concrete does not fail in tension, calculate the maximum tensile and compressive stresses in concrete at the crown section Find also the tensile and compressive stresses in steel. Take m = 18.*

Solution. Equivalent concrete area of the arch section
$A_e = 30 \times 30 + (18-1) \times 2 \times 6{\cdot}03$ *cm.*^{2}
$= 1105$ *cm.*^{2}

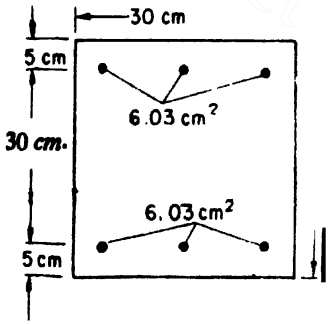

Fig. 79

Equivalent moment of Inertia.

$$= I_e = \frac{30 \times 30^3}{12} + (18-1)2 \times 6{\cdot}03 \text{ cm.}^4$$

$= 88000$ *cm.*^{4}

Extreme stresses in concrete

$$= \frac{11000}{1105} \pm \frac{100,000}{88000} \times 15 \text{ kg./cm}^2.$$

$= 9{\cdot}95 \pm 17{\cdot}05$ *kg./cm*^{2}.

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$\therefore$ Maximum *compressive stress* in concrete

$= 9{\cdot}95 + 17{\cdot}05$ *kg./cm.*^{2}

$= 27$ *kg./cm.*^{2}

Maximum *tenstle stress* in concrete

$= 17{\cdot}05 - 9{\cdot}95 = 7{\cdot}10$ *kg./cm.*^{2}

Compressive stress in steel $= 18\left[9{\cdot}95 + \frac{10}{15} \times 17{\cdot}05\right]$ *kg./cm.*^{2}

$= 383{\cdot}76$ *kg./cm.*^{2}

Tensile stress in steel $= 18\left[\frac{10}{15} \times 17{\cdot}05 - 9{\cdot}95\right]$ *kg./cm.*^{2}

$= 25{\cdot}6$ *kg./cm.*^{2}

Problem 55. *An R.C. column 30 cm. × 30 cm. is reinforced with four bars of 26 mm. diameter placed at an effective cover of 5 cm. Find how far from the centre, the line of thrust may pass along the YY axis without causing tension in concrete. Take m = 18.*

Solution. Area of 4 bars of 26 *mm.* diameter $= 21{\cdot}24$ *cm*^{2}.

Let the maximum eccentricity by *e cm.*

(*See* Fig. 80)

Equivalent concrete area $= 30 \times 30 + (18-1)\, 21{\cdot}24$ *cm.*^{2}

$= 1261$ *cm.*^{2}

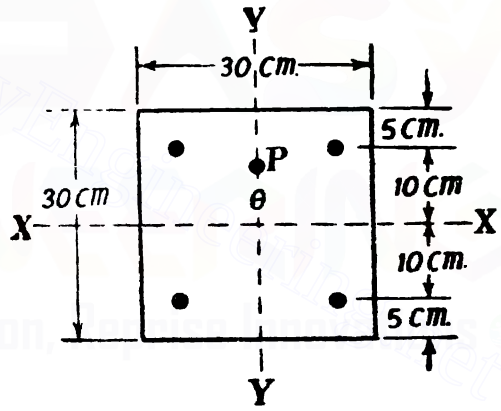

Fig. 80

Equivalent moment of Inertia.

$= \frac{30 \times 30^3}{12} + (18-1)\, 21{\cdot}24 \times 10^2$ *cm.*^{4}

$= 103600$ *cm.*^{4}

If tension in concrete should be avoided the direct and bending stresses should be equal for the section.

Let P be the applied load

$$\therefore \quad \frac{P}{1261} = \frac{P.\,e}{103600} \times \frac{30}{2}$$

$$\therefore \quad e = 5{\cdot}48 \text{ cm.}$$

Problem 56. *A two-hinged arched rib has a section 30 cm. wide and 90 cm. deep, at the crown. The section is reinforced with six bars of 22 mm. diameter at the top and an equal reinforcement at the bottom. The reinforcements are placed at an effective cover of 5 cm.*

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from the respective edges. If the resultant thrust on the section is 100,000 kg. inclined at 3° with the tangent to the arch centre line and acting on the vertical axis and 10 cm. from the centre line of the section, determine the stresses in concrete and steel at the top and bottom of the section ; modular ratio may be taken as 15. [*UPSC*]

Solution. *See* Fig. 81.

Since $e = 10$ *cms* which is less than 90/6 the stresses will remain compressive.

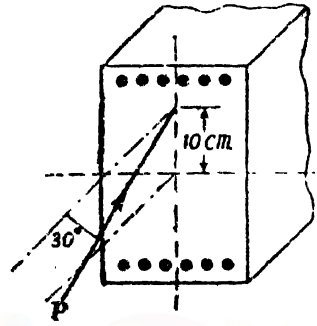

Fig. 81

Thrust normal to the cross-section

$= 100{,}000 \cos 3°$

$= 99860$ *kg.*

Equivalent area of the section

$= 30 \times 90 + (15-1) \times 12 \times 3{\cdot}80$

$= 3338{\cdot}4$ *cm.*^{2}

Equivalent Moment of Inertia

$$\frac{30 \times 90^3}{12} + 14 \times 12 \times 3{\cdot}8\,(45-5)^2 \text{ cm.}$$

$= 18,22,500 + 10,21,440$ *cm.*^{4}

$= 28,43,940$ *cm.*^{4}

$\therefore$ Extreme stresses in concrete

$$= -\frac{99860}{3338{\cdot}4} \pm \frac{99860 \times 10}{28.43,940} \times \frac{90}{2} \text{ kg./cm.}^2$$

$= 29{\cdot}92 \pm 15{\cdot}81$ *kg./cm.*^{2}

$\therefore$ Maximum compressive stress $= 45{\cdot}73$ *kg./cm.*^{2}

Minimum compressive stress $= 14{\cdot}11$ *kg./cm.*^{2}

Stress in steel $= 1{\cdot}5\left[29{\cdot}92 \pm \frac{45-5}{45}(15{\cdot}81)\right]$ *kg./cm.*^{2}

$= 659{\cdot}55$ *kg./cm.*^{2} compressive in steel at top and $238{\cdot}95$ *kg./cm.*^{2} compressive in steel at bottom.

Case 2. When the eccentricity is so large that excessive tensile stresses are developed. In this case the concrete on the tension zone is ignored and the position of the neutral axis is found from first principles

Fig. 82 shows the section of an *R.C.* member subjected to an eccentric load. The stress at a point is proportional to its distance from the neutral axis.

Let the stress at any point be given by $c = ry$

where y = distance of the point from the neutral axis

and r = a constant of proportionality

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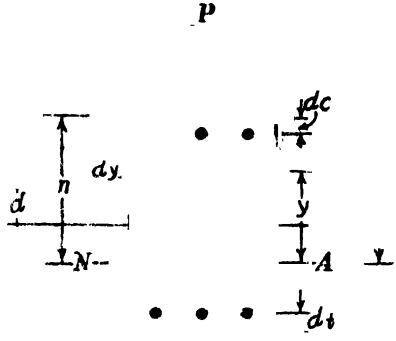

Fig. 82

Total compression − Total tension = Thrust on the section

$$\therefore \int_0 b\,dy\,ry + (m-1)A_c\,r(n-d_c) - mA_t\,r(d-n) = P$$

$$\therefore \quad P = r\left[\frac{bn^2}{2} + (m-1)\,A_c(n-d_c) - mA_t(d-n)\right]$$

Similarly, Moment of resistance = bending moment

or Total moment of the induced compressive and tensile stresses about the neutral axis } = bending moment.

$$\therefore \quad bn \cdot \frac{n\,r}{2} \cdot \tfrac{2}{3}n + (m-1)A_c\,r\,(n-d_c)^2 + mA_t\,r\,(d-n)^2 = Pe_n$$

$$\therefore Pe_n = r\left[\frac{bn^3}{3} + (m-1)A_c(n-d_c)^2 + mA_t\,(d-n)^2\right]$$

$$\therefore \quad e_n = \frac{Pe_n}{P} = \frac{\dfrac{bn^3}{3} + (m-1)A_c(n-d_c)^2 + mA_t(d-n)^2}{\dfrac{bn^2}{2} + (m-1)A_c(n-d_c) - mA_t(d-n)}$$

But $e_n = e + n - \dfrac{D}{2}$

$$\therefore e + n - \frac{D}{2} = \frac{\dfrac{bn^3}{3} + (m-1)A_c(n-d_c)^2 + mA_t(d-n)^2}{\dfrac{cn^2}{2} + (m-1)A_c(n-d_c) - mA_t(d-n)}$$

Hence, for a given position of the eccentric load the depth of neutral axis can be determined from the above relation.

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Problem 57. *The haunch section of a bowstring girder bridge is subjected to a bending moment of 1,931,500 kg. cm. and a normal thrust of 81,260 kg. The section is 40 cm. wide and 80 cm. deep. Eight bars of 18 mm. dia. are provided at the top and an equal rein. forcement at the bottom. The reinforcements are placed at an effective cover of 6 cm. from the respective edges. Determine the maximum stress in concrete and steel. The modular ratio may be taken as 15.*

Solution. $A_c = A_t = 20{\cdot}36\ cm.^2$

See Fig. 83.

Eccentricity from the centre of the section

$$= e = \frac{M}{P} = \frac{1,93,15,000}{81260}\ cm.$$

$$= 23{\cdot}77\ cm.$$

The eccentricity is considerably large to produce tensile stresses.

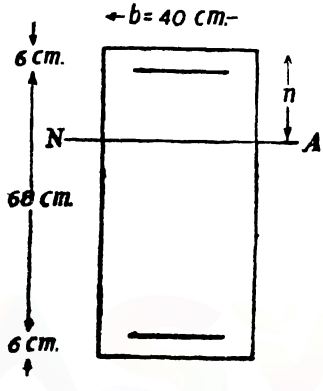

Fig. 83

Hence the depth of the neutral axis is given by

$$e + n - \frac{D}{2} = \frac{\frac{bn^3}{3} + (m-1)A_c(n-d_c)^2 + mA_t(d-n)^2}{\frac{bn^2}{2} + (m-1)A_c(n-d_c) - mA_t(d-n)}$$

$$23{\cdot}77 + n - \frac{80}{2} = \frac{\frac{40n^3}{3} + 14 \times 20{\cdot}36(n-6)^2 + 15 \times 20{\cdot}36(74-n)^2}{\frac{40n^2}{2} + 14 \times 20{\cdot}36(n-6) - 15 \times 20{\cdot}36(74-n)}$$

Simplifying, we get

$$n^3 - 48{\cdot}69n^2 + 2209{\cdot}05n - 193212{\cdot}471 = 0$$

or $$n[n^2 - 48{\cdot}59n + 2209{\cdot}05] = 193212{\cdot}471$$

Solving by trial and error, we have

When $n = 63$, $LHS = 195966{\cdot}54$

$n = 62$, $LHS = 188124{\cdot}74$

$n = 62{\cdot}7$ $LHS = 193593{\cdot}77$

$n = 62{\cdot}6$ $LHS = 192796{\cdot}73$

$n = 62{\cdot}66$ $LHS = 193269{\cdot}13$

$n = 62{\cdot}652$ $LHS = 193206{\cdot}24$

$n = 62{\cdot}653$ $LHS = 193213{\cdot}71$

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Taking moments about the tensile steel,

$$40n\,\frac{c}{2}\left(74-\frac{n}{3}\right)+(15-1)\times 20\cdot 36\left(\frac{n-6}{n}\right)c\times 68$$

$$=81260\times(34+23\cdot 77)$$

$$\therefore\quad 20\times 62\cdot 653c\left(74-\frac{62\cdot 653}{3}\right)+14\times 20\cdot 36\times\frac{56\cdot 653}{62\cdot 653}\times 68$$

$$=81260\times 57$$

$$66557\cdot 5c+17526\cdot 5c=46,94,390\cdot 2$$

$$\therefore\quad c=55\cdot 83\ kg./c$$

Corresponding stress in tensile steel is given by

$$\frac{15\times 55\cdot 83}{t}=\frac{62\cdot 653}{74-62\cdot 653}$$

$$\therefore\quad t=\frac{15\times 55\cdot 83\times 11\cdot 347}{62\cdot 653}\ kg./cm.^2$$

$$t=151\cdot 7\ kg./cm.^2$$

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10
Design of Slabs

§99. Simply Supported Rectangular Slabs Spanning in One Direction

These slabs are generally designed on the assumption that they consist of a number of beams of breadth one *metre*. Strictly, the slab is monolithic and is not composed of individual separate beams. It is only from the point of view of easy computations we consider the slab to be formed of a number of beams of width one *metre*.

Effective span. The effective span of a simply supported slab shall be taken as the lesser of the following :

(*i*) Distance between the centres of bearings.
(*ii*) Clear span plus the effective depth.

Thickness of a slab. The following table gives the maximum values of the ratio of span to depth.

Type of slab	*Ratio of span to depth*
Simply supported and spanning in one direction.	30
Continuous and spanning in one direction.	35
Simply supported and spanning in two directions.	35
Continuous and spanning in two directions.	40
Cantiever slabs.	12

For normal conditions of loading the thickness of the slab may be taken as 4 *cms.* per *metre* run of the span. A slab shall not ordinarily be less than 10 *cms.* in thickness.

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§100. Reinforcement

The reinforcement for a slab spanning in one direction consists of main bars and distribution or temperature bars. These two sets of bars form a mesh of reinforcement. The minimum reinforcement in either direction shall be 0·15 percent of the total cross-sectional area. (However, this value can be reduced to 0·12 percent when high strength deformed bars, or welded fabric are used)

Main bars. The reinforcement constituting the main bars shall be based on the maximum bending moment. The reinforcement shall not be less than 0·15 percent of the gross sectional area. The pitch of the main bars shall not exceed the following :

(*i*) Three time the effective depth of slab, and

(*ii*) 45 *cms.*

Distribution or temperature reinforcement. These are reinforcements provided running at right angles to the main steel in order to distribute the load and the temperature and shrinkage stresses. This steel shall have an area at least equal to 0·15 percent of the gross area of the slab. The pitch of the distribution bars shall not exceed the following :

(*ii*) Five times the effective depth of the slab, and

(*ii*) 45 *cms.*

Diameter of bars. The diameter of the main bars may be from 8 *mm.* to 14 *mm.* generally. However, the diameter of main bars shall not exceed one-eighth of the total thickness of the slab. For distribution steel 6 *mm.* to 8 *mm.* diameter bars are generally used.

§101. Cover to Reinforcement

The minimum cover to outside of the main bars shall not be less than the following :

(*i*) 15 *mm.*, and

(*ii*) Diameter of the main bar.

§102. Dead Load

The dead load to which a slab is subjected to, consists of the weight of the slab, floor finishes, and ceiling finish when provided. In all calculations of loading, unless otherwise mentioned the weight of reinforced concrete shall be taken as 2500 $kg./metre^3$ or 25000 $N/metre^3$ and that of plain concrete as 2400 $kg./metre^3$ or 24000 $N/metre^3$. The weight of the finishes depends on the type of finish provided. In most cases before the final finish is applied it is usual to apply a screening coat 18 to 20 *mm.* thick so as to form a good flat surface on which the finishes can be applied. The finishes may consist of Terrazo, Granolithic, Mosaic, saw dust filler, rubber sheets, deal boards, hard wood blocks, *etc.*

The following table gives approximately the weight per square *metre* of various types of finish.

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Floor finish	*Weight in kg./metre²*	*Weight in N/metre²*
Terrazo 20 *mm.*	46	460
Granolithic 25 *mm.*	58	580
Mosaic 7 *mm.* bedded in mortar	51	510
Saw dust filler 20 *mm.*	31	310
Rubber sheets 7 *mm.*	12	120
Hard-wood blocks 25 *mm.*	22	220

§103. Steps to be followed in the Design of the Slab

The following steps may be adopted in the design of the slab :

1. Assuming suitable bearings (not less than 10 *cm.*) find the span of the slab between the centres of bearing.

2. Assume the thickness of the slab (say at 4 *cm.* per *metre* run of the span).

3. Now find the effective span which is the lesser of :

(*a*) Distance between centres of bearing, and

(*b*) Clear span+Effective depth.

4. Now find the dead load and the live load per *square metre* of the slab.

5. Determine the maximum bending moment for a one *metre* wide strip of the slab. The maximum bending moment per *metre* width $M=\frac{wl^2}{8}$

where w=total load intensity per *square metre* of the slab.

6. Equate the balanced moment of resistance to the maximum bending moment.

$\therefore \quad Q\,bd^2=Q\times 100\,d^2=M.$

Find the effective depth d from the above relation and compare the value obtained with what is available. The effective depth available should not be less than the effective depth required.

7. Now calculate the main steel required per *metre* width

$$A_t=\frac{M}{\left(\begin{matrix}\text{Safe stress}\\ \text{in steel}\end{matrix}\right)\times\left(\begin{matrix}\text{Lever}\\ \text{arm}\end{matrix}\right)}$$

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For M 150 concrete in M.K.S. designs, and for M 15 concrete in S.I. designs take lever arm $a = 0{\cdot}87\ d$.

Select a suitable diameter for the main reinforcement and calculate the spacing.

$$\text{Spacing} = \frac{\text{Area of 1 bar} \times 100}{A_t}$$

Shear and bond stresses are usually very low in the slabs which we usually come across (In bridge slabs, however, check for shear and bond stresses may be made).

Design 1 **Simply supported slab.** *Design a simply supported slab supported on masonry walls to the following requirements.*

(*i*) *Clear span* $= 2{\cdot}75$ *metres.*

(*ii*) *Live load* $= 370\ kg./m.^2$

Use M 150 concrete.

Solution Let us provide a bearing of 12 *cms.* at each end.

Span between centres of bearing $= 2{\cdot}75 + 0{\cdot}12 = 2{\cdot}87\ m.$

Thickness of slab $= 4$ *cm.* per *metre* of span

$= 4 \times 2{\cdot}87 = 11{\cdot}48$ *cm.* say 12 *cm.*

Loads. The load per *square metre* of the slab consists of the following :

D L. of slab (12 *cm.* thick) $= 12 \times 25$	$= 300\ kg./m^2$
Live load	$= 370\ kg./m^2$
Total	$670\ kg./m^2$

Providing 12 *mm.* bars at a clear cover of 15 *mm.* effective cover $= 15 + 6 = 21$ *mm.* $= 2{\cdot}1$ *cm.*

Effective depth $= d = 12 - 2{\cdot}1 = 9{\cdot}9$ *cm.*

Effective span This is taken as the lesser of the following :

(*i*) Centre to centre distance between bearings $= 2{\cdot}87\ m.$

(*ii*) Clear span + effective depth $= 2{\cdot}75 + 0{\cdot}099 = 2{\cdot}849\ m.$ say $2{\cdot}85\ m.$

$\therefore$ Effective span $= 2{\cdot}85\ m$

Consider a 1 m wide strip of the slab.

$$\text{Maximum bending moment} = M = \frac{wl^2}{8}$$

$$= \frac{670 \times 2{\cdot}85^2}{8} \times 100\ kg.\ cm.$$

$$= 68026\ kg.\ cm.$$

Equating the moment of resistance to the maximum bending moment,

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$8{\cdot}5 \times 100\ d^2 = 68026 \qquad \therefore\ d = 9\ cm.$

But, effective depth available $= 9{\cdot}9\ cm.$

$$\therefore \quad A_t = \frac{68026}{1400 \times 0{\cdot}87 \times 9{\cdot}9} = 5{\cdot}64\ cm.^2$$

$$\text{Spacing of 12 } mm. \text{ diameter bars} = \frac{1{\cdot}13 \times 100}{5{\cdot}64}$$

$$= 20\ cm.\ c/c.$$

Provide 12 mm @ 20 cm. c/c.

Alternate bars may be bent-up at one-seventh of the span.

Distribution steel $= 0{\cdot}15\%$ of gross area

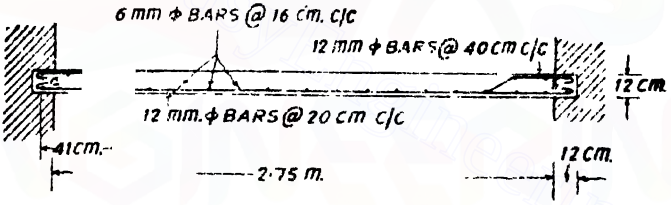
$$= \frac{0{\cdot}15}{100} \times 100 \times 12 = 1{\cdot}80\ cm.^2$$

Spacing of 6 *mm.* ϕ diameter bars

$$= \frac{0{\cdot}28 \times 100}{1{\cdot}8} = 16\ cm.$$

Provide 6 mm. ϕ @ 16 *cm. c/c.*

Fig. 84 shows details of reinforcement.



12 mm. BARS @ 40 cm c/c a

12 mm. φ BARS @ 40 cm. c/c

The bars may also be arranged as shown below :

12 mm @ 40 cm c/c a

12 mm @ 40 cm c/c b

Fig. 84.

Design 2. (S.I.). *Design a simply supported slab on masonry walls to the following requirements.*

(*i*) *Clear span* $= 2{\cdot}75$ *metres, and*

(*ii*) *Live load* $= 3700\ N/metre.^2$

Use M 15 concrete.

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Solution.

Let us provide a bearing of 12 *cm.* at each end.

Span between centres of bearing $= 2{\cdot}75 + 0{\cdot}12 = 2{\cdot}87$ *m.*

Thickness of slab : 4 *cm.* per *metre* run of the span

$= 4 \times 2{\cdot}87 = 11{\cdot}48$ *cm. say* 12 *cm.*

Loads. The load per *square metre* of the slab consists of the following :

D.L. of slab (12 *cm.* thick)	$= 12 \times 250$	$= 3000$ *N/metre*^{2}
L.L.		$= 3700$ *N/metre*^{2}
	Total	6700 *N/metre*^{2}

Provide 12 *mm.* ϕ bars at a clear cover of 15 *mm.*, effective cover $= 15 + 6 = 21$ *mm.* $= 2{\cdot}1$ *cm.*

Effective depth $= 12 - 2{\cdot}1 = 9{\cdot}90$ *cm.*

Effective span. This is taken as the lesser of the following :

(*i*) Centre to centre distance between bearings $= 2{\cdot}87$ *m*

(*ii*) Clear span + effective depth $= 2{\cdot}75 + 0{\cdot}099$

$= 2{\cdot}849$ *m* say $2{\cdot}85$ *m*

$\therefore$ Effective span $= 2{\cdot}85$ *m*

Consider a 1 *metre* wide strip of the slab.

Maximum bending moment $= M = \frac{wl^2}{8}$

$= 6700 \times \frac{2{\cdot}85^2}{8} \times 100$ *N cm.*

$= 680260$ *N cm.*

Taking $c = 5$ *N/mm.*$^2 = 500$ *N/cm.*^{2}, $t = 140$ *N/mm.*^{2}

$= 14000$ *N/cm.*^{2}

and $m = 18$ $M.R. = 85\, bd^2$ *N cm.*

Equating the moment of resistance to the maximum bending moment,

$$85 \times 100\, d^2 = 680260$$

$$\therefore \quad d = 9 \text{ cm.}$$

But effective depth available $= 9{\cdot}90$ *cm.*

$$\therefore \quad A_t = \frac{680260}{14000 \times 0{\cdot}87 \times 9{\cdot}90} = 5{\cdot}64 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars $= \frac{1{\cdot}13 \times 100}{5{\cdot}64} = 20$ *cm.*

Provide 12 mm. ϕ *bars @ 20 cm. c/c*

Alternate bars may be bent up at one-seventh of the sp

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Distribution steel $= 0{\cdot}15$ percent of gross area

$$= \frac{0{\cdot}15}{100} \times 100 \times 12 = 1{\cdot}80 \text{ cm.}^2$$

Spacing of 6 *mm.* ϕ bars $$= \frac{0{\cdot}28 \times 100}{1{\cdot}8} = 16 \text{ cm.}$$

Provide 6 mm. ϕ *@ 16 cm. c/c.*

Fig. 84 shows the reinforcement details.

Design 3. *Design a cantilever slab to carry a live load of 200 kg /m*^{2}. *The overhang of the slab is 1·20 metre.*

Solution. Assume an average thickness of 6 *cm.*

Dead load	$= 25 \times 6 = 150$	*kg./m.*^{2}
Live load	200	*kg./m.*^{2}
Total	350	*kg /m.*^{2}

Consider a 1 *metre* wide strip of the slab

Maximum bending moment

$$= M = \frac{wl^2}{2}$$

$$= 350 \times \frac{1{\cdot}2^2}{2} \times 100 \text{ kg. cm.}$$

$$= 25200 \text{ kg. cm.}$$

Equating the moment of resistance to the bending moment,

$$8{\cdot}5 \times 100\, d^2 = 25200$$

$$d = \sqrt{\frac{25200}{850}} = 5{\cdot}44 \text{ cm.}$$

Providing 8 *mm.* diameter bars at a clear cover of 15 *mm.*,

Effective cover $= 15 + 4 = 19$ *mm.* $= 1{\cdot}90$ *cm.*,

$\therefore$ Overall depth $= 5.44 + 1{\cdot}90 = 7{\cdot}34$ *cm. say 8 cm.*

Actual effective depth $= d = 8 - 1{\cdot}90 = 6{\cdot}10$ *cm.*

The thickness may be reduced to 5 *cm.* at the free end

$$A_t = \frac{25200}{1400 \times 0{\cdot}87 \times 6{\cdot}10} = 3{\cdot}39 \text{ cm.}^2$$

Spacing of 8 *mm* bars $= \frac{0{\cdot}5 \times 100}{3\,39} = 14{\cdot}75$ *cm.*

Provide 8 mm. ϕ *bars @ 14 cm. c/c.*

Distribution steel $= \frac{0{\cdot}15}{100} \times 100 \times 8 = 1{\cdot}20$ *cm.*^{2}

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Spacing of 6 *mm.* bars $= \frac{0{\cdot}28 \times 100}{1{\cdot}20} = 23$ *cm.*

Provide 6 mm ϕ bars @ 20 cm. c/c.

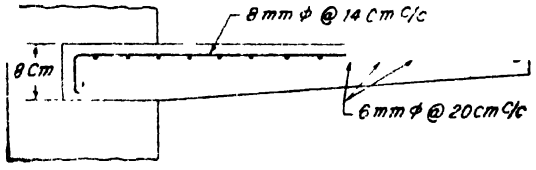

Fig. 85

Fig. 85 shows the details of reinforcement.

Design 4. (S.I.). *Design a cantilever slab to carry a live load of 2000 N/metre². The overhang of the slab is 1·20 metre.*

Solution. Assume an average thickness of 6 *cm.*

Dead load $= 250 \times 6 = 1500$ *N/metre²*

Live load $= 2000$ *N/metre²*

Total $= 3500$ *N/metre²*

Consider a 1 *metre* wide strip of the slab.

Maximum bending moment

$$= M = \frac{wl^2}{2}$$

$$= 3500 \times \frac{1{\cdot}2^2}{2} \times 100 \text{ } N \text{ } cm.$$

$$= 252000 \text{ } N \text{ } cm.$$

Taking $C = 5$ *N/mm.²* $= 500$ *N/cm.²,*

$t = 140$ *N/mm.²* $= 14000$ *N/cm.²* and $m = 18$,

and equating the moment of resistance to the maximum bending moment

$$85 \text{ } bd^2 = 85 \times 100 \text{ } d^2 = 252000$$

$$\therefore \sqrt{\frac{252000}{8500}} = 5{\cdot}44 \text{ } cm.$$

providing 8 *mm.* diameter bars at a clear cover of 15 *mm.*

Effective cover $= 15 + 4 = 19$ *mm.* $= 1{\cdot}90$ *cm.*

$\therefore$ Overall depth $= 5{\cdot}44 + 1{\cdot}90 = 7{\cdot}34$ *cm. say 8 cm.*

Actual effective depth

$$= d = 8 - 1{\cdot}90 = 6{\cdot}10 \text{ } cm.$$

The thickness may be reduced to 5 *cm.* at the free end.

$$A_t = \frac{252000}{14000 \times 0{\cdot}87 \times 6{\cdot}10} = 3{\cdot}39 \text{ } cm.^2$$

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Spacing of 8 *mm.* bars

$$= \frac{0{\cdot}5 \times 100}{3{\cdot}39} = 14{\cdot}75 \text{ cm}$$

Provide 8 mm. ϕ bars @ 14 cm. c/c.

Distribution steel $= \frac{0{\cdot}15}{100} \times 100 \times 8 = 1{\cdot}20 \text{ cm.}^2$

Spacing of 6 *mm.* bars

$$= \frac{0{\cdot}28 \times 100}{1{\cdot}20} = 23 \text{ cm.}$$

Provide 6 mm. ϕ bars @ 20 cm. c/c.

Fig. 85 shows the details of reinforcement.

Design 5. *The roof of a cycle-stand consists of a reinforced concrete slab which cantilevers 3·50 metres on each side of a central reinforced concrete beam, supported on columns. Allowing a live load of 175 kg /metre^2, design the cantilevering slab.*

Solution. Thickness of the slab near the beam

$$= \frac{1}{12} \times \text{cantilevering length}$$

$$= \frac{3{\cdot}5 \times 100}{12} \text{ cm.}$$

$$= 29{\cdot}2 \text{ cms. say } 30 \text{ cms.}$$

Let the thickness of the slab at the ends be 10 *cms.*

$\therefore$ Mean thickness of the slab

$$= \frac{30+10}{2} = 20 \text{ cm.}$$

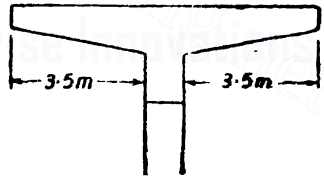

Fig. 86

Loads. The load per square *metre* of the slab consists of the following :

Dead load (20 *cms.*)	
$=20 \times 25 = 500$ *kg./m.*^{2}	
finishing $= 50$ *kg./m.*^{2}	
Live load	$=175$ *kg./m.*^{2}
Total	$=725$ *kg./m.*^{2}

Consider a one *metre* wide strip of the slab.

Maximum bending moment

$$= \frac{wl^2}{2}$$

$$= \frac{725 \times 3{\cdot}5^2}{2} \times 100 \text{ kg. cm.}$$

$$= 444{,}100 \text{ kg. cm.}$$

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Adopting $c = 50\ kg./cm.^2$

$t = 1400\ kg./cm.^2$

and $m = 18$

Equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}50 \times 100\ d^2 = 444{,}100$$

$$\therefore \quad d = 23\ cm.$$

Using 12 *mm.* bars at a clear cover of 15 *mm.*, effective depth available

$$= 30 - 2{\cdot}10 = 27{\cdot}90\ cm.$$

$$\therefore \quad A_s = \frac{444{,}100}{1400 \times 0{\cdot}87 \times 27{\cdot}9}\ cm.^2$$

$$= 13{\cdot}07\ cm^2$$

$\therefore$ Spacing of 12 *mm* diameter bars

$$= \frac{1{\cdot}13 \times 100}{13{\cdot}07}\ cm.$$

$$= 8{\cdot}6\ cms\ say\ 8\ cms\ centres$$

Distribution steel $= 0{\cdot}15\%$ of gross area

$$= \frac{0{\cdot}15}{100} \times 30 \times 100\ cm.^2$$

$$= 4{\cdot}50\ cm.^2$$

$\therefore$ Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{4{\cdot}5}\ cms.$$

$$= 11\ cms.$$

Curtailment of Bars

Let alternate bars be curtailed at half the cantilevering length.

Area of steel per *metre* width available

$$= \frac{100}{16} \times 1{\cdot}13\ cm.^2 = 7{\cdot}06\ cm.^2$$

B.M. at this section $= \frac{wx^2}{2}$

$$= \frac{725}{2} \times \left(\frac{3{\cdot}5}{2}\right)^2 \times 100\ kg.\ cm.$$

$$= 111{,}000\ kg.\ cm.$$

Depth of slab $= 20\ cms.$ overall.

$\therefore$ Requirement of steel

$$= \frac{111{,}000}{1400 \times 0{\cdot}87 \times 17{\cdot}9}\ cm.$$

$$= 5{\cdot}1\ cm.^2$$

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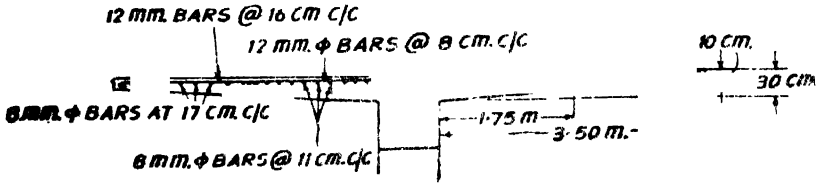

Fig. 87

Hence the section is safe.

Distribution steel at this section

$$\frac{0{\cdot}15}{100} \times 20 \times 100 \ cm.^2 = 3 \ cm.^2$$

∴ Spacing of 8 *mm.* diameter bars

$$\frac{0{\cdot}5 \times 100}{3} \ cm. = 17 \ cm.$$

Design 6. (S.I.). *The roof of a cycle stand consists of a reinforced concrete slab which cantilevers 1·5 metres on each side of a central reinforced concrete beam supported on columns. Allowing a live load of 1750 N/metre², design the cantilevering slab.*

Solution Thickness of the slab near the beam

$$= \frac{1}{12} \times \text{cantilevering length}$$

$$\frac{3{\cdot}5 \times 100}{12} = 29{\cdot}2 \ cm. \ say \ 30 \ cm.$$

Let the thickness of the slab at the ends be 10 *cm.*

∴ Mean thickness of the slab

$$= \frac{30+10}{2} = 20 \ cm.$$

Loads. The load per *square metre* of the slab consists of the following :

Dead load (20 *cm*) : $250 \times 20 = 5000$ N/*metre*²	
Finishing	= 500 N/*metre*²
Live load	= 1750 N/*metre*²
Total	= 7250 N/*metre*²

Consider a one *metre* wide strip of the slab.

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Maximum bending moment

$$= \frac{wl^2}{2} = \frac{7250 \times 3{\cdot}5^2}{2} \times 100 \ N\ cm.$$

$$= 4{,}441{,}000 \ N\ cm.$$

Taking $c = 5\ N/mm.^2 = 500\ N/cm.^2$

$t = 140\ N/mm.^2 = 14000\ N/cm.^2$

and $m = 18$

and, equating the moment of resistance to the maximum bending moment,

$$85\ bd^2 = 85 \times 100\ d^2 = 4{,}441{,}000$$

$$\therefore \quad d = 23\ cm.$$

Using 12 *mm.* bars at a clear cover of 15 *mm.*,

Effective depth available

$$= 30 - 2{\cdot}10 = 27{\cdot}90\ cm.$$

$$\therefore \quad A_t = \frac{4{,}441{,}000}{14000 \times 0{\cdot}87 \times 27{\cdot}90} = 13{\cdot}07\ cm.^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{13{\cdot}07} = 8{\cdot}60\ cm.\ say\ 8\ cm.\ centres$$

Distribution steel $= 0{\cdot}15\%$ of gross area

$$= \frac{0{\cdot}15}{100} \times 30 \times 100\ cm.^2 = 4{\cdot}50\ cm.^2$$

Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}50 \times 100}{4{\cdot}50}\ cm. = 11\ cms.$$

Curtailment of bars

Let alternate bars be curtailed at half the cantilevering length.

Area of steel per *metre* width available,

$$= \frac{100}{16} \times 1{\cdot}13\ cm.^2 = 7{\cdot}06\ cm.^2$$

B.M. at this section $= \frac{wx^2}{2} = \frac{7250}{2}\left(\frac{3{\cdot}5}{2}\right)^2 \times 100\ N\ cm.$

$$= 1{,}110{,}000\ N\ cm.$$

Depth of the slab $= 20\ cm.$

$$\therefore \quad \text{Steel required} = \frac{1{,}110{,}000}{14000 \times 0{\cdot}87 \times 17{\cdot}90}\ cm.^2 = 5{\cdot}10\ cm.^2$$

But steel area available

$$= 7{\cdot}06\ cm.^2$$

Hence the design is safe.

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Distribution steel at this section

$$= \frac{0{\cdot}15}{100} \times (20 \times 100) = 3 \text{ cm.}^2$$

$\therefore$ Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{3} = 17 \text{ cm.}$$

Design 7. *Fig. 88 represents the cross-section of a roof slab supported on walls. Allowing a live load of 200 kg./metre², design the slab. Use M 150 concrete. Assume weight of R.C.C. = 2400 kg./metre³.*

Solution

Thickness of slab = about 4 *cms.* per *metre* run of span

$= 4 \times 4{\cdot}5 = 18$ *cms.*

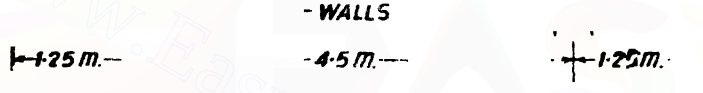

Fig. 88

Loads. The load per square *metre* on the slab consists of the following :

Dead load (18 *cm*) $18 \times 24 = 432$ *kg./metre²*,

Floor finish $= 50$ *kg./metre²*,

Total dead load $= 482$ *kg./metre²*.

Live load $= 200$ *kg./metre²*.

Design of the Span between the Two Supports

The maximum positive bending moment for this part of the slab will occur when the live load covers only this portion. Hence the loading on this part will be taken as $482 + 200 = 682$ *kg./m.²* while on the cantilevering parts the loading should be taken at 482 *kg/m²*.

Consider a one-*metre* wide strip of the slab.

Maximum positive bending moment at the centre

$$= \frac{682 \times 4{\cdot}5^2}{8} - \frac{482 \times 1{\cdot}25^2}{2} \text{ kg-m.}$$

$= 1{,}350$ *kg.-m.*

$= 135{,}000$ *kg. cm.*

Adopting $c = 50$ *kg./cm.²*, $t = 1400$ *kg./cm.²*, $m. = 18$

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and equating the moment of resistance to the maximum bending moment, we have

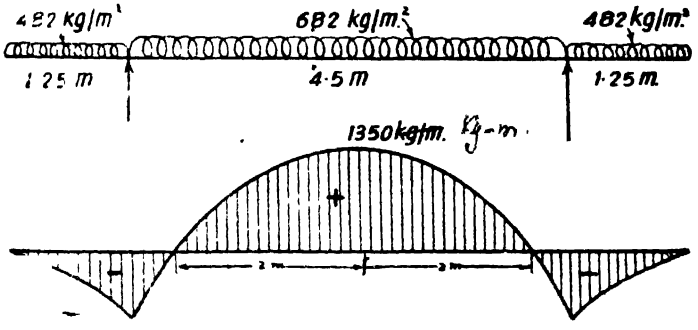

Fig. 89

$$8{\cdot}50 \times 100\, d^2 = 135{,}000$$

$$d = 13 \text{ cms.}$$

But, effective depth available

$$= 18 - 2 = 16 \text{ cms.}$$

$$A_t = \frac{135{,}000}{1400 \times 0{\cdot}87 \times 16} \text{ cm.}$$

$$= 6{\cdot}93 \text{ cm.}^2$$

Spacing of 10 *mm.* diameter bars

$$= \frac{0{\cdot}79 \times 100}{6{\cdot}93} \text{ cms.}$$

$$\simeq 11 \text{ cms. centres}$$

Distribution steel $= 0{\cdot}15\%$ of gross area

$$= \frac{0{\cdot}15}{100} \times 18 \times 100 \text{ cm.}^2$$

$$= 2{\cdot}70 \text{ cm.}^2$$

Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{2{\cdot}7} \text{ cms.}$$

$$= 18{\cdot}52 \text{ cms, say } 18 \text{ cms. centres}$$

Points of contraflexure

B.M. at any section distant x *metres* from the centre

$$= M = 1350 - 682 \times \frac{x^2}{2}$$

$\therefore$ For points of contraflexure

$$1350 - \frac{682 \times x^2}{2} = 0$$

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$x = 1{\cdot}99$ *metres* say 2 *metres* from the centre

or $2{\cdot}25 - 2 = 0{\cdot}25$ *m.* from the support.

Design of Cantilevering Part of the Slab.

This will be subjected to the maximum hogging bending moment when the live load is applied on the cantilevering parts.

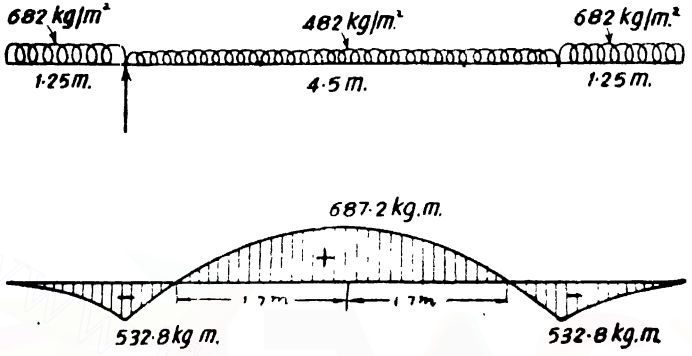

Fig. 90

Maximum negative bending moment

$$= -\frac{682 \times 1{\cdot}25^2}{2} \text{ kg. m.} = -532{\cdot}8 \text{ kg. m.}$$

$$A_s' \quad \frac{53280}{1400 \times 0{\cdot}87 \times 16} \text{ cm.}^2 = 2{\cdot}73 \text{ cm.}^2$$

Spacing of 10 *mm.* diameter bars

$$\frac{0{\cdot}79 \times 100}{2{\cdot}73} \text{ cm.} = 28{\cdot}94 \text{ cm.}$$

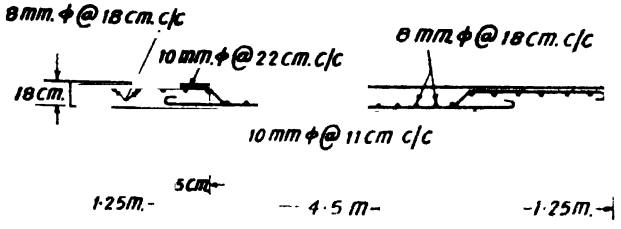

Fig. 87

(Let alternate bars of the bottom bars of the supported span be bent-up to provide 10 *mm.* diameter bars at 22 *cms.* centres).

B.M. at centre $= \frac{482 \times 4{\cdot}5^2}{8} = 532{\cdot}8 \text{ kg. m.}$

$= 687{\cdot}2 \text{ kg. m.}$

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Point of contraflexure

Let the bending moment at a distance x from the centre be zero.

$$\therefore \quad 687{\cdot}2 - 482 \times \frac{x^2}{2} = 0$$

$\therefore$ $x = 1{\cdot}7$ *m.* from the centre

or $2{\cdot}25 - 1{\cdot}7 = 0{\cdot}55$ *m.* from the support.

10.4. Slabs Carrying Concentrated Load

The I.S. code has made the following recommendations in the analysis of slabs carrying point loads (clause 23·3·2—I.S. 456).

If a solid slab supported on two opposite edges, carries concentrated loads, the maximum bending moment caused by the concentrated loads shall be assumed to be resisted by an effective width of slab (measured parallel to the supporting edges) as follows :

(*a*) For a single concentrated load, the effective width shall be calculated in accordance with the following equation provided that it shall not exceed the actual width of the slab :

$$b_{ef} = Kx\left(1 - \frac{x}{l_{ef}}\right) + a$$

where, b_{ef} = effective width of slab,

K = constant having the values given in the table below, depending upon the ratio of the width of the slab l' to the effective span l_{ef},

x = distance of the centroid of the concentrated load from nearer support,

l_{ef} = effective span, and

a = width of the contact area of the concentrated load measured parallel to the supported edge.

And provided further that in case of a load near the unsupported edge of a slab, the effective width shall not exceed the above value nor half the above value plus the distance of the load from the unsupported edge.

(*b*) For two or more concentrated loads placed in a line in the direction of the span, the bending moment per *metre* width of the slab shall be calculated separately for each load according to its appropriate effective width of slab calculated as in (*a*) above and added together for design calculations.

(*c*) For two or more loads not in a line in the direction of the span.

If the effective width of slab for one load does not overlap the effective width of slab for another load, both calculated as in (*a*) above, then the slab for each load can be designed separately. If the effective width of slab for one load overlaps the effective width of

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slab for an adjacent load, the overlapping portion of the slab shall be designed for the combined effect of the two loads.

Values of K for Simply Supported and Continuous Slabs

r' / l_{ef}		K for simply supported slabs		K for continuous slabs
0·1	...	0·4	...	0·4
0·2	...	0·8	...	0·8
0·3	...	1·16	...	1·16
0·4	...	1·48	...	1·44
0·5	...	1·72	...	1·68
0·6	...	1·96	...	1·84
0·7	...	2·12	...	1·96
0·8	...	2·24	...	2·08
0·9	...	2·36	...	2·16
1·0 and above	...	2·48	...	2·24

(*d*) For cantilever solid slabs, the effective width shall be calculated in accordance with the following formula :

$$b_{ef} = 1{\cdot}2\, a_1 + a$$

where, b_{ef} = effective width

a_1 = distance of the concentrated load from the face of the cantilever support, and

a = width of contact area of the concentrated load measured parallel to the supporting edge.

Provided that the effective width of the cantilevering slab shall not exceed one-third the length of the cantilever slab measured parallel to the fixed edge.

And provided further that when the concentrated load is placed near the extreme ends of the length of cantilever slab in the direction parallel to the fixed edge, the effective width shall not exceed the above value, nor shall it exceed half the above value plus the distance of the concentrated load from the extreme end measured in the direction parallel to the fixed edge.

For slabs other than solid slabs, the effective width shall depend on the ratio of the transverse and longitudinal flexural rigidities of the slab. When this ratio is one, that is, when the transverse and longitudinal flexural rigidities are approximately equal, the value of the effective width as found for solid slabs may be used. But as the ratio decreases a proportionately small value should be taken.

Any other recognised method of analysis for cases of slabs men-

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tioned above and for other cases of slabs may be used with the approval of the engineer-in-charge.

§105. Continuous Slabs

Suppose a slab is supported at the ends and also at intermediate points on beams, the maximum sagging and hogging moments to which the slab is subjected to, due to a uniformly distributed load can be computed approximately by the following methods.

First method

Let the load intensity on the slab be w per *square metre*.

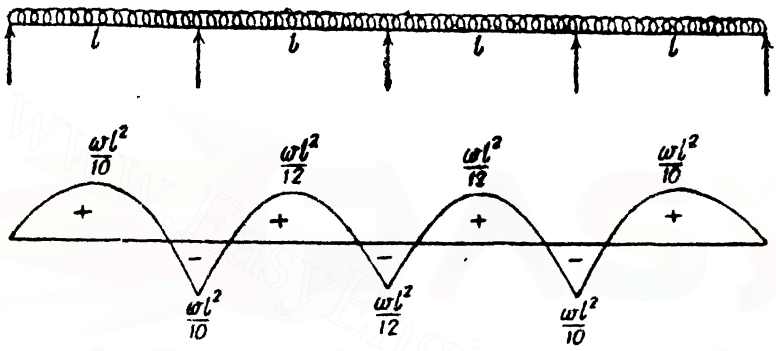

Fig. 92

Maximum sagging bending moment for the end spans $= +\frac{wl^2}{10}$

Maximum hogging bending moment over penultimate supports

$$= -\frac{wl^2}{10}$$

Maximum sagging bending moment for the interior spans $= +\frac{wl^2}{12}$.

Maximum hogging bending moment over interior supports other than penultimate supports $= -\frac{wl^2}{12}$.

The above values may be adopted (*i*) when the loading is uniformly distributed on the whole length and (*ii*) the spans are equal.

When the end span is made equal to 0·9 of the interior spans the above bending moments are nearly equal to $\frac{wl^2}{12}$.

Second method. (Suggested by I.S. code)

Let w_d = Intensity of dead load per *metre*^{2}

w_l = Intensity of live load per *metre*^{2}

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Bending moments due to dead load and live load may be taken as given in the table below :

	At middle of end span	*Over penultimate support*	*At middle of interior span*	*Over interior support*
B.M. due to dead load	$+\frac{wal^2}{12}$	$-\frac{wl^2}{10}$	$+\frac{wal^2}{24}$	$-\frac{wal^2}{12}$
B.M. due to live load	$+\frac{wl^2}{10}$	$-\frac{wl^2}{9}$	$+\frac{wl^2}{12}$	$-\frac{wl^2}{9}$

The values of the bending moments given above are reasonably satisfactory when the following conditions are satisfied :

(*i*) The slab is continuous over three or more spans.

(*ii*) The spans do not differ by more than 15 percent of the longest span.

B.M. at the end support of a slab. Where a slab is built into a masonry wall which develops only partial restraint, the member shall be designed to resist a negative moment at the face of the

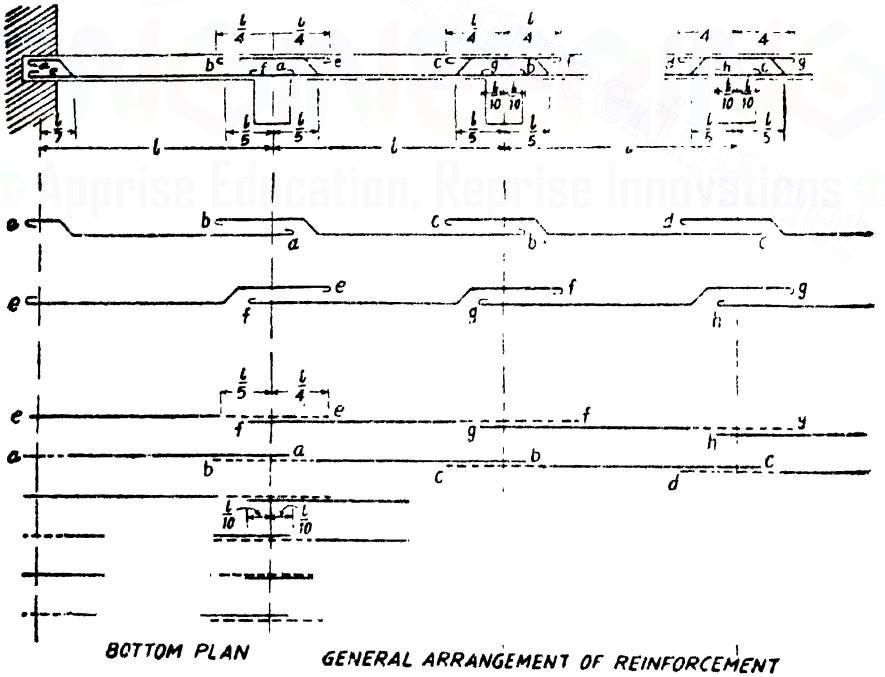

Fig. 93

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support, equal to $\frac{wl^2}{24}$ where w is the total design load intensity and l is the effective span.

Arrangement of reinforcement. Fig. 93 shows the usual arrangement of placing the reinforcement.

Design. 8. *Design a continuous floor slab for an office building 15 metres × 5 metres supported on R.C. floor beams. The live load on the slab is 300 kg/metre² Weight of R.C.C. and plain concrete may be taken as 2400 kg./metre³ and 2300 kg./metre³ respectively.*

Solution. Let us provide 15 *cm* bearing at each end of the continuous slab.

Hence, the distance between the centres of end bearings

$$= 15 + 0{\cdot}15 + 15{\cdot}15 \text{ metres.}$$

It is usual to make the end span length at 0·9 times the intermediate span length. Fig. 94 shows the arrangement of the beams.

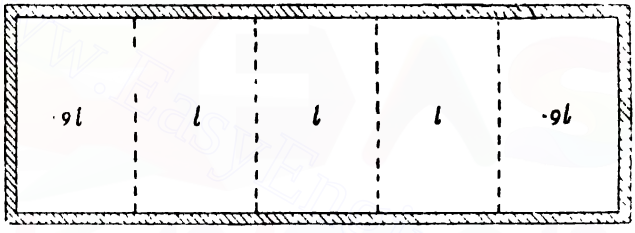

Fig. 94

If l be the intermediate span of the slab, we have

$$3l + 2 \times 0{\cdot}9l = 15{\cdot}15 \text{ metres}$$

$$l = \frac{15{\cdot}15}{4{\cdot}8} = 3{\cdot}16 \text{ metres}$$

Hence each end span $= 0{\cdot}9 \times 3{\cdot}16$ *metres*

$$= 2{\cdot}84 \text{ metres}$$

Loads

Dead load of slab (12 *cm.* thick) $= 12 \times 24 = 288$ *kg./m.²*

Floor finish (4 *cm*). $= 4 \times 23 = 92$ *kg./m.²*

Live load $= 300$ *kg./m.²*

Total $= 680$ *kg./m.²*

Consider a one *metre* wide strip of the slab.

Maximum bending moment for an end span $= \frac{wl^2}{10}$

$$= \frac{680 \times 2{\cdot}84^2}{10} \times 100 \text{ kg cm.}$$

$$= 54{,}840 \text{ kg cm.}$$

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Maximum bending moment for an intermediate span $= \frac{wl^2}{12}$

$$= \frac{680 \times 3{\cdot}16^2}{12} \times 100 \text{ kg. cm.}$$

$$= 56580 \text{ kg. cm.}$$

We will, therefore, design the slab for a bending moment of 56580 *kg. cm.*

Adopting $c = 50$ *kg./cm.*^{2}, $t = 1400$ *kg./cm.*^{2} and $m = 18$.

We have $a = 0{\cdot}87\,d$ and $Q = 8{\cdot}50$

Equating the moment of resistance to the maximum bending moment, we have

$$8{\cdot}50 \times 100\, d^2 = 56580$$

$$d = 8{\cdot}16 \text{ cm.}$$

Effective depth available $= 12 - 2 = 10$ *cm.*

$$\therefore \quad A_t = \frac{56580}{1400 \times 0{\cdot}87 \times 10} \text{ cm}^2.$$

$$= 4{\cdot}64 \text{ cm}^2.$$

Spacing of 10 *mm.* diameter bars

$$= \frac{0{\cdot}79 \times 100}{4{\cdot}64} \text{ cm.}$$

$$= 17 \text{ cm.}$$

Distribution steel $= \frac{0{\cdot}15}{100} \times 100 \times 12 \text{ cm.}^2$

$$= 1{\cdot}8 \text{ cm}^2.$$

Spacing of 6 *mm.* diameter bars $= \frac{0{\cdot}28 \times 100}{1{\cdot}8}$ cm.

$= 15{\cdot}5$ *cm., say 15 cm. centres.*

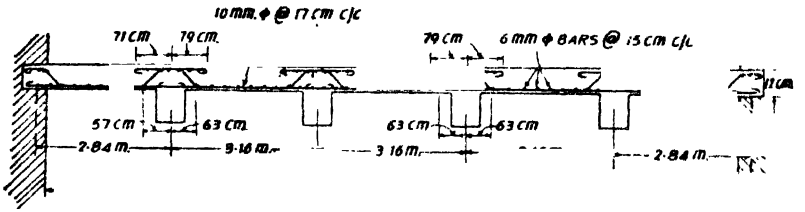

Fig. 95

Design 9. (S.I.). *Design a continuous floor slab for an office building 15 metres × 5 metres supported on R.C. floor beams. The live load on the slab is 3000 N/metre*^{2}

Solution. Provide 15 *cm.* bearing at each end of the slab. Providing four intermediate beams and making the end span 0·9 times the intermediate span *l*, we have

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$2 \times 0{\cdot}9\, l + 3\, l = 15{\cdot}15 \quad \therefore \quad l = 3{\cdot}16\ m.$

End span $= l_1 = 0{\cdot}9 \times 3{\cdot}16 = 2{\cdot}84$ *metres*

Loads

Dead load of slab (12 *cm.* thick) $= 12 \times 250 = 3000\ N/metre^2$

Floor finish (4 *cm*). $= 4 \times 240 = 960\ N/metre^2$

Total dead load $= w_d = 3960\ N/metre^2$

Live load $= w_l = 3000\ N/metre^2$

Consider a one *metre* wide strip of the continuous slab

B.M. for end span

B.M. at the centre of end span

$$= \frac{wal_1^2}{12} + \frac{wl_1^2}{10}$$

$$= \left[\frac{3960 \times 2{\cdot}84^2}{12} + \frac{3000 \times 2{\cdot}84^2}{10}\right] 100\ N\ cm.$$

$$= 508133\ N\ cm.$$

B.M. over the penultimate support

$$= -\left[\frac{wal_1^2}{10} + \frac{wl_1^2}{9}\right]$$

$$= -\left[\frac{3960 \times 2{\cdot}84^2}{10} + \frac{3000 \times 2{\cdot}84^2}{9}\right] 100\ N\ cm.$$

$$= -588251\ N\ cm.$$

B.M. for intermediate span

B.M. at the centre of the span

$$= \frac{wal^2}{24} + \frac{wl^2}{12}$$

$$= \left[\frac{3960 \times 3{\cdot}16^2}{24} + \frac{3000 \times 3{\cdot}16^2}{12}\right] 100\ N\ cm.$$

$$= 414402\ N\ cm.$$

B.M. over interior support

$$= -\left[\frac{wal^2}{12} + \frac{wl^2}{9}\right]$$

$$= -\left[\frac{3960 \times 3{\cdot}16^2}{12} + \frac{3000 \times 3{\cdot}16^2}{9}\right] 100\ N\ cm.$$

$$= -662378\ N\ cm.$$

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Maximum bending moment

= Greatest of the above four bending moments

= 662378 *N cm*

Equating the moment of resistance to the bending moment,

$$85 \times 100 \quad d^2 = 662378$$

$$\therefore \qquad d = 8{\cdot}83 \text{ cm.}$$

Providing 10 *mm.* diameter bars at a clear cover of 15 *mm.* effective depth available = 12 − 2 = 10 *cm.*

$$\therefore \qquad A_s = \frac{662378}{14000 \times 0{\cdot}87 \times 10} = 5{\cdot}44 \text{ cm}^2.$$

Spacing of 10 *mm* diameter bars

$$= \frac{0{\cdot}79 \times 100}{5{\cdot}44} = 14 \text{ cm}$$

Provide 10 mm. ϕ bars @ 14 cm. c/c.

Distribution steel

$$= \frac{0{\cdot}15}{100} \times 100 \times 12 \text{ cm}^2 = 1{\cdot}80 \text{ cm}^2$$

Spacing of 6 *mm.* diameter bars

$$= \frac{0{\cdot}28 \times 100}{1{\cdot}8} = 15{\cdot}5 \text{ cm.}$$

Provide 6 mm. ϕ bars @ 15 cm. c/c.

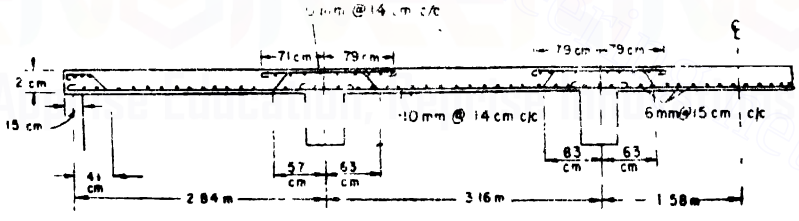

Fig. 96

Fig. 96 shows the details of reinforcement for the slab.

§107. Two Way Slabs (Slabs Spanning in Two Directions at Right Angles)

A simply supported slab may be supported on all its four edges. The load on the slab is then distributed to the four supports. In this case the load is carried by the slab in two directions to the supports. The bending moment in each direction is considerably less than what it would be if the slab had been spanning in one direction only.

The bending moments for short and long span strips can be calculated by various methods given below.

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§107. Rankine-Grashoff Theory

Fig. 97 shows a simply supported slab, supported on all the four edges. Let the long and short spans be l_y and l_x respectively.

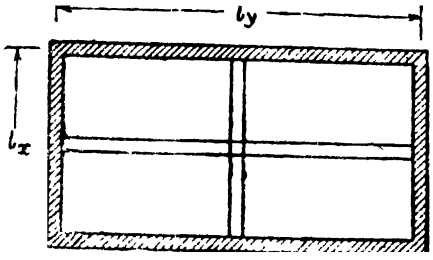

Fig. 97

Let the load on the slab be w per unit area. This load can be split into w_x and w_y acting on strips parallel to l_x and l_y.

Since the central deflection of the mid-strips spanning in the two directions should be the same, we have,

$$\frac{5}{384}\frac{w_x l_x^4}{EI} = \frac{5}{384}\frac{w_y l_y^4}{EI}$$

$$\frac{w_x}{w_y} = \left(\frac{l_y}{l_x}\right)^4$$

Let $\frac{l_y}{l_x} = r$ $\quad\therefore\ w_x = r^4 w_y$

But $w_x + w_y = w$ $\quad\therefore\ r^4 w_y + w_y = w$

$$w_y(1+r^4) = w \quad\therefore\ w_y = \frac{w}{(1+r^4)}$$

and $$w_x = \frac{r^4}{1+r^4} w$$

∴ Max. B.M. for the short span

$$= \frac{w_x l_x^2}{8} = \frac{r^4}{1+r^4}\frac{wl_x^2}{8} = Z_x \frac{wl_x^2}{8}$$

where $$Z_x = \frac{r^4}{1+r^4}$$

Max. B.M. for long span

$$\frac{w_y l_y^2}{8} = \frac{w}{1+r^4}\frac{l_y^2}{8} = Z_y \frac{wl_y^2}{8}$$

where $$Z_y = \frac{1}{1+r^4}$$

The values of Z_x and Z_y for various values of $r = \frac{l_y}{l_x}$ are given in the table on page 208.

If Z_y is such that the B.M. on this long span is so small that the reinforcement requirement for the B.M. of $Z_y\frac{wl_y^2}{8}$ is less than the usual amount of distribution steel, than at least 0·15% of the gross area shall be provided along the larger span.

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B.M. co-efficient Z_x and Z_y for two-way simply supported slabs (Based on Rankine Grashoff Theory)

$\frac{l_y}{l_x} = r \rightarrow$	1·0	1·1	1·2	1·3	1·4	1·5	1·75	2·0	2·5	3·0
	0·50	0·59	0·68	0·74	0·79	0·84	0·90	0·94	0·97	0·98
	0·50	0·41	0·32	0·26	0·21	0·16	0·10	0·06	0·03	0·02

In the above discussion the slab has been assumed as simply supported on all its four edges and no provision has been made to resist torsion at the corners of the slab and no provision is made to prevent the corners from lifting.

Design 10. *A slab over a room is 4·75 m × 4·75 m. The edges of the slab are simply supported and the corners are not held down. The live load on the slab is 300 kg./m². The slab has a bearing of 15 cm on the supporting walls. Design the slab as per Rankine-Grashoff theory. Assume weight of RCC = 2400 kg./m³*

Solution.

Span between centres of bearings
$= 4·75 + 0·15 = 4·90$ *m.*

Overall depth of the slab
$= \frac{1}{30}$ of the span
$= \frac{1}{30}(4·90 \times 100)$
$= 16·33$ *cm* say 16 *cm.*

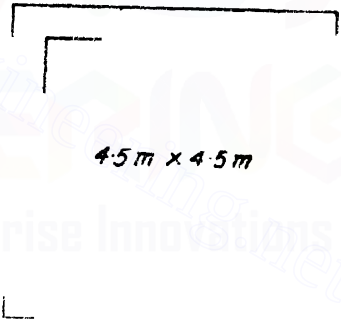

Fig. 98

Providing 12 *mm.* bars at a clear cover of 15 *mm* to the lower layer of bars.

Effective depth to the centre of the upper layer of bars
$= 16 - 2·1 - 1·2 = 12·7$ *cm.*

Effective Span :

This is taken as the lesser of the following :

(*i*) Centre to centre of bearings $= 4·90$ *m.*

(*ii*) Clear span + effective depth $= 4·75 + 0·127$
$= 4·877$ *m.* say 4·88 *m.*

∴ Effective span $= 4·88$ *m.*

i.e., $l_x = l_y = 4·88$ *m.*

∴ $z_x = z_y = 0·5$

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Loads. D.L. of slab (16 *cm.* thick) $= 16 \times 24 = 384$ *kg./m.*^{2}
2 *cm.* thick granolithic finishing $= 2 \times 23 = 46$ *kg./m.*^{2}
L.L. $= 300$ *kg./m.*^{2}
Total $= 730$ *kg./cm.*^{2}

Consider a 1 *m* wide strip of slab

$$M_0 = M_0 = 0{\cdot}5 \frac{wl_0^2}{8}$$

$$= 0{\cdot}5 \times 730 \times \frac{4{\cdot}88^2}{8} \times 100 \text{ kg. cm.}$$

$$= 108650 \text{ kg cm}$$

Adopting $c = 50$ *kg./cm.*^{2}, $t = 1400$ *kg./cm.*^{2} and $m = 18$

Equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}5 \times 100\, d^2 = 108650$$

$$\therefore \quad d = 11{\cdot}30 \text{ cm.}$$

But effective depth available $= 12{\cdot}7$ *cm.*

$$A_t = \frac{108650}{1400 \times 0{\cdot}87 \times 12{\cdot}7} = 7{\cdot}02 \text{ cm.}$$

$\therefore$ Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{7{\cdot}02} = 16 \text{ cm.}$$

Provide 12 mm. ϕ bars 16 cm. c/c in each principal direction.

Fig. 99

Design 11. *Design a slab over a room 5 metres × 7 metres by Rankine-Grashoff theory. The edges of the slab are simply supported and the corners are not held down. The live load on the slab may be taken as 300 kg./metre*^{2}*. The slab has a bearing of 15 cms on the supporting walls.*

Solution.

Short span between centres of bearings $= 5{\cdot}15$ *metres*

Long span between centres of bearings $= 7{\cdot}15$ *metres*

Overall depth of the slab

$\frac{1}{30}$ of short span

Fig. 100

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$$= \frac{5{\cdot}15 \times 100}{30} = 17{\cdot}2 \text{ cm.}$$

Provide a thickness of 18 *cm.*

Providing 12 *mm.* diameter bars, and providing a clear cover of 15 *mm.* to the lower layer of bars,

Effective depth to the centre of short span steel

$$= 18 - 2{\cdot}10 = 15{\cdot}90 \text{ cm.}$$

Effective depth to the centre of long span steel

$$= 15{\cdot}90 - 1{\cdot}20 = 14{\cdot}7 \text{ cm.}$$

Clear span + effective depth for short span $= 5 + 0{\cdot}159 = 5{\cdot}159$ *m.*

Clear span + effective depth for long span $= 7 + 0{\cdot}147 = 7{\cdot}147$ *m.*

$\therefore$ **Effective short span** $= l_x = 5{\cdot}150$ *metre.*

Effective long span $= l_y = 7{\cdot}147$ *say* $7{\cdot}150$ *metre.*

$$r = \frac{l_y}{l_x} = \frac{7{\cdot}15}{5{\cdot}15} = 1{\cdot}388$$

$$Z_x = \frac{r^4}{1+r^4} = \frac{1{\cdot}388^4}{1+1{\cdot}388^4} = 0{\cdot}788$$

$$Z_y = \frac{1}{1+r^4} = \frac{1}{1+1{\cdot}388^4} = 0{\cdot}212$$

Loads

D.L. of slab	$= 18 \times 25$	$= 450$ *kg./m.*^{2}
2 *cm.* granolithic finishing	$= 2 \times 24$	$= 48$ *kg./m.*^{2}
L.L. on the slab		$= 300$ *kg./m.*^{2}
Total		$= 798$ *kg./m.*^{2}

Maximum short span bending moment per *metre* width

$$= Z_x \frac{wl_x^2}{8}$$

$$= 0{\cdot}788 \times \frac{798 \times 5{\cdot}15^2}{8} \times 100 \text{ kg. cm}$$

$$= 208475 \text{ kg. cm.}$$

Equating the moment of resistance to the bending moment,

$$8{\cdot}5 \times 100\, d^2 = 208475$$

$\therefore$ $d = 15{\cdot}7$ *cm.*

But the effective depth available $= d = 18 - 2{\cdot}10 = 15{\cdot}90$ *cm.*

$\therefore$ $$A_t = \frac{208475}{1400 \times 0{\cdot}87 \times 15{\cdot}90} = 10{\cdot}76 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{10{\cdot}76} = 10{\cdot}50 \text{ cm.}$$

Provide 12 mm. diameter bars @ 10 cm. c/c.

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Maximum bending moment for long span per *metre* width

$$= Z_y \frac{wl_y^2}{8}$$

$$= 0{\cdot}212 \times \frac{798 \times 7{\cdot}15^2}{8} \times 100 \text{ kg. cm.}$$

$$= 108108 \text{ kg. cm.}$$

$$A_t = \frac{108108}{1400 \times 0{\cdot}87 \times 14{\cdot}7} = 6 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars $= \frac{1{\cdot}13 \times 100}{6} = 19$ *cm.*

Provide 12 *mm.* ϕ bars @ 19 *cm.* c/c.

Details of reinforcement are shows in given figure on pages 212-213.

Design 12. (S.I.). *Design a slab over a room 5 metres × 7 metres by Rankine-Grashoff theory. The edges of the slab are simply supported and the corners are not held down. The live load on the slab may be taken as 3000 N/metre². The slab has a bearing of 15 cms. on the supporting walls.*

Solution.

Short span between centres of bearings = 5·15 *m*

Long span between centres of bearings = 7·15 *m*

Overall depth of the slab

$= \frac{1}{30}$ of short span

$= \frac{5{\cdot}15 \times 100}{30} = 17{\cdot}2$ *cm. say 18 cm.*

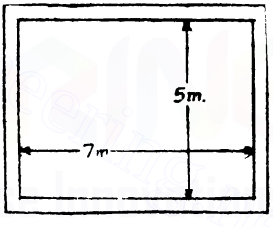

Fig. 101

Providing 12 *mm.* diameter bars, and providing a clear cover of 15 *mm.* to the lower layer of bars.

Effective depth to the centre of short span steel

$= 18 - 2{\cdot}10 = 15{\cdot}90$ *cm.*

Effective depth to the centre of long span steel

$= 15{\cdot}90 - 1{\cdot}20 = 14{\cdot}70$ *cm.*

Clear span + effective depth for short span = 5 + 5·159 = 5·159 *m.*

Clear span + effective depth for long span = 7 + 0·147 = 7·147 *m.*

∴ Effective short span $= l_x = 5{\cdot}150$ *m.*

Effective long span $= l_y = 7{\cdot}147$ *m say* 7·15 *m.*

$$\therefore \quad r = \frac{l_y}{l_x} = \frac{7{\cdot}15}{5{\cdot}15} = 1{\cdot}388$$

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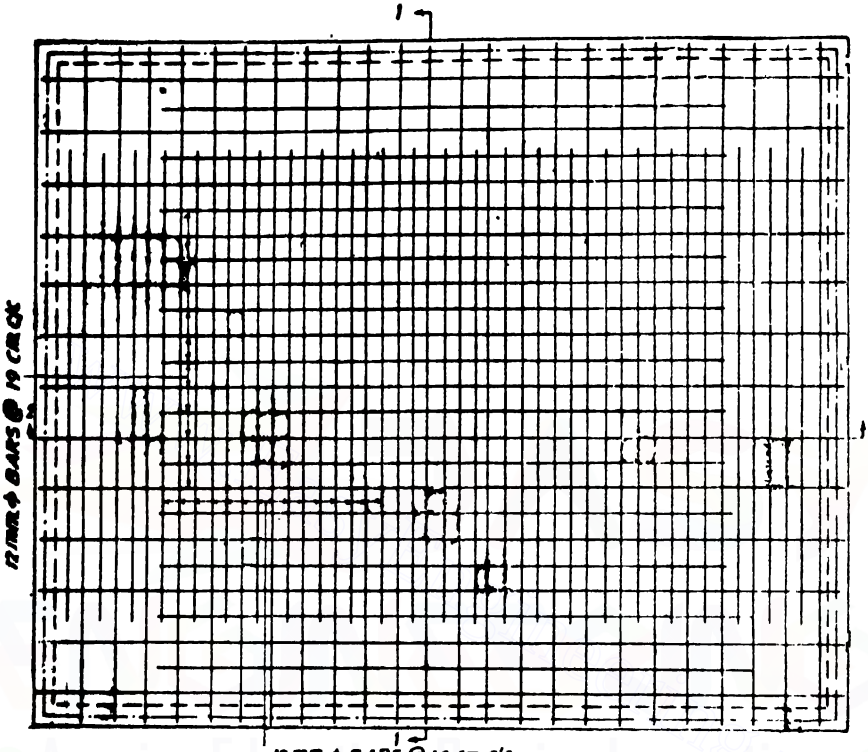
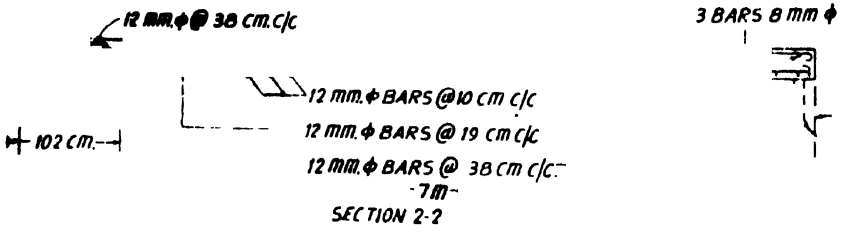
SIMPLY SUPPORTED



BOTTOM PLAN



SECTION 2-2

NOTE

1. SLAB THICKNESS HAS BEEN EXAGGERATED
2. COVER TO SHORT SPAN REINFORCENT = 13 CM.
3. BEARING FOR SLAB = 15 CM.
4. CORNERS OF SLAB ARE NOT HELD DOWN

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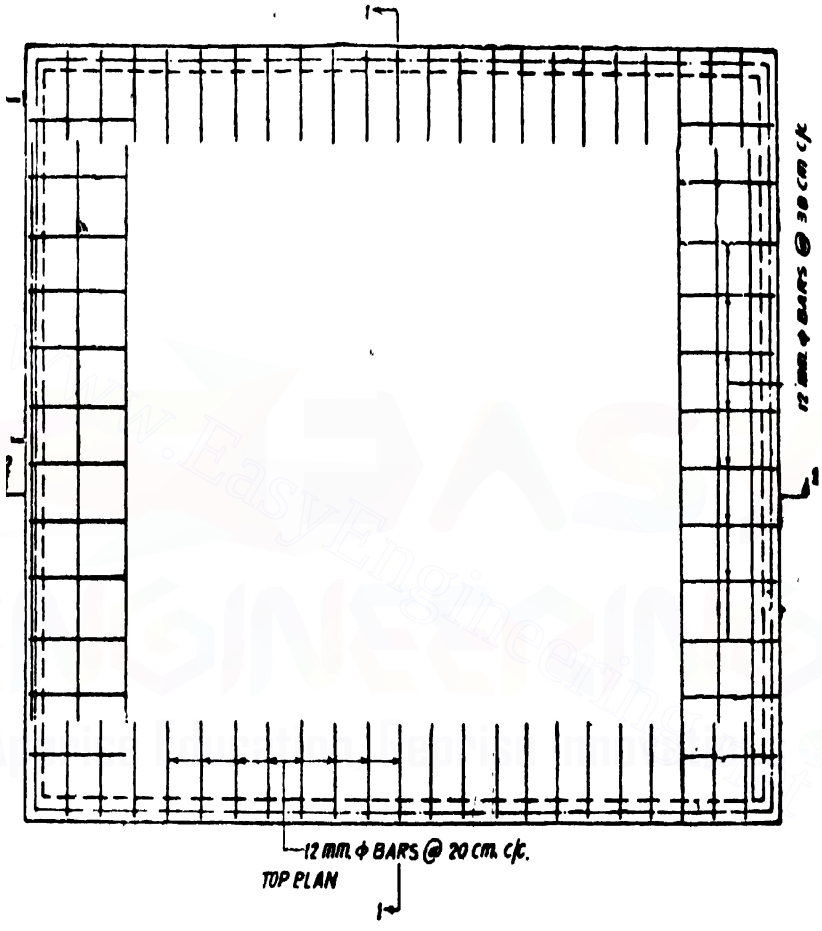
TWO-WAY SLAB



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$$Z_x = \frac{r^4}{1+r^4} = \frac{1{\cdot}388^4}{1+1{\cdot}388^4} = 0{\cdot}788$$

$$Z_y = \frac{r^4}{1+r^4} = \frac{1}{1+1{\cdot}388^4} = 0{\cdot}212$$

Loads. D.L. of the slab $= 18 \times 250 = 4500\ N/metre^2$
2 cm. granolithic finishing $= 2 \times 240 = 480\ N/metre^2$
L.L. on the slab $= 3000\ N/metre^2$
Total $= 7980\ N/metre^2$

Maximum short span bending moment per *metre* width $= Z_x \frac{wl_x^2}{8}$

$$= 0{\cdot}788 \times \frac{7980 \times 5{\cdot}15^2}{8} \times 100\ N\ cm. = 2084750\ N\ cm.$$

Taking $c = 5\ N/mm.^2 = 500\ N/cm.^2$

$t = 140\ N/mm^2 = 14000\ N/cm.^2$ and $m = 18$, and equating the moment of resistance to the maximum bending moment.

$85\ bd^2 = 85 \times 100\ d^2 = 2084750 \quad \therefore \quad d = 15{\cdot}7\ cm.$

But, effective depth available $= d = 18 - 2{\cdot}10 = 15{\cdot}90\ cm.$

$$\therefore \quad A_t = \frac{2084750}{14000 \times 0{\cdot}87 \times 15{\cdot}90} = 10{\cdot}76\ cm.^2$$

Spacing of 12 *mm.* diameter bars $= \frac{1{\cdot}13 \times 100}{10{\cdot}76} = 10{\cdot}50\ cm.$

Provide 12 mm. ϕ bars @ 10 cm. c/c.

Maximum long span bending moment per *metre* width $= Z_y \frac{wl_y}{8}$

$$= 0{\cdot}212 \times \frac{7980 \times 7{\cdot}15^2}{8} \times 100\ N\ cm. = 1081080\ N\ cm.$$

$$\therefore \quad A_t = \frac{1081080}{14000 \times 0{\cdot}87 \times 14{\cdot}70} = 6\ cm.^2$$

Spacing of 12 *mm.* diameter bars $= \frac{1{\cdot}13 \times 100}{6} = 19\ cm.$

Provide 12 mm ϕ bars @ 19 cm. c/c.

§108. Two Way Slabs with Corners Held Down. Marcus Correction

In this case the bending moments determined by the simple theory of Rankine-Grashoff are not exactly correct. This is because no portion of a strip spanning parallel to the short span can bend without causing torsion in a strip transverse to it.

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Fig. 102 shows a slab supported on all its four edges with corners prevented from getting up.

Let l_y and l_x be the long and short spans of the slab.

Let the load on the slab be w per unit area. Consider a strip AB of unit width spanning on the long span l_y. Its deflected form is given by AEB. Let the central deflection of this strip be δ.

It will be assumed that the deflected shape of the strip is parabolic.

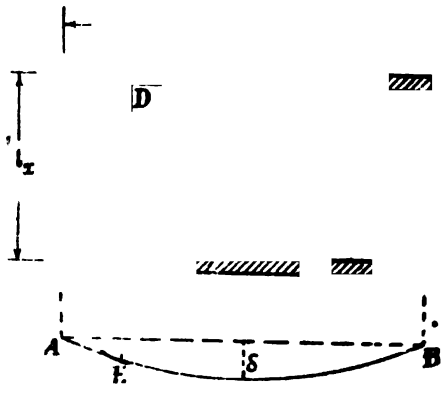

Fig. 102

Hence with A or B as origin the equation to the elastic AEB will be $y=\frac{4\delta}{l_y^2}\,x(l_y-2x)$.

Hence the slope of the elastic curve at any section is given by

$$\frac{dy}{dx}=\frac{4\delta}{l_y^2}(l_y-2x)$$

The slope at $x=\frac{l_y}{4}$ will be $\frac{dy}{dx}=\frac{4\delta}{l_y^2}\left[l_y-\frac{l_y}{2}\right]=\frac{2\delta}{l_y}$

Now consider a strip CD The section at E of the strip has been twisted by the angle $\frac{2\delta}{y}$ with respect to the section at C and D.

Let N be the Modulus of rigidity. Hence, the torsional moment in the strip CD,

$$T=\frac{2N\,\theta\,I}{l_x}=\frac{4N\,\theta\,I}{l_x}$$

But we know the Young's Modulus E is given by

$$E=2N\left(1+\frac{1}{m}\right)$$

where $\frac{1}{m}$ is the Poisson's ratio. Assuming the Poisson's ratio to be zero, $E=2N$.

Substituting for θ and N in the expression for torsion, we have

$$T=\frac{EI}{l_y}\cdot\frac{4\delta}{l_x}$$

∴ Total torsion on all the parallel strips parallel to the strip CD

$$=\frac{EI}{l_y}\cdot\frac{4\delta}{l_x}\cdot l_y=EI\cdot\frac{4\delta}{l_x}$$

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Hence, the torsional resistance on the strip AB per unit width

$$= EI \cdot \frac{4\delta}{l_x^2}.$$

Hence, it is evident that part of the load on the strip AB will cause twisting of the transverse strip CD and only the remaining part of the load on the strip AB will cause its bending.

Let w_v be the load per unit area causing bending of the slab on long span as determined from Rankine-Grashoff rule.

$$\therefore \quad w_v = \frac{w l_x^4}{l_x^4 + l_y^4}$$

$$= \frac{w}{1+r^4} \qquad \text{where } r = \frac{l_y}{l_x}$$

Let the max. B.M. on long span as per the Rankine-Grashoff rule be M_v.

$$\therefore \quad M_v = \frac{w}{1+r^4} \cdot \frac{l_y^2}{8}$$

This moment is accompanied by a torsion of $T = \frac{EI4\delta}{l_x^2}$

where $$\delta = \frac{5}{384} \cdot \frac{w_v \cdot l_y^4}{EI} = \frac{5}{384EI} \cdot \frac{w}{1+r^4} \cdot l_y^4$$

$$\therefore \text{ Torsion } T = \frac{EI}{l_x^2} \cdot 4 \times \frac{5}{384EI} \frac{w l_y^4}{(1+r^4)}$$

$$\therefore \quad T = \frac{5}{96} \cdot \frac{w l_y^4}{(1+r^4) l_x^2}$$

But $$M_v = \frac{w}{1+r^4} \cdot \frac{l_y^2}{8}$$

$$\frac{5}{12} \cdot M_v \cdot \frac{l_y^2}{l_x^2} \qquad \therefore \quad T = \frac{5}{12}$$

This is the torsional moment which will cause the reduction of bending moment in the strip AB.

To allow for the corners being held down *i.e.*, to include the effect of corner restraint the reduction of the B.M. in the strip AB determined from the above relation is further increased by Marcus by multiplying by a ratio $\frac{w_v}{w}$.

Hence, the final reduction of the B.M.

$$= \frac{5}{12} \cdot M_v r^2 \cdot 2 \frac{w_v}{w}$$

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$$= \frac{5}{6} \cdot M_y r^2 \frac{w_y}{w} \qquad \text{But} \quad w_y = \frac{w}{1+r^4}$$

$$\therefore \quad \frac{w_y}{w} = \frac{1}{1+r^4}$$

$$\therefore \text{ Actual reduction of B.M.} = \frac{5}{6} \cdot \frac{r^2}{1+r^4} M_y.$$

Hence, the net B.M. in the strip AB

$$= M_y - \frac{5}{6} \frac{r^2}{1+r^4} M_y$$

$$= M_y \left[1 - \frac{5}{6} \frac{r^2}{1+r^4} \right]$$

Similarly it can be shown that the B.M. in the transverse strip CD.

$$= M_x \left[1 - \frac{5}{6} \frac{r^2}{1+r^4} \right]$$

Hence, this is just a modification of the Rankine-Grashoff method. The bending moments M_x and M_y calculated by the Rankine-Grashoff method are multiplied by a correcting co-efficient $\left[1 - \frac{5}{6} \frac{r^2}{1+r^4} \right]$.

Thus we have,

$$\text{Actual B.M. for short span strip} = \left[1 - \frac{5}{6} \frac{r^2}{1+r^4} \right] \frac{r^4}{1+r^4} \frac{wl_x^2}{8}$$

$$= C_x \frac{wl_x^2}{8}$$

$$\text{and, actual B.M. for long span strip} = \left[1 - \frac{5}{6} \frac{r^2}{1+r^4} \right] \frac{1}{1+r^4} \frac{wl_y^2}{8}$$

$$= C_y \frac{wl_y^2}{8}$$

Values of C_x and C_y for different values of r are given in the table on page 218.

Now the reinforcements for short and long span strips can be calculated. Besides these reinforcements, additional reinforcements are also provided near each corner to allow for torsion. This additional reinforcement at each corner consists of a top and a bottom mesh, covering a rectangular area $\frac{l_x}{5} \times \frac{l_y}{5}$. In each mesh in each principal direction the area of steel is equal to that required at the centre of the panel.

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r	$C_x=\left[1-\frac{5}{6}\frac{r^2}{1+r^4}\right]\left[\frac{r^2}{1+r^4}\right]$	$C_y=\left[1-\frac{5}{6}\frac{r^2}{1+r^4}\right]\left[\frac{1}{1+r^4}\right]$
1·0	0·292	0·292
1·1	0·348	0·243
1·2	0·415	0·195
1·3	0·470	0·165
1·4	0·524	0·139
1·5	0 580	0·111
1·75	0·680	0·075
2·0	0·756	0·048
2·5	0·844	0·026
3·0	0·891	0·018

Design 13. *For the data of design 11, design the slab by applying Marcus correction.*

Solution. $l_x = 5{\cdot}150\ m.$

$l_y = 7{\cdot}150\ m.$

$$r=\frac{7{\cdot}15}{5{\cdot}15}=1{\cdot}388$$

$r^2=1{\cdot}926,\ r^4=3{\cdot}709$

$w=7980\ N/metre^2$

$$C_x=\left[1-\frac{5}{6}\frac{r^2}{1+r^4}\right]\left[\frac{r^4}{1+r^4}\right]$$

$$=\left[1-\frac{5}{6}\times\frac{1{\cdot}926}{1+3{\cdot}709}\right]\left[\frac{3{\cdot}709}{1+3{\cdot}709}\right]=0{\cdot}519$$

$$C_y=\left[1-\frac{5}{6}\frac{r^2}{1+r^4}\right]\left[\frac{1}{1+r^4}\right]$$

$$=\left[1-\frac{5}{6}\times\frac{1{\cdot}926}{1+3{\cdot}709}\right]\left[\frac{1}{1+3{\cdot}709}\right]=0{\cdot}140$$

B.M. for short span strip $=M_x=C_x\cdot\frac{wl_x^2}{8}$

$$=0{\cdot}519\times\frac{7980\times5{\cdot}15^2}{8}\times100\ N\ cm.=1373080\ N\ cm.$$

Equating the moment of resistance to the maximum bending moment,

$$85\ bd^2=85\times100\ d^2=1373080 \qquad \therefore\quad d=12{\cdot}71\ cm.$$

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Effective depth available $= 18 - 2{\cdot}10 = 15{\cdot}9$ *cm.*

$$A_t = \frac{1373080}{14000 \times 0{\cdot}87 \times 15{\cdot}9} = 7{\cdot}09 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars $= \frac{1{\cdot}13 \times 100}{7{\cdot}09} = 15{\cdot}9$ *cm.*

Provide 12 mm. ϕ bars @ 15 cm. c/c.

Bending moment for long span strip $= M_y = C_y \frac{wl_y^2}{8}$

$$= 0{\cdot}140 \times \frac{0{\cdot}78 \times 7{\cdot}15^2}{8} \times 100 \text{ N cm.} = 713930 \text{ N cm}$$

$$\therefore \quad A_t = \frac{713930}{14000 \times 0{\cdot}87 \times 14{\cdot}7} = 3{\cdot}99 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars $= \frac{1{\cdot}13 \times 100}{3{\cdot}99} = 28{\cdot}3$ *cm.*

Provide 12 mm ϕ bars @ 25 cm. c/c.

Besides these reinforcements, we will also provide corner reinforcement in the form of a mesh covering a rectangular area $\frac{l_x}{5} \times \frac{l_y}{5}$.

In each mesh, in the direction of short span provide 12 mm. @ 15 cm. c/c and in the direction of long span provide 12 mm. @ 25 cm. c/c.

§109. Analysis of Two Way Slabs by I.S. Code

The slabs are classified into two types :

(*i*) *Simply supported slabs which do not have adequate provision to resist torsion at corners, and to prevent the corners from lifting.*

In this case the maximum bending moments per unit width are given by the following equations :

$$M_x = \alpha_x w l_x^2$$

and $M_y = \alpha_y w l_x^2$

where, M_x = B.M. for for short span strip.

M_y = B M for long span strip.

α_x and α_y are moment coefficients depending on the ratio l_y/l_x as given below :

Bending Moment Coefficients for Slabs Spanning in Two Directions at Right Angles—Simply Supported on Four Sides (Corners Not Held Down)

$\frac{l_y}{l_x} = r$	1·0	1.1	1·2	1·3	1·4	1·5	1·75	2·0	2·5	3·0
α_x	0·062	0·074	0·084	0·093	0·099	0·104	0·113	0·118	0·122	0·124
α_y	0·062	0·061	0·059	0·055	0·051	0·046	0·037	0·029	0·020	0·014

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Disposition of reinforcement. At least 50 percent of the tension reinforcement provided at midspan should extend to the supports. The remaining 50 percent should extend to within 0·1 l_x or 0·1 l_y of the support, as approximate.

Design 14. *Design a slab over a room 5 metres × 7 metres as per I.S. code. The edges of the slab are simply supported and the corners are not held down. The live load on the slab is 300 kg./metre². The slab has a bearing of 15 cm. on the supporting walls.*

Solution.

Short span between centres of bearing
$= 5{\cdot}15$ *metres*

Long span between centres of bearing
$= 7{\cdot}15$ *metre*

Overall depth of the slab

$$= \frac{1}{30} \times \text{short span}$$

$$\frac{1}{30} \times 5{\cdot}15 \times 100 = 17{\cdot}2 \text{ cm.}$$

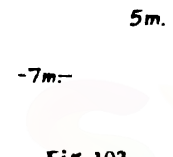

Fig. 103

Provide a thickness of 18 cm.

Providing 12 *mm.* diameter bars, and providing a clear cover of 15 *mm* to the lower layer of bars,

Effective depth to the centre of short span steel
$= 18 - 2{\cdot}10 = 15{\cdot}90$ *cm.*

Effective depth to the centre of long span steel
$= 15{\cdot}90 - 1{\cdot}20 = 14{\cdot}70$ *cm.*

Clear span + effective depth for short span
$= 5 + 0{\cdot}159 = 5{\cdot}159$ *m.*

Clear span + effective depth for long span
$= 7 + 0{\cdot}147 = 7{\cdot}147$ *m.*

∴ Effective short span $= l_x = 5{\cdot}150$ *m.*

Effective long span $= l_y = 7{\cdot}147$ *m. say* $7{\cdot}15$ *m.*

$$r = \frac{l_y}{l_x} = \frac{7{\cdot}15}{5{\cdot}15} = 1{\cdot}388 \text{ say } 1{\cdot}4$$

Referring to I.S. tables, for $r = 1{\cdot}4$, $\alpha_x = 0{\cdot}099$ and $\alpha_y = 0{\cdot}051$

Loads

D.L. of the slab	$= 18 \times 25$	$= 450$ kg./m.²
2 *cm.* granolithic finishing	$= 2 \times 24$	$= 48$ kg./m.²
L.L. on the slab		$= 300$ kg./m.²
	Total	798 kg./m.²

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Maximum short span bending moment per *metre* width

$$= \alpha_x w l_x^2$$
$$= 0{\cdot}099 \times 798 \times 5{\cdot}15^2 \times 100 \text{ kg. cm.}$$
$$= 209530 \text{ kg. cm.}$$

Equating the moment of resistance to the maximum bending moment $= 8{\cdot}5 \times 100\, d^2 = 209530$

$\therefore$ $d = 15{\cdot}70$ *cm.*

But, effective depth available

$$= d = 18 - 2{\cdot}10 = 15{\cdot}90 \text{ cm.}$$

$\therefore$ $$A_t = \frac{209530}{1400 \times 0{\cdot}87 \times 15{\cdot}90} = 10{\cdot}82 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{10{\cdot}82} = 10{\cdot}4 \text{ cm.}$$

Provide 12 mm. ϕ bars @ 10 cm. c/c.

Maximum bending moment for long span per *metre* width

$$= \alpha_y w l_x^2$$
$$= 0{\cdot}051 \times 798 \times 5{\cdot}15^2 \times 100 \text{ kg. cm.}$$
$$= 107940 \text{ kg. cm.}$$

$$A_t = \frac{107940}{1400 \times 0{\cdot}87 \times 14{\cdot}7} = 6 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{6} = 19 \text{ cm.}$$

Provide 12 mm. ϕ bars @ 19 cm. c/c.

Reinforcement details are similar to those in the design based on Rankine-Grashoff Theory.

(ii) Slab supported on the four edges and the corners of the slab are held down.

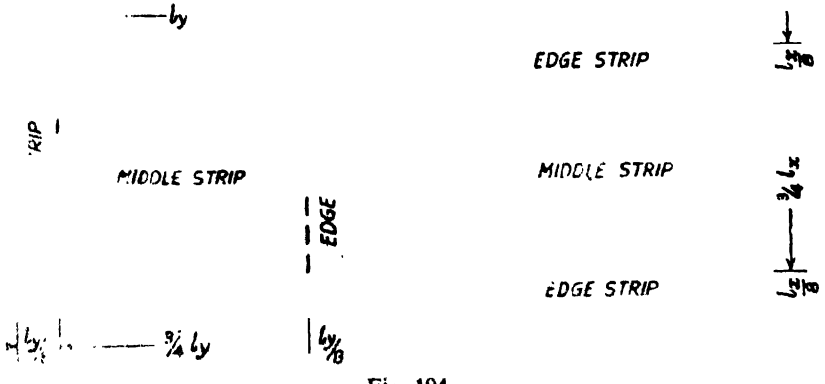

Fig. 104

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In this case, the slab is divided into middle and edge strips as per specifications shown in Fig 105.

The middle strips have a width equal to three-fourth of the width of the slab and each edge strip will have a width of one-eighth of the slab.

If M_x and M_y are the maximum bending moments per unit width in the middle strip of the slab, in the short and long spans respectively, then,

$$M_x = \alpha_x w l_x^2$$

and,

$$M_y = \alpha_y w l_x^2$$

where, α_x and α_x are coefficients depending on the ratio $\frac{l_y}{l_x}$ as given below :

Bending Moment Coefficients for Slab Spanning in Two Directions and With Corners Held Down

$\frac{l_y}{l_x} = r$	1·0	1·1	1·2	1·3	1·4	1·5	1·75	2·0
α_x	0·056	0·064	0·072	0·079	0·085	0·089	0·100	0·107
α_y	For all values of r, $\alpha_y = 0·056$							

Reinforcements are provided in the respective middle strips calculated for the respective bending moments.

At the corners top and bottom reinforcement should be provided for torsion. Both top and bottom reinforcement should consist of two layers of bars placed parallel to the sides of the slab and extending in these directions for a distance of one-fifth of the shorter span. The area of the bars in each of the four layers per unit width of the slab should be three-quarters of the area required for the maximum positive moment in the slab.

No reinforcement parallel to the adjacent edges of the slab need be inserted in the edge strips above that required to comply with the above clause.

In each direction of the main bars alternate bars will be bent up at one-seventh of the approximate span.

Design 15. *Design a slab over a room 5 metres × 7 metres as per I S. code The slab is supported on masonry walls all round with adequate restraint and the corners are held down. The live load on the slab is 300 kg./metre². The slab has a bearing of 15 cm. on the supporting walls.*

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Solution.

Short span between centres of bearing $= 5{\cdot}15\ m.$

Long span between centres of bearing $= 7{\cdot}15\ m.$

Fig. 105

Overall depth of the slab $= \frac{1}{30}$ of short span $= \frac{5{\cdot}15 \times 100}{30} = 17{\cdot}2\ cm.$

say 18 cm.

Providing 12 *mm.* diameter bars, and providing a clear cover of 15 *mm.* to the lower layer of bars,

Effective depth to the centre of short span steel
$= 18 - 2{\cdot}10 = 15{\cdot}90\ cm.$

Effective depth to the centre of long span steel
$= 15{\cdot}90 - 1{\cdot}20 = 14{\cdot}70\ cm.$

Clear span + effective depth for short span steel
$= 5 + 0{\cdot}159 = 5{\cdot}159\ metre$

Clear span + effective depth for long span steel
$= 7 + 0{\cdot}147 = 7{\cdot}147\ metre.$

$\therefore$ Effective short span $= l_x = 5{\cdot}150\ metre$

Effective long span $= l_y = 7{\cdot}147\ metre\ say\ 7{\cdot}15\ metre$

$$r = \frac{l_y}{l_x} = \frac{7{\cdot}15}{5{\cdot}15} = 1{\cdot}388\ say\ 1{\cdot}4\ metre$$

Referring to I S. tables, for $r = 1{\cdot}4$, $\alpha_x = 0{\cdot}085$ and $\alpha_y = 0{\cdot}056$.

Loads

D.L. of slab	$= 18 \times 25$	$= 450\ kg./m.^2$
2 *cm.* granolithic finishing	$= 2 \times 24$	$= 48\ kg./m.^2$
L.L. on the slab		$= 300\ kg./m.^2$
Total		$= 798\ kg./m.^2$

Maximum short span bending moment per *metre* width
$= \alpha_x\ w l_x^2$
$= 0{\cdot}085 \times 798 \times 5{\cdot}15^2 \times 100\ kg.\ cm.$
$= 179900\ kg.\ cm.$

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Maximum long span bending moment per *metre* width

$$= \alpha_y \, w l_x^2$$
$$= 0{\cdot}056 \times 798 \times 5{\cdot}15^2 \times 100 \ kg.\ cm.$$
$$= 118520 \ kg.\ cm.$$

Equating the moment of resistance to the maximum bending moment,

$$8{\cdot}50 \times 100 \, d^2 = 179900$$

$$\therefore \quad d = 14{\cdot}5 \ cm.$$

Effective depth available

$$= 18 - 2{\cdot}10 = 15{\cdot}90 \ cm.$$

$\therefore$ Steel for short span

$$= A_t = \frac{179900}{1400 \times 0{\cdot}87 \times 15{\cdot}9} = 9{\cdot}29 \ cm^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{9{\cdot}29} = 12{\cdot}2 \ cm.$$

Provide 12 mm. diameter bars @ 12 cm. c/c

Steel for long span bending moment

$$= A_t = \frac{118520}{1400 \times 0{\cdot}8 \times 14{\cdot}7} cm.^2 = 6{\cdot}62 \ \ cm.^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{6{\cdot}62} = 17 \ cm.$$

Provide 12 mm. ϕ bars @ 17 cm. c/c

Reinforcement at corners

At each corner a top mesh and a bottom mesh of reinforcement shall be provided covering a square area $\frac{l_x}{5} \times \frac{l_x}{5}$.

In each mesh in each principal direction steel required per *metre* width $= \frac{3}{4} \times$ steel for short span bending moment $= \frac{3}{4} \times 9{\cdot}29$

$$= 6{\cdot}97 \ cm.^2$$

Spacing of 10 *mm.* bars in the mesh

$$= \frac{0{\cdot}79 \times 100}{6{\cdot}97} = 11{\cdot}3 \ cm.$$

Provide 10 mm. ϕ bars @ 11 cm. c/c.

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Bending Moment Coefficients for Rectangular Panels Supported on Four Sides with Provision for Torsion at Corners

(Clause C-1-1)

Case No	Type of panel and moments considered	Short span Coefficient α_x (Values of l_y/l_x)								Long span coefficient α_y for all values of l_y/l_x
		1·0	1·1	1·2	1·3	1·4	1·5	1·75	2	
1.	*Interior panels* :									
	Negative moment at continuous edge	0·032	0·037	0·043	0·047	0·051	0·053	0·060	0·065	0·032
	Positive moment at mid span	0·024	0·028	0·032	0·036	0·039	0·041	0·045	0·049	0·024
2.	*One short edge discontinuous* :									
	Negative moment at continuous edge	0·037	0·043	0·048	0·051	0·055	0·057	0·064	0·068	0·037
	Positive moment at mid span	0·028	0·032	0·036	0·039	0·041	0·044	0·048	0·052	0·028
3.	*One long edge discontinuous* :									
	Negative moment at continuous edge	0·037	0·044	0·052	0·057	0·063	0·067	0·077	0·085	0·037
	Positive moment at mid span	0·028	0·033	0·039	0·044	0·047	0·051	0·059	0·065	0·028
4.	*Two Adjacent edge discontinuous* :									
	Negative moment at continuous edge	0·047	0·053	0·060	0·065	0·071	0·075	0·084	0·091	0·047
	Positive moment at mid span	0·035	0·040	0·045	0·049	0·053	0·056	0·063	0·069	0·035

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Case No.	Type of panel and moments considered	Short span coefficient α_x (values of l_y/l_x) 1·0	1·1	1·2	1·3	1·4	1·5	1·75	2	Long span coefficient α_y for all values of l_y/l_x
5.	*Two short edges discontinuous* :									
	Negative moment at continuous edge	0·045	0·049	0·052	0·056	0·059	0·060	0·065	0·069	—
	Positive moment at mid span	0·035	0·037	0·040	0·043	0·044	0·045	0·049	0·052	0·035
6.	*Two long edges discontinuous* :									
	Negative moment at continuous edge	—	—	—	—	—	—	—	—	0·045
	Positive moment at mid span	0·035	0·043	0·051	0·057	0·063	0·068	0·080	0·088	0·035
7.	*Three edges discontinuous*									
	(*One long edge continuous*)									
	Negative mon ent at continuous edge	0·057	0·064	0·071	0·076	0·080	0·084	0·091	0·097	
	Positive moment at mid span	0·043	0·048	0·053	0·057	0·060	0·064	0·069	0·073	0·043
8.	*Three edges discontinuous*									
	(*One short edge discontinuous*)									
	Negative moment at continuous edge	—	—	—	—	—	—	—	—	0·057
	Positive moment at mid span	0·043	0·051	0·059	0·065	0·071	0·076	0·087	0·096	0·043
9.	*Four edges discontinuous* : Positive moment at mid span	0·056	0·064	0·072	0·079	0·085	0·089	0·100	0·107	0·056

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§110. Flat Slabs

A flat slab consists of an R.C. slab built monolithically with the supporting columns and reinforced in two or more directions. Beams are not provided to support the slabs. The load is carried by the slab which is directly supported by the columns. The columns are generally provided with enlarged heads called *column heads* or *capitals*. To support heavy loads the thickness of the slab over the column is increased. This thickened part of the slab is called a *drop panel*. Hence the various elements in a flat slab construction are : columns, column heads, slabs and drop panels. When the loading is light, the drop panels and sometimes even the column heads are omitted.

§111. Advantages of Flat Slab Construction

From the point of view of utilization of concrete, it may be noted that concrete is more logically used in flat slab construction than in a conventional beam and slab construction. The advantages of a flat slab are many and include the following :

(1) The floor systems require lesser depth and hence there will be a reduction in storey height.

(2) There is a reduction of dead loads and foundation loads since the overall weight and height of the structure are decreased.

(3) Form work for this work is simple.

(4) Flat slab construction has improved fire resistance as compared with other types of systems of floors.

(5) Since beams are absent illumination is better, and the plain ceiling gives an attractive appearance having the best property of diffusing light.

(6) Since beams are absent it is easier to install sprinkler systems and other piping and utilities.

(7) It is relatively easier to point and provide acoustical treatment to the underside of the slab.

(8) A flat slab is capable of withstanding concentrated loads.

§112. Drop of a Flat Slab

Sometimes the thickness of the slab is increased by 25% to 50% over the column heads. This part of the slab of greater thickness is called the Drop.

The drops shall be rectangular in plan and shall have a length in each direction not less than one-third of the panel length in that direction. This is usually taken at one-half the panel length. For exterior panels the width of the drop at right angles to the non-continuous edge measured from the centre line of the columns shall be equal to half the width of the drop for the interior panels.

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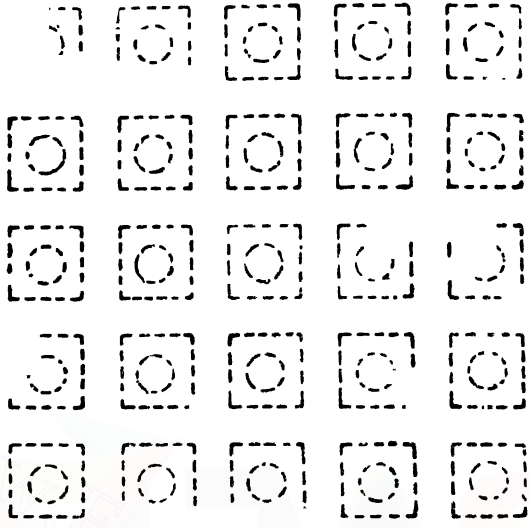

Fig. 106. Flat Slabs

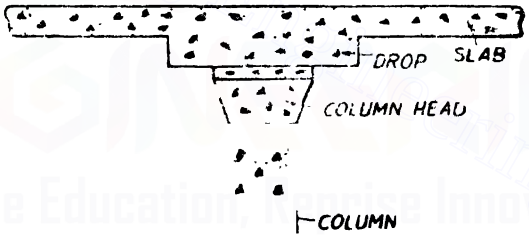

Fig. 107. Components of a flat slab.

§113. Thickness of the Slab

This may be taken not less than the following :

(*i*) 125 *mm*.

(*ii*) $\frac{L}{32}$ for end panels without drops.

(*iii*) $\frac{L}{36}$ for end panels with drops.

(*iv*) $\frac{L}{36}$ for interior panels without drops.

(*v*) $\frac{L}{40}$ for interior panels with drops.

where L=average length of a panel$=\frac{L_1+L_2}{2}$.

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§114. Panel Sub-divisions

The flat slab is taken to consist of the following panel sub-divisions in each principal direction.

(*i*) *Column strip* This is a design strip which has a width of 0·25 l_2 but not greater than 0·25 l_1 on each side of the column centre line, where l_1 is the span in the direction moments are being determined, measured centre to centre of supports and l_2 is the span transverse to l_1, measured centre to centre of supports.

(*ii*) *Middle strip.* This is a design strip bounded on each of its opposite sides by the column strip.

§115. Column Head

The upper supporting part of a column is enlarged to form the column head. The diameter or the column head is made 0·20 to 0·25 of the span length. As per the code, when column heads are provided, that portion of the column head which lies within the largest right circular cone or pyramid that has a vertex angle of 90° and can be included entirely within the outlines of the column and the column head, shall be considered for design purposes. See Fig. 108.

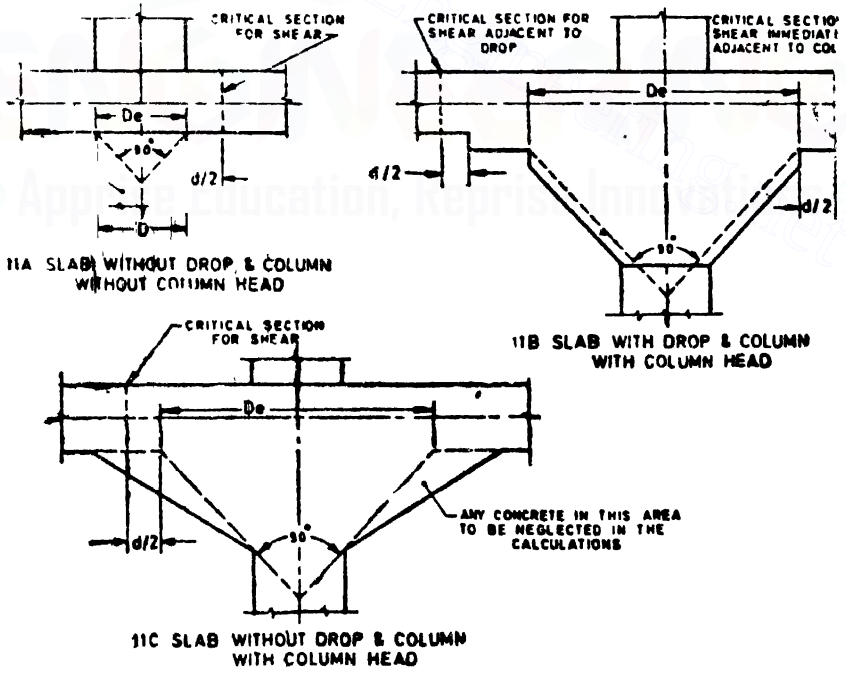

Fig. 108. Critical sections for shear in flat slabs.

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§116. Bending Moment for a Flat Slab

The I.S. code has suggested the following two methods of determining the bending moments in a flat slab :

(*i*) The direct design method, and

(*ii*) The equivalent frame method.

(*i*) *The direct design method*

Limitations. Slab systems designed by the direct design method shall fulfil the following conditions :

(*a*) There shall be minimum of three continuous spans in each direction,

(*b*) The panels shall be rectangular, and the ratio of the longer span to the shorter span within a panel shall not be greater than 2·0,

(*c*) It shall be permissible to offset columns to a maximum of 10 percent of the span in the direction of the offset notwithstanding the provision in (*b*),

(*d*) The successive span lengths in each direction shall not differ by more than one-third of the longer span. The end spans may be shorter, but not longer than the interior spans, and

(*e*) The design live load shall not exceed three times the design dead load.

Total design moment for a span. The total design moment for a span shall be determined for a strip bounded laterally by the centre line of the panel on each side of the centre line of the supports.

The absolute sum of the positive and average negative bending moments in each direction shall be taken as,

$$M_o = \frac{Wl_n}{8}$$

where, M_0 = total moment ;

W = design load on area $l_2 l_n$;

l_n = clear span extending from face to face of columns, capitals, brackets or walls, but not less than 0·65 l_1 ;

l_1 = length of span in the direction of M_o ; and

l_2 = length of span transverse to l_1.

Circular supports shall be treated as square supports having the same area.

When the transverse span of the panels on either side of the centreline of supports varies, l_2 shall be taken as the average of the transverse spans.

When the span adjacent and parallel to an edge is being considered, the distance from the edge to the centre-line of the panel shall be substituted for l_2 in the above expression.

Negative and positive design moments. The negative design moment shall be located at the face of rectangular supports, circular supports being treated as square supports having the same area.

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In an interior span, the total design moment M_o shall be distributed in the following proportions:

Negative design moment 0·65

Positive design moment 0·35

In an end span, the total design moment M_0 shall be distributed in the following proportions :

Interior negative design moment $0{\cdot}75-\dfrac{0{\cdot}10}{1+\dfrac{1}{\alpha_c}}$

Positive design moment $0{\cdot}63-\dfrac{0{\cdot}28}{1+\dfrac{1}{\alpha_c}}$

Exterior negative design moment : $\dfrac{0{\cdot}65}{1+\dfrac{1}{\alpha_c}}$

α_c is the ratio of flexural stiffness of the exterior columns to the flexural stiffness of the slab at a joint taken in the direction of moments are being determined and is given by,

$$\alpha_c=\frac{\Sigma K_c}{K_s}$$

where, ΣK_c = Sum of the flexural stiffnesses of the columns meeting at the joint ; and

K_s = Flexural stiffness of the slab, expressed as moment per unit rotation

It shall be permissible to modify these design moments by up to 10 percent, so long as the total design moment M_o for the panel in the direction considered is not less than that required by the value given earlier.

The negative design moment section shall be designed to resist the larger of the two negative design moments determined for the spans framing into a common support unless an analysis is made to distribute the unbalanced moment in accordance with the stiffness of the adjoining parts.

Distribution of bending moment across the panel width. Bending moments at critical cross-section shall be distributed to the column strips and middle strips as follows :

Column strip

Negative moment at interior supports. At an interior support the column strip shall be designed to resist 75 percent of the total negative moment in the panel at that support.

Negative moment at an exterior support :

(*a*) At an exterior support, the column strip shall be designed to resist the total negative moment in the panel at that support.

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(*b*) Where the exterior support consists of a column or a wall extending for a distance equal to or greater than three quarters of the value of l_2, the length of span transverse to the direction moments are being determined, the exterior negative moment shall be considered to be uniformly distributed across the length l_2.

Positive moment for each span. For each span, the column strip shall be designed to resist 60 percent of the total positive moment in the panel.

Middle strip

The middle strip shall be designed on the following bases :

(*a*) That portion of the design moment not resisted by the column strip shall be assigned to the adjacent middle strips.

(*b*) Each middle strip shall be proportioned to resist the sum of the moments assigned to its two half middle strips.

(*c*) The middle strip adjacent and parallel to an edge supported by a wall shall be proportioned to resist twice the moment assigned to half the middle strip corresponding to the first row of interior columns.

Shear in flat slab. The critical section for shear shall be

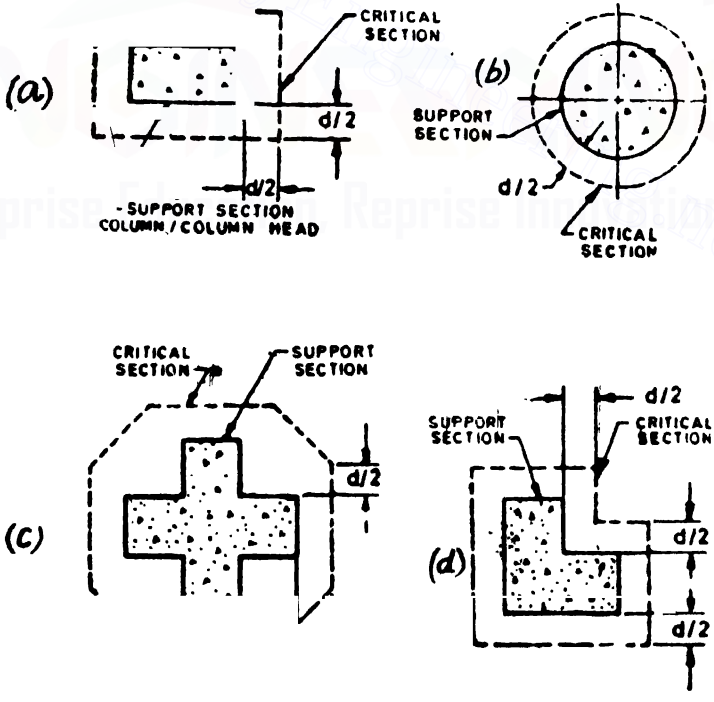

Fig. 109

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at a distance $\frac{d}{2}$ from the periphery of the column/drop panel, perpendicular to the plane of the slab where, d is the effective depth of the section (See Fig. 108). The shape in plan is geometrically similar to the support immediately below the slab [See Figs. 109 (*a*) and (*b*).]

For column sections with re-entrant angles, the critical section shall be taken as shown in Figs. 109 (*c*) and (*d*). In the case of columns near the free edge of a slab, the critical section shall be taken as shown in Fig. 110. When openings in flat slabs are located at a

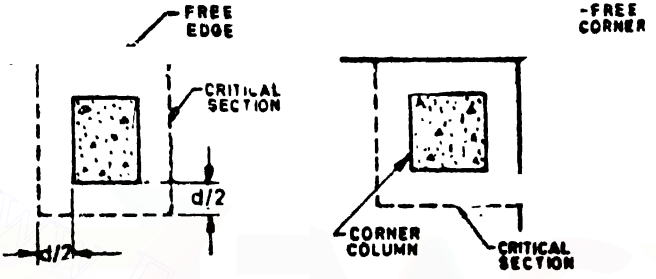

Fig. 110

distance less than ten times the thickness of the slab from a concentrated reaction or when the openings are located within the column

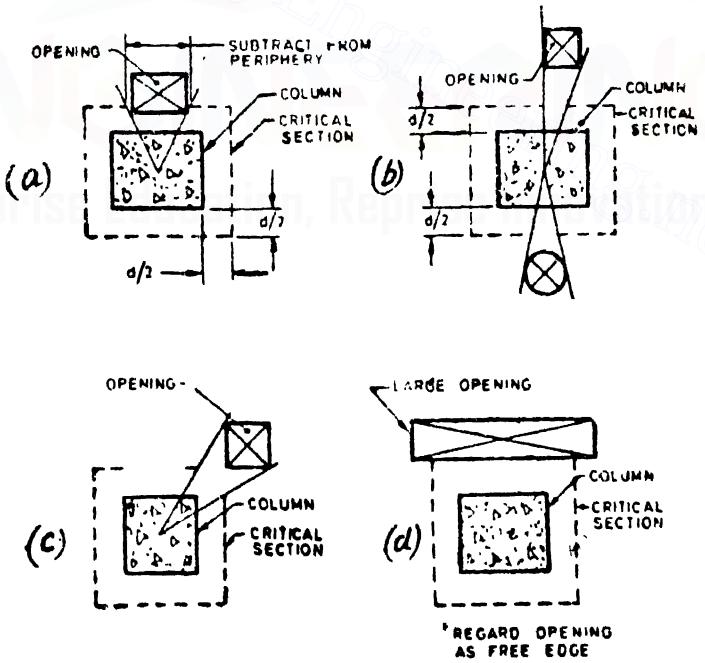

Fig. 111

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strips, the critical sections specified above shall be modified so that part of the periphery of the critical section which is enclosed by radial projections of the openings to the centroid of the reaction area shall be considered ineffective (see Fig. 111), and openings shall not encroach upon column head.

Calculation of shear stresses. The shear stress q_v shall be the sum of the values calculated according to (*a*) and (*b*) below :

(*a*) The nominal shear stress in flat slabs shall be taken as $\frac{V}{b_o d}$ where V is the shear force due to design load, b_o is the periphery of the critical section and d is the effective depth

(*b*) When unbalanced gravity load, wind, earthquake or other forces cause transfer of bending moment between the slab and column, a fraction $(1-\alpha)$ of the moment shall be considered transferred by eccentricity of the shear about the centroid of the critical section. Shear stresses shall be taken as varying linearly about the centroid of the critical section. The value of α shall be obtained by the formula

$$\frac{1}{1+\frac{2}{3}\sqrt{\frac{a_1}{a_2}}}$$

where a_1 = overall dimension of the critical section for shear in the direction in which moment acts, and

a_2 = overall dimension of the critical section for shear transverse to the direction in which moment acts

§117. Permissible Shear Stresses

When shear reinforcement is not provided, the calculated shear stress at the critical section shall not exceed $K_s q_c$

where, $K_s = (0{\cdot}5+\beta_c)$ but not greater than 1, β_c being the ratio of short side to long side of the column/capital ; and

$$q_c = 0{\cdot}16\sqrt{f_{ck}}$$

[In M.K.S. designs $q_c = 0{\cdot}5\sqrt{f_{ck}}$].

When the shear stress at the critical section exceeds the value given above, but less than $1{\cdot}5\,q_c$ shear reinforcement shall be provided. If the shear stress exceeds $1{\cdot}5\,q_c$, the flat slab shall be redesigned. Shear stresses shall be investigated at successive section more distant from the support and shear reinforcement shall be provided upto a section where the shear stress does not exceed $0{\cdot}5\,q_c$. While designing the shear reinforcement, the shear stress carried by the concrete shall be assumed to be $0{\cdot}5\,q_c$ and reinforcement shall carry the remaining shear.

§118. Reinforcement in a flat slab

Spacing. The spacing of bars in a flat slab shall not exceed

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2 times the slab thickness, except where a slab is of cellular or ribbed construction.

When drop panels are used, the thickness of the drop panel for determination of area of reinforcement shall be the lesser of the following :

(*a*) Thickness of drop, and

(*b*) Thickness of slab plus one quarter the distance between edge of drop and edge of capital.

Minimum length of reinforcement

(*a*) Reinforcement in flat slab shall have the minimum lengths specified in Fig. 112. Larger lengths of reinforcement shall be provided when required by analysis.

(*b*) Where adjacent spans are unequal, the extension of negative reinforcement beyond each face of the common column shall be based on the longer span

Anchoring Reinforcement

(*a*) All positive reinforcement perpendicular to a discontinuous edge shall have anchorage (straight, bent or otherwise anchored) past the internal face of the spandrel beam, wall or column, of an amount :

(1) *For positive reinforcement* not less than 15 *cm.* except that with fabric reinforcement having a fully welded transverse wire directly over the support, it shall be permissible to reduce this length to one-half of the width of the support or 5 *cm.* whichever is greater ; and

(2) *For negative reinforcement* such that the design stress is developed at the internal face.

(*b*) Where the slab is not supported by a spandrel beam or wall, or where the slab cantilevers beyond the support, the anchorage shall be obtained within the slab

§119. Openings in Flat Slabs

Openings of any size may be provided in the flat slab if it is shown by analysis that the requirements of strength and serviceability are met However, for openings conforming to the following no special analysis is required.

(*a*) Openings of any size may be placed within the middle half of the span in each direction provided the total amount of reinforcement required for the panel without the opening is maintained.

(*b*) In the area common to two column strips not more than one-eighth of the width of strip in either span shall be interrupted by the openings.

(*c*) In the area common to one column strip and one middle strip, not more than one-quarter of the reinforcement

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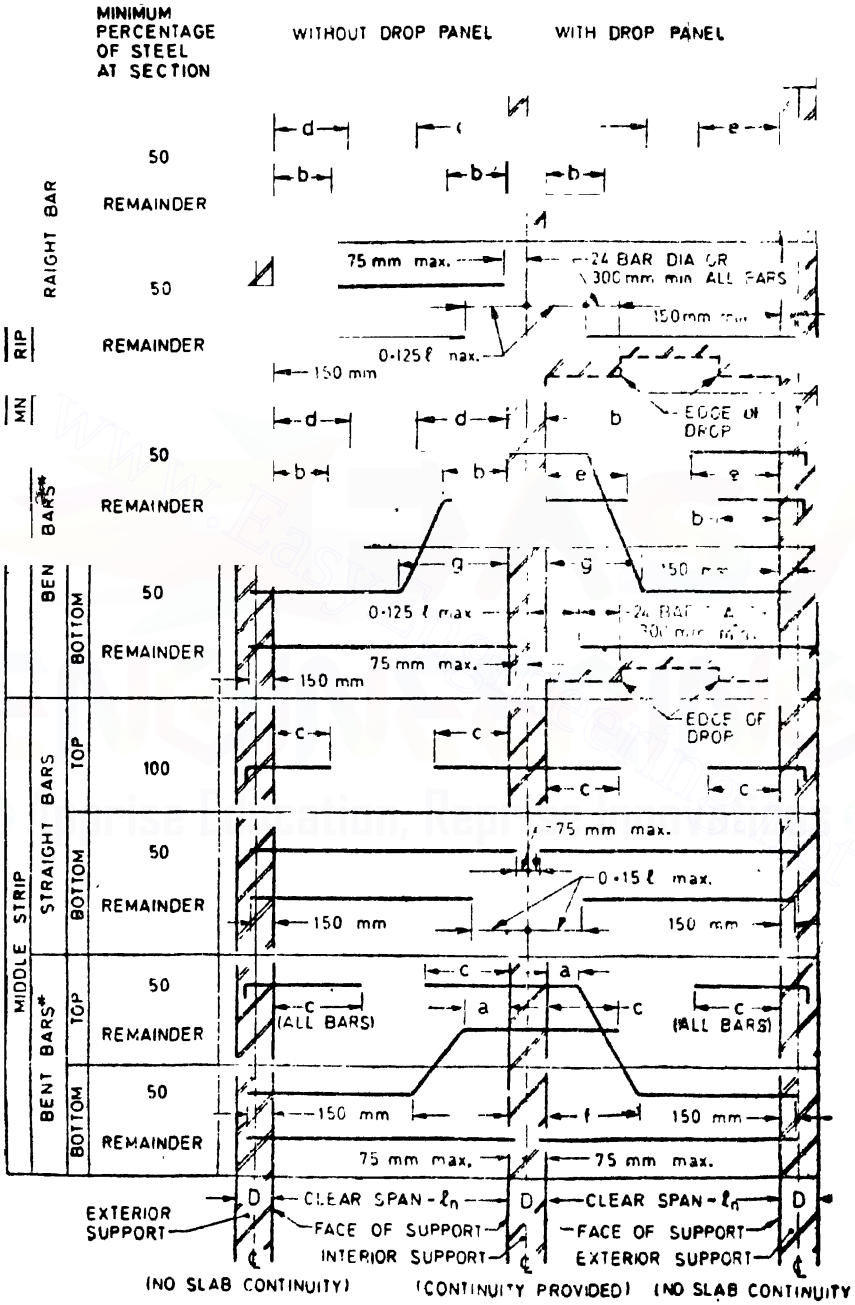

Fig. 112

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Minimum Bend Joint Locations and Extensions for Reinforcement in Flat Slabs

	Bar Length From Face or Support						
	Minimum Length					*Maximum Length*	
Mark	*a*	*b*	*c*	*d*	*e*	*f*	*g*
Length	0·14 l_n	0·20 l_n	0·22 l_n	0·30 l_n	0·33 l_n	0·20 l_n	0·24 l_n

* ent bars at exterior supports may be used if a general analysis is made.

NOTE—D is the diameter of the column and the dimension of the rectangular column in the direction under consideration.

in either strip shall be interrupted by the openings. The equivalent of reinforcement interrupted shall be added on all sides of the openings.

(*d*) The shear requirements mentioned above shall be satisfied.

§120. Moments in Columns

Columns built integrally with the slab system shall be designed to resist moments arising from loads on the slab system.

At an interior support, the supporting members above and below the slab shall be designed to resist the moment M given by the following equation, in direct proportion to their stiffnesses unless a general analysis is made :

$$M = 0{\cdot}08 \frac{(w_d + 0{\cdot}5 w_l) l_2 l_n^2 - w_d' l_2' l_n'^2}{1 + \frac{1}{\alpha_c}}$$

where w_d, w_l = design dead and live loads respectively, per unit area,

l_2 = length of span transverse to the direction of M,

l_n = length of the clear span in the direction of M, measured face to face of supports,

$\alpha_c = \frac{\Sigma K_c}{\Sigma K_s}$ where K_c and K_s are as defined earlier and,

w_d', l_2' and l_n' refer to the shorter span.

Effects of pattern loading. In the direct design method, when the ratio of live load to dead load exceeds 0·5,

(*a*) The sum of the flexural stiffnesses of the columns above and below the slab, ΣK_c, shall be such that α_c is not less than the

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appropriate minimum value $\alpha_{c\ min}$ specified in the table below.

Minimum Permissible Values of α_c

Live load / Dead load	$\frac{l_2}{l_1}$	*Value of* $\alpha_{c\ min}$
0·5	0·5 to 2	0
1·0	0·5	0·6
1·0	0·8	0·7
1·0	1·0	0·7
1·0	1·25	0·8
1·0	2·0	1·2
2·0	0·5	1·3
2·0	0·8	1·5
2·0	1·0	1·6
2·0	1·25	1·9
2·0	2·0	1·9
3·0	0·5	1·8
3·0	0·8	2·0
3·0	1·0	2·3
3·0	1·25	2·8
3·0	2·0	3·0

(*b*) If the sum of the flexural stiffnesses of the columns ΣK_c, does not satisfy (*a*). The positive design moments for the panel shall be multiplied by the coefficient β_s given by the following equations :

$$\beta_s = 1 + \frac{2 - \frac{w_d}{w_l}}{4 + \frac{w_d}{w_l}} \left[1 - \frac{\alpha_c}{\alpha_{c\ min}} \right]$$

α_c is the ratio of flexural stiffness of columns above and below the slab to the flexural stiffness of the slabs at a joint taken in the direction moments are being determined, and given by,

$$\alpha_c = \frac{\Sigma K_c}{\Sigma K_s}$$

where K_c and K_s are flexural stiffnesses of column and slab respectively.

§121. Equivalent Frame Method

The bending moments and shear forces may be determined by an analysis of the structure as a continuous frame and the following assumptions may be made :

(*a*) The structure shall be considered to be made up of equivalent frames on column lines taken longitudinally and transversely through the building. Each frame consists of a row of equivalent columns or supports, bounded laterally by the centre line of the panel on each side of the centre line of the columns or supports.

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Frames adjacent and parallel to an edge shall be bounded by the edge and the centre line of the adjacent panel.

(*b*) Each such frame may be analysed in its entirety, or, for vertical loading, each floor thereof and the roof may be analysed separately with its columns being assumed fixed at their remote ends. Where slabs are thus analysed separately, it may be assumed in determining the bending moment at a given support that the slab is fixed at any support two panels distant therefrom provided the slab continues beyond that point.

(*c*) For the purpose of determining relative stiffness of members, the moment of inertia of any slab or column may be assumed to be that of the gross section of the concrete alone.

(*d*) Variations of moment of inertia along the axis of the slab on account of provision of drops shall be taken into account. In the case of recessed or coffered slab which is made solid in the region of the columns, the stiffening effect may be ignored provided the solid part of the slab does not extend more than $0{\cdot}15\,l_e$, into the span measured from the centre line of the columns. The stiffening effect of flared columns heads may be ignored.

Loading pattern

When the loading pattern is known, the structure shall be analysed for the load concerned.

When the live load is variable but does not exceed three-quarters of the dead load, or the nature of the live load is such that all panels will be loaded simultaneously, the maximum moments may be assumed to occur at all sections when full design live load is on the entire slab system.

For other conditions of live load/dead load ratio and when all panels are not loaded simultaneously :

(*a*) maximum positive moment near midspan of a panel may be assumed to occur when three-quarters of the full design live load is on the panel and on alternate panels ; and

(*b*) maximum negative moment in the slab at a support may be assumed to occur when three-quarters of the full design live load is on the adjacent panels only.

In no case shall design moments be taken to be less than those occurring with full design live load on all panels.

Negative design moment. At interior supports, the critical section for negative moment, in both the column strip and the middle strip, shall be taken at the face of rectilinear supports, but in no case at a distance greater than $0{\cdot}175\,l_1$ from the centre of the column where l_1 is the length of the span in the direction moments are being determined, measured centre to centre of supports.

At exterior supports provided with brackets or capitals, the critical section for negative moment in the direction perpendicular to the edge shall be taken at a distance from the face of the supporting

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element not greater than one-half the projection of the bracket or capital beyond the face of the supporting element.

Modification of Maximum Moment. Moments determined by means of the equivalent frame method, for slabs which fulfil the limitations mentioned earlier, may be reduced in such proportion that the numerical sum of the positive and average negative moments is not less than the value of total design moment M_o given by,

$$M_o = \frac{Wl_n}{8} \text{ (given in the direct design method)}$$

The bending moment distribution across the panel width shall be done as explained in the direct design method.

Design 16 (S I.). *Design an interior panel of a flat slab for a live load of 5000 N/metre². Drops shall be provided. All panels are 6 m × 6 m. Use M 15 concrete.*

Solution.

DL of the slab (20 *cm.* thickness) $= 250 \times 20 = 5{,}000$ *N/metre²*

LL on the slab $= 5{,}000$ *N/metre²*

Total $= 10{,}000$ *N/metre²*

Diameter of the column head

$$= D = \frac{L}{4} = 1{\cdot}50 \text{ metre}$$

Size of Drop $= \frac{L}{2} \times \frac{L}{2} = 3 \text{ m.} \times 3 \text{ m}$

B.M. Calculations Absolute sum of the positive and average negative bending moments in each direction

$$= M_o = \frac{WL}{8}$$

where, W = Design load on the area $L \times L$

$= 10{,}000 \times 6 \times 6 = 360{,}000$ *N*

$$\therefore \quad M_o = \frac{360{,}000 \times 6}{8} \times 100 \text{ N cm.}$$

$= 27{,}000{,}000$ *N cm.*

Negative design moment

$= 0{\cdot}65\ M_o = 0{\cdot}65 \times 27{,}000{,}000$

$= 17{,}550{,}000$ *N cm.*

Positive design moment

$= 0{\cdot}35\ M_o = 0{\cdot}35 \times 27{,}000{,}000$

$= 9{,}450{,}000$ *N cm.*

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B.M. for column strip

Negative moment for column strip

$= 75\%$ of total negative moment in the panel
$= 0{\cdot}75 \times 17{,}550{,}000 = 13{,}162{,}500$ *N cm.*

Positive moment for column strip

$= 60\%$ of total positive moment in the panel
$= 0\ 60 \times 9{,}450{,}000 = 5{,}670{,}000$ *N cm.*

B.M. for middle strip

Negative moment for middle strip

$= 25\%$ of total negative moment in the panel
$= 0{\cdot}25 \times 17{,}550{,}000 = 4{,}387{,}500$ *N cm.*

Positive moment for middle strip

$= 40\%$ of total positive moment in the panel
$= 0{\cdot}40 \times 9{,}450{,}000 = 3{,}780{,}000$ *N cm.*

The depth of the slab must be determined for a bending moment of 5,670,000 *N cm.*

Equating the moment of resistance to the bending moment,

$$85\ bd^2 = 85 \times 300\ d^2 = 5{,}670{,}000$$

$$\therefore \quad d = 14{\cdot}90\ cm.$$

Using 12 *mm.* diameter bars, and providing a clear cover of 15 *mm.* to the lower layer of bars.

effective cover to the centre of the upper layer of bars
$= 15 + 12 + 6 = 33\ mm. = 3{\cdot}3\ cm.$

$\therefore$ Thickness of slab $= 14{\cdot}90 + 3{\cdot}3 = 18{\cdot}20\ cm.$

Provide a thickness of 20 cm.

Depth of drop. Equating the moment of resistance to the maximum negative bending moment for the column strip,

$$85\ bd^2 = 85 \times 300\ d^2 = 13162500$$

$$\therefore \quad d = 22{\cdot}70\ cm.$$

$\therefore$ Thickness required for the drop

$$= 22{\cdot}70 + 3{\cdot}3 = 26\ cm.$$

Provide a thickness of 30 cm. for the drop.

Reinforcement for bending moment

Column strip

Steel for negative moment

$$\frac{13162500}{14{,}000 \times 0{\cdot}87 \times 26{\cdot}7} = 40{\cdot}50\ cm.^2$$

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This is the steel requirement for the entire width of 3 *metres.*

∴ Steel required per *metre* width

$$= \frac{40{\cdot}5}{3} = 13{\cdot}5 \text{ cm.}^2$$

∴ Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{13{\cdot}5} = 8{\cdot}37 \text{ cm.}$$

Provide 12 mm. diameter bars @ 8 cm c/c.

Steel for positive moment

$$= \frac{5670000}{14000 \times 0{\cdot}87 \times 16{\cdot}7} = 27{\cdot}9 \text{ cm.}^2$$

∴ Steel per *metre* width

$$= \frac{27{\cdot}9}{3} = 9{\cdot}3 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{9{\cdot}3} = 12{\cdot}15 \text{ cm}^2$$

Provide 12 mm. diameter bars @ 12 cm. c/c.

Middle strip

Steel for negative moment

$$= \frac{4387500}{14000 \times 0\,87 \times 16{\cdot}7} = 21{\cdot}6 \text{ cm.}^2$$

∴ Steel per *metre* width

$$= \frac{21{\cdot}6}{3} = 7{\cdot}2 \text{ cm.}^2$$

Spacing of 12 *mm* diameter bars

$$= \frac{1{\cdot}13 \times 100}{7{\cdot}2} = 15{\cdot}7 \text{ cm.}^2$$

Provide 12 mm diameter bars @ 15 cm. c/c

Steel for positive moment

$$= \frac{3780000}{14000 \times 0{\cdot}87 \times 16{\cdot}7} = 18{\cdot}6 \text{ cm.}^2$$

∴ Steel per *metre* width

$$= \frac{18{\cdot}6}{3} = 6{\cdot}2 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{6{\cdot}2} = 18{\cdot}2 \text{ cm.}$$

Provide 12 mm. diameter bars @ 18 cm. c/c.

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Check for shear stresses. It is necessary to check for shear stresses at the following critical sections specified by the I.S code.

(*a*) *Critical section in the drop.* This critical section is taken at a distance of half the effective depth of the drop from the column head.

Effective depth of the drop

$$= 30 - 3{\cdot}3 = 26{\cdot}7 \text{ cm.}$$

Diameter of the critical circle

$$= 1{\cdot}50 + 0{\cdot}267 = 1{\cdot}767 \text{ m.}$$

Load on one column

Load from the slab : $10000 \times 6 \times 6 = 360000$ *N*

Additional load due to 10 *cm.* additional thickness : $250 \times 10 \times 3 \times 3 = 22500$ *N*

Total $= 382500$ *N*

Total shear force on the critical section

= Total load transferred to one column − Total load acting within the critical circle.

$$= 382500 - \frac{\pi}{4} \times 1{\cdot}767^2 \, [250 \times 30 + 5000]$$

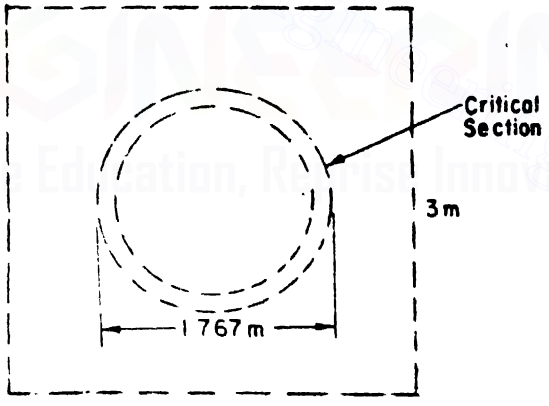

Fig. 113

$$= 382500 - 30600 = 351900 \text{ N}$$

$\therefore$ Nominal shear stress

$$= q_v = \frac{S}{bd},$$

where, S = Total shear force on the critical section
$= 351900$ *N*.

b = Perimeter of the critical circle

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$= \pi \times 1{\cdot}767 \times 100 = 555{\cdot}12$ *cm.*

d = effective depth $= 26{\cdot}7$ *cm.*

$$\therefore \quad q_v = \frac{351900}{555{\cdot}12 \times 26{\cdot}7} = 23{\cdot}7 \text{ } N/cm.^2$$

$$= 0{\cdot}237 \text{ } N/mm.^2$$

Permissible shear stress

$$= q_c = 0{\cdot}16\sqrt{f_{ck}} = 0{\cdot}16\sqrt{15} = 0{\cdot}61 \text{ } N/mm.^2$$

$\therefore \quad q_v < \quad \therefore$ The design is safe.

(*b*) *Critical section for the slab.* This critical section is taken at a distance of one half the effective depth of the slab from the edge of the drop. (See Fig. 114).

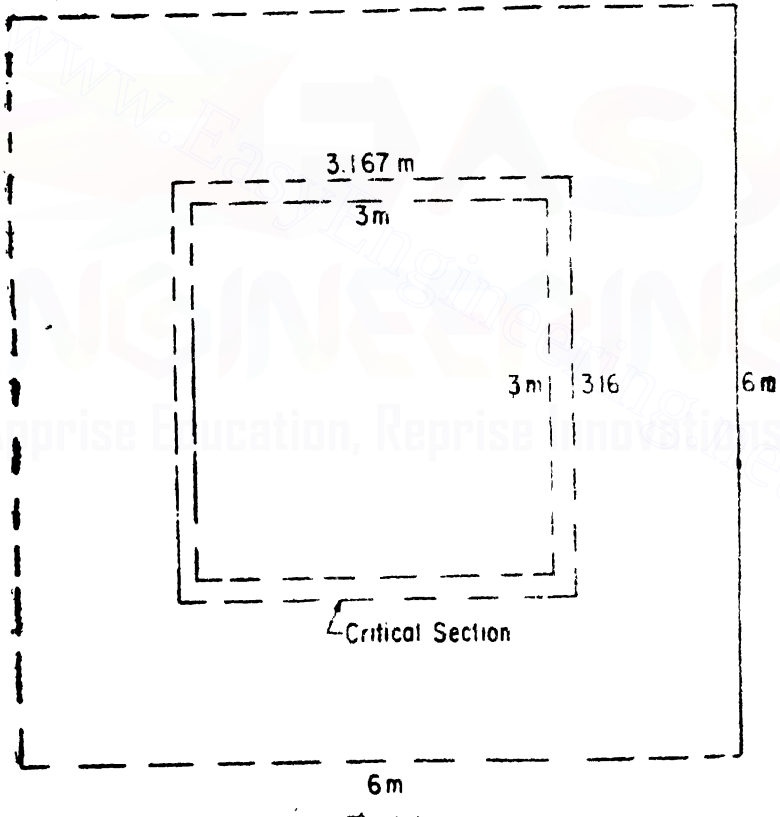

Fig. 114

Total length of the critical section

$= 4(3 + 0{\cdot}167) = 12{\cdot}668$ *m.* $= 1266{\cdot}8$ *cm.*

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Total shear force on the critical section

$=$Load acting on an area of $(6^2-3{\cdot}167^2)=25{\cdot}97$ *metre*^{2}

$\therefore$ Shear force at the critical section

$=10{,}000\times25{\cdot}97=259700\ N$

$\therefore$ Nominal shear stress

$$=q_v=\frac{S}{bd}=\frac{259700}{1266{\cdot}8\times16{\cdot}7}=12{\cdot}2\ N/cm.^2$$

$$=0{\cdot}122\ N/mm.^2$$

Permissible nominal shear stress

$$=q_c=0{\cdot}16\sqrt{f_{ck}}=0{\cdot}16\sqrt{15}=0{\cdot}61\ N/mm.^2$$

$\therefore$ $q_v<q_c$ $\therefore$ The design is safe.

Design 17. *Design an interior panel of a flat slab for a live load of 500 kg./metre*^{2}*. Drops shall be provided. All panels are 6 m.*$\times$*6 m. Use M 150 concrete.*

Solution.

Loading on the slab

D.L. of the slab (20 *cm.* thickness) $=25\times20$ $=$ 500 *kg./metre*^{2}

L.L. on the slab $=$ 500 *kg./metre*^{2}

Total $=1000$ *kg./metre*^{2}

Diameter of the column head

$$=D=\frac{L}{4}=\frac{6}{4}=1{\cdot}50\ metre.$$

Size of Drop $\frac{L}{2}\times\frac{L}{2}=3\ m.\times3\ m.$

B.M. Calculations

Absolute sum of the positive and average negative bending moments in each direction $=M_o=\frac{WL}{8}$

where, $W=$Design load on the area $L\times L$

$=1000\times6\times6=36000\ kg.$

$\therefore$ $$M_o=\frac{36000\times6}{8}\times100\ kg.\ cm.$$

$$=2{,}700{,}000\ kg.\ cm.$$

Negative design moment $=0{\cdot}65\ M_o$

$=0{\cdot}65\times2{,}700{,}000=1755000\ kg.\ cm.$

Positive design moment $=0{\cdot}35\ M_o$

$=0{\cdot}35\times2{,}700{,}000=945000\ kg.\ cm.$

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B.M. for column strip

Negative moment for column strip
$= 75\%$ of total negative moment in the panel.
$= 0{\cdot}75 \times 1755000 = 1316250$ *kg. cm.*

Positive moment for column strip
$= 60\%$ of total positive moment in the panel.
$= 0{\cdot}60 \times 945000 = 567000$ *kg. cm.*

B.M. for middle strip

Negative moment for middle strip
$= 25\%$ of total negative moment in the panel
$= 0{\cdot}25 \times 1755000 = 438750$ *kg cm.*

Positive moment for middle strip
$= 40\%$ of total positive moment in the panel
$= 0{\cdot}40 \times 945000 = 378000$ *kg. cm.*

The depth of the slab must be determined for a bending moment of 567000 *kg. cm.*

Equating the moment of resistance to the bending moment,

$$8{\cdot}5\, bd^2 = 8{\cdot}5 \times 300\, d^2 = 567000$$

$$\therefore \quad d = 14{\cdot}90 \text{ cm.}$$

Using 12 *mm.* diameter bars, and providing a clear cover of 15 *mm.* to the lower layer of bars,

Effective cover to the centre of the upper layer of bars
$= 15 + 12 + 6 = 33$ *mm.* $= 3{\cdot}3$ *cm.*

$\therefore$ Thickness of slab $= 14{\cdot}90 + 3{\cdot}30 = 18{\cdot}2$ *cm.*

Provide a thickness of 20 cm.

Depth of drop. Equating the moment of resistance to the maximum negative bending moment for the column strip

$$8{\cdot}50\, bd^2 = 8{\cdot}50 \times 300\, d^2 = 1316250$$

$$\therefore \quad d = 22{\cdot}7 \text{ cm.}$$

$\therefore$ Thickness required for the drop
$= 22{\cdot}7 + 3{\cdot}3 = 26$ *cm.*

Provide a thickness of 30 cm. for the drop.

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Reinforcement for bending moment

Column strip

Steel for negative moment

$$= \frac{1316250}{1400 \times 0{\cdot}87 \times 26{\cdot}7} = 40{\cdot}50 \text{ cm.}^2$$

This is the steel requirement for the entire width of 3 *metres*

$\therefore$ Steel required per *metre* width

$$= \frac{40{\cdot}5}{3} = 13{\cdot}5 \text{ cm.}^2$$

$\therefore$ Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{13{\cdot}5} = 8{\cdot}37 \text{ cm.}$$

Provide 12 mm. diameter bars @ 8 cm. c/c.

Steel for positive moment

$$= \frac{567000}{1400 \times 0{\cdot}87 \times 16{\cdot}7} = 27{\cdot}9 \text{ cm.}^2$$

Steel per *metre* width

$$= \frac{27{\cdot}9}{3} = 9{\cdot}3 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{9{\cdot}3} = 12{\cdot}15 \text{ cm.}$$

Provide 12 mm diameter bars @ 12 cm. c/c.

Middle Strip

Steel for negative moment

$$= \frac{438750}{1400 \times 0{\cdot}87 \times 16{\cdot}7} = 21{\cdot}6 \text{ cm.}^2$$

$\therefore$ Steel per *metre* width

$$= \frac{21{\cdot}6}{3} = 7{\cdot}2 \text{ cm.}^2$$

Spacing of 12 *mm* diameter bars

$$= \frac{1{\cdot}13 \times 100}{7{\cdot}2} = 15{\cdot}7 \text{ cm.}$$

Provide 12 mm. diameter bars @ 15 cm. c/c.

Steel for positive moment

$$\frac{378000}{1400 \times 0{\cdot}87 \times 16{\cdot}7} = 18{\cdot}6 \text{ cm.}^2$$

$\therefore$ Steel per *metre* width

$$= \frac{18{\cdot}6}{3} = 6{\cdot}2 \text{ cm.}^2$$

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Spacing of 12 *mm.* diameter bars

$$\frac{1{\cdot}13\times100}{6{\cdot}2}=18{\cdot}2 \text{ cm.}$$

Provide 12 mm. diameter bars @ 18 cm. c/c.

Check for shear stress. It is necessary to check for shear stresses at the following critical sections specified by the I.S. code.

(*a*) *Critical section in the drop.* This critical section is taken at a distance of half the effective depth of the drop from the column head.

Effective depth of the drop

$$=30-3{\cdot}3=26{\cdot}7 \text{ cm.}$$

Diameter of the critical circle

$$=1{\cdot}50+0{\cdot}267=1{\cdot}767 \text{ m.}$$

Load on one column :

Load from slab : $1000\times6\times6$		$=36000$ kg.
Additional load due to 10 *cm.* additional thickness	$=25\times10\times3\times3$	$=2250$ kg.
	Total	$=38250$ kg.

Total shear force on the critical section = Total load transferred to one column − total load acting within the critical circle

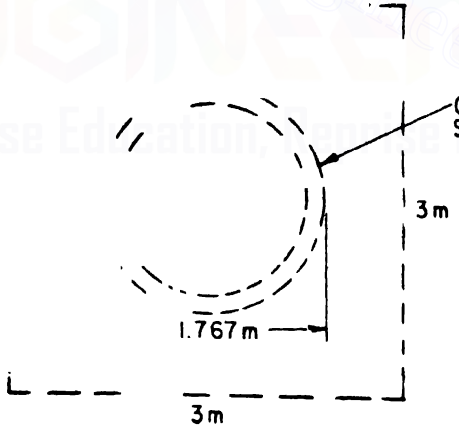

Fig. 115

$$=38250-\frac{\pi}{4}\times1{\cdot}767^2\,[25\times30+500]$$

$$=38250-3060=35190 \text{ kg.}$$

Nominal shear stress

$$=q_v=\frac{S}{bd}$$

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where, S=Total shear force on the critical section
$=35190$ *kg.*
b=perimeter of the critical circle
$=\pi \times 1{\cdot}767 \times 100 = 555{\cdot}12$ *cm.*
d=effective depth $=26{\cdot}7$ *cm.*

$$\therefore \quad q_v = \frac{35190}{555{\cdot}12 \times 26{\cdot}7} = 2{\cdot}37 \text{ kg./cm.}^2$$

Permissible nominal shear stress

$$= q_c = 0{\cdot}5\sqrt{f_{ck}} = 0{\cdot}5\sqrt{150} = 6{\cdot}1 \text{ kg./cm.}^2$$

$\therefore$ $q_v < q_c$. Hence the design is safe.

(*b*) *Critical section for the slab.* This critical section is taken at a distance of one half the effective depth of the slab from the edge of the drop. (See Fig. 116).

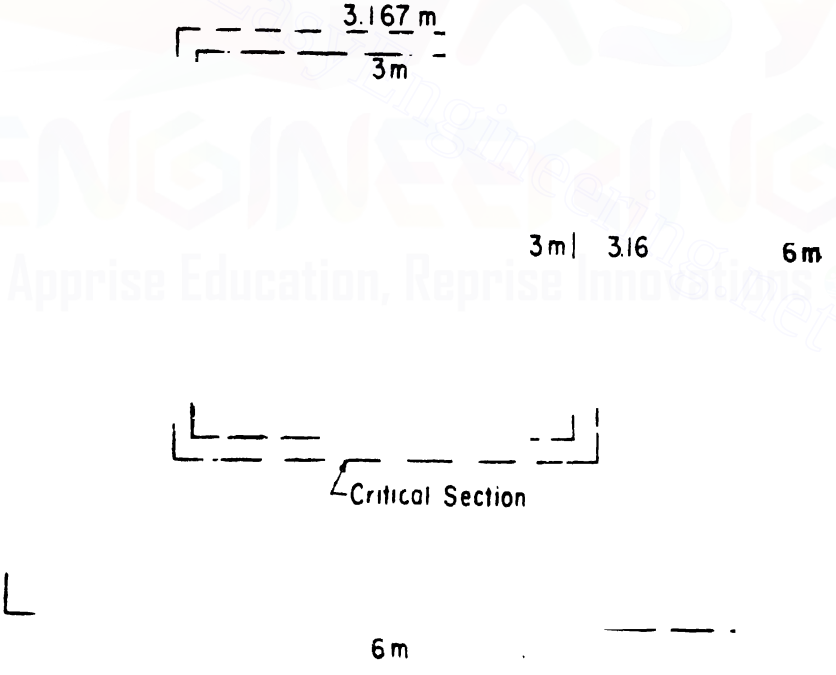

Fig. 116

Total length of the critical section

$$= 4(3 + 0{\cdot}167) = 12{\cdot}668 \text{ m.} = 1266{\cdot}8 \text{ cm.}$$

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Total shear force on the critical section

$$= \text{load acting on an area of } (6^2 - 3{\cdot}167^2)$$

$$= 25{\cdot}97 \text{ metre}^2$$

$\therefore$ Shear force at the critical section

$$= 1000 \times 25{\cdot}97 = 25970 \text{ kg.}$$

$\therefore$ Nominal shear stress

$$= q_v = \frac{S}{bd}$$

$$= \frac{25970}{1266{\cdot}8 \times 16{\cdot}7} = 1{\cdot}22 \text{ kg./cm.}^2$$

Permissible nominal shear stress

$$= 0{\cdot}5\sqrt{f_{ck}} = 0{\cdot}5\sqrt{150} = 6{\cdot}1 \text{ kg./cm.}^2$$

$\therefore$ $q_v < q_c$. Hence the design is safe.

§122. Circular Slabs

Circular slabs are uncommon in buildings ; but such slabs are used in circular water tanks. Circular slabs when subjected to external loading will deflect to a saucer shape. Hence, they develop circumferential as well as radial stresses. In order to prevent circumferential and radial cracks it is, therefore, necessary to provide radial and circumferential reinforcement. But this arrangement of placing the reinforcement will be found inconvenient near the centre due to congestion of radial bars. Near the centre the reinforcement should, therefore, consist of a mesh so that the reinforcement provided in each principal direction is at least equal to the greater of the requirements of circumferential and radial bending moments.

Near the periphery of the slab, however, it is found convenient to provide the radial and circumferential system of reinforcement. In simply supported slabs the above reinforcements may be necessary when the stresses are considerable These reinforcements may be found absolutely necessary when the slab is fixed along the edges.

We come across the following cases of circular slabs :

Case 1 *Simply supported circular slab and carrying a uniformly distributed loading.*

For this case the maximum circumferential and radial moments occur at the centre Let R be the radius of the slab. Let w be the load on the slab per unit area.

Radial moment M_r

The radial moment increases parabolically from zero at the support to $\frac{3}{16} wr^2$ per unit width of the slab, at the centre.

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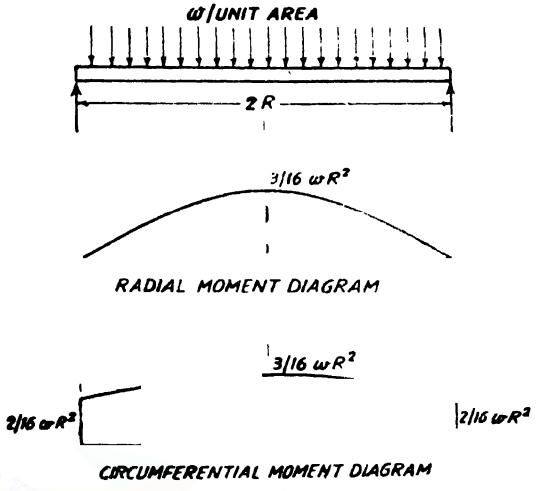

Fig. 117. Simply supported slab.

Circumferential moment M_θ

This moment increases parabolically from $\frac{2}{16}wR^2$ at the support to $\frac{3}{16}wR^2$ at the centre. Radial shear force at any radius x is $\frac{wx}{2}$ per unit width.

There will be no circumferential shear. [See Fig 117]

Case 2. *Slab fixed at the ends and carrying a uniformly distributed loading.*

Radial moment M_r

Maximum positive radial moment occurs at the centre while the maximum negative radial moment occurs at the ends.

$$\text{Maximum positive moment} = \frac{wR^2}{16}$$

$$\text{Maximum negative moment} = \frac{2}{16}wR^2$$

Circumferential moment M_θ

The circumferential moment increases from zero at the edges to $\frac{wR^2}{16}$ at the centre. (See Fig. 118)

Case 3. *Simply supported slab carrying a total load W distributed on the circumference of a concentric circle.*

Let R be the radius of the slab and r the radius of the load circle. (See Fig. 119).

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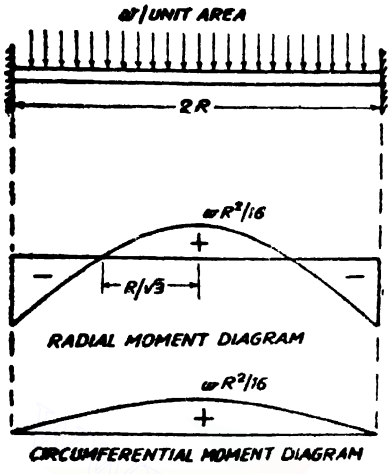

Fig. 118

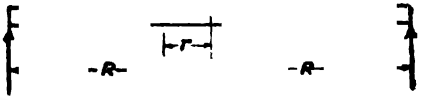

Fig. 119

At any distance $x<r$ from the centre,

Radial moment $M_r = \frac{W}{8\pi}\left[2 \log_e \frac{R}{r} + 1 - \left(\frac{r}{R}\right)^2\right]$ per unit width

Circumferential moment $M_\theta = M_r$

Shear force $= 0$

At any distance $x>r$ from the centre

$$M_r = \frac{W}{8\pi}\left[2 \log_e \frac{R}{x} - \left(\frac{r}{R}\right)^2 + \left(\frac{r}{x}\right)^2\right]$$ per unit width

$$M_\theta = \frac{W}{8\pi}\left[2 \log_e \frac{R}{x} - \left(\frac{r}{x}\right)^2 + 2 - \left(\frac{r}{R}\right)^2\right]$$ per unit width

Shear force $= \frac{W}{2\pi x}$

Case 4. *Simply supported slab carrying a total load W uniformly applied in a concentric circular area.*

Let R be the radius of the circular slab. (See Fig. 120).

Let r be the radius of the loaded circular area.

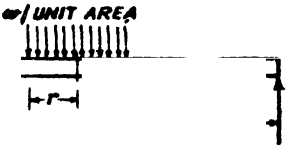

Fig. 120

Let the loading be w per unit area.

At any distance $x<r$ from the centre.

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$$M_r = -\frac{3}{16} wx^2 + \frac{1}{4} wr^2 \left[1 - \log_e \frac{r}{R} - \frac{r^2}{4R^2}\right]$$

per unit width

$$M_\theta = -\frac{1}{16} wx^2 + \frac{1}{4} wr^2 \left[1 - \log_e \frac{r}{R} - \frac{r^2}{4R^2}\right]$$

per unit width

Shear force $= wx$

At any distance $x > r$ from the centre,

$$M_r = -wr^2 \left[\frac{1}{4} \log_e \frac{x}{R} + \frac{r^2}{16}\left(\frac{1}{R^2} - \frac{1}{x^2}\right)\right]$$

per unit width

$$M\theta = -wr^2 \left[\frac{1}{4} \log_e \frac{x}{R} - \frac{1}{4} + \frac{r^2}{16}\left(\frac{1}{R^2} + \frac{1}{x^2}\right)\right]$$

per unit width

Shear force $= \frac{1}{2} \frac{wr^2}{x}$

Case 5. *Simply supported slab with a central hole and loaded with load W distributed on the circumference of a concentric circle.*

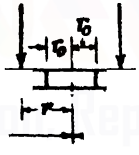

Fig. 121

Let R be the radius of the slab.

Let r be the radius of the load circle.

Let r_o be the radius of the central hole.

At any distance $x < r$

$$M_r = \frac{W}{4\pi} \frac{R^2}{(R^2 - r_o^2)} \left(1 - \frac{r_o^2}{x^2}\right)\left[\log_e \frac{R}{r} + \frac{1}{2} - \frac{r^2}{2R^2}\right]$$

per unit width

Shear force $= 0$

$$M_\theta = \frac{W}{4\pi} \frac{R^2}{(R^2 - r_o^2)} \left(1 + \frac{r_o^2}{x^2}\right)\left[\log_e \frac{R}{r} + \frac{1}{2} - \frac{r}{2R^2}\right]$$

per unit width

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At any distance $x > r$

$$M = \frac{W}{4\pi}\left[\log_e \frac{r}{x} - \frac{1}{2} + \frac{R^2}{(R^2-r_o^2)}\cdot\frac{x^2-r_o^2}{x^2}\left(\log_e \frac{R}{r} + \frac{1}{2} - \frac{r^2}{2R^2}\right) + \frac{r^2}{2x^2}\right] \quad \text{per unit width}$$

Shear force $= \frac{W}{2\pi x}$ per unit width

$$M_\theta = \frac{W}{4\pi}\left[\log_e \frac{r}{x} + \frac{1}{2} + \frac{R^2}{(R^2-r_o^2)}\cdot\frac{x^2+r_o^2}{x^2}\left(\log_e \frac{R}{r} + \frac{1}{2} - \frac{r^2}{2R^2}\right) - \frac{r^2}{2x^2}\right] \quad \text{per unit width.}$$

(See Fig. 121)

Case 6. *Simply supported slab with a central hole and carrying a uniformly distributed loading.*

Let R be the radius of the slab.

Let r_o be the radius of the central hole.

Let the loading be w per unit area.

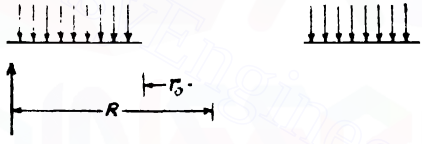

Fig. 122

At any distance x from the centre,

$$M_{r\,x} = -\frac{3}{16}wx^2 + \frac{wr_o^2}{4}\left[\log_e \frac{x}{R} + \frac{3}{4}\left(1 + \frac{R^2}{r_o^2} - \frac{R^2}{x^2}\right) + \frac{R^2-x^2}{R^2-r_o^2}\cdot\frac{r_o^2}{R^2}\log_e \frac{R}{r_o}\right] \quad \text{per unit width.}$$

$$\text{S.F.} = \frac{W}{2\pi x} \text{ per unit width.}$$

$$M_\theta = -\frac{1}{16}wx^2 + \frac{wr_o^2}{4}\left[\log_e \frac{x}{R} + \frac{3}{4}\left(\frac{1}{3} + \frac{R^2}{r_o^2} + \frac{R^2}{x^2}\right) - \frac{R^2+x^2}{x^2}\cdot\frac{r_o^2}{R^2-r_o^2}\log_e \frac{R}{r_o}\right] \quad \text{per unit width.}$$

(See Fig. 122)

Design 18 (S.I.). *A hall is circular with an internal diameter of 6·5 metres. Design a simply supported slab for the hall allowing a live load of 3800 N/metre^2 Use M 15 concrete.*

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Solution. Let the bearing of slab on the supporting wall be 15 *cm.*

Effective diameter $D = 6\cdot50 + 0\cdot15 = 6\cdot65$ *metre*

$\therefore$ Effective radius $= R = 3\cdot325$ *metre.*

Thickness of slab. This may be taken at about $\frac{1}{40}$ of the diameter *i.e.*, $\frac{1}{40} \times 6\cdot65 \times 100 = 16\cdot6$ *cm say* 17 *cm.*

Loads :

DL of the slab : $250 \times 17 =$	$4250\ N/metre^2$
Live Load on the slab :	$3800\ N/metre^2$
Total	$8050\ N/metre^2$

Maximum bending moment per *metre* width

$$= \frac{3}{16} wR^2 = \frac{3}{16} \times 8050 \times 3\cdot325^2 \times 100\ N\ cm.$$

$$= 1668710\ N\ cm.$$

Equating the moment of resistance to the maximum bending moment, $85\ bd^2 = 85 \times 100\ d^2 = 1668710$

$\therefore$ $d = 14$ *cm.*

Using 10 *mm.* diameter bars at a clear cover of 15 *mm.*, effective cover to the centre of the upper layer of bars

$$= 15 + 10 + 5 = 30\ mm. = 3\ cm.$$

$\therefore$ Effective depth available $= 17 - 3 = 14$ *cm.*

$$A_t = \frac{1668710}{14000 \times 0\cdot87 \times 14} = 9\cdot79\ cm.^2$$

Spacing of 10 *mm.* diameter bars

$$\frac{0\cdot79 \times 100}{9\cdot79} = 8\ cm.\ centres$$

Provide 10 mm. ϕ bars @ 8 cm. c/c.

Circumferential moment at the end

$$= \frac{2}{16} wR^2 = \frac{2}{16} \times 8050 \times 3\cdot325^2 \times 100\ N\ cm.$$

$$= 1112470\ N\ cm.$$

$$A_t = \frac{1112470}{14000 \times 0\cdot87 \times 14} = 6\cdot52\ cm.^2$$

Spacing of 10 *mm.* diameter bars

$$= \frac{0\cdot79 \times 100}{6\cdot52} = 12\cdot1\ cm.$$

Sag 10 mm. ϕ bars @ 12 cm. c/c.

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This reinforcement is placed in the form of rings. The rings should be provided in a width equal to $\frac{1}{6}$ the radius of the slab, *i.e.*, within a distance equal to $\frac{3{\cdot}325}{6} = 0{\cdot}55m. = 55\ cm.$ from the support.

At top also, near the edges provide two rings of 10 *mm.* diameter and provide also radial bars to form a mesh with the rings.

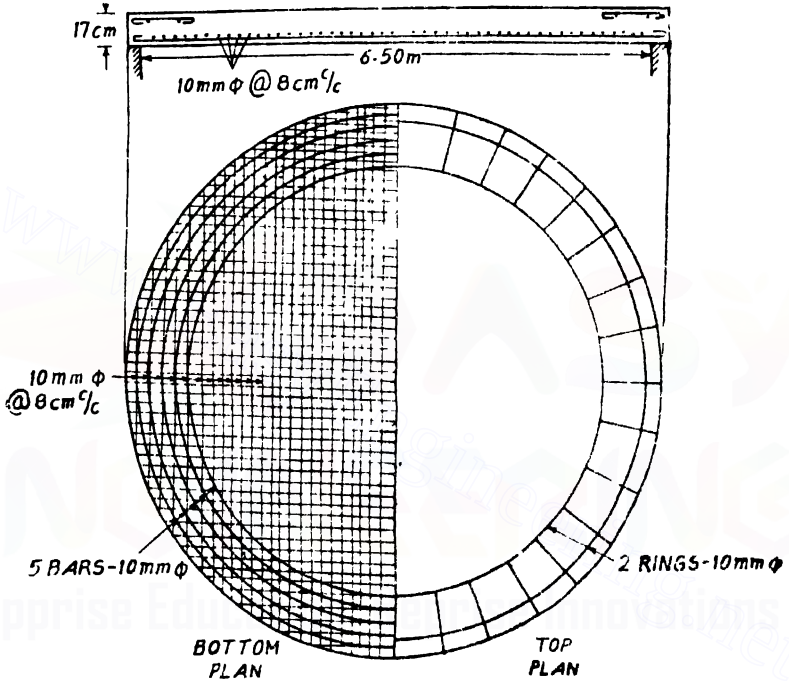

Fig. 123

Design 19. *A hall is circular with an internal diameter of 6·5 metres. Design a simply supported slab for the hall allowing a live load of 380 kg./metre².*

Solution. Let the bearing of slab on the supporting wall be 15 *cm.*

$\therefore$ Effective diameter $= D = 6{\cdot}50 + 0{\cdot}15 = 6{\cdot}65\ m.$

$\therefore$ Effective radius $= R = 3{\cdot}325\ m.$

Thickness of slab

This may be taken at about $\frac{1}{40}$ of the diameter.

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Loads :

DL of the slab $=25\times17$ $=425$ *kg./metre*^{2}
LL on the slab $=380$ *kg./metre*^{2}
Total $=805$ *kg./metre*^{2}

Maximum bending moment per *metre* width

$$=\frac{3}{16}wR^2=\frac{3}{16}\times805\times3{\cdot}325^2\times100\ kg.\ cm.$$
$$=166870\ kg.\ cm.$$

Equating the moment of resistance to the maximum bending moment,

$$8{\cdot}5\ bd^2=8{\cdot}5\times100\ d^2=166870$$
$$\therefore\quad d=14\ cm.$$

Using 10 *mm.* diameter bars at a clear cover of 15 *mm.*, effective cover to the centre of the upper layer of bars

$$=15+10+5=30\ mm.=3\ cm.$$

Effective depth available $=17-3=14$ *cm.*

$$\therefore\quad A_t=\frac{166870}{1400\times0{\cdot}87\times14}=9{\cdot}79\ cm.^2$$

Spacing of 10 *mm.* diameter bars

$$=\frac{0{\cdot}79\times100}{9{\cdot}79}=8\ cm.$$

Provide 10 mm. diameter bars @ 8 cm. c/c.

Circumferential moment at the end

$$=\frac{2}{16}\ wR^2=\frac{2}{16}\times805\times3{\cdot}325^2\times100\ kg.\ cm.$$
$$=111250\ kg.\ cm.$$
$$\therefore\quad A_t=\frac{111250}{1400\times0{\cdot}87\times14}=6{\cdot}52\ cm.^2$$

Spacing of 10 *mm* diameter bars $=\dfrac{0{\cdot}79\times100}{6{\cdot}52}=12{\cdot}1\ cm.$

Provide 10 mm. ϕ bars @ 12 cm. c/c.

This reinforcement is placed in the form of rings. The rings should be provided in a width equal to $\frac{1}{6}$ the radius of the slab *i.e.*, within a distance equal to $\dfrac{3{\cdot}325}{6}=0{\cdot}55\ m.=55\ cm.$ from the support. At top also, near the edges provide two rings of 10 *mm.* diameter and provide also radial bars to form a mesh with the rings.

For details of reinforcement see previous design.

Design 20. *A circular slab is to be provided for a room circular in plan having a diameter of 6 metre. The live load on the slab is 380 kg./metre*^{2}.

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Solution. Thickness of slab

$$= \frac{1}{40} \text{ of diameter} = \frac{6 \times 100}{40} = 15 \text{ cm.}$$

Loads :

DL of the slab $= 25 \times 15$	$= 375 \text{ kg./m.}^2$
LL on the slab	$= 380 \text{ kg./m.}^2$
Total w	$= 755 \text{ kg./m.}^2$

Since the edges are neither simply supported nor absolutely fixed the moments for the slab will have values intermediate to the above two conditions.

For simply supported condition,

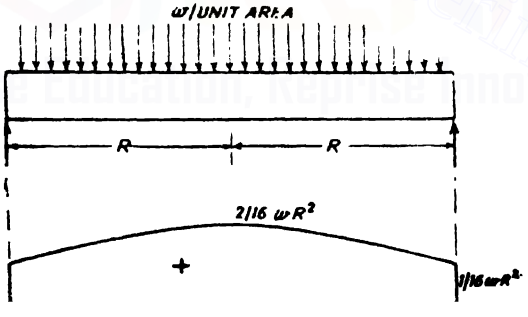
maximum circumferential and radial moments

$$= \frac{3}{16} wR^2$$

But for the fixed supported condition these moments

$$= \frac{1}{16} wR^2$$

Hence, as the slab is partially restrained, the moments at the centre may be taken as $\frac{2}{16} wR^2$



CIRCUMFERENTIAL MOMENT DIAGRAM

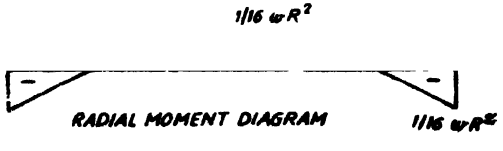


Fig. 124

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Similarly circumferential moment at the ends

$$= + \frac{1}{16} wR^2$$

and Radial moment at the ends $= -\frac{1}{16} wR^2$

Fig. 124 shows that B.M. diagram for the partially restrained case.

In our case, maximum moment at centre

$$= \frac{2}{16} wR^2.$$

$$= \frac{2}{16} \times 755 \times 3^2 \times 100 = 84940 \text{ kg. cm.}$$

Equating the moment of resistance to the maximum bending moment,

$$8{\cdot}5\, bd^2 = 8{\cdot}5 \times 100\, d^2 = 84940$$

$$\therefore \quad d = 10 \text{ cm.}$$

Providing 12 *mm.* diameter bars and providing a clear cover of 15 *mm.* to the lower layer of bars, the effective cover to the centre of the upper layer of bars

$$= 15 + 12 + 6 = 33 \text{ mm.} = 3{\cdot}3 \text{ cm.}$$

Effective depth available

$$= 15 - 3{\cdot}3 = 11{\cdot}7 \text{ cm.}$$

Radial and circumferential steel requirement at centre

$$= A_s = \frac{84940}{1400 \times 0{\cdot}87 \times 11{\cdot}7} = 5{\cdot}96 \text{ cm.}^2$$

Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{5{\cdot}96} = 18{\cdot}9 \text{ cm.}$$

Provide 12 mm. ϕ bars @ 18 cm. c/c.

Circumferential moment at the ends

$$= \frac{1}{16} wR^2$$

= half the maximum moment at the centre

$$= \frac{84940}{2} = 42470 \text{ kg. cm.}$$

$$\text{Steel required} = \frac{5{\cdot}96}{2} = 2{\cdot}98 \text{ cm.}^2$$

Spacing of 10 *mm.* ϕ bars

$$= \frac{0{\cdot}79 \times 100}{2{\cdot}98} = 26{\cdot}5 \text{ cm.}$$

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This reinforcement in the form of rings is needed only near the supports. These rings may be provided for a distance

$$\frac{R}{4} = \frac{300}{4} = 75 \text{ cm.}$$

Radial moment at the ends

$$\frac{1}{16} wR^2.$$

Hence provide 10 *mm.* ϕ bars @ 25 *cm.* c/c. Since the radial moment varies parabolically from $-\frac{1}{16} wR^2$ at the ends to $+\frac{1}{16} wR^2$ at the centre, the point of contraflexure for radial moment may be taken at a distance $\sqrt{\frac{2}{3}} R$ from the centre, *i.e.*, at $\sqrt{\frac{2}{3}} \times 300$ $= 245$ *cm.* from the centre ; or $300 - 245 = 55$ *cm.* from the ends.

It is usual to provide the radial bars for a radial length equal to at least up to the point of contraflexure from the ends.

Design 21. *A traffic control post 2·5 metres diameter is supported centrally by a 30 cm. diameter reinforced concrete column. Allowing a live load of 140 kg /m². design the circular slab. Weight of RCC = 2400 kg./m³* :

Solution. Let the slab be 12 *cm.* thick.

Loads

Dead load $= 24 \times 12 =$	288 *kg./m.²*
Live load	$= 140$ *kg./m.²*
Total load	$= 428$ *kg./m.²*

Total load on the slab

$$= 428 \times \frac{\pi}{4} \times 2{\cdot}5^2 \text{ kg.} = 2100 \text{ kg.}$$

Fig. 125 shows the circular slab supported on the central column.

For analysis purpose the slab may be taken to be subjected to the following conditions.

The slab may be considered simply supported round the edges and subjected to (*i*) a downward load of 428 *kg /m.²* and (*ii*) an upward central load of 2100 *kg.*

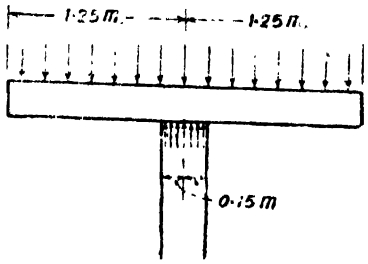

Fig. 125

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(i) Simply supported slab carrying a distributed loading of 428 kg./m².

Radial or circumferential moment at the centre $= \frac{3}{16} wR^2$.

Let M' be the moment at a distance r from the centre.

(See Fig. 126)

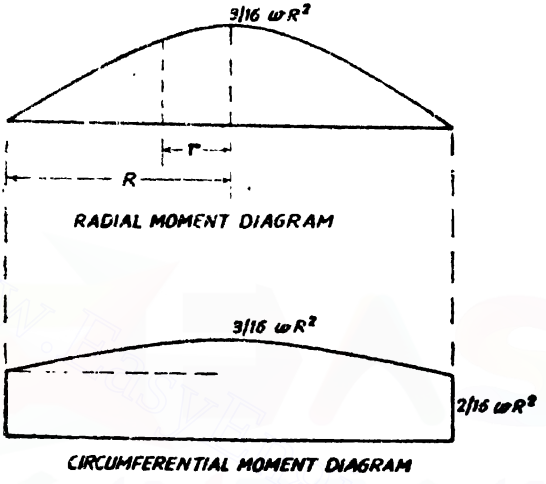

Fig. 126

$$\therefore \quad \frac{\frac{3}{16} wR^2 - M'}{\frac{3}{16} wR^2} = \frac{r^2}{R^2}$$

$$\therefore \quad \frac{3}{16} wR^2 - M' = \frac{r^2}{R^2} \cdot \frac{3}{16} wR^2$$

$$\therefore \quad M' = \left[1 - \frac{r^2}{R^2} \right] \frac{3}{16} wR^2$$

$\therefore$ Radial moment at the edges of the column

$$= \left[1 - \left(\frac{3}{25} \right)^2 \right] \frac{3}{16} wR^2$$

$$= \frac{616}{625} \cdot \frac{3}{16} wR^2$$

$$= \frac{616}{625} \times \frac{3}{16} \times 428 \times 1{\cdot}25^2 \text{ kg. m.}$$

$$= 123{\cdot}6 \text{ kg. m.}$$

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Circumferential moment at the edges of the column

$$= \frac{2}{16} wR^2 + \left(1 - \frac{r^2}{R^2}\right)\frac{1}{16} wR^2$$

$$= \frac{2}{16} wR^2 + \left[1 - \left(\frac{3}{25}\right)^2\right]\frac{1}{16} wR^2$$

$$= \frac{wR^2}{16}\left[2 + 1 - \frac{9}{625}\right]$$

$$= \frac{1866}{625} \cdot \frac{wR^2}{16}$$

$$\frac{1866}{625} \times \frac{428 \times 1{\cdot}25^2}{16} \text{ kg. m.} = 124{\cdot}7 \text{ kg. m.}$$

Circumferential moment at the ends $= \frac{2}{16} wR^2$

$$= \frac{2}{16} \times 428 \times 1{\cdot}25^2 \text{ kg. m.} = 83{\cdot}60 \text{ kg. m.}$$

(ii) Simply supported slab carrying a uniformly distributed upward load of 2100 kg. acting in an area of a concentric circle of radius 0·15 m.

For this case $w = \frac{2100}{\pi \times (0{\cdot}15)^2}$ kg./m.^{2}

$= 29700$ kg./m.^{2} (*upwards*)

∴ The moments due to this loading will be nagative.

At any radius $x > r$ the radial moment is given by

$$M_r = wr^2 \left[\frac{1}{4} \log_e \frac{x}{R} + \frac{r^2}{16}\left(\frac{1}{R^2} - \frac{1}{x^2}\right)\right]$$

per unit width

When $r = x = 0{\cdot}15$ m. and $R = 1{\cdot}25$ m.

$$M_r = 29700 \times 0{\cdot}15^2 \left[\frac{1}{4} \log_e \frac{0{\cdot}15}{1{\cdot}25} + \frac{(0{\cdot}15)^2}{16}\left(\frac{1}{1{\cdot}25^2} - \frac{1}{0{\cdot}15^2}\right)\right] \text{ kg. m.}$$

$$= 29700 \times 0{\cdot}15^2(-0{\cdot}5294 - 0{\cdot}0616) \text{ kg. m.}$$

$$= -395 \text{ kg. m.}$$

At the ends, the radial moment = 0

At any radius $x > r$ the radial moment is given by

$$M_r = wr^2\left[\frac{1}{4} \log_e \frac{x}{R} + \frac{r^2}{16}\left(\frac{1}{R^2} - \frac{1}{x^2}\right)\right]$$

per unit width

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When $r = x = 0{\cdot}1^{r}$ *m.* and $R = 1{\cdot}25$ *m.*

$$M_r = 29700 \times 0{\cdot}15^2 \left[\frac{1}{4} \log_e \frac{0{\cdot}15}{1{\cdot}25} + \frac{(0{\cdot}15)^2}{16}\left(\frac{1}{1{\cdot}25^2} + \frac{1}{0{\cdot}15^2}\right)\right] kg.\ m.$$

$$= 29700 \times 0{\cdot}15^2\ [-0{\cdot}5294 - 0{\cdot}0616]\ kg.\ m.$$

$$= -395\ kg.\ m.$$

At the ends the radial moment = 0

At any radius $x > r$ the circumferential moment is given by

$$M_\theta = wR^2\left[\frac{1}{4} \log_e \frac{x}{R} - \frac{1}{4} + \frac{r^2}{16}\left(\frac{1}{R^2} + \frac{1}{x^2}\right)\right]$$

per unit width

When $r = x = 0{\cdot}15$ *m.* and $R = 1{\cdot}25$ *m.*

$$M_\theta = 29700 \times 0\ 15^2 \left[\frac{1}{4} \log_e \frac{0{\cdot}15}{1{\cdot}25} - \frac{1}{4} + \frac{0\ 15^2}{16}\left(\frac{1}{1{\cdot}25^2} + \frac{1}{0{\cdot}15^2}\right)\right]\ kg.\ m.$$

$$= 29700 \times 0{\cdot}15^2[-0{\cdot}5294 - 0{\cdot}2500 + 0{\cdot}0634]$$

$$= -478{\cdot}5\ kg.\ m.$$

Circumferential moment at the ends *i.e.*, when $x = 1{\cdot}25$ *m.*, is given by

$$M_\theta = 29700 \times 0{\cdot}15^2 \left[0 - 0{\cdot}2500 + \frac{0{\cdot}15^2}{16}\left(2 \times \frac{1}{1{\cdot}25^2}\right)\right] kg.\ m.$$

$$= -165{\cdot}9\ kg\ m.$$

The following is the abstract of the moments computed above :

Case	Radial moment (kg. m.)		Circumferential moment (kg. m.)	
	$x = 0{\cdot}15$ *m.*	$x = 1{\cdot}25$ *m*	$x = 0{\cdot}15$ *m.*	$x = 1{\cdot}25$ *m.*
(*i*)	+123·6	0	+124·7	+83·6
(*ii*)	−395·0	0	−478·5	−165·9
Net values	−271·4	0	−353 8	− 82·3

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Hence, the greatest bending moment for the slab is 353·8 *kg. m.* Adopting $c = 50$ *kg./cm.*^{2}, $t = 1400$ *kg./cm* ^{2} and $m = 18$, and equating the moment of resistance to the maximum bending moment,

$$8{\cdot}50\ bd^2 = 8{\cdot}5 \times 100\ d^2 = 353{\cdot}8 \times 100 \quad \therefore\ d = 6{\cdot}45\ cm.$$

Providing 8 *mm.* diameter bars, and allowing a clear cover of 15 *mm.* to the outer layer of bars, effective cover to the centre of the inner layer of bars

$$= 15 + 8 + 4 = 27\ mm. = 2{\cdot}7\ cm.$$

Effective depth available $= 12 - 2{\cdot}7 = 9{\cdot}3$ *cm.*

$$A_t = \frac{353{\cdot}8 \times 100}{1400 \times 0{\cdot}87 \times 9{\cdot}3} = 3{\cdot}1\ cm.^2$$

Spacing of 8 *mm.* ϕ bars $= \dfrac{0{\cdot}5 \times 100}{3{\cdot}1}$ 16·1 *cm.*

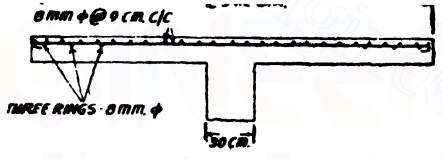
Provide 8 mm. ϕ *bars @ 16 cm. c/c*

Maximum shear force at the column edges

$$= \frac{2100}{\pi \times 0{\cdot}3} - 428 \times \frac{0{\cdot}15}{2}\ kg/m.$$

$$= 2228\ kg.\ \text{per}\ metre.$$

$\therefore$ Nominal Shear stress $= q = \dfrac{S}{bd}$



TOP PLAN

Fig. 127

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$$= \frac{2228}{100 \times 9 \cdot 3} \text{ kg./cm.}^2$$

$$= 2 \cdot 5 \text{ kg./cm.}^2$$

§123. Lift Slab Construction

This is one of the newly introduced methods of construction. In this method, the various slabs meant for the various floors and the roof slab are cast one upon the other on the ground and are subsequently lifted one after another to their final levels. This method of construction has been used in Great Britain and America and may soon find its place in our country. The construction is carried out as follows :

First, structural steel columns are erected having rigid base connections so as to enable them to stand as cantilevers. Concreting is done for the ground floor slab and an even finish is given to the surface The ground floor slab is then covered with a medium (say paraffin solution) to prevent adhesion between the ground floor slab and the first floor slab A specially made steel collar threaded to the columns is lowered on to the ground floor slab. Now concreting is done for the first floor slab. Concrete poured will also incorporate the steel collars. The same procedure is repeated and a number of floor slabs are cast one above the other.

By using hydraulic jacks on the tops of the columns the uppermost slab *i e.*, the roof slab is lifted Lifting rods connecting the jacks and the collars are used for this purpose. Usually the stroke of the jacks is 8 *cms* and hence after several strokes the roof slab is lifted stage by stage to its final position at the top of the columns.

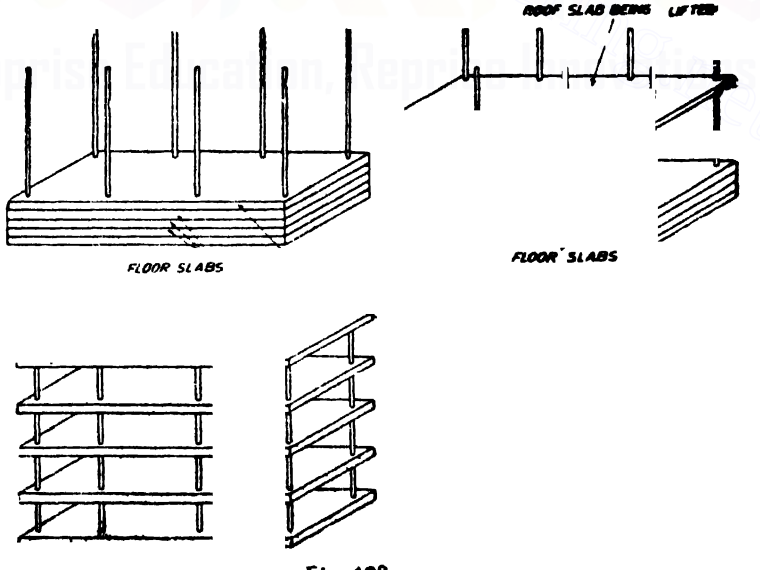

Fig. 128

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The steel sealing plates are now welded to the columns immediately below the lifting collars. Now the lifting rods are released and connected to the lifting collars on the uppermost floor slab and shifted in a similar manner to its final level. Fig. 128 shows the various stages of construction.

In order to limit the quantity of work lifted in any one operation the slab at each floor or roof level is divided into separate section in plan. Usually the divisions in plan are so made that not more than twelve columns are present in any one section, so that not more than twelve jacks are to be controlled and regulated at a time.

Lift slab construction has the following advantages :

(*i*) Since all slabs are connected at ground level there is economy of normal hoisting or pumping equipment.

(*ii*) Damage due to exposure is reduced.

(*iii*) Soffit shuttering and propping are avoided.

(*iv*) Considerable saving of time can be attained.

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11
Beams

Beams are meant for supporting slabs, walls or secondary beams.

§124. Some Important Practical Rules

(*a*) *Effective span.* Unless specified otherwise, the effective span of a beam shall be taken as follows :

1. For a simply supported beam or slab the effective span shall be taken as clear span plus effective depth or the distance between the centres of bearing whichever is less.

2. In the case of continuous beam or slab, if the width of the support is less than 1/12 of the clear span, the effective span should be taken as mentioned above. If the supports are wider than 1/12 of the clear span or 600 *mm.* whichever is less, the effective span shall be taken as follows :

(*i*) For end span with one end fixed and the other continuous, or for intermediate spans, the effective span shall be the clear span between supports ; and

(*ii*) for end span with one end free and the other continuous, the effective span shall be equal to the clear span plus half the width of the discontinuous support, whichever is less.

NOTE—In the case of spans with roller or rocker bearings, the effective span shall always be the distance between the centres of bearings.

3. In the analysis of a continuous frame, the effective span is the distance between the centres of supporting members.

(*b*) *Stiffness of beams and control on deflection.* Beams must possess adequate stiffness to prevent such deflection as might impair the strength, appearance or efficiency of the structure or produce cracks in finishes and partitions. The deflection shall generally be limited to the following :

(*i*) The final deflection due to all loads including the effects of temperature, creep and shrinkage measured from the as cast-level of the supports of floors, roofs and all other horizontal members, should not normally exceed span/250.

(*ii*) The deflection including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes should not normally exceed span/350 or 20 *mm.* whichever is less.

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For beams and slabs, the vertical deflection limits may generally be assumed to be satisfied provided that the span to depth ratios are not greater than the values obtained as below :

Basic values of span to effective depth ratios for spans up to 10 m.

Contilevers	7
Simply supported beams	20
Cantinuous beams	26

For spans above 10 *m*. the values given above may be multiplied by $\left[\frac{10}{\text{span in metres}}\right]$ except for cantilever in which case deflection calculations should be made.

(*c*) *Slenderness limits for beams to ensure lateral stability.* A simply supported or continuous beam shall be so proportioned that the clear distance between the lateral restraints does not exceed 60 *b* or $\frac{250\ b^2}{d}$ whichever is less, where, *d* is the effective depth of the beam and *b* the breadth of the compression face midway between the lateral restraints.

For a cantilever, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed 25 *b* or $\frac{100\ b^2}{d}$ whichever is less.

(*d*) *Selection of breadth.* The breadth of the beam must be such as to accommodate the necessary reinforcement. Often this is selected from architectural considerations. Generally a beam is satisfactory when its breadth is between half the effective depth to two-thirds the effective depth.

(*e*) *Loading imposed on a beam.* The load imposed on a beam comprises of

(*i*) Its external live load, (*ii*) Any external dead load.

(*iii*) The load of the beam itself.

When a beam is designed, the external loads namely the external dead and live loads are known ; but the dead load of the beam itself is not known, depending as it does on the cross-section finally selected for the beam. An initial estimate may be made by assuming the width of the beam as $\frac{1}{30}$ th of the span and that its overall depth is slightly more than twice the breadth.

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§125. Loads Transmitted to Supporting Beams from Slabs

As per the I.S. code the loads on the supporting beams may be assumed to be in accordance with Fig. 129

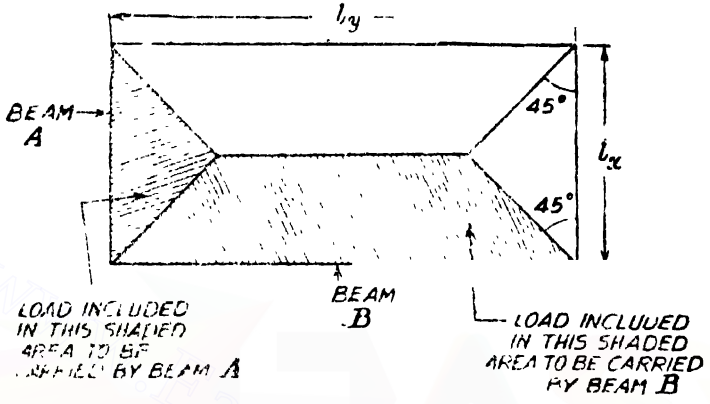

Fig. 129. Load distribution to supporting beams.

§126. Design Procedure

The following procedure should be followed in the design of singly reinforced beams

(*a*) After estimating the breadth and depth of the beam section (by rules mentioned earlier) compute the effective span.

(*b*) Estimate the load on the beam which consists of the external live load, the external dead load and the dead load of the beam itself.

(*c*) Determine the maximum bending moment.

Unless more exact estimates are made, the bending moments in uniformly loaded beams may be taken at the following values.

Single Span Beams

(*i*) *Beam simply supported at its ends*

Maximum positive bending moment at centre

$$= + \frac{wl^2}{8}$$

B.M. at ends

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(*ii*) *Beams with partial fixity at ends*

Maximum positive bending moment at centre $= +\frac{wl^2}{8}$

Maximum negative bending moment at ends $= -\frac{wl^2}{20}$

(*iii*) *Beam fixed at both ends*

Maximum positive bending moment at centre $= +\frac{wl^2}{12}$

Maximum negative bending moment at centre $= -\frac{wl^2}{12}$

(*iv*) *Two span beams*

Maximum positive bending moment near centre of span $= \frac{wl^2}{10}$

Maximum negative bending moment over interior support $= -\frac{wl^2}{8}$

Negative bending moment at end supports $= -\frac{wl^2}{20}$

(*v*) *Beams continuous for more than two spans*

Let w_d be the dead load per unit run of the beam.

Let w_l be the live load per unit run of the beam.

B.M.	*Near middle of end pad*	*At middle of interior spans*	*At support next to the end support*	*At other interior supports*
Moment due to dead load	$+\frac{w_d l^2}{12}$	$+\frac{w_d l^2}{24}$	$-\frac{w_d l^2}{10}$	$-\frac{w_d l^2}{12}$
Moment due to live load	$+\frac{w_l l^2}{10}$	$+\frac{w_l l^2}{12}$	$-\frac{w_l l^2}{9}$	$-\frac{w_l l^2}{9}$

The above formulae may be used when the spans are nearly equal. Two spans may be considered as nearly equal when they do not differ by more than 15% of the longer span.

(*d*) Adopting suitable working stresses, equate the moment of resistance to the maximum bending moment and find the effective depth. If the M.R. of the beam section is Qbd^2 and M the maximum bending moment.

$$Qbd^2 = M$$

$$d = \sqrt{\frac{M}{Qb}}$$

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In S.I. designs, if $c = 500\ N/cm^2$ $(5\ N/mm.^2)$

$t = 14000\ N/cm^2$ $(140\ N/mm.^2)$

$m = 18$

Then $Q = 85$

In MKS designs, if $c = 50\ kg./cm.^2$

$t = 1400\ kg./cm.^2$

$m = 18$

Then $Q = 8{\cdot}5$

Check the value of the computed depth with the depth provided and see that the depth as found necessary is not greater than what had been estimated earlier.

(*d*) Determine the area of tensile steel required from the relation, $A_t = \frac{M}{a_t\, d_t}$

where a_t = lever arm factor = 0·87

Now find the suitable number of bars of a selected diameter. Place the reinforcement in one row if the reinforcement can be accommodated in one row within the width of the beam. If the bars cannot be accommodated in one row, place the bars in two (or more) rows.

(*f*) Find the maximum shear force S for the beam.

This may be taken at the following values.

Let w be the uniformly distributed load per unit run.

(*i*) *Simply supported beams*

$$S = \frac{WL}{2}$$

where L = clear span.

(*i*) *Continuous beams.* For a continuous beam the shearing forces at the supports may be assumed to have the following values. Let w_d and w_l be the dead load and live load per unit run.

S	*At exterior face of support next to end support*	*All faces of all other supports*
Shearing force due to dead load	0·6 $w_d l$	0·5 $w_d l$
Shearing force due to live load	0·5 $w_l l$	0·5 $w_l l$

(*g*) Determine the maximum nominal shear stress induced from

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the relation $q_v = \frac{S}{bd}$ and verify if q_v is less than the permissible nominal shear stress q_c. If $q_v < q_c$ provide nominal shear reinforcement in the form of vertical stirrups at a spacing not exceeding 0 75 d In no case this spacing shall exceed 45 *cm.*

Shear reinforcement is designed when $q_v > q_c$. When shear reinforcement is provided q_v shall not be greater than $q_{c\,max}$ specified by the I.S. code. Shear reinforcement may be in the form of stirrups or in the form of bent up bars.

$$\text{Spacing of stirrups} = \frac{A_w\, t_w\, d}{V_s}$$

where
V_s = total S.F. $- q_c\, bd$
A_w = area of the legs of the stirrup
t_w = permissible tensile stress in the stirrup.

In a simply supported beam, at least one-third the positive reinforcement shall extend along the same face of the member into the support, to a length equal to $\frac{L_d}{3}$. If the beam is continuous over a support, at least one-fourth the positive reinforcement shall extend along the same face of the member into the support. to a length to $\frac{L_d}{3}$. L_d is the development length given by

$$L_d = \frac{t\, d}{4\, S_b}$$

where
t = permissible stress in the bar
S_b = bond stress.

The bond stress shall be taken as follows :

S.I. UNITS

Grade of concrete	M 15	M 20	M 25	M 30	M 35	M 40
Design bond stress in N/mm.²	0·6	0·8	0·9	1·0	1·1	1·2

M.K.S. UNITS

Grade of concrete	M 150	M 200	M 250	M 300	M 350	M 400
Design bond stress in kg./cm.²	6	8	9	10	11	12

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When bars are bent up to resist shear, their contribution towards shear resistance shall not be more than half that of the total shear reinforcement.

Shear value of bars bent up at one section $= A_w t_w \sin \theta$

where A_w = area of the bent-up bars at the section

t_w = permissible tensile stress in the bars

θ = angle of bar with the longitudinal axis which shall not be less than 45°.

Design 22 (S.I.). *Design a singly reinforced beam to carry a live load of 14500 N/metre run. The clear span of the beam is 5·50 metre. The bearing at each end is 30 cm Use M 15 concrete.*

Solution. Span between centres of bearings $= 5{\cdot}5 + 0{\cdot}30 = 5{\cdot}80$ *m.* Since the width as well as depth are to be chosen, we may choose the dimensions so that the width of the beam may be nearly half the effective depth. The convenient width and depth are given by the thumb rules :

Width of beam $= \frac{\text{span}}{30} + 4$ *cm.*

Overall depth of beam

D = twice the width + 4 *cm.*

In our case, $b = \frac{5{\cdot}8 \times 100}{30} + 8 = 27{\cdot}33$ *cm.*

and $D = (2 \times 27{\cdot}33) + 4 = 58{\cdot}66$ *cm.*

Provide 30 *cm.* × 60 *cm.*

Providing 20 *mm.* diameter bars at a clear cover of 25 *mm.* effective cover to the centre of the tensile steel :

$= 25 + 10 = 35$ *mm.* $= 3{\cdot}5$ *cm.*

$\therefore$ Effective depth $= d = 60 - 3{\cdot}5 = 56{\cdot}5$ *cm.*

Effective span. This is taken as the lesser of the following :

(*i*) Centre to centre distance between bearings $= 5{\cdot}80$ *m.*

(*ii*) Clear span + effective depth $= 5{\cdot}50 + 0{\cdot}565 = 6{\cdot}065$ *m.*

$\therefore$ Effective span $= 5{\cdot}80$ *metre.*

Consider one *metre* run of the beam. The load on the beam consists of the following :

DL of the beam $= 0{\cdot}30 \times 0{\cdot}60 \times 25000$	$= 4500$ *N/metre*
LL on the beam	$= 14500$ *N/metre*
Total load	$= 19000$ *N/metre*

Maximum bending moment

$$= M = \frac{wl^2}{8} = 19000 \times \frac{5{\cdot}8^2}{8} \times 100 \text{ N cm.}$$

$$= 7989500 \text{ N cm.}$$

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Equating the moment of resistance to the maximum bending moment,

$$8{\cdot}5\ bd^2 = 8{\cdot}5 \times 30\ d^2 = 7989500$$

$$\therefore \quad d = 56\ cm.$$

Effective depth available

$$= 56{\cdot}5\ cm.$$

$$A_t = \frac{7989500}{14000 \times 0{\cdot}87 \times 56{\cdot}5} = 11{\cdot}61\ cm.^2$$

Provide 4 bars of 20 mm. diameter (12 56 cm.²)

Design for shear

Clear span $= L = 5{\cdot}50\ m.$

Maximum shear force

$$= S = \frac{wL}{2} = \frac{19000 \times 5{\cdot}5}{2} = 52250\ N$$

Nominal shear stress at support

$$= q_v = \frac{S}{bd} = \frac{52250}{30 \times 56{\cdot}5} = 30{\cdot}8\ N/cm.^2$$

$$= 0{\cdot}308\ N/mm.^2$$

For M 15 concrete,

$$q_{c\,max} = 1{\cdot}60\ N/mm.^2$$

With 50% curtailment of steel,

area of steel available at end

$$= \frac{12{\cdot}56}{2} = 6{\cdot}28\ cm.^2$$

Percentage of steel $= \frac{6{\cdot}28}{30 \times 56{\cdot}5} 100 = 0{\cdot}37\%$

As per code, for 0·25% steel,

$$q_c = 0{\cdot}22\ N/mm.^2$$

and, for 0·50% steel, $q_c = 0{\cdot}29\ N/mm.^2$

∴ for 0·37% steel,

$$q_c = 0{\cdot}22 + \left[\frac{0{\cdot}29 - 0{\cdot}22}{0{\cdot}25}\right] \times 0{\cdot}12$$

$$= 0{\cdot}22 + 0{\cdot}03 = 0{\cdot}25\ N/mm.^2$$

$$= 25\ N/cm.^2$$

Shear capacity of concrete

$$= q_c\ bd = 25 \times 30 \times 56{\cdot}5 = 42375\ N$$

Shear to be carried by shear reinforcement

$$= V_s = 52250 - 42375 = 9875\ N$$

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Spacing of two-legged 8 *mm.* ϕ stirrups

$$= p = \frac{A_w\, t_w\, d}{V_s} = \frac{(2 \times 0{\cdot}5) 14000 \times 56{\cdot}5}{9875}$$

$$= 80 \text{ cm.}$$

Maximum allowable pitch is the lesser of the following :

(*i*) $0{\cdot}75\, d = 0{\cdot}75 \times 56{\cdot}5 = 42{\cdot}4$ *cm.*

(*ii*) **45 *cm.***

Provide 2-legged 8 mm. ϕ stirrups @ 40 cm. c/c.

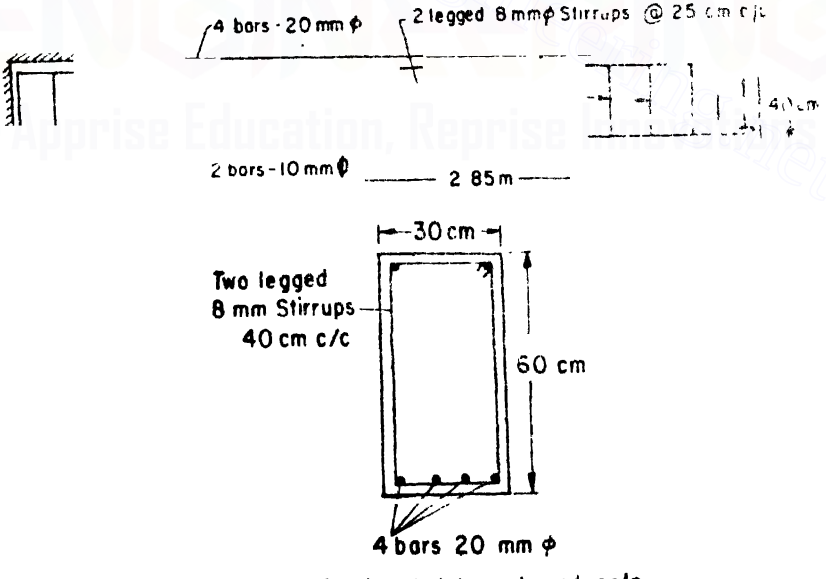
Curtailment of the tensile steel

Half the number of bars may be curtailed at a distance of $\frac{1}{7}$ of the span $= \frac{1}{7} \times 5{\cdot}5 \times 100 = 79$ *cm.* from the centre of bearing. As per code the bars have to be extended from the theoretical cut off point for a distance of 12 diameters or effective depth whichever is greater,

12 diameters $= 12 \times 2 = 24$ *cm.*

Effective depth $= 56{\cdot}5$ *cm.*

Hence the bars must be extended from the theoretical cut off point for a distance of 56·5 *cm.*



Section 1-1 to enlarged scale

Fig. 130

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$\therefore$ Distance of the actual point of cut off from the centre of bearing $= 79 - 56{\cdot}5 = 22{\cdot}5$ *cm* say 22 *cm*.

Design 23. *Design a singly reinforced beam to carry a live load of 1450 kg/metre run. The clear span of the beam is 5·50 m The bearing at each end is 30 cm. Use M 150 concrete.*

Solution. Span between centres of bearings

$$= 5{\cdot}5 + 0{\cdot}30 = 5\ 80 \text{ metre.}$$

Since the width as well as depth are to be chosen, we may choose the dimensions so that the width of the beam may be nearly half the effective depth. The convenient width and depth are given by the following thumb rules :

Width of beam $b = \frac{\text{span}}{30} + 8$ *cm*.

Overall depth of beam D = twice the width + 4 *cm*.

In our case, $b = \frac{5{\cdot}8 \times 100}{30} + 8 = 27{\cdot}33$ *cm*.

and $D = (2 \times 27{\cdot}33) + 4 = 58{\cdot}66$ *cm*

Provide 30 *cm*. × 60 *cm*.

Providing 20 *mm*. diameter bars at a clear cover of 25 *mm* effective cover to the centre of the tensile steel

$$= 25 + 10 = 35 \text{ mm.} = 3{\cdot}5 \text{ cm.}$$

$\therefore$ Effective depth $= d = 60 - 3{\cdot}5 = 56{\cdot}50$ *cm*.

Effective span. This is taken as the lesser of the following :

(*i*) Centre to centre distance between bearings = 5 80 *m*

(*ii*) Clear span + effective depth $= 5{\cdot}50 + 0{\cdot}565 = 6{\cdot}065$ *m*.

$\therefore$ Effective span = 5·80 *metre*

Consider one *metre* run of the beam. The load on the beam consists of the following :

D.L. of the beam $= 0{\cdot}30 \times 0{\cdot}60 \times 2500 =$ 450 *kg./m.*

L.L. on the beam $= 1450$ *kg./m.*

Total load = 1900 *kg./m.*

Maximum bending moment

$$= M = \frac{wl^2}{8} = 1900 \times \frac{5{\cdot}8^2}{8} \times 100 \text{ kg. cm.} = 798950 \text{ kg. cm.}$$

Equating the moment of resistance to the maximum bending moment,

$8\ 5\ bd^2 = 8{\cdot}5 \times 30\ d^2 = 798950$

$\therefore$ $d = 56$ *cm*.

Effective depth available = 56·5 *cm*.

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$$A_t = \frac{798950}{1400 \times 0{\cdot}87 \times 56{\cdot}5} = 11{\cdot}61 \text{ cm.}^2$$

Provide 4 bars of 20 mm. diameter (12·56 cm.²).

Design for shear

Clear span $= L = 5{\cdot}5$ m.

Maximum shear force $= S = \frac{wL}{2} = \frac{1900 \times 5{\cdot}5}{2} = 5225$ kg.

Nominal shear stress at support

$$= q_v = \frac{S}{bd} = \frac{5225}{30 \times 56{\cdot}5} = 3{\cdot}08 \text{ kg./cm.}^2$$

For *M* 150 concrete, $q_{c\,max} = 16$ kg./cm.²

With 50% curtailment of steel,

area of steel available at end $= \frac{12{\cdot}56}{2} = 6{\cdot}28$ cm.²

Percentage of steel $\frac{6{\cdot}28}{30 \times 56{\cdot}5} \times 100 = 0{\cdot}37\%$

As per code, for 0·25% steel, $q_c = 2{\cdot}2$ kg./cm.²

and for 0·50% steel, $q_c = 2{\cdot}9$ kg./cm.²

$\therefore$ for 0·37% steel, $q_c = 2{\cdot}2 + \frac{2{\cdot}9 - 2{\cdot}2}{0{\cdot}25} \times 0{\cdot}12$

$= 2{\cdot}2 + 0{\cdot}3 = 2{\cdot}5$ kg./cm²

Shear capacity of concrete $= q_c\, bd$

$= 2{\cdot}5 \times 30 \times 56{\cdot}5 = 4238$ kg.

$\therefore$ Shear to be carried by shear reinforcement

$$V_s = 5225 - 4238 = 987 \text{ kg.}$$

Spacing of two-legged 8 mm. ϕ stirrups

$$p = \frac{A_{sv}\, d}{V_s} = \frac{(2 \times 0{\cdot}5) 1400 \times 56{\cdot}5}{987} = 80 \text{ cm.}$$

Maximum allowable pitch is the lesser of the following :

(*i*) $0{\cdot}75\, d = 0{\cdot}75 \times 56{\cdot}5 = 42{\cdot}4$ cm.

(*ii*) 45 cm.

Provide 2-legged 8 mm. ϕ stirrups @ 40 cm. c/c.

Curtailment of the tensile steel.

Half the number of bars may be curtailed at a distance of

× span

$$= \frac{1}{7} \times 5{\cdot}5 \times 100 = 79 \text{ cm.}$$

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from the centre of bearing. As per the code the bars have to be extended, from the theoretical cut off point for a distance of 12 diameters or effective depth whichever is greater.

12 diameters $= 12 \times 2 = 24$ *cm.*

Effective depth $= 56{\cdot}5$ *cm.*

Hence the bars must be extended from the theoretical cut off point for a distance of 56·5 *cm.*

∴ Distance of the actual point of cut off from the centre of bearing

$$= 79 - 56{\cdot}5 = 22{\cdot}5 \text{ cm. say } 22 \text{ cm.}$$

Design 24. *Design a rectangular singly reinforced beam having a clear span of 4·5 metres for a superimposed load of 6000 kg /m. run. The beam has 30 cms. bearing at the ends.*

Solution. Let the beam section be 30 *cm.* × 90 *cm.*

Loads

Consider one *metre* run of the beam. The load on the beam consists of the following :

Dead load	$= 0{\cdot}30 \times 0{\cdot}90 \times 2500 =$	675 *kg./m.*
Live load	$=$	6000 *kg./m.*
Total load	$=$	6675 *kg./m.*

Span between centres of bearing

$$= 4{\cdot}50 + 0{\cdot}30 = 4{\cdot}80 \text{ m.}$$

Maximum bending moment

$$= \frac{6675 \times 4{\cdot}8^2}{8} \times 100 \text{ kg. cm.}$$

$$= 1{,}922.400 \text{ kg. cm.}$$

Adopting $c = 50$ *kg./cm.*^{2}, $t = 1400$ *kg,/cm.*^{2}, $m = 18$ and equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}50\, bd^2 = 8{\cdot}50 \times 30\, d^2 = 1{,}922{,}400 \text{ kg. cm.}$$

$$\therefore \quad d = 86{\cdot}8 \text{ cm.}$$

Providing 25 *mm.* diameter bars at a clear cover of 25 *mm.* effective cover $= 25 + 12{\cdot}5 = 37{\cdot}5$ *mm* $= 3{\cdot}75$ *cm.*

Overall depth required $= 86{\cdot}80 + 3{\cdot}75 = 90{\cdot}55$ *cm. say* 91 *cm.*

Actual effective depth $= 91 - 3{\cdot}75 = 87{\cdot}25$ *cm.*

$$A_t = \frac{1922400}{1400 \times 0{\cdot}87 \times 87{\cdot}25} = 18{\cdot}09 \text{ cm.}^2$$

Provide 4 bars of 25 mm. ϕ (19·63 cm.^{2}*)*

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Design for shear

Maximum shear force at the edge of support

$$= S = \frac{6675 \times 4{\cdot}5}{2} = 15019 \text{ kg.}$$

Nominal shear stress at support

$$= q_v = \frac{S}{bd} = \frac{15019}{30 \times 87{\cdot}25} = 5{\cdot}74 \text{ kg./cm.}^2$$

For M 150 concrete $q_{c\ max} = 16 \text{ kg./cm.}^2$

$$\therefore \quad q_v < q_{c\ max}$$

Assuming 50% curtailment of tensile steel,

percentage of tensile steel at support

$$= \frac{9{\cdot}815}{30 \times 87{\cdot}25} \times 100 = 0{\cdot}37\%$$

As per code, for 0·25% steel, $q_c = 2{\cdot}2 \text{ kg./cm.}^2$

and for 0·50% steel, $q_c = 2{\cdot}9 \text{ kg./cm.}^2$

$$\therefore \quad \text{for } 0{\cdot}37\% \text{ steel, } q_c = 2{\cdot}2 + \frac{2{\cdot}9 - 2{\cdot}2}{0{\cdot}25} \times 0{\cdot}12$$

$$= 2{\cdot}5 \text{ kg./cm.}^2$$

Shear capacity of concrete

$$= q_c\, bd = 2{\cdot}5 \times 30 \times 87{\cdot}25 = 6544 \text{ kg.}$$

Shear to be carried by stirrups

$$= V_s = 15019 - 6544 = 8475 \text{ kg.}$$

Spacing of two-legged 8 *mm* diameter stirrups

$$= p = \frac{A_{sv} f d}{V_s} = \frac{(2 \times 0{\cdot}5) 1400 \times 87{\cdot}25}{8475} = 14 \text{ cm. c/c.}$$

Bend two bars at one-seventh of the span to resist any hogging moment due to partial fixity.

In the region where all the four bars are available as tension steel,

$$\text{percentage of steel} = \frac{19{\cdot}63}{30 \times 87{\cdot}25} \times 100 = 0{\cdot}75\%$$

As per code for 0·75% steel $q_c = 3{\cdot}4 \text{ kg./cm.}^2$

Let the nominal shear stress be 3·4 *kg./cm.*^{2} at a distance x *metres* from the centre.

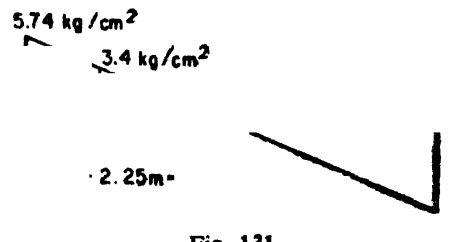

Fig. 131

$$\therefore \quad x = \frac{3{\cdot}4}{5{\cdot}74} \times 2{\cdot}25 = 1{\cdot}33 \text{ m.}$$

Hence the two-legged stirrups of 8 *mm.* ϕ @ 14 *cm. c/c* are necessary for a distance 2·25 − 1·33 = 0·92 *m* from the

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support. For the middle $2x$ *metre* or $2 \times 1.33 = 2.66$ *m.* nominal stirrups shall be provided.

Spacing of nominal stirrups = lesser of the following :

(*i*) $0.75\,d = 0.75 \times 87.25 = 65.4$ *cm.*

(*ii*) 45 *cm.*

Provide 2-legged 8 mm. diameter stirrups @ 45 cm. c/c.

Design 25. (S I). *Design a rectangular singly reinforced beam having a clear span of 4·50 metres far a superimposed load of 60 kN per metre run. The beam has 30 cm. bearing at the ends. Use M 15 concrete.*

Solution. Let the beam section be 30 *cm.* × 90 *cm.*

Loads. Consider one *metre* run of the beam. The load on the beam consists of the following :

D.L. $= 0.30 \times 0.90 \times 25000 =$ 6750 *N/m.*

L.L. ... ... $=$ 60000 *N/m.*

Total load $=$ 66750 *N/m.*

Span between centres of bearing $= 4.50 + 0.30 = 4.80$ *m.*

Maximum bending moment

$$= \frac{66750 \times 4.8^2}{8} \times 100 \; N\,cm. = 19224000 \; N\,cm.$$

Equating the moment of resistance to the bending moment,

$$85\,bd^2 = 85 \times 30\,d^2 = 19224000$$

$$\therefore \quad d = 86.8 \; cm$$

Providing 25 *mm.* diameter bars at a clear cover of 25 *mm.* effective cover $= 25 + 12.5 = 37.5$ *mm.* $= 3.75$ *cm.*

Overall depth required

$$= 86.80 + 3.75 = 90.55 \; cm. \; say \; 91 \; cm.$$

Actual effective depth $= 91 - 3.75 = 87.25$ *cm.*

$$\therefore \quad A_t = \frac{1922400}{14000 \times 0.87 \times 87.25} = 18.09 \; cm.^2$$

Provide 4 bars of 25 mm. ϕ (19·63 cm²)

Design for shear

Maximum shear force at the edge of the support

$$= S = \frac{66750 \times 4.5}{2} = 150190 \; N$$

$\therefore$ Nominal shear stress at the support

$$= q_v = \frac{S}{bd} = \frac{150190}{30 \times 87.25} = 57.4 \; N/cm.^2 = 0.574 \; N/mm^2.$$

For *M* 15 concrete, $q_{c\,max} = 1.60 \; N/mm.^2$

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$\therefore \quad q_v < q_{c\,max}$

Assuming 50% curtailment of tensile steel,

percentage of tensile steel at support $= \frac{9{\cdot}815}{30 \times 87{\cdot}25} \times 100 = 0{\cdot}37\%$.

As per code, for 0·25% steel, $q_c = 0{\cdot}22\ N/mm.^2$

and for 0·50% steel, $q_c = 0{\cdot}29\ N/mm.^2$

$\therefore$ for 0·37% steel, $q_c = 0{\cdot}22 + \frac{0{\cdot}29 - 0{\cdot}22}{0{\cdot}25} \times 0{\cdot}12$

$= 0{\cdot}25\ N/mm.^2 = 25\ N/cm.^2$

Shear capacity of concrete

$= q_c bd = 25 \times 30 \times 87{\cdot}25 = 65440\ N$

$\therefore$ Shear to be carried by stirrups

$= V_s = 150190 - 65440 = 84750\ N$

Spacing of two-legged 8 *mm.* diameter stirrups

$$= p = \frac{A_{wt} w d}{V_s}$$

$$= \frac{(2 \times 0{\cdot}5) 14000 \times 87{\cdot}25}{84750} = 14\ cm.\ c/c.$$

Bend two bars at one-seventh of the span to resist any hogging moment due to partial fixity.

In the region where all the four bars are available as tension steel, percentage of steel $= \frac{19{\cdot}63}{30 \times 87{\cdot}25} \times 100 = 0{\cdot}75\%$. For 0·75% steel, as per code, the nominal shear stress permissible

$$q_c = 0{\cdot}34\ N/mm.^2 = 34\ N/cm.^2$$

Let the nominal shear stress be 0·34 $N/mm.^2$ at a distance x *metres* from the centre.

$\therefore\ x = \frac{0{\cdot}34}{0{\cdot}574} \times 2{\cdot}25 = 1{\cdot}33$ *metre*

Hence, the two-legged stirrups of 8 *mm.* ϕ @ 14 *cm.*

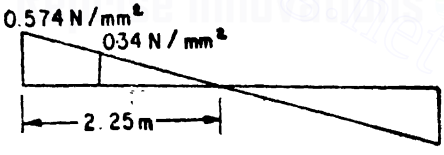

Fig. 132

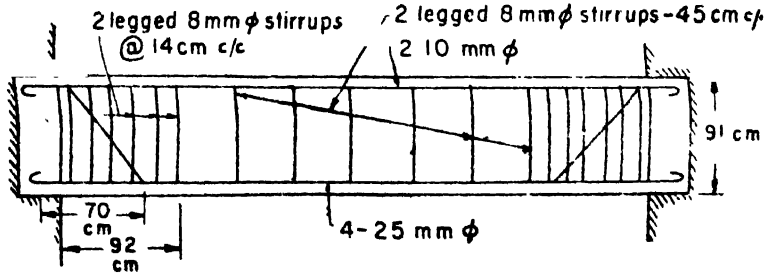

Fig. 133

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c/c are necessary for a distance $2.25 - 1.33 = 0.92$ *m* from the support. For the middle $2x$ *metre* or $2 \times 1.33 = 2.66$ *metre* nominal stirrups shall be provided

Spacing of nominal stirrups = lesser of the following :

(*i*) $0.75d = 0.75 \times 87.25 = 65.4$ *cm.*

(*ii*) 45 *cm.*

Provide 2-legged 8 mm. diameter stirrups @ 45 cm. c/c.

Design 26. (S.I.). *Design a cantilever 3·50 metre carrying a superimposed load of 10600 N/metre. Use M 15 concrete.*

Solution. Let the section of the cantilever be 30 *cm.* × 60 *cm.* at the fixed end and 30 *cm.* × 20 *cm* at the free end.

Weight of the cantilever

$$= \frac{0.6 + 0.2}{2} \times 3.50 \times 0.3 \times 25000 = 10500 \text{ } N$$

This load acts at a distance of $\frac{0.6 + 2 \times 0.2}{0.6 + 0.2} \times \frac{3.5}{3}$ *metres* from the fixed end

$= 1.458$ *metre* from the fixed end

D.L. moment $= 10500 \times 1.458 \times 100 = 1530900$ *N cm.*

L.L. moment $= 10600 \times \frac{3.5^2}{2} \times 100 = 6492500$ *N cm*

Total B.M. $= 8023400$ *N cm*

Adopting $c = 5$ *N/mm.*$^2 = 500$ *N/cm.*^{2} $t = 140$ *N/mm.*$^2 = 14000$ *N/cm.*^{2} and $m = 18$ and equating the moment of resistance to the maximum bending moment. We have,

$$85\,bd^2 = 85 \times 30\,d^2 = 8023400$$

$$\therefore \quad d = 56.09 \text{ } cm.$$

Providing 20 *mm.* ϕ bars at a clear cover of 25 *mm.*

Effective cover $= 25 + 10 = 35$ *mm.* $= 3.5$ *cm.*

Effective depth available $= 60 - 3.5 = 56.5$ *cm.*

$$A_t = \frac{8023400}{14000 \times 0.87 \times 56.5} = 11.66 \text{ } cm^2$$

Provide 4 bars of 20 mm. diameter (12·56 cm.^{2})

Maximum shear force

$$= S = 10500 + 10600 \times 3.5 \text{ } N.$$

$$= 47600 \text{ } N.$$

Nominal shear stress $= q_v = \frac{S}{bd}$

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$$= \frac{47600}{30 \times 56\cdot5} = 28\cdot1 \ N/cm.^2 = 0\cdot28 \ N/mm.^2$$

For M 15 concrete $q_{c\ max} = 1\cdot6 \ N/mm.^2$

Percentage of steel provided

$$= \frac{12\cdot56}{30 \times 56\cdot5} \times 100 = 0\cdot74\%.$$

As per I.S. code for 0·70% steel $q_c = 3\cdot3 \ kg./cm.^2$

Hence $q_v < q_c$.

Hence, provide nominal shear reinforcement.

Provide 2-legged 8 mm. ϕ stirrups @ 12 cm. c/c.

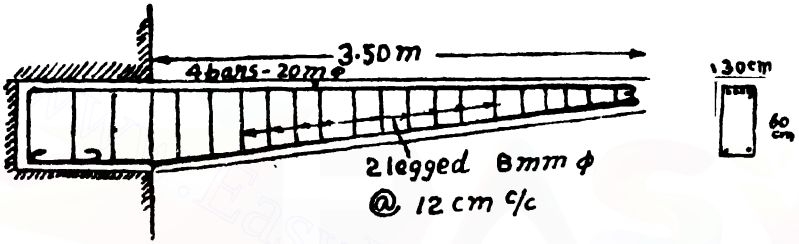

Fig. 134

Design 27. *Design a cantilever 3·50 metre long carrying a superimposed load of 1060 kg /m. run Use M 150 concrete.*

Solution Let the section of the cantilever be 30 *cm.* × 60 *cm.* at the fixed end and 30 *cm.* × 20 *cm.* at the free end.

Weight of the cantilever

$$= \frac{0\cdot6+0\cdot2}{2} \times 3\cdot50 \times 0\cdot3 \times 2500 = 1050 \ kg.$$

This load acts at a distance of

$$\frac{0\cdot6+2\times0\cdot2}{0\cdot6+0\cdot2} \times \frac{3\cdot50}{3} \ metres \text{ from the fixed end}$$

$$= 1\cdot458 \ metre \text{ from the fixed end.}$$

D.L. moment $= 1050 \times 1\cdot458 \times 100 = 153090 \ kg. \ cm.$

L.L. moment $= 1060 \times \frac{3\cdot5^2}{2} \times 100 = 649250 \ kg. \ cm.$

Total B.M. $= 802340 \ kg. \ cm.$

Adopting $c = 50 \ kg./cm.^2$, $t = 1400 \ kg./cm.^2$ and $m = 18$ and equating the moment of resistance to the bending moment, we have

$$8\cdot5 \times 30 \ d^2 = 802340$$

$$\therefore \quad d = 56\cdot09 \ cm.$$

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Providing 20 *mm.* ϕ bars at a clear cover of 25 *mm*,

Effective cover $= 25 + 10 = 35\ mm. = 3{\cdot}50\ cm.$,

Effective depth available

$$= 60 - 3{\cdot}5 = 56{\cdot}5\ cm.$$

$$A_t = \frac{802340}{1400 \times 0{\cdot}87 \times 56{\cdot}5} = 11{\cdot}66\ cm.^2$$

Provide 4 bars of 20 mm ϕ ($12{\cdot}56\ cm.^2$)

Maximum shear force

$$= S = 1050 + 1060 \times 3{\cdot}5\ kg. = 4760\ kg.$$

Nominal shear stress

$$\therefore q_v = \frac{S}{bd} = \frac{4760}{30 \times 56{\cdot}5} = 2{\cdot}8\ kg./cm.^2$$

For M 150 concrete,

$$q_{c\ max} = 16\ kg/cm.^2$$

$$\therefore \quad q_v < q_{c\ max}$$

Percentage of steel provided

$$= \frac{12{\cdot}56}{30 \times 56{\cdot}5} \times 100 = 0{\cdot}74\%$$

As per I.S. code, for 0·70% steel, $q_c = 3{\cdot}3\ kg./cm.^2$

Hence $q_v < q_c$. Hence provide nominal shear reinforcement

Provide 2-legged 8 mm ϕ *nominal stirrups @ 12 cm. c/c.*

Design 28. (S I.). *A beam and slab floor of R.C.C. has a 10 cm. thick slab forming part of T-beams, which are 6 metres clear span having 40 cm. end bearings. The beams are spaced at 2·75 metres centres. The live load on the slab is 3000 N/metre². Use M 15 concrete. Design the slab and an intermediate T-beam.*

Solution.

Design of the slab

Loading on the slab D.L. of the slab : $250 \times 10 = 2500\ N/metre^2$

Floor finish (2 *cm*) : $240 \times 2 = 480\ N/metre^2$

L.L. on the slab $= 3000\ N/metre^2$

Total $= 5980\ N/metre^2$

say $6000\ N/metre^2$

Consider a 1 *metre* wide strip of the slab.

Maximum bending moment

$$= \frac{wl^2}{12} = \frac{6000 \times 2{\cdot}75^2}{12} \times 100\ N\ cm.$$

$$= 378125\ N\ cm.$$

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Adopting $c=5\ N/mm.^2=500\ N/cm.^2$, $t=140\ N/mm.^2=14000\ N/cm.^2$ and $m=18$ and equating the moment of resistance to the maximum bending moment, we have

$$85\ bd^2=85\times 100\ d^2=378125$$

$$\therefore \quad d=6{\cdot}67\ cm.$$

Providing a clear cover of 15 *mm.* for 10 *mm.* diameter bars,

Effective depth available

$$=d=10-2=8\ cm.$$

$$A_t=\frac{378125}{14000\times 0{\cdot}87\times 8}=3{\cdot}88\ cm^2$$

Spacing of 10 *mm.* diameter bars

$$=\frac{0{\cdot}79\times 100}{3{\cdot}88}=20{\cdot}4\ cm.$$

Provide 10 mm. ϕ bars @ 20 cm c/c.

Distribution steel $\quad \frac{0{\cdot}15}{100}\times(100\times 10)=1{\cdot}50\ cm.^2$

Spacing of 6 *mm.* diamater bars

$$=\frac{0{\cdot}28\times 100}{1{\cdot}5}=18{\cdot}7\ cm.$$

Provide 6 mm. ϕ bars @ 18 cm. c/c.

Design of the intermediate T-beam

Overall depth of beam

$$=\frac{1}{12}\ \text{to}\ \frac{1}{20}\ \text{of the span}$$

Provide an overall depth of 45 *cm.*

Width of rib $\quad =\frac{2}{3}$ (depth of rib) $=\frac{2}{3}(45-10)=23{\cdot}3\ cm.$

Provide a width of 25 *cm.*

Loading on the beam

From the slab :	$6000\times 2{\cdot}75$	$=16500\ N/metre$
D.L. of the rib :	$0{\cdot}25\times 0{\cdot}35\times 25000=$	$2190\ N/metre$
	Total	$18690\ N/metre$

Providing 20 *mm.* diameter bars at a clear cover of 25 *mm.*

Effective cover $\quad =25+10=35\ mm.=3{\cdot}5\ cm.$

Effective depth $\quad =d=45-3{\cdot}5=41{\cdot}50\ cm.$

Effective span. This is taken as the lesser of

(*i*) centre to centre of bearings $=6{\cdot}40\ metre$

(*ii*) clear span + effective depth $=6+0{\cdot}415=6{\cdot}415\ metre$

$\therefore$ Effective span $=6{\cdot}40\ metre.$

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Maximum bending moment

$$= \frac{18690 \times 6{\cdot}4^2}{8} \times 100 \ N\ cm.$$

$$= 9569280 \ N\ cm.$$

Approximate lever arm

$$= a = d - \frac{d_s}{2} = 41{\cdot}50 - 5 = 36{\cdot}50 \ cm.$$

Approximate steel required

$$= A_s = \frac{9569280}{14000 \times 36{\cdot}5} = 18{\cdot}73 \ cm.^2$$

Provide 6 bars of 20 mm diameter (18·84 cm.²)

These bars will be arranged in two rows with three bars per row.

Clear space between the two rows of bars = greater of the following :

(*i*) 15 *mm.*

(*ii*) $\frac{2}{3}$ maximum size of aggregate $= \frac{2}{3}(20) = 13{\cdot}3$ *mm.*

(*iii*) Maximum size of bar $= 20$ *mm.*

Provide a clearance of 20 *mm.* between the two rows of bars.

Effective cover to the C.G. of steel

$$= 25 + 20 + 10 = 55 \ mm = 5{\cdot}5 \ cm.$$

$\therefore$ Actual effective depth

$$= d = 45 - 5{\cdot}50 = 39{\cdot}50 \ cm.$$

Actual effective span = clear span + effective depth

$$= 6 + 0{\cdot}395 \ m.\ \text{say}\ 6{\cdot}40 \ metre.$$

Width of flange $= B = \frac{l}{6} + b_r + 6d_s$

$$= \frac{640}{6} + 25 + (6 \times 10) = 192 \ cm.$$

Position of neutral axis

Assuming the neutral axis to lie below the flange and taking moments about the neutral axis,

$$192 \times 10\,(n - 5) = 18 \times 18{\cdot}84\,(39{\cdot}5 - n)$$

$\therefore$ $n = 10{\cdot}18 \ cm.$

$$y = \frac{3n - 2d_s}{2n - d_s} \cdot \frac{d_s}{3}$$

$$= \frac{3(10{\cdot}18) - (2 \times 10)}{2(10{\cdot}18) - 10} \times \frac{10}{3} = 3{\cdot}39 \ cm.$$

$\therefore$ Actual lever arm $= a = d - y = 39{\cdot}50 - 3{\cdot}39 = 36{\cdot}11 \ cm.$

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∴ Actual stress in steel

$$= t = \frac{9569280}{18{\cdot}84 \times 36{\cdot}1} = 14066 \ N/cm^2$$

$$= 140{\cdot}66 \ N/mm.^2$$

Percentage overstress $= \frac{0{\cdot}66}{140} \times 100 = 0{\cdot}47\%$. This is very small.

Maximum stress in concrete

$$= c = \frac{n}{d-n} \cdot \frac{t}{m}$$

$$\frac{10{\cdot}18}{39{\cdot}5-10{\cdot}18} \times \frac{14066}{18} = 271 \ N/cm.^2$$

$$= 2{\cdot}71 \ N/mm.^2$$

Design for shear

Clear span $= 6$ *metres*

Maximum shear force

$$= S = \frac{18690 \times 6}{2} = 56070 \ N$$

Nominal shear stress at support

$$= q_v = \frac{S}{bd} = \frac{56070}{25 \times 39{\cdot}5} = 56{\cdot}8 \ N/cm.^2$$

$$= 0{\cdot}568 \ N/mm.^2$$

For M 15 concrete,

$$q_{c\,max} = 1{\cdot}6 \ N/mm.^2$$

$$\therefore \quad q_v < q_{c\,max}.$$

Assuming 50% curtailment of tensile steel,

Percentage of tensile steel at support

$$= \frac{9{\cdot}42}{25 \times 39{\cdot}5} \times 100 = 0{\cdot}95\%.$$

As per code, for 0·9% steel,

$$q_c = 0{\cdot}36 \ N/mm.^2$$

and for 1% steel, $q_c = 0{\cdot}37 \ N/mm.^2$

∴ For 0·95 % steel,

$$q_c = 0{\cdot}365 \ N/mm.^2 = 36{\cdot}5 \ N/cm.^2$$

∴ Shear capacity of concrete

$$= q_c bd = 36{\cdot}5 \times 25 \times 39{\cdot}5 = 36040 \ N$$

∴ Shear to be carried by stirrups

$$= V_s = 56070 - 36040 = 20030 \ N$$

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Spacing of 2-legged 8 *mm.* diameter stirrups

$$= p = \frac{A_{wt_v} d}{V_s} = \frac{(2 \times 0{\cdot}5) 14000 \times 39{\cdot}5}{20030} = 27{\cdot}6 \text{ cm.}$$

Provide 2-legged 8 mm ϕ stirrups @ 25 cm. c/c.

Bend the three bars of the upper layer at one-seventh of the span, to resist any hogging moment due to partial fixity.

In the region where all the six bars are available, percentage of tensile steel $= \frac{18{\cdot}84}{25 \times 39{\cdot}5} \times 100 = 1{\cdot}9\%$

For 1·9% steel, as per code $q_c = 0{\cdot}44$ *N/mm.*$^2 = 44$ *N/cm.*^{2}

Let the nominal shear stress be 0·44 *N/mm.*^{2} at a distance of *x metres* from the centre.

$$\therefore \quad x = \frac{0{\cdot}44}{0{\cdot}568} \times 3 = 2{\cdot}33 \text{ m.}$$

Hence the stirrups as calculated above, will be provided for a distance $\left(\frac{l}{2} - x\right) = 3 - 2{\cdot}33 = 0{\cdot}67$ *metre* from the support. For the middle $2x$ *metres* or 4·66 *metres* we will provide nominal stirrups.

Spacing of the nominal stirrups is equal to the lesser of the following :

(*i*) $0{\cdot}75\, d = 0{\cdot}75 \times 39{\cdot}5 = 29{\cdot}63$ *cm.*

(*ii*) 45 *cm.*

Provide 2-legged 8 mm ϕ stirrups @ 30 cm. c/c.

Design 29. *A beam and slab floor of R.C.C. has a 10 cm. thick slab forming part of T-beams, which are 6 metres clear span having 40 cm end bearings. The beams are spaced at 2·75 metres centres. The live load on the slab is 300 kg./metre*^{2}*. Design the slab and an intermediate T-beam.*

Solution.

Design of the slab

Loading on the slab :

DL of slab : 25×10	= 250 *kg./metre*^{2}
Floor finish (2 *cm.*) : 24×2	= 48 *kg./metre*^{2}
LL on the slab	= 300 *kg./metre*^{2}
Total	= 598 *kg./metre*^{2}
	say 600 *kg./metre*^{2}

Consider a 1 *metre* wide strip of the slab.

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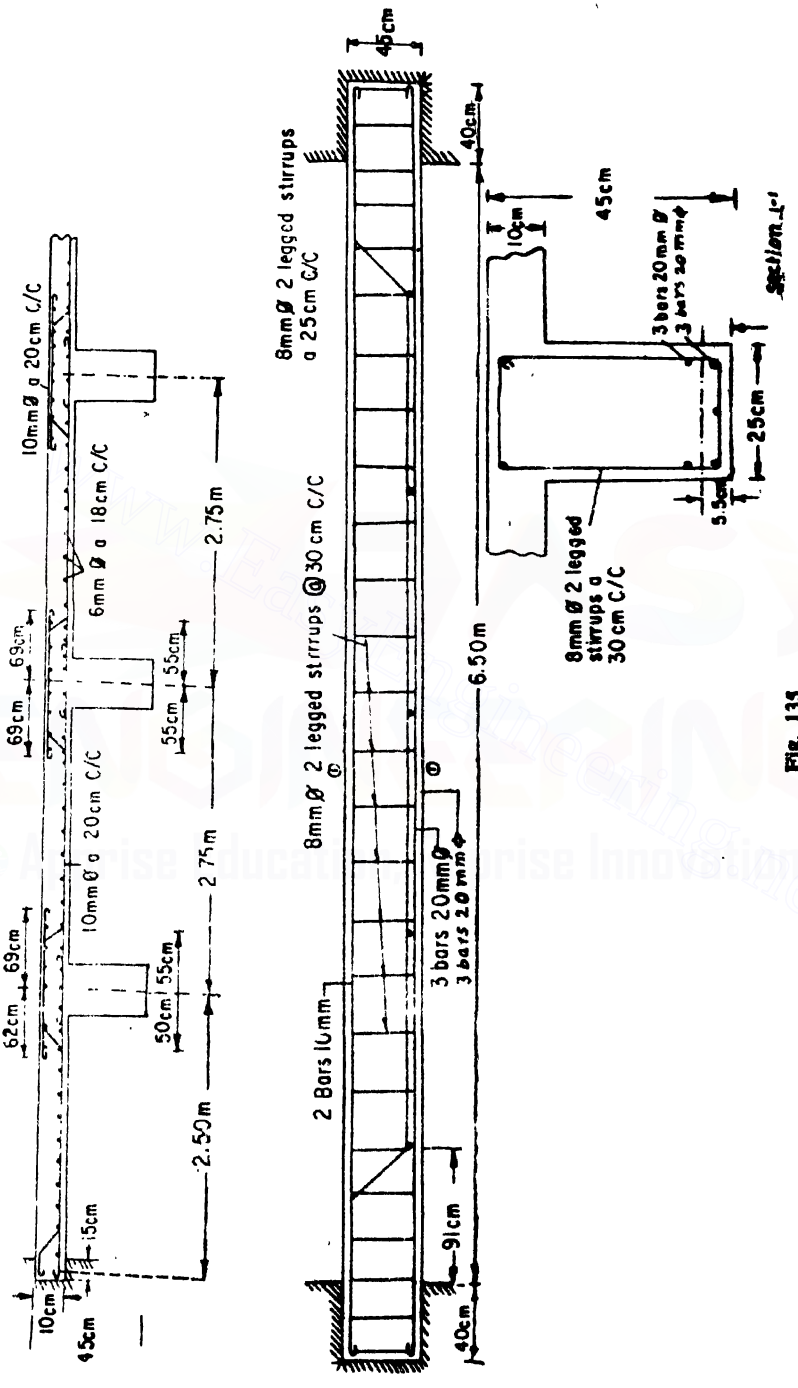

Fig. 135

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Maximum bending moment

$$= \frac{wl^2}{12} = \frac{600 \times 2{\cdot}75^2}{12} \times 100 \text{ kg. cm.}$$

$$= 37810 \text{ kg. cm.}$$

Adopting $c = 50$ kg./cm.^{2}, $t = 1400$ kg./cm.^{2} and $m = 18$, and equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}5\, bd^2 = 8{\cdot}5 \times 100 \times d^2 = 37810$$

$$\therefore \quad d = 6{\cdot}67 \text{ cm.}$$

Providing a clear cover of 15 mm. for 10 mm. diameter bars

Effective depth available $= d = 10 - 2{\cdot}0 = 8$ cm.

$$\therefore \quad A_t = \frac{37810}{1400 \times 0{\cdot}87 \times 8} = 3{\cdot}88 \text{ cm.}^2$$

Spacing of 10 mm. diameter bars

$$= \frac{0{\cdot}79 \times 100}{3{\cdot}88} = 20{\cdot}4 \text{ cm.}$$

Provide 10 mm. ϕ bars @ 20 cm. c/c.

Distribution steel $= \frac{0{\cdot}15}{100} \times (100 \times 10) = 1{\cdot}50$ cm.^{2}

Spacing of 6 mm. diameter bars

$$= \frac{0{\cdot}28 \times 100}{1{\cdot}5} = 18{\cdot}7 \text{ cm.}$$

Provide 6 mm. ϕ bars @ 18 cm. c/c.

Design of the intermediate T-beam

Overall depth of beam $= \frac{1}{12}$ to $\frac{1}{20}$ of the span.

Provide an overall depth of 45 cm.

Width of rib $= \frac{2}{3}$ (depth of rib) $= \frac{2}{3}(45 - 10) = 23{\cdot}3$ cm.

Provide a width of 25 cm.

Loading on the beam

From slab : $600 \times 2{\cdot}75$	$= 1650$ kg./m.
DL of rib : $0{\cdot}25 \times 0{\cdot}35 \times 2500$	$= 220$ kg./m.
Total	$= 1870$ kg./m.

Providing 20 mm. diameter bars at a clear cover of 25 mm.
effective cover $= 25 + 10 = 35$ mm. $= 3{\cdot}50$ cm.
Effective depth $= d = 45 - 3{\cdot}5 = 41{\cdot}50$ cm.

Effective span. This is taken as the lesser of

(*i*) Centre to centre of bearings $= 6{\cdot}40$ metre

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(*ii*) Clear span + effective depth = 6 + 0·415 = 6·415 *metre*

∴ Effective span = 6·40 *metre*

Maximum bending moment

$$= \frac{1870 \times 6{\cdot}4^2}{8} \times 100 \text{ kg. cm.}$$

$$= 957440 \text{ kg. cm.}$$

Approximate lever arm

$$= a = d - \frac{ds}{2} = 41{\cdot}50 - 5 = 36{\cdot}50 \text{ cm.}$$

Approximate steel required

$$= A_s = \frac{957440}{1400 \times 36{\cdot}5} = 18\ 74 \text{ cm.}^2$$

Provide 6 bars of 20 mm. diameter (18 84 cm.²)

These bars will be arranged in two rows with three bars per row.

Clear space between the two rows of bars = greater of the following :

(*i*) 15 *mm.*

(*ii*) $\frac{2}{3}$ maximum size of aggregate = $\frac{2}{3}$ (20) = 13·3 *mm.*

(*iii*) Maximum size of bar = 20 *mm.*

Provide a clearance of 20 *mm.* between the two rows of bars.

Effective cover to C G of steel = 25 + 20 + 10 = 55 *mm.* = 5·50 *cm.*

∴ Actual effective depth = d = 45 − 5·50 = 39·50 *cm.*

Actual effective span = clear span + effective depth

= 6 + 0·395 = 6·395 *m.* say 6·40 *m.*

Width of flange $B = \frac{l}{6} + b_r + 6d_s$

$$= \frac{640}{6} + 25 + 6 \times 10 = 192 \text{ cm.}$$

Position of neutral axis

Assuming the neutral axis to lie below the flange, and taking moments about the neutral axis.

$$192 \times 10\ (n - 5) = 18 \times 18{\cdot}84\ (39{\cdot}5 - n)$$

$$\therefore \quad n = 10{\cdot}18 \text{ cm.}$$

$$y = \frac{3n - 2ds}{2n - ds} \cdot \frac{ds}{3} = \frac{3\ (10{\cdot}18) - (2 \times 10)}{2\ (10{\cdot}18) - 10} \times \frac{10}{3}$$

$$= 3{\cdot}39 \text{ cm.}$$

∴ Actual lever arm = $a = d - y$ = 39·50 − 3·39 = 36·11 *cm.*

∴ Actual stress in steel

$$= t = \frac{957440}{18{\cdot}84 \times 36{\cdot}11} = 1407 \text{ kg./cm.}^2$$

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Overstress $= 7\ kg./cm.^2$

Percentage overstress $= \frac{7}{1400} \times 100 = 0{\cdot}5\%$. This is very small.

Maximum stress in concrete

$$= c = \frac{n}{d-n}\ \frac{t}{m} = \frac{10{\cdot}18}{39{\cdot}5-10{\cdot}18} \times \frac{1407}{18}$$
$$= 27{\cdot}1\ kg./cm.^2$$

Design for shear

Clear span $= 6$ *metres*

Maximum shear force

$$= S = \frac{1870 \times 6}{2} = 5610\ kg.$$

Nominal shear stress at support

$$= q_v = \frac{S}{b_r d} = \frac{5610}{25 \times 39\ 5} = 5{\cdot}68\ kg./cm.^2$$

For M 150 concrete, $q_{c\ max} = 16\ kg./cm.^2$

$\therefore \quad q_v < q_{c\ max}$

Assuming 50% curtailment of tensile steel

Percentage of tensile steel at support

$$= \frac{9{\cdot}42}{25 \times 39{\cdot}5} \times 100 = 0{\cdot}95\%$$

As per code for 0·9% steel, $q_c = 3{\cdot}6\ kg./cm.^2$.

and for 1% steel, $q_c = 3{\cdot}7\ kg./cm^2$

$\therefore$ for 0·95% steel, $q_c = 3{\cdot}65\ kg./cm.^2$

Shear capacity of concrete $= q_c\ b_r\ d$

$$= 3{\cdot}65 \times 25 \times 39{\cdot}5 = 3604\ kg.$$

$\therefore$ Shear to be carried by stirrups

$$V_s = 5610 - 3604 = 2006\ kg.$$

Spacing of 2-legged 8 *mm.* diameter stirrups

$$= p = \frac{A_w\ t_w\ d}{V_s} = \frac{(2 \times 0{\cdot}5) 1400 \times 39{\cdot}5}{2006} = 27{\cdot}6\ cm.$$

Provide 2-legged 8 mm. ϕ stirrups @ 25 cm. c/c.

Bend the three bars of the upper layer at one seventh of the span to resist any hogging moment due to partial fixity.

In the region where all the six bars are available as tension steel,

percentage of steel $= \frac{18{\cdot}84}{25 \times 39{\cdot}5} \times 100 = 1{\cdot}9\%$

For 1·9% steel, as per code, $q_c = 4{\cdot}4\ kg./cm.^2$

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Let the nominal shear stress be 4·4 *kg./cm.*^{2} at a distance of x *metres* from the centre.

$$\therefore \qquad x = \frac{4{\cdot}4}{5{\cdot}68} \times 3 = 2{\cdot}33 \ m.$$

Hence the stirrups as calculated above, will be provided for a distance $\left(\frac{l}{2} - x\right) = 3 - 2{\cdot}33 = 0{\cdot}67$ *m.* from the support. For the middle $2x$ *metres* or 4·66 *metres*, we will provide nominal stirrups.

Spacing of nominal stirrups is equal to the lesser of,

(*i*) $0{\cdot}75\ d = 0{\cdot}75 \times 39{\cdot}5 = 29{\cdot}63$ *cm.* (*ii*) 45 *cm.*

Provide 2-legged 8 mm. diameter stirrups @ 30 cm. c/c.

§127. Beam and Slab Floor

In the *T*-beam and slab designed above, the slab was monolithic with the beams. Suppose the beams are separately cast and the slab is not monolithic with them, then we should design the beams as rectangular beams. The slab may be of course designed as a continuous slab. Concreting for the slab should be done in one operation. Construction joints may be located over the beams. It is also possible to cast the slab in separate panels spanning from beam to beam with a clear joint over the beams. In such a case the slab should be designed as a simply supported slab spanning from beam to beam.

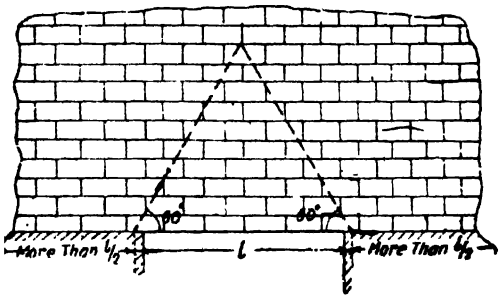
§128 Lintel

It becomes necessary to provide openings in walls for doors, windows, cupboards, *etc.* Such openings must be bridged over so as to support the load of the wall above them. This is accomplished by providing a lintel Hence a lintel serves to span over an opening like a beam.

Loading for which a lintel must be designed

Case (*i*). *When the length of wall on each side of opening is at least equal to half the effective span.*

In well-bonded brick work extending on each side of the opening for a distance equal to at least half the span of the beam, it is usual to consider the loading on the lintel as the weight of brickwork contained in an equilateral triangle (See Fig. 136).



LINTEL - CASE I

Fig. 136

If W be the weight of brick work the bending moment due to the weight of brickwork

$$\frac{Wl}{}$$

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Case (*ii*). ***When one end of the lintel is close to the end of the wall.***

Suppose the length of the wall on one side of opening does not extend to half the effective span, the loading must be determined allowing for a height equal to the effective span (Fig. 137).

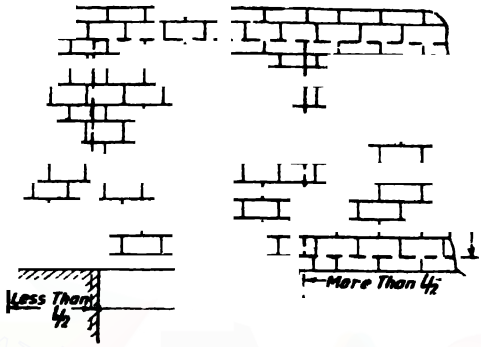

Fig. 137

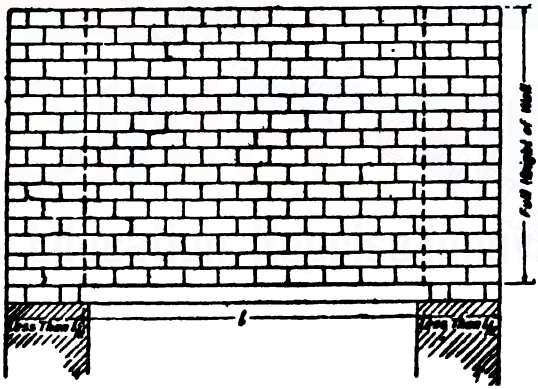

Fig. 138

Case (*iii*) ***When both ends are close to the ends of the walls.***

If the length of the wall on each side of the opening is less than half the effective span, the loading must be computed for the full height of the wall above the lintel (Fig. 138).

Case (*iv*). ***When the wall carries a slab.***

If a slab is provided at a level within the equilateral triangle mentioned in case (*i*) the loading on the lintel will consist of the load of masonry between the lintel and the slab and a triangular loading above the slab level (See Fig 139).

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Case (v). ***When there are openings above the lintel.***

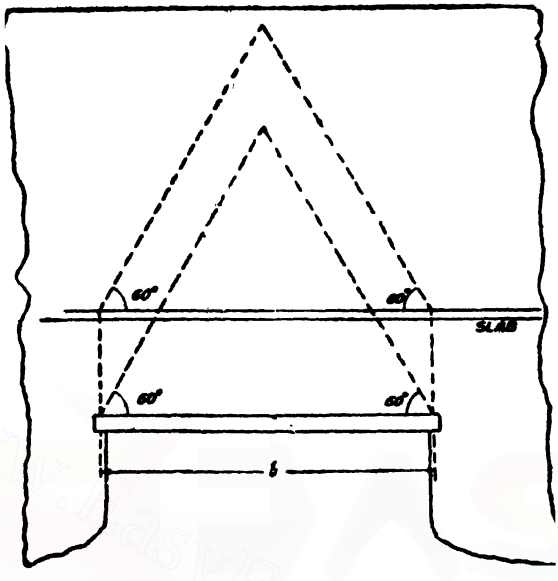

Fig. 139

In this case the loading is computed allowing dispersion lines at 60° as shown in Fig. 140.

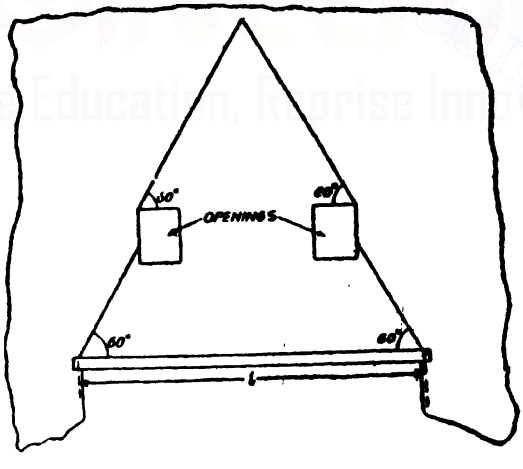

Fig. 140

Design 30. *Design an R.C. lintel over a door opening 2·5 metres wide, and 2·75 metres deep. The height of brick work above the lintel is 3·5 metres. Masonry weighs 1920 kg. per cubic metre. Brick walls are 20 cm. thick.*

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Solution. Let the bearing for the lintel be 20 *cm.*

∴ Span between centres of bearing

$$= 2{\cdot}50 + 0{\cdot}20 \ m. = 2{\cdot}70 \ metres.$$

Let the size of the lintel be 20 *cm.* wide and 18 *cm.* deep. Providing an effective cover of 3 *cm.* to the reinforcement, effective depth

$$= 18 - 3 = 15 \ cm.$$

Effective span. This is taken as the lesser of the following :

(*i*) Distance between centres of bearings

$$= 2{\cdot}70 \ metres.$$

(*ii*) Clear span + Effective depth

$$= 2{\cdot}50 + 0{\cdot}15 = 2{\cdot}65 \ m.$$

∴ Effective span = 2·65 *metres*

Figs. 141 (*a*) and (*b*) show the lintel supporting the wall. Weight of the wall to be supported

= weight of the equilateral triangular portion of wall.

$$= \tfrac{1}{2} \times 2{\cdot}65 \times 2{\cdot}65 \sin 60^\circ \times 0{\cdot}20 \times 1920 \ kg.$$

$$= 1168 \ kg.$$

B.M. due to weight of brick masonry

$$= \frac{Wl}{1168 \times 2{\cdot}65} \times 100 \ kg. \ cm.$$

$$= 51580 \ kg. \ cm.$$

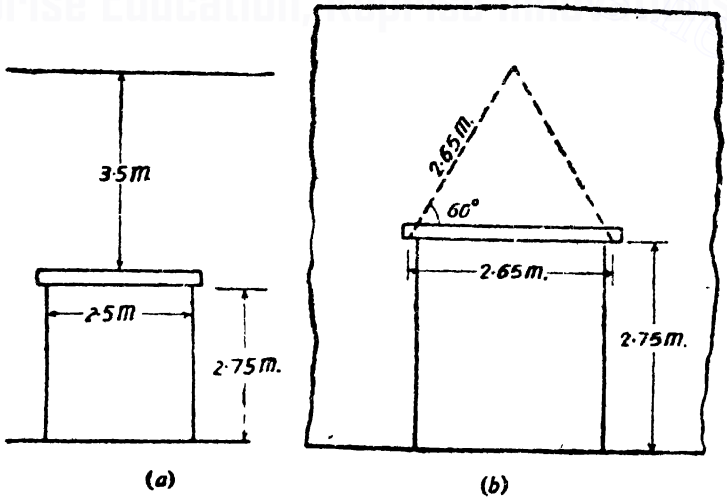

Fig. 1·41

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D.L. of the lintel $= 0{\cdot}20 \times 0{\cdot}18 \times 2500 = 90$ *kg./m.*

B.M. due to D.L. $= \frac{wl^2}{8} = \frac{90 \times 2{\cdot}65^2}{8} \times 100$ *kg. cm.*

$= 7900$ *kg. cm.*

Total B.M. $= 51580 + 7900 = 59480$ *kg. cm.*

Adopting $c = 50$ *kg/cm.*^{2}, $t = 1400$ *kg./cm.*^{2} and $m = 18$ and equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}5 \times 20\, d^2 = 59480 \quad \therefore \quad d = 18{\cdot}7 \text{ cm.}$$

Overall depth required

$= 18{\cdot}7 + 3 = 21{\cdot}7$ *cm. say* 22 *cm.*

Additional load due to 4 *cm.* additional thickness

$= 0{\cdot}20 \times 0{\cdot}04 \times 2500 = 20$ *kg./m.*

Additional B.M. $= \frac{20 \times 2{\cdot}65^2}{8} \times 100$ *kg. cm.*

$= 1756$ *kg cm.*

Total B.M. $= 59480 + 1756 = 61236$ *kg. cm.*

Equating the moment of resistance to the maximum bending moment,

$$8{\cdot}5 \times 20\, d^2 = 61236$$

$\therefore \quad d = 19$ *cm.*

$\therefore \quad A_t = \frac{61236}{1400 \times 0{\cdot}87 \times 19} = 2{\cdot}65$ *cm.*^{2}

Provide 4 bars of 10 mm. diameter (3·14 cm.^{2}*)*

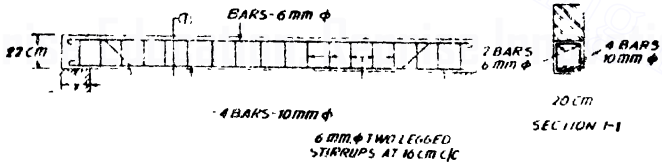

Fig. 142

Maximum shear force

$$S = \frac{W}{2} + \frac{wl}{2} = \frac{1168}{2} + (90 + 20)\ \frac{2{\cdot}65}{2},$$

$$S = 584 + 146 = 730 \text{ kg.}$$

Nominal shear stress

$$= q_v = \frac{S}{bd} = \frac{730}{20 \times 19} = 1{\cdot}92 \text{ kg./cm.}^2$$

Assuming 50% curtailment of reinforcement,

percentage of tensile steel available at support

$$= \frac{(2 \times 0{\cdot}785)}{20 \times 19} \times 100\% = 0{\cdot}40\%.$$

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For M 150 concrete for 0·4% steel permissible nominal shear stress $q_c = 2{\cdot}7$ *kg./cm*^{2}.

$\therefore \quad q_v < q_c$

Hence provide two-legged 8 *mm.* diameter nominal stirrups.

Spacing of the stirrups = lesser of the following :

(*i*) $0{\cdot}75\,d = 0{\cdot}75 \times 19 = 14{\cdot}25$ *cm.* (*ii*) 45 *cm.*

Provide 2 legged 8 mm. ϕ stirrups @ 14 cm. c/c.

Design 31 (S.I.). *Design an R.C. lintel over a door opening 2·5 metre wide and 2·75 metre deep. The height of brickwork above the lintel is 3·5 metre. Masonry weighs 19200 N per metre*^{3}*. Brick walls are 20 cm. thick.*

Solution. Let the bearing for the lintel be 20 *cm.*

$\therefore$ Span between centres of bearing $= 2{\cdot}50 + 0{\cdot}20 = 2{\cdot}70$ *metre.*

Let the size of the lintel be 20 *cm.* wide and 18 *cm.* deep.

Providing an effective cover of 3 *cm.* to the reinforcement, effective depth $= 18 - 3 = 15$ *cm.*

Effective span. This is taken as the lesser of the following :

(*i*) Distance between centres of bearings $= 2{\cdot}70$ *metre.*

(*ii*) Clear span + effective depth $= 2{\cdot}50 + 0{\cdot}15 = 2{\cdot}65$ *metre.*

$\therefore$ Effective span $= 2{\cdot}65$ *metre.*

Weight of the wall to be supported

= weight of the equilateral triangular portion of the wall

$$= \tfrac{1}{2} \times 2{\cdot}65 \times 2{\cdot}65 \sin 60° \times 0{\cdot}20 \times 19200\ N$$

$$= 11680\ N.$$

B.M. due to weight of brick masonry

$$= \frac{Wl}{6}$$

$$= \frac{11680 \times 2{\cdot}65}{6} \times 100\ N\ cm.$$

$$= 515800\ N\ cm.$$

D.L. of the lintel $= 0{\cdot}20 \times 0{\cdot}18 \times 25000 = 900\ N/metre.$

B.M. due to D.L. $= \frac{wl^2}{8} = \frac{900 \times 2{\cdot}65^2}{8} \times 100\ N\ cm.$

$$= 79000\ N\ cm.$$

Total B.M. $= 515800 + 79000 = 594800\ N\ cm.$

Adopting $c = 5\ N/mm.^2 = 500\ N/cm^2$; $t = 140\ N/mm.^2 = 14000\ N/cm.^2$ and $m = 18$, and equating the moment of resistance to the maximum bending moment, we have,

$$85\,bd^2 = 85 \times 20\,d^2 = 594800 \qquad \therefore\ d = 18{\cdot}7\ cm.$$

Overall depth required $= 18{\cdot}7 + 3 = 21{\cdot}7$ *cm. say* 22 *cm.*

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Additional load due to 4 *cm.* additional thickness

$$= 0{\cdot}20 \times 0{\cdot}04 \times 25000 = 200 \ N/metre$$

Additional B M. $= \dfrac{200 \times 2{\cdot}65^2}{8} \times 100 \ N\ cm.$

$= 17560 \ N\ cm.$

Total B.M. $= 594800 + 17560 = 612360 \ N\ cm.$

Equating the moment of resistance to the maximum bending moment,

$$85\ bd^2 = 85 \times 10\ d^2 = 612360 \qquad \therefore\ d = 19\ cm.$$

$$A_t = \frac{612360}{14000 \times 0{\cdot}87 \times 19} = 2{\cdot}65\ cm.^2$$

Provide 4 bars of 20 mm. diameter (3·14 cm.²).

Maximum S.F. $= S = \dfrac{W}{2} + \dfrac{wl}{2}$

$$= \frac{11680}{2} + (900 + 200)\ \frac{2{\cdot}65}{2}$$

$$= 5840 + 1460 = 7300\ N$$

Nominal shear stress $= q_v = \dfrac{S}{bd} = \dfrac{7300}{20 \times 19} = 19{\cdot}2\ N/cm.^2$

$= 0{\cdot}192\ N/mm.^2$

Assuming 50% curtailment of reinforcement,

percentage of tensile steel available at support

$$= \frac{(2 \times 0{\cdot}785)}{20 \times 19}$$

For M 15 concrete, for 0·4% steel permissible nominal shear stress

$$= q_c = 0{\cdot}27\ N/mm^2$$

$\therefore \qquad q_v < q_c$

Hence provide two-legged 8 *mm.* diameter nominal stirrups.

Spacing of the nominal stirrups = lesser of the following :

(*i*) $0{\cdot}75\ d = 0{\cdot}75 \times 19 = 14{\cdot}25\ cm.$ (*ii*) 45 *cm.*

Provide 2-legged 8 mm. ϕ stirrups @ 14 cm. c/c.

Design 32 (S.I.). *An R.C. lintel is to be provided over an opening 2 metres wide. The lintel carries a 40 cm. thick wall, the height of the wall above the lintel being 4 metres. Design the lintel with a 0·60 metre wide sun shade. The live load on the sun shade may be taken as 2000 N/metre². Use M 15 concrete.*

Solution. *Design of the sun shade.*

This is designed as a cantilever. Let the thickness of the sun shade be 6 *cm.*

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Loads on the sun shade :

Dead load $= 250 \times 6$ $= 1500$ *N/metre*^{2}

Live Load $= 2000$ *N/metre*^{2}

Total load $= 3500$ *N/metre*^{2}

Consider a one *metre* wide strip of the cantilever.

Maximum B.M. $= 3500 \times \frac{0{\cdot}6^2}{2} \times 100$ *N cm.* $= 63000$ *N cm.*

Adopting $c = 5$ *N/mm.*$^2 = 500$ *N/cm*^{2} ; $t = 140$ *N/mm.*$^2 = 14000$ *N/cm.*^{2} and $m = 18$, and equating the moment of resistance to the maximum bending moment, we have,

$85\ bd^2 = 85 \times 100\ d^2 = 63000$ $\therefore\ d = 2{\cdot}72$ *cm.*

Providing 5 *mm.* diameter bars at a clear cover of 15 *mm.*, effective cover $= 15 + 2{\cdot}5 = 17{\cdot}5$ *mm.* $= 1{\cdot}75$ *cm.*

Effective depth available $= 6 - 1{\cdot}75 = 4{\cdot}25$ *cm.*

$\therefore$ $A_t = \frac{63000}{14000 \times 0{\cdot}87 \times 4{\cdot}25} = 1{\cdot}22$ *cm.*^{2}

Spacing of 5 *mm.* diameter bars

$= \frac{0{\cdot}20 \times 100}{1{\cdot}22} = 16{\cdot}4$ *cm.*

But the spacing of main bars should not exceed three times the effective depth.

In our case three times the effective depth $= 3 \times 4{\cdot}25 = 12{\cdot}75$ *cm.*

$\therefore$ *Provide 5 mm* ϕ *bars @ 12 cm. c/c.*

Spacing of 5 *mm.* diameter distribution bars

= not greater than five times the effective depth.

= not greater than $5 \times 4{\cdot}25 = 21{\cdot}25$ *cm.*

Since the projecting length of the sunshade is only 0·6 *m.*

Provide 4 bars of 5 mm. ϕ *as distribution steel.*

Design of the lintel

Let the lintel be 40 *cm.* wide and 20 *cm.* deep.

Effective depth $= 20 - 3 = 17$ *cm.*

Let the bearing at each end be 20 *cm.*

Effective span. This is taken as the lesser of the following :

(*i*) Distance between centres of bearings $= 2{\cdot}20$ *metre.*

(*ii*) Clear span + effective depth $= 2 + 0{\cdot}17 = 2{\cdot}17$ *metre.*

$\therefore$ Effective span $= 2{\cdot}17$ *metre.*

Height of an equilateral triangle of side 2·17 *metre*

$= 2{\cdot}17 \sin 60° = 1{\cdot}88$ *metre*

Height of wall above the lintel $= 4$ *metre*

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Hence only triangular loading of brick masonry will be transferred to the lintel.

Loads on the lintel

Triangular load due to masonry

$$= W = \tfrac{1}{2} \times 2{\cdot}17 \times 1{\cdot}88 \times 0{\cdot}4 \times 19200 \; N = 15665 \; N$$

Distributed load :

D.L. of lintel $= 0{\cdot}4 \times 0{\cdot}20 \times 25000$ $= 2000$ *N/metre*

Load from sun shade $= 3500 \times 0{\cdot}6$ $= 2100$ *N/metre*

Total distributed load $= w$ $= 4100$ *N/metre*

Maximum bending moment

$$= \frac{Wl}{6} + \frac{wl^2}{8}$$

$$= \frac{15665 \times 2{\cdot}17}{6} + \frac{4100 \times 2{\cdot}17^2}{8} \; Nm.$$

$$= 5667 + 2413 = 8080 \; N\,m. = 808000 \; N \; cm.$$

Equating the moment of resistance to the maximum bending moment, we have,

$$85 \; bd^2 = 85 \times 40 \; d^2 = 808000 \quad \therefore \; d = 15{\cdot}41 \; cm.$$

Effective depth available $= 20 - 3 = 17$ *cm.*

$$\therefore \quad A_t = \frac{808000}{14000 \times 0{\cdot}87 \times 17} = 3{\cdot}90 \; cm.^2$$

Provide 5 bars of 10 mm. ϕ (3·93 cm.^{2})

Maximum shear force

$$= S = \frac{W}{2} + \frac{wL}{2}$$

$$= \frac{15665}{2} + \frac{4100 \times 2}{2} \; N = 11933 \; N$$

Nominal shear stress

$$= q_v = \frac{S}{bd} = \frac{11933}{40 \times 17} = 17{\cdot}5 \; N/cm.^2$$

$$= 0{\cdot}175 \; N/mm.^2$$

Percentage of tensile steel provided

$$= \frac{3{\cdot}93}{40 \times 17} \times 100 = 0{\cdot}58\%$$

For 0·50% steel, $q_c = 0{\cdot}29 \; N/mm.^2$

$\therefore \quad q_v < q_c$

Provide nominal stirrups.

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Spacing of nominal stirrups=lesser of the following :

(*i*) 0·75 d=0·75×17=12·75 *cm.* (*ii*) 45 *cm.*

Provide two-legged 8 mm. ϕ stirrups @ 12 cm. c/c.

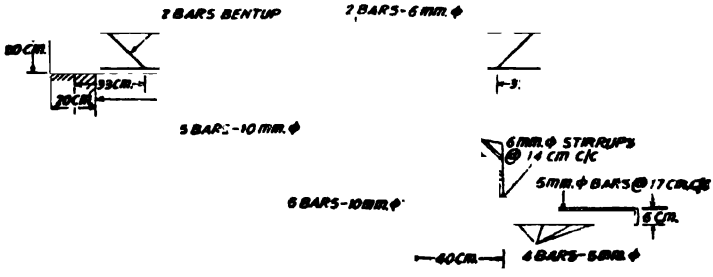

Fig. 143

Design 33. *An R.C. lintel is to be provided over an opening 2 metres wide. The lintel carries a 40 cm. thick wall, the height of the wall above the lintel being 4 metres. Design the lintel with a 0·6 metre wide sun shade. The live load on the sun shade may be taken at 200 kg./m.²*

Solution. Design of sun shade.

This is designed as a cantilever. Let the thickness of the sun-shade be 6 *cm.*

Loads on the sun shade

Dead load=25×6 =150 *kg./metre²*

Live load =200 *kg./metre²*

Total load =w=350 *kg./metre²*

Consider a one *metre* wide strip of the cantilever.

$$\text{Maximum B.M.} = \frac{350 \times 0{\cdot}6^2}{2} \times 100 \text{ kg. cm.}$$

$$= 6300 \text{ kg. cm.}$$

Addopting c=50 *kg./cm.²*, t=1400 *kg./cm.²* and m=18 and equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}50 \times 100d^2 = 6300$$

$$\therefore \quad d = 2{\cdot}72 \text{ cm.}$$

Let 5 *mm.* diameter bars be used at a clear cover of 15 *mm.*

∴ Effective cover to the centre of reinforcement

=15+2·5=17·5 *mm.*=1·75 *cm.*

∴ Effective depth available

=6−1·75=4·25 *cm.*

$$A_t = \frac{6300}{1400 \times 0{\cdot}87 \times 4{\cdot}25} \text{ cm.}^2 = 1{\cdot}22 \text{ cm.}^2$$

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Spacing of 5 *mm.* dia. bars

$$= \frac{0{\cdot}20 \times 100}{1{\cdot}22} \text{ cms.} = 16{\cdot}4 \text{ cm.}$$

But the spacing of main bars should not exceed three times the effective depth.

In our case, three times the effective depth

$$= 3 \times 4{\cdot}25 = 12{\cdot}75 \text{ cm.}$$

Provide 5 mm ϕ stirrups @ *12 cm. c/c.*

Spacing of 5 *mm.* diameter distribution bars

= not greater than five times the effective depth

= not greater than $5 \times 4\ 25 = 21{\cdot}25$ *cm.*

Since the projecting length of the sun shade is only 0·6 *m.*

Provide 4 bars of 5 mm. ϕ @ *as distribution steel.*

Let the lintel be 40 *cm.* wide and 20 *cm.* deep.

Effective depth $= 20 - 3 = 17$ *cm.*

Let the bearing at each end be 20 *cm.*

Effective span. This is taken as the lesser of the following :

(*i*) Distance between centres of bearings $= 2{\cdot}20$ *metre*

(*ii*) Clear span + effective depth $= 2 + 0{\cdot}17 = 2{\cdot}17$ *metre*

$\therefore$ Effective span $= 2{\cdot}17$ *metre*

Height of an equilateral triangle of side 2·17 *metre*

$= 2{\cdot}17 \sin 60°$ $= 1{\cdot}88$ *metre*

Height of wall above the lintel $= 4$ *metre*

Hence only triangular loading of brick masonry will be transferred to the lintel.

Loads on the lintel

Triangular load due to masonry

$$= W = \frac{1}{2} \times 2{\cdot}17 \times 1{\cdot}88 \times 0{\cdot}4 \times 1920 \text{ kg.}$$

$$= 1567 \text{ kg.}$$

Distributed load

Dead load of lintel $= 0{\cdot}4 \times 0{\cdot}2 \times 2500$ $= 200$ *kg./m.*

Load from sun shade $= 350 \times 0{\cdot}6$ $= 210$ *kg./m.*

Total distributed load $w = 410$ *kg./m.*

Max. B.M.

$$= \frac{1567 \times 2{\cdot}17}{6} + \frac{410 \times 2{\cdot}17^2}{8} \text{ kg. m.}$$

$$= 567 + 241 \text{ kg. m.} = 808 \text{ kg. m.}$$

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Equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}50 \times 40d^2 = 80800 \text{ kg. cm.}$$

$$\therefore \quad d = 15{\cdot}41 \text{ cm.}$$

Effective depth available

$$= 20 - 3 = 17 \text{ cm.}$$

$$\therefore \quad A_t = \frac{80800}{1400 \times 0{\cdot}87 \times 17} \text{ cm.}^2$$

$$= 3{\cdot}90 \text{ cm}^2$$

Provide five bars of 10 mm. diameter (3·93 cm.²)

Maximum shear force

$$= \frac{W}{2} + \frac{wL}{2} = \frac{1567}{2} + \frac{410 \times 2}{2} \text{ kg.}$$

$$= 784 + 410 \text{ kg.} = 1194 \text{ kg.}$$

Nominal shear stress

$$= q_v = \frac{S}{bd} = \frac{1194}{40 \times 17} = 1{\cdot}76 \text{ kg./cm.}^2$$

Percentage of tensile steel provided

$$= \frac{3{\cdot}93}{40 \times 17} \times 100 = 0{\cdot}58\%$$

For 0·5% steel $q_c = 2{\cdot}9$ kg./cm.²

$\therefore \quad q_v < q_c$

Hence provide nominal stirrups.

Spacing of nominal stirrups = lesser of the following :

(*i*) $0{\cdot}75d = 0{\cdot}75 \times 17 = 12{\cdot}75$ cm.

(*ii*) 45 cm

Provide two-legged 8 mm. ϕ stirrups @ 12 cm. c/c.

Design 34. *Design an R.C.C. lintel over a 2 m wide window opening (See Fig. 144). The window is provided in 30 cm. thick wall of the room 5 m. wide and 16 m long. Allow a superimposed load of 150 kg/metre² on the slab.*

Solution. Provide a depth of 30 cm. for the lintel,

Providing a bearing of 25 cm. at each end of the lintel,

Effective span of the lintel = 2 + 0·25 = 2·25 m.

Considering the 60° dispersion lines, the altitude of the load triangle = $2{\cdot}25 \sin 60° = 2{\cdot}25 \frac{\sqrt{3}}{2} = 1{\cdot}95$ m.

But the height of the slab above the lintel = 1·70 − 0·3 = 1·40 m. Since the height of the load triangle is greater than the height of the slab above the lintel, the slab load will also be transmitted to the

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lintel. Again considering the 60° dispersion lines above the slab we find that the apex of the equilateral triangle is above the parapet. This means the weight of the parapet also will be transmitted to the lintel. (*see* Fig 144).

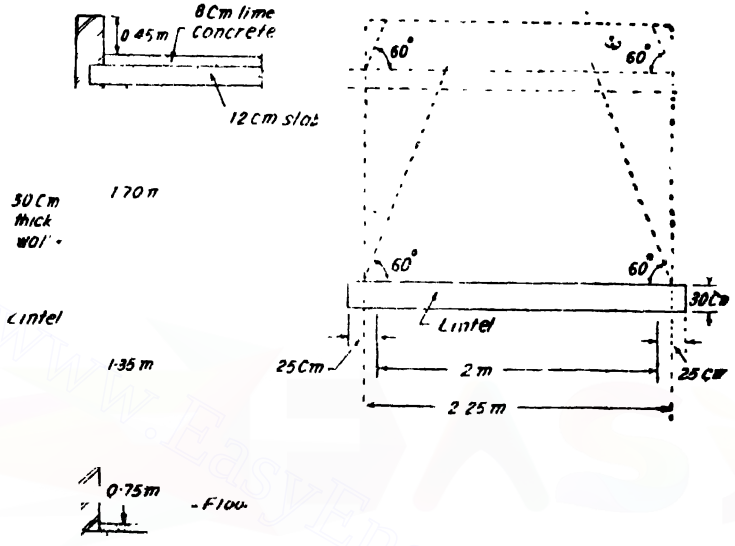

Fig. 144

Now let us estimate the loading on the lintel per *metre* run.

(*i*) *Load due to weight of wall* — *kg/m.*

$= 0{\cdot}3 \times (1{\cdot}40 + 0{\cdot}12 + 0{\cdot}08 + 0{\cdot}45)\ 1900 = 1169$

(*ii*) *Load from slab*

(*i*)	Live load $= \frac{1}{2}\ (5 \times 1)\ 150$	..	375
(*ii*)	D.L. of slab $= \frac{1}{2}\ (5 \times 1 \times 0{\cdot}12 \times 2500)$	..	750
(*iii*)	D.L. of lime concrete $= \frac{1}{2}\ (5 \times 1 \times 0{\cdot}08 \times 1900)$	..	= 380
(*iii*)	*D.L. of lintel* $= 0{\cdot}30 \times 0{\cdot}30 \times 2500$	..	= 225
		Total	= 2899

Maximum bending moment $= M = \dfrac{wl^2}{8}$

$= 2899 \times \dfrac{2{\cdot}25^2}{8} \times 100\ kg.\ cm.$

$= 183450\ kg.\ cm.$

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Equating the M.R. to the bending moment

$$8{\cdot}5 \times 30d^2 = 183450$$

$$d = 26{\cdot}82 \text{ cm.}$$

Providing 12 *mm.* ϕ bars at a clear cover of 25 *mm*, effective cover $= 31 \text{ mm.} = 3{\cdot}10 \text{ cm.}$

Effective depth available $= 30 - 3{\cdot}10 = 26{\cdot}90 \text{ cm.}$

$$\therefore \quad A_s = \frac{183450}{1400 \times 0{\cdot}87 \times 26{\cdot}90} = 5{\cdot}60 \text{ cm.}^2$$

Provide 5 bars of 12 mm. ϕ (5·65 cm.²)

2 bars may be bent up at $\frac{l}{7} = \frac{2{\cdot}25}{7}$

$= 0{\cdot}32$ *m.* say 0·30 *m.* from each end.

Check for shear

Maximum S.F. $= S = \frac{wl}{2} = 2899 \times \frac{2}{2} = 2899 \text{ kg.}$

Nominal shear stress at support

$$= q_v = \frac{S}{bd} = \frac{2899}{30 \times 26{\cdot}9} = 3{\cdot}59 \text{ kg./cm.}^2$$

Percentage of steel available after bending 2 bars

$$= \frac{3 \times 1{\cdot}13}{30 \times 26{\cdot}9} \times 100 = 0{\cdot}42\%$$

As per code for 0·4% steel, $q_c = 2{\cdot}7 \text{ kg./cm.}^2$

and for 0·5% steel, $q_c = 2{\cdot}9 \text{ kg./cm.}^2$

$$\therefore \quad \text{for } 0{\cdot}42\% \text{ steel, } q_c = 2{\cdot}7 + \frac{2{\cdot}9 - 2{\cdot}7}{0{\cdot}1} \times 0{\cdot}02$$

$$= 2{\cdot}74 \text{ kg./cm.}^2$$

Shear capacity of concrete

$$= q_c\, bd = 2{\cdot}74 \times 30 \times 26{\cdot}9 = 2211 \text{ kg.}$$

$\therefore$ Shear to be resisted by stirrups

$$= V_s = 2899 - 2211 = 688 \text{ kg.}$$

Spacing of 2-legged stirrups of 6 *mm.* diameter

$$= \frac{A_w\, t_w\, d}{V_s} = \frac{2 \times 0{\cdot}28 \times 1400 \times 26{\cdot}9}{688}$$

$$= 30{\cdot}6 \text{ cm.}$$

Maximum spacing of stirrups

$$= 0{\cdot}75d = 0{\cdot}75 \times 26{\cdot}9 = 20{\cdot}2 \text{ cm.}$$

Provide 2-legged 6 mm. ϕ stirrups @ 20 cm. c/c.

Fig. 145 shows the details of reinforcement.

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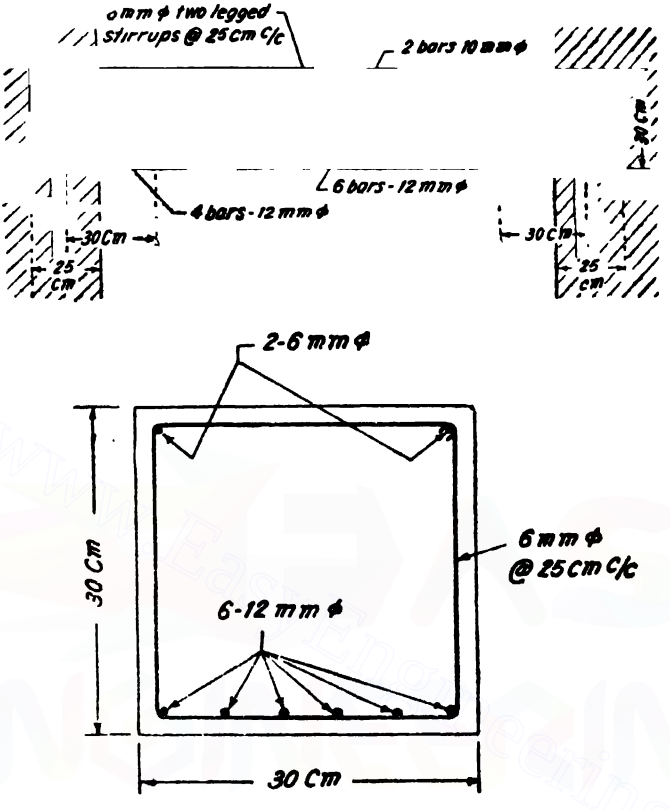

Fig. 145

Design 35 (S.I.). *Design an R.C.C. lintel over a 2 metre wide window opening (See Fig. 144). The window is provided in 30 cm. thick wall of the room 5 m. wide and 16 m. long. Allow a superimposed load of 1500 N/metre² on the slab. Use M 15 concrete.*

Solution. Provide a depth of 30 *cm.* for the lintel.

Providing a bearing of 25 *cm.* at each end of the lintel,

Effective span of the lintel = 2 + 0·25 = 2·25 *metre*

Considering the 60° dispersion lines, the altitude of the load triangle = 2·25 sin 60° = 1·95 *metre.*

But the height of the slab above the lintel = 1·70 − 0·30

= 1·40 *metre.*

Since the height of the load triangle is greater than the height of the slab above the lintel, the slab load will also be transmitted to the lintel. Again considering the 60° dispersion lines above the slab we find that the apex of the equilateral triangle is above the

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parapet. This means the weight of the parapet also will be transmitted to the lintel.

Now let us estimate the loading on the lintel per *metre* run.

		N/metre
(*i*)	*Load due to weight of wall*	
	$=0{\cdot}3\times(1{\cdot}40+0{\cdot}12+0{\cdot}08+0{\cdot}45)\ 19000$	$=11690$
(*ii*)	*Load from slab*	
	(*i*) Live load $=\frac{1}{2}(5\times1)\ 1500$	$=3750$
	(*ii*) D.L. of slab $=\frac{1}{2}(5\times1\times0{\cdot}12\times25000)$	$=7500$
	(*iii*) D.L. of lime concrete $=\frac{1}{2}(5\times1\times0{\cdot}08\times19000)$	$=3800$
(*iii*)	*D.L. of lintel* $=0{\cdot}3\times0{\cdot}3\times25000$	$=2250$
	Total	28990

$$\text{Maximum B.M.} = M = \frac{wl^2}{8} = \frac{28990\times2{\cdot}25^2}{8}\times100$$

$$= 1834500\ N\ cm.$$

Equating the moment of resistance to the maximum bending moment,

$$8\ bd^2 = 85\times30d^2 = 1834500$$

$$\therefore \quad d = 26{\cdot}82\ cm.$$

Providing 12 *mm.* diameter bars at a clear cover of 25 *mm.*, effective cover $= 25+6=31\ mm. = 3{\cdot}1\ cm.$

Effective depth available

$$= 30-3{\cdot}1 = 26{\cdot}90\ cm.$$

$$A_s = \frac{1834500}{14000\times0{\cdot}87\times26{\cdot}90} = 5{\cdot}60\ cm.^2$$

Provide 5 bars of 12 mm. ϕ (5·65 cm.^{2})

2 bars may be bent up at $\frac{l}{7} = \frac{2{\cdot}25}{7}$

$= 0{\cdot}32\ m.$ **say** $0{\cdot}3\ m.$ **from each end.**

Analysis for shear

Maximum $\text{S.F.} = S = \frac{wl}{2} = 28990\times\frac{2}{2} = 28990\ N$

Nominal shear stress at support

$$v = \frac{S}{bd} = \frac{28990}{30\times26{\cdot}9} = 35{\cdot}9\ N/cm.^2$$

$$= 0{\cdot}359\ N/mm.^2$$

Percentage of tensile steel available after bending 2 bars

$$= \frac{3\times1{\cdot}13}{30\times26{\cdot}9}\times100 = 0{\cdot}42\%$$

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As per code, for 0·4% steel, $q_c = 0{\cdot}27\ N/m.^2$

and for 0·5% steel $q_c = 0{\cdot}29\ N/m.^2$

$\therefore$ for 0·42% steel, $q_c = 0{\cdot}27 + \frac{0{\cdot}29 - 0{\cdot}27}{0{\cdot}1} \times 0{\cdot}02$

$= 0{\cdot}274\ N/mm.^2 = 27{\cdot}4\ N/cm.^2$

Shear capacity of concrete

$= q_c\, bd = 27{\cdot}4 \times 30 \times 26{\cdot}9 = 22110\ N$

$\therefore$ Shear to be resisted by stirrups

$= V_s = 28990 - 22110 = 6880\ N$

Spacing of two-legged stirrups of 6 *mm.* diameter

$$= \frac{A_w\, f_w\, d}{V_s} = \frac{2 \times 0{\cdot}28 \times 14000 \times 26{\cdot}9}{6880}$$

$= 30{\cdot}6\ cm.$

Maximum spacing of stirrups

$= 0{\cdot}75d = 0{\cdot}75 \times 26{\cdot}9 = 20{\cdot}2\ cm.$

Provide 2-legged 6 mm. ϕ stirrups @ 20 cm. c/c.

Fig. 145 shows details of reinforcement.

Design 36. (S.I.) *Fig. 146 shows part plan of a 6 metre wide room Design a lintel which shall be continuous over three window openings shown in the figure. The height of 20 cm. wall above the lintel may be taken as 1·25 metre. Provide also 0·6 metre wide sun shade integrally cast with the lintel. The live load on the sun shade is 1800 N/metre^2. Use M 15 concrete.*

Solution. *Design of the sun shade*

Let the thickness of the sun shade be 5 *cm.*

Load on the sun shade

(*i*) D.L. $= 5 \times 250 = 1250\ N/metre^2$

(*ii*) L.L. $= 1800\ N/metre^2$

Total $= 3050\ N/metre^2$

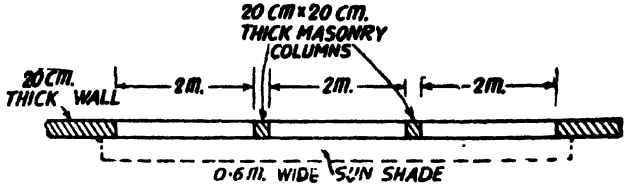

Fig. 146

Consider a one *metre* strip of the sun shade.

Maximum bending moment

$$= \frac{wl^2}{2} = \frac{3050 \times 0{\cdot}6^2}{2} \times 100\ N\ cm. = 54900\ N\ cm.$$

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Equating the moment of resistance to the maximum bending moment,

$$85\ bd^2 = 85 \times 100\ d^2 = 54900 \qquad \therefore\ d = 2{\cdot}54\ cm.$$

Providing 6 *mm.* diameter bars at a clear cover of 15 *mm.*, effective cover $= 15 + 3 = 18\ mm. = 1{\cdot}8\ cm.$

$\therefore$ Effective depth available $= 5 - 1{\cdot}8 = 3{\cdot}20\ cm.$

$$\therefore \qquad A_t = \frac{54900}{14000 \times 0{\cdot}87 \times 3{\cdot}2} = 1{\cdot}41\ cm.^2$$

Spacing of 6 *mm.* diameter bars $= \dfrac{0{\cdot}28 \times 100}{1{\cdot}41} = 19{\cdot}85\ cm.$

Maximum spacing of main bars $= 3 \times$ effective depth

$= 3 \times 3{\cdot}2 = 9{\cdot}6\ cm.$ *say* 10 *cm.*

Provide 6 mm. ϕ bars @ 10 cm. c/c.

Distribution steel : Provide four bars of 6 mm. diameter.

Design of the lintel. Let the lintel be 20 *cm.* $\times$ 18 *cm.*

Since the height of brickwork above the lintel is small, the entire weight of brick work above the lintel will be transferred to the lintel.

The loading on the lintel consists of the following :

Dead load of lintel $= 0{\cdot}20 \times 0{\cdot}18 \times 25000 = 900\ N/metre$

Weight of brick masonry $= 0{\cdot}20 \times 1{\cdot}25 \times 19200 = 4800\ N/metre$

Load from sun shade $= 3050 \times 0{\cdot}6 = 1830\ N/metre$

Total $= 7530\ N/metre$

Maximum bending moment

$$= \frac{wl^2}{10} = \frac{7530 \times 2{\cdot}2^2}{10} \times 100\ N\ cm. = 364450\ N\ cm.$$

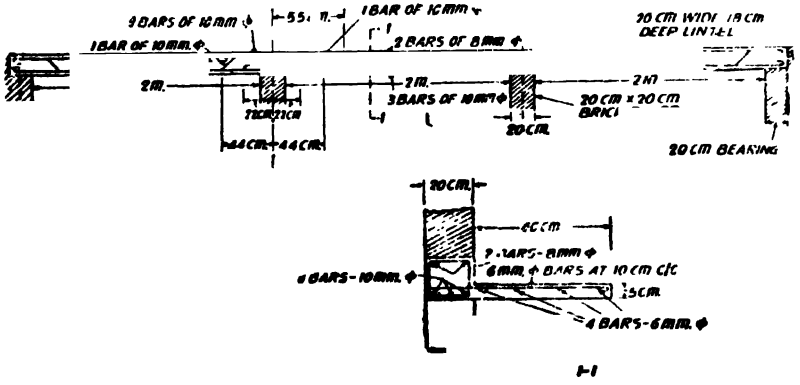

Fig. 147

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Equating the moment of resistance to the maximum bending moment, we have,

$$85\,bd^2 = 85 \times 20\,d^2 = 364450 \qquad \therefore \quad d = 14{\cdot}64\ cm.$$

Let 10 *mm.* diameter bars be provided at a clear cover of 25 *mm.*

$\therefore$ Effective cover to the centre of steel

$$= 25 + 5 = 30\ mm. = 3\ cm.$$

$\therefore$ Effective depth available $= 18 - 3 = 15\ cm.$

$$A_t = \frac{364450}{14000 \times 0{\cdot}87 \times 15} = 1{\cdot}99\ cm.^2$$

Provide 3 bars of 10 mm. ϕ (2·36 cm.^{2})

Maximum shear force for the lintel

$$= S = 0{\cdot}6\ wl = 0{\cdot}6 \times 7530 \times 2 = 9040\ N$$

Nominal shear stress at the support

$$= q_v = \frac{S}{bd} = \frac{9040}{20 \times 15} = 30{\cdot}1\ N/cm.^2 = 0{\cdot}301\ N/mm.^2$$

Percentage of tensile steel after curtailing 1 bar,

$$\frac{2 \times 0{\cdot}79}{20 \times 15} \times 100 = 0{\cdot}53\%$$

As per code, for 0·5% steel, $q_c = 0{\cdot}29\ N/mm^2$

and for 0·6% steel, $q_c = 0{\cdot}31\ N/mm.^2$

$\therefore$ for 0·53% steel, $q_c = 0{\cdot}29 + \frac{0{\cdot}02}{0{\cdot}1} \times 0{\cdot}03 = 0{\cdot}296\ N/mm.^2$

$$= 29{\cdot}6\ N/cm.^2$$

Shear resistance of concrete

$$= q_c\,bd = 29{\cdot}6 \times 20 \times 15 = 8880\ N$$

Shear force to be resisted by stirrups $= V_s = 9040 - 8880 = 160\ N$

This is very small. *Provide 2-legged 8 mm ϕ stirrups @ 11 cm/c.*

Design 37. *Fig. 146 shows part plan of a 6 metre wide room. Design a lintel which shall be continuous over three window openings shown in the figure. The height of 20 cm. wall above the lintel may be taken as 1·25 metres. Provide also 0·6 metre wide sun shade integrally cast with the lintel. The live load on the sun shade is 180 kg./m^2.*

Solution. Design of sun shade

Let the thickness of the sun shade be 5 *cm.*

Load on the sun shade

(*i*) Dead Load	$= 5 \times 25 = 125\ kg./m^2$
(*ii*) Live load	$= 180\ kg./m^2$
Total	$= 305\ kg./m.^2$

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Consider a one *metre* strip of the sun shade

Max. B.M. $= \frac{wl^2}{2} = \frac{305 \times 0.6^2}{2} \times 100$ *kg. cm.* $= 5490$ *kg. cm.*

Adopting $c = 50$ *kg./cm.*^{2}, $t = 1400$ *kg. cm.*^{2}, $m = 18$, and equating the moment of resistance to the maximum bending moment, we have, $8.5 \times 100\, d^2 = 5490$

$\therefore \quad d = 2.54$ *cm.*

Providing 6 *mm.* dia. bars at a clear cover of 13 *mm.* the effective depth available $= 5 - 1.5 - 0.3 = 3.20$ *cm.*

$\therefore$ Steel requirement $= A_t = \frac{5490}{0.87 \times 1400 \times 3.20}$ *cm.*^{2}

$= 1.41$ *cm.*^{2}

$\therefore$ Spacing of 6 *mm.* dia. bars

$= \frac{0.28 \times 100}{1.41} = 19.85$ *cm*

But maximum spacing of main bars

$= 3 \times$ effective depth

$= 3 \times 3.2 = 9.6$ *cm say* 10 *cm.*

Hence, let us provide 6 mm. dia. bars at 10 cm. c/c.

Distribution steel. Provide four bars of 6 mm. dia. as distribution steel.

Design of lintel. Let the lintel be 20 *cm* $\times$ 18 *cm.*

Since the height of brickwork above the lintel is small, the entire weight of brickwork above the lintel will be transferred to the lintel.

The load on the lintel will therefore consist of the following :

Dead load of lintel	$= 0.20 \times 0.18 \times 2500 =$	90 *kg./m*
Weight of brick masonry	$= 0.20 \times 1.25 \times 1920 =$	480 *kg /m*
Load from sun shade	$= 305 \times 0.6$	$= 183$ *kg./m*
	Total load	$= 753$ *kg./m*

Max. B.M. $= \frac{wl^2}{10} = \frac{753 \times (2 + 0.2)^2}{10}$ *kg. m*

$= 364$ *kg. m* $= 36400$ *kg. cm.*

Equating the moment of resistance to the maximum bending moment, we have,

$8.50 \times 20\, d^2 = 36400$

$\therefore \quad d = 14.64$ *cm.*

Let 10 *mm.* dia. bars be used at a clear cover of 25 *mm.*

$\therefore$ Effective cover to the centre of steel

$= 2.5 + 0.5 = 3$ *cm.*

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$\therefore$ Effective depth available $= 18 - 3 = 15$ *cm.*

$$A_t = \frac{36400}{1400 \times 0{\cdot}87 \times 15} = 1{\cdot}99 \text{ cm.}^2$$

Provide 3 bars of 10 mm. ϕ (2·36 cm.^{2})

Maximum shear force for the lintel

$$= S = 0{\cdot}6\ wl = 0{\cdot}6 \times 753 \times 2 = 904 \text{ kg.}$$

Nominal shear stress at the support

$$= q_v = \frac{S}{bd} = \frac{904}{20 \times 15} = 3{\cdot}01 \text{ kg./cm.}^2$$

Percentage of tensile steel after curtailing 1 bar

$$= \frac{2 \times 0{\cdot}79}{20 \times 15} = 0{\cdot}53\%$$

As per code, for 0·5% steel, $q_c = 2{\cdot}90$ *kg./cm.*^{2}

and for 0·6% steel, $q_c = 3{\cdot}10$ *kg./cm.*^{2}

$\therefore$ for 0·53% steel, $q_c = 2{\cdot}9 + \frac{0{\cdot}2}{0{\cdot}1} \times 0{\cdot}03 = 2{\cdot}96$ *kg./cm.*^{2}

$\therefore$ Shear resistance of concrete

$$= q_c\ bd = 2{\cdot}96 \times 20 \times 15 = 888 \text{ kg.}$$

$\therefore$ S.F. to be resisted by stirrups $= V_s = 904 - 888 = 16$ *kg.*

This is very small.

Provide 2-legged 8 mm ϕ stirrups @ 11 cm. c/c.

Design 38 (S.I.) (Canopy). *An R.C. cantilevered canopy 2·85 metres effective overhang and 6 metres wide is supported on two beams cantilevered out from walls and spaced at 4 metres centres so that the slab overhangs each beam by 1 metre on each side. If the ribs of the beams are to be integrated with the slab so as to give a flat soffit, design the slab and beams and draw details of reinforcement in beams and slab. Live load on the slab may be taken as 1400 N/metres^2. Use M 15 concrete.*

Solution. *Design of the slab*

The loading on the slab consists of the following :

D.L. of slab (10 *cm.*) : $250 \times 10 = 2500$ *N/metre*^{2}

L.L. on the slab $= 1400$ *N/metre*^{2}

Total load $w = 3900$ *N/metre*^{2}

Consider a one *metre* wide strip of the slab.

End reaction $= 3900 \times \frac{6}{2} N = 11700\ N$

$\therefore$ *B.M. at centre* $= 11700 \times 2 - 3900 \times \frac{3^2}{2}$

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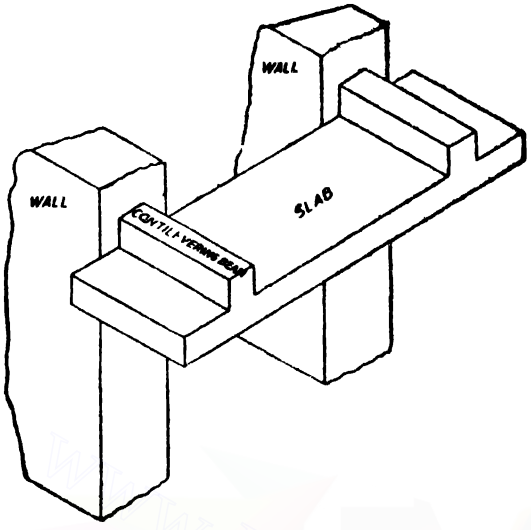

Fig. 148

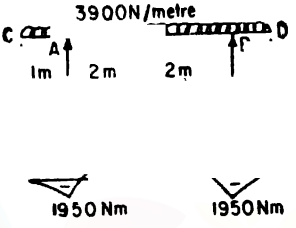

Fig. 149

$= 23400 - 17550 = 5850\ Nm = 585000\ N\ cm.$

B.M. over the support $= 3900 \times \frac{1^2}{2} Nm$

$= 1950\ Nm = 195000\ N\ cm.$

Points of contraflexure

Let at a distance x *metres* from A the bending moment be zero. Equating the bending moment to zero, we have,

$$11700\ x - 3900\ \frac{(x+1)^2}{2} = 0$$

$x^2 - 4x + 1 = 0 \quad \therefore \quad x = 0{\cdot}27\ m$ and $3{\cdot}73\ m$.

$\therefore$ The points of contraflexure are $0{\cdot}27\ m$ from the nearer support.

Adopting $c = 5\ N/mm^2 = 500\ N/cm.^2$; $t = 140\ N/mm.^2 = 14000\ N/cm.^2$ and $m = 18$, and equating the moment of resistance to the maximum bending moment, we have,

$$85\ bd^2 = 85 \times 100\ d^2 = 585000 \quad \therefore\ d = 8{\cdot}3\ cm.$$

Providing 8 *mm.* diameter bars at a clear cover of 15 *mm.*, effective cover to the centre of steel $= 15 + 4 = 19\ mm. = 1{\cdot}90\ cm.$

Effective depth available

$$= 10 - 1{\cdot}9 = 8{\cdot}1\ cm.$$

Steel for maximum sagging moment

$$= \frac{585000}{14000 \times 0{\cdot}87 \times 8{\cdot}1} = 5{\cdot}9\ cm.^2$$

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Spacing of 8 *mm.* ϕ bars

$$= \frac{0{\cdot}5 \times 100}{5{\cdot}9} = 8{\cdot}5 \text{ cm.}$$

Steel for maximum hogging bending moment

$$= \frac{195000}{14000 \times 0{\cdot}87 \times 8{\cdot}1} = 1{\cdot}98 \text{ cm.}^2$$

Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{1{\cdot}98} = 25{\cdot}3 \text{ cm.}$$

It will be convenient to bend up alternate bottom bars.

Design of cantilevers. Since the soffit is given to be flat, the cantilevers must be designed as inverted *T*-beams. But the flanges being in compression, the beams have *T*-beam action.

Consider the loading on one beam.

Let the depth be 40 *cm.* overall. Let the width of the rib be 20 *cm.*

Load from the slab	$= 3900 \times \frac{6}{2}$	$= 11700$ *N/metre*
D L. of rib	$= 0{\cdot}30 \times 0{\cdot}20 \times 25000$	$= 1500$ *N/metre*
	Total	$= 13200$ *N/metre*

Length of cantilever $= 2{\cdot}85$ *metre*

Maximum bending moment

$$= \frac{13200 \times 2{\cdot}85^2}{2} \times 100 \text{ N cm.}$$

$$= 5360900 \text{ N cm.}$$

Let 20 *mm.* diameter bars be provided at a clear cover of 25 *mm.*

Effective cover to the centre of steel

$$= 25 + 10 = 35 \text{ mm.} = 3{\cdot}5 \text{ cm.}$$

$\therefore$ Effective depth $= d = 40 - 3{\cdot}5 = 36{\cdot}5$ *cm.*

Approximate lever arm

$$= d - \frac{d_s}{2} = 36{\cdot}5 - 5{\cdot}0 = 31{\cdot}5 \text{ cm.}$$

Approximate steel requirement

$$= \frac{5360900}{14000 \times 31{\cdot}5} = 12{\cdot}16 \text{ cm.}^2$$

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Provide 4 bars of 20 mm. ϕ (12·57 cm.²)

Width of flange. This will be taken at the least of the following :

(*i*) $\frac{2}{3}$ length of cantilever $= \frac{2}{3} \times 2{\cdot}85 \times 100 = 190$ *cm.*

(*ii*) Cantilever projection of slab + half the spacing of beams

$= 1 + 2 = 3$ *m* $= 300$ *cm.*

(*iii*) $12d_s + b_r = (12 \times 10) + 20 = 140$ *cm.*

Assuming the neutral axis to lie outside the flange and taking moments about the neutral axis, we have,

$$140 \times 10(n - 5) = 18 \times 12\ 57(36{\cdot}5 - n)$$

Solving, we get $n = 9{\cdot}38$ *cm.*

But, the thickness of the slab $= 10$ *cm.*

∴ Neutral axis is within the flange.

Taking moments about the neutral axis,

$$140\ \frac{n^2}{2} = 18 \times 12{\cdot}57(36{\cdot}5 - n)$$

Solving, we get,

$n = 9{\cdot}36$ *cm.*

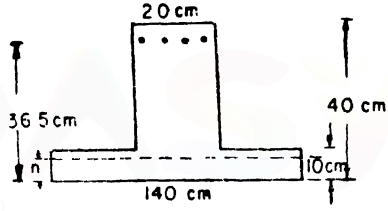

Fig. 150

∴ Actual lever arm

$$= a = d - \frac{n}{3} = 36{\cdot}5 - \frac{9{\cdot}36}{3} = 33{\cdot}38 \text{ cm.}$$

Actual stress in steel

$$= \frac{5360900}{12{\cdot}57 \times 33{\cdot}38} = 12780 \text{ N/cm.}^2$$

$$= 127{\cdot}8 \text{ N/mm.}^2 \text{ (less than 140 N/mm.}^2\text{)}$$

Compressive stress in concrete

$$= \frac{12780}{18} \times \frac{9{\cdot}36}{36{\cdot}5 - 9{\cdot}36} = 250 \text{ N/cm.}^2$$

$$= 2{\cdot}50 \text{ N/mm.}^2$$

Analysis for shear

Maximum S.F. $= S = 13200 \times 2{\cdot}85 = 37620$ *N*

Nominal shear stress

$$= q_v = \frac{S}{b_r d} = \frac{37620}{20 \times 36{\cdot}5} \quad 51{\cdot}5 \text{ N/cm.}^2$$

$$= 0{\cdot}515 \text{ N/mm.}^2$$

Percentage of tensile steel provided

$$= \frac{12{\cdot}57}{20 \times 36{\cdot}5} \times 100 = 1{\cdot}72\%$$

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As per code, for 1·7% steel, $q_c = 0{\cdot}44\ N/mm.^2$

and for 1·8% steel, $q_c = 0{\cdot}44\ N/mm.^2$

$\therefore$ for 1·72% steel, $q_c = 0{\cdot}44\ N/mm.^2 = 44\ N/cm.^2$

Shear resistance of concrete

$$= q_c\, b_r d = 44 \times 20 \times 36{\cdot}5 = 32120\ N$$

$\therefore$ Shear resistance to be provided by the stirrups

$$= V_s = 37620 - 32{\cdot}20 = 5500\ N$$

Spacing of two-legged 8 *mm.* diameter stirrups

$$= \frac{A_w\, f_w\, d}{V_s} = \frac{(2 \times 0{\cdot}5)14000 \times 36{\cdot}5}{5500}$$

$$= 93\ cm.$$

Maximum permissible spacing

$$= 0{\cdot}75d = 0{\cdot}75 \times 36{\cdot}5 = 27{\cdot}4\ cm.$$

Provide 2-legged 8 mm. ϕ stirrups @ 25 cm. c/c.

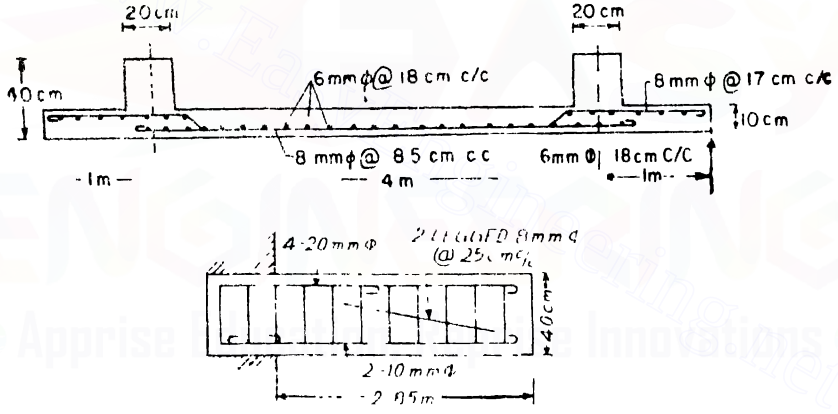

Fig. 151

Curtailment of reinforcement. Let at x *metres* from the free end, two bars be curtailed.

$$\frac{A_{t1}}{A_{t2}} = \frac{1}{2} = \frac{x^2}{2{\cdot}85^2}$$

$\therefore$ $x = 2$ *m.* from the free end or 0·85 *m.* from the support.

As per code, the reinforcement shall extend beyond the theoretical cutoff point for a distance equal to the effective depth of the member or 12 times the diameter whichever is greater.

In our case, effective depth $= 36{\cdot}5\ cm.$

12 diameters $= 12 \times 2 = 24\ cm.$

Greater of the two $= 36{\cdot}5\ cm.$

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Hence, the two bars can be curtailed at a distance of $0{\cdot}85+0{\cdot}365=1{\cdot}215$ *m.* say $1{\cdot}25$ *m.* from the fixed end.

Design 39. (Canopy). *An R.C. cantilevered canopy 2 85 metres effective overhang and 6 metres wide is supported on two beams cantilevered out from walls and spaced at 4 metres centre to centre so that the slab overhangs each beam by 1 metre on each side. If the ribs of the beams are to be integrated with the slab so as to give a flat soffit, design the slab and beams and draw details of reinforcements in beams and slabs. Live load on the slab may be taken as 140 kg./m.²*

Solution. Design of canopy slab

The loading on the slab consists of the following :

Dead load (10 *cm.* thick slab) $= 25\times 10 = 250$ *kg./m.²*

Live load $= 140$ *kg./m.²*

Total load $= w = 390$ *kg./m.²*

Consider a one *metre* wide strip of the slab.

Each reaction

$$= 390\times\frac{6}{2} = 1170 \text{ kg.}$$

$\therefore$ *B.M. at centre*

$= 1170\times 2 - 1170\times 1{\cdot}5 \text{ kg.m}$

$= 585$ *kg. m.*

$= 58500$ *kg. cm.*

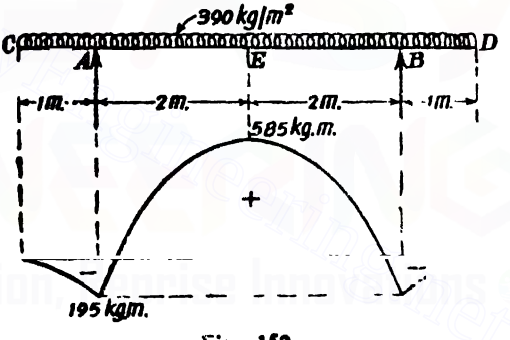

Fig. 152

Fig. 152 shows the loading on the slab.

$$B.M. \text{ over supports} = \frac{390\times 1^2}{2} \text{ kg. m.} = 195 \text{ kg. m.}$$

$$= 19500 \text{ kg. cm.}$$

Points of Contraflexure. Let at a distance of x *metres* from A the bending moment be zero. Equating the bending moment to zero, we have,

$$1170x - 390\,\frac{(x+1)^2}{2} = 0$$

$$\therefore \quad x^2 - 4x + 1 = 0$$

$$\therefore \quad x = 0{\cdot}27 \text{ m. or } 3{\cdot}73 \text{ m.}$$

$\therefore$ Each point of contraflexure is at $0{\cdot}27$ *m.* from the support.

Adopting $c = 50$ *kg./cm.²*, $t = 1400$ *kg./cm.²* and $m = 18$ and

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equating the moment of resistance to the maximum bending moment. we have,

$$8{\cdot}50 \times 100\, d^2 = 58500$$

$$\therefore \quad d = 8{\cdot}3 \text{ cm.}$$

Using 8 *mm.* diameter bars at a clear cover of 15 *mm.*, effective cover to the centre of steel

$$1{\cdot}5 + 0{\cdot}40 = 1{\cdot}90 \text{ cm.}$$

Effective depth available

$$= 10 - 1{\cdot}9 = 8{\cdot}1 \text{ cm.}$$

Steel for maximum sagging moment

$$= \frac{58500}{1400 \times 0{\cdot}87 \times 8{\cdot}1} \text{ cm.}^2 = 5{\cdot}9 \text{ cm.}^2$$

$\therefore$ Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{5{\cdot}90} \text{ cm.} = 8{\cdot}5 \text{ cm. c/c}$$

Steel for maximum hogging moment

$$= \frac{19500}{1400 \times 0{\cdot}87 \times 8{\cdot}1} \text{ cm.}^2 = 1{\cdot}98 \text{ cm.}^2$$

$\therefore$ Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{1{\cdot}98} = 25{\cdot}3 \text{ cms.}$$

It will be convenient to bend up alternate bottom bars.

Design of Cantilevers. Since the soffit is given to be flat, the cantilevers must be designed as inverted *T*-beams. But the flanges being in compression, the beams have *T*-beam action.

Consider the loading on one beam.

Let the depth be 40 *cm.* overall. Let the width of the rib be 20 *cm.*

$$\text{Load from the slab} = 390 \times \frac{2}{6} = 1170 \text{ kg./m.}$$

$$\text{Dead load of rib} = 0{\cdot}30 \times 0{\cdot}20 \times 2500 = \frac{150 \text{ kg./m.}}{1320 \text{ kg./m.}}$$

Length of cantilever = 2·85 *metre*

$\therefore$ Maximum bending moment

$$= \frac{1320 \times 2{\cdot}85^2}{2} \times 100 \text{ kg. cm.}$$

$$= 536090 \text{ kg. cm.}$$

Let 20 *mm.* diameter bars be provided at a clear cover of 25 *mm.*

$\therefore$ Effective cover to the centre of steel

$$= 20{\cdot}5 + 1{\cdot}00 = 3{\cdot}50 \text{ cm.}$$

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$\therefore$ Effective depth $= 40 - 3{\cdot}5 = 36{\cdot}5$ *cm.*

Approximate lever arm

$$= d - \frac{d_s}{2}$$

$$= 36{\cdot}5 - 5{\cdot}0 = 31{\cdot}5 \text{ cm.}$$

$\therefore$ Approximate steel requirement

$$= \frac{536090}{1400 \times 31{\cdot}5} \text{ cm.}^2$$

$$= 12{\cdot}16 \text{ cm.}^2$$

Let us provide 4 bars of 20 *mm.* diameter ($12{\cdot}57$ *cm.*^{2})

Width of flange. This will be taken at the least of the following :

(*i*) $\frac{2}{3}$ length of cantilever

$= \frac{2}{3} \times 2{\cdot}85 \times 100 = 190$ *cm.*

(*ii*) Cantilever projection of slab + half the spacing of beams

$= 1 + 2 = 3\ m = 300$ *cms.*

(*iii*) $12 d_s + b_r$

$= 12 \times 10 + 20 = 140$ *cms.*

$\therefore$ **Width of flange = 140** *cms.*

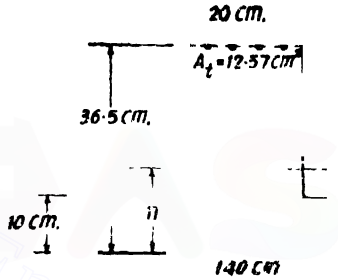

Fig. 153

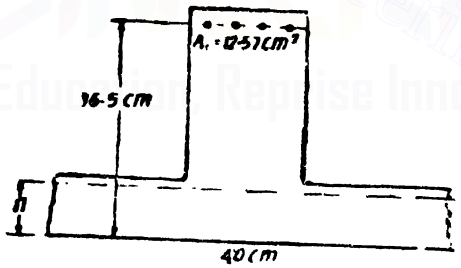

Fig. 154

Let us assume that the neutral axis is outside the flan Taking moments about the neutral axis, we have,

$$140 \times 10(n - 5) = 18 \times 12{\cdot}57(36{\cdot}5 - n)$$

$$n = 9{\cdot}38 \text{ cm.}$$

But, the thickness of slab = 10 *cm.*

$\therefore$ Neutral axis is within the flange.

Taking moments about the neutral axis, we have

$140 \frac{n^2}{2} = 18 \times 12{\cdot}57\ (36{\cdot}5 - n)$. Solving we get, $n = 9{\cdot}36$ *cm.*

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Actual lever arm

$$= a - d - \frac{n}{3} = 36{\cdot}50 - \frac{9{\cdot}36}{3} = 33{\cdot}38 \text{ cm.}$$

Actual stress in steel

$$\frac{536090}{12{\cdot}57 \times 33{\cdot}38} = 1278 \text{ kg./cm.}^2$$

Compressive stress in concrete

$$= \frac{1278}{18} \times \frac{9{\cdot}36}{36{\cdot}5 - 9{\cdot}36} = 25 \text{ kg./cm.}^2$$

Analysis for shear

Maximum S.F. $= S = 1320 \times 2{\cdot}85 = 3762$ kg.

Nominal shear stress $= q_v = \frac{S}{b_r d} = \frac{3762}{20 \times 36{\cdot}5} = 5{\cdot}15$ kg./cm.^{2}

Percentage of tensile steel provided

$$= \frac{12{\cdot}57}{20 \times 36{\cdot}5} \times 100 = 1{\cdot}72\%$$

As per code, for 1·70% steel, $q_c = 4{\cdot}4$ kg /cm.^{2}

and for 1·8% steel, $q_c = 4{\cdot}4$ kg./cm.^{2}

$\therefore$ for 1·72% steel, $q_c = 4{\cdot}4$ kg./cm.^{2}

Shear resistance of concrete,

$$= q_c b_r d$$
$$= 4{\cdot}4 \times 20 \times 36{\cdot}5 = 3212 \text{ kg.}$$

$\therefore$ Shear resistance to be provided by the stirrups

$$= V_s = 3762 - 3212 = 550 \text{ kg.}$$

Spacing of two-legged 8 *mm.* diameter stirrups

$$= \frac{A_{sv} f_v d}{V_s} = \frac{(2 \times 0{\cdot}5)\ 1400 \times 30{\cdot}5}{550} = 93 \text{ cm.}$$

Maximum permissible spacing

$$= 0{\cdot}75\ d.$$
$$= 0{\cdot}75 \times 36{\cdot}5 = 27{\cdot}4 \text{ cm.}$$

Provide 2-legged 8 mm. ϕ stirrups @ 25 cm. c/c.

Curtailment of reinforcement. Let at x *metres* from the free end two bars be curtailed,

$$\frac{A_{s1}}{A_{s2}} = \frac{1}{2} \quad \frac{x^2}{2{\cdot}85}$$

$\therefore$ $x = 2$ *m* from the free end or 0·85 *m* from the support.

As per code, the reinforcement shall extend beyond the theoretical cut off point for a distance equal to the effective depth of the member or 12 times the diameter whichever is greater.

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In our case, Effective depth = 36·5 *cm.*
12 diameters = 12 × 2 = 24 *cm.*
Greater of the two = 36·5 *cm.*

Hence the two bars can be curtailed at a distance of 0·85 *m.* + 0·365 *m.* = 1·215 *m.* say 1·25 *m.* from the fixed end.

§129. Beams Subjected to Torsion

The I.S. code has made the following recommendations in the design of an R C. member subjected to torsion. (See clause 48)

General. In general, where the torsional resistance or stiffness of members has not been taken into account, in the analysis of the structure, no specific calculations for torsion will be necessary; adequate control of any torsional cracking being provided by the required nominal shear reinforcement. Where, the torsional resistance or stiffness of members is taken into account, in the analysis, the members will be designed for torsion

The approach to design in this clause for torsion is as follows :

Torsional reinforcement is not calculated separately from that required for bending and shear Instead, the total longitudinal reinforcement is determined for a fictitious bending moment which is a function of actual bending moment and torsion. Similarly, web reinforcement is determined for a fictitious shear which is a function of actual shear and torsion.

1. The design rules laid down in paragraphs below shall apply to beams of solid rectangular cross-section. However, these clauses may also be applied to flanged beams by substituting b_f for b, in which case they are generally conservative ; therefore specialist literature may be referred to.

2. *Shear and torsion*

(*a*) *Equivalent shear.* Equivalent shear S_e shall be calculated from the formula,

$$S_e = S + 1{\cdot}6\ \frac{T}{b}$$

where, S_e = Equivalent shear,
S = Shear,
T = Torsional moment, and
b = breadth of beam.

The equivalent nominal shear stress q_{ve} is given by

$$q_{ve} = \frac{S_e}{bd}$$

The equivalent nominal shear stress q_{ve} shall not exceed $q_{c\,max}$ given in the tables earlier.

If the equivalent nominal shear stress q_{ve} does not exceed q_c given earlier, minimum shear reinforcement shall be provided.

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(*b*) *Reinforcement in members subjected to torsion.* The reinforcement for torsion, when required, shall consist of longitudinal and transverse reinforcement.

Longitudinal Reinforcement. The longitudinal reinforcement shall be designed to resist an equivalent bending moment M_{e1} given by

$$M_{e1} = M + M_t$$

where, M = bending moment at the cross-section, and

$$M_t = T \frac{\left(1 + \frac{D}{b}\right)}{1{\cdot}7}$$

where T is the torsional moment, D is the overall depth of the beam and b is the breadth of the beam.

If the numerical value of M_t exceeds the numerical value of the moment M, longitudinal reinforcement shall be provided on the flexural compression face, such that the beam can also withstand an equivalent moment $M_{e2} = M_t - M$, the moment M_2 being taken as acting in the opposite sense to the moment M.

Transverse Reinforcement. Two-legged closed hoops enclosing the corner longitudinal bars shall have an area of cross-section A_v given by

$$A_v = \frac{T\,p}{b_1\,d_1\,t_w} + \frac{S p}{2{\cdot}5\,d_1\,t_w}$$

but, the total transverse reinforcement shall not be less than

$$\frac{(q_{ve} - q_c)\,bp}{t_w}$$

where,

T = torsional moment,

S = shear force,

p = spacing of the stirrup reinforcement,

b_1 = centre to centre distance between corner bars in the direction of the width,

d_1 = centre to centre distance between corner bars in the direction of the depth,

b = breadth of the member,

t_w = permissible tensile stress in shear reinforcement,

q_{ve} = equivalent shear stress,

q_c = shear strength of concrete,

Design 40 (S.I.). *A rectangular beam 45 cm. wide is subjected to a bending moment of 50 kNm, a shear force of 30 kN and a torsion of 25 kNm. Design the section. Use M 15 concrete.*

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Solution. *Design of the beam section for B.M.*

Adopting $c=5\ N/mm.^2=500\ N/cm.^2$, $t=140\ N/mm.^2=14000\ N/cm.^2$ and $m=18$, and equating the moment of resistance to the bending moment, we have,

$$85\ bd^2=85\times 45\ d^2=50\times 1000\times 100 \quad \therefore\ d=36{\cdot}2\ cm.$$

Providing 18 *mm.* bars at a clear cover of 25 *mm.*, effective cover $=25+9=34\ mm=3\,4\ cm.$

$$\therefore \quad D=36\,2+3{\cdot}4=39{\cdot}6\ cm.\ \text{say}\ 40\ cm.$$

Actual effective depth $=40-3\,4=36{\cdot}6\ cm.$

Equivalent shear force

$$S_e=S+1{\cdot}6\frac{T}{}$$

$$=30{,}000+1{\cdot}6\times\frac{25\times 1000\times 100}{45}$$

$$=30{,}000+88890=118890\ N$$

$\therefore$ Equivalent nominal shear stress

$$=q_{ve}=\frac{S_e}{bd}=\frac{118890}{45\times 36\,6}\ 72{\cdot}2\ N/cm.^2$$

$$=0{\cdot}722\ N/mm.^2$$

For M 15 concrete $q_{c\,max}=1{\cdot}6\ N/mm.^2$

Longitudinal reinforcement

Equivalent B.M. $=M_{e1}=M+M_t$

where, $$M_t=T\frac{\left[1+\frac{D}{b}\right]}{1{\cdot}7}$$

$$=25000\times\frac{\left[1+\frac{40}{45}\right]}{1{\cdot}7}=27780\ Nm.$$

$$\therefore \quad M_{e1}=50{,}000+27780=77780\ Nm.$$

$$=77710\times 100\ N\ cm.$$

$$\therefore \quad A_t=\frac{77780\times 100}{14000\times 0{\cdot}87\times 36{\cdot}6}=17{\cdot}45\ cm.^2$$

Provide 8 bars of 18 mm. ϕ (20 36 cm.^{2})

Transverse reinforcement

Fig. 155 shows the section of the beam. 8 bars of 18 *mm.* diameter are provided at bottom and 2 bars of 18 *mm.* ϕ are provided at top to support the stirrups.

Distance between centres of corner bars parallel to the width $=b_1=45-2\times 3{\cdot}4$

$=38{\cdot}2\ cm.$

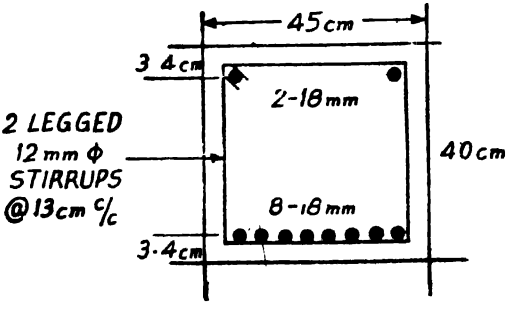

Fig. 155

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Distance between centres of corner bars parallel to the depth

$$= d_1 = 40 - 2 \times 3{\cdot}4 = 33{\cdot}2 \text{ cm.}$$

Area of the section of stirrups

$$= A_w = \frac{T\,p}{b_1\,d_1\,t_w} + \frac{S\,p}{2{\cdot}5\,d_1\,t_w}$$

Let 12 *mm.* ϕ 2 legged stirrups be provided.

$$\therefore \quad A_w = 2 \times 1{\cdot}13 = 2{\cdot}26 \text{ cm.}^2$$

$$2{\cdot}26 = \left[\frac{25 \times 1000 \times 100}{38{\cdot}2 \times 33{\cdot}2 \times 14000} + \frac{30{,}000}{2{\cdot}5 \times 33{\cdot}2 \times 14000}\right] p$$

$$\therefore \quad 2{\cdot}26 = [0{\cdot}1408 + 0{\cdot}0258]\, p$$

$$\therefore \quad p = \frac{2{\cdot}26}{0{\cdot}1666} = 13\ 56 \text{ cm.}$$

Provide 12 mm. ϕ *2 legged stirrups at 13 cm. c/c.*

Design 41. *A rectangular beam 45 cm. wide is subjected to a bending moment 5000 kg. m., a shear force of 3000 kg., and a torsion of 2500 kg. m. Design the section. Use M 150 concrete.*

Solution. *Design of the beam section for B M.*

Adopting $c = 50$ *kg./cm.*^{2}, $t = 1400$ *kg./cm.*^{2} and $m = 18$, and equating the moment of resistance to the bending moment, we have,

$$8{\cdot}5\ bd^2 = 8{\cdot}5 \times 45\ d^2 = 5000 \times 100 \qquad \therefore \quad d = 36{\cdot}20 \text{ cm.}$$

Providing 18 *mm.* diameter bars at a clear cover of 25 *mm.*, effective cover

$$= 25 + 9 = 34 \text{ mm.} = 3{\cdot}4 \text{ cm.}$$

$$\therefore \quad D = 36{\cdot}2 + 3{\cdot}4 = 39{\cdot}6 \text{ cm. say } 40 \text{ cm.}$$

Actual effective depth $= d = 40 - 3{\cdot}4 = 36{\cdot}6$ *cm.*

Equivalent shear force $= S_e = S + 1{\cdot}6\,\frac{T}{b}$

$$= 3000 + \frac{1{\cdot}6\,(2500 \times 100)}{45} = 11890 \text{ kg.}$$

Equivalent nominal shear stress

$$= q_{ve} = \frac{S_e}{bd} = \frac{11890}{45 \times 36{\cdot}6} = 7{\cdot}22 \text{ kg./cm.}^2$$

For M 150 concrete, $q_{c\ max} = 16$ *kg./cm.*^{2}

Longitudinal reinforcement

Equivalent B.M. $= M_{e1} = M + M_t$

where

$$M_t = T\,\frac{\left[1 + \frac{D}{b}\right]}{1{\cdot}7}$$

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$$= 2500 \times \frac{1 + \frac{40}{45}}{17} = 2780 \text{ kg. m.}$$

$\therefore M_{t1} = 5000 + 2780 = 7780 \text{ kg. m.} = 778000 \text{ kg. cm.}$

$$\therefore A_t = \frac{77800}{1400 \times 0.87 \times 36.6} = 17.45 \text{ cm.}^2$$

Provide 8 bars of 18 mm. ϕ (20.36 cm.^{2})

Transverse reinforcement

Fig. 156 shows the section of the beam. 8 bars of 18 *mm.* ϕ are provided at bottom and 2 bars of 18 *mm* ϕ are provided at top to support the stirrups.

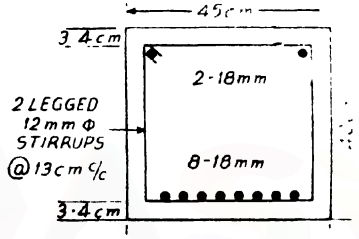

Fig. 156

Distance between centres of corner bars parallel to the width

$= b_1 = 45 - 2 \times 3.4$

$= 38.2 \text{ cm.}$

Distance between centres of corner bars parallel to the depth

$$= d_1 = 40 - 2 \times 3.4 = 33.2 \text{ cm.}$$

Area of the section of stirrups

$$= A_w = \frac{T p}{b_1 d_1 t_w} + \frac{S p}{2.5 d_1 t_w}$$

Let two-legged 12 *mm.* ϕ stirrups be provided.

$\therefore A_w = 2 \times 1.13 = 2.26 \text{ cm}^2$

$$2.26 = \left[\frac{2500 \times 100}{38.2 \times 33.2 \times 1400} + \frac{3000}{2.5 \times 33.2 \times 1400} \right] p$$

$$2.26 = (0.1408 + 0.0258)\, p$$

$$p = \frac{2.26}{0.1666} = 13.56 \text{ cm}$$

Provide 12 mm. ϕ two-legged stirrups @ 13 cm. c/c.

Design 42. *Design a canopy beam and slab over a 4.50 m. wide opening as per details shown in Fig. 157. The live load on the canopy may be taken to be 75 kg./metre^2. Use 150 concrete.*

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Solution.

Thickness of the slab

$= \frac{1}{12}$ cantilevering length

$= \frac{175}{12} = 14{\cdot}6$ *cm. say* 15 *cm.*

The thickness may be reduced to 10 *cm.* near the end.

Loads

D.L. $= 25\left[\frac{15+10}{2}\right] = 313$

Finishing and plastering 40

L.L. = ... 75

Total 428 $kg./m^2$

Considering a 1 *metre* wide strip of the canopy slab, maximum bending moment

$= M = \frac{wl^2}{2}$

$= \frac{428 \times 1{\cdot}75^2}{2} \times 100$ *kg. cm.*

$= 65540$ *kg. cm.*

Equating the moment of resistance to the maximum bending moment,

$8{\cdot}5\ bd^2 = 8{\cdot}5 \times 100\ d^2 = 65540$

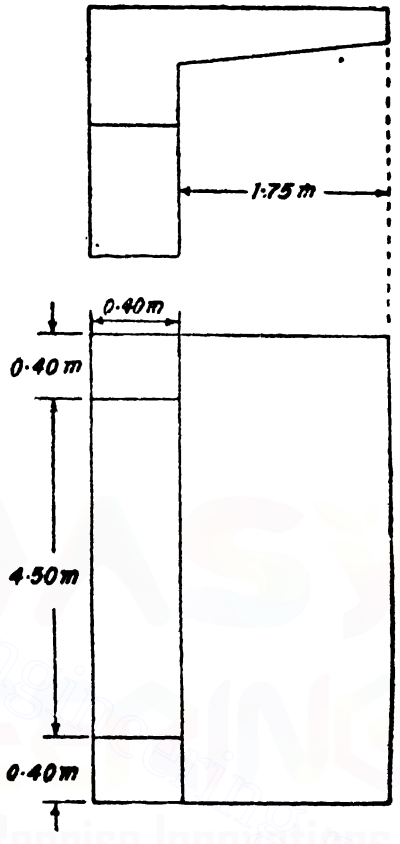

Fig. 157.

$\therefore\ d = 8{\cdot}78$ *cm.*

Providing 10 *mm.* diameter bars at a clear cover of 15 *mm.*, effective cover $= 15 + 5 = 20$ *mm.* $= 2$ *cm.*

Actual effective depth available $= 15 - 2 = 13$ *cm.*

$\therefore\quad A_t = \frac{65540}{1400 \times 0{\cdot}87 \times 13} = 4{\cdot}14$ *cm.*^{2}

Spacing of 10 *mm.* diameter bars $= \frac{0{\cdot}79 \times 100}{4{\cdot}14} = 19{\cdot}08$ *cm.*

Provide 10 mm. ϕ bars @ 18 cm. c/c.

Distribution steel $= \frac{0{\cdot}15}{100} \times 15 \times 100 = 2{\cdot}25$ *cm.*^{2}

Spacing of 8 *mm.* diameter bars $= \frac{0{\cdot}5 \times 100}{2{\cdot}25} = 22{\cdot}2$ *cm.*

Provide 8 mm. ϕ bars @ 20 cm. c/c.

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Design of the canopy beam

The canopy beam is an isolated L beam. The beam should be designed for bending moment as well as torsion.

Let us provide 40 *cm.* × 40 *cm.*

Effective span = 4·50 + 0·40 = 4·90 *metre*

Flange width of the isolated L beam

$$= B = \frac{0{\cdot}5\,l}{+4} + b_r$$

where, B = effective width of flange

l = effective span = 4·90 *m.* = 490 *cm.*

b_r = breadth of the web = 40 *cm.*

b = actual width of the flange = 215 *cm.*

$$B = \frac{0{\cdot}5 \times 490}{\frac{490}{215}+4} + 40 = 79 \text{ cm.}$$

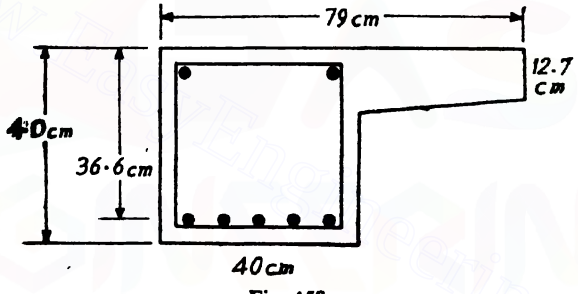

Fig. 158

Fig. 158 shows the section of the L beam.

Loading on the beam per metre run

Load from the canopy slab = 428 × 1·75 = 749 *kg./m.*

D.L. of the beam = 0·4 × 0·4 × 2500 = 400 *kg./m.*

Total 1149 *kg./m.*

Assuming partial fixity at the ends

maximum bending moment $M = \frac{wl^2}{10} = 1149 \times \frac{4{\cdot}95^2}{10} = 2816$ *kg. m.*

maximum shear force $S = 1149 \times \frac{4{\cdot}95}{2} = 2844$ *kg.*

maximum torsion $T = 428 \times 1{\cdot}75 \times 4{\cdot}95\left[\frac{1{\cdot}75}{2} + \frac{0{\cdot}4}{2}\right]\frac{1}{2}$ *kg. m.*

$= 1993$ *kg. m.*

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Providing 18 *mm.* diameter bars at a clear cover of 25 *mm.* effective cover $=25+9=34$ *mm.* $=3{\cdot}4$ *cm.*

Effective depth $=40-3{\cdot}4=36{\cdot}6$ *cm.*

Equivalent shear force $S_e=S+1{\cdot}6\,\dfrac{T}{b_r}$

$$=2844+\frac{1{\cdot}6\times1993}{0{\cdot}4}=10816\ kg.$$

Equivalent nominal shear stress

$$=q_{ve}=\frac{}{b_r d}\quad\frac{10816}{40\times36{\cdot}6}=7{\cdot}39\ kg./cm.^2$$

For M 150 concrete $q_{c\,max}=16\ kg./cm.^2$

Longitudinal reinforcement

Equivalent B.M. $=M_{e1}=M+M_t$

where $M_t=T\dfrac{\left[1+\dfrac{D}{b_r}\right]}{1{\cdot}7}$

$$=1993\left[\frac{1+\frac{40}{40}}{1{\cdot}7}\right]=2345\ kg.\ m.$$

$M_{e1}=2816+2345=5161\ kg.\ m.\ 516100\ kg.\ cm.$

Approximate lever arm $a=d-\dfrac{d_s}{2}=36{\cdot}6-\dfrac{12{\cdot}7}{2}=30{\cdot}25$ *cm.*

Approximate steel required $=\dfrac{516100}{1400\times30{\cdot}25}=12{\cdot}20\ cm.^2$

Provide 5 bars of 18 mm. ϕ (12 72 cm.^{2})

Taking moments about the *NA*

$$79\,\frac{n^2}{2}=18\times12{\cdot}72\ (36{\cdot}6-n)$$

$$n^2+5{\cdot}8\,n-212{\cdot}15=0$$

$$\therefore\qquad n=12{\cdot}35\ cm.$$

Actual lever arm $=a=d-\dfrac{n}{3}$

$$=36{\cdot}6-\frac{12{\cdot}35}{3}=32{\cdot}48\ cm.$$

Actual stress in steel $=\dfrac{516100}{12{\cdot}72\times32\,48}=1249\ kg./cm.^2$

Corresponding maximum stress in concrete

$$=c=\frac{t}{m}\cdot\frac{n}{d-n}=\frac{1249}{18}\times\frac{12\,35}{36{\cdot}6-12{\cdot}35}$$

$$=35{\cdot}34\ kg./cm.^2$$

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Transverse reinforcement

Besides the 4 bars of 18 *mm.* diameter as tensile steel, we will also provide 2 bars of 18 *mm.* diameter at top to support the stirrups.

b_1 = Distance between centres of corner bars parallel to the width
$= 40 - 2 \times 3{\cdot}4 = 33{\cdot}20$ *cm.*

d_1 = Distance between centres of corner bars parallel to the depth
$= 40 - 2 \times 3{\cdot}4 = 33{\cdot}20$ *cm.*

Area of section of stirrups

$$= A_w = \frac{Tp}{b_1 d_1 t_w} + \frac{Sp}{2{\cdot}5\, d_1 t_w}$$

Let two-legged 12 *mm.* ϕ stirrups be provided

$\therefore \quad A_w = 2 \times 1{\cdot}13 = 2{\cdot}26$ *cm.*^{2}

$$\therefore \quad A_w = 2{\cdot}26 = \left[\frac{199300}{33{\cdot}2 \times 33\ 2 \times 1400} + \frac{2844}{2{\cdot}5 \times 33{\cdot}2 \times 1400}\right]$$

$$\therefore \quad 2{\cdot}26 = [0{\cdot}1292 + 0{\cdot}0245]p$$

$$\therefore \quad p = \frac{2{\cdot}26}{0{\cdot}1\ 37} = 14{\cdot}7 \text{ cm.}$$

Provide 2-legged 12 mm. ϕ stirrups @ 14 cm. c/c.

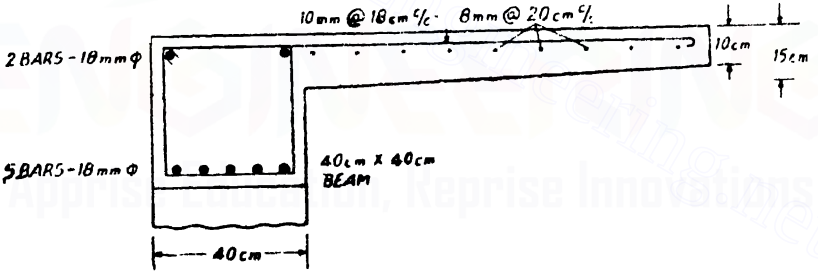

Fig. 159

§130. Hollow Tile Floors (Hollow-Block Slabs)

We know in the case of *T*-beam and slab arrangement, the tensile resistance of concrete below the neutral plane is ignorable and ineffective The concrete in this zone only serves the purposes of resisting shear surrounding the tensile reinforcement and connecting the bars to the compression zone It is therefore worthy to reduce this amount of concrete and thereby reduce some dead load providing only as much concrete as may be necessary to serve the purposes mentioned above. This principle is the basis of ribbed slab or *T*-beam slab construction. In the case of floors over rooms *T*-beam construction may not be pleasing in appearance and a plane ceiling may be felt architecturally desirable. For the shuttering of the cavities between the ribs is also costly. Hence an alternative

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arrangement would be to incorporate hollow blocks (hollow blocks of burnt clay) as permanent shuttering. Thus the concrete in the tension zone of the slab is substituted by hollow blocks placed side by side making room for ribs of concrete for containing the reinforcing bars. A clay slip-tile may be provided at the bottom of the rib to provide continuity of surface similar to that of the soffit of the block. The concrete slab over the hollow blocks acts as the compression flange of the *T*-beam. With this arrangement not only a plane ceiling is provided but at the same time the tiles provide satisfactory thermal and acoustic insulation. Since the tiles are hollow, the dead load of the floor is considerably reduced. Fig. 160 shows the section through a hollow tile roof. This type of roof is suitable where the loads are distributed loads and there are no heavy concentrated loads. In cases where concentrated loads are also likely it is advisable to provide reinforced ribs in both the principal directions.

The thickness of the (topping) slab is very small (5 *cm* to 8 *cm*). The hollow tiles are available in various sizes of widths 22·5 *cm*. to 30 *cm*. and depths 11·5 *cm*. to 15 *cm*. For special works tiles may

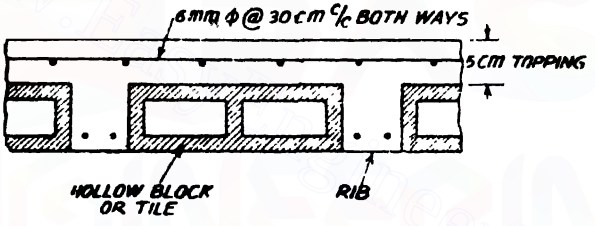

Fig. 160

be specially manufactured to any required size. In some cases instead of using burnt clay tiles, the hollow blocks are made of a lean mix of cinder concrete and may be cast at the site itself. These blocks are usually made having a trapezoidal outline. The thickness of the block is 12 *mm*. The side of greater width is placed upwards so that it remains held by concrete properly. A light mesh of reinforcement say 6 *mm*. bars at 30 *cm* centres is provided in the slab to prevent any cracking due to shrinkage and temperature.

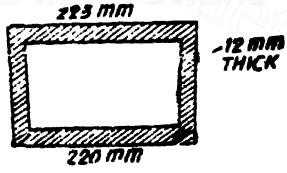

Fig. 161

Breadth of rib. This should not be less than 6 *cm*. or 7·5 *cm*. when one bar per rib is used and not less than 8 *cm*. to 10 *cm*. when two bars per rib are provided.

Size of aggregate. The size of the large aggregate may be limited to 10 *mm* to facilitate placing of the concrete in the narrow ribs.

Size of steel bars. The maximum diameter of bar used for this type of work is 22 *mm*. diameter.

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Concrete cover. The clear cover to reinforcement shall be at least 12 *mm.* It is usual to provide small slip tiles 12 *mm.* thick in the bottom of the ribs so as to provide an unbroken tile surface for plastering. In such a case the overall cover to outside of bar will be taken as a minimum of 24 *mm.*

Design details

Moment of resistance. The moment of resistance should be determined neglecting the blocks The thickness of the flange d_s of the Tee beam is equal to the thickness of the concrete topping.

Dimensions. The clear spacing between the ribs shall not be greater than 45 *cm.* The width of the rib shall not be less than 7·5 *cm.* The overall depth of the slab shall not exceed four times the breadth of the rib. The topping shall be at least 5 *cm.* thick and shall be provided with a light mesh of reinforcement.

Breadth of Flange (B)

This is equal to the spacing of centres of ribs.

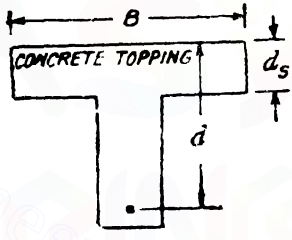

Fig. 162

Shear Stresses. We know that in solid slabs the shear stresses are low whereas in hollow-tile floors shear stresses of considerable intensity are induced since the shear is taken by the ribs alone In some cases, the tiles are stopped off some distance from the supports where the shear can be taken by the ribs. From these points up to the supports the floor will thus be a solid slab which is strong enough to resist the shear.

Depth of the beam. This may be assumed about 1/30 of the span.

Design 43. (S.I.) *An office floor has to be provided with hollow tiles slab, the span of the ribs being 4 metres. The ribs are spaced at 30 cm. centres. Allow a live load of 2000 N/metre².*

Solution.

The span is 4 *metres*

$\therefore$ Approximate depth of *T*-beam $= \frac{1}{30}$ span

$$\frac{4 \times 100}{30} \quad 13 \text{ cm.}$$

Let us provide 15 *cm.* deep beam.

Let us use 22·5 *cm.* × 22·5 *cm.* × 10 *cm.* tiles. See Fig. 163.

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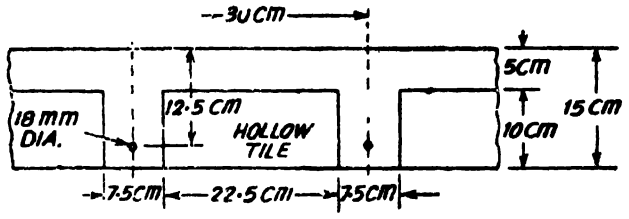

Fig. 163

Loads per sq. m.

Floor finish (1·5 *cm*)	$240 \times 1{\cdot}5 = 360$ *N/metre*^{2}
Ceiling finish (1 *cm.*)	$240 \times 1 = 240$ *N/metre*^{2}
Topping (5 *cm*)	$250 \times 5 = 1250$ *N/metre*^{2}
Tiles	$= 600$ *N/metre*^{2}
Live Load	$= 2000$ *N/metre*^{2}
Total	4450 *N/metre*^{2}

Loading per metre run of the beam

From slab $4450 \times 0{\cdot}3 = 1335$ *N/m.*

rib $\frac{7{\cdot}5}{100} \times \frac{10}{100} \times 25000 = 188$ *N/m.*

Total 1523 *N/m.*

Maximum B M. $= M = \frac{1523 \times 4^2}{8} \times 100$ *N cm.*

$= 304600$ *N cm.*

Consider the beam section shown in Fig. 164.

Providing an effective cover of 2·5 *cm.*

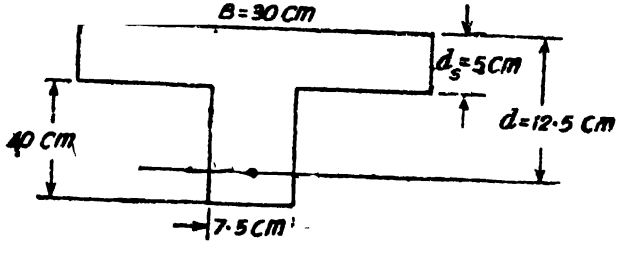

Fig. 164

Effective depth $= d = 15 - 2{\cdot}5$

$= 12{\cdot}5$ *cm.*

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Approximate lever arm

$$= a = d - \frac{d_s}{2}$$

$$= 12.5 - 2.5 = 10 \text{ cm.}$$

approximate steel required

$$= \frac{304600}{14000 \times 10} \text{ cm.}^2$$

$$= 2.17 \text{ cm.}^2$$

Provide 1 bar of 18 mm. dia. (2.54 cm².)

Position of actual neutral axis

Assuming the neutral axis to lie below the flange, taking moments about the neutral axis,

$$30 \times 5(n - 2.5) = 18 \times 2.54 (12.5 - n)$$

$$\therefore \quad n = 4.84 \text{ cm.} \text{ (less than 5 cm.)}$$

Hence the N.A. lies within the flange itself

Hence the depth of neutral axis is given by,

$$\frac{30n^2}{2} = 18 \times 2.54 (12.5 - n)$$

$$\therefore \quad n^2 + 3.048n = 38.1$$

Solving we get $n = 4.83$ cm.

$$\therefore \text{ Correct lever hand} = a = d - \frac{n}{3} = 12.5 - \frac{4.83}{3} = 10.89 \text{ cm.}$$

$$\therefore \text{ Actual stress in steel} = \frac{304600}{2.54 \times 10.89} \text{ N/cm.}^2$$

$$= 11010 \text{ N/cm.}^2 = 1.01 \text{ N/mm.}^2$$

Corresponding compressive stress in concrete

$$= \frac{4.83}{12.5 - 4.83} \times \frac{11010}{18} \text{ N/cm}^2$$

$$= 385 \text{ N/cm.}^2 = 3.85 \text{ N/mm}^2$$

Maximum shear force $S = 1523 \times \frac{4}{2} = 3046$ N

$$\text{Nominal shear stress} = q_v = \frac{S}{b_r d} = \frac{3046}{7.5 \times 12.5} = 32.5 \text{ N/cm.}^2$$

$$= 0.325 \text{ N/mm.}^2$$

$$\text{Percentage of steel provided} = \frac{2.54}{7.5 \times 12.5} \times 100 = 2.7\%$$

For 2.7% steel, as per code prmissible nominal shear stress

$$= q_c = 0.44 \text{ N/mm.}^2$$

$$\therefore \quad q_v < q_c$$

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Design 44. *An office floor has to be provided with hollow tiles slab, the span of the ribs being 4 metres. The ribs are spaced at 30 cm. centres Allow a live load of 200 kg /metre².*

Solution The span is 4 *metres.*

$$\text{Approximate depth of } T\text{-beam} = \frac{1}{30} \text{ span} = \frac{4 \times 100}{30} = 13 \text{ cm.}$$

Let us use 22·5 *cm* × 22·5 *cm.* × 10 *cm.* tiles. See Fig. 163.

Loads per sq. metres.

Floor finish (1·5 *cm*)	= 24 × 1·5 =	36 *kg /metre²*
Ceiling finish (1 *cm*)	= 24 × 1 =	24 *kg /metre²*
Topping (5 *cm.*)	= 25 × 5 =	125 *kg./metre²*
Tiles	=	60 *kg./metre²*
Live load	=	200 *kg./metre²*
	Total	445 *kg.metre²*

Loading per metre run of the beam

From slab 445 × 0·3 = 133·50 *kg./metre*

D.L. of rib $\frac{7.5}{100} \times \frac{10}{100} \times 2500$ = 18·80 *kg./metre*

Total = 152·30 *kg./metre*

Maximum B.M. $= M = \frac{152.3 \times 4^2}{} \times 100$ *kg. cm.*

$= 30460$ *kg. cm.*

Consider the beam section shown in Fig. 164.

Effective depth $= d = 15 - 2.5 = 12.5$ *cm.*

Approximate lever arm $= a = d - \frac{ds}{2} = 12.5 - 2.5 = 10$ *cm.*

Approximate steel required $= \frac{30460}{1400 \times 10} = 2.17$ *cm.²*

Provide 1 bar of 18 mm. ϕ (2·54 cm.²)

Position of actual neutral axis

Assuming the actual neutral axis to lie below the flange, taking moments about the neutral axis,

$$30 \times 5 (n - 2.5) = 18 \times 2.54 (12.5 - n)$$

$$n = 4.84 \text{ cm. (less than 5 cm.)}$$

Hence, the neutral axis lies within the flange itself.

Hence the depth of neutral axis is given by,

$$\frac{30 n^2}{2} = 18 \times 2.54 (12.5 - n)$$

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$$n^2+3{\cdot}048\,n=38{\cdot}1$$

Solving we get $n=4{\cdot}83$ *cm.*

$$\therefore \text{ Correct lever arm} = a = d-\frac{n}{3} = 12{\cdot}5-\frac{4{\cdot}38}{3} = 10{\cdot}89 \text{ cm.}$$

$$\text{Actual stress in steel} = \frac{30460}{2{\cdot}54\times 10{\cdot}89} = 1101 \text{ kg./cm.}^2$$

Corresponding compressive stress in concrete

$$= \frac{4{\cdot}83}{12{\cdot}5-4{\cdot}83}\times\frac{1101}{8} = 38{\cdot}5 \text{ kg./cm.}^2$$

$$\text{Maximum shear force} = S = 152{\cdot}3\times\frac{4}{2} = 304{\cdot}6 \text{ kg.}$$

$$\text{Nominal shear stress} = q_v = \frac{S}{bjd} = \frac{304{\cdot}6}{7{\cdot}5\times 12{\cdot}5} = 3{\cdot}25 \text{ kg /cm.}$$

$$\text{Percentage of steel provided} = \frac{2{\cdot}54}{7{\cdot}5\times 12{\cdot}5}\times 100 = 2{\cdot}7\%.$$

For 2·7% steel, as per code permissible nominal shear stress $q_c = 4{\cdot}40$ *kg /cm.*^{2} $\quad\therefore\ q_v < q_c$

§131. Reinforced Brickwork

Brick masonry has a permissible compressive strength of 2·5 *N/mm.*^{2} (25 *kg./cm*^{2}). Hence, brickwork can be used in combination with steel reinforcement as in reinforced concrete construction. Reinforced brickwork is being used for slabs of small span and also for lintels. Since the strength in diagonal tension is low, it is safe to use reinforced brickwork for beams. Reinforced brick slabs are designed following the same principles as for reinforced cement concrete slabs.

Permissible stresses adopted for reinforced brickwork

(*i*) Permissible compressive stress in brickwork : 2 *N/mm.*^{2} to 3·5 *N/mm.*^{2} (20 *kg./cm.*^{2} to 35 *kg./cm.*^{2}) (the strength increases with the coarseness of sand used in mortar and the corresponding decrease in water cement ratio).

(*ii*) Permissible tensile stress in steel : 140 *N/mm.*^{2} (1400 *kg./cm.*^{2})

(*iii*) Modular ratio $= m = 40$

(*iv*) Safe diagonal tension $= 0{\cdot}1$ *N/mm.*^{2} (1 *kg./cm.*^{2})

(*v*) Local bond stress $= 1$ *N/mm.*^{2} (10 *kg /cm.*^{2})

(*vi*) Average bond stress $= 0{\cdot}6$ *N/mm*^{2} (6 *kg./cm.*^{2})

(*vii*) Lever arm factor $= at = 0{\cdot}86$

(*viii*) M R. factor $Q = 45$ in S.I. calculations and 4·5 in M K.S. calculations

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In case the induced diagonal tensile stress exceeds 0·1 $N/mm.^2$ (1 $kg./cm.^2$) suitable shear reinforcement shall be provided. Even with shear reinforcements the total diagonal tension shall not exceed 0·3 N/mm^2 (3 $kg./cm.^2$). Bricks used shall be first class bricks and the mortar used shall be of 1 : 3 proportion (1 part of cement to 3 parts of clean coarse sand). The quantity of water used forming should be the minimum possible, so as to render it workable in the joints The bricks should be thoroughly soaked in water before using them, otherwise they will absorb the water from the mortar and seriously affect the curing process. Small size bars are used and the brick joints are made wide enough so that the cement mortar may fill all round the bars. Bars shall not come directly into contact with brick to prevent rusting. As far as possible, it is better to break the joints when the bricks are being laid. Care should be taken to see that there shall not be any joint vertically right above the edge of the supporting wall. At such places the bricks should lie half on the wall and half projecting beyond it.

Reinforced brickwork is found to be economical for small spans requiring lesser supervision and control. Thermal conductivity of brickwork is lower than that of concrete and so this construction provides relatively better insulation to heat. But they have the defect that they are liable to absorb moisture leading to the corrosion of steel. It is therefore advisable to provide 10 *cm.* thick lime concrete terracing over R B roof slabs.

Fig. 165 shows a section of a reinforced concrete brick slab.

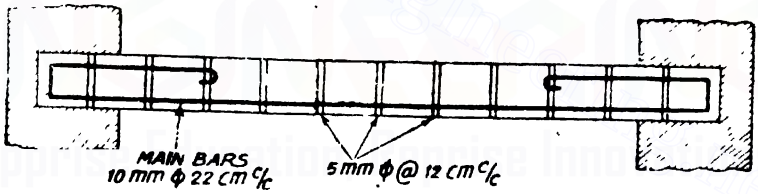

Fig 165

As in hollow tile construction, sometimes the strength of R.B. work can be increased by providing wider jonts filled with concrete and with a layer of topping concrete over the bricks The thickness of the joints in such cases may be 4 *cm.* and the thickness of the topping layer of concrete may be 2·5 *cm.* to 5 *cm.* See Fig. 166.

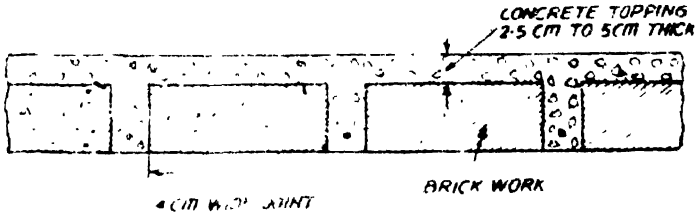

Fig. 166

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In this arrangement the slab is designed in a manner similar to hollow-tile floors.

Design 45. *A reinforced brick slab is to be provided for a verandah 2·25 m. wide. Design the slab for a live load of 2000 N/metre².*

Solution.

Clear span between supporting walls = 2·25 *m*

Providing a bearing of 8 *cm.*,

Effective span between centres of bearing = 2·33 *m.*

Loads

D.L. of slab (10 *cm.*) $= \frac{10}{100} \times 1 \times 19200 = 1920$ *N/metre²*

Floor finish (2 *cm.*)	= 460 *N/metre²*
Live load	= 2000 *N/metre²*
Total	= 4380 *N/metre²*
Say	= 4400 *N/metre²*

Consider a 1 *metre* wide strip of the slab

$\therefore$ Max. B.M. $= M = \frac{4400 \times 2{\cdot}33^2}{8} \times 100$ *N cm* = 298600 *N cm.*

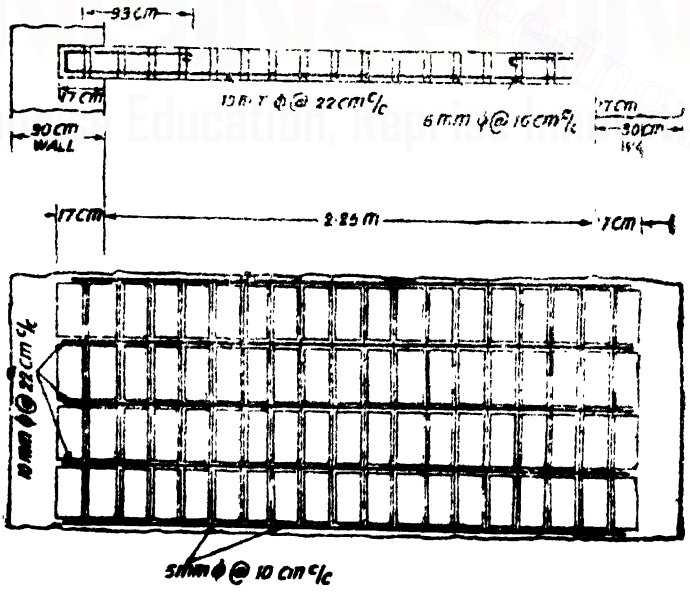

Fig. 167

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Equating the moment of resistance to the maximum bending moment, we get,

$$45\ bd^2 = 45 \times 100d^2 = 298600 \qquad \therefore\ d = 8{\cdot}10\ cm.$$

Provide an overall thickness of 10 *cm.* and allowing an effective cover of 2 *cm.*, effective depth available $10-2=8$ *cm.*

$$A_t = \frac{298600}{14000 \times 0{\cdot}86 \times 8}\ cm^2. = 3{\cdot}10\ cm.^2$$

$$\therefore \text{ Spacing of 10 } mm. \text{ bars} = \frac{0{\cdot}79 \times 100}{3{\cdot}1} = 25{\cdot}5\ cm.$$

A brick being 20 *cm.* long and since the joint will be 2 *cm.* thick let us *provide 10 mm ϕ bars @ 22 cm. c/c.*

Check for shear

$$\text{Max. S.F. (based on clear span)} = \frac{4400 \times 2{\cdot}25}{2} = 4950\ N$$

$$\text{Shear stress} = q = \frac{S}{ab} = \frac{4950}{0{\cdot}86 \times 8 \times 100}\ N/cm.^2$$

$$= 7{\cdot}2\ N/cm.^2 = 0{\cdot}072\ N/mm^2.$$

This is less than the permissible limit of 0·1 $N/mm.^2$

$$\text{Local bond stress} = \frac{S}{a\Sigma o} = \frac{4950}{0{\cdot}86 \times 8 \times \frac{100}{22} \times 3{\cdot}14}\ N/cm.^2$$

$$= 0{\cdot}5\ N/mm^2$$

This is less than the permissible limit of 1 $N/mm.^2$

Design 46. *A reinforced brick slab is to be provided for a verandah 2·5 metres wide. Design the slab for a live load of 200 kg./metre².*

Solution.

Clear span between supporting walls = 2·25 *m.*

Providing a bearing of 8 *cm.*, Effective span between centres of bearing = 2·33 *m.*

Loads

$$\text{D.L. of slab (10 } cm) = \frac{10}{100} \times 1 \times 1920 = 192\ kg./m.^2$$

Floor finish (2 *cm.*) = 46 $kg./m.^2$

Live load = 200 $kg./m^2$

Total = 438 kg/m^2

Say = 440 $kg./m.^2$

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Consider a 1 *metre* wide strip of the slab

$$\therefore \text{ Max. B.M.} = M = \frac{440 \times 2{\cdot}33^2}{8} \times 100 \text{ kg cm.} = 29860 \text{ kg. cm.}$$

Equating the moment of resistance to the maximum bending moment, we get,

$$4{\cdot}5\, bd^2 = 4{\cdot}5 \times 100 d^2 = 29860 \qquad \therefore\ d = 8{\cdot}10 \text{ cm}$$

Provide an overall thickness of 10 *cm* and allowing an effective cover of 2 *cm.*, effective depth available $10-2=8$ *cm.*

$$\therefore \qquad A_t = \frac{29860}{1400 \times 0{\cdot}86 \times 8} \text{ cm.}^2 = 3{\cdot}10 \text{ cm}^2$$

$$\therefore \text{ Spacing of 10 mm bars} = \frac{0{\cdot}79 \times 100}{3{\cdot}1} = 25{\cdot}5 \text{ cm.}$$

A brick being 20 *cm* long and since the joint will be 2 *cm.* thick, let us *provide 10 mm. ϕ bars @ 22 cm c/c.*

Check for shear

$$\text{Max. S.F. (based on clear span)} = \frac{440 \times 2{\cdot}25}{2} = 495 \text{ kg.}$$

$$\text{Shear stress} = q = \frac{S}{ab} = \frac{495}{0{\cdot}86 \times 8 \times 100} = 0{\cdot}72 \text{ kg./cm.}^2$$

This is less than the permissible shear stress of 1 $kg./cm^2$

Check for bond

$$\text{Local bond stress} = S_b = \frac{S}{a\Sigma o} = \frac{495}{0{\cdot}85 \times 8 \times \frac{100}{22} \times 3{\cdot}14}$$

$$= 5 \text{ kg/cm}^2. \qquad \text{(less than 10 kg./cm.}^2\text{)}$$

§132. Slabs Formed with Precast Sections

We may construct slabs by using precast sections of various shapes These slabs possess some advantages. They do not need any extensive staging and can be erected quickly by crane. The precast units can be cast in a factory These units are designed for that part of the uniformly distributed floor load each unit carries in the finished slab. Fig 168 shows different types of precast units used. The box sections and the inverted U-sections may be considered as T-beams in which a topping of 3 *cm.* or more may be taken to act in conjunction with the precast units In the case of U-sections in *in-situ* reinforced concrete may be taken to provide additional strength. But for this latter case due consideration shall

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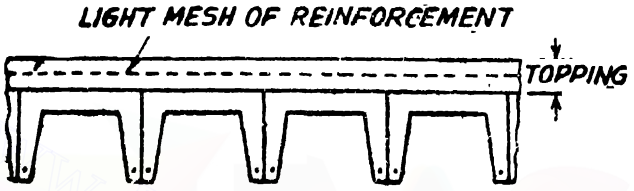
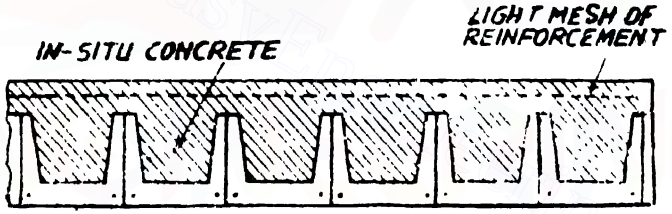
be given to the initial stresses induced in the precast units to the dead load of the *in situ* concrete.

(*a*) Box Sections



(*b*) Inverted U-Sections.



(*c*) U-Sections.

Fig. 168

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12
Foundations

Introduction. The *foundation* of a structure is that part of the structure which transfers the load to the soil on which it rests. It forms a very important part of the structure. The ground surface in contact with the lower surface of the foundation is called the *base of the foundation*. The ground on which the foundation rests is called the *subgrade* or *foundation soil*. A foundation should be designed to safely transmit the load of the structure on to a sufficient area of the soil so that the stresses induced in the soil are within the safe limit. If a soil is overstressed, it may lead to a shear failure resulting in the sliding of the soil along a plane of rupture and thus result in the collapse of the structure. Still another point of importance is the need to bring about uniform settlements which should be within the tolerance limits of the superstructure.

Due to the load of the structure the soil below is compressed. The settlement of the soil depends on various factors like the intensity of the loading, the quality of the soil and the depth below the surface level It is very necessary that the settlement of the various components of a structure should be the same to prevent serious secondary stresses To ensure uniform settlement it is necessary that the foundation area is so provided the intensity of soil reaction is the same under all the footings of a structure.

A foundation should be designed such that there is no possibility of tilting of the structure. If the foundation area for a structure is such that the centre of gravity of the loads does not coincide (in plan) with the centre of gravity of the foundation area the consequent bearing reaction will be non-uniform. At the edge closer to the centre of gravity of the loads the pressure intensity will be higher resulting in a greater settlement of the soil at this edge. This will result in tilting of the structure in this direction. Hence it is better to design the foundation area such that the centre of gravity of the loads (in plan) will coincide with the centre of gravity of the foundation area so that the soil reaction will be of uniform intensity. Foundations may be broadly classified into *shallow foundations* and *deep foundations*. The *depth of foundation* means the difference of level between the ground surface and the base of the foundation. If the depth of foundation is equal to or less than its width, the foundation is classified as a shallow foundation. If the depth of foundation is greater than its width the foundation is classified as a deep foundation.

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We come across the following types of shallow foundations.

(*i*) *Footings.* In this type of foundation the base of a wall or column is sufficiently enlarged to act as the individual support. The widened base not only provides stability but is useful in distributing the load on sufficient area of the soil.

(*ii*) *Mats or rafts.* These are provided when a number of column loads are supported by single slab.

(*iii*) *Combined footings, strap and strip footings.* A combined footing consists of a common footing provided to two columns. A strap footing consists of two column footings connected by a strap beam. A strip footing is a common footing provided for a number of columns in a row.

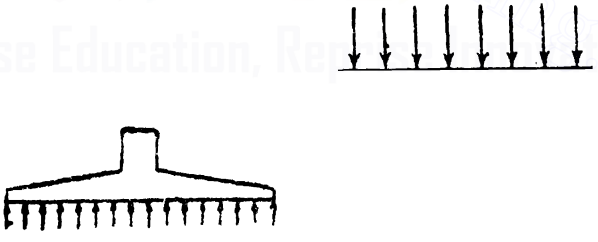
A deep foundation consists of a *pile* or a *pier*.

A pier is cylindrical (or prismatic) body of concrete which transfers the load to the soil at a depth greater than its width.

A pile is a slender member which transfers the load either through its lower end into a strong strata or may transfer its load to the surrounding soil by friction.

§133. Pressure Distribution Under a Foundation

The law of distribution of pressure under a foundation depends on the extent of homogeneity of the soil and the flexibility of the base. If really the soil is homogeneous and the base of the foundation is flexible, the pressure distribution under the foundation will be uniform. See Fig 169 (*a*) On the contrary if the foundation base is absolutely rigid, the pressure distribution will not be uniform but may follow such pattern as shown in Fig. 169 (*b*).



(*a*) Pressure distribution under a flexible base. (*b*) Pressure distribution under a rigid base.

Fig 169

In our designs it is usual to assume a flexible base and hence to regard the pressure distribution to be uniform. This can be achieved by gradually decreasing the thickness of the base towards the edges so that the base is only as much thick as it is regarded to resist the induced moments and shears.

A foundation shall not be provided directly on such soil which may contain organic elements like loam, loose filled earth *etc.* It shall be built on a firm soil which may have a reliable bearing

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capacity. The approximate safe bearing capacities of various types of soils are given in the following table.

Type of soil	*Safe bearing capacity*	
	kN/metre^2	*t/metre*
1. Hard dry clay	350	35
2. Sand and clay mixed	200	20
3. Firm clay	200	20
4. Fine confined wet sand	200	20
5. Fine dry sand	350	35
6. Coarse sand	450	45
7. Gravel	650	65
8. Soft rock	900	90
9 Hard rock (mixture of gravel and clay)	1100	110
10. Medium hard rocks	2750	275

To know the exact bearing capacity of a soil it is advisable to actually determine it by load tests at the site. A loose soil may have such a low bearing capacity of 80 *kN/metre*^{2} (8 *t/metre*^{2}) to 100 *kN/metre*^{2} (10 *t/metre*)^2, while very sound hard rock may have a bearing capacity of 4500 *kN/metre*^{2} (450 *t/metre*^{2})

§134. General Rules of Design

While designing a foundation the following points must be borne in mind.

When a soil is a yielding soil a certain amount of settlement of the foundation is sure to occur. But the extent of the settlement must be reduced as much as possible by bringing down the pressure intensities. It is necessary that a foundation shall be designed so that if at all a settlement should occur, it will be uniform. In other words, the settlement of all the footings must be more or less the same. This is a very important point in reinforced concrete structures due to the rigid connection between the different components of the structure.

§135 Design of Footings

A footing comprises of the lower end of a column, pillar or wall which is enlarged with projecting courses so as to distribute the

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The design formulae in S.I. and M.K.S. Units are obtained below :

S.I. Units

Let the upward pressure on the slab be p *N/metre*^{2}

Consider 1 *metre* length of the foundation

Total upward load on the cantilever

$$= p.\ \frac{a}{100}\ N$$

Maximum bending moment

$$= M = \frac{pa}{100} \times \frac{a}{2} = \frac{pa^2}{200}\ N\ cm.$$

Plain concrete may be stressed to a bending stress equal to $\frac{1}{30}$ of the 28 day cube strength. For M 15 concrete permissible bending stress $= f' = \frac{15}{30}$

$= 0{\cdot}5\ N/mm^2 = 50\ N/cm.^2$

Equating the moment of resistance to the maximum bending moment,

$$\frac{1}{6} f'\ bD^2 = \frac{1}{6} \times 50 \times 100 \times D^2 = \frac{pa^2}{200}$$

$$\therefore\ D = 0{\cdot}00245\ a\sqrt{p}$$

For a bearing capacity of soil of 100 *kN/m.*^{2}

$$p = 100{,}000\ N/metre^2$$

$$D = 0{\cdot}00245\ \sqrt{100{,}000}\ a$$

$$\therefore\ D = 0{\cdot}775\ a$$

For M 15 concrete, the relation between D and a for different values of safe bearing capacity of soil are given in the table below.

Safe bearing capacity of soil	*Relation between D and a*
50 *kN m*$^{-2}$	$D = 0{\cdot}548\ a$
100 *kN/m.*^{2}	$D = 0{\cdot}775\ a$
150 *kN/m.*^{2}	$D = 0{\cdot}949\ a$
200 *kN/m.*^{2}	$D = 1{\cdot}096\ a$
250 *kN/m.*^{2}	$D = 1{\cdot}225\ a$

M.K.S. Units

Let the upward pressure on the slab be p *kg./metre*^{2}

Consider 1 *metre* length of the foundation

Total upward load on the cantilever

$$= p \times \frac{a}{100}\ kg.$$

Maximum bending moment

$$= M = \frac{pa}{100} \times \frac{a}{2} = \frac{pa^2}{200}\ kg.\ cm$$

Plain concrete may be stressed to a bending stress equal to $\frac{1}{30}$ of the 28 day cube strength. For M 150 concrete permissible bending stress

$$= f' = \frac{150}{30} = 5\ kg/cm^2$$

Equating the moment of resistance to the maximum bending moment,

$$\frac{1}{6} f'\ bd^2 = \frac{1}{6} \times 5 \times 100 \times D^2 = \frac{pa^2}{200}$$

$$\therefore\ D = 0{\cdot}00775\ a\sqrt{p}$$

For a bearing capacity of soil of 10 *t/metre*^{2}

$$p = 10{,}000\ kg/metre^2$$

$$D = 0{\cdot}00775\ \sqrt{10{,}000}\ a$$

$$\therefore\ D = 0{\cdot}775\ a$$

For M 150 concrete, the relation between D and a for different values of safe bearing capacity of soil are given in the table below.

Safe bearing capacity of soil	*Relation between D and a*
5 *t/m.*^{2}	$D = 0{\cdot}548\ a$
10 *t/m.*^{2}	$D = 0{\cdot}775\ a$
15 *t/m.*^{2}	$D = 0{\cdot}949\ a$
20 *t/m.*^{2}	$D = 1{\cdot}096\ a$
25 *t/m.*^{2}	$D = 1{\cdot}225\ a$

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The following table shows the thickness of plain concrete foundation slab in *cms.* for different lengths of projections :

Maximum Depth of Unreinforced Concrete Foundation Slab in Centimetres

Projection a (cms)	*Safe bearing capacity of soils*			
	50 kN/metre² *5 t/metre²*	*100 kN/metre²* *10 t/metre*	*150 kN/metre²* *15 t/metre²*	*200 kN/metre²* *20 t/metre²*
10	5·48	7·75	9·49	10·96
12	6·58	9·30	11·34	13·15
14	7·67	10·85	13·29	15·34
16	8·77	12·40	15·18	17·54
18	9·86	13·95	17·08	19·73
20	10·96	15·50	18·93	21·92
22	12·06	17·05	20·88	24·11
24	13·15	18·60	22·78	26·30
26	14·25	20·15	24·67	28·50
28	15·34	21·70	26·57	30·69
30	16·44	23·25	28·47	32·88
32	17·54	24·80	30·37	35·07
34	18·63	26·35	32·27	37·26
36	19·73	27·90	34·16	39·46
38	20·82	29·45	36·06	41·65
40	21·92	1·00	37·96	43·84
42	23·02	32·55	39·86	46·03
44	24·17	34·10	41·76	48·22
46	25·21	35·65	43·65	50·42
48	26·30	37·20	44·55	52·61
50	27·40	38·75	47·45	54·80

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load over a large area of the soil so that stress induced immediately under the footing is less than the safe bearing capacity of the soil.

§136. Wall Footing

Plain concrete footing

This consists of a slab projecting as cantilevers from both sides of the wall, as shown in Fig. 170.

When the projection is small it is economical to use plain concrete wall footing. Let the projection of the wall footing be *a cms.*

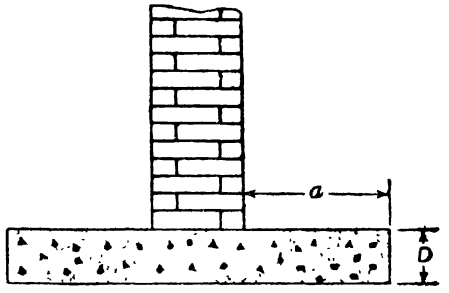

Fig 170. Unreinforced concrete wall footings.

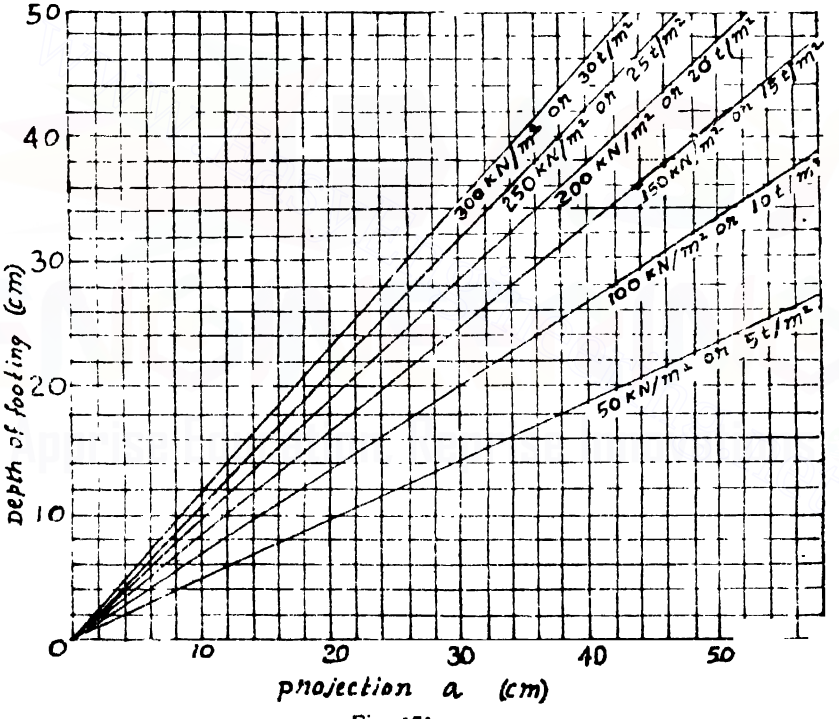

Fig. 171

Design 47. (S.I.) *Design an unreinforced concrete wall footing for a wall 45 cm. thick carrying a load of 300 kN per metre run. The safe bearing capacity of the soil is 200 kN per metre.*[2]

Solution. Consider one *metre* run of the wall.

Load on the wall $= 300$ *kN/m.*	$= 300{,}000$ *N/m.*
Weight of footing at 10% of load on the wall	30,000 *N/m.*
Total load	$= 330{,}000$ *N/m.*

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Area of footing for one *metre* run

$$= \frac{330,000}{200,000} = 1{\cdot}65 \text{ metre.}^2$$

Width of footing $= \frac{1{\cdot}65}{1} = 1{\cdot}65 \text{ metre} = 165 \text{ cm.}$

Thickness of wall $= 45$ cm.

Cantilever projection of footing

$$= a = \frac{165 - 45}{2} = 60 \text{ cm.}$$

Net upward pressure $p = \frac{300,000}{1{\cdot}65} = 181800 \text{ N/metre}^2$

Depth of footing $D = 0{\cdot}00245\, a\sqrt{p}$

$$= 0{\cdot}00245 \times 60\sqrt{181800}$$

$$= 62{\cdot}70 \text{ cm. say } 63 \text{ cm.}$$

Design 48. *Design an unreinforced concrete wall footing for a wall 45 cm. thick carrying a load of 30,000 kg. per metre run. The safe bearing capacity of the soil is 20 tonnes per square metre.*

Solution. Consider one *metre* run of wall.

Load on the wall $= 30,000$ kg

Weight of footing at 10% of load on wall $= 30,00$ kg.

Total load $= 33,000$ kg.

Area of footing for one *metre* run

$$= \frac{33,000}{20,000} \text{ m.}^2 = 1{\cdot}65 \text{ m.}^2$$

Width of footing

$$= \frac{1{\cdot}65}{1} m = 1{\cdot}65 m$$

$$= 165 \text{ cm.}$$

Thickness of wall $= 45$ cm.

Cantilever projection of footing

$$= a = \frac{165 - 45}{2} \text{ cm.} = 60 \text{ cm.}$$

Net upward pressure $= \frac{30,000}{1{\cdot}65}$ kg./m.^{2}

$$= 18180 \text{ kg./m.}^2$$

$\therefore$ Depth of footing $D = 0{\cdot}00775\sqrt{p}\, a$ cm.

$$= 0{\cdot}00775\sqrt{18180} \times 60 \text{ cm.}$$

$$= 62{\cdot}7 \text{ cm. say } 63 \text{ cm.}$$

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§137 Reinforced Concrete Footings

Our code of practice has given the following specifications in connection with reinforced concrete footings :

1. General. Footings shall be designed to sustain the applied loads, moments and forces and the induced reactions and to ensure that any settlement which may occur shall be as nearly uniform as possible, and the safe bearing capacity of the soil is not exceeded

In sloped or stepped footings the effective cross-section in compression shall be limited by the area above the neutral plane, and the angle of slope or depth and location of steps shall be such that the design requirements are satisfied at every section. Sloped and stepped footings that are designed as a unit shall be constructed to assure action as a unit.

Thickness at the edge of the footing In reinforced and plain concrete footings, the thickness at the edge shall be not less than 15 *cm.* for footings on soils, nor less than 30 *cm.* above the tops of piles for footings on piles.

In the case of plain concrete pedestals, the angle α between the plane passing through the bottom edge of the pedestal and the corresponding junction edge of the column with pedestal and the horizontal plane (See. Fig 172) shall be governed by the expression

$$\tan \alpha \leq 0{\cdot}9\sqrt{\frac{100q_o}{f_{ck}}+1}$$

where, q_o = calculated maximum bearing pressure at the base of the pedestal,

f_{ck} = characteristic strengths of concrete at 28 days.

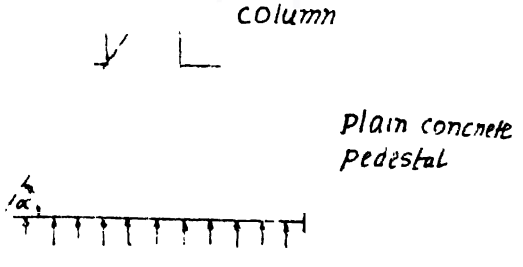

Fig. 172

2. Moments and Forces. In the case of footings on piles computation for moments and shears may be based on the assumption that the reaction from any pile is concentrated at the centre of the pile.

For the purpose of computing stresses in footings which support round or octagonal concrete column or pedestal, the face of the

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column or pedestal shall be taken as the side of a square inscribed within the perimeter of the round or octagonal column or pedestal.

Bending Moment. The bending moment at any section shall be determined by passing through the section a vertical plane which extends completely across the footing and computing the moment of the forces acting over the entire area of the footing on one side of the said plane.

The greatest bending moment to be used in the design of an isolated concrete footing which supports a column, pedestal or wall, shall be the moment computed in the manner described above, at sections located as follows :

(*a*) At the face of the column, pedestal or wall for supporting a concrete column, pedestal or wall ;

(*b*) Halfway between the centreline and the edge of the wall, for footings under masonry walls ; and

(*c*) Halfway between the face of the column or pedestal and the edge of the gussetted base, for footings under gussetted bases.

Shear and bond. The shear strength of footings is governed by the more severe of the following two conditions :

(*a*) The footing acting essentially as a wide beam, with a potential diagonal crack extending in a plane across the entire width ; the critical section for this condition shall be assumed as a vertical section located from the face of the column, pedestal or wall at a distance equal to the effective depth of the footing in case of footings on soils, and a distance equal to half the effective depth of footing for footings on piles.

(*b*) Two-way action of the footing, with potential diagonal cracking along the surface of a truncated cone or pyramid around the concentrated load ; in this case, the footing shall be designed for shear in accordance with appropriate provisions specified for flat slabs

In computing the external shear on any section through a footing supported on piles, the entire reaction from any pile of diameter D_p whose centre is located $D_p/2$ or more outside the section shall be assumed as producing shear on the section : the reaction from any pile whose centre is located $D_p/2$ or more inside the section shall be assumed as producing no shear on the section. For intermediate positions of the pile centre, the portion of the pile reaction to be assumed producing shear on the section shall be based on straight line interpolation between full value at $D_p/2$ outside the section and zero at $D_p/2$ inside the section.

The critical section for checking the development length in a footing shall be assumed at the same planes as those described for bending moment and also at all other vertical planes where abrupt changes of section occur If reinforment is curtailed, the anchorage requirements shall he satisfied as follows.

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The reinforcement shall extend beyond the point at which it is no longer required to resist flexure, for a distance equal to the effective depth of the member or 12 times the bar diameter whichever is greater.

3. **Tensile reinforcement.** The total tensile reinforcement at any section shall provide a moment of resistance at least equal to the calculated bending moment of the section and the total reinforcement thus determined shall be distributed across the corresponding resisting section as given below :

(*a*) In one-way reinforced footing, the reinforcement shall be distributed uniformly across the full width of the footing.

(*b*) In two-way reinforced square footing the reinforcement extending in each direction shall be distributed uniformly across the full width of the footing.

(*c*) In two-way reinforced concrete rectangular footing, the reinforcement in the long direction shall be distributed uniformly across the full width of the footing. For reinforcement in the short direction, a central band equal to width of the footing shall be marked along the length of the footing and a portion of the reinforcement determined in accordance with the equation given below shall be uniformly distributed across the central band :

$$\frac{\text{Reinforcement in central band width}}{\text{Total reinforcement in short direction}} = \frac{2}{\beta+1}$$

where, β = Ratio of the long side to the short side of the footing. The remainder of the reinforcement shall be uniformly distributed in the outer portions of the footing

4. **Transfer of load at base of column.** The compressive stress in concrete at the base of the column or pedestal shall be considered as being transferred by bearing to the top of the supporting pedestal or footing. The bearing pressure on the loaded area shall not exceed the permissible bearing stress in direct compression multiplied by a value equal to $\sqrt{\frac{A_1}{A_2}}$ but not greater than 2 ;

where, A_1 = supporting area for bearing of footing, which in sloped or stepped footing may be taken as the area of the lower base of the largest frustrum of a pyramid or cone contained wholly within the footing and having for its upper base, the area actually loaded and having side slope of one vertical to two horizontal ; and

A_2 = loaded area at the column base.

The permissible bearing stress on full area of concrete shall be taken as $0{\cdot}25\ f_{ck}$.

Where the permissible bearing stress on the concrete in the supporting or supported member would be exceeded, reinforcement,

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shall be provided for developing the excess force either by extending the longitudinal bars into the supporting member, or by dowels.

Where transfer of force is accomplished by reinforcement, the development length of the reinforcement shall be sufficient to transfer the compression or tension to the supporting member.

Extended longitudinal reinforcement of dowels of atleast 0·5 percent of the cross-sectional area of the supported column or pedestal and a minimum of four bars shall be provided Where dowels are used, their diameter shall not exceed the diameter of the column bars by more than 3 *mm*.

Column bars of diameters larger than 36 *mm*., in compression only can be dowelled at footings with bars of smaller size of the necessary area. The dowel shall extend into the column, a distance equal to the development length of the column bar and into the footing, a distance equal to the development length of the dowel.

§138. Square Column Footing

General principles of design :

Let the column section be of side b and the foundation plan be of side l. Let the bearing capacity of the soil be p' per unit area.

Let the load on the column be W, including the weight of the column Let the weight of the footing be W_f The weight of the footing may be approximately estimated at 10% of the load on the column.

Area required for the foundation $= l^2 = \dfrac{W+W_f}{p'}$

$$l = \sqrt{\frac{W+W_f}{p'}}$$

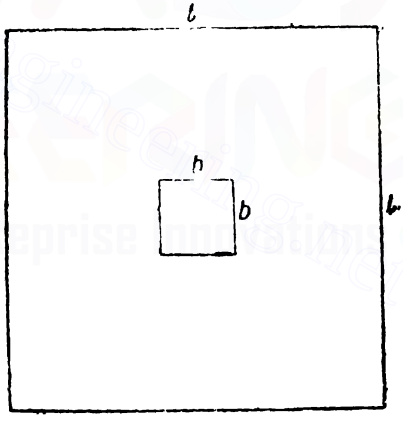

Fig. 173

Hence the side of the square footing is determined.

Depth of the footing. The depth of the footing is determined from the following considerations :

(*i*) B M. consideration.

(*ii*) Punching shear consideration.

(*i*) **Depth from B.M. consideration.** The critical section for calculating the maximum bending moments is corresponding to the column face (Fig. 174).

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Let p be the net upward pressure.

$$p = \frac{W}{l^2}$$

(The selfweight of the footing should not be considered in finding the net upward pressure on the footing).

Max. B.M.

$$= pl\frac{(l-b)}{2}\cdot\frac{(l-b)}{4}$$

$$= \frac{pl(l-b)^2}{8}$$

Equating the moment of resistance, Qbd^2 to the maximum bending moment the effective depth may be computed.

(*ii*) **Depth from punching shear consideration.** Let W be the load on the column including its weight.

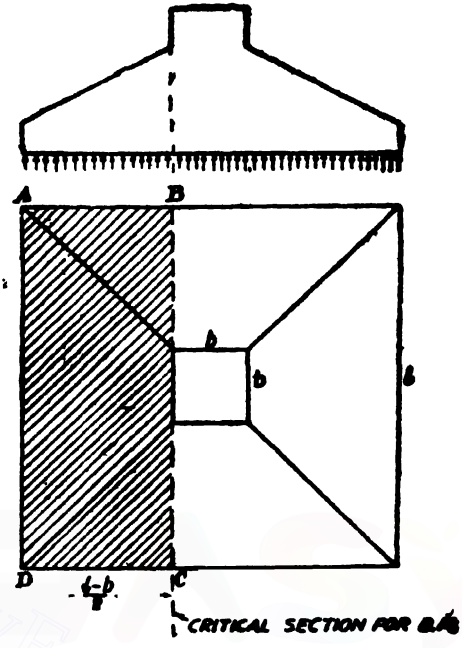

Fig. 174

Let p be the net upward intensity of pressure on the footing slab.

If the footing should fail by punching shear, the type of failure will be as shown in Fig. 175.

Let D be the depth of the footing.

Load causing the punching shear

$$= \frac{W}{l^2}(l^2-b^2)$$

If the punching shear resistance be q per unit area, we have

$$q \times 4bD \quad \frac{W}{l^2}(l^2-b^2)$$

$$\therefore \quad D = \frac{W(l^2-b^2)}{4l^2bq}$$

Thus, the depth of the slab is determined from the above two considerations and the greater of the two depths is provided.

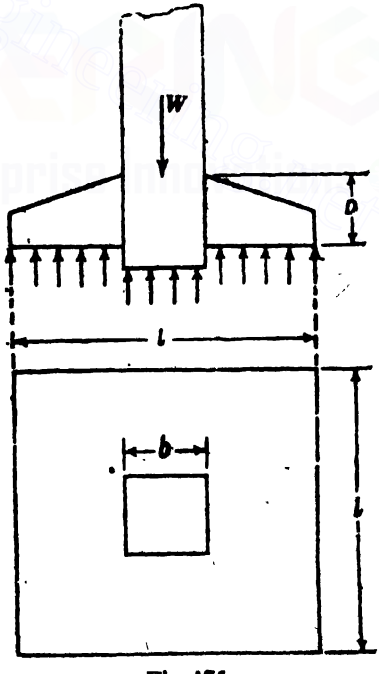

Fig. 175

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Check for diagonal tension

The critical section to check for the diagonal tensile stress is at a distance of effective depth from the face of the column.

Shear force at the critical section

$$S = pl\left\{\left[\frac{(l-b)}{2}\right]-d\right\}$$

Nominal shear stress at the critical section

$$q_v = \frac{S}{b'd'}$$

where b' = width of footing at top at the critical section

and d' = Effective depth at the critical section.

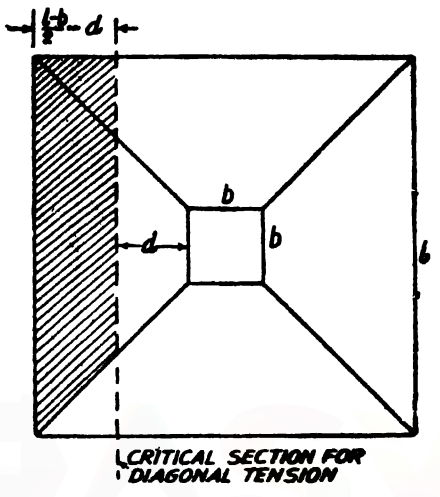

Fig. 176

q_v shall not exceed the permissible nominal shear stress q_c.

Design 49 (S.I.). *A square column 40 cm. side carries an axial load of 900 kN. Design the column and a square footing for the column. The safe bearing capacity of the soil is 100 kN per square metre. Use M 15 concrete.*

Solution. Design of the column

Safe stress in steel $= 130\ N/mm^2$
$= 13000\ N/cm.^2$

Safe stress in concrete
$= 4\ N/mm.^2$
$= 400\ N/cm.^2$

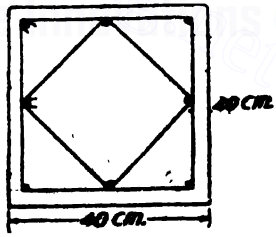

Fig. 177

Safe load on column = area of steel × safe stress in steel + area of concrete × safe stress in concrete

$= A_s \times 13000 + (1600 - A_s)\ 400\ N$

$= 900{,}000\ N$; $A_s = 20{\cdot}60\ cm.^2$

Provide 8 bars of 18 mm. ϕ

Lateral Ties

Diameter of lateral ties should not be less than one-fourth the diameter of main reinforcement, *i.e.*, in our case the diameter of lateral ties shall not be less than $\frac{18}{4} = 4{\cdot}5\ mm$. The diameter of ties

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should also be not less than 5 *mm*. Hence, in our case the diameter of lateral ties may be taken at 5 *mm*.

Pitch of Ties

This shall not be greater than the following :

(*i*) Least lateral dimension

$= 40$ *cm*.

(*ii*) 16 times the diameter of longitudinal bar

$= 16 \times 1{\cdot}8 = 28{\cdot}8$ *cm*.

(*iii*) 48 times the diameter of transverse reinforcement

$= 48 \times 0{\cdot}5 = 24$ *cm*.

Provide 5 mm. ϕ lines @ 24 cm. c/c.

Design of the foundation

Load on the column $= 900{,}000$ *N*

Weight of footing at 10% of the column load $= 90{,}000$ *N*

Total load $= 990{,}000$ *N*

Safe bearing capacity of the soil

$= 100$ *kN/metre*^{2}

$= 100{,}000$ *N/metre*^{2}

$\therefore$ Area of foundation

$$= \frac{990{,}000}{100{,}000} = 9{\cdot}90 \text{ metre}^2$$

Side of footing $= \sqrt{9{\cdot}90} = 3{\cdot}15$ *m*

Provide 3·20 m. × 3·20 m.

Net upward pressure intensity

$$= \frac{900{,}000}{3{\cdot}2 \times 3{\cdot}2} = 87890 \text{ N/metre}^2$$

Depth from B.M. consideration

Critical section for bending moment is shown in Fig. 178, projection beyond the critical section for bending moment

$$\frac{320 - 40}{2} = 140 \text{ cm.}$$

Maximum bending moment

$$= 87 \times 90 \times 3{\cdot}2 \times 1{\cdot}4 \times \frac{1{\cdot}4}{2} \times 100 \text{ N cm.}$$

$$= 27562300 \text{ N cm.}$$

Adopting $C = 5$ *N/mm.*$^2 = 500$ *N/cm*^{2} and $t = 140$ *N/mm.*$^2 = 14000$ *N/cm.*^{2}, and $m = 18$ and equating the moment of resistance to the bending moment at the critical section,

$$85\, bd^2 = 85 \times 40\, d^2 = 27562300$$

$\therefore$ $d = 90$ *cm*.

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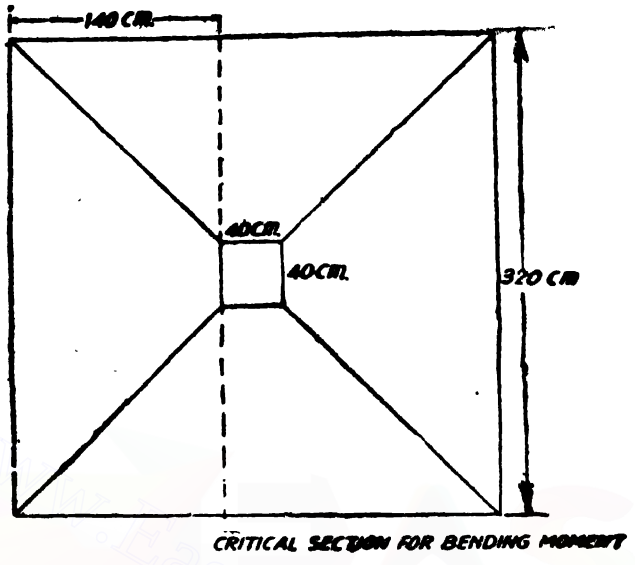

Fig. 178

Let 12 *mm.* diameter bars be provided. Since the reinforcement in the form of a mesh, if a clear cover of 6 *cm.* be provided to the outer layer of bars, then the effective depth to the centre of the upper layer of bars

$$=6{\cdot}00+1{\cdot}20+0{\cdot}60=7{\cdot}8 \text{ cm.}$$

Overall depth required

$$=90+7{\cdot}8=97{\cdot}8 \text{ cm.}$$

Provide a depth of 98 *cm.*

Depth from punching shear consideration

Punching load = Column load − Reaction of soil on column area

$$=900{,}000-87890\times0{\cdot}40\times0{\cdot}40=885940 \text{ N}$$

Permissible punching shear stress

$$=1 \text{ N/mm}^2=100 \text{ N/cm.}^2$$

Equating the punching resistance to the punching load,

$4\times40\times D\times100=885940$ $\quad\therefore\quad D=55{\cdot}4$ *cm.*

Hence let us provide an overall depth of 98 *cm.* as determined from bending moment consideration

Actual effective depth to the centre of the upper layer of bars

$$=98-7{\cdot}8=90{\cdot}20 \text{ cm.}$$

$$A_t=\frac{27562300}{14000\times0{\cdot}87\times90{\cdot}2}=25{\cdot}10 \text{ cm.}^2$$

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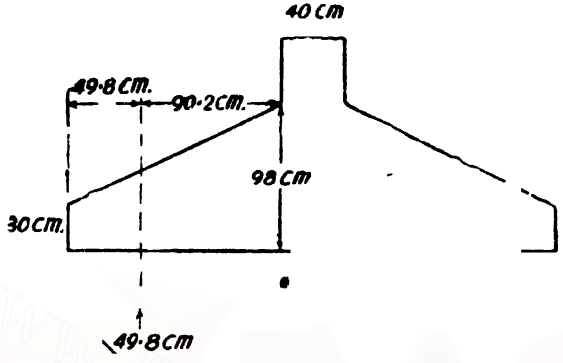
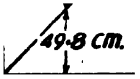
Number of bars of 12 *mm.* diameter

$$\frac{25{\cdot}10}{1{\cdot}13} = 23 \text{ bars.}$$

Similarly provide the same amount of steel, *i.e. 23 bars of 12 mm.* ϕ in the other principal direction also.





CRITICAL SECTION FOR DIAGONAL TENSILE STRESS

Fig. 179

Check for diagonal tension

The critical section for diagonal tension is taken at a distance equal to the effective depth from the column face. Let the depth of footing at the edges be 30 *cm.*

∴ Overall depth at the critical section

$$= 98 - \frac{98-30}{140} \times 90{\cdot}2 \text{ cm.}$$

$$= 54{\cdot}20 \text{ cms.}$$

∴ Effective depth at the critical section

$$d' = 54{\cdot}20 - 7{\cdot}80 \text{ cm.}$$

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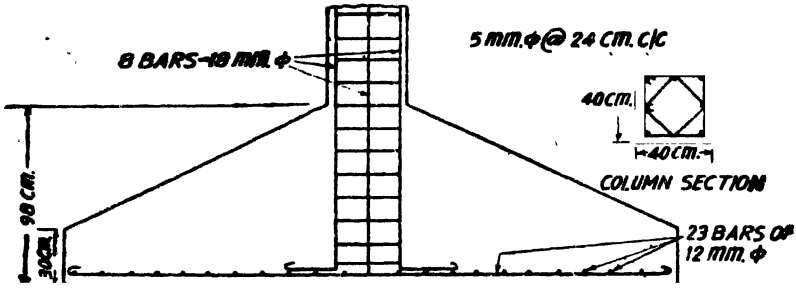
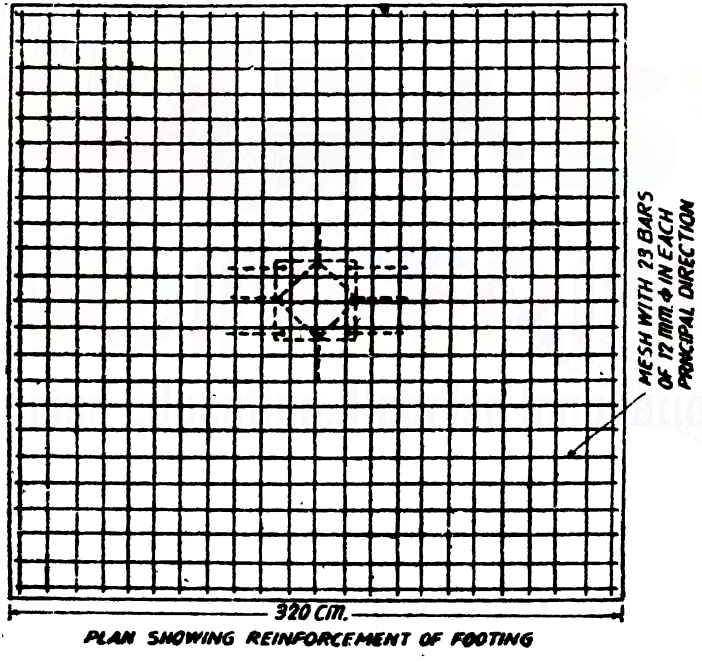
$= 46{\cdot}40$ *cm.*

Shear force at the critical section

$= 87890 \times 3{\cdot}20 \times 0{\cdot}498$ *N*

$= 140060$ *N*





PLAN SHOWING REINFORCEMENT OF FOOTING

Fig. 1a0

Breadth of footing at the top at the critical section

$b' = 40 + 2 \times 90{\cdot}2$ *cm.*

$= 220{\cdot}4$ *cm.*

Nominal shear stress at the critical section

$$= q_v = \frac{w}{b'd'} = \frac{140060}{220{\cdot}4 \times 46{\cdot}4} = 13{\cdot}7 \ N/cm.^2$$

$= 0{\cdot}137$ *N/mm.*^{2}

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Percentage of steel at critical section

$$= \frac{23 \times 1{\cdot}13}{220{\cdot}4 \times 46{\cdot}4} \times 100 = 0{\cdot}25\%$$

For 0·25% steel, $q_c = 0{\cdot}22\ N/mm.^2$

$\therefore$ $q_v < q_c$.

Design 50 (S.I.). *A rectangular column 60 cm. × 40 cm. carries a load of 800 kN. Design a rectangular footing to support the column. The safe bearing capacity of the soil is 200 kN per square metre. Use M 15 concrete.*

Solution.

Load on the column = 800,000 *N*

Weight of foundation @ 10% of the column load = 80,000 *N*

Total load transmitted to the soil = 880,000 *N*

Safe bearing capacity of the soil

$= 200{,}000\ N/metre^2$

$\therefore$ Area of the foundation

$$= \frac{880{,}000}{200{,}000} = 4{\cdot}40\ metre^2$$

The dimensions of the foundation in plan must be such that the projections beyond the column faces are nearly equal.

Let the length of the foundation be *L metres*.

$\therefore$ Breadth $= B = \frac{4{\cdot}450}{L}$ *metre*

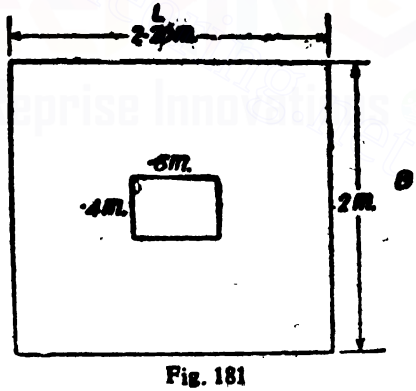

Fig. 181

Equating the projections beyond the column faces,

$$\frac{1}{2}\left[\frac{4{\cdot}40}{L} - 0{\cdot}4\right] = \frac{1}{2}[L - 0{\cdot}6]$$

Solving, we get $L = 2{\cdot}20$ *metre*

$\therefore$ $$B = \frac{4{\cdot}40}{2{\cdot}20} = 2{\cdot}00\ metre$$

Projection beyond column faces

$$= \frac{2{\cdot}20 - 0{\cdot}60}{2} = 0{\cdot}8\ metre$$

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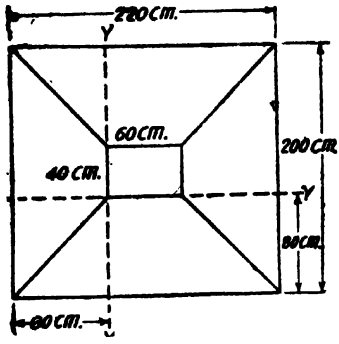

Fig. 182

Net upward pressure on the foundation $= \frac{800,000}{4 \cdot 40}$

$= 181820\ N/metre^2$

B.M. at section XX

$= M_x = 181820 \times 2 \cdot 20 \times \frac{0 \cdot 8^2}{2} \times 100\ N\ cm.$

$= 12800130\ N\ cm.$

Adopting $c = 5\ N/mm.^2$

$= 500\ N/cm.^2$

$t = 140\ N/mm.^2$

$= 1400\ N/cm.^2$

and $m = 18$,

and equating the moment of resistance to the bending moment at the critical section XX, we have,

$$85bd^2 = 85 \times 60\ d^2 = 12800130$$

$$\therefore \quad d = 50 \cdot 10\ cm.$$

B.M. at section $YY = M_y = 181820 \times 2 \times \frac{0 \cdot 8^2}{2} \times 100\ N\ cm.$

$= 11636480\ N\ cm.$

Equating moment of resistance to the bending moment at the critical section YY, we have,

$$85bd^2 = 85 \times 40d^2 = 11636480$$

$$\therefore \quad d = 58 \cdot 50\ cm.$$

Suppose 10 *mm.* diameter bars be provided with a clear cover of 6 *cm.*, the effective cover to the centre of lower layer of bars

$= 6 + 0 \cdot 5 = 6 \cdot 5\ cm.$

Effective cover to the centre of upper layer of bars

$= 6 \cdot 5 + 1 \cdot 0 = 7 \cdot 5\ cm.$

Overall depth required

$= 58 \cdot 5 + 7 \cdot 5 = 66\ cm.$

Steel required parallel to long direction *i.e.* for the bending moment M_y $= \frac{11636480}{14000 \times 0 \cdot 87 \times 58 \cdot 5}\ cm.^2 = 16 \cdot 33\ cm.^2$

Provide 21 bars of 10 mm. diameter.

Reinforcement required parallel to long direction *i.e.* for the bending moment M_x $= \frac{12800130}{14000 \times 0 \cdot 87 \times 59 \cdot 5}\ cm.^2 = 17 \cdot 66\ cm.^2$

Hence, reinforcement required in a central band of 200 *cm.*

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$$= \frac{2}{\beta+1} \times 17{\cdot}66 \text{ cm.}^2$$

$$\frac{220}{200}+1 \times 17{\cdot}66 \text{ cm.}^2 = 16{\cdot}82 \text{ cm.}^2$$

Hence let us provide 22 bars of 10 mm. ϕ within the central band and provide one more bar of 10 mm. ϕ at each end.

Check for diagonal tension.

Critical section $X_1\ X_1$ is taken at a distance equal to the effective depth from the face of the column *i.e.* at a distance 59·5 *cm.* from the face of the column.

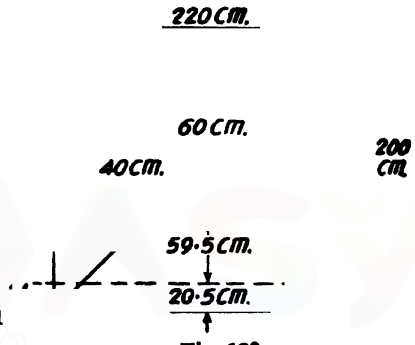

Fig. 183

Shear force at the critical section $X_1\ X_1$

$= 181820 \times 2{\cdot}20 \times 0{\cdot}205 = 82000\ N$

Let the depth of the footing at the ends be 30 *cm.*

Depth of the footing at X_1X_1

$$= 66 - \frac{66-30}{80} \times 59{\cdot}9 \text{ cm.}$$

$= 39{\cdot}20$ *cm.*

Effective depth at X_1X_1

$$= 39{\cdot}20 - 7{\cdot}50 = 31{\cdot}70 \text{ cm.}$$

Breadth of the footing at X_1X_1

$$= b' = 60 + \frac{220-60}{80} \times 59{\cdot}5 = 179 \text{ cm.}$$

Nominal shear stress at this critical section

$$= q_v = \frac{S}{b'd'} = \frac{82000}{179 \times 31{\cdot}7} = 14{\cdot}45\ N/cm.^2$$

$$= 0{\cdot}14\ N/mm.^2$$

Percentage of steel provided

$$= \frac{24 \times 1{\cdot}13}{179 \times 31{\cdot}7} \times 100\% = 0{\cdot}5\%$$

For 0·5% steel, $q_c = 0{\cdot}29\%\ N/mm.^2$ Hence $q_v < q_c$.

Similarly the shear stress can also be checked at a critical section $Y_2\ Y_2$ at a distance of effective depth from the 40 *cm.* face of the column.

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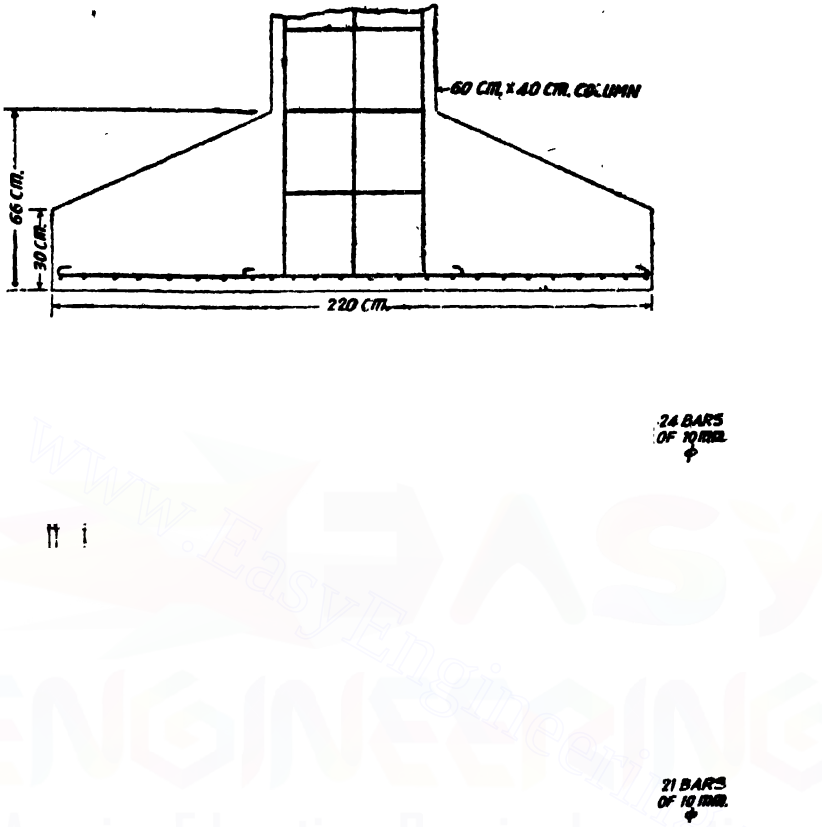

Fig. 184

Design 51. (S.I.). *An R.C.C. footing 50 cm. × 50 cm. has to support an all inclusive load of 550 kN and is also subjected to a moment of 27·5 kNm. Design a footing for the column. The bearing capacity of the soil is 250 kN per sq. metre. Use M 15 concrete.*

Solution.

Total column load = 550 *kN*

Approximate weight of footing (10% of column load) = 55 *kN*

Total = 605 *kN*

= 605000 *N*

Moment = 27·5 *kNm.* = 27500 *Nm.*

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Eccentricity $= \frac{27500}{605000} m = 0{\cdot}0455\ m. = 4{\cdot}55\ cm.$

Area of footing $= \frac{\text{Total load}}{\text{safe bearing capacity}}$

$= \frac{605000}{250000} = 2{\cdot}42\ metre^2$

$\therefore$ Side of foundation $= \sqrt{2{\cdot}42} = 1{\cdot}56\ metre^2$

Provide $1{\cdot}6\ m \times 1\ 6\ m$ footing.

The footing will be so provided that the centre of gravity of the column load will coincide with the centre of gravity of the footing area. The footing is placed symmetrical with respect to the XX axis of the column. See Fig. 185. The projections of the footing beyond the column faces will be

$80 - 4{\cdot}55 - 25$

$= 50{\cdot}45\ cm.$

and $80 + 4{\cdot}55 - 25$

$= 59{\cdot}55\ cm.$ respectively

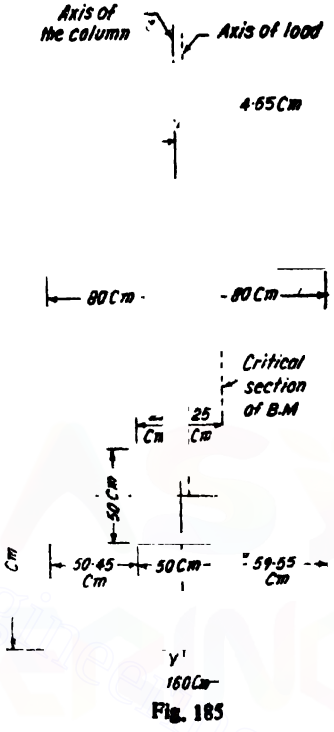

Fig. 185

Net upward pressure intensity

$$= \frac{550000}{1{\cdot}6 \times 1{\cdot}6} = 214840\ N/metre^2$$

Depth of footing

Punching shear consideration

Punching resistance = punching load

$$4 \times 50 \times D \times 100 = 550000 - 214840 \times 0{\cdot}5^2$$

$\therefore$ $D = 24{\cdot}8\ cm.$

Bending moment consideration

B.M. at the critical section

$$= (1{\cdot}6 \times 0{\cdot}5955 \times 214840)\frac{0{\cdot}5955}{2} \times 100\ N\ cm$$

$$= 6094900\ N\ cm.$$

Equating the moment of resistance to the bending moment

$$85 \times 50d^2 = 6094900$$

$\therefore$ $d = 37{\cdot}9\ cm.$

Providing 10 *mm.* diameter bars at a clear cover of 6 *cm.*

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Effective cover $= 0{\cdot}5 + 6 = 6{\cdot}5$ *cm.*

Overall depth $= 37{\cdot}9 + 6{\cdot}5 = 44{\cdot}4$ *cm.*

Provide an overall depth of 45 *cm.*

Actual effective depth

$$= 45 - 6{\cdot}5 = 38{\cdot}5 \text{ cm.}$$

$$\therefore \quad A_t = \frac{6094900}{14000 \times 0{\cdot}87 \times 38{\cdot}5} = 13 \text{ cm.}^2$$

Provide 12 bars of 12 mm. diameter.

We will provide the same reinforcement in the other principal direction also.

Check for diagonal tension

The critical section for diagonal tension is taken at a distance equal to the effective depth from the face of the column, *i.e.* at a distance of 38·5 *cm.* from the column face.

Width of footing at critical section

$$= 50 + \frac{(160 - 50)}{59{\cdot}55} \times 38{\cdot}50$$

$$= 50 + 71{\cdot}12 = 121{\cdot}12 \text{ cm.}$$

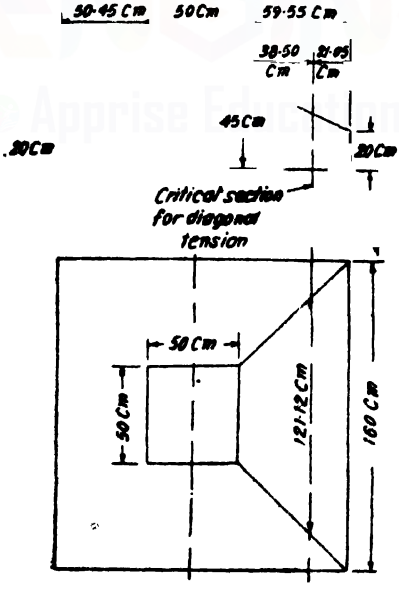

Fig. 186

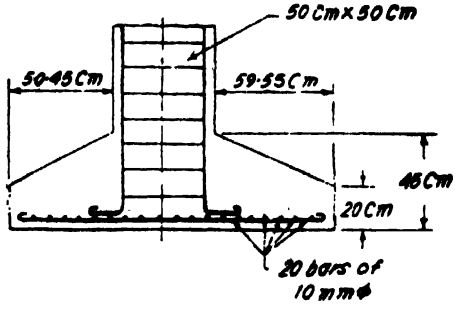

Fig. 187

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Overall depth at this section

$$= 45 - \frac{45-20}{59\ 55} \times 38{\cdot}50$$

$$= 45 - 16{\cdot}16 = 28{\cdot}84\ cm.$$

$\therefore$ Actual effective depth at this section

$$= 28{\cdot}84 - 6{\cdot}50\ cm.$$

$$= 22{\cdot}34\ cm.$$

S.F. at this critical section

$$= S' = 214840 \times 1{\cdot}6 \times 0{\cdot}2105$$

$$= 72360\ N$$

$\therefore$ Nominal shear stress

$$= q_v = \frac{72360}{121{\cdot}12 \times 22{\cdot}34} = 26{\cdot}7\ N/cm.^2$$

$$= 0\ 27\ N/mm.^2$$

Percentage of steel provided

$$= \frac{13{\cdot}93}{121{\cdot}12 \times 22{\cdot}34} \times 100 = 0{\cdot}5\%$$

For 0·5% steel, $q_c = 0{\cdot}29\ N/mm^2$

$\therefore$ $q_v < q_c$

Design 52. *A reinforced concrete column 40 cm. × 40 cm. carries an axial load of 900 kN. Design a stepped foundation for the column. The bearing capacity of soil is 100 kN/metre².*

Solution.

Total column load	$= 900{,}000\ N$
Approximate weight of footing	$= 90{,}000\ N$
Total	$= 990{,}000\ N$

Area of foundation $= \frac{990{,}000}{100{,}000}\ m.^2 = 9{\cdot}9\ m.^2$

$\therefore$ Side of the foundation

$$= \sqrt{9{\cdot}9} = 3{\cdot}15\ m$$

Provide $3{\cdot}20\ m \times 3{\cdot}20\ m$

Net upward pressure intensity

$$p = \frac{900{,}000}{3{\cdot}2 \times 3{\cdot}2} = 87880\ N/metre^2$$

Maximum depth of the footing

Punching shear consideration

Punching resistance = Punching load

$$4 \times 40 \times D \times 100 = 900{,}000 - 87880 + 0{\cdot}4^2$$

$\therefore$ $D = 55{\cdot}4\ cm.$

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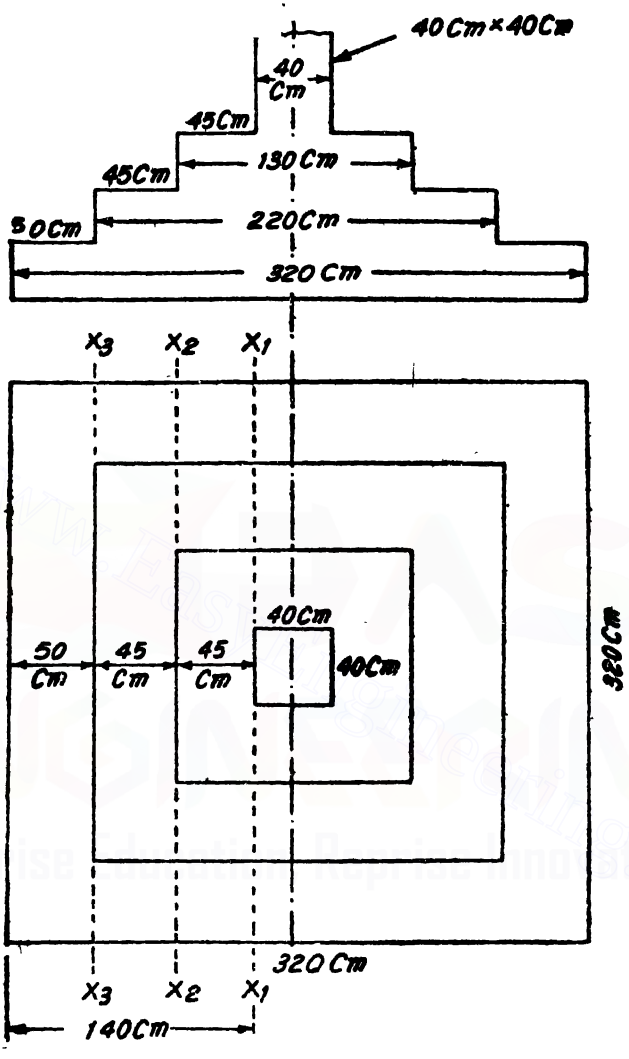

Fig. 188

B.M. Consideration. Let the first step be given at a distance of 45 *cm.* from the column face and the second step given at 90 *cm.* from the column face.

B.M. at the critical section X_1X_1

$$= 87880 \times 3{\cdot}2 \times \frac{1{\cdot}4}{2} \times 1{\cdot}4 \times 100 \ N \ cm.$$

$$= 27560000 \ N \ cm.$$

Equating the moment of resistance to the bending moment

$$85 \times 130 d^2 = 27560000 \quad \therefore \ d = 49{\cdot}9 \ cm.$$

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Providing 12 *mm.* ϕ bars and providing a clear cover of 6 *cm.* for the lower layer of bars,

Effective cover to the centre of the upper layer of bars

$$= 6 + 1{\cdot}2 + 0{\cdot}6 = 7{\cdot}8 \text{ cm.}$$

$\therefore$ Overall depth $= 49{\cdot}9 + 7{\cdot}8 = 57{\cdot}7$ *cm.* say 58 *cm.*

Actual effective depth

$$= 58 - 7{\cdot}8 = 50{\cdot}2 \text{ cm.}$$

$$\therefore \quad A_t = \frac{2756000}{14000 \times 0{\cdot}87 \times 50{\cdot}2} = 45{\cdot}07 \text{ cm.}^2$$

Provide 40 bars of 12 mm. dia (*45·2 cm.*^{2})

Check for diagonal tension. Diagonal tensile stress should be checked at a distance of 50·2 *cm.* from the face of the column. But at this section the depth is less, since this section lies after the first step. Let us therefore determine the depth of the footing after the first step.

Width of the footing after the first step $= 220$ *cm.*

B.M. at $X_2X_2 = 87880 \times 3{\cdot}2 \times 0{\cdot}95 \times \frac{0{\cdot}95}{2} \times 100$ *kg. cm.*

$= 12690000$ *N cm.*

Equating the M.R. to the B.M.

$85 \times 220 \times d^2 = 12690000 \quad \therefore \quad d = 26{\cdot}1$ *cm.*

$\therefore$ Overall depth required $= 26{\cdot}1 + 7{\cdot}8 = 33{\cdot}9$ *cm.*

Provide 35 *cm.* thickness.

To check for diagonal tension, S.F. at the critical section 50·2 *cm.* from the face of the column

$$S = 87880 \times 3{\cdot}2 \times 0{\cdot}898 = 252500 \text{ N}$$

Nominal shear stress

$$q_v = \frac{S}{bd} = \frac{252500}{220 \times 27{\cdot}2} = 40 \text{ N/cm.}^2$$

$$= 0{\cdot}40 \text{ N/mm.}^2$$

Percentage of steel $= \frac{45{\cdot}2 \times 100}{220 \times 27{\cdot}2} = 0{\cdot}75\%$

For 0·75% steel, $q_c = 0{\cdot}35$ *N/mm.*^{2} Excess shear is small.

Depth of footing after the second step

B.M. at the critical section X_3X_3

$$87880 \times 3{\cdot}20 \times 0{\cdot}50 \times \frac{0{\cdot}5}{2} \times 100 \text{ N cm.}$$

$$= 3515200 \text{ N cm.}$$

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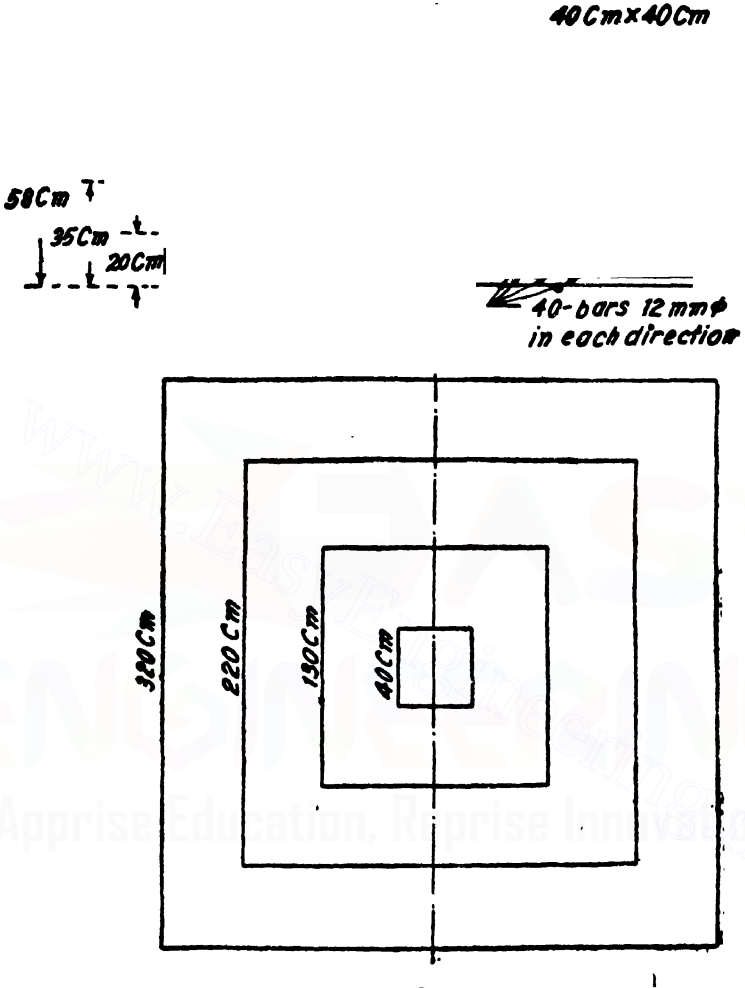

Fig. 189

Equating the M.R. to the B.M.

$$85 \times 320d^2 = 3515200$$

$$\therefore \quad d = 11{\cdot}4 \text{ cm.}$$

$\therefore$ Overall depth required

$$= 11{\cdot}4 + 7{\cdot}8 = 19{\cdot}2 \text{ cm. say } 20 \text{ cm.}$$

Design 53. *A square column 40 cm. side carries an axial load of 90,000 kg. Design the column and a square footing for the column. The bearing capacity of the soil is 10 tonnes per square metre.*

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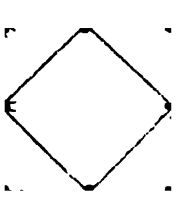
Solution. Design of the column



Safe stress in steel = 1300 *kg./cm.*²
Safe stress in concrete = 40 *kg./cm.*²
Safe load on column
= area of steel × safe stress in steel
+ area of concrete
× safe stress in concrete
$= A_s \times 1300 + (1600 - A_s)\ 40$ *kg.*
= 90,000

$A_s = 20{\cdot}60$ *cm.*²

Provide 8 bars of 18 mm. diameter.

Lateral Ties

Diameter of lateral ties should not be less than one-fourth the diameter of main reinforcement, *i.e.*, in our case the diameter of lateral ties shall not be less than $\frac{18}{4} = 4{\cdot}5$ *mm.* The diameter of ties should also be not less than 5 *mm.* Hence, in our case the diameter of lateral ties may be taken at 5 *mm.*

Pitch of Ties

This shall not be greater than the following :

(*i*) Least lateral dimension
= 40 *cm.*

(*ii*) 16 times the diameter of longitudinal bar
$= 16 \times 1{\cdot}8 = 28{\cdot}8$ *cm.*

(*iii*) 48 times the diameter of transverse reinforcement
$= 48 \times 0{\cdot}5 = 24$ *cm.*

Hence, pitch of 5 mm. diameter links will be 24 cm. c/c.

Design of foundation

Load on the column = 90,000 *kg.*
Weight of footing at 10% of the column load = 9,000 *kg.*

Total load = 99,000 *kg.*

$\therefore$ Area of footing $= \frac{99{,}000}{10{,}000}$ *metre*²
$= 9{\cdot}9$ *metre*²

$\therefore$ Side of footing $= \sqrt{9{\cdot}9} = 3{\cdot}15$ *metre.*

Net upward pressure $= \frac{90{,}000}{3{\cdot}2 \times 3{\cdot}2}$ *kg./metre*²
$= 8788$ *kg./metre.*²

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Depth from B.M. Consideration

Critical section for bending moment is at the column face.

Projection beyond the critical section for bending moment

$$= \frac{320-40}{2} \text{ cm.} = 140 \text{ cms.}$$

Maximum bending moment

$$= 8788 \times 3{\cdot}2 \times 1{\cdot}4 \times \frac{1{\cdot}4}{2} \times 100 \text{ kg. cm.}$$

$$= 2{,}756{,}000 \text{ kg. cm.}$$

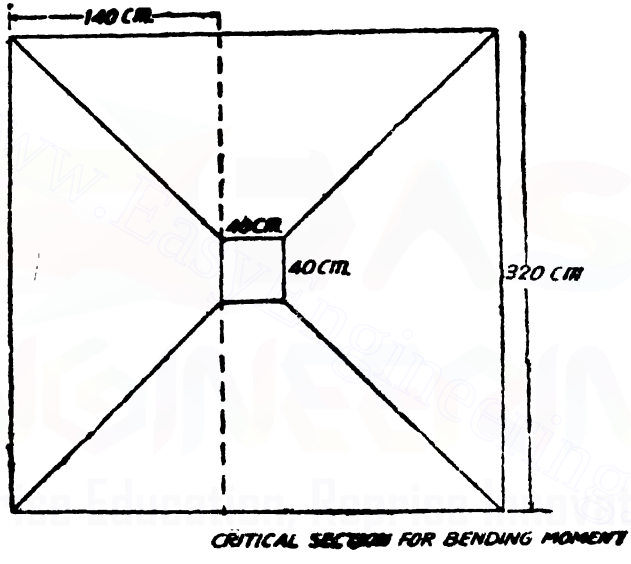

Fig. 191

Adopting $c = 50$ kg./cm.^{2}, $t = 1400$ kg./cm.^{2} and $m = 18$ and equating the moment of resistance to the maximum bending moment, we have

$$8{\cdot}5\, bd^2 = 8{\cdot}5 \times 40\, d^2 = 2{,}756{,}000$$

$$\therefore \quad d = 90 \text{ cms.}$$

Let 12 *mm.* diameter bars be provided. Since the reinforcement is provided in the form of a mesh if a clear cover of 6 *cm.* be provided to the outer layer of bars, the effective cover to the centre of the upper layer of bars

$$= 6{\cdot}00 + 1{\cdot}20 + 0{\cdot}60 \text{ cm.}$$

$$= 7{\cdot}8 \text{ cm.}$$

$\therefore$ Overall depth required

$$= 90 + 7{\cdot}8 \text{ cm.}$$

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$= 97{\cdot}8$ *cm.* say 98 *cm.*

Depth from punching shear consideration

Punching load $= 90{,}000 - 8788 \times 0{\cdot}40 \times 0{\cdot}40$ *kg.*

$= 88{,}594$ *kg.*

Permissible punching shear stress

$= 10$ *kg./cm.*^{2}

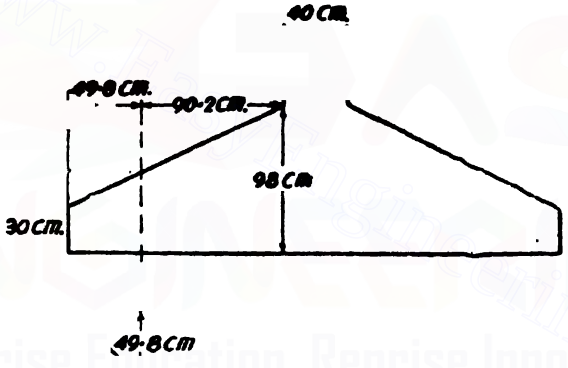
Let the depth of footing be D *cm.*

Equating the punching resistance to the punching load, we have,

$$4 \times 40 \times D \times 10 = 88594$$

$$D = 55{\cdot}4 \text{ cm.}$$

Hence, let us provide an overall depth of 98 *cm.* as determined from bending moment consideration.



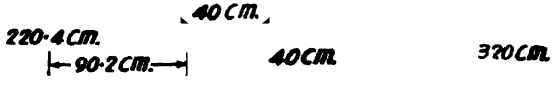


Fig. 192

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Actual effective depth to centre of the upper layer of bars

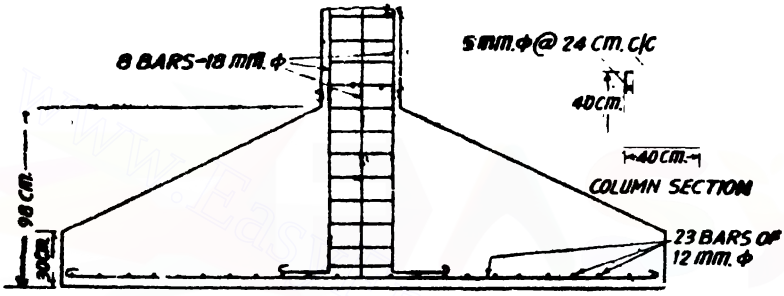
$$= 98 - 7{\cdot}8 = 90{\cdot}2 \text{ cms.}$$

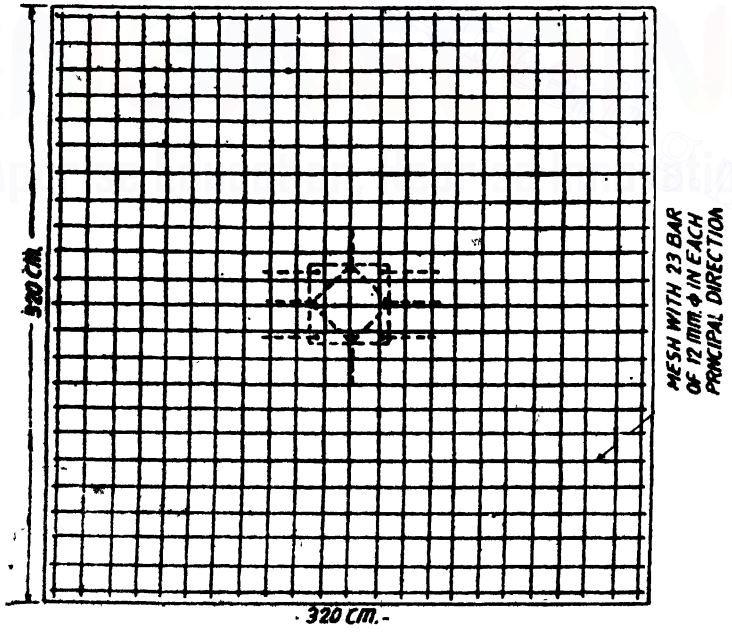
$$\therefore \quad A_s = \frac{2{,}756{,}000}{1400 \times 0{\cdot}87 \times 90{\cdot}2} = 25{\cdot}10 \text{ cm.}^2$$

$\therefore$ Number of bars of 12 *mm.* diameter

$$= \frac{25{\cdot}10}{1{\cdot}13} = 23 \text{ bars.}$$

Similarly, provide the same amount of steel, *i.e., 23 bars of 12 diameter* in the other principal direction also.





PLAN SHOWING REINFORCEMENT OF FOOTING

Fig. 193

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Check for diagonal tension. The critical section for diagonal tension is taken at a distance equal to the effective depth from the column face. Let the depth of the footing at the edges be 30 *cm.*

∴ Overall depth at the critical section

$$= 98 - \frac{98-30}{140} \times 90{\cdot}2 \text{ cm.}$$

$$= 54{\cdot}20 \text{ cm.}$$

Effective depth at the critical section

$$= d' = 54{\cdot}20 - 7{\cdot}80 = 46{\cdot}40 \text{ cm.}$$

Shear force at the critical section

$$= 8788 \times 3{\cdot}20 \times 0{\cdot}498 \text{ kg.}$$

$$= 14010 \text{ kg.}$$

Breadth of the footing at the top of the critical section

$$= b' = 40 + 2 \times 90{\cdot}2 = 220{\cdot}4 \text{ cm.}$$

Nominal shear stress $= q_v = \frac{S}{b'd'} = \frac{14010}{220{\cdot}4 \times 46{\cdot}4}$ kg./cm.^{2}

$$= 1{\cdot}37 \text{ kg./cm.}^2$$

Percentage of steel at critical section

$$= \frac{23 \times 1{\cdot}13}{220{\cdot}4 \times 46{\cdot}4} \times 100\% = 0{\cdot}25\%$$

For 0·25% steel, $q_c = 2{\cdot}20$ kg./cm.^{2}

∴ $q_v < q_c$

Design 54. *A rectangular column 60 cm. × 40 cm. carries a load of 80,000 kg. Design a rectangular footing to support the column. The safe bearing capacity of the soil is 20 tonnes per square metre. Use M 150 concrete.*

Solution. Load on the column = 80,000 *kg.*

Weight of foundation @ 10% column load = 8,000 *kg.*

Total load transmitted to the soil = 88,000 *kg.*

∴ Area of foundation $= \frac{88{,}000}{20{,}000} = 4{\cdot}40$ m.^{2}

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The dimensions of the foundation in plan must be such that the projections beyond the column faces are nearly equal.

Let the length of foundation be L *metres.*

$\therefore$ Breadth $= B = \frac{4 \cdot 0}{L}$ *m.*

Equating the projections beyond the column faces,

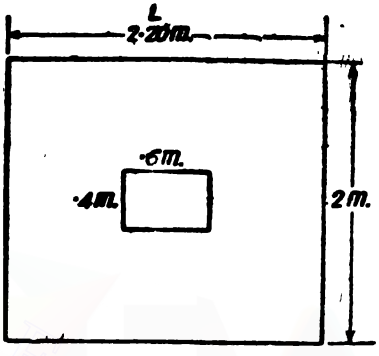

Fig. 194

$$\therefore \quad \frac{1}{2}\left[\frac{4 \cdot 40}{L} - 0 \cdot 4\right] = \frac{1}{2}\left[L - 0 \cdot 6\right]$$

Solving, we get $L = 2 \cdot 20$ *m.*

$$\therefore \quad B = \frac{4 \cdot 40}{2 \cdot 20} = 2 \cdot 00 \text{ metres}$$

Projection beyond column faces

$$= \frac{2 \cdot 20 - 0 \cdot 60}{2} = 0 \cdot 80 \text{ metre.}$$

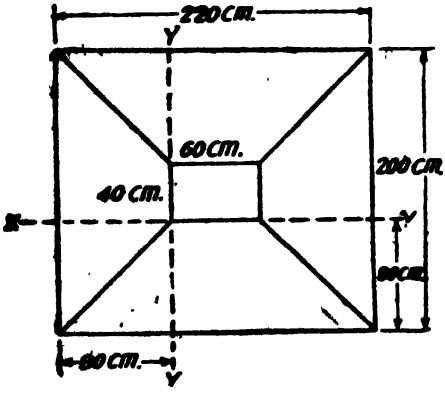

Fig. 195

Net upward pressure on foundation

$$= \frac{80,000}{4 \cdot 40} \text{ kg./m.}^2$$

$$= 18,200 \text{ kg./m.}^2$$

$$= 1 \cdot 82 \text{ kg /cm.}^2$$

Depth from B.M. consideration

Critical section XX and YY for bending moment are shown in Fig. 195.

B.M. at section XX

$$= M_x = 1 \cdot 82 \times 220 \times \frac{80^2}{2} \text{ kg. cm.}$$

$$= 1,281,280 \text{ kg. cm.}$$

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Adopting $c=50$ *kg./cm.*^{2}, $t=1400$ *kg./cm.*^{2} and $m.=18$ and equating the moment of resistance to the maximum bending moment, we have

$$8{\cdot}50 \times 60 \times d^2 = 1{,}281{,}280$$

$$\therefore \quad d = 50{\cdot}12 \text{ cm.}$$

B.M. at section $YY = M_v$

$$= 1{\cdot}82 \times 200 \times \frac{80^2}{2} \text{ kg. cm.}$$

$$\therefore \quad M_v = 1{,}164{,}800 \text{ kg. cm.}$$

Equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}50 \times 40 \times d^2 = 1{,}164{,}800$$

$$\therefore \quad d = 58{\cdot}5 \text{ cm.}$$

Suppose 10 *mm.* dia. bars be provided with a clear cover of 6 *cm.*, the effective cover to the centre of lower layer of bars

$$= 6 + 0{\cdot}5 = 6{\cdot}5 \text{ cm.}$$

Effective cover to the centre of upper layer of bars

$$= 6{\cdot}5 + 1{\cdot}0 = 7{\cdot}5 \text{ cm.}$$

$\therefore$ Overall depth required

$$= 58{\cdot}5 + 7{\cdot}50 \text{ cm.}$$

$$= 66 \text{ cm.}$$

Steel required parallel to long direction, *i.e.* for the bending moment M_v

$$= \frac{1{,}164{,}800}{1400 \times 0{\cdot}87 \times 58{\cdot}5} \text{ cm.}^2$$

$$= 16{\cdot}35 \text{ cm.}^2$$

Provide 21 bars of 10 mm diameter.

Reinforcement required parallel to short direction, *i.e.*, for the bending moment M_u

$$= \frac{1{,}281{,}280}{1400 \times 0{\cdot}87 \times 59{\cdot}5} \text{ cm.}^2$$

$$= 17{\cdot}68 \text{ cm.}^2$$

Hence, reinforcement required in the central band of 200 *cms.*

$$= \frac{2}{\beta + 1} \times 17{\cdot}68$$

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$$\frac{2}{\frac{220}{200}+1} \times 17{\cdot}68$$

$$= 16{\cdot}84 \text{ cm.}^2$$

Hence, let us provide 22 bars of 10 mm. dia. within the central band and *provide one more bar of 10 mm. dia. at each end.*

Check for diagonal tensile stress

Critical section X_1X_1 is taken at a distance equal to the effective depth from the column face, *i.e.*, at a distance 59·5 *cm.* from column face.

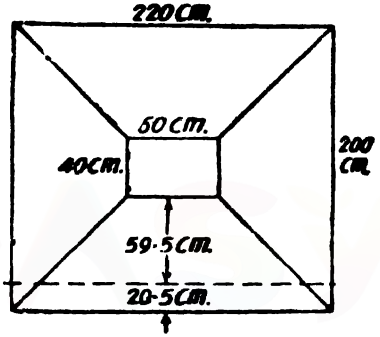

Fig. 196.

Shear force at the critical section X_1X_1

$= 1{\cdot}82 \times 20{\cdot}5 \times 220$ *kg.*

$= 8210$ *kg.*

Let the depth of footing at ends be 30 *cm.*

∴ Depth of footing at X_1X_1

$$= 66 - \frac{(66-30)}{80} \times 59{\cdot}5 \text{ cm.}$$

$$= 39{\cdot}2 \text{ cm.}$$

∴ Effective depth at X_1X_1

$$= 39{\cdot}20 - 7{\cdot}50 = 31{\cdot}70 \text{ cm.}$$

Breadth of footing at X_1X_1

$$= b' = 60 + 2 \times 59{\cdot}5 = 179 \text{ cm.}$$

Nominal shear stress at this critical section

$$= q_v = \frac{S}{b'd'} = \frac{8210}{179 \times 31{\cdot}7} \quad 1{\cdot}45 \text{ kg./cm.}^2$$

Percentage of steel provided

$$= \frac{24 \times 1{\cdot}13}{179 \times 31{\cdot}7} \times 10 = 0{\cdot}5\%$$

For 0·5% steel, $q_c = 2{\cdot}90$ *kg./cm.*^{2} Hence $q_v < q_c$

Similarly the shear stress can also be checked at a critical section Y_2Y_2 at a distance of effective depth from the 40 *cm.* face of the column.

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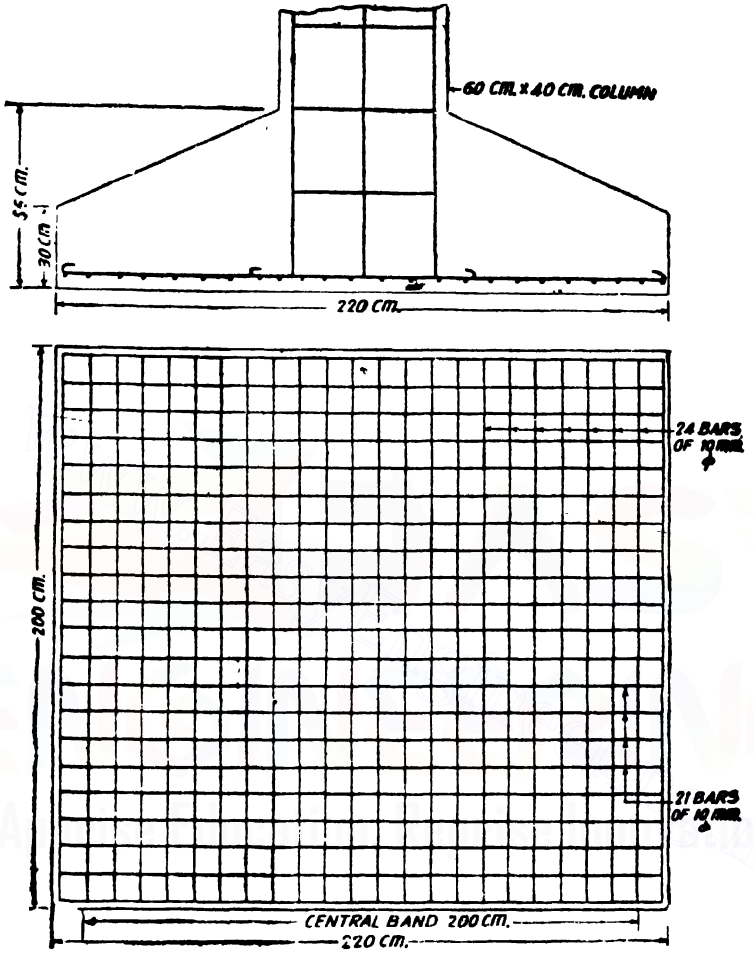

Fig. 197

Design 55. *An R.C.C. footing 50 cm. × 50 cm. has to support an all inclusive load of 55000 kg. and is also subjected to a moment of 275000 kg. cm. Design a footing for the column. The bearing capacity of the soil is 25 tonnes per sq. m.*

Solution.

Total column load		= 55000 *kg.*
Approximate weight of Footing (10% of column load)		= 5500 *kg.*
	Total	=60500 *kg.*
Moment		=275000 *kg. cm.*

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$$\text{Eccentricity} = \frac{275000}{60500} = 4{\cdot}55 \text{ cm.}$$

$$\text{Area of the footing} = \frac{\text{Total load}}{\text{Safe bearing capacity of soil}}$$

$$= \frac{60500}{25000} = 2{\cdot}42 \text{ metre}^2$$

$\therefore$ side of foundition

$$\sqrt{2{\cdot}42} = 1{\cdot}56 \text{ metre}$$

Provide 1·6 *m.* × 1·6 *m.* footing.

The footing will be so provided that the centre of gravity of the column load will coincide with the centre of gravity of the footing area. The footing is placed symmetrical with respect to the *XX* axis of the column. See Fig. 198. The projections of the footing beyond the column faces will be

$$80 - 4{\cdot}55 - 25$$
$$= 50{\cdot}45 \text{ cm.}$$

and
$$80 + 4{\cdot}55 - 25$$
$$= 59{\cdot}55 \text{ cm. respectively}$$

Net upward pressure intensity

$$= \frac{55000}{1{\cdot}6 \times 1{\cdot}6}$$
$$= 21484 \text{ kg./m.}^2$$

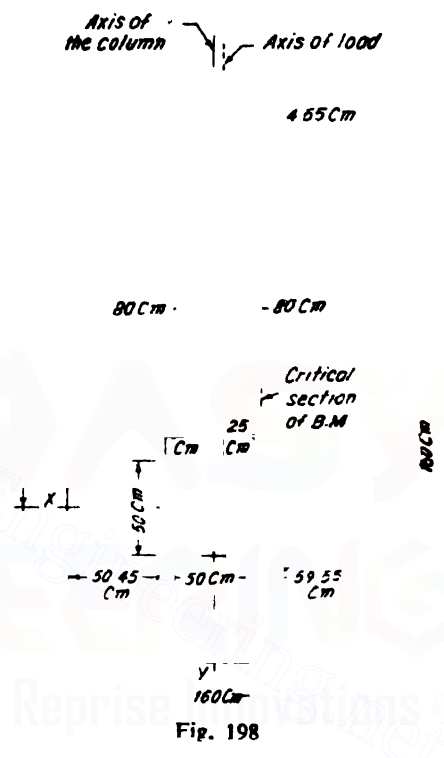

Fig. 198

Depth of footing

Punching shear consideration

Punching resistance = punching load

$$4 \times 50 \times D \times 10 = 55000 - 21484 \times 0{\cdot}5^2$$
$$D = 24{\cdot}8 \text{ cm.}$$

Bending moment consideration

B.M. at the critical section

$$= (1{\cdot}6 \times 0{\cdot}5955 \times 21484) \frac{0{\cdot}5955}{2} \times 100 \text{ kg. cm.}$$
$$= 609490 \text{ kg. cm.}$$

Equating the moment of resistance to the bending moment

$$8{\cdot}5 \times 50\, d^2 = 609490$$
$$d = 37{\cdot}9 \text{ cm.}$$

Providing 10 *mm.* diameter bars at a clear cover of 6 *cm.*

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Effective cover $= 0{\cdot}5 + 6 = 6{\cdot}5$ *cm.*
Overall depth $= 37{\cdot}9 + 6{\cdot}5 = 44{\cdot}4$ *cm.*
Provide an overall depth of 45 *cm.*
Actual effective depth
$$= 45 - 6{\cdot}5 = 38{\cdot}5 \text{ cm.}$$
$$\therefore \quad A_t = \frac{609490}{1400 \times 0{\cdot}87 \times 38{\cdot}5} = 13 \text{ cm.}^2$$

Provide 20 bars of 10 mm. diameter (13·93 cm.²)

We will provide the same reinforcement in the other principal direction also.

Check for diagonal tension

The critical section for diagonal tension is taken at a distance equal to the effective depth from the face of the column, *i.e.*, at a distance of 38 5 *cm.* from the column face.

Width of footing at this critical section
$$= 50 + \frac{(160 - 50)}{59{\cdot}55} \times 38{\cdot}50$$
$$= 50 + 71{\cdot}12 \text{ cm.} = 121{\cdot}12 \text{ cm}$$

Overall depth at this section
$$= 45 - \frac{45 - 20}{59{\cdot}55} \times 38{\cdot}50$$

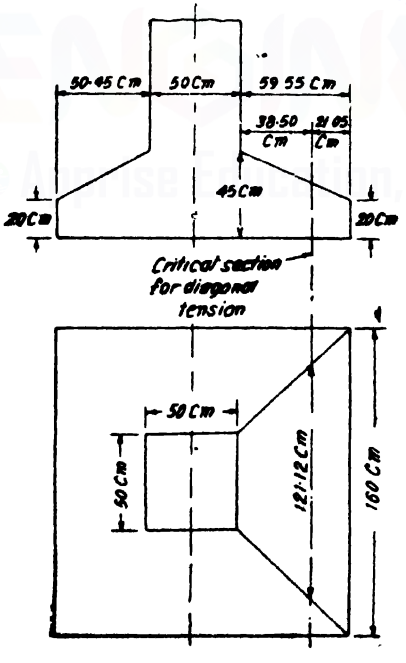

Fig. 199

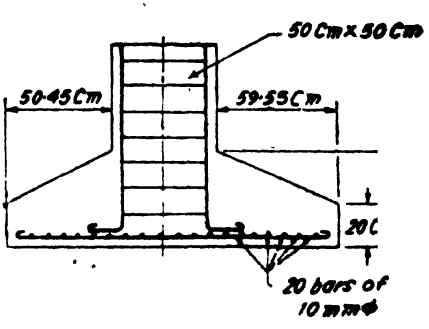

Fig. 200

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$$= 45 - 16{\cdot}16 = 28{\cdot}84 \text{ cm.}$$

$\therefore$ Actual effective depth at this section

$$= 28{\cdot}84 - 6{\cdot}50 \text{ cm.}$$

$$= 22{\cdot}34 \text{ cm.}$$

S.F. at this critical section $= S' = 21484 \times 1{\cdot}60 \times 0{\cdot}2105$ kg.

$$= 7236 \text{ kg.}$$

$\therefore$ Nominal shear stress $= q_v = \frac{S'}{b'd'} = \frac{7236}{121{\cdot}12 \times 22{\cdot}34}$

$$= 2{\cdot}61 \text{ kg./cm.}^2$$

Percentage of steel provided $= \frac{13{\cdot}93}{121{\cdot}12 \times 22{\cdot}34} \times 100 = 0{\cdot}5\%$.

For 0·5% steel, $q_c = 2{\cdot}90$ kg./cm.^{2}

$\therefore$ $q_v < q_c$

Design 56. *A reinforced concrete column 40 cm. × 40 cm. carries an axial load of 90,000 kg. Design a stepped foundation for the column. The bearing capacity of the soil is 10 t per sq. m.*

Solution.

Total column load = 90,000 kg.

Approximate weight of footing. = 9,000 kg.

Total = 99,000 kg.

Area of foundation $= \frac{99{,}000}{10{,}000}$ m.^{2}

$$= 9{\cdot}9 \text{ m.}^2$$

$\therefore$ Side of the foundation $= \sqrt{9{\cdot}9} = 3{\cdot}15$ m.

Provide 3·20 m. × 3·20 m.

Net upward pressure intensity

$$p = \frac{90{,}000}{3{\cdot}2 \times 3{\cdot}2} = 8788 \text{ kg./m.}^2$$

Maximum depth of the footing

Punching shear consideration

Punching resistance = Punching load

$$4 \times 40 \times D \times 10 = 90{,}000 - 8788 \times 0{\cdot}4^2$$

$$D = 55{\cdot}4 \text{ cm.}$$

B.M. Consideration : Let the first step be given at a distance of 45 *cm.* from the column face and the second step be given at 90 *cm.* from the column face.

B.M. at the critical section $x_1x_1 = 8788 \times 3{\cdot}2 \times \frac{1{\cdot}4}{2} \times 1{\cdot}4 \times 100$ kg. cm.

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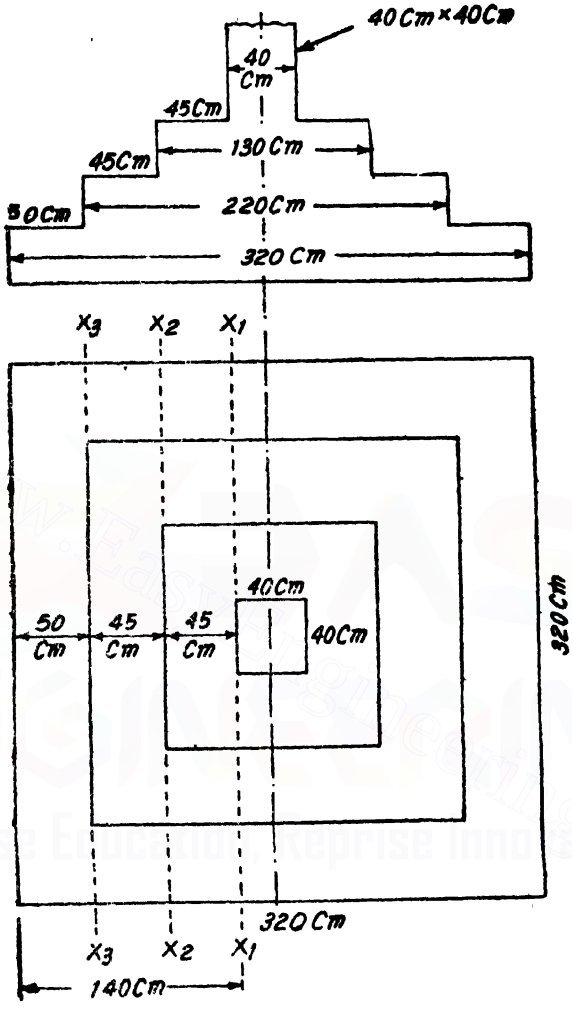

Fig. 201

$= 2756000$ *kg. cm.*

Equating the moment of resistance to the bending moment

$8{\cdot}50 \times 130 \times d^2 = 2,756,000 = d = 49{\cdot}9$ *cm.*

Providing 12 *mm.* ϕ bars and providing a clear cover of 6 *cm.* for the lower layer of bars,

Effective cover to the centre of the upper layer of bars

$= 6 + 1\,2 + 0{\cdot}6 = 7{\cdot}8$ *cm.*

$\therefore$ Overall depth $= 49{\cdot}9 + 7{\cdot}8 = 57{\cdot}7$ *cm.* say 58 *cm.*

Actual effective depth $= 58 - 7{\cdot}8 = 50{\cdot}2$ *cm.*

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$$A_t = \frac{2,756.000}{1400 \times 0.87 \times 50.2} = 45.07 \text{ cm.}^2$$

Provide 40 bars of 12 *mm.* dia (45.2 *cm.*²)

Check for diagonal tension. Diagonal tensile stress should be checked at a distance of 50.2 *cm.* from the face of the column. But at this section the depth is less since this section lies after the first step. Let us therefore determine the depth of the footing after the first step.

Width of the footing after the first step = 220 *cm.*

B.M. at $x_2x_2 = 8,788 \times 3.2 \times 0.95 \times \frac{0.95}{2} \times 100$ *kg. cm.*

$= 1,268,987.2$ *say* 1,269,000 *kg. cm.*

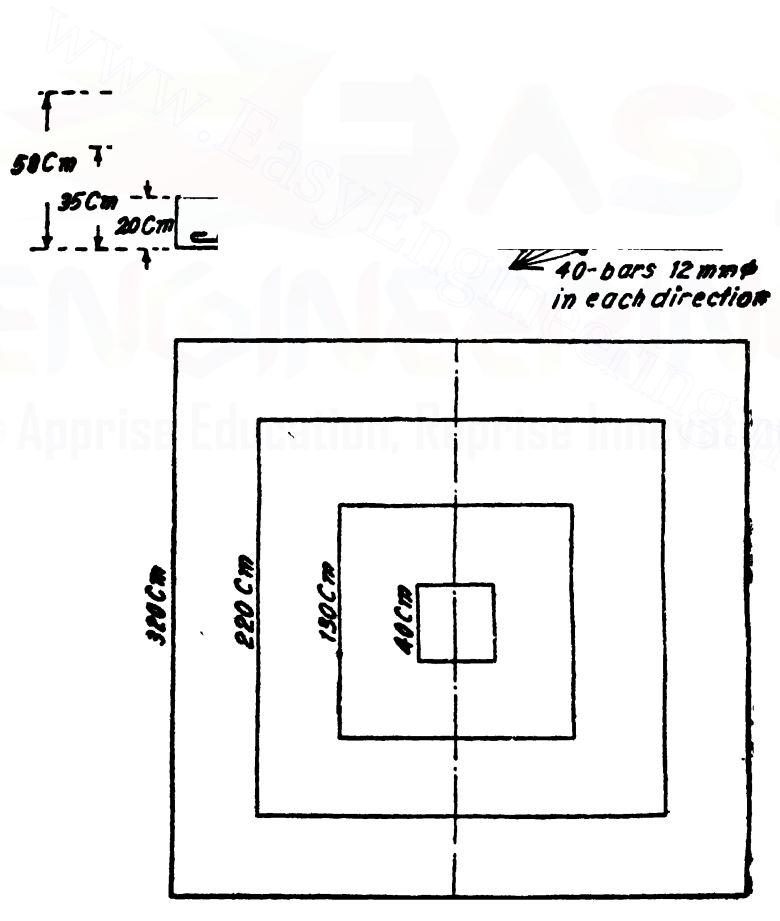

Fig. 202

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Equating the M.R. to the B.M.

$$8{\cdot}5 \times 220 \times d^2 = 1{,}269{,}000 \qquad \therefore \quad d = 26{\cdot}1 \text{ cm.}$$

$\therefore$ Overall depth required $= 26{\cdot}1 + 7{\cdot}8 = 33\,9$ *cm.*

Provide 35 *cm.* thickness.

To check for diagonal tension, S F. at the critical section 50·2 *cm.* from the face of the column

$$S = 8{,}788 \times 3{\cdot}2 \times 0{\cdot}898 = 25{,}250 \text{ kg.}$$

Nominal shear stress $q_v = \dfrac{S}{bd} = \dfrac{25250}{220 \times 27{\cdot}2} = 4 \text{ kg./cm.}^2$

Percentage of steel $= \dfrac{45{\cdot}2 \times 100}{220 \times 27{\cdot}2} = 0{\cdot}75\%$

For 0·75% steel $q_c = 3{\cdot}5 \text{ kg./cm.}^2$

Excess shear stress is small.

Depth of footing after the second step

B.M. at the critical section x_3x_3

$$\therefore \quad 8{,}788 \times 3{\cdot}20 \times 0{\cdot}50 \times \frac{0{\cdot}50}{2} \times 100 \text{ kg. cm.}$$

$$= 351{,}520 \text{ kg. cm.}$$

Equating the M.R. to the B.M.

$$8{\cdot}5 \times 320 \times d^2 = 3{,}51{,}520$$

$$d = 11{\cdot}4 \text{ cm.}$$

Overall depth required $= 11{\cdot}4 + 7{\cdot}8 = 19{\cdot}2$ *cm.* say 20 *cm.*

Design 57. (S.I.) *Design a circular column and a circular footing to the following requirements :*

Diameter of the column $= 50$ *cm.*

Load on the column $= 1200$ *kN.*

Safe bearing capacity of soil $= 180$ *kN/metre*^{2}

Use M 15 concrete.

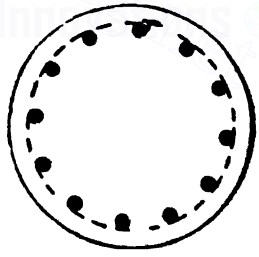

Fig. 203

Solution. Design of the column

Diameter of column $= 50$ *cm.*

Clear cover to longitudinal bars $= 4$ *cm.*

Let us provide 8 *mm* helical bars.

$\therefore$ Clear cover to helical bar $= 4 - 0{\cdot}8 = 3{\cdot}2$ *cm.*

$\therefore$ Core diamater $= 50 - 2 \times 3{\cdot}2 = 43{\cdot}6$ *cm.*

Diameter of column to centre of helical bars

$$= 43{\cdot}6 - 0{\cdot}8 = 42{\cdot}8 \text{ cm.}$$

Gross area of column $= A_g = \dfrac{\pi}{4}(50)^2 = 1963{\cdot}5 \text{ cm.}^2$

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Let the area of longitudinal steel $= A_t\ cm.^2$
Safe stress in concrete $= 4\ N/mm.^2 = 400\ N/cm.^2$
Safe stress in steel $= 130\ N/mm.^2 = 13000\ N/cm.^2$
As per code,

Safe load = 1·05[concrete area × safe stress in concrete + steel area × safe stress in steel]

$$1{\cdot}05[400(1963{\cdot}50 - A_t) + 13000\ A_t] = 1200000\ N$$

$$\therefore \quad A_t = 28{\cdot}36\ cm.^2$$

Number of 18 *mm.* ϕ bars

$$= \frac{28{\cdot}36}{2{\cdot}54} = 12\ bars$$

Provide 8 *mm.* helical reinforcement at a pitch of 8 *cm.*

Length of helix per pitch length

$$= \sqrt{(\pi \times 42{\cdot}8)^2 + 8^2} = 134{\cdot}7\ cm.$$

Volume of helical steel per pitch length

$$= 0{\cdot}50 \times 134{\cdot}7 = 67{\cdot}35\ cm^3$$

Volume of core per pitch length

$$= \frac{\pi}{4}(43{\cdot}6)^2 \times 8 = 11944{\cdot}11\ cm.^3$$

Ratio of volume of helical steel to volume of core

$$= \frac{67{\cdot}35}{11944{\cdot}11} = 0{\cdot}0056$$

This should not be less than $0{\cdot}36\left[\frac{A_g}{A_k} - 1\right]\frac{f_{ck}}{f_y}$

$$= 0{\cdot}36\left[\frac{1963{\cdot}5}{1493} - 1\right]\frac{15}{415} = 0{\cdot}0041$$

Hence, provision of helical reinforcement is satifactory.

Design of the footing

Total column load	$= 1200\ kN$
Approximate weight of footing	$= 120\ kN$
Total	$= 1320\ kN$
Safe bearing capacity of soil	$= 180\ kN\ metre^2$

$$\therefore \text{ Area of foundation} = \frac{1320}{180} = 7{\cdot}33\ metre^2$$

Let the diameter of the footing be x *metre*

$$\therefore \quad \frac{\pi x^2}{4} = 7{\cdot}33 \quad \therefore \quad x = 3{\cdot}05\ m \quad \text{say } 3{\cdot}10\ m.$$

Net upward pressure intensity

$$= \frac{1200 \times 1000}{\pi \times 1{\cdot}55^2} = 159000\ N/metre^2$$

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Depth of footing from B.M. considerations

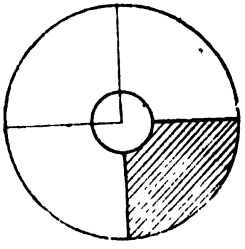

Fig. 204

Consider the shaded area (Fig. 204)

Distance of the centroid of the shaded area from the column axis

$$=0{\cdot}6\left[\frac{R^2+r^2+Rr}{R+r}\right]$$

where, R = radius of the footing

and r = radius of the column

$$=0{\cdot}6\ \frac{155^2+25^2+155\times25}{155+25}\quad 95\ cm.$$

Area shaded: $(R^2-r^2)\ \frac{\pi}{4}\ (1{\cdot}55^2-0{\cdot}25^2)\ metre$

$$=1{\cdot}84\ metre^2$$

Load on the shaded area

$$=159000\times1{\cdot}84=292560\ N$$

Maximum B.M. $=292560(95-25)=20479200\ N\ cm.$

Breadth of the shaded part of the column face

$$=\pi\times\frac{50}{4}=39{\cdot}27\ cm.$$

Adopting $c=5\ N/mm.^2=500\ N/cm.^2$, $t=140\ N/mm.^2=14000\ N/cm.^2$ and $m=18$ and equating the moment of resistance to the maximum bending moment, we have,

$$85\ bd^2=85\times39{\cdot}27\ d^2=20479200$$

$$\therefore\quad d=78\ cm.$$

Providing a clear cover of 30 *mm.* to the lower layer of bars and using 12 *mm.* ϕ bars, overall depth required

$$=78+0{\cdot}6+1{\cdot}2+3{\cdot}0=78+4{\cdot}8=82{\cdot}8$$

say 83 cm.

Depth from punching shear consideration

Let D be the depth from punching shear consideration.

Safe punching shear stress

$$=1\ N/mm.^2=100\ N/cm.^2$$

Equating the punching shear resistance to the punching load,

$$\pi\times50\times100\quad 159000\times\frac{\pi}{4}\ (3{\cdot}10^2-0{\cdot}50^2)$$

$$D=74{\cdot}4\ cm.$$

Hence the depth of the footing is governed by the bending moment consideration.

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Effective depth to the centre of the upper layer of bars

$$= 83 - 4{\cdot}8 = 78{\cdot}2 \text{ cm.}$$

$$\therefore \quad A_t = \frac{20479200}{14000 \times 0{\cdot}87 \times 78{\cdot}2} = 21{\cdot}5 \text{ cm.}^2$$

Provide 19 bars of 12 mm. ϕ.

The reinforcement to the above extent should be provided in each of the two principal directions and in a width equal to the side of a square inscribed in a circle of radius 155 *cm* (radius of the footing).

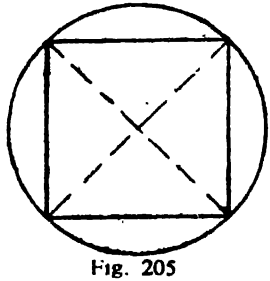

Fig. 205

Length of the side of the inscribed square $= R\sqrt{2} = 155\sqrt{2} = 220$ *cm.*

Let the footing be 30 *cm* deep at the ends.

The critical section for diagonal tension is at a distance equal to the effective depth from the column face, *i.e.* at a distance of 78·2 *cm.* from the column face.

Overall depth at the critical section

$$= 83 - \frac{83-30}{130} \times 78{\cdot}2 \text{ cm.} = 51{\cdot}12 \text{ cm.}$$

Effective depth at the critical section

$$= 51{\cdot}12 - 4{\cdot}80 = 46{\cdot}32 \text{ cm.}$$

Radius of the critical circle

$$= 25 + 78{\cdot}2 = 103{\cdot}2 \text{ cm.}$$

Nominal shear stress at the critical section

$$= q_v = \frac{V}{bd}$$

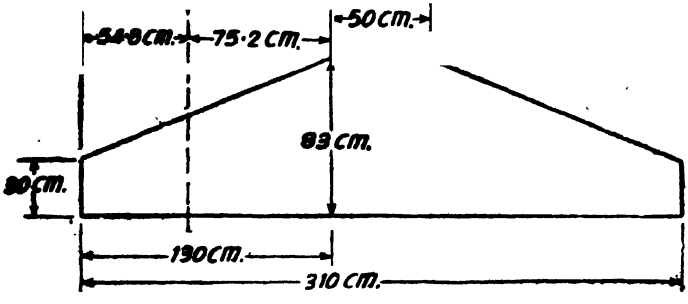

Fig. 206

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$$= \frac{159000\ \pi(1{\cdot}55^2 - 1{\cdot}032^2)}{2\pi \times 103{\cdot}2 \times 46{\cdot}32} = 22{\cdot}2\ N/cm.^2$$

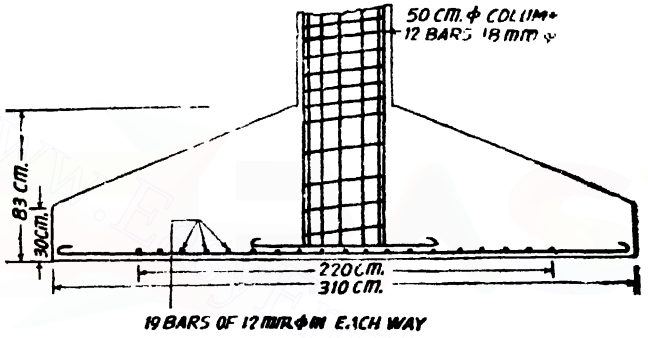
$$= 0{\cdot}222\ N/mm.^2$$

Percentage of steel at this section,

$$= \frac{19 \times 1{\cdot}13 \times 100}{\frac{\pi \times 103{\cdot}2}{2} \times 46{\cdot}32} = 0{\cdot}3\%$$

For 0·3% steel, $q_c = 0{\cdot}24\ N/mm.^2$

$\therefore$ $q_v < q_c$



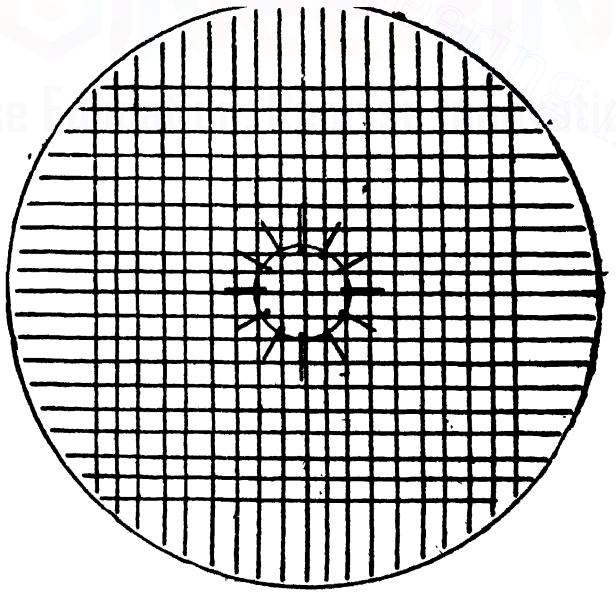


Fig. 207

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Design. 58. *Design a circular column and a circular footing to the following requirements :*

Diameter of the column $= 50$ *cm.*

Load on the column $= 120000$ *kg.*

Safe bearing capacity of soil $= 18$ *tonnes/metre*^{2},

Use M 150 concrete.

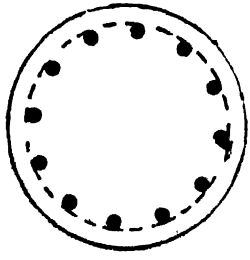

Fig. 208

Solution.

Design of the column

Diameter of column $= 50$ *cm.*

Clear cover to longitudinal bars $= 4$ *cm.*

Let us provide 8 *mm.* helical bars.

$\therefore$ Clear cover to helical bar

$$= 4 - 0{\cdot}8 = 3{\cdot}2 \text{ cm.}$$

$\therefore$ Core diameter $= 50 - 2 \times 3{\cdot}2 = 43{\cdot}6$ *cm.*

Diameter of column to centre of helical bar $= 43{\cdot}6 - 0{\cdot}8$

$$= 42{\cdot}8 \text{ cm.}$$

Gross area of column

$$= A_g = \frac{\pi}{4} (50)^2 = 1963{\cdot}5 \text{ cm}^2$$

Let the area of longitudinal steel $= A_t$ *cm.*^{2}

Safe stress in concrete $= 40$ *kg./cm.*^{2}

Safe stress in steel $= 1300$ *kg./cm.*^{2}

As per I.S. code,

Safe load $= 1{\cdot}05$ [concrete area $\times$ safe stress in concrete + steel area $\times$ safe stress in steel]

$$1{\cdot}05 [40(1963{\cdot}50 - A_t) + 1300 A_t] = 120{,}000 \text{ kg.}$$

$$A_t = 28{\cdot}36 \text{ cm.}^2$$

Number of 18 *mm.* ϕ bars

$$= \frac{28{\cdot}36}{2{\cdot}54} = 12 \text{ bars.}$$

Provide 8 *mm.* helical reinforcement at a pitch of 8 *cm.*

Length of helix per pitch length

$$= \sqrt{(\pi \times 42{\cdot}8)^2 + 8^2} = 134{\cdot}7 \text{ cm.}$$

Volume of the helical steel per pitch length

$$= 0{\cdot}50 \times 134{\cdot}7 = 67{\cdot}35 \text{ cm.}^3$$

Volume of core per pitch length

$$= \frac{\pi}{4} (43{\cdot}6)^2 \times 8 = 11944{\cdot}11 \text{ cm.}^3$$

Ratio of volume of helical steel to volume of core

$$= \frac{67{\cdot}35}{11944{\cdot}11} = 0{\cdot}0056$$

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This should not be less than,

$$0.36\left[\frac{A_g}{A_k}-1\right]\frac{f_{ck}}{f_y}$$

$$=0.35\left[\frac{1963.5}{1493}-1\right]\frac{150}{4150}=0.0041$$

Hence provision of helical reinforcement is satisfactory.

Design of the footing

Total column load = 120,000 *kg.*

Approximate weight of footing = 12,000 *kg.*

Total = 132,000 *kg.*

Safe bearing capacity of soil

$$=18000\ kg./metre^2$$

∴ Area of the footing

$$=\frac{132000}{18000}=7.33\ metre^2$$

Let the diameter of the footing be x *metre*

$$\frac{\pi x^2}{4}=7.33 \qquad x=3.05\ m.\ say\ 3.10\ metre$$

Net upward pressure intensity

$$=\frac{120,000}{\pi\times 1.55^2}=15900\ kg./m.^2$$

Depth of footing from B.M. consideration

Consider the shaded area (Fig. 209).

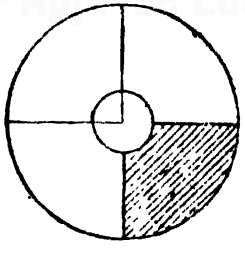

Fig. 209

Distance of the centroid of the shaded area from the column axis

$$=0.6\left(\frac{R^2+r^2+Rr}{R+r}\right)$$

where, R = radius of the footing

r = radius of the column

$$=0.6\,\frac{155^2+25^2+155\times 25}{155+25}=95\ cm$$

Area shaded $=\frac{\pi}{4}(R^2-r^2)=\frac{\pi}{4}(1.55^2-0.25)^2\ metre^2$

$$=1.84\ metre^2$$

Load on the shaded area

$$=15900\times 1.84=29256\ kg.$$

Maximum B.M. $=29256\times(95-25)=2047920$ kg. *cm.*

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Breadth of shaded part of the column face

$$= \pi \times \frac{50}{4} = 39{\cdot}27 \text{ cm.}$$

Adopting $c = 50$ *kg./cm.*^{2}, $t = 1400$ *kg./cm.*^{2} and $m = 18$ and **equating the moment of resistance to the maximum bending moment, we have,**

$$8{\cdot}5\, bd^2 = 8{\cdot}5 \times 39{\cdot}27\, d^2 = 2047920 \quad \therefore \quad d = 78 \text{ cm.}$$

Providing a clear cover of 30 *mm.* **to the lower layer of bars and using** 12 *mm.* **diameter bars, overall depth required**

$$= 78 + 0{\cdot}6 + 1{\cdot}2 + 3{\cdot}0 = 78 + 4{\cdot}8 = 82{\cdot}8 \text{ cm.}$$

say 83 *cm.*

Depth from punching shear consideration

Let D **be the depth from punching shear consideration.**

Equating the punching re istance to the punching load,

$$\pi \times 50 \times 10 = 15900 \times \frac{\pi}{4} (3{\cdot}10^2 - 0{\cdot}50^2)$$

$$D = 74{\cdot}4 \text{ cm.}$$

Hence the depth of the footing is governed by the bending moment consideration.

Effective depth to the centre of the upper layer of bars

$$= 83 - 4\,8 = 78{\cdot}2 \text{ cm.}$$

$$A_t = \frac{2047920}{1400 \times 0{\cdot}87 \times 78{\cdot}2} = 21{\cdot}5 \text{ cm.}^2$$

Provide 19 bars of 12 mm ϕ.

The reinforcement to the above extent should be provided in each of the two principle directions and in a width equal to the side of a square inscribed in a circle of radius 155 *cm.* (radius of the footing).

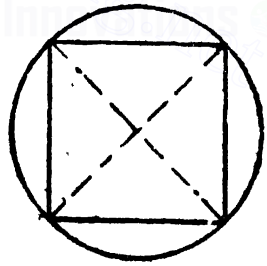

Fig. 210

Length of the side of the inscribed square $= R\sqrt{4} = 155\sqrt{2} = 220$ *cm*

Let the footing be 30 *cm.* **deep at the**

The critical section for diagonal tension is at a distance equal to effective depth from the column face, *i.e.* **at a distance of** 78·2 *cm* **from the column face.**

Overall depth at the critical section

$$= 83 - \frac{83 - 30}{130} \times 78{\cdot}2 = 51{\cdot}12 \text{ cm.}$$

Effective depth at the critical section

$$= 51{\cdot}12 - 4{\cdot}80 = 46{\cdot}32 \text{ cm.}$$

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Radius of the critical circle

$$=25+78{\cdot}2=103{\cdot}2 \ cm.$$

Nominal shear stress at the critical section

$$=q_v=\frac{S}{l\,d}=\frac{15900\times\pi(1{\cdot}55^2-1{\cdot}032^2)}{2\pi\times103{\cdot}2\times46{\cdot}32}=2{\cdot}22 \ kg./cm.^2$$

Percentage of steel at this section

$$=\frac{19\times1{\cdot}13\times100}{\frac{\pi\times103{\cdot}2}{2}\times46{\cdot}32}=0{\cdot}3\%$$

For 0 3% steel, $q_c=2-40 \ kg./cm.^2$

$\therefore \quad q_v<q_c$

§139. Combined Footings

Very often a column is to be provided near the edge of some property and it may not be permissible to extend the footing beyond a certain limit. In such a case the load on the footing will be eccentric, and hence this will result in uneven distribution of load to the supporting soil. Hence, an alternative design would be to provide a common footing to the edge column and to an interior column close to it. The common footing provided to the two columns is called a combined footing. The shape of the combined footing in plan shall be such that the centroid of the foundation plan shall coincide with the centroid of the loads on the two columns. With this arrangement the load on the two columns will be evenly distributed to the soil.

Combined footings are either rectangular or trapezoidal. A rectangular footing is provided in the following circumstances.

(*i*) The width of the foundation is restricted.

or (*ii*) The projection of the footing parallel to the length of the footing is restricted on one side.

A trapezoidal footing is adopted when the projection parallel to the length of the footing is restricted on both the sides, *i.e.*, the trapezoidal footing is adopted when the length of the footing itself is restricted.

Rectangular combined footing. Let A and B represent two columns carrying loads W_a and W_b respectively. Let the left column A be the exterior column and let B be the interior column. Let the columns be l apart between centres. Due to boundary limitations, let the offset on the left side of the exterior column A be restricted to x_a. Now the plan dimensions of the footing can be fixed as follows :

Total column load $= W_a+W_b=(\Sigma W)$

Weight of the foundation (this may be taken equal to 10% of ΣW approximately) $= W_f$

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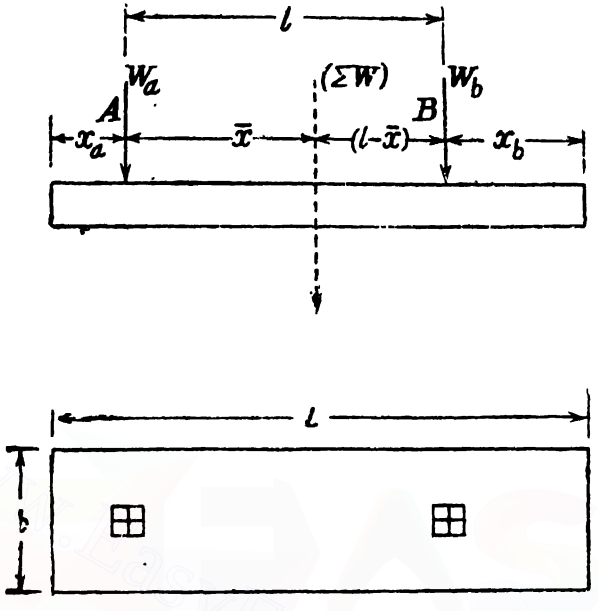

Fig. 211

Total load transmitted to the soil $=(\Sigma W)+W_f$

Area of foundation $= A_f = \dfrac{(\Sigma W)+W_f}{\text{safe bearing capacity of soil}}$

Distance of centre of gravity of column loads from the axis of column $A = \bar{x} = \dfrac{W_b \times l}{(\Sigma W)}$

$\therefore$ Distance of C.G. of column loads from the left edge of the footing

$$= x_a + \bar{x}$$

In order the intensity of the soil reaction on the footing may be uniform, the condition to be satisfied is that the C G. of column loads (in plan) and the C.G. of the foundation plan must coincide.

To satisfy this condition,

Length of the footing $= L = 2[x_a + \bar{x}]$

After determining the length L of the footing the offset x_b on the right side of the column B can be easily determined.

$$x_b = \frac{L}{2} - (l - \bar{x})$$

Since the area of the foundation is already calculated, we can determine the width b of the footing

$$b = \frac{\text{area of foundation}}{}$$

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Thus the plan dimensions of the footing and the exact positions of the columns on the footing are established. Net upward pressure on the footing

$$= p = \frac{\text{Total column load}}{\text{Area of footing}} = \frac{(\Sigma W)}{A_f}$$

Thickness of the footing This is determined from punching shear and bending moment considerations.

Punching shear consideration under column A

If D be the overall depth required, equating the punching resistance to the punching load, we get,

[Perimeter of column A] $\times D \times$ safe punching shear stress $= W_a - p \times$ area of column A.

We can find D from the above relation.

Punching shear consideration under column B

Equating the punching resistance to the punching load, we get,

[Perimeter of column B] $\times D \times$ safe punching shear stress $= W_b - p \times$ area of column B.

B M. Consideration

We can now draw the B.M. diagram for the footing.

Net upward reaction per unit length of the footing.

$$= w = pb$$

B.M. under the column $A = M_a = + \frac{wx_a^2}{2}$

B.M. under the column $B = M_b = + \frac{wx_b^2}{2}$.

Maximum hogging moment will occur at a section between the columns, where the shear force is zero. Let the S.F be zero at a distance x from the left edge. Equating the S.F to zero, we get,

$$wx = W_a$$

$$\therefore \quad x = \frac{W_a}{w}$$

$\therefore$ Max. hogging B.M. $= M_{max} = \frac{wx^2}{2} - W_a (x - x_a)$ where $x = \frac{W_a}{w}$.

This B.M. will be generally the greatest B.M. for the footing.

Equating the M.R. to the max. B.M. we get

$$Qbd^2 = M_{max}$$

$$d = \sqrt{\frac{M_{max}}{Qb}}$$

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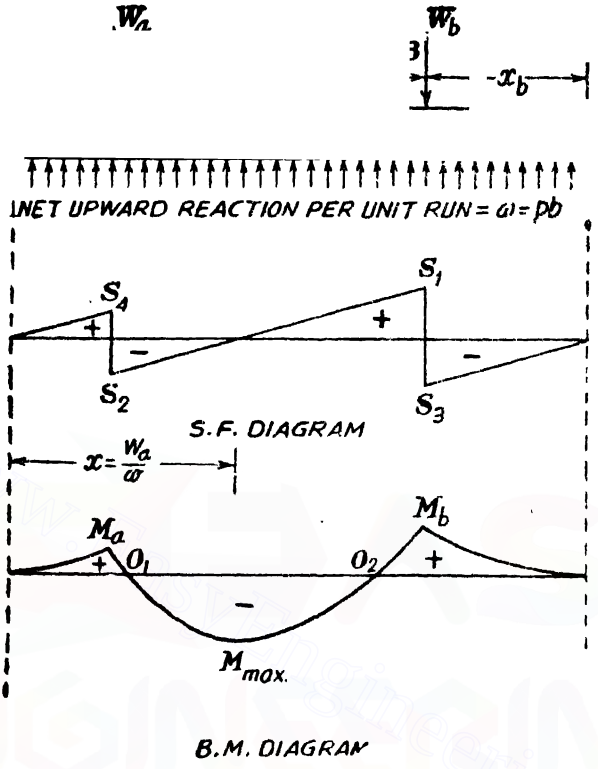

Fig. 212

Providing a clear cover of 60 *mm* and using suitable diameter (20 *mm.* to 22 *mm.*) of bars, overall thickness $= D = d +$ half dia of bar + clear cover.

The greater of overall depths calculated from the punching shear and B.M. considerations should be adopted.

Reinforcement from B M. consideration

Now we can determine the steel required for the above critical bending moments.

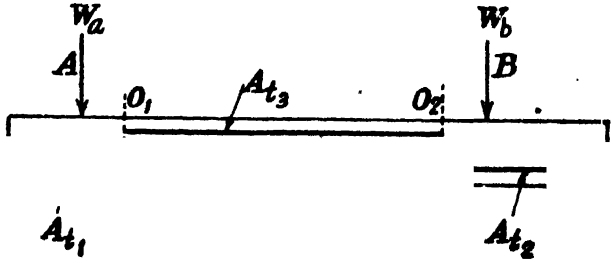

Fig. 213

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Bottom steel required under column $A = A_{t1} = \dfrac{M_a}{t \times 0.87d}$

Bottom steel required under column $B = A_{t2} = \dfrac{M_b}{t \times 0.87d}$

Total steel required for max. hogging moment $= A_{t3}$

$$= \frac{\text{Max. Hogging moment}}{t \times 0.87d}$$

Shear analysis. Now we should determine the shear forces at all critical sections

S.F. at the left edge $= 0$

S F. just on L H.S of column $A = S_4 = +wx_a$

S.F. just on R.H.S. of column $A = S_2 = +wx_a - W_a$

S F. just on R.H.S. of column $B = S_3 = -wx_b$

S.F. just on L.H.S. of column $B = S_1 = -wx_b + W_b$

These shear forces are named as S_1, S_2, S_3 and S_4 in the order of their magnitudes

Nominal shear stresses due to the above shear forces are given by,

$$q_1 = \frac{S_1}{bd}$$

$$q_2 = \frac{S_2}{bd}$$

$$q_3 = \frac{S_3}{bd}$$

and, $$q_4 = \frac{S_4}{bd}$$

Let q_c = permissible shear stress In the zones where the nominal shear stress is less than q_c, nominal eight-legged stirrups will be provided at a nominal spacing of 30 *cm* centres. Where the nominal shear stresses are greater than q_c 12-legged stirrups may be provided. Sufficient longitudinal steel shall also be provided to support the stirrups. To support eight-legged stirrups at least five longitudinal bars are needed at the top and at the bottom. See Fig 214.

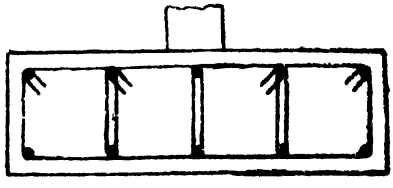

Fig. 214
Section showing eight-legged stirrups.

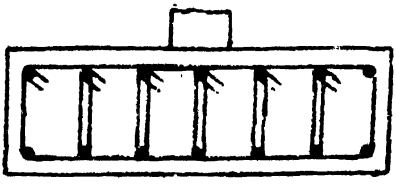

Fig. 215
Section showing twelve-legged stirrups.

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To support 12-legged stirrups *at least 7 longitudinal bars* are needed at the top and the bottom. See Fig. 216.

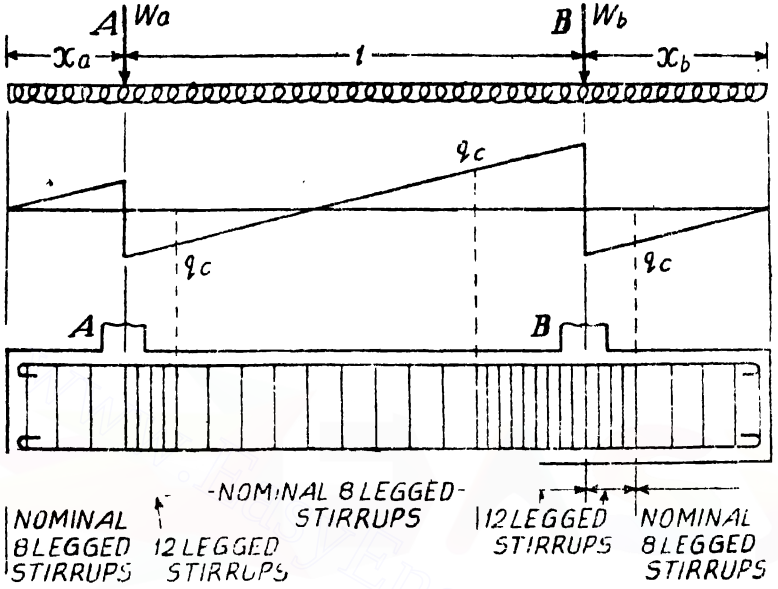

Fig. 216 Disposition of 8-legged and 12-legged stirrups.

Disposition of top and bottom longitudinal bars The following points may be noted while deciding the provision and disposition of the longitudinal reinforcement.

1. Minimum steel requirement for flexure is 0 15 percent of gross area. However, in the case of footings to cover other considerations a minimum of 0·3% of gross area shall be provided. If the steel reinforcement calculated from B.M. consideration is less than the above limit, then at least the minimum requirement specified above may be provided.

2. *Disposition of top bars*

The top bars which have been determined for the maximum hogging bending moment may be provided between outer face of one column to outer face of the other column. Beyond this region as

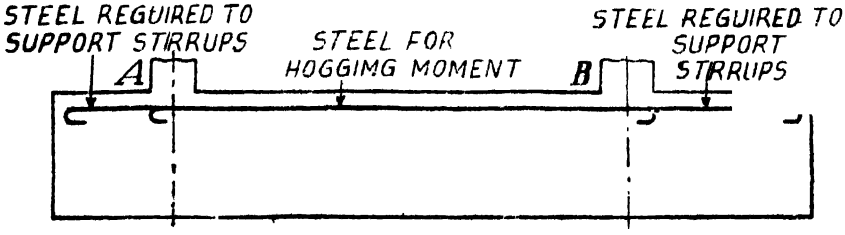

Fig. 217. Disposition of top bars.

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much longitudinal steel as may be needed to support the stirrups may be provided.

3. *Disposition of bottom bars*

Bottom bars under the column A

The bottom bars under the column *A* shall be extended into the interior of the footing to the greater of the following distances

(*i*) 42 diameters from the outer face of the column *A*

(*ii*) up to the point of contraflexure.

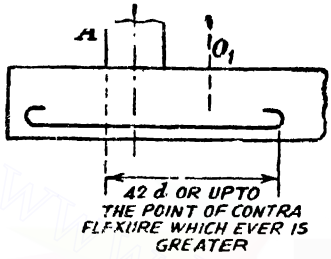

Fig. 218. Disposition of bottom bars under column *A*.

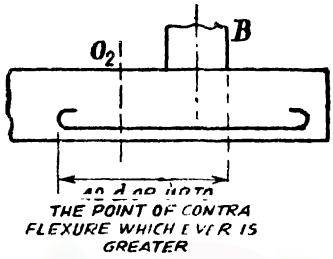

Fig. 219. Disposition of bottom bars under column *B*.

Bottom bars under the column B

The bottom bars under the column *B* should be extended into the interior of the slab for the greater of the following distances :

(*i*) 42 diameters from the outer face of the column *B*

(*ii*) up to the point of contraflexure.

Transverse bending. The footing has also a tendency to bend in a transverse direction. However, such bending is likely to occur for a certain region in the neighbourhood of each column. For design

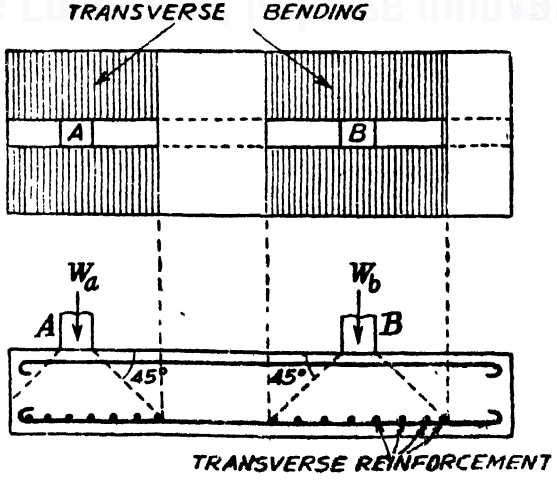

Fig. 220. Region of transverse bending.

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purposes it may be assumed that such transverse bending will occur for a distance equal to the effective depth measured from the face of the column.

Consider a strip of unit width bending in a transverse direction.

Maximum transverse bending moment per unit width.

$$= M = \frac{pf^2}{2}$$

where f = Maximum transverse projection. Now we can calculate the transverse steel required for this bending moment. The transverse steel is placed over the bottom longitudinal bars in the zone of transverse bending.

Design 59 (S I.). *Design a reinforced concrete combined rectangular footing for two columns located 3 60 metres apart. The overall sizes of the columns are 40 cm × 40 cm. and 60 cm × 60 cm. and the loads on them are 1000 kN and 1500 kN respectively The space available for the width of footing is restricted to 180 cm. The safe bearing capacity of the soil is 280 kN per sq. metre. Use M 15 concrete.*

Solution.

Total load on the two columns A and B

$= 1000 + 1500 = 2500$ *kN.*

Approximate weight of footing at 10% of column loads $= 250$ *kN*

Total load transmitted to soil $= 2750$ *kN*

Safe bearing capacity of the soil $= 280$ *kN/m*^{2}

$\therefore$ Area required for the footing $= \frac{2750}{280} = 9{\cdot}82$ *metre*^{2}

Width of footing $= 180$ *cm.* $= 1{\cdot}80$ *metre*

$\therefore$ Length of footing $= \frac{9{\cdot}82}{1{\cdot}80} = 5{\cdot}46$ *m say* $5{\cdot}50$ *metre*

Distance of the resultant of the two column loads from the axis of column $A = \frac{1500 \times 3{\cdot}6}{2500}$ *metre* $= 2{\cdot}16$ *metre*

Projection on the left side of the axis of column A

$= 2{\cdot}75 - 2{\cdot}16 = 0{\cdot}59$ *m.*

Projection on the right side of the axis of column B

$= 2{\cdot}75 - 1{\cdot}44 = 1{\cdot}31$ *m.*

Net upward pressure intensity on the foundation

$$\frac{2500}{5{\cdot}5 \times 1{\cdot}8} = 252{\cdot}5 \text{ kN/metre}^2$$

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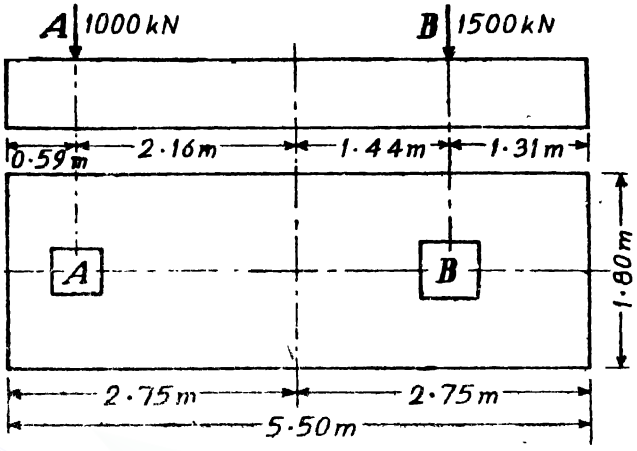

Fig. 221

Net upward load per *metre* length of footing
$= 252{\cdot}5 \times 1{\cdot}8 = 454{\cdot}5$ *kN/metre*

Depth of footing

(*a*) *From punching shear stress consideration under the column A*

Safe punching shear stress $= 1$ *N/mm.*$^2 = 100$ *N/cm.*^{2}

Let D be the depth of the footing in *cm.*

Punching resistance = Punching load,

$$4 \times 40 \times D \times 100 = [1000 - 252{\cdot}5 \times 0{\cdot}4^2]\ 1000$$

$$\therefore \quad D = 60 \text{ cm.}$$

(*b*) *From punching shear consideration under column B*

$$4 \times 60 \times D \times 100 = [1500 - 252{\cdot}5 \times 0{\cdot}60^2]\ 1000$$

$$\therefore \quad D = 58{\cdot}7 \text{ cm.}$$

(*c*) *From bending moment consideration*

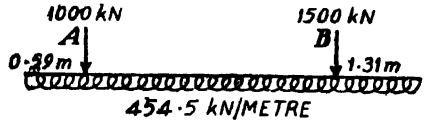

Fig. 222

B.M. under column $A = 454{\cdot}5 \times \frac{0{\cdot}59^2}{2} = 79{\cdot}11$ *kNm (sagging)*

B.M. under column $B = 454{\cdot}5 \times \frac{1{\cdot}31^2}{2} = 389{\cdot}98$ *kNm (sagging)*

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The maximum hogging moment will occur where the shear force is zero.

Let this section be x *metres* from the left end.

Equating the S.F. to zero

$$454{\cdot}5\,x - 1000 = 0$$

$$x = \frac{1000}{454{\cdot}5} \quad 2{\cdot}20 \text{ metre}$$

B.M. at 2·20 *metre* from the left end

$$= M_{max} = 454{\cdot}5 \times \frac{2{\cdot}2^2}{2} - 1000\,(2{\cdot}2 - 0{\cdot}59)\ kNm$$

$$= -510\ kNm$$

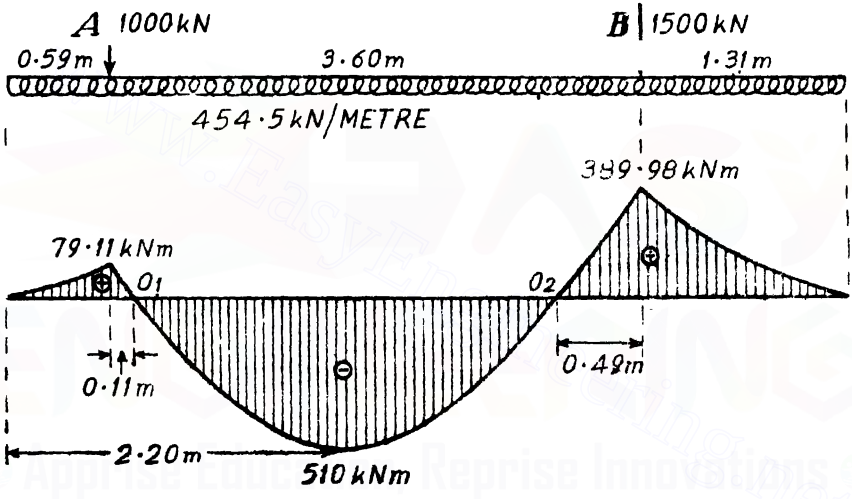

Fig. 223

Taking $c = 5\ N/mm^2 = 500\ N/cm.^2$, $t = 140\ N/mm.^2 = 14000\ N/cm.^2$ and $m = 18$ and equating the moment of resistance to the maximum bending moment, we have,

$$85\,bd^2 = 85 \times 180\,d^2 = 510 \times 1000 \times 100$$

$$\therefore \quad d = 57{\cdot}8\ cm.$$

Let 20 *mm.* diameter bars be used at a clear cover of 6 *cm.*

∴ Overall depth required = 57·80 + 1·00 + 6·00 = 64·8 *cm., say 65 cm.*

Hence the depth of the footing should be 65 *cm.* which is the maximum requirement from all the considerations.

∴ Actual effective depth = 65 − 7 = 58 *cm.*

Bottom steel under column A

$$\frac{79{\cdot}11 \times 1000 \times 100}{14000 \times 0{\cdot}87 \times 58} = 11{\cdot}2\ cm.^2$$

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Minimum steel requirement = 0·3% of gross area

$$= \frac{0{\cdot}3}{100} \times 180 \times 65 = 35{\cdot}10 \ cm.^2$$

Provide 12 bars of 20 mm. ϕ.

Bottom steel under the column B

$$= \frac{389{\cdot}98 \times 1000 \times 100}{14000 \times 0{\cdot}87 \times 58} = 55{\cdot}20 \ cm.^2$$

Number of 20 *mm.* ϕ bars $= \frac{55{\cdot}20}{3{\cdot}14} = 18$ bars.

Top steel for maximum hogging moment

$$\frac{510 \times 1000 \times 100}{14000 \times 0{\cdot}87 \times 58} = 72{\cdot}2 \ cm.^2$$

Number of 20 *mm.* ϕ bars $= \frac{72{\cdot}2}{3{\cdot}14} = 24$ *bars*

Shear force analysis

S.F. on LHS of column $A = +454{\cdot}5 \times 0{\cdot}59 = +268{\cdot}2$ *kN* (S_4)

S.F. on RHS of column $A = +268{\cdot}2 - 1000 = -731{\cdot}8$ *kN* (S_2)

S.F. on RHS of column $B = -454{\cdot}5 \times 1{\cdot}31 = -595{\cdot}4$ *kN* (S_3)

S.F. on LHS of column $B = -595{\cdot}4 + 1500 = +904{\cdot}6$ *kN* (S_1)

Fig. 224 shows the S.F. diagram for the footing.

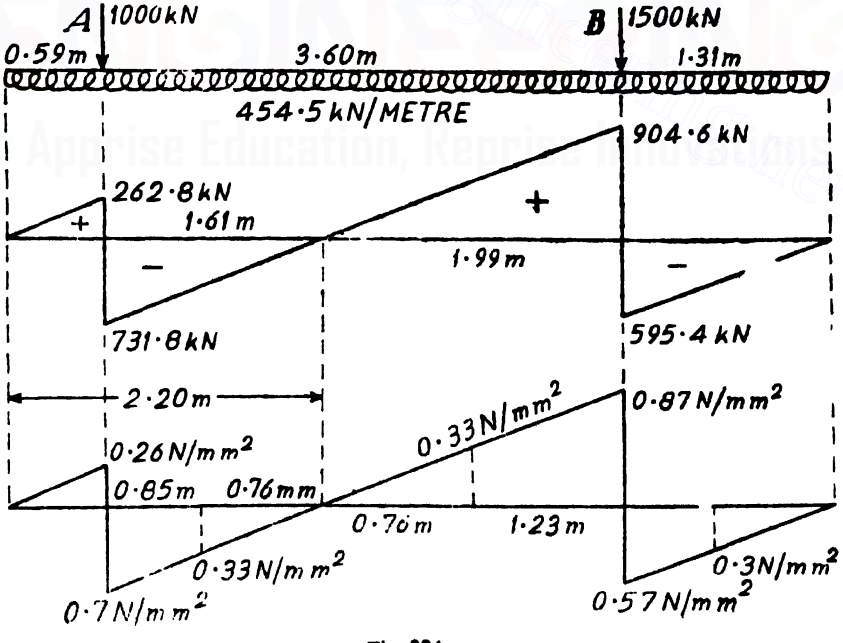

Fig. 224

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The nominal shear stresses corresponding to these shear forces are,

$$q_1 = \frac{S_1}{bd} = \frac{904{\cdot}6 \times 1000}{180 \times 58} = 86{\cdot}6 \ N/cm.^2 = 0{\cdot}87 \ N/mm.^2$$

$$q_2 = \frac{S_2}{bd} = \frac{731{\cdot}8 \times 1000}{180 \times 58} = 70{\cdot}1 \ N/cm.^2 = 0{\cdot}70 \ N/mm.^2$$

$$q_3 = \frac{S_3}{bd} = \frac{595{\cdot}4 \times 1000}{180 \times 58} = 57 \ N/cm^2 = 0 \ 57 \ N/mm.^2$$

$$q_4 = \frac{S_4}{bd} = \frac{268{\cdot}2 \times 1000}{180 \times 58} = 25{\cdot}7 \ N/cm.^2 = 0{\cdot}26 \ N/mm^2.$$

In the region of hogging bending moment

$$\text{percentage of steel provided} = \frac{24 \times 3{\cdot}14}{180 \times 58} \times 100 = 0{\cdot}72\%$$

For 0·72% steel, $q_c = 0{\cdot}33 \ N/mm.^2$

From the point of zero shear, distance of the section where the nominal shear stress is 0·33 $N/mm.^2$ is equal to,

$$\frac{0{\cdot}33}{0{\cdot}7} \times 1{\cdot}61 = 0{\cdot}76 \ m.$$

In the region of sagging moment under the column A,

$$\text{percentage of steel provided} = \frac{12 \times 3{\cdot}14}{180 \times 58} \times 100 = 0{\cdot}36\%$$

For 0·36% steel, $q_c = 0{\cdot}26 \ N/mm.^2$

In the region of sagging moment under the column B

$$\text{percentage of steel provided} = \frac{18 \times 3{\cdot}14}{180 \times 58} \times 100 = 0{\cdot}54\%$$

For 0·54% steel, $q_c = 0{\cdot}3 \ N/mm.^2$

Distance from the right end at which the normal shear stress is 0·3 $N/mm.^2$

$$= \frac{0{\cdot}3}{0{\cdot}57} \times 1{\cdot}31 = 0{\cdot}70 \ m.$$

Shear reinforcement between the two columns

On RHS of column A,

Shearing strength of concrete $= q_c \ bd$

$$= (0{\cdot}33 \times 100) \ 180 \times 58 = 344520 \ N$$

Shear to be resisted by stirrups

$$= V_s = 731800 - 344520 = 387280 \ N$$

Spacing of 12-legged 12 $mm.$ ϕ stirrups $= \frac{A_w \ t_w \ d}{V_s}$

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$$= \frac{12 \times 1{\cdot}13 \times 14000 \times 58}{387280} = 28 \text{ cm.}$$

Provide a spacing of 25 cm. c/c.

On LHS of column B

Shear to be resisted by stirrups $= 904600 - 344520 = 560080$ *N*

Spacing of 12-legged 12 *mm.* ϕ stirrups

$$= \frac{12 \times 1{\cdot}13 \times 14000 \times 58}{560080} = 19{\cdot}7 \text{ cm.}$$

Provide a spacing of 18 cm. c/c.

Shear reinforcement on RHS of column B

Shearing strength of concrete $= q_c\, bd$

$$= (0{\cdot}3 \times 100)\ 180 \times 58 = 313200\ N$$

Shear to be resisted by stirrups

$$= V_s = 595400 - 313200 = 282200\ N$$

Spacing of 12 legged 12 *mm.* ϕ stirrups

$$= \frac{12 \times 1{\cdot}13 \times 14000 \times 58}{282200} = 39 \text{ cm.}$$

Provide a spacing of 30 *cm.* c/c.

S.F. on LHS of column A

= In this case, $q_c = 0{\cdot}26\ N/mm.^2$ and $q_4 = 0{\cdot}26\ N/mm.^2$

$q_4 = q_c$

Provide 8-legged 12 mm. ϕ *stirrups @ 30 cm. c/c.*

Disposition of longitudinal bars

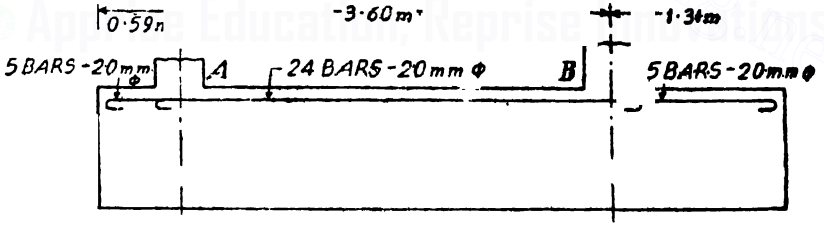

Fig. 225. Disposition of top bars.

The top 24 bars of 20 *mm.* ϕ which have been determined from the maximum hogging moment may be provided between the outer face of one column to outer face of the other column. Beyond this, only as many longitudinal bars as may be necessary to support the stirrups will be provided.

Disposition of bottom bars under the column A

These bottom bars under the column *A* shall be extended into the interior of the footing for the greater of the following distances :

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(*i*) *42d from the outer face of the column A,*

$= 42 \times 2 = 84$ *cm.* from the outer face of the column *A*

$= 84 - 20 = 64$ *cm.* from the centre of the column *A*.

(*ii*) *up to the point of contraflexure* $= 11$ *cm. from the centre of the column A.*

Further beyond, only as many bars as may be necessary to support the stirrups will be provided. Note that where 12-legged stirrups exist at least 7 longitudinal bars are needed to support them. In the zones of 8-legged stirrups five longitudinal bars are sufficient to support the stirrups.

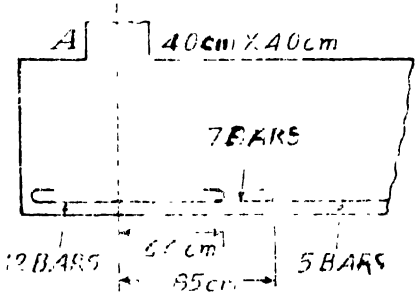

Fig. 226

Disposition of bottom bars under the column B.

These bottom bars under the column *B* shall be extended into the interior of the footing for the greater of the following distances :

(*i*) 42*d* from the outer face of the column *B*

$= 42 \times 2 = 84$ *cm* from the outer face of the column *B*

$= 84 - 30 = 54$ *cm.* from the centre of the column *B*.

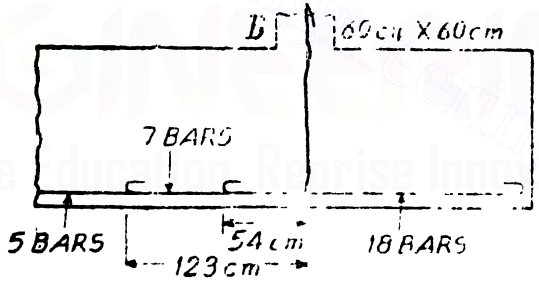

Fig. 227

Beyond this point, only as many longitudinal bars as may be needed to support the stirrups will be provided. Note that where 12-legged stirrups exist, we need at least 7 longitudinal bars to support them. In the regions where 8-legged stirrups exist, 5 longitudinal bars are sufficient to support them.

Transverse reinforcement

The footing slab has also a tendency to bend in a transverse direction. However, such bending is likely to occur for a certain region in the neighbourhood of each column. For design purposes it may be assumed that such a transverse bending will occur for a distance equal to the effective depth measured from the face of the column.

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Transverse projection under the column A

$$= \frac{1{\cdot}80 - 0{\cdot}4}{2} = 0{\cdot}70 \text{ m.}$$

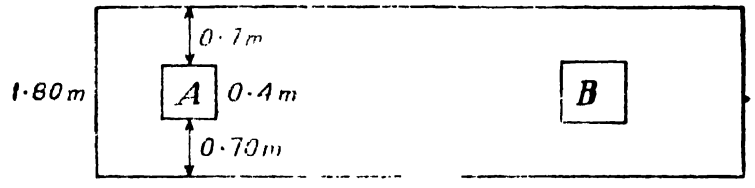

Fig. 228

Transverse B.M. per *metre* width

$$= 252{\cdot}5 \times \frac{0{\cdot}70^2}{2} \text{ kNm} = 61{\cdot}86 \text{ kNm}$$

$$= 61{\cdot}86 \times 1000 \times 100 \text{ N cm.}$$

The reinforcement for the transverse bending moment will be placed just above the main reinforcement of the footing. Hence, the effective depth to the centre of 12 *mm.* diameter transverse steel.

$$= 58 - 1{\cdot}00 - 0{\cdot}6 = 56{\cdot}40 \text{ cm.}$$

$$A_t = \frac{61{\cdot}86 \times 1000 \times 100}{14000 \times 0{\cdot}87 \times 56{\cdot}40} = 9 \text{ cm.}^2$$

Spacing of 12 *mm.* ϕ bars $= \frac{1{\cdot}13 \times 100}{9}$

$= 12{\cdot}5$ *cm. say 12 cm. c/c.*

Similarly the transverse reinforcement under the column B can also be determined. Since the transverse bending moment under the column B is only slightly less than that under the column A, let us provide the same amount of transverse steel under the column B.

Design 60. *Design a reinforced concrete combined rectangular footing for two columns located 3 60 metres apart. The overall sizes of the columns are 40 cm. × 40 cm and 60 cm. × 60 cm. and the loads on them are 100 tonnes and 150 tonnes respectively. The space available for the width of footing is restricted to 180 cms. The safe bearing capacity of the soil is 28 tonnes per sq. metre. Use M 150 concrete.*

Solution.

Total load on the two columns A and $B = 100 + 150 = 250$ *tonnes*

Approximate weight of footing at 10% of colums loads = 25 *tonnes*

Total load transferred to the soil = 275 *tonnes*

Safe bearing capaicty = 28 *tonnes/metre*^{2}

Area required for the footing

$$= \frac{275}{28} \text{ metre}^2$$

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$= 9{\cdot}82$ *metre*^{2}

Width of footing $= 180$ *cm.*

$= 1{\cdot}80$ *metre*

$\therefore$ **Length of footing**

$$= \frac{9{\cdot}82}{1{\cdot}80} = 5{\cdot}46 \text{ metre}$$

say $5{\cdot}50$ *metres*

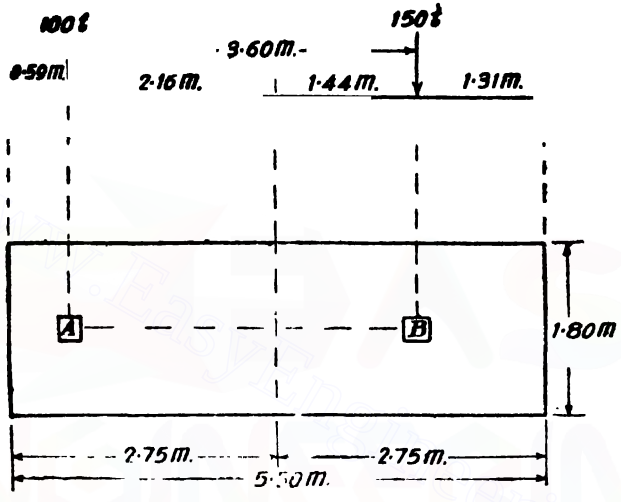

Fig. 229

Distance of centre of gravity of the two loads from the axis of column *A*

$$= \frac{150 \times 3{\cdot}6}{250} \text{ metres.}$$

$= 2{\cdot}16$ *m.*

Projection on the left side of the axis of column *A*

$= 2{\cdot}75 - 2{\cdot}16 = 0{\cdot}59$ *m.*

Projection on the right side of axis of column *B*

$= 2{\cdot}75 - 1{\cdot}44 = 1{\cdot}31$ *m.*

Net upward pressure on the footing

$$= \frac{250}{5{\cdot}5 \times 1{\cdot}8} \text{ tonnes/metre}^2$$

$= 25{\cdot}25$ *tonnes/metre*^{2}

$\therefore$ **Net upward load per** *metre* **length of footing**

$= 25{\cdot}25 \times 1{\cdot}8$

$= 45{\cdot}45$ *tonne/metre*

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Depth of footing

(*a*) *From punching shear consideration under column A*

Let D be the depth of footing in *cms*.

$\therefore \quad (100 - 25{\cdot}25 \times 0{\cdot}40^2)\ 1000 = 4 \times 40 \times D \times 10$

$\therefore \quad D = 60\ cm.$

(*b*) *From punching shear consideration under column B*

$(150 - 25{\cdot}25 \times 0{\cdot}60^2)\ 1000 = 4 \times 60 \times D \times 10$

$D = 58{\cdot}7\ cm.$

(*c*) *From bending moment consideration*

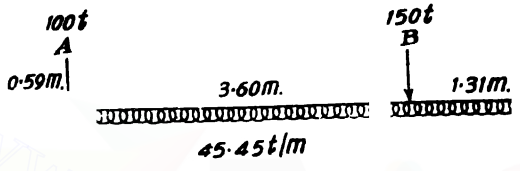

Fig. 230

B.M. under column *A*

$$= 45{\cdot}45 \times \frac{0{\cdot}59^2}{2}\ \textit{tonne metre}$$

$$= 7{\cdot}916\ \textit{tonne metre}$$

$$= 791{,}600\ kg.\ cm\ (\textit{sagging})$$

B.M under column *B*

$$= \frac{45{\cdot}45 \times 1{\cdot}31^2}{2}\ \textit{tonne metre}$$

$$= 39{\cdot}01\ \textit{tonne metre}$$

$$= 3{,}901{,}000\ kg.\ cm.\ (\textit{sagging})$$

The maximum hogging bending moment will occur where the shear force will be zero If this section be x *metres* from the left end, we have, for the condition of zero shear,

$$45{\cdot}45\,x - 100 = 0$$

$$x = \frac{100}{45{\cdot}45}\ m.$$

$$= 2{\cdot}2\ m.$$

B.M. at x $= 2{\cdot}2\ m.$

$$45{\cdot}45 \times 2{\cdot}2^2 - 100(2{\cdot}2 - 0{\cdot}59)\ \textit{tonne metre}$$

$$= -51\ \textit{tonne metre}$$

$$= -5{,}100{,}000\ kg.\ cm.\ (\textbf{hogging})$$

Hence, the greatest bending moment $= 5{,}100{,}000\ kg.\ cm.$

Let the effective depth required be d *cm.*

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Equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}50\ bd^2 = 8{\cdot}50 \times 180\ d^2$$

$$= 5{,}100{,}000\ kg.\ cm.$$

$$d = 57{\cdot}8\ cm.$$

Let 20 *mm.* diameter be used at a clear cover of 6 *cm.*

∴ Overall depth required

$$= 57{\cdot}80 + 1{\cdot}00 + 6{\cdot}00\ cm.$$

$$= 64{\cdot}8\ cm.\ \text{say}\ 65\ cm.$$

Hence, the thickness of the footing should be 65 *cm.* which is the maximum requirement from all the considerations.

∴ Actual effective depth

$$= 65 - 7 = 58\ cm.$$

Reinforcement under the column A

$$= \frac{791{,}600}{1400 \times 0{\cdot}87 \times 58}\ cm.^2 = 11{\cdot}2\ cm.^2$$

Minimum steel requirement = 0·30% of gross area

$$= \frac{0{\cdot}30}{100} \times 180 \times 65 = 35{\cdot}10\ cm.^2$$

Provide 12 bars of 20 mm. ϕ.

Reinforcement under the column B

$$= \frac{3{,}901{,}000}{1400 \times 0{\cdot}87 \times 58}\ cm.^2$$

$$= 55{\cdot}24\ cm.^2$$

∴ Number of 20 *mm.* dia. bars

$$= \frac{55{\cdot}24}{3{\cdot}14} = 18\ bars$$

Reinforcement for maximum hogging bending moment.

$$= \frac{5{,}100{,}000}{1400 \times 0{\cdot}87 \times 58}\ cm.^2$$

$$= 72{\cdot}21\ cm.^2$$

∴ Number of 20 *mm.* dia. bars

$$= \frac{72{\cdot}21}{3{\cdot}14} = 24\ bars$$

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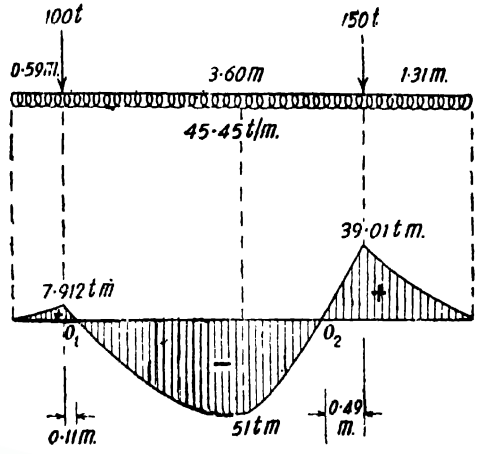

Fig. 231

Shear force analysis

S F. on *LHS* of column *A* (S_4)

$$= +45{\cdot}45 \times 0{\cdot}59 \text{ tonnes}$$

$$= +26{\cdot}82 \text{ tonnes}$$

S.F. on *RHS* of column *A* (S_2)

$$= +26{\cdot}82 - 100$$

$$= -73{\cdot}18 \text{ tonnes}$$

S.F. on *RHS* of column *B* (S_3)

$$= -45{\cdot}45 \times 1{\cdot}31 \text{ tonnes}$$

$$= -59{\cdot}55 \text{ tonnes}$$

S F. on *LHS* of column *B* (S_1)

$$= -59{\cdot}55 + 150 \text{ tonnes}$$

$$= +90{\cdot}45 \text{ tonnes}$$

Fig 232 shows the S.F. diagram for the footing.

The nominal shear stresses corresponding to these shear forces are,

$$q_1 = \frac{S_1}{bd} = \frac{90{\cdot}45 \times 1000}{180 \times 58} = 8{\cdot}66 \text{ kg/cm.}^2$$

$$q_2 = \frac{S_2}{bd} = \frac{73{\cdot}18 \times 1000}{180 \times 58} = 7 \text{ kg./cm.}^2$$

$$q_3 = \frac{S_3}{bd} = \frac{59{\cdot}55 \times 1000}{180 \times 58} = 5{\cdot}70 \text{ kg./cm.}^2$$

$$q_4 = \frac{S_4}{bd} = \frac{26{\cdot}82 \times 1000}{180 \times 58} = 2{\cdot}57 \text{ kg./cm.}^2$$

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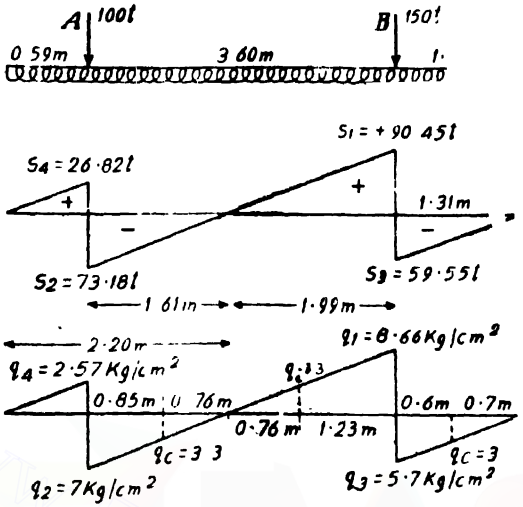

Fig 232

In the region of hogging bending moment

Percentage of steel provided $= \frac{24 \times 3{\cdot}14}{180 \times 58} \times 100 = 0{\cdot}72\%$

For 0·72% steel, $q_c = 3{\cdot}3\ kg./cm.^2$

From the point of zero shear distance of the section where the nominal shear stress is $3{\cdot}3\ kg./cm.^2$ is equal to $\frac{3{\cdot}3}{7} \times 1{\cdot}61 = 0{\cdot}76\ m.$

In the region of sagging moment under the column A,

percentage of steel provided $= \frac{12 \times 3\ 14}{180 \times 58} \times 100 = 0\ 36\%$

For 0·36% steel, $q_c = 2\ 6\ kg./cm.^2$

In the region of sagging moment under the column B,

percentage of steel provided $= \frac{18 \times 3{\cdot}14}{180 \times 58} \times 100 = 0{\cdot}54\%$

For 0·54% steel $q_c = 3\ kg./cm.^2$

Distance from this right end at which the nominal shear stress is $3\ kg./cm.^2$

$$= \frac{3}{5{\cdot}7} \times 1{\cdot}31 = 0{\cdot}70\ m.$$

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Shear reinforcement between the two columns

On RHS coulmn A,

Shearing strength of concrete

$$= q_c\, bd = 3{\cdot}3 \times 180 \times 58 = 34450\ kg.$$

Shear to be resisted by stirrups

$$= V_s = 73180 - 34450 = 38730\ kg.$$

Spacing of 12-legged 12 *mm.* ϕ stirrups

$$= \frac{A_{wt}\, d}{V_s} = \frac{12 \times 1{\cdot}13 \times 1400 \times 58}{38730} = 28\ cm.$$

Provide a spacing of 25 cm c/c

On LHS of column *B*,

Shear to be resisted by stirrups

$$= V_s = 90450 - 34450 = 56000\ kg.$$

Spacing of 12-legged 12 *mm.* ϕ stirrups

$$= \frac{12 \times 1{\cdot}13 \times 1400 \times 58}{56000} = 19{\cdot}7\ cm.$$

Provide a spacing of 18 cm. c/c.

Shear reinforcement on RHS column B

Shearing strength of concrete

$$= q_c\, bd = 3 \times 180 \times 58 = 31320\ kg.$$

Shear to be resisted by stirrups

$$= V_s = 59550 - 31320 = 28230\ kg.$$

Spacing of 12-legged 12 *mm.* ϕ stirrups

$$= \frac{12 \times 1{\cdot}33 \times 140 \times 58}{28230} = 39\ cm.$$

Provide a spacing of 30 cm. c/c.

Shear reinforcement on LHS of column A

In this region $q_c = 2{\cdot}6\ kg./cm.^2$ but $q_4 = 2{\cdot}57\ kg./cm.^2$

$q_4 < q_c$

Hence provide 8-legged 12 mm. ϕ stirrups at 30 cm. c/c.

Disposition of longitudinal bars

Disposition of top bars

The top 24 bars of 20 *mm.* diameter which have been determined for the maximum hogging moment may be provided between the outer face of one column to the outer face of the other column. Beyond this only as many longitudinal bars as are necessary to support the stirrups, will be provided.

Disposition of bottom bars under the column A

These bottom bars under the column *A* shall be extended into the interior of the footing for the greater of the following distances :

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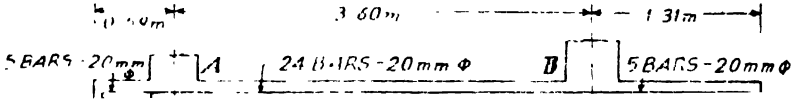

Fig. 233. Disposition of top bars.

(i) 42 d from the outer face of the column A

= 42 × 2 = 84 cm. from the outer face of the column A

= 84 − 20 = 64 cm. from the centre of the column A

(ii) up to the point of contraflexure

= 11½ cm. from the centre of the column A.

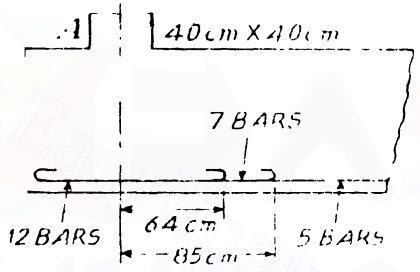

Fig. 234 Disposition of the bottom bars under the column A.

Further beyond only as many bars as may be necessary to support the stirrups will be provided. Note that where 12-legged stirrups exist we need 7 longitudinal bars, to support them. In the zones of 8-legged stirrups 5 longitudinal bars are sufficient to support the stirrups.

Disposition of bottom bars under the column B.

These bottom bars under the column *B* shall be extended into interior of the footing for the greater of the following distances :

(i) 42 d from the outer face of the column B

= 42 × 2 = 84 cm. from the outer face of the column B

= 84 − 30 = 54 cm. from the centre of the column B

(ii) up to the point of contraflexure

= 49 cm. from the centre of the column B

Beyond this point, only as many bars as may be necessary to support the stirrups will be provided. Note that where 12-legged stirrups exist we need at least 7 longitudinal bars to support them. In the zones of 8-legged stirrups, 5 longitudinal bars are sufficient to support the stirrups.

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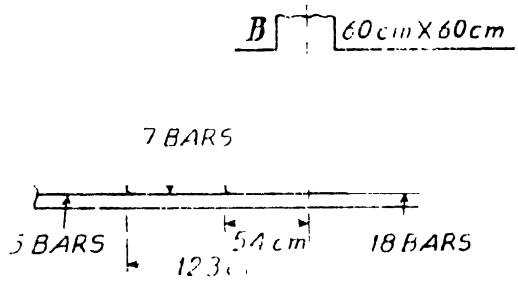

Fig. 235. Disposition of bottom bars under the column B.

Transverse reinforcement

The footing slab has also a tendency to bend in a transverse direction. However, such bending is likely to occur for a certain region in the neighbourhood of each column. For design purposes it may be assumed that such a transverse bending will occur for a distance equal to the effective depth measured from the face of the column.

Transverse projection under the column *A*

$$= \frac{1{\cdot}80 - 0{\cdot}4}{2} \ m. = 0{\cdot}70 \ m.$$

∴ Transverse B.M. per *metre* width

$$= 25{\cdot}25 \times \frac{0{\cdot}70^2}{2} \ \textit{tonne metre}$$

$$= 6{\cdot}186 \ \textit{tonne metre} = 618600 \ \textit{kg. cm.}$$

The reinforcement for the transverse bending moment will be placed just above the main reinforcement of the footing. Hence, the effective depth to centre of 12 *mm.* diameter transverse steel

$$= 58 - 1{\cdot}00 - 0{\cdot}6 = 56{\cdot}4 \ \textit{cm.}$$

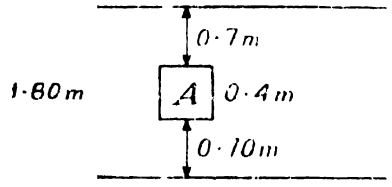

Fig. 236

$$A_t = \frac{618600}{1400 \times 0{\cdot}87 \times 56{\cdot}4} \ cm^2.$$

$$= 9 \ cm.^2$$

∴ Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{9} \ cm.$$

$$= 12 \ cm.$$

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Similarly, the transverse reinforcement under the column *B* can also be determined. Since the transverse bending moment under the column *B* is only slightly less than that under the column *A*, let us provide the same amount of transverse steel under the column *B*.

Design 61 (S I) *Two columns 5 metres apart between centres carry loads of 1600 kN and 2400 kN respectively The sizes of the columns are 60 cm. × 60 cm. and 75 cm. × 75 cm. Design a combined footing for the columns. The projections beyond the centres of the columns parallel to the length of the footing are limited to 0·8 metre and 1·4 metre respectively. Approximate weight of the footing may be taken as 320 kN. Safe bearing capacity of the soil is equal to 300 kN/metre². Use M 15 concrete*

Solution. This is a case where the length of the footing is restricted. Hence a trapezoidal footing should be provided

Total length of the footing

$$= 5 + 0{\cdot}80 + 1{\cdot}40 = 7{\cdot}20 \text{ m}.$$

Fig. 237 shows the footing supporting the two columns.

Let the widths of the footing be *a* and *b* at the left and the right extremities.

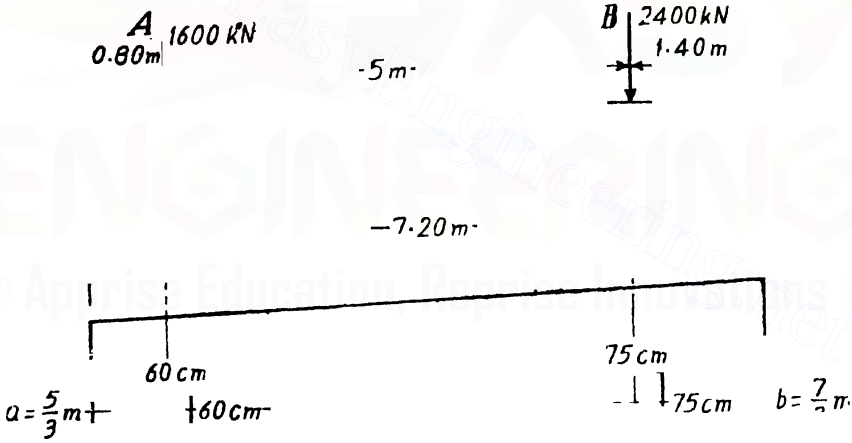

Fig. 237

The dimensions *a* and *b* should be selected such that, not only the necessary area of the footing is provided to transmit the loads to the soil safely, but also, the resultant of the column loads must pass through the centroid of the foundation plan.

Total column load $= 1600 + 2400 = 4000$ *kN*
Weight of footing $= 320$ *kN*

Total load transmitted to soil $= 4320$ *kN*

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Safe bearing capacity of soil = 300 $kN/metre^2$

$\therefore$ Area of footing $= \frac{4320}{300} = 14{\cdot}40\ metre^2$

$\therefore \quad \frac{a+b}{2} \times 7{\cdot}2 = 14{\cdot}40$

$\therefore \quad a+b=4 \qquad ...(i)$

Distance of the resultant of the column loads from the axis of column $A = \frac{2400 \times 5}{4000} = 3\ metre$

$\therefore$ Distance of the centroid of the trapezoidal footing from the left edge $= \frac{a+2b}{a+b} \cdot \frac{7{\cdot}2}{3} = 3{\cdot}8$

$\therefore \quad \frac{a+2b}{a+b} = \frac{19}{12}$

$a+2b = \frac{19}{12}(a+b)$

$\therefore \quad a+2b = \frac{19}{12}(4)$

$\therefore \quad a+2b = \frac{19}{3} \qquad ...(ii)$

But, $a+b=4 \qquad ...(i)$

Solving we get,

$a = \frac{5}{3}\ metre$ and $b = \frac{7}{3}\ metre$

Net upward pressure intensity

$= \frac{4000}{14{\cdot}4} = \frac{2500}{9}\ kN/metre^2$

Some useful general expressions. It is worth obtaining certain general expressions which will be useful in further computations.

(*i*) ***Width of the footing at any section distant x metres from the left edge.*** Let the width be *b' metre* at a section distant *x metre* from the left edge.

$$b' = \frac{5}{3} + \left(\frac{7}{3} - \frac{5}{3}\right)\frac{x}{7{\cdot}2}$$

$$b' = \frac{5}{3} + 0{\cdot}0926\,x$$

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(ii) Net upward soil pressure on the footing for a distance x metre from the left edge.

Area of footing for a distance of *x metres* from the left edge

$$= A = \int_0^x b\,dx = \int_0^x \left(\frac{5}{3} + 0{\cdot}0926\,x\right) dx$$

$$\therefore \qquad A = \frac{5}{3}\,x + 0{\cdot}0463x^2$$

$\therefore$ Net upward soil pressure for a distance of x *metre* from the left end,

$$= V = \frac{2500}{9}\left(\frac{5}{3}\,x + 0{\cdot}0463\,x^2\right)$$

$$\therefore \qquad V = \frac{12500}{27}\,x + 12{\cdot}86\,x^2$$

(iii) Moment of upward soil pressure acting for x metres from the left edge at the section x metres from the left edge.

If M is this moment, then

$$M = V \cdot \frac{b'+2a}{b'+a} \cdot \frac{x}{3}$$

$$= \frac{2500}{9} \cdot \frac{b'+a}{2} \cdot x \cdot \frac{b'+2a}{b'+a} \cdot \frac{x}{3} = \frac{2500}{9\times 2\times 3}\,(b'+2a)\,x^2$$

$$= \frac{2500}{9\times 2\times 3}\left[\frac{5}{3} + 0{\cdot}0926x + \frac{10}{3}\right] x^2 = \frac{6250}{27}\,x^2 + 4{\cdot}29x^3$$

Alternatively, $V = \dfrac{dM}{dx} \quad \therefore \quad M = \displaystyle\int_0^x V\,dx$

$$= \int_0^x \left[\frac{12500}{27}\,x + 12{\cdot}86x^2\right] dx$$

$$\therefore \qquad M = \frac{6250}{27}\,x^2 + 4{\cdot}29x^3$$

The expressions obtained above are the following .

$$b' = \frac{5}{3} + 0{\cdot}0926x \text{ metre}$$

$$V = \frac{12500}{27}\,x + 12{\cdot}86x^2 \text{ N}$$

$$M = \frac{6250}{27}\,x^2 + 4{\cdot}29x^3 \text{ Nm.}$$

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Depth of the footing slab

The depth of the footing slab should be determined from consideration of bending moment and punching shear.

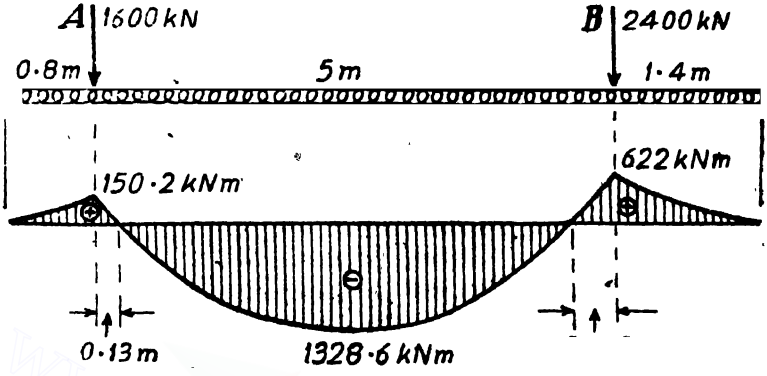

Fig. 238. B.M. diagram.

(i) Depth from B M. consideration

B M. under the column *A*

$$= M = \frac{6250}{27}(0{\cdot}8)^2 + 4{\cdot}29(0{\cdot}8)^3 \ kN\ m$$

$$= 150{\cdot}2\ kN\ m\ (sagging\ moment)$$

B M. under the column *B*

$$= \frac{6250}{27}(5{\cdot}8)^2 + 4{\cdot}29(5{\cdot}8)^3 - 1600 \times 5$$

$$= 622\ kN\ m\ (sagging\ moment)$$

Maximum hogging moment will occur at a section where the S.F. is zero.

Let the S.F. be zero at a distance *x metre* from the left edge.

$$\frac{12500}{27}x + 12{\cdot}86\,x^2 - 1600 = 0$$

$$x^2 + 35{\cdot}99\,x - 124{\cdot}4 = 0$$

Solving, we get, $x = 3{\cdot}175\ m$

B.M. at $x = 3{\cdot}175\ m$,

$$= M_{max} = \frac{6250}{27}(3{\cdot}175)^2 + 4{\cdot}29(3{\cdot}175)^3 - 1600(3{\cdot}175 - 0{\cdot}8)$$

$$= -1328{\cdot}6\ kN\ m\ (hogging\ moment).$$

Fig. 238 shows the B.M. diagram for the footing.

Points of contraflexure

Equating the general expression for bending moment to zero,

$$\frac{6250}{27}(0{\cdot}8)^2 + 4{\cdot}29(0{\cdot}8)^3 - 1600\,(x - 0{\cdot}8) = 0.$$

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Solving by trial and error, $x = 0{\cdot}93\ m$ and $x = 5{\cdot}35\ m$.

Let E and F be the points of contraflexure. See Fig. 238.

Distance of E from the axis of column A

$$= 0\,93 - 0\,80 = 0{\cdot}13\ \textit{metre}$$

Distance of F from the axis of column B

$$= 5{\cdot}80 - 5{\cdot}35 = 0{\cdot}45\ \textit{metre}$$

Width of footing at $x = 3{\cdot}175\ m$,

$$= \frac{5}{3} + 0{\cdot}0926 \times 3{\cdot}175 = 1{\cdot}96\ m = 196\ cm.$$

Equating the M.R. to the maximum B.M.,

$$85 \times 196\ d^2 = 1328{\cdot}6 \times 1000 \times 100 \quad \therefore\ d = 90\ cm.$$

(ii) Depth from punching shear consideration

Allowable punching shear stress $= 1\ N/mm.^2 = 100\ N/cm.^2$

The depth requirement under the column A is given by,

$$\left[1600 - \frac{2500}{9} \times 0{\cdot}6^2\right] 1000 = 4 \times 60 \times D \times 100 \quad \therefore\ D = 62{\cdot}50\ cm.$$

The depth requirement under the column B is given by,

$$\left[2400 - \frac{2500}{9} \times 0\ 75^2\right] 1000 = 4 \times 75 \times D \times 100 \ \therefore\ D = 74{\cdot}80\ cm.$$

We will therefore provide an effective depth of 90 *cm.* and an overall depth of 96 *cm.*

Reinforcement from B.M. consideration

A_s under column $A = \dfrac{150\ 2 \times 1000 \times 100}{14000 \times 0{\cdot}87 \times 90} = 13{\cdot}70\ cm.^2$

This quantity of steel is very small. It is advisable to provide a minimum steel of 0·3% of gross area.

Width of footing at this section

$$= \frac{5}{3} + 0{\cdot}0926 \times 0{\cdot}8 = 1{\cdot}74\ m = 174\ cm.$$

Minimum steel requirement $= \dfrac{0{\cdot}3}{100} \times 174 \times 96 = 50{\cdot}11\ cm.^2$

Number of bars of 22 *mm* $\phi = \dfrac{50{\cdot}11}{3{\cdot}8} = 14\ bars.$

A_s under column $B = \dfrac{622 \times 1000 \times 100}{14000 \times 0{\cdot}87 \times 90} = 56{\cdot}75\ cm.^2$

Width of footing at this section

$$= \frac{5}{3} + 0{\cdot}0926 \times 5{\cdot}8 = 2{\cdot}20\ m = 220\ cm.$$

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Minimum steel requirement $= \frac{0.3}{100} \times 220 \times 96 = 63.36$ *cm.*^{2}

Number of 22 *mm.* ϕ bars $= \frac{63.36}{3.8} = 17$ *bars*

A_t for maximum hogging moment

$$= \frac{1328.6 \times 1000 \times 100}{14000 \times 0.87 \times 90} = 121.3 \text{ cm.}^2$$

Width of footing at this section $= 1.96$ *m* $= 196$ *cm.*

Minimum steel required $= \frac{0.3}{100} \times 196 \times 96 = 56.45$ *cm.*^{2}

Number of 22 *mm* ϕ bars $= \frac{121.3}{3.8} = 32$ *bars.*

Shear Analysis

S.F. just on L.H.S of column A

$$= -\frac{12500}{27} x + 12.86 x^2$$

$$= \frac{12500}{27} \times 0.8 + 12.86(0.8)^2$$

$= -378.5$ *kN* (S_4)

S F. just on R.H.S. of column A

$= +378.5 - 1600 = -1221.5$ *kN* (S_2)

S F. just on L.H.S. of column B

$$= -\frac{12500}{27} \times 5.8 + 12.86 \times 5.8^2 - 1600 \text{ kN}$$

$= +1517.8$ *kN* (S_1)

S.F. just on R.H.S. of column B

$= +1517.8 - 2400 = -882.2$ *kN* (S_3)

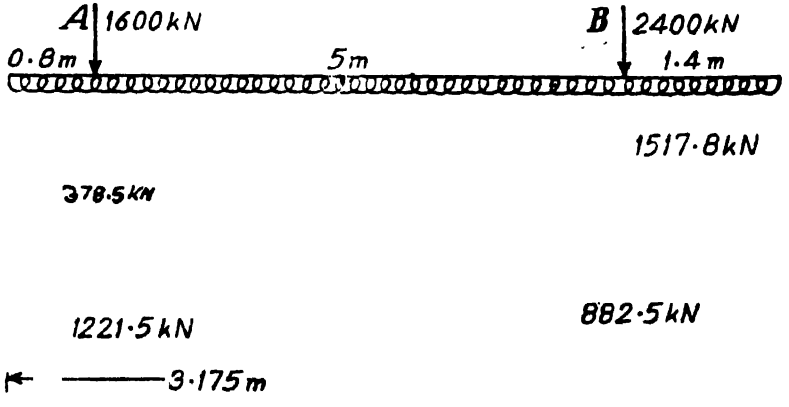

Fig. 239. S.F. diagram.

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Width of footing under the column $A = 174$ *cm.*

Width of footing under the column $B = 220$ *cm.*

Nominal shear stress at any section $= q = \dfrac{S}{b'd}$

where b' = width of the footing at the section considered.

Nominal shear stress due to $S_1 = q_1 = \dfrac{1517.8 \times 1000}{220 \times 90}$

$= 76.7\ N/cm.^2 = 0.77\ N/mm.^2$

Nominal shear stress due to $S_2 = q_2 = \dfrac{1221.5 \times 1000}{174 \times 90}$

$78\ N/cm^2 = 0.78\ N/mm.^2$

Nominal shear stress due to $S_3 = q_3 = \dfrac{882.2 \times 1000}{220 \times 90}$

$= 44.5\ N/cm^2 = 0.45\ N/mm.^2$

Nominal shear stress due to $S_4 = q_4 = \dfrac{378.5 \times 1000}{174 \times 90}$

$= 24.2\ N/cm.^2 = 0.24\ N/mm.^2$

Shear reinforcement for the footing between the columns

Under the column A percentage of steel $= \dfrac{32 \times 3.8 \times 100}{196 \times 90} = 0.69\%$

For 0.69% steel, $q_c = 0.33\ N/mm.^2 = 33\ N/cm.^2$

Let in the zone of negative shear, the nominal shear stress be 33 $N/cm.^2$ at a distance x *m* from the left end.

$$\frac{S}{b'd} = -33 \quad S = -33\, b'd$$

$$-33 \times 90\left[\frac{5}{3} + 0.0926\, x\right] 100 = \left[\frac{12500}{27}\, x + 12.86\, x^2 - 1600\right] 1000$$

$\therefore\ x^2 + 38.14\ x - 85.92 = 0.$

Solving we get $x = 2.10$ *m.* from the left end

or $2.10 - 0.80 = 1.3$ *m.* from the axis of column *A*.

Shear resistance of concrete

$= q_c\, b'd = 33 \times 174 \times 90 = 516780\ N$

Shear force $= 1221500\ N$

$\therefore$ Shear to be resisted by stirrups

$= V_s = 1221500 - 516780 = 704720\ N$

Spacing of 12-legged 12 *mm* ϕ stirrups $= \dfrac{A_{sv} f_{sd}}{V_s}$

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$$= \frac{12 \times 1{\cdot}13 \times 14000 \times 90}{704720}$$

$$= 24 \text{ cm. say } 20 \text{ cm. c/c.}$$

Under the column B, percentage of steel $= \frac{32 \times 3{\cdot}8 \times 100}{220 \times 90} = 0{\cdot}61\%$

For 0·61% steel, $q_c = 0{\cdot}31 \ N/mm.^2 = 31 \ N/cm.^2$

Let in the zone of positive shear, the nominal shear stress be 31 $N/cm.^2$ at a distance of x *metres* from the left end.

$$\therefore \quad \frac{S}{b'd} = +31$$

$$\therefore \quad S = +31 \ b'd$$

$$+31 \times 90 \left[\frac{5}{3} + 0{\cdot}0926 \ x\right] 100 = \left[\frac{12500}{27} x + 12{\cdot}86 \ x^2 - 1600\right] 1000$$

$$x^2 + 34{\cdot}05 \ x - 160{\cdot}76 = 0$$

Solving, we get, $x = 4{\cdot}20$ *metre* from the left end

or $5{\cdot}8 - 4{\cdot}2 = 1{\cdot}60$ *metre* from the axis of column *B*.

Shear resistance of concrete

$$= q_c \ b'd = 31 \times 220 \times 90 = 613800 \ N$$

Shear force $= 1517800 \ N$

.. Shear to be resisted by stirrups

$$= V_s = 1517800 - 613800 = 904000 \ N$$

∴ Spacing of 12-legged 12 *mm* ϕ stirrups

$$= \frac{A_{sw} f_s d}{V_s} = \frac{12 \times 1{\cdot}13 \times 14000 \times 90}{904000}$$

$$= 18{\cdot}9 \text{ cm. say } 18 \text{ cm. c/c.}$$

Shear reinforcement on L.H.S. of column A

Percentage of steel $= \frac{14 \times 3{\cdot}8}{174 \times 90} \times 100 = 0{\cdot}34\%$

For 0·34% steel, $q_c = 0{\cdot}25 \ N/mm.^2$

But $q = 0{\cdot}24 \ N/mm.^2$

∴ $q < q_c$

Provide nominal 8-legged stirrups of 12 mm. ϕ @ 30 cm. c/c.

Shear reinforcement on R.H.S. of column B

Percentage of steel $= \frac{17 \times 3{\cdot}8}{220 \times 90} \times 100 = 0{\cdot}33\%$

For 0·33% steel, $q_c = 0{\cdot}25 \ N/mm.^2 = 25 \ N/cm.^2$

But $q = 0{\cdot}45 \ N/mm.^2$

Shear resistance of concrete

$$= q_c \ b'd = 25 \times 220 \times 90 = 495000 \ N$$

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But, shear force $= S = 882200\ N$

$\therefore$ Shear to be resisted by concrete

$$= V_c = 882200 - 495000 = 387200\ N$$

Spacing of 12-legged 12 *mm.* ϕ stirrups

$$= \frac{A_v t_w d}{V_c} = \frac{12 \times 1{\cdot}13 \times 14000 \times 90}{387200} = 44\ cm.$$

Provide 12-legged 12 mm. ϕ *stirrups @ 30 cm. c/c.*

Disposition of longitudinal bars

Disposition of top bars

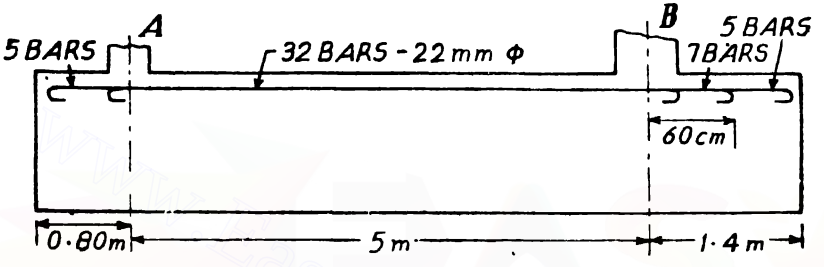

Fig. 240. Disposition of top bars.

The top 32 bars which have been determined for the maximum hogging moment, may be provided between the outer face of one column to the outer face of the other column. Beyond this, only as many longitudinal bars as are necessary to support the stirrups, will be provided.

Disposition of bottom bars under the column A

These bottom bars under the column *A* shall be extended into the interior of the footing for the greater of the following distances :

(*i*) *42d from the outer face of the column A*

$= 42 \times 2{\cdot}2 = 93$ *cm. from the outer face of the column A*

$= 93 - 30 = 63$ *cm. from the axis of column A.*

(*ii*) *up to the point of contraflexure*

$= 13$ *cm. from the axis of column A.*

Beyond this point, only as many bars as may be needed to support the stirrups will be provided. See Fig 241.

Disposition of bottom bars under the column B

These bottom bars under the column *B* shall be extended into the interior of the footing for the greater of the following distances :

(*i*) *42 d from the outer face of the column B*

$= 42 \times 2{\cdot}2 = 93$ *cm. from the outer face of the column B*

$= 93 - 37{\cdot}5 = 55{\cdot}5$ *cm. say 56 cm. from the axis of column B.*

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60cm X 60cm

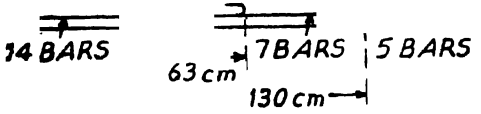

Fig. 241 Disposition of bottom bars under column *A*.

(*ii*) *up to the point of contraflexure*

=45 cm. from the axis of column B.

Beyond this point, only as many bars as may be needed to support the stirrups will be provided.

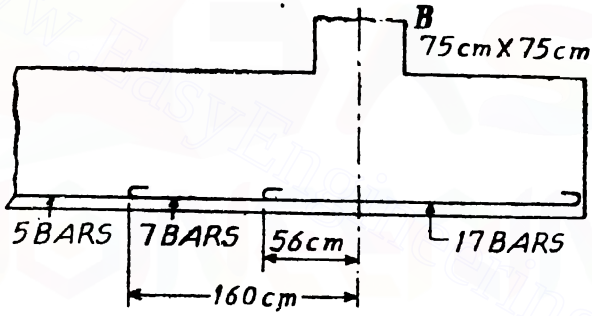

Fig. 242. Disposition of bottom bars under column *B*.

Transverse bending

Width of the footing under the column $A = 1{\cdot}74$ *m*.

Width of the footing under the column $B = 2{\cdot}20$ *m*.

Transverse projection under the column *A*

$$= \frac{1{\cdot}74 - 0{\cdot}6}{2} = 0{\cdot}57 \text{ m.}$$

Transverse projection under the column *B*

$$= \frac{2{\cdot}20 - 0{\cdot}75}{2} = 0{\cdot}725 \text{ m.}$$

Maximum transverse B.M. per *metre* width

$$= \frac{2500}{9} \times \frac{0{\cdot}725^2}{2} \times 1000 \times 100 \text{ N cm.}$$

$$= 7298000 \text{ N cm.}$$

Effective depth to the centre of 12 *mm*. ϕ transverse steel

$$= 90 - 1{\cdot}1 - 0{\cdot}6 = 88{\cdot}3 \text{ cm.}$$

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$$A_t = \frac{7298000}{14000 \times 0{\cdot}87 \times 88{\cdot}3} = 6{\cdot}78 \text{ cm.}^2$$

Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{6{\cdot}78} = 16{\cdot}7 \text{ cm. say } 16 \text{ cm. c/c.}$$

Design 62. *Two columns 5 metres apart between centres carry loads of 160 and 240 tonnes respectively. The sizes of the columns are 60 cm. × 60 cm. and 75 cm. × 75 cm. Design a combined footing for the columns The projections beyond the centres of the columns parallel to the length of the footing are 0 8 m. and 1·4 m. respectively. Approximate weight of the footing may be taken as 32 tonnes. Safe bearing capacity of the soil is equal to 30t/metre². Use M150 concrete.*

Solution. This is a case where the length of the footing is restricted.

Total length of footing = 5 + 0·8 + 1·4 = 7·20 *m.* Fig. 243 shows the footing supporting the two columns. Let the widths of the footing be *a* and *b* at the left and right extremities.

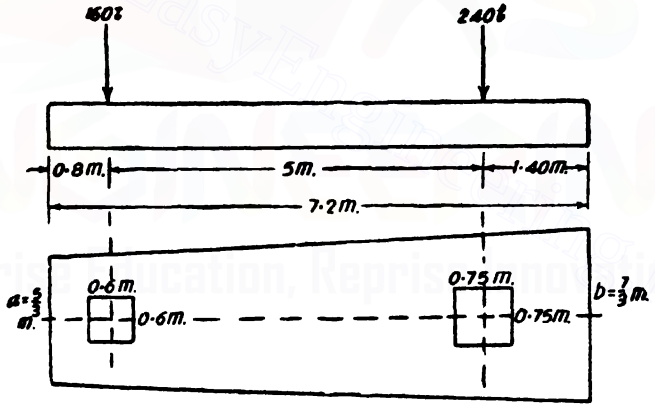

Fig. 243

The dimensions *a* and *b* should be so as to provide the necesary rea to transmit the loads safely to the soil and to satisfy the condiaions that the centre of gravity of the column loads shall coincide twith the centroid of the trapezoidal footing.

Distance of centre of gravity of column loads from the axis of column *A*

$$\frac{240 \times 5}{400} = 3 \text{ m.}$$

∴ Distance of centre of gravity of the column loads from the left edge of the footing = 0·8 + 3 = 3·8 *m.*

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But the distance of the centroid of the footing area from the left edge

$$= \frac{a+2b}{a+b} \cdot \frac{7{\cdot}2}{3}$$

$$\therefore \quad \frac{a+2b}{a+b} \times 2{\cdot}4 = 3{\cdot}8$$

$$\therefore \quad \frac{a+2b}{a+b} = \frac{3{\cdot}8}{2{\cdot}4} = \frac{19}{12} \qquad \ldots(i)$$

Total column loads $= 160 + 240 = 400$ *tonnes*

Weight of footing $= 32$ *tonnes*

Total load transferred to soil $= 432$ *tonnes*

$$\therefore \quad \text{Area of footing} = \frac{432}{30} m.^2$$

$$= 14{\cdot}4 \ m.^2$$

$$\frac{a+b}{2} \times 7{\cdot}2 = 14{\cdot}4$$

$$\therefore \quad a+b = \frac{14{\cdot}4}{3{\cdot}6} = 4$$

$$\therefore \quad a+b = 4 \qquad \ldots(ii)$$

But $$\frac{a+2b}{a+b} = \frac{19}{12}$$

$$\therefore \quad \frac{a+2b}{4} = \frac{19}{12}$$

$$\therefore \quad a+2b = \frac{19}{3} \qquad \ldots(iii)$$

and $$a+b = 4 \qquad (ii)$$

Solving the above equations, we get,

$$b = \frac{7}{3} \ m.$$

and $$a = \frac{5}{3} \ m.$$

Net upward pressure $= \frac{400}{14{\cdot}4} = \frac{250}{9}$ *tonne*/$m.^2$

Certain general expressions. It is worth obtaining certain general expressions which will be useful in further computations.

(*i*) *Width of the footing at any section distant x metres from the left edge.* Let the width be b' *m.* at a section distant x from the left edge.

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$$b' = \frac{5}{3} + \left(\frac{7}{3} - \frac{5}{3}\right)\frac{x}{7{\cdot}2}$$

$$b' = \frac{5}{3} + 0{\cdot}0926\,x$$

(ii) Net upward soil pressure on the footing for a distance of x metres from the left end.

Area of footing for a distance of x m. from the left end

$$= A = \int_0^x b'\,dx = \int_0^x \left(\frac{5}{3} + 0{\cdot}0926\,x\right) dx$$

$$\therefore \quad A = \frac{5}{3}x + 0{\cdot}0463\,x^2$$

$\therefore$ Net upward soil pressure for a distance of x m. from the left end

$$= V = \frac{250}{9}\left(\frac{5}{3}x + 0{\cdot}0463\,x^2\right)$$

$$\therefore \quad V = \frac{1250}{27}x + 1{\cdot}286\,x^2.$$

(iii) Moment of upward soil pressure acting for x metres from the left edge ot the section x m. from the left edge.

If M is this moment, then

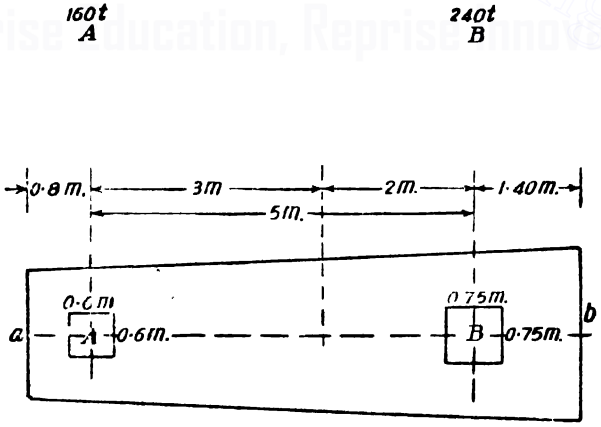

Fig. 244

$$M = V \times \frac{b' + 2a}{b' + a} \cdot \frac{x}{3}$$

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$$= \frac{250}{9}\left(\frac{5}{3}x + 0{\cdot}0463\,x^2\right)\frac{\frac{5}{3} + 0{\cdot}0926x + \frac{10}{3}}{\frac{5}{3} + 0{\cdot}0926x + \frac{5}{3}} \cdot \frac{x}{3}$$

$$= \frac{250}{9} \cdot \frac{x}{2}\left(\frac{10}{3} + 0{\cdot}0926x\right)\frac{5 + 0{\cdot}0926\,x}{\frac{10}{3} + 0{\cdot}0926\,x} \cdot \frac{x}{3}$$

$$= \frac{250}{9} \cdot \frac{x}{2}\left(5 + 0{\cdot}0926\,x\right)\frac{x}{3}$$

$$M = \frac{625}{27}x^2 + 0{\cdot}429\,x^3$$

Alternatively $V = \dfrac{dM}{dx}$

$$M = \int_0^x V\,dx$$

$$= \int_0^x \left(\frac{1250}{27}x + 1{\cdot}286\,x^2\right) dx$$

$$\therefore \quad M = \frac{625}{27}x^2 + 0{\cdot}429\,x^3$$

The expressions obtained above are rewritten below

$$b' = \frac{5}{3} + 0\,0926\,x \text{ metres}$$

$$V = \frac{1250}{27}x + 1{\cdot}286\,x^2 \text{ tonnes}$$

$$M = \frac{625}{27}x^2 + 0{\cdot}429\,x^3 \text{ tonnes metre.}$$

Depth of the footing slab

The depth of the footing slab should be determined from consideration of bending moment and punching shear.

(*i*) **Depth from B.M. consideration.** B.M. under the column *A*

$$= M = \frac{625}{27}(0{\cdot}8)^2 + 0\,429(0{\cdot}8)^3 \text{ tonne metre}$$

$$= 15\,02 \text{ tonne metre (sagging moment)}$$

B.M. under the column *B*

$$= \frac{625}{27}(5{\cdot}8)^2 + 0\,429(5{\cdot}8)^3 - 160 \times 5$$

tonne metre

$$= 62{\cdot}2 \text{ tonne metre (sagging moment)}$$

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Section where the S.F. is zero

Let the S.F. be zero at x *metres* from the left edge.

$\therefore$ x is given by the condition

$$\frac{1250}{27}x + 1{\cdot}286x^2 = 160$$

$$\therefore \quad x^2 + 35{\cdot}99x = 124{\cdot}4$$

Solving $x = 3{\cdot}175$ *m.*

B.M. at x $= 3{\cdot}175$ *m.*

$$= \frac{625}{27} \times 3{\cdot}175^2 + 0{\cdot}429(3{\cdot}175)^3 - 160(3{\cdot}175 - 0{\cdot}8)$$

tonne metre.

$$= -132{\cdot}86 \text{ tonne metre (hogging moment)}$$

Width of footing at $x = 3{\cdot}175$ *m.*

$$= b' = \frac{5}{} + 0{\cdot}0926 \times 3{\cdot}175 \text{ metre}$$

$$= 1{\cdot}96 \text{ metre}$$

Adopting $c = 50$ *kg./cm.*^{2}, $t = 1400$ *kg./cm.*^{2} and $m = 18$ and equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}50 \times 196d^2 = 132{\cdot}86 \times 1000 \times 100$$

$$\therefore \quad d = 89\ 32 \text{ cm. say } 90 \text{ cm.}$$

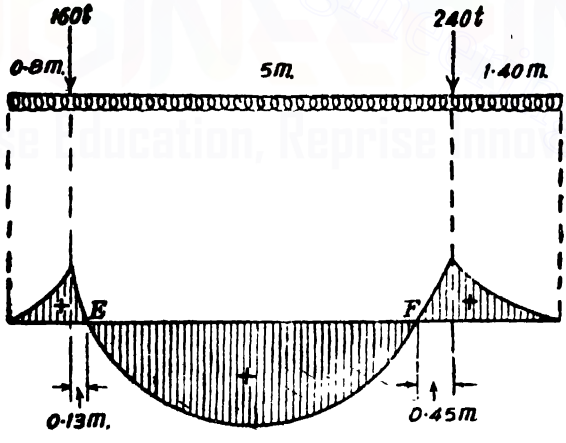

Fig. 245

Fig. 245 shows the B.M. diagram for the footing.

Points of contraflexure

Equating the general expression for bending moment to zero,

$$\frac{625}{27}x^2 + 0{\cdot}429x^3 - 160\ (x - 0{\cdot}8) = 0$$

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Solving, by trial and error $x = 0.93$ *m.*
and $x = 5.35$ *m.*

Let E and F be the points of contraflexure.

Distance of E from the axis of column $A = 0.93 - 0.8 = 0.13$ *m.*

Distance of F from the axis of column $B = 5.80 - 5.35 = 0.45$ *m.*

(*ii*) **Depth from punching shear consideration.** Let D be the depth required in *centimetres*. Let us allow a punching shear stress of 10 *kg./cm.*^{2}. The requirement under the column A is given by

$$\left[160 - \frac{250}{9} \times 0.6^2\right] 1000 = 4 \times 60 \times D \times 10$$

$$\therefore \quad D = 62.5 \text{ cm.}$$

The requirement under the column B is given by

$$\left[240 - \frac{250}{9} \times 0.75^2\right] 1000 = 4 \times 75 \times D + 10$$

$$\therefore \quad D = 74.8 \text{ cm}$$

We will, therefore, provide an effective depth of 90 *cm.* and an overall depth of 96 *cm.*

Reinforcement from B.M. consideration

A_s under column $A = \dfrac{15.02 \times 1000 \times 100}{1400 \times 0.87 \times 90}$ *cm.*^{2}

$= 13.7$ *cm.*^{2}

This quantity of steel is very small. It is advisable to provide a minimum steel of 0.3% of gross area.

= Width of footing at this section

$$= \frac{5}{3} + 0.0926 \times 0.8 = 1.74 \text{ m.} = 174 \text{ cm.}$$

Minimum steel requirement

$$= \frac{0.3}{100} \times 174 \times 96 = 50.11 \text{ cm.}^2$$

Number of bars of 22 *mm.* ϕ

$$= \frac{50.11}{3.8} = 14 \text{ bars.}$$

A_s under column $B = \dfrac{62.2 \times 1000 \times 100}{1400 \times 0.87 \times 90}$ *cm.*^{2}

$= 56.75$ *cm.*^{2}

Width of footing at this section

$$= \frac{5}{3} + 0.0926 \times 5.8 = 2.20 \text{ m.} = 220 \text{ cm.}$$

Minimum steel requirement

$$= \frac{0.3}{100} \times 220 \times 96 = 63.36 \text{ cm.}^2$$

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Number of bars of 22 *mm.* ϕ

$$\frac{63{\cdot}36}{3{\cdot}8} = 17 \text{ bars}$$

A_s for maximum hogging moment

$$= \frac{132{\cdot}86 \times 1000 \times 100}{1400 \times 0{\cdot}87 \times 90} = 121{\cdot}3 \text{ cm.}^2$$

Width of footing at this section

$$= 1{\cdot}96 \text{ m.} = 196 \text{ cm.}$$

Minimum steel required

$$= \frac{0{\cdot}3}{100} \times 196 \times 96 = 56{\cdot}45 \text{ cm.}^2$$

Number of bars of 22 *mm.* ϕ

$$= \frac{121{\cdot}3}{3{\cdot}8} = 32 \text{ bars.}$$

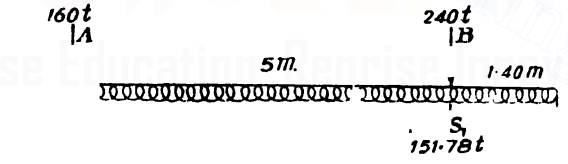
Shear analysis

S.F. just on LHS of column *A*

$$\frac{1250}{27} x + 1{\cdot}286\, x^2 \text{ tonne}$$

$$\frac{1250}{27} \times 0{\cdot}8 + 1{\cdot}286\, (0{\cdot}8)^2 \text{ tonne}$$

$$+37{\cdot}85 \text{ tonne } (S_4)$$



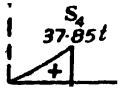


Fig.246

S.F. just on *RHS* of column *A*

$$= +37{\cdot}85 - 160 = -122{\cdot}15 \text{ tonnes } (S_2)$$

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S.F. just on *LHS* of column *B*

$$= -\frac{1250}{27} \times 5{\cdot}8 + 1{\cdot}286 \times 5\ 8^2 - 160$$

$$= +151{\cdot}78 \text{ tonnes } (S_1)$$

S.F. just on *RHS* of column *B*

$$= +151{\cdot}78 - 240 = -88{\cdot}22 \text{ tonnes } (S_2)$$

Width of footing under the column *A*

$$= 174 \text{ cm.}$$

Width of footing under the column *B*

$$= 220 \text{ cm.}$$

Nominal shear stsess at any section

$$= q = \frac{S}{b'd}$$

where b' = width of the footing at the section considered.

Nominal shear stress due to

$$S_1 = q_1 = \frac{151{\cdot}78 \times 1000}{220 \times 90} = 7{\cdot}67 \text{ kg./cm.}^2$$

Nominal shear stress due to

$$S_2 = q_2 = \frac{122{\cdot}15 \times 1000}{174 \times 90} = 7{\cdot}80 \text{ kg./cm.}^2$$

Nominal shear stress due to

$$S_3 = q_3 = \frac{88{\cdot}22 \times 1000}{220 \times 90} = 4{\cdot}45 \text{ kg./cm.}^2$$

Nominal shear stress due to

$$S_4 = q_4 = \frac{37{\cdot}85 \times 1000}{174 \times 90} = 2{\cdot}42 \text{ kg./cm.}^2$$

Shear reinforcement for the footing between the columns near the column A

$$\text{Percentage of steel} = \frac{32 \times 3{\cdot}8 \times 100}{196 \times 90} = 0{\cdot}69\%$$

For 0·69% steel, $q_c = 3{\cdot}3 \text{ kg./cm.}^2$

Let in the zone of negative shear, the nominal shear stress be 3·3 *kg./cm.*^{2} at a distance of x *m.* from the left end.

$$\frac{S}{b'd} = -3{\cdot}3 \qquad \therefore \quad S = -3\ 3\ b'd$$

$$-3{\cdot}3 \times 90\left[\frac{5}{3} + 0{\cdot}0926\ x\right]100 = \left[\frac{1250}{27}x + 1{\cdot}286\ x^2 - 160\right]1000$$

$$\therefore \qquad x^2 + 38{\cdot}14\ x - 85{\cdot}92 = 0$$

Solving, we get, $x = 2{\cdot}10$ *m.* from the left end.

or $2{\cdot}10 - 0{\cdot}80 = 1\ 3$ *m.* from column *A*.

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Shear resistance of concrete

$$= q_c\, b'd = 3{\cdot}3 \times 174 \times 90 = 51678 \text{ kg.}$$

Shear force $= 122150$ *kg.*

$\therefore$ Shear to be resisted by stirrups

$$= V_s = 122150 - 51678 = 70472 \text{ kg.}$$

Spacing of 12-legged 12 *mm.* ϕ stirrups

$$= \frac{A_w t_w d}{V_s} = \frac{12 \times 1{\cdot}13 \times 1400 \times 90}{70472}$$

$$= 24 \text{ cm. say } 20 \text{ cm. c/c.}$$

Under the column B,

$$\text{Percentage of steel} = \frac{32 \times 3{\cdot}8 \times 100}{220 \times 90} = 0{\cdot}61\%$$

For 0·61% steel, $q_c = 3{\cdot}10$ *kg./cm.*^{2}

Let in the zone of positive shear, the nominal shear stress be 3·10 *kg./cm.*^{2} at a distance of x *metres* from the left end.

$$\therefore \quad \frac{S}{b'd} = +3{\cdot}1 \qquad \therefore \quad S = +3{\cdot}1\, b'd$$

$$\therefore \quad +3{\cdot}1 \times 90\left[\frac{5}{3} + 0{\cdot}0926\, x\right]100 = \left[\frac{1250}{27}\, x + 1{\cdot}286\, x^2 - 160\right]1000$$

$$\therefore \quad x^2 + 34{\cdot}05\, x - 160{\cdot}76 = 0$$

Solving, we get, $x = 4{\cdot}20$ *metre* from the left end

or $5{\cdot}8 - 4{\cdot}2 = 1{\cdot}6$ *m.* from the column *B.*

Shear resistance of concrete

$$= q_c\, b'd = 3{\cdot}1 \times 220 \times 90 = 61380 \text{ kg.}$$

Shear force $= 151780$ *kg.*

$\therefore$ Shear to be resisted by stirrups

$$= V_s = 151780 - 61380 = 90400 \text{ kg.}$$

$\therefore$ Spacing of 12-legged 12 *mm.* ϕ stirrups

$$= \frac{A_w t_w d}{V_s} = \frac{12 \times 1{\cdot}13 \times 1400 \times 90}{90400}$$

$$= 18{\cdot}9 \text{ cm. say } 18 \text{ cm. c/c.}$$

Shear reinforcement on LHS of column A

$$\text{Percentage of steel} = \frac{14 \times 3{\cdot}8}{174 \times 90} \times 100 = 0{\cdot}34\%$$

For 0·34% steel, $q_c = 2{\cdot}5$ *kg./cm.*^{2}

But $q = 2{\cdot}42$ *kg./cm.*^{2}

$q < q_c$.

Provide nominal 8-legged 12 mm. ϕ stirrups @ 30 cm. c/c.

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Shear reinforcement on RHS of column B

Percentage of steel $= \frac{17 \times 3{\cdot}8}{220 \times 90} \times 100 = 0{\cdot}33\%$

For 0·33% steel, $q_c = 2{\cdot}5$ *kg./cm.*^{2}

But $q = 4{\cdot}45$ *kg./cm.*^{2}

Shear resistance of concrete

$= q_c\, b'd = 2{\cdot}50 \times 220 \times 90 = 49500$ *kg.*

But shear force $= 88220$ *kg.*

$\therefore$ Shear to be resisted by concrete

$= V_s = 88220 - 49500 = 38720$ *kg.*

Spacing of 12-legged 12 *mm.* ϕ stirrups

$$= \frac{A_w t_w d}{V_s} = \frac{12 \times 1{\cdot}13 \times 1400 \times 90}{38720} = 44 \text{ cm.}$$

Provide 12-legged 12 mm. ϕ *stirrups @ 30 cm. c/c.*

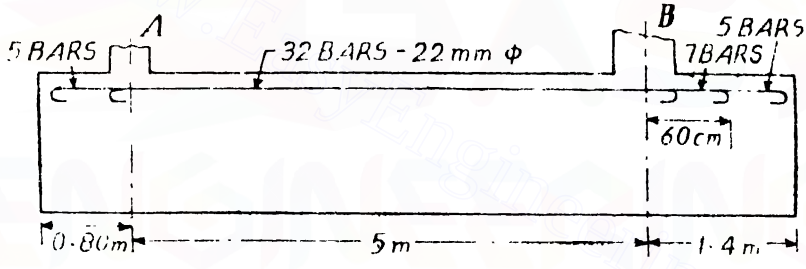

Fig. 247. Disposition of top bars.

Disposition of top bars

The top 32 bars which have been determined for the maximum hogging moment may be provided between the outer face of one column to the outer face of the other column. Beyond this, only as many longitudinal bars as are necessary to support the stirrups, will be provided.

Disposition of bottom bars under the column A

These bottom bars under the column *A* shall be extended into the interior of the footing for the greater of the following distances :

(*i*) *42 d from the outer face of the column A*

$= 42 \times 2{\cdot}2 = 93$ *cm. from the outer face of the column A*

$= 93 - 30 = 63$ *cm. from the axis of the column A*

(*ii*) *Up to the point of contraflexure*

$= 13$ *cm. from the axis of the column A.*

Beyond this point, only as many bars as may be needed to support the stirrups will be provided.

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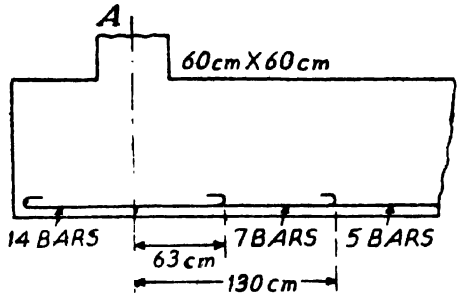

Fig. 248. Disposition of bottom bars under column A.

Disposition of bottom bars under the column B

These bottom bars under the column *B* shall be extended into the inferior of the footing for the greater of the following distances.

(i) *42 d from the outer face of the column B*

$= 42 \times 2{\cdot}2 = 93$ *cm. from the outer face of the column B*

$= 93 - 37{\cdot}5 = 55{\cdot}5$ *cm. say 56 cm. from the axis of column B.*

(ii) *Up to the point of contraflexure*

$= 45$ *cm from the axis of column B.*

Beyond this point, only as many bars as may be needed to support the stirrups will be provided.

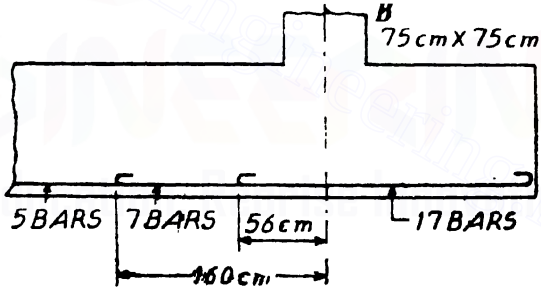

Fig. 249. Disposition of bottom bars under column B

Transverse bending

Width of footing under column *A*

$= 1{\cdot}74$ *m.*

Width of footing under column *B*

$= 2{\cdot}20$ *m.*

Transverse projection under column *A*

$$= \frac{1{\cdot}74 - 0{\cdot}6}{2} = 0{\cdot}57 \text{ m.}$$

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Transverse projection under column B

$$\frac{2{\cdot}20-0{\cdot}75}{2} = 0{\cdot}725 \text{ m.}$$

Maximum transverse bending moment per *metre* width

$$= \frac{250}{9} \times \frac{0{\cdot}725^2}{2} \times 1000 \times 100 \text{ kg. cm.}$$

$$= 729800 \text{ kg. cm.}$$

Effective depth to the centre of 12 *mm.* ϕ transverse steel

$$= 90 - 1{\cdot}1 - 0{\cdot}6 = 88{\cdot}3 \text{ cm.}$$

$$A_s = \frac{729800}{1400 \times 0{\cdot}87 \times 88{\cdot}3} = 6{\cdot}78 \text{ cm.}^2$$

Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{6{\cdot}78} = 16{\cdot}7 \text{ cm. say 16 cm. c/c.}$$

Design 63 (S.I.) (Ribbed footing). *Design a ribbed footing for a 30 cm. × 45 cm. column to sustain a load of 500 kN and a moment of 95 kNm. A base of 2·50 m. × 150 m. is to be provided. Use M 15 concrete.*

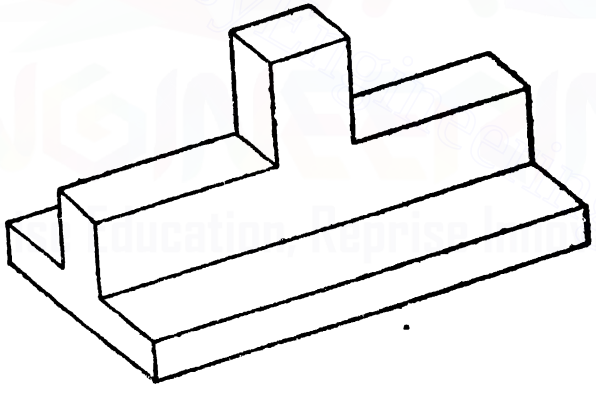

Fig. 250

Solution. *Design of the Footing Slab*

Moment of inertia of the foundation plan

$$I = \frac{1{\cdot}5 \times 2{\cdot}5^3}{12} = 1{\cdot}953 \text{ metre}^4$$

Area of foundation $= 1{\cdot}5 \times 2{\cdot}5 = 3{\cdot}75 \text{ metre}^2$

$\therefore$ Extreme pressure intensities on the base are given by

$$p = \frac{500}{3{\cdot}75} \pm \frac{95}{1{\cdot}953} \times \frac{2}{2{\cdot}5} \text{ kN/metre}^2$$

$$= 133{\cdot}3 \pm 60{\cdot}8 \text{ kN/metre}^2$$

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$p_{max} = 194{\cdot}1\ kN/metre^2$

$p_{min} = 72{\cdot}5\ kN/metre^2$

Let the width of the rib be 30 *cm.*

$\therefore$ Maximum projection of the slab $= \dfrac{1{\cdot}50 - 0{\cdot}30}{2} = 0{\cdot}6$ *metre*

Depth of footing slab

(i) *B.M. consideration*

Maximum load on cantilever slab per *metre* width

$= 194{\cdot}1 \times 0{\cdot}6 = 116{\cdot}46\ kN$

Maximum bending moment $= 116{\cdot}46 \times \dfrac{0{\cdot}6}{2} = 34{\cdot}94\ kNm$

Equating the moment of resistance of the bending moment,

$85\ bd^2 = 85 \times 100\ d^2 = 34{\cdot}94 \times 1000 \times 100 \quad \therefore\ d = 20{\cdot}3\ cm$

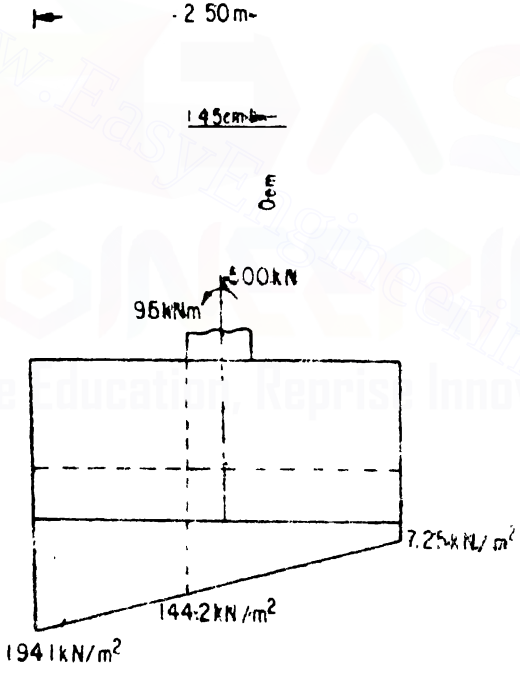

Fig. 251

Providing 16 *mm.* ϕ bars at a clear cover of 6 *cm.* effective cover

$= 6 + 0{\cdot}8 = 6{\cdot}8\ cm.$

Overall depth $= 20{\cdot}3 + 6{\cdot}8 = 27{\cdot}1\ cm.$

Provide 30 *cm.* overall

Effective depth $= 30 - 6{\cdot}8 = 23{\cdot}2\ cm.$

$$A_t = \frac{34{\cdot}94 \times 1000 \times 100}{14000 \times 0{\cdot}87 \times 23{\cdot}2} = 12{\cdot}36\ cm.^2$$

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Spacing of 16 *mm.* ϕ bars $= \dfrac{2{\cdot}01 \times 100}{12{\cdot}36} = 16{\cdot}3$ *cm.*

Provide 16 mm. ϕ *bars @ 16 cm. c/c (12·56 cm.*^{2})

Check for shear

Percentage of steel $= \dfrac{12{\cdot}56 \times 100}{100 \times 23{\cdot}2} = 0{\cdot}54\%$

For 0·54% $\quad q_c = 0{\cdot}3\ N/mm^2 = 30\ N/cm.^2$

S.F. at a critical section at a distance equal to effective depth from the face of the rib

$$= 194{\cdot}1 \times (0{\cdot}6 - 0{\cdot}232) = 71{\cdot}43\ kN$$

Nominal shear stress $= q_v = \dfrac{S}{bd} = \dfrac{71{\cdot}43 \times 1000}{100 \times 23{\cdot}2} = 30\ N/mm.^2$

$q_v = q_c$

Design of the rib

Maximum upward load on the cantilevering part

$$= 1{\cdot}5 \times 1{\cdot}025 \times \left(\frac{194{\cdot}1 + 144{\cdot}2}{2} \right) kN$$

$$= 260{\cdot}1\ kN$$

This acts at $\dfrac{144{\cdot}2 + 2 \times 194{\cdot}1}{144{\cdot}2 + 194{\cdot}1} \times \dfrac{1{\cdot}025}{3} = 0{\cdot}538$ *metre* from the column face.

Maximum bending moment $= 260{\cdot}1 \times 0{\cdot}538 = 140\ kNm$

Equating the moment of resistance to the maximum bending moment,

$$85 \times 30\ d^2 = 140 \times 1000 \times 100$$

$$d = 74{\cdot}1\ cm.$$

Effective cover to the centre of reinforcement

Let 20 *mm.* ϕ bars be provided,

Clear cover to slab bars	$= 6{\cdot}00$ *cm.*
Diameter of slab bars	$= 1{\cdot}60$ *cm.*
Half diameter of rib bars	$= 1{\cdot}00$ *cm.*
$\therefore$ Effective cover	$= 8{\cdot}60$ *cm.*

$\therefore$ Overall depth required $= 74{\cdot}10 + 8{\cdot}60 = 82{\cdot}7$ *cm.*

Provide an overall depth of 85 *cm.*

Actual effective depth $= d = 85 - 8{\cdot}6 = 76{\cdot}40$ *cm.*

$\therefore \quad A_t = \dfrac{140 \times 1000 \times 100}{14000 \times 0{\cdot}87 \times 76{\cdot}4} = 15{\cdot}04\ cm.^2$

Provide 5 bars of 20 mm. ϕ *(15·71 cm.*^{2})

Check for shear

Maximum shear force $= S = 260{\cdot}1\ kN = 260{\cdot}1 \times 1000\ N$

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Nominal shear stress $= \frac{S}{bd} = \frac{260{\cdot}1 \times 1000}{30 \times 76{\cdot}4} = 113\ N/cm.^2$

$= 1{\cdot}13\ N/mm.^2$

Percentage of steel $= \frac{15{\cdot}71}{30 \times 76{\cdot}4} \times 100 = 0{\cdot}70\%$

For 0·7% steel $q_c = 0{\cdot}33\ N/mm.^2 = 33\ N/cm.^2$

Shearing resistance of concrete $= q_c\ bd$

$= 33 \times 30 \times 76{\cdot}4 = 75636\ N$

to be resisted by stirrups

$V_s = 260100 - 75636 = 184464\ N$

Spacing of 4-legged 12 *mm.* ϕ stirrups

$$= \frac{A_w\ f_w\ d}{} = \frac{4 \times 1{\cdot}13 \times 14000 \times 76{\cdot}4}{184464} = 26\ cm.$$

Provide 4-legged 12 mm. ϕ stirrups @ 25 cm. c/c.

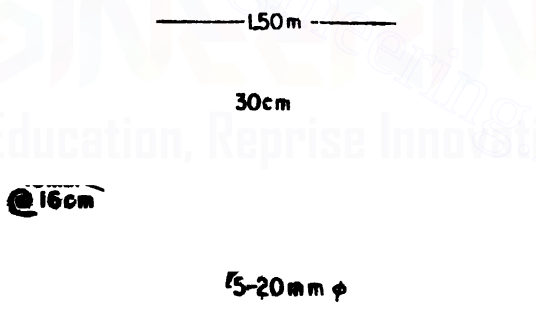

Fig. 252

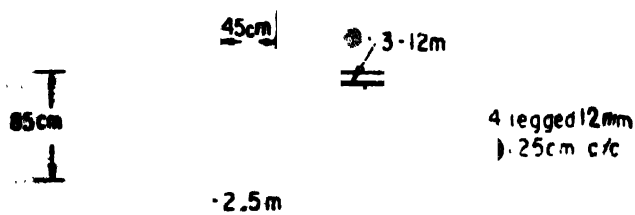

Fig. 253

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Design 64. *Two columns 4·50 metres apart between their centres are to be provided with a combined footing. Each column carries a load of 100 tonnes. The projection of the footing beyond the centre of each column is 1·25 metre. The columns are 40 cm.×40 cm. each. Design the footing. The safe bearing capacity of the soil is 20 t/m². (Use M 150 concrete).*

Solution.

Total column load $= 2 \times 100 = 200$ *tonnes*

Approximate weight of footing = (10% of 200) $= 20$ *tonnes*

Total $= 220$ *tonnes*

Area of foundation $= \frac{220}{20} = 11$ *metres*²

Length of footing $= 4{\cdot}5 + (2 \times 1{\cdot}25) = 7$ *metres*

Width of the footing $= \frac{11}{7} = 1{\cdot}57$ *m.*

Let us provide a width of 1·60 *m*

Net upward pressure intensity $= \frac{200{,}000}{7 \times 1{\cdot}6} = 17{,}860$ *kg./m.*²

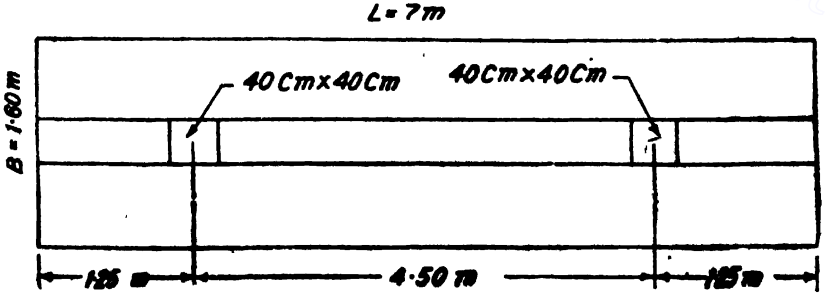

Fig. 254

Width of the central beam = 40 *cm.*

Cantilever projection beyond the face of the beam

$= \frac{1{\cdot}60 - 0{\cdot}4}{2} = 0{\cdot}6$ *m.*

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Design of the cantilevering slab

Consider a 1 *m.* wide strip of the cantilevering slab

Max. B.M. $= M = 17{,}860 \times \frac{0{\cdot}6^2}{2} \times 100$ *kg. cm.*

$= 321{,}480$ *kg. cm.*

Equating the moment of resistance to the maximum bending moment,

$8{\cdot}50 \times 100\, d^2 = 321480 \quad \therefore\ d = 19{\cdot}4$ *cm.*

Providing 12 *mm.* bars at a clear cover of 6 *cm.*

Effective cover $= 6 + 0{\cdot}6 = 6{\cdot}6$ *cm.*

$\therefore$ Overall depth of slab $= 19{\cdot}4 + 6{\cdot}6 = 26$ *cm.*

Provide an overall depth of 30 *cm.*

Actual effective depth $= 30 - 6{\cdot}6 = 23{\cdot}40$ *cm.*

$$A_t = \frac{321480}{1400 \times 0{\cdot}87 \times 23{\cdot}40} = 11{\cdot}28 \text{ cm.}^2$$

Spacing of 12 *mm.* ϕ bars $= \frac{1{\cdot}13 \times 100}{11{\cdot}28} = 10{\cdot}02$ *cm.*

Provide 12 mm. ϕ bars @ 10 cm. c/c.

Check for diagonal tension

Let the depth of the footing slab be reduced to 10 *cm.* near the edges. Critical section for diagonal tension is taken at a distance equal to the effective depth of the slab measured from the face of the central beam.

Overall depth at the critical section

$$= 30 - \frac{30 - 10}{60} \times 26{\cdot}4 = 21{\cdot}2 \text{ cm.}$$

Effective depth at the critical section $21{\cdot}2 - 6{\cdot}6 = 14{\cdot}6$ *cm.*

S.F. at the critical section $= 17860 \times \left(\frac{60 - 23{\cdot}4}{100}\right) = 6537$ *kg.*

Nominal shear stress $= q_v = \frac{6537}{100 \times 14{\cdot}6} = 4{\cdot}47$ *kg./cm.*^{2}

Permissible nominal shear stress $= k\, q_c$

For a thickness of 21·2 *cm.* $k = 1{\cdot}18$

Percentage of steel $= \frac{11{\cdot}28}{100 \times 14{\cdot}6} \times 100 = 0{\cdot}77\%$

For 0·77% steel, $q_c = 3{\cdot}4$ *kg./cm.*^{2}

$\therefore \quad k\, q_c = 1{\cdot}18 \times 3{\cdot}4 = 4$ *kg./cm.*^{2}

$\therefore \quad q_v > k\, q_c$

Hence let us increase the depth of the footing to 33 *cm.*

$\therefore$ Effective depth $= 33 - 6{\cdot}6 = 26{\cdot}4$ *cm.*

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Overall depth at the critical section

$$= 33 - \frac{33-10}{60} \times 26{\cdot}4 = 22{\cdot}88 \text{ cm.}$$

Effective depth at the critical section $= 22{\cdot}88 - 6{\cdot}60 = 16{\cdot}28$ cm.

S.F. at the critical section $= 17860 \left(\frac{60-26{\cdot}4}{100} \right) = 6000$ kg.

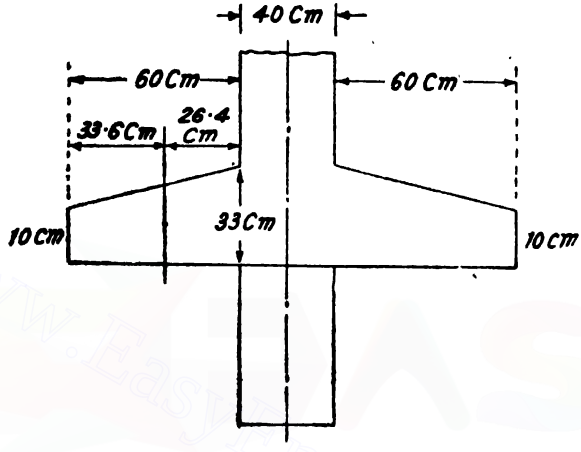

Fig. 255

Nominal shear stress $= q_v = \frac{6000}{100 \times 16{\cdot}28} = 3{\cdot}68$ kg./cm.^{2}

Permissible nominal shear stress $= k\, q_c$

For a thickness of 22·88 cm. $k = 1{\cdot}14$

Percentage of steel $= \frac{11{\cdot}28 \times 100}{100 \times 16{\cdot}28} = 0{\cdot}69\%$

For 0·69% steel, $q_c = 3{\cdot}3$ kg /cm.^{2}

$$k\, q_c = 1{\cdot}14 \times 3{\cdot}3 = 3{\cdot}76 \text{ kg./cm.}^2$$

$\therefore$ $q_v < k\, q_c$. Hence the design is safe.

Design of the central longitudinal beam

Upward *udl* on the beam

$$= \frac{200}{7} \text{ tm}$$

B.M at A or B $= + \frac{200}{7} \times \frac{1{\cdot}25^2}{2}$ tm.

$= 22{\cdot}32$ tm.

B.M. at centre $= \frac{200}{7} \times \frac{3{\cdot}5^2}{2} - 100 \times 2{\cdot}25$

$= 175 - 225 = -50$ tm.

$8{\cdot}50 \times 40 \times d^2 = 50 \times 1000 \times 100$

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$\therefore$ $d = 121{\cdot}3$ *cm.*

Providing 20 *mm.* ϕ bars at a clear cover of 6 *cm.*

Effective cover $= 6 + 1 = 7$ *cm.*

$\therefore$ Overall depth required

$= 121{\cdot}3 + 7 = 128{\cdot}3$ *cm.* say 130 *cm.*

Punching shear consideration

Punching effect will be considered on two sides for the beam and two sides for the slab.

Equating the punching resistance to the punching load,

$[2 \times 40 \times D + 2 \times 40 \times 33]\ 10 = 100{,}000 - 17{,}860 \times 0{\cdot}4^2$

$D = 88{\cdot}5$ *cm.*

Let us provide an overall depth of 130 *cm.*

Actual effective depth $= 130 - 7 = 123$ *cm.*

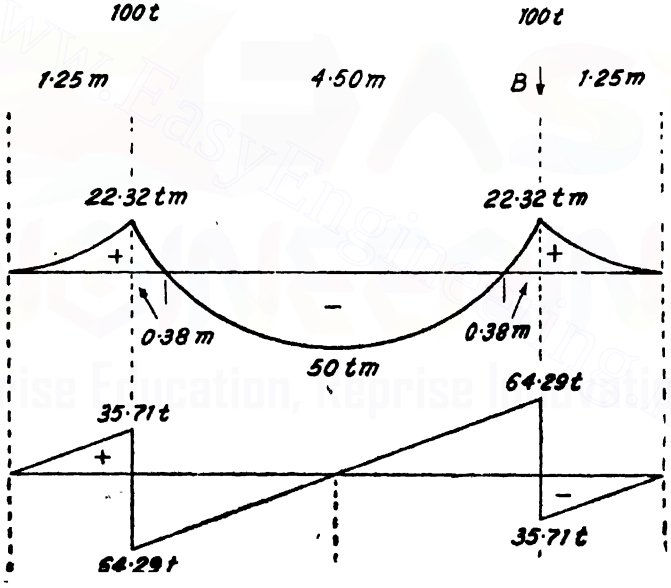

Fig. 256

Shear reinforcemet in the cantilevering parts

Percentage of tensile steel $= \dfrac{15{\cdot}71 \times 100}{40 \times 123} = 0{\cdot}32\%$

For 0·32% steel, $q_c = 2{\cdot}5$ *kg./cm.*^{2} but $q_v = 7{\cdot}25$ *kg./cm.*^{2}

Shear resistance of concrete

$= q_c \cdot bd = 2{\cdot}5 \times 40 \times 123 = 12300$ *kg.*

$\therefore$ Shear to be resisted by the stirrups

$= V_s = 35710 - 12300 = 23410$ *kg.*

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Spacing of 4-legged 12 *mm.* ϕ stirrups

$$\frac{A_w f_{wd}}{V_s} = \frac{4 \times 1{\cdot}13 \times 1400 \times 123}{23410} = 33{\cdot}2 \text{ cm.}$$

Provide 4-legged 12 mm. ϕ *stirrups @ 30 cm. c/c.*

Distance of the section from the end where the nominal shear stress is 2·5 *kg./cm.*^{2}

$$= \frac{2{\cdot}5}{7{\cdot}25} \times 1{\cdot}25 = 0{\cdot}43 \text{ m.}$$

Hence the shear reinforcement as calculated above will be provided in the cantilevering parts for a distance 1·25 − 0·43 = 0·82 *m* say 0·85 *m* from the axis of the column. In the remaining region,

provide 2-legged 12 mm. ϕ *stirrups @ 30 cm. c/c.*

Disposition of top bars

The eleven bars of 20 *mm.* ϕ provided at the top are provided from outer face of one column to the outer face of the other column.

Beyond this, provide only as many bars as needed to support the stirrups.

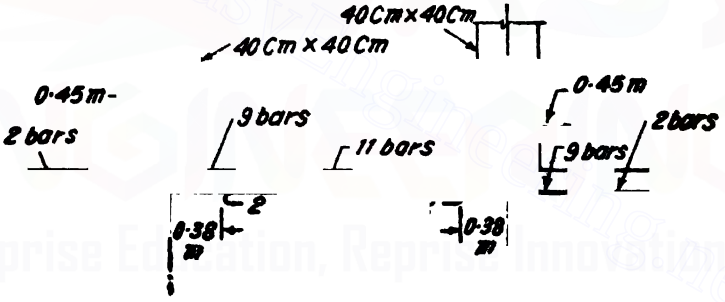

Fig. 257. Disposition of top bars.

Disposition of bottom bars

The 5 bars at the bottom should be extended into the interior of the footing for the greater of the following distances :

(*i*) up to point of contraflexure = 38 *cm.* from column centre.

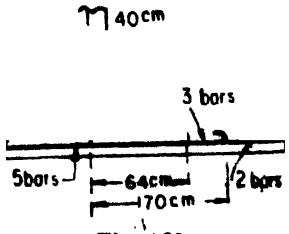

Fig. 258

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(*ii*) 42 diameter $= 42 \times 2 = 84$ *cm.* from the outer face of the column or $84 - 20 = 64$ *cm* from the column axis.

Nominal shear stress is 3·3 *kg /cm.*^{2} at 170 *cm.* from column axis.

Top steel for maximum hogging bending moment

$$= \frac{50 \times 1000 \times 100}{1400 \times 0{\cdot}87 \times 123} = 33\,37 \text{ cm.}^2$$

Provide 11 bars of 20 mm. diameter (*34·56 cm.*^{2})

Bottom steel for cantilever moment

$$= \frac{22{\cdot}32 \times 1000 \times 100}{1400 \times 0{\cdot}87 \times 123} = 14{\cdot}90 \text{ cm.}^2$$

Provide 5 bars of 20 mm. diameter (*15·71 cm.*^{2})

Shear analysis

S.F. just on L.H.S. of column $A = \frac{200}{7} \times 1{\cdot}25 = +35{\cdot}71$ *t*

S.F. just on R.H.S. of column $A = +35{\cdot}71 - 100 = -64{\cdot}29$ *t*

S.F. just on L.H.S. of column $B = +64{\cdot}29$ *t*

S F. just on R.H.S. of column $B = -35{\cdot}71$ *t*

Nominal shear stress on L.H.S. of column *A*

$$= \frac{35{\cdot}71 \times 1000}{40 \times 123} = 7{\cdot}25 \text{ kg /cm.}^2$$

Nominal shear stress on R.H.S. of column *A*

$$= \frac{64{\cdot}29 \times 1000}{40 \times 123} = 13{\cdot}07 \text{ kg./cm.}^2$$

Shear reinforcement between the two columns

Percentage of tensile steel

$$= \frac{34{\cdot}56 \times 100}{40 \times 123} = 0{\cdot}7\%$$

For 0 7% steel, $q_c = 3{\cdot}30$ *kg./cm.*^{2} and $q_{c\,max} = 16$ *kg./cm.*^{2}

Shear resistance of concrete

$$= q_c bd = 3{\cdot}30 \times 40 \times 123 = 16236 \text{ kg.}$$

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Shear to be resisted by stirrups

$$= V_s = 64290 - 16236 = 48054 \text{ kg.}$$

Spacing of 4-legged 12 *mm.* ϕ stirrups

$$= \frac{A_{sv} f_s d}{V_s} = \frac{4 \times 1{\cdot}13 \times 1400 \times 123}{48054} = 16{\cdot}2 \text{ cm.}$$

Provide 4-legged 12 mm. ϕ *stirrups @ 16 cm. c/c.*

Distance from the centre at which the nominal shear stress is 3 3 $kg./cm.^2$

$$= \frac{3{\cdot}3}{13{\cdot}07} \times 2{\cdot}25 = 0{\cdot}57 \text{ m}$$

Hence the shear reinforcement as calculated above will be provided for a distance

$2{\cdot}25 - 0{\cdot}57 = 1{\cdot}68$ *m say* $1{\cdot}70$ *m* from the column centre.

In the central region where the nominal shear stress is less than q_c provide 2-legged 12 *mm.* ϕ stirrups at 30 *cm. c/c.*

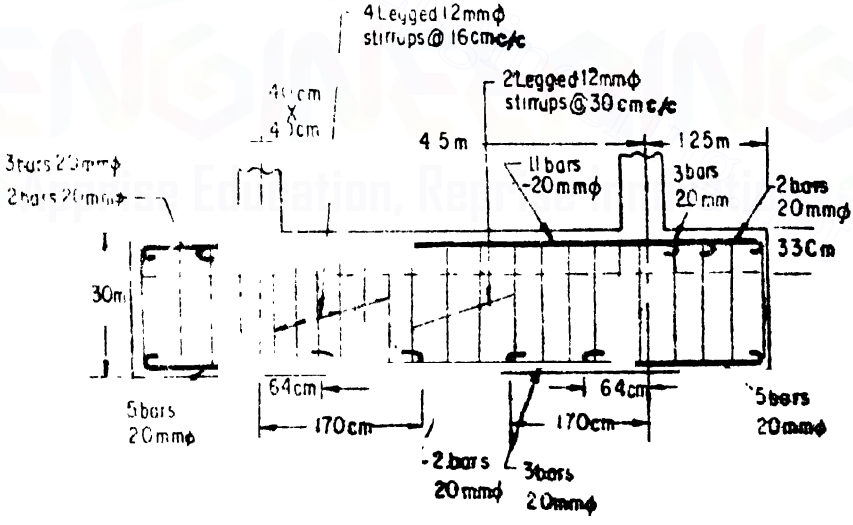

Fig. 260

§140. Strap Footing

This is a special type of footing provided for two columns. Two separate footings are provided under two columns and the two

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footings are connected by a strap beam. The areas provided by the two footings and the strap beam are such that the pressure distribution on the footing and strap becomes uniform.

Fig. 261 shows a strap foundation. The column on the left

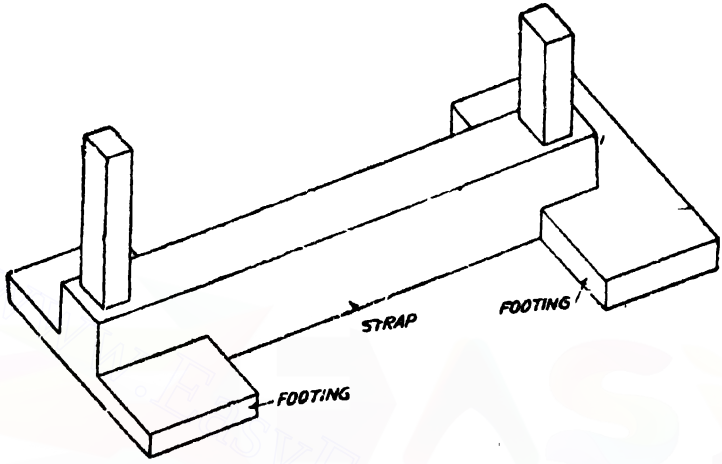

Fig. 261. Strap foundation.

end is the wall column (end column) and the other column is the interior column.

Let A be the end column and B the interior column.

Let W_a and W_b be the loads on the columns A and B.

Total column loads $= W_a + W_b = (\Sigma W)$.

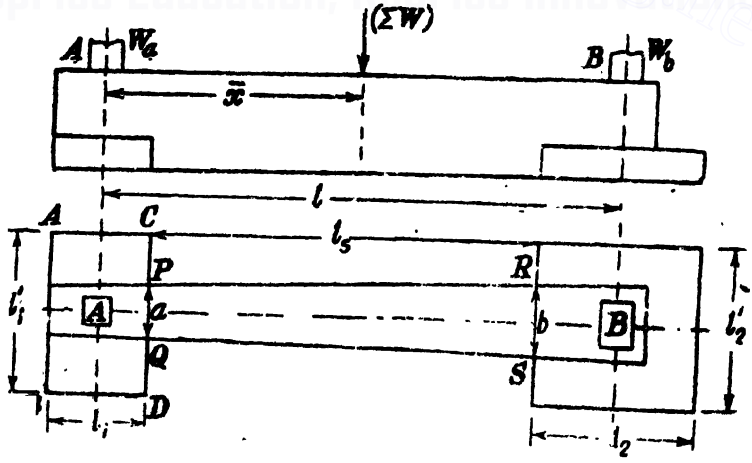

Fig. 262. Plan of strap foundation.

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Approximate weight of foundation $= W_f = 10\%$ of (ΣW).

Total load transmitted to the soil $= (\Sigma W) + W_f$.

$$\therefore \text{ Area of foundation} = A_f = \frac{(\Sigma W) + W_f}{\text{safe bearing capacity of soil}}$$

Let a_{f1} and a_{f2} be the available footing areas under the columns A and B.

$\therefore$ Balance area to be provided by the strap

$$= A_{strap} = A_f - a_{f1} - a_{f2}$$

Let the widths of the strap be a and b where it meets the end column footing and the interior column footing. Let l_s be the length of the strap beam between the two footings.

Area of the strap between the footings

$$= \frac{a+b}{2} l_s = A_{strap} \qquad ...(i)$$

Distance of C.G. of column loads from the axis of column

$$= \bar{x} = \frac{W_b l}{(\Sigma W)}$$

Let the sizes of the footings under the columns A and B be $l_1 \times l_1'$ and $l_2 \times l_2'$ (*See* Fig 262).

Distance of C.G. of the foundation plan from the left edge AB

$$= \left(\bar{x} + \frac{l_1}{2} \right)$$

This distance should also be equal to

$$\frac{\text{Moment of the total area about } AB}{\text{Total area of the foundation}}$$

$$\left(\bar{x} + \frac{l_1}{2} \right) = \frac{\left(a_{f1} \cdot \frac{l_1}{2} \right) + \left[a_{f2} \times \left(l + \frac{l_1}{2} \right) \right] + A_{strap} \left(l_1 + \frac{a+2b}{a+b} \cdot \frac{l_s}{3} \right)}{A_f}$$

$$...(ii)$$

From equations (i) and (ii) we can determine the widths of the strap a and b at the edges where it meets the footings of the columns A and B.

Net upwards pressure on the foundation

$$= p = \frac{(\Sigma W)}{A_f}$$

Design of the footings under the columns

(Note the strap extends up to the left edge of the column A)

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Footing under column A

B M. consideration. Consider a strip of footing slab of unit width. Max. cantilevering moment $= M = p\left(\frac{l_1' - a}{2}\right)$

Equating the M.R. to the B.M.

$$Qbd^2 = M$$

We can find d the effective depth of footing.

The effective depth however may not be less than 0·4 times the cantilevering projection to avoid excessive shear stresses.

Reinforcement may now be calculated for the maximum B.M. Shear stress shall be checked at a critical section taken at a distance equal to the effective depth from the face of the strap beam.

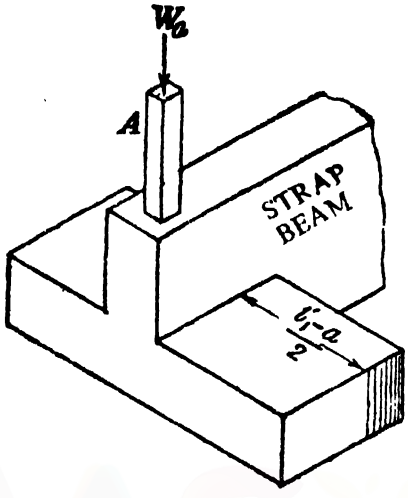

Fig. 263

Footing under the column B

This may also be designed similar to the footing under the

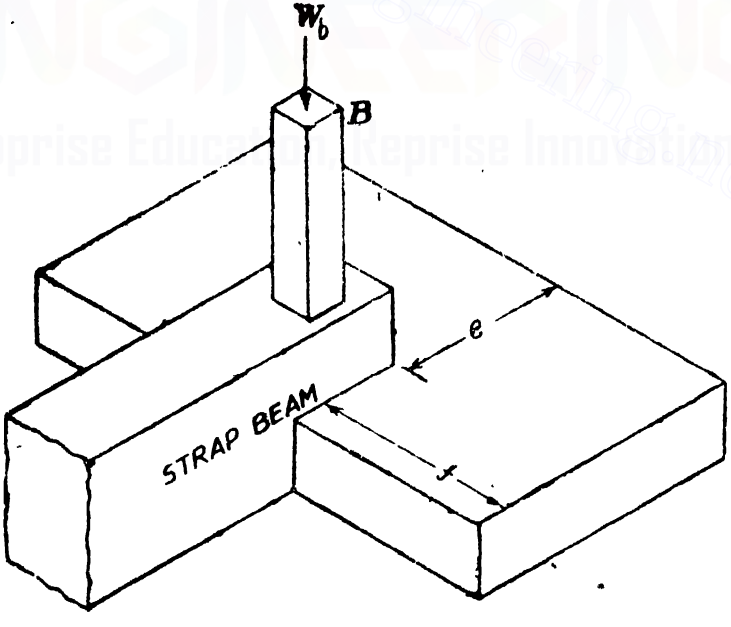

Fig. 264

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column *A*. The strap beam will be taken for a distance of 10 *cm.* to 15 *cm.* beyond the outer edge of the column *B*.

B.M. consideration. We can now determine the bending moment due to cantilevering projections *e* and *f* of the footing beyond the strap. See Fig. 264.

Max. B M. per unit wide strip

$$= p \times [\text{greater of } e \text{ or } f]^2$$

The effective depth necessary for the above bending moment can be calculated.

The effective depth however may not be less than 0·4 times the greater cantilevering projection, to avoid excessive shear stresses. Reinforcement may now be calculated for the bending moment.

Shear stress shall be checked at a critical section taken at a distance equal to the effective depth from the face of the strap beam.

Design of the strap beam Consider the strap beam between the inner edges of the two footings.

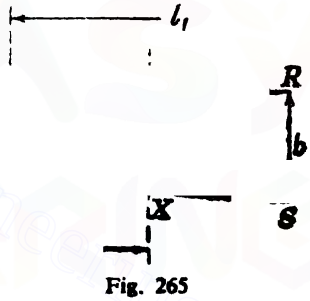

Fig. 265

Consider any section distant x from the edge PQ. Width of the beam $XX = b' = a + \left(\frac{b-a}{l_1}\right) x$. Area of the beam between PQ and $XX = \int b'\, dx = ax + \frac{b-a}{l_1} \cdot \frac{x^2}{2}$

Upward reaction between section PQ and $XX = p\left[ax \cdot \frac{b-a}{l_1} \cdot \frac{x^2}{2}\right]$

Moment of this force about $XX = \int p\left[ax + \frac{b-a}{l_1} \cdot \frac{x^2}{2}\right] dx$

$$= p\left[ax^2 + \frac{b-a}{l_1} \cdot \frac{x^3}{6}\right]$$

The strap beam is subjected to a hogging bending moment. The maximum hogging moment will occur at a section where the shear force equals zero. If this section is at a distance x from the edge PQ, equating the S.F. at the section XX to zero,

$$\left(p \times a_{/1}\right) + p\left[ax - \frac{b-a}{l_1} \cdot \frac{x^2}{2}\right] - W_a = 0$$

From this equation we can determine x.

Now we can find the width b' at the section XX.

Since the section where the S.F. is zero is now determined we can find the B.M. at this section.

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Max. Hogging B.M.

$$M = \left[W_s - pa_{/1} \right] \left(x + \quad \right) - p\left[\frac{ax^2}{2} + \frac{b-a}{l_s} \cdot \frac{x^3}{6} \right]$$

Equating the M.R. to the max. B.M.

$$Qb'\, d^2 = M.$$

From the above equation we can find d **the effective depth required for the strap beam. Assuming suitable cover the overall depths can be calculated. The depth of the beam may also be calculated from punching shear considerations under the columns** A **and** B**. The greater of the two depths calculated above for the strap beam should be provided. Now we can calculate the reinforcement required for the B.M. calculated above.**

Check for shear. Shear force should be calculated at the sections corresponding to the inner edges of the two footings. Necessary shear reinforcement in the form of 8-legged stirrups will be provided.

Design 65. *Design a strap footing to carry two column loads of 112,500 kg. and 202,500 kg. The columns are 6 metres between centres. The sizes of columns are 50 cm. × 50 cm and 60 cm. × 60 cm. respectively. The footing areas under the columns are respectively 1·20 m × 3 m. and 3 m. × 3 m. and connected by a suitable strap. The safe bearing capacity of the soil is 20500 kg./metre².*

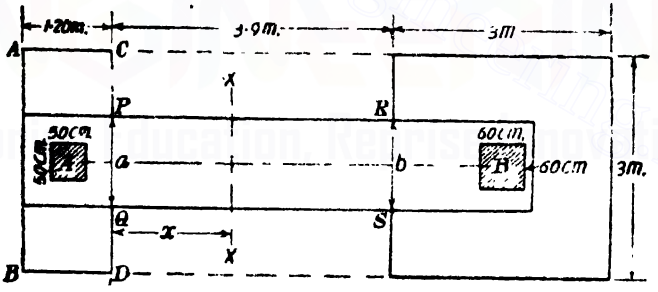

Fig. 266

Solution. Total load on the column = 315,000 *kg.*

Approximate weight of foundation = 31,500 *kg.*

Total load transmitted to the soil = 346,500 *kg.*

$\therefore$ Area required for the foundation $= \frac{346,500}{20500}$ *metre²*

$= 16{\cdot}90$ *metre²*

Area provided by the footings under the two columns

$= 3 \times 1{\cdot}2 + 3 \times 3 = 12{\cdot}60$ *metre²*

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$\therefore$ Balance area to be provided by the strap

$= 16{\cdot}90 - 12{\cdot}60 = 4{\cdot}30$ *metre*^{2}

Let the widths of the trapezoidal strap be a and b respectively (See Fig. 266).

$$\therefore \quad \frac{3{\cdot}9}{2}(a+b) = 4{\cdot}30$$

$$\therefore \quad a+b = 2{\cdot}205 \text{ metre.} \qquad \ldots(i)$$

Let the left and right columns be designated as columns A and B.

Distance of the C.G. of column loads from the axis of column A

$$= \frac{202500 \times 6}{315{,}000} = 3{\cdot}857 \text{ m.}$$

Distance of C.G. of column loads from the left edge AB

$= 0{\cdot}6 + 3{\cdot}857$ m.

$= 4{\cdot}457$ m.

Distance of centroid of the whole area from the edge AB

$$\frac{3 \times 1{\cdot}2 \times 0{\cdot}6 + 3 \times 3 \times 6{\cdot}60 + 4{\cdot}30\left(1{\cdot}20 + \frac{a+2b}{a+b} \times \frac{3{\cdot}9}{3}\right)}{16{\cdot}90}$$

$$\frac{66{\cdot}72 + 5{\cdot}59\,\frac{a+2b}{a+b}}{16{\cdot}9}$$

$$= 3{\cdot}948 + 0{\cdot}3308\left(\frac{a+2b}{a+b}\right)$$

$$= 3{\cdot}948 + \frac{0{\cdot}3308\,(a+2b)}{2{\cdot}205}$$

(since $a+b = 2{\cdot}205$)

$$= 3{\cdot}948 + 0{\cdot}15\,(a+2b).$$

In order the load distribution may be uniform the centre of gravity of column loads and the centroid of the foundation area must coincide.

$$3{\cdot}948 + 0{\cdot}15\,(a+2b) = 4{\cdot}457$$

$$\therefore \quad 0{\cdot}15\,(a+2b) = 9{\cdot}509$$

$$\therefore \quad a+2b = 3{\cdot}393 \qquad \ldots(ii)$$

But $\quad a+b = 2{\cdot}205 \qquad \ldots(i)$

From equation (i) and (ii) $\quad a = 1{\cdot}017$ m. and $b = 1{\cdot}188$ m.

Let us provide $\quad a = 1{\cdot}02$ m. $= 102$ cm.

and $\quad b = 1{\cdot}19$ m. $= 119$ cm.

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Actual areas adopted

Under column A $3 \times 1{\cdot}2 = 3{\cdot}600$ *metre*^{2}

Under column B $3 \times 3 = 9{\cdot}000$ *metre*^{2}

Strap $\frac{1}{2} \times 3{\cdot}9\ (1{\cdot}02 + 1{\cdot}19) = 4{\cdot}310$ *metre*^{2}

Total area provided $= 16{\cdot}910$ *metre*^{2}.

$$\text{Net upward pressure} = \frac{315{,}000}{16{\cdot}91} \quad kg./metre^2$$

$$= 18620\ kg./metre^2.$$

Footing under the column A

Cantilever projection of footing slab

$$= \frac{3{\cdot}00 - 1{\cdot}02}{2} = 0{\cdot}99\ m.$$

Maximum bending moment per *metre* width

$$= 18620 \times \frac{0{\cdot}99^2}{2} \times 100\ kg.\ cm. = 912400\ kg.\ cm.$$

Adopting $c = 50\ kg./cm.^2$, $t = 1400\ kg./cm.^2$ and $m = 18$, and equating the moment of resistance to the bending moment,

$$8{\cdot}5\ bd^2 = 8{\cdot}5 \times 100\ d^2 = 912400$$

$$\therefore \quad d = 32{\cdot}8\ cm.$$

To avoid excessive shear stresses d may be chosen not less than $0{\cdot}4 \times$ cantilever projection

$$= 0{\cdot}4 \times 99 = 39{\cdot}6\ cm.$$

Providing 20 *mm* ϕ bars at a clear cover of 6 *cm*.

Effective cover $= 6 + 1 = 7\ cm.$

Overall depth required

$$= 39{\cdot}6 + 7 = 46{\cdot}7\ cm.$$

Provide an overall depth of 50 *cm*.

Actual effective depth

$$= d = 50 - 7 = 43\ cm.$$

$$A_t = \frac{912400}{1400 \times 0{\cdot}87 \times 43} = 17{\cdot}42\ cm.^2$$

Spacing of 20 *mm*. ϕ bars

$$= \frac{3{\cdot}14 \times 100}{17{\cdot}42} = 18\ cm.\ c/c.$$

Provide also suitable distribution steel.

Check for shear

The critical section for shear is taken at a distance of effective depth from the face of the strap beam.

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In our case the critical section is at a distance of 43 *cm.* from the face of the strap beam.

S.F. at this critical section per *metre* width

$$S = 18620\,(0{\cdot}99 - 0{\cdot}43) = 10427\ kg.$$

Nominal shear stress

$$q_v = \frac{S}{bd} = \frac{10427}{100 \times 43} = 2{\cdot}42\ kg./cm.^2$$

Percentage of steel provided

$$= \frac{17{\cdot}42 \times 100}{100 \times 43} = 0{\cdot}4\%$$

For 0·4% steel, $q_c = 2{\cdot}7\ kg./cm.^2$ $\therefore$ $q_v < q_c$

Footing under column B

(*i*) *Depth from B.M. consideration*

Let the strap project beyond the column be 10 *cm.*

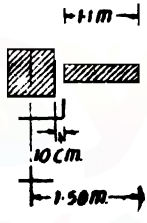

Fig. 267

$\therefore$ Projection of footing

$= 1{\cdot}50 - 0{\cdot}33 - 0{\cdot}10$ *metre*

$= 1{\cdot}10$ *m.*

B.M. *per metre* width

$$= \frac{18620 \times 1{\cdot}10^2}{2} \times 100\ kg.\ cm.$$

$= 1{,}127{,}000\ kg\ cm.$

Equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}50 \times 100\ d^2 = 1{,}127{,}000$$

$$d = 36{\cdot}40\ cm.$$

Minimum effective depth

$= 0{\cdot}4$ times cantilever projection.

$= 0{\cdot}4 \times 110 = 44\ cm.$

Using 20 *mm.* ϕ bars at a clear cover of 6 *cm.*

Effective cover to the centre of upper layer of bars

$= 6 + 2 + 1 = 9\ cm.,$

$\therefore$ Overall depth required

$= 44 + 9 = 53\ cm.$

Provide an overall depth of 55 *cm.*

Actual effective depth

$= 55 - 9 = 46\ cm.$

$$A_t = \frac{1{,}127{,}000}{1400 \times 0{\cdot}87 \times 46} = 20{\cdot}1\ cm.^2$$

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Spacing of 20 *mm.* ϕ bars

$$= \frac{3{\cdot}14 \times 100}{20{\cdot}1} = 15{\cdot}6 \text{ cm.}$$

Provide 20 mm ϕ *bars @ 15 cm. c/c.*

Check for shear

S.F. per *metre* width at the critical section width is at a distance of 46 *cm.* from the face of the strap beam

$$S = 18620\ (1{\cdot}10 - 0{\cdot}46) = 11917 \text{ kg.}$$

Nominal shear stress $= q_v = \frac{S}{bd} = \frac{11917}{100 \times 46} = 2{\cdot}59 \text{ kg./cm.}^2$

Percentage of steel provided

$$\frac{20{\cdot}1 \times 100}{100 \times 46} = 0{\cdot}44\%$$

For 0·44% steel, $q_c = 2{\cdot}80$ *kg./cm.*^{2} $\therefore$ $q_v < q_c$

Design of the strap beam

Maximum bending moment for the strap beam will occur where the shear force is zero.

Let the shear force be zero at a section XX, at a distance of x *metres* from the edge CD.

Width of the strap at XX

$$b' = 1{\cdot}02 + \frac{1{\cdot}19 - 1{\cdot}02}{3{\cdot}9} \ x \text{ metre}$$

$$= 1{\cdot}02 + 0{\cdot}044\ x$$

Area of the strap upto XX

$$= A = \tfrac{1}{2}x[1{\cdot}02 + 1{\cdot}02 + 0{\cdot}044x]$$

$$= 1{\cdot}02x + 0{\cdot}044\frac{x^2}{2}$$

$$= 1{\cdot}02x + 0{\cdot}022x^2$$

For the condition of zero shear at XX

$$112500 = 18620 \times 3 \times 1{\cdot}20 + 18620(1{\cdot}02x + 0{\cdot}022x^2)$$

$$\therefore \quad 112500 = (67032 + 18620(1{\cdot}02x + 0{\cdot}022x^2)$$

$$\therefore \quad 18620(1{\cdot}02x + 0{\cdot}022x^2) = 45468$$

$$\therefore \quad 0{\cdot}022x^2 + 1{\cdot}02x = 2{\cdot}432$$

$$\therefore \quad x^2 + 46{\cdot}36x = 110{\cdot}55$$

Solving, we get, $x = 2{\cdot}27$ *m.*

Width at the section XX

$$= b' = 1{\cdot}02 + 0{\cdot}044 \times 2{\cdot}27 \text{ metre}$$

$$= 1{\cdot}12 \text{ metre}$$

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Distance of the C.G. of the trapezium $PQXX$ from XX,

$$= \frac{1{\cdot}12+2\times1{\cdot}02}{1{\cdot}12+1{\cdot}02} \times \frac{2{\cdot}27}{3} \text{ metre}$$

$$= 1{\cdot}118 \text{ metre}$$

$\therefore$ Maximum bending moment at XX,

$= [112{,}500 - 18620 \times 3 \times 1{\cdot}20][0{\cdot}6 + 2{\cdot}27 - 1{\cdot}118]$ *kg. m.*

$= 79680$ *kg. m.*

$= 7{,}968{,}000$ *kg. cm.* (*hogging*)

Equating the moment of resistance to the maximum bending moment, we have,

$8{\cdot}50 \times (1{\cdot}12 \times 100)d^2 = 7{,}968{,}000$ *kg. cm.*

$d = 91{\cdot}5$ *cm.*

Providing 25 *mm.* ϕ bars at a clear cover of 6 *cm.*

Effective cover $= 6 + 1{\cdot}25 = 7{\cdot}25$ *cm.*

$\therefore$ Overall depth required

$= 91{\cdot}50 + 7{\cdot}25 = 98{\cdot}75$ *cm.*

Provide on overall depth of 100 *cm.*

Actual effective depth

$$= d = 100 - 7{\cdot}25 = 92{\cdot}75 \text{ cm.}$$

$$A_t = \frac{7968000}{1400 \times 0{\cdot}87 \times 92{\cdot}75} = 70{\cdot}53 \text{ cm.}^2$$

Provide 15 bars of 25 *mm.* ϕ (73·63 *cm.*^{2})

Check for shear

Shear force at the left edge PQ of the strap

$$= 112500 - 18620 \times 3 \times 1{\cdot}2 = 45468 \text{ kg.}$$

Width of the strap at this section

$$= 1{\cdot}02 \text{ m.} = 102 \text{ cm.}$$

Nominal shear stress $= q_v = \frac{45468}{102 \times 92{\cdot}75} = 4{\cdot}8$ *kg./cm.*^{2}

Percentage of steel provided

$$= \frac{73{\cdot}63 \times 100}{102 \times 92{\cdot}75} = 0{\cdot}78\%$$

For 0·78% steel, $q_c = 3{\cdot}4$ *kg./cm.*^{2}

Shear resistance of concrete $= q_c\, bd$

$$= 3{\cdot}4 \times 102 \times 92{\cdot}75 = 32166 \text{ kg.}$$

$\therefore$ Shear to be resisted by stirrups

$$= V_s = 45468 - 32166 = 13302 \text{ kg.}$$

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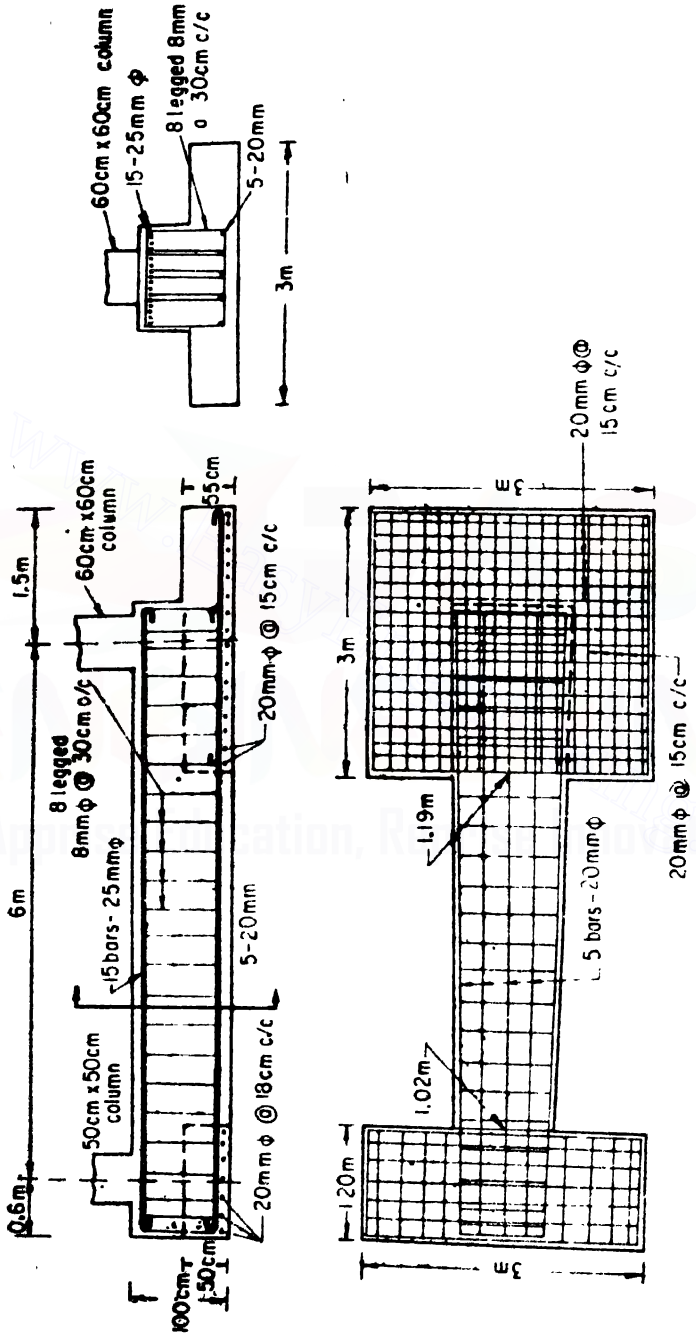

Fig. 268

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Spacing of 8-legged 8 *mm.* ϕ stirrups

$$= \frac{8 \times 0{\cdot}5 \times 1400 \times 92{\cdot}75}{13302} = 39 \text{ cm.}$$

Provide a spacing of 30 cm. c/c.

Shear force at the section *RS* of the strap

$$= 202500 - 18620 \times 3^2 = 34920 \text{ kg.}$$

This is less than the S.F. at the section *PQ*.

Hence we may provide 8-legged 8 mm. ϕ *stirrups @ 30 cm. c/c.*

§141. Strip Footing

Suppose a number of columns in a row are to be supported on a soil of such low bearing strength, that separate bases if provided would overlap a practical solution would be to provide a column footing to support all the columns of the row. Such a footing is called a *strip footing*.

Design 66. (S.I.). *Fig. 269 shows an arrangement for a footing to support five columns. The footing is chosen to be rectangular 2 metres wide and 15 metres long. Design the footing. Use M 15 concrete. All columns are 30 cm.* $\times$ *30 cm.*

Solution. Let us first determine the position of the resultant of all the loads with respect to the right end. Let the resultant load be $\bar{x}$ *metres* from the right end.

$$2000\bar{x} = (320 \times 2) + (360 \times 5) + (400 \times 8) + (440 \times 10{\cdot}5) + (480 \times 13)$$

$$= 8{\cdot}25 \text{ m.}$$ from the right end.

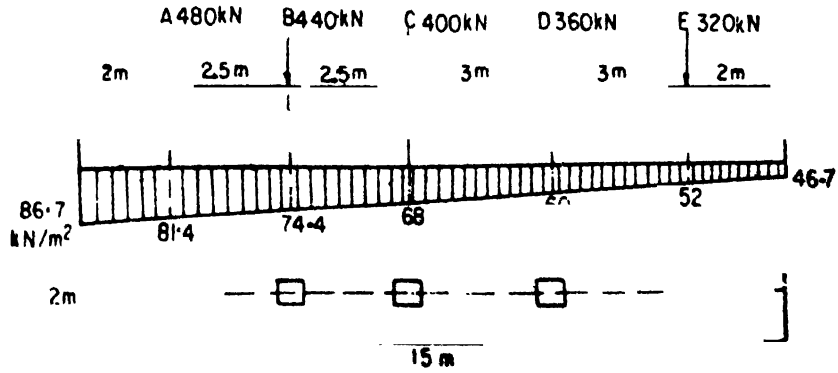

Fig. 269

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Eccentricity of the resultant load

$$= e = 8{\cdot}25 - 7{\cdot}50 = 0{\cdot}75 \ m.$$

$\therefore$ Extreme pressure intensities at the base

$$= \frac{W}{A}\left(1 \pm \frac{6e}{l}\right) = \frac{2000}{2 \times 15}\left(1 \pm \frac{6 \times 0{\cdot}75}{15}\right) \ kN/metre^2$$

$$p_{max} = \frac{260}{3} \ kN/m.^2 \text{ and } p_{min} = \frac{140}{3} \ kN/m.^2$$

The bending moments and shear forces at the various sections are now determined. These computations are shown in the table on page 459.

Maximum bending moment per *metre* width $= 237{\cdot}3 \ kN \ m.$

Equating the moment of resistance to the maximum bending moment per *metre* width

$$85 \times 100 \ d^2 = 237{\cdot}3 \times 1000 \times 100$$

$\therefore$ $$d = 52{\cdot}8 \ cm.$$

Providing 24 *mm.* ϕ bars at a clear cover of 6 *cm.*,

Effective cover $= 6 + 1{\cdot}2 = 7{\cdot}2 \ cm$

Overall depth $= 52{\cdot}8 + 7{\cdot}2 = 60 \ cm.$

$$A_t = \frac{237{\cdot}3 \times 1000 \times 100}{14000 \times 0{\cdot}87 \times 52{\cdot}8} = 36{\cdot}9 \ cm.^2$$

This is the steel required per *metre* width. Hence the total steel required in the whole width of 2 *metres* $= 2 \times 36{\cdot}9 = 73{\cdot}8 \ cm.^2$

$\therefore$ *Provide 17 bars of 24 mm.* ϕ *(76·84 cm.²)*

Maximum shear force $= S = 168{\cdot}1 \ kN$ per *metre* width.

Nominal shear stress due to the above shear force

$$= q_v = \frac{S}{bd} = \frac{168{\cdot}1 \times 1000}{100 \times 52{\cdot}8}$$

$$= 31{\cdot}8 \ N/cm.^2 = 0{\cdot}32 \ N/mm.^2$$

Percentage of steel provided $= \frac{\quad}{100 \times 52{\cdot}8} = 0{\cdot}72\%$

For 0·72% steel, $q_c = 0{\cdot}33 \ N/mm.^2$ $\therefore$ $q_v < q_c$

Hence provide nominal 12-legged 8 mm. ϕ *stirrups @ 30 cm. c/c.*

Transverse Reinforcement

Transverse projection $= \frac{2 - 0{\cdot}3}{2} = 0{\cdot}85$ *metre.*

Maximum transverse B.M. per *metre* width

$$= 81{\cdot}4 \times \frac{0{\cdot}85^2}{2} \ kN \ m = 29{\cdot}41 \ kN \ m.$$

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Section	Bending moment per metre width (kN m)		Shearing force per metre width (kN)	
A Loads on *L.H.S.*	$+81{\cdot}4\times\frac{2^2}{2}$	$=+\ 162{\cdot}8$	$+81{\cdot}4\times 2$	$=+\ 162{\cdot}8$
	$+\ 5{\cdot}3\times\frac{2^2}{3}$	$=+\ 7{\cdot}1$	$+\ 5{\cdot}3\times\frac{2}{2}$	$=+\ 5{\cdot}3$
		$+\ 169{\cdot}9$		$+\ 168{\cdot}1$
B Loads on *L.H.S.*	$+74{\cdot}7\times\frac{4{\cdot}5^2}{2}$	$=+\ 756{\cdot}3$	$+74{\cdot}7\times 4{\cdot}5$	$=+\ 336{\cdot}2$
	$+\ 12\times\frac{4{\cdot}5^2}{3}$	$=+\ 81{\cdot}0$	$+\ 12\times\frac{4{\cdot}5}{2}$	$=+\ 27{\cdot}0$
		$=+\ 837{\cdot}3$		$+\ 363{\cdot}2$
	$-\frac{480}{2}\times 2{\cdot}5$	$=-\ 600{\cdot}0$	$-\frac{480}{2}$	$=-\ 240{\cdot}0$
		$+\ 237{\cdot}3$		$+\ 123{\cdot}2$
C Loads on *L.H.S.*	$+\ 68\times\frac{7^2}{2}$	$=+1666{\cdot}0$	$+68\times 7$	$=+\ 476{\cdot}0$
	$+18{\cdot}7\times\frac{7^2}{3}$	$=+\ 305{\cdot}4$	$+18{\cdot}7\times\frac{7}{2}$	$=+\ 65{\cdot}5$
		$+1971{\cdot}4$		$+\ 541{\cdot}5$
	$-\frac{480}{2}\times 5 = -1200$	$-1750{\cdot}0$	$-\frac{480}{2} = -240$	$-\ 460{\cdot}0$
	$-\frac{440}{2}\times 2{\cdot}5 = -550$	$+\ 22{\cdot}4$	$-\frac{440}{2} = -220$	$+\ 81{\cdot}5$
	-1750		-460	
D Loads on *R.H.S.*	$+46{\cdot}7\times\frac{5^2}{2}$	$=+\ 583{\cdot}8$	$-46{\cdot}7\times 5$	$=-\ 233{\cdot}5$
	$+13{\cdot}3\times\frac{5^2}{6}$	$=+\ 55{\cdot}4$	$-13{\cdot}3\times\frac{5}{2}$	$=-\ 33{\cdot}3$
		$=+\ 639{\cdot}2$		$-\ 266{\cdot}8$
	$-\frac{320}{2}\times 3$	$=-\ 480{\cdot}0$	$+\frac{320}{2}$	$=+\ 160{\cdot}0$
		$+\ 159{\cdot}2$		$-\ 106{\cdot}8$
E Loads on *R.H.S.*	$+46{\cdot}7\times\frac{2^2}{2}$	$=+\ 93{\cdot}4$	$-46{\cdot}7\times 2$	$=-\ 93{\cdot}4$
	$+\ 5{\cdot}3\times\frac{2^2}{6}$	$=+\ 3{\cdot}5$	$-\ 5{\cdot}3\times\frac{2}{2}$	$=-\ 5{\cdot}3$
		$+\ 96{\cdot}9$		$-\ 98{\cdot}7$

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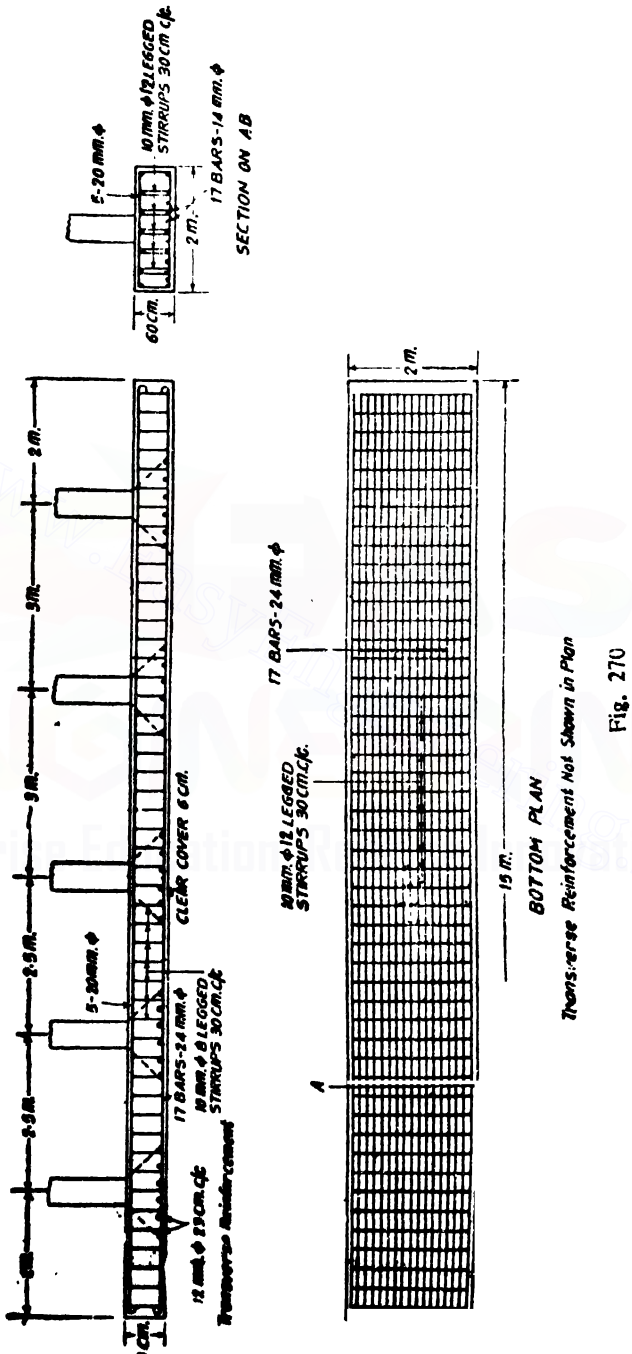

Fig. 270

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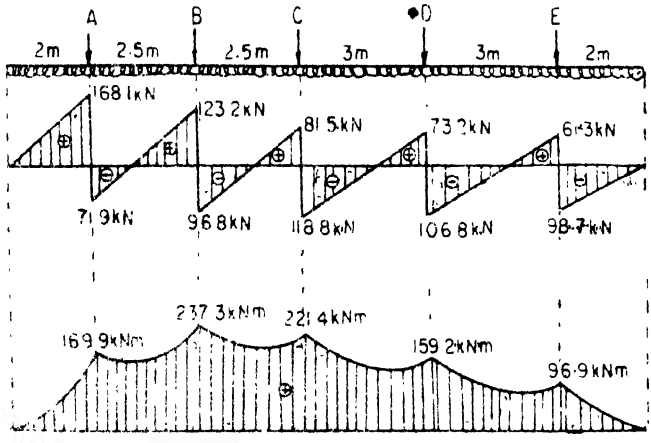

Fig. 271

If 12 *mm.* ϕ bars be used as transverse steel,

Effective depth $= 52{\cdot}8 - 1{\cdot}2 - 0{\cdot}6 = 51$ *cm.*

$$A_t = \frac{29{\cdot}41 \times 1000 \times 100}{14000 \times 0{\cdot}87 \times 51}\ cm.^2 = 4{\cdot}73\ cm.^2$$

$\therefore$ Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{4{\cdot}73} = 23\ cm.\ c/c.$$

Design 67. *Fig. 272 shows an arrangement for a column footing to be provided for five columns. The footing is chosen to be rectangular 2 metre wide and 15 metres long. Design the footing. All columns are 30 cm. × 30 cm.*

Solution. Let us first determine the position of the resultant of all the loads with respect to the right end. Let the resultant load be at $\bar{x}$ *metres* from the right end.

$\therefore$ $200\ \bar{x} = 32 \times 2 + 36 \times 5 + 40 \times 8 + 44 \times 10{\cdot}5 = 48 \times 13$

$\bar{x} = 8{\cdot}25\ m$ from the right end

$\therefore$ Eccentricity of the resultant load

$= e = 8{\cdot}25 - 7{\cdot}50 = 0{\cdot}75\ m.$

$\therefore$ Maximum and minimum pressure at the base

$$= \frac{W}{A}\left(1 \pm \frac{6e}{l}\right) = \frac{200}{2 \times 15}\left(1 \pm \frac{6 \times 0{\cdot}75}{15}\right)\ tonnes/metre^2$$

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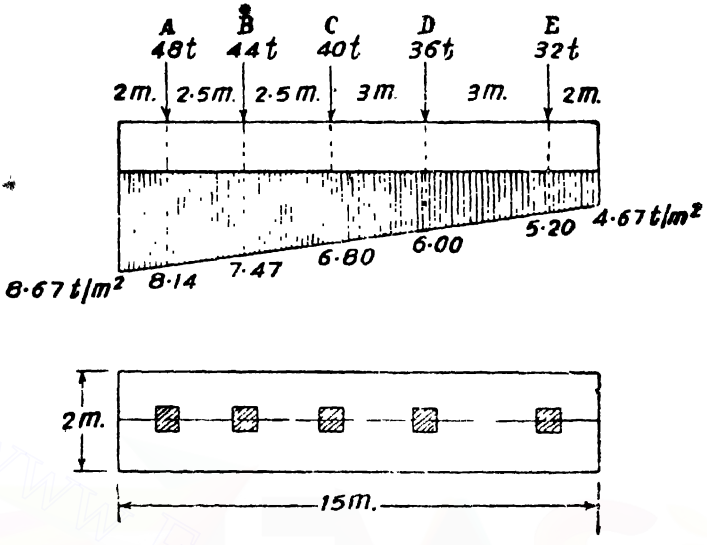

Fig. 272

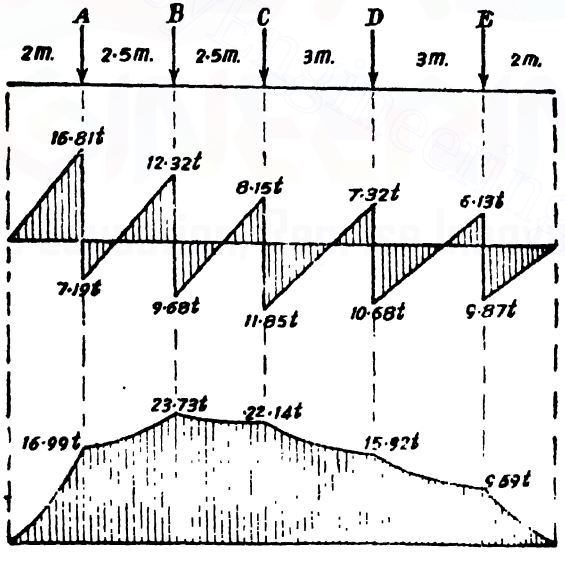

Fig. 273

$$\therefore \quad p_{max} = \frac{26}{3} \text{ t/metre}^2 \text{ and } p_{min} = \frac{14}{3} \text{ t/metre}^2$$

The bending moments and shear forces at the various sections are now determined. This computation is shown in the table below :

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Section	Bending moment per metre width (tonne metre)		Shearing force per metre width (tonnes)	
A Loads on *L.H.S.*	$+8{\cdot}14\times\frac{2^2}{2}$	$=+\ 16{\cdot}28$	$+8{\cdot}14\times 2$	$=+16{\cdot}28$
	$+0{\cdot}53\times\frac{2^2}{3}$	$=+\ 0{\cdot}71$	$+0{\cdot}53\times\frac{2}{2}$	$=+\ 0{\cdot}53$
		$+\ 16{\cdot}99$		$+16{\cdot}81$
B Loads on *L.H.S*	$+7{\cdot}47\times\frac{4\cdot5^2}{2}$	$=+\ 75{\cdot}63$	$+7{\cdot}47\times 4{\cdot}5$	$=+33{\cdot}62$
	$+1{\cdot}20\times\frac{4\cdot5^2}{3}$	$=+\ 8{\cdot}10$	$+1{\cdot}20\times\frac{4{\cdot}5}{2}$	$=+\ 2{\cdot}70$
		$+\ 83{\cdot}73$		$+36{\cdot}32$
	$-\frac{48}{2}\times 2{\cdot}50$	$=-\ 60{\cdot}00$	$-\frac{48}{2}$	$-24{\cdot}00$
		$+\ 23{\cdot}73$		$+12{\cdot}32$
C Loads on *L.H.S.*	$+6{\cdot}80\times\frac{7^2}{2}$	$=+166{\cdot}60$	$+6{\cdot}80\times 7$	$=+47{\cdot}60$
	$+1{\cdot}87\times\frac{7^2}{3}$	$=+\ 30{\cdot}54$	$+1{\cdot}87\times\frac{7}{2}$	$=+\ 6{\cdot}55$
		$+197{\cdot}14$		$+54{\cdot}15$
	$-\frac{48}{2}\times 5=-120{\cdot}00$	$-175{\cdot}00$	$-\frac{48}{2}=-24{\cdot}00$	$-46{\cdot}00$
	$-\frac{44}{2}\times 2{\cdot}5=-\ 55{\cdot}00$	$+\ 22{\cdot}14$	$-\frac{44}{2}=-22{\cdot}00$	$+\ 8{\cdot}15$
		$-175{\cdot}00$		$-46{\cdot}00$
D Loads on *R.H.S.*	$+4{\cdot}67\times\frac{5^2}{2}$	$=+\ 58{\cdot}38$	$-4{\cdot}67\times 5$	$=-23{\cdot}35$
	$+1{\cdot}33\times\frac{5^2}{6}$	$=+\ 5{\cdot}54$	$-1{\cdot}33\times\frac{5}{2}$	$=-\ 3{\cdot}33$
		$+\ 63{\cdot}92$		$-26{\cdot}68$
	$-\frac{32}{2}\times 3$	$=-\ 48{\cdot}00$	$+\frac{32}{2}$	$=+16{\cdot}00$
		$+\ 15{\cdot}92$		$-10{\cdot}68$
E Loads on *R.H.S.*	$+4{\cdot}67\times\frac{2^2}{2}$	$=+\ 9{\cdot}34$	$-4{\cdot}67\times 2$	$=-\ 9{\cdot}34$
	$+0{\cdot}53\times\frac{2^2}{6}$	$=+\ 0{\cdot}35$	$-0{\cdot}53\times\frac{2}{2}$	$=-\ 0{\cdot}53$
		$+\ 9{\cdot}69$		$-\ 9{\cdot}87$

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Maximum bending moment per *metre* width

$$= 23{\cdot}73 \ tm.$$

Equating the moment of resistance to the maximum bending moment,

$$8{\cdot}50 \times 100 \ d^2 = 23{\cdot}73 \times 1000 \times 100 \ kg.\ cm.$$

$$\therefore \quad d = 52{\cdot}8 \ cm.$$

If 24 *mm.* diameter bars be used at a clear cover of 6 *cm.* the overall depth required

$$= 52{\cdot}8 + 1{\cdot}2 + 6 = 60 \ cm.$$

$$\therefore \quad A_s = \frac{23{\cdot}73 \times 1000 \times 100}{1400 \times 0{\cdot}87 \times 52{\cdot}8} \ cm.^2$$

$$= 36{\cdot}91 \ cm.^2 \text{ per } m. \text{ width}$$

$\therefore$ Total steel required in the whole width of 2 *m.*

$$= 2 \times 36{\cdot}91 \ cm.^2 = 73{\cdot}82 \ cm^2.$$

Provide 17 bars of 24 mm. diameter (76·84 cm.²)

Maximum shear force per *m.* width

$$= 16{\cdot}81 \ tonnes.$$

Nominal shear stress due to the above shear force

$$= q_v = \frac{S}{bd} = \frac{16{\cdot}81 \times 1000}{100 \times 52{\cdot}8} = 3{\cdot}18 \ kg./cm.^2$$

$$\text{Percentage of steel provided} = \frac{\frac{76{\cdot}84}{2}}{100 \times 52{\cdot}8} = 0{\cdot}72\%$$

For 0·72% steel, $q_c = 3{\cdot}30 \ kg./cm.^2$ $\therefore$ $q_v < q_c$.

Hence provide nominal 12-legged 8 mm. ϕ stirrups @ 30 cm. c/c.

Transverse reinforcement. Assuming the columns to be 30 *cm.* × 30 *cm.*

$$\text{Transverse projection} = \frac{2 - 0{\cdot}3}{2} = 0{\cdot}85 \ m.$$

$\therefore$ Maximum transverse bending moment per *metre* width

$$= \frac{8{\cdot}14 \times 0{\cdot}85^2}{2} \ tonne\ m. = 2{\cdot}941 \ tonne\ m.$$

If 12 *mm.* bars be used as transverse steel

Effective depth $= 52{\cdot}8 - 1{\cdot}2 - 0{\cdot}6 \ cm. = 51 \ cm.$

$$A_s = \frac{2{\cdot}941 + 1000 \times 100}{1400 \times 0{\cdot}87 \times 51} \ cm.^2 = 4{\cdot}73 \ cm.^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{4{\cdot}73} \ cm. = 23 \ cms. \ c/c.$$

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§142. Raft Foundation

If the loads transmitted by the columns are such that they need large footings requiring a plan area more than half the area covered by the building it is preferable to provide a raft foundation.

A raft foundation consists of an R.C. slab acted upon by upward soil pressure and supported by secondary beams and main beams.

Design 68. *A building rests on six columns 45 cm. × 45 cm. arranged as shown in Fig 274. Each central column carries a load of 80 tonnes and the end columns carry 50 tonnes each. Design a raft foundation for the column. The design shall also allow for a wind load moment of 120 tonnes metres about the base of the raft.*

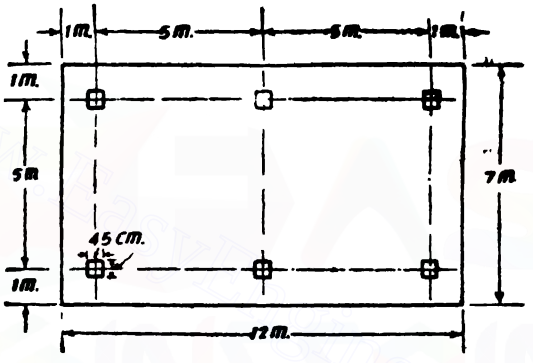

Fig. 274

Solution.

Total load on the columns $= 2 \times 80 + 4 \times 50 = 360$ *tonnes*

Approximate weight of foundation

@ 10% column loads = 36 *tonnes*

Total 396 *tonnes*

Total moment about the base

$= 120$ *tonne metres*

$\therefore$ Eccentricity of the load

$$= \frac{120}{396} = 0{\cdot}303 \ m.$$

Extreme pressure intensity at the base

$$= \frac{396}{12 \times 7}\left\{1 \pm \frac{6 \times 0{\cdot}303}{7}\right\} \ tonnes/metre^2$$

$\therefore$ $p_{max} = 5{\cdot}94 \ t/m.^2$

and $p_{min} = 3{\cdot}49 \ t/m.^2$

Pressure due to wt. of foundation

$$= \frac{36}{12 \times 7} = 0{\cdot}43 \ t/m.^2$$

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∴ Net pressure,

$$p'_{max} = 5{\cdot}94 - 0{\cdot}43 = 5{\cdot}51 \ t/m.^2$$

$$p'_{min} = 3{\cdot}49 - 0{\cdot}43 = 3{\cdot}06 \ t/m.^2$$

Since the columns are 45 *cm.* × 45 *cm.*, let the width of the main beams be 50 *cm.*

Cantilevering projection

$$= 1{\cdot}00 - 0{\cdot}25 = 0{\cdot}75 \ m.$$

Upward pressure at 0·75 *m.* from the edge

$$= 5{\cdot}51 - \frac{(5{\cdot}51 - 3{\cdot}06)}{7} \times 0{\cdot}75 \ t/m.^2$$

$$= 5{\cdot}25 \ t/m.^2$$

Consider a one *metre* wide strip of the cantilever slab.

Upward load on the cantilever

$$= \frac{(5{\cdot}51 + 5{\cdot}25)}{2} \times 0{\cdot}75 \ tonnes$$

$$= 4{\cdot}04 \ tonnes$$

This acts at $\frac{5{\cdot}25 + 2 \times 5{\cdot}51}{5{\cdot}25 + 5{\cdot}51} \times \frac{0{\cdot}75}{3}$ *metre* from the edge of beam

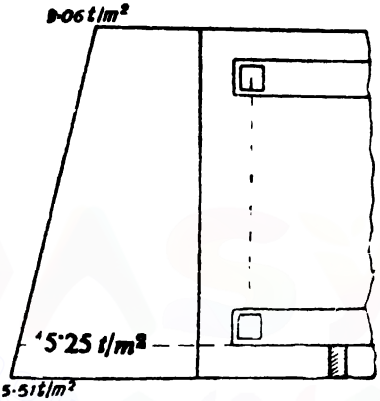

Fig. 275

$$= 0{\cdot}39 \ m. \text{ from the edge of beam}$$

∴ Maximum bending moment for the cantilever slab per *metre* width

$$= 4{\cdot}04 \times 0{\cdot}39 \ tonne \ metre$$

$$= 1{\cdot}57 \ tonne \ metre$$

Equating the moment of resistance to the maximum bending moment, we have

$$8{\cdot}50 \times 100 d^2 = 1{\cdot}57 \times 1000 \times 100 \ kg. \ cm.$$

$$\therefore \quad d = 13 \ cm.$$

If 12 *mm.* diameter bars be used at a clear cover of 6 *cm.* overall depth required

$$= 13 + 0{\cdot}6 + 6 \ cm.$$

$$= 19{\cdot}6 \ cm. \text{ say } 20 \ cm.$$

∴ Actual effective depth

$$= 20 - 6{\cdot}6 = 13{\cdot}4 \ cm.$$

$$\therefore \quad A_t = \frac{1{\cdot}57 \times 1000 \times 100}{1400 \times 0{\cdot}87 \times 13{\cdot}4} \ cm.^2 = 8{\cdot}75 \ cm.^2$$

∴ Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{8{\cdot}75} \ cm. = 12 \ cms. \ c/c.$$

Distribution steel $= \frac{0{\cdot}15}{100} \times 100 \times 20 \ cm.^2 = 3{\cdot}00 \ cm.^2$

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Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{3} \text{ cms.} = 16 \text{ cms. c/c.}$$

slab. It is proposed to provide secondary beams at 5/3 *metres* centres. The slab should be designed for an upward

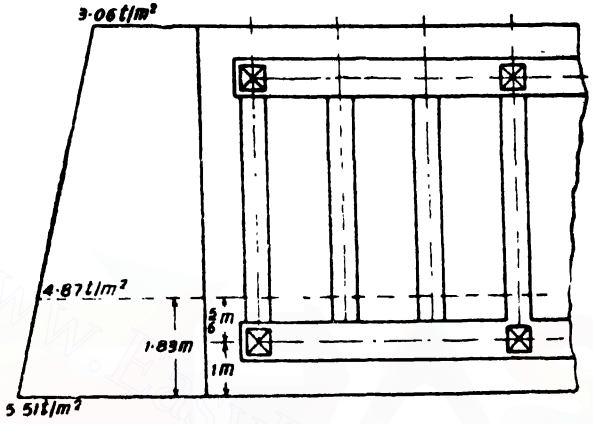

Fig. 276

pressure corresponding to a section at a distance of half the spacing of the secondary beams from the centre of main beams.

In our case the pressure is calculated at a distance of $1 + \frac{5}{6} = 1{\cdot}83$ *m.* from the edge of the foundation.

The pressure at this section

$$= 5{\cdot}51 - \frac{(5{\cdot}51 - 3\,06)}{7} \times 1{\cdot}83 = 4{\cdot}87 \text{ t/m.}^2$$

$\therefore$ Maximum bending moment per *metre* width

$$= \frac{4{\cdot}87}{12} \times \left(\frac{5}{3}\right)^2 \text{ tonne metre}$$

$$= 1{\cdot}127 \text{ tonnes metre}$$

$$\therefore \quad A_t = \frac{1{\cdot}127 \times 1000 \times 100}{1400 \times 0{\cdot}87 \times 13{\cdot}4} \text{ cm.}^2 = 6{\cdot}90 \text{ cm.}^2$$

$\therefore$ Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{6{\cdot}90} \text{ cm.} = 16{\cdot}4 \text{ cm.}$$

say 16 cms. c/c.

Secondary beams. These are designed as T-beams.

The loading on an intermediate secondary beam is less than that on the secondary beam.

The loading on the end secondary beam varies from

$$5{\cdot}16 \times 1{\cdot}\frac{5}{2} = 9{\cdot}46 \text{ t/m.}$$

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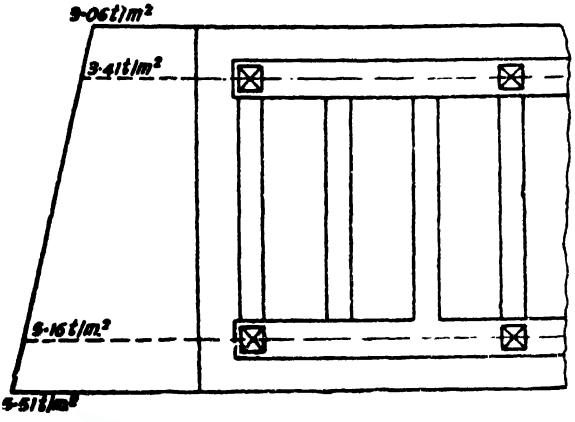

Fig. 277

to $3{\cdot}41 \times 1\frac{5}{6} = 6{\cdot}25\ t/m$

Total load

$$= \frac{(6{\cdot}25+9{\cdot}46)}{2} \times 5 \text{ tonnes}$$

$$= 39{\cdot}28 \text{ tonnes}$$

This acts at

$$\left(\frac{6{\cdot}26+2\times 9{\cdot}46}{6{\cdot}25+9{\cdot}46}\right) \times \frac{5}{3} \text{ m.}$$

from A

$= 2{\cdot}67$ m. from A

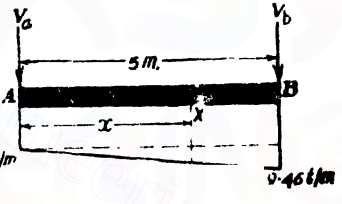

Fig. 278

Taking moments about A, we have

$$V_b \times 5 = 39{\cdot}28 \times 2{\cdot}67$$

$$\therefore \quad V_b = 20{\cdot}97 \text{ tonnes}$$

$$\therefore \quad V_a = 39{\cdot}28 - 20{\cdot}97 = 18{\cdot}31 \text{ tonnes}$$

Shear force at any section X distant x m. from A

$$= S = 6{\cdot}25\,x + \frac{1}{2}x \times \frac{3{\cdot}21\,x}{5} - 18{\cdot}31 \text{ tonnes}$$

$$S = 6{\cdot}25x + 0{\cdot}321x^2 - 18{\cdot}3_1 \text{ tonnes}$$

B.M. at section $X = M$

$$= 6\ 25\,\frac{x^2}{2} + 0{\cdot}321\,\frac{x^3}{3} - 18{\cdot}31\,x \text{ tonne m.}$$

[Expression for M is obtained easily by integrating the expression for S].

Maximum bending moment. This occurs at a section where the shear force is zero. Equating the general expression for shear force to zero, we have,

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$$6{\cdot}25x+0{\cdot}321x^2-18{\cdot}31=0$$

$$x^2+19{\cdot}47x-57{\cdot}05=0$$

Solving, we get, $x=2{\cdot}58$ *m.*

$\therefore$ B.M. at $x=2{\cdot}58$ *m.*

$$=6{\cdot}25\times\frac{2{\cdot}58^2}{2}+0{\cdot}321\times\frac{2{\cdot}58^3}{3}-18{\cdot}31\times 2{\cdot}58 \text{ tonne metre}$$

$$=-24{\cdot}602 \text{ tonne metre}$$

Overall depth of beam = about $\frac{1}{6}$ of span

$$=\frac{500}{6} \text{ cm. say } 80 \text{ cm.}$$

Let the effective cover to reinforcement be 8 *cm.*

Effective depth $= d - 80 - 2$

$= 72$ *cm.*

Approximate lever arm

$$= a = d - \frac{d_1}{2}$$

$$= 72 - \frac{20}{2} = 62 \text{ cm.}$$

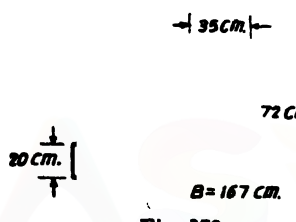

Fig. 279

Approximate steel requirement

$$A_s = \frac{24{\cdot}602\times 1000\times 100}{1400\times 62} \text{ cm.}^2$$

$$=28{\cdot}34 \text{ cm.}^2$$

Let us try *6 bars of 24 mm. diameter* ($27{\cdot}14$ *cm.*^{2}).

Breadth of rib to accommodate the above reinforcement

6 bars of 24 *mm.* dia. $= 6\times 2{\cdot}4 = 14{\cdot}4$ *cm.*

5 spacings	$=2\times 2{\cdot}5$	$=12{\cdot}5$ *cm.*
2 end covers	$=2\times 4$	$=8{\cdot}0$ *cm.*
	Total	$34{\cdot}9$ *cm.*

Let us provide a width of 35 *cm.*

Breadth of flange

This shall be taken at least of the following :

(*i*) $1+\frac{5}{6}=1{\cdot}83$ *m.* $=183$ *cm.*

(*ii*) $\frac{1}{6}\times 5=1{\cdot}67$ *m.* $=167$ *cm.*

(*iii*) $12\times 20+35=275$ *cm.*

Hence width of flange will be taken as $B=167$ *cm.*

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Position of neutral axis

Assuming the neutral axis to be outside the flange, taking moments about neutral axis, we have,

$$167 \times 20(n-10) = 18 \times 27{\cdot}14(72-n)$$

$$n = 17{\cdot}9 \text{ cm.}$$

But the thickness of the slab itself is 20 *cm.*

∴ Neutral axis must lie within the flange

Hence the neutral axis is given by

$$167\frac{n^2}{2} = 18 \times 27{\cdot}14(72-n)$$

Solving, we get, $n = 17{\cdot}8$ *cm.*

∴ Actual lever arm

$$= a = d - \frac{n}{3} = 72 - \frac{17{\cdot}8}{3} = 66{\cdot}0 \text{ cm.}$$

∴ Actual stress in steel

$$= \frac{24\,602 \times 1000 \times 100}{27\,14 \times 66{\cdot}07} \text{ kg./cm.}^2$$

$$= 1378 \text{ kg./cm.}^2 \text{ (less than 1400 kg./cm.}^2\text{)}$$

Stress in concrete $= \frac{1378}{18} \times \frac{17{\cdot}8}{(72-17{\cdot}8)}$ kg /cm.^{2}

$$= 25{\cdot}1 \text{ kg./cm.}^2$$

Shear Stress

Shear stress due to a S.F. of V_b

$$= q_b = \frac{S}{ab}$$

$$= \frac{20{\cdot}97 \times 1000}{66{\cdot}07 \times 35} \text{ kg./cm.}^2$$

$$= 9{\cdot}07 \text{ kg./cm.}^2$$

Shear stress due to a S.F. of V_a

$$= \frac{18{\cdot}31 \times 1000}{66{\cdot}07 \times 35} \text{ kg./cm.}^2$$

$$= 7{\cdot}92 \text{ kg./cm.}^2$$

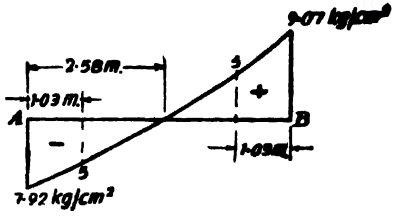

Fig. 280

Section where the shear stress will be exactly 5 kg./cm.^{2}

S.F. corresponding to this condition $= \pm 5\,ab$ kg.

Section in the zone of negative shear

The position of this section is given by the condition

$$6{\cdot}25x + 0{\cdot}321x^2 - 18{\cdot}31 = -5 \times \frac{66{\cdot}07 \times 35}{1000} \text{ tonnes}$$

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$$\therefore \quad x^2+19{\cdot}47x=21{\cdot}03$$
$$\therefore \quad x=1{\cdot}03 \ m.$$

Section in the zone of positive shear

The position of this section is given by the condition

$$6{\cdot}25x+0{\cdot}321x^2-18{\cdot}31 = +\frac{5\times 66{\cdot}07\times 3{\cdot}5}{1000} \ tonnes$$

$$\therefore \quad x^2+19{\cdot}47x=93{\cdot}05$$
$$\therefore \quad x=3{\cdot}97 \ m.$$

Main beams

Each main beam is subjected to the following loads :

(*i*) Upward concentrated loads transmitted by secondary beams.

(*ii*) Upward uniformly distributed load transferred by slab cantilevering from the main beams.

(*iii*) Downward column loads.

Maximum load transmitted by an end secondary beam

$$= 20{\cdot}97 \ tonnes$$

Maximum load transmitted by an intermediate secondary beam

$$\left[\frac{\frac{5}{3}}{1\frac{5}{6}}\right]\times 20{\cdot}97 \ tonnes$$

$$= 19{\cdot}06 \ tonnes$$

Uniformly distributed load on the main beam

$$= \tfrac{1}{2}[5{\cdot}51+5{\cdot}16]\times 1 \ tonne \ per \ metre$$
$$= 5{\cdot}335 \ tonne \ per \ metre$$

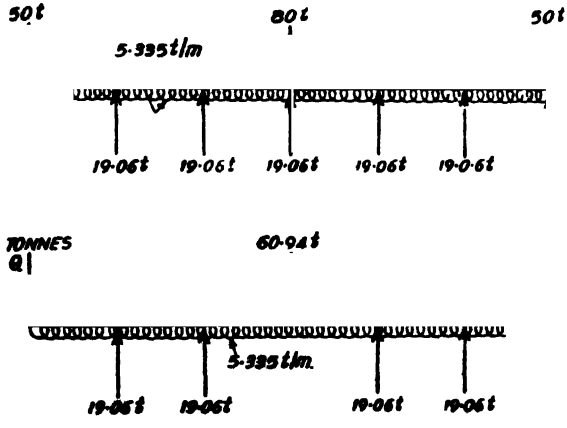

Fig. 281

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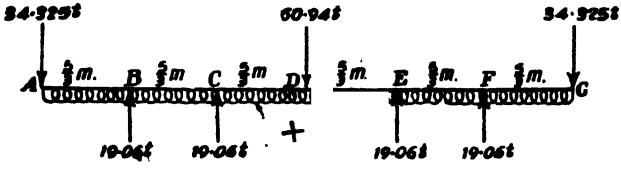
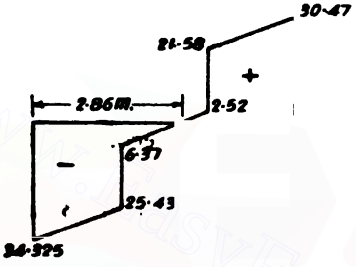
The load system acting on the main beam is shown in Fig. 281. Let the resultant point load at each end be Q *tonnes*. Resolving the forces on the main beam vertically, we have,

$$2Q+60{\cdot}94=19{\cdot}06\times4+5{\cdot}335\times10$$

$$\therefore \quad Q=34{\cdot}325 \text{ tonnes}.$$





S.F. DIAGRAM FOR HALF THE LENGTH OF MAIN BEAM

Fig. 282

S.F. Calculations

$$S_a=-34\ 325\ t$$

$$S_{ba}=-34{\cdot}325+5{\cdot}335\times\frac{5}{3}\ t$$

$$=-25{\cdot}43\ t$$

$$S_{bc}=-25{\cdot}43+19{\cdot}06=-6{\cdot}37\ t$$

$$S_{cb}=-6{\cdot}37+5{\cdot}335\times\frac{5}{3}=+2{\cdot}52\ t$$

$$S_{cd}=+2{\cdot}52+19{\cdot}06=+21{\cdot}58\ t$$

$$S_{dc}=21{\cdot}58+5{\cdot}335\times\frac{5}{3}=+30{\cdot}47\ t.$$

B.M. Calculations

$$M_a=0$$

$$M_b=-34{\cdot}325\times\frac{5}{3}+\frac{5{\cdot}335}{2}\times\left(\frac{5}{3}\right)^2=-49{\cdot}8\ tm.$$

$$M_c=-34{\cdot}325\times\frac{10}{3}+19{\cdot}06\times\frac{5}{3}+\frac{5{\cdot}335}{2}\left(\frac{10}{3}\right)^2$$

$$=-53{\cdot}01\ tm.$$

$$M_d=-34{\cdot}375\times5+19{\cdot}06\times\frac{10}{3}+19{\cdot}06\times\frac{5}{3}+\frac{5{\cdot}335}{3}\times5^2$$

$$=-9{\cdot}65\ tm.$$

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Shear stress due to a S.F. of 25·43 *tonnes*

$$= \frac{25{\cdot}43 \times 1000}{101{\cdot}6 \times 50} kg./cm.^2$$

$$= 5{\cdot}03\ kg./cm.^2$$

Shear stress due to a S.F. of 30·47 *tonnes*

$$= \frac{30{\cdot}47 \times 1000}{101{\cdot}6 \times 50} kg./cm^2$$

$$= 6\,04\ kg/cm.^2$$

Spacing of 4-legged 12 *mm.* dia. stirrups betwen *A* and *B* and *F* and *G*.

$$= \frac{4 \times 1{\cdot}13 \times 1400 \times 101{\cdot}6}{34{\cdot}325 \times 1000}\ cm.$$

$$= 18\ cm.$$

Spacing of 4-legged 12 *mm.* dia. stirrups between *C* and *D*, and *D* and *E*.

$$= \frac{4 \times 1{\cdot}13 \times 1400 \times 101{\cdot}6}{30{\cdot}47 \times 1000}\ cm.$$

$$= 21\ cms.$$

Design 69. *It is proposed to provide a common foundation for eight columns carrying loads as indicated in Fig. 283. Design the foundation. The safe bearing capacity of the soil is 10 t/metre². All columns are 40 cm. × 40 cm.*

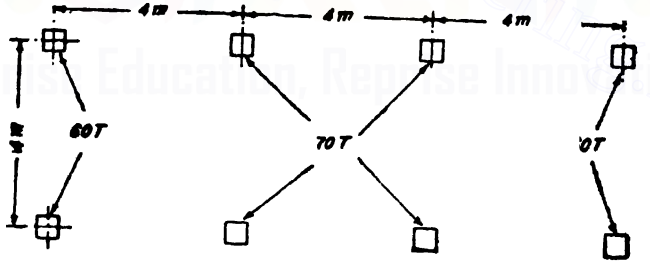

Fig. 283

Solution.

Total load on the columns $= (4 \times 60) + (4 \times 70)$ $= 520\ t$

Approximate weight of foundation (10% of column loads) $= 52\ t$

Total $= 572\ t$

Area of foundaitio $= \frac{572}{10} = 57{\cdot}2\ mtere^2$

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Provide 13 *m.* × 5 *m.* foundation as shown in Fig. 284.

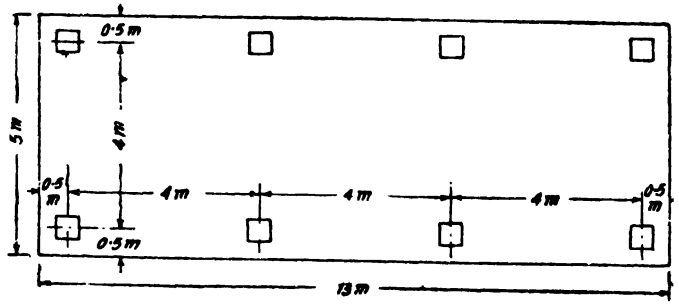
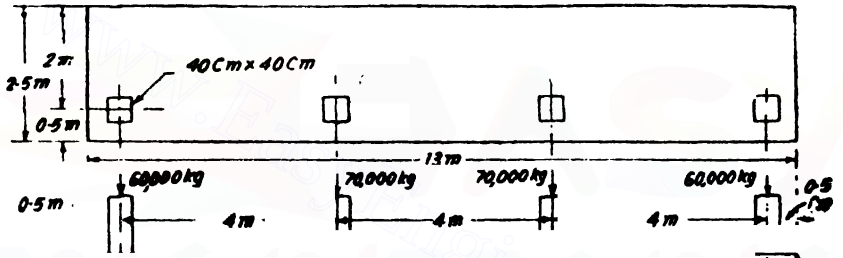


Fig. 284

Net upward pressure intensity $= \frac{250 \times 1000}{13 \times 5} = 8000$ *kg./m*^{2}.



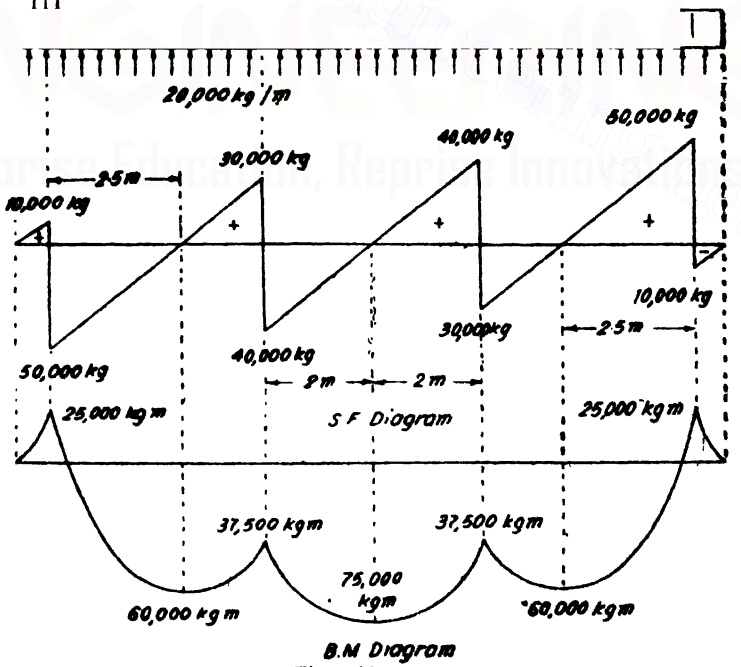


Fig. 285

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The footing may be considered as a continuous slab in two directions. A reduction coefficient of 0·2 will be adopted for shear force and bending moment.

Considering one-half the foundation in the longitudinal direction upward reaction per *metre* run $= 8000 \times 2{\cdot}5 = 20{,}000$ *kg./metre*

Fig. 286 shows the S F. and B M diagrams.

Maximum logitudinal bending moment

$$= 0{\cdot}8 \times 77500 = 62{,}000 \text{ kg. m}$$

Equating the moment of resistance to the bending moment.

$$8{\cdot}50 \times 250\, d^2 = 6{,}200{,}000 \qquad \therefore \quad d = 54\,02 \text{ cm.}$$

Providing an effective cover of 10 *cm.*

Overall depth required

$$= 54{\cdot}02 + 10 = 64\,02 \text{ cm.}$$

Provide 65 *cm.* overall and 55 *cm.* effective.

Punching shear consideration

Equating the punching resistance to the punching load

$$4 \times 40 \times D \times 10 = 70{,}000 - 8000 \times 0{\cdot}4 \times 0\,4$$

$$\therefore \qquad D = 42\,95 \text{ cm.}$$

Hence provide 65 *cm.* overall depth as decided by B.M. consideration

$$A_t = \frac{6{,}200.000}{1400 \times 0{\cdot}87 \times 55} = 92{\cdot}6 \text{ cm}^2.$$

Provide 25 bars of 22 mm. ϕ (95 cm.^{2})

Maximum S F. $= 0\,8 \times 50{,}000 - 40{,}000$ *kg.*

$$\therefore \text{ Shear stress} = \frac{S}{ab} = \frac{40{,}000}{0{\cdot}87 \times 55 \times 250} = 3{\cdot}34 \text{ kg./cm.}^2$$

(less than 5 *kg /cm*^{2})

Transverse bending

Now consider the entire length of the footing. Fig 286 shows the loading and the corresponding S.F. and B.M. diagrams.

Maximum transverse bending moment

$= 0{\cdot}8 \times 195{,}000$ *kg. m.*

$= 156{,}000$ *kg. m.*

Equating the moment of resistance to the bending moment,

$$8{\cdot}50 \times 1300\, d^2 = 156{,}000 \times 100$$

$$\therefore \qquad d = 37\,6 \text{ cm.}$$

But available effective depth

$= 55$ *cm.*

$$A_t = \frac{156{,}000 \times 100}{1400 \times 0{\cdot}87 \times 55}$$

$$232\,9 \text{ cm.}^2$$

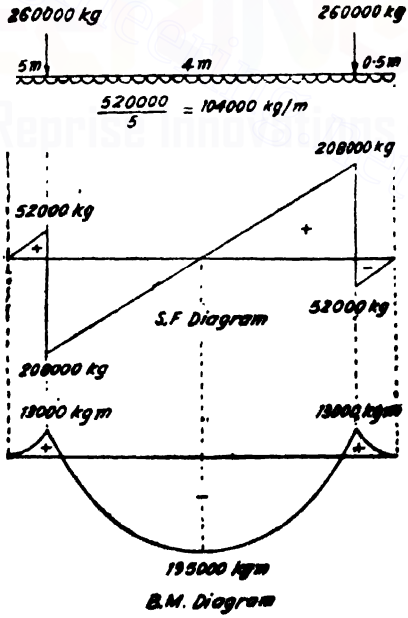

Fig. 286

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Provide 62 bars of 22 mm. ϕ (235·6 cm²)

Maximum transverse shear

$$= 0{\cdot}8 \times 208{,}000 = 166{,}400\ kg.$$

$$\text{Shear stress} = \frac{S}{ab}$$

$$= \frac{166{,}400}{0{\cdot}87 \times 55 \times 1300} = 2{\cdot}68\ kg./cm.^2$$

(less than 5 *kg./cm.*²)

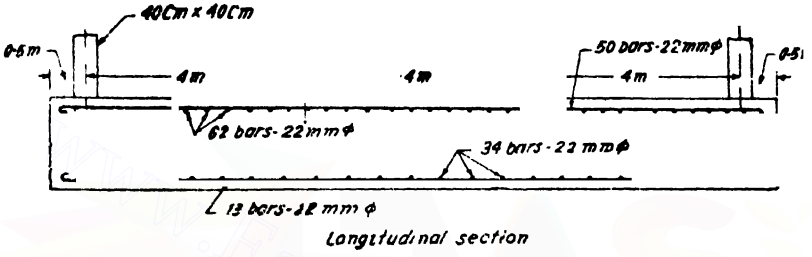

Fig. 287

Transverse bottom steel

$$= 0{\cdot}15\%\ \text{of gross area}$$

$$= \frac{0{\cdot}15}{100} \times 1300 \times 65 = 126{\cdot}75\ cm^2.$$

Provide 34 bars of 22 mm. ϕ (129·2 cm.²)

Longitudinal bottom steel

$$= 0{\cdot}15\%\ \text{of gross area}$$

$$= \frac{0{\cdot}15}{100} \times 500 \times 65 = 48{\cdot}75\ cm.^2$$

Provide 13 bars of 22 mm. dia.

§143. Pile Foundations

Sometimes, it becomes necessary to transfer the load of a structure to underground strata due to the poor bearing capacity of the surface soil. In such cases a pile foundation is a satisfactory solution. A pile foundation may also be used where the presence of water makes it difficult or impossible to provide normal foundations without the use of caissons or coffer dams From the point of view of their function, piles may be classified into bearing piles and friction piles Bearing piles are piles driven up to a hard stratum. These piles support the load of the structure and transfer the same to the hard stratum. Sometimes a hard stratum may not be available at a reasonable depth In such cases the piles which carry the load of the structure are in equilibrium entirely by the friction offered by the soil surrounding the pile. Such piles are called friction piles.

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The frictional resistance can be increased by increasing the depth of the pile or diameter. The surface of the pile may also be made rough to increase the frictional resistance Piles may be of wood or concrete Timber piles serve well if the water table at the site is near the surface. But if there is any fear that the level of water table will be lowered by any future drainage, concrete piles should be used. A timber pile may be reliable as long as it is in contact with wet soil Of the timbers available in India, the *Deodar*, *Sal*, *Teak* and *Rayani* are some of the suitable types. It is necessary that the tops of timber piles should be cut off at a level below the lowest water level. If this is not done, the top part of the pile will be subjected to alternate wetting and drying and this will result in the rotting of the pile. Timber piles are usually most economical.

Concrete piles. Piles may also be of plain or reinforced concrete. They have the advantage of not being liable to decay. They are not subject to destruction or deterioration under the influence of termites *etc.* They are unrestricted in their length. The concrete piles have higher bearing capacity and hence the number of piles required will be less than the number of timber piles, and hence it requires a foundation of lesser size They need not be cut below the low water mark. They are immune to decay. Concrete piles may be classified into—

(*i*) *Pre-cast piles*, (*ii*) *Piles cast in-situ*, and (*iii*) *Caisson piles*.

Pre-cast piles. These piles are cast in a yard before the construction is started. The pile may be square, circular or octagonal in section. Often square piles with corners chamfered are used. It is usual to provide the pile with a cast iron shoe to prevent the end of the pile from breaking particularly when it strikes a boulder underground. A pile shoe should be coaxial with the pile and firmly fixed to the concrete.

Concrete used may be preferably of *M* 200 grade (say 1 : 1½ : 3 mix or richer mix) with well-graded aggregate. The slump of concrete should be about 38 *mm*.

After piles are driven, the concrete must be carefully stripped off from the pile heads so that the longitudinal reinforcement should be exposed for a distance sufficient for bonding with the pile cap.

Pile driving This part of the job requires careful supervision and piles should therefore be driven under the supervision of men experienced in this work. Piles shall be driven vertically and straight. The depth to which a pile has to be driven should be determined by preliminary borings This will prevent over-driving of piles. If a pile is overdriven, that is driven by force to a depth greater than the depth at which the hard stratum is reached, it will result in buckling or breakling or splitting of the piles at its end. Usually piles are driven using any one of the following types of hammer : (*i*) Drop hammer, (*ii*) Single acting steam hammer,

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(*iii*) Diesel operated hammer. In the case of the simple drop hammer the weight is lifted by a strong cable taken over a pulley at the top of the pile frame and extended back to a winch, which is operated by a winch operator. A single acting steam hammer is operated by steam raising the movable mass of the hammer and then released. The hammer drops by gravity. But in a double acting steam hammer, steam pressure is utilised not only in lifting the hammer but also in dropping it with greater energy.

A heavy hammer with a small drop is preferred to a light hammer with a large drop Precast piles are more liable to be shattered due to a large number of light blows than a small number of heavy blows. The weight of a hammer shall be at least equal to half the weight of the pile to be driven. It is regarded that the best drop is 120 *dms*.

The head of the pile should be protected by providing a steel helmet with soft packed materials inside it so as to form cushion between the top of the pile and the helmet. The cushion helps in distributing the load uniformly on the top of the pile. About 8 *centimetres* of compressed saw dust, provided in a bag and put inside the helmet, is found to be effective. Felt or any such material may also be used in place of saw dust.

Fig. 288 shows a usual arrangement of dolly, cushion and helmet

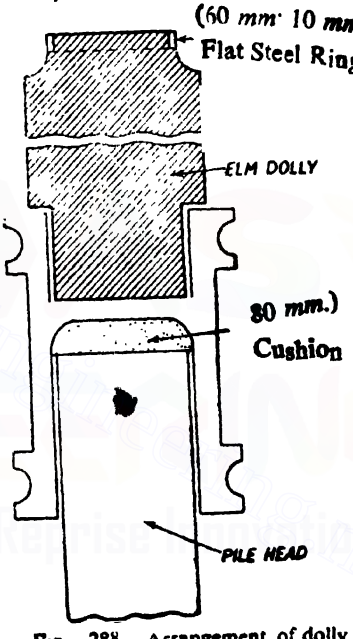

Fig. 288. Arrangement of dolly, cushion and helmet.

A pile should be provided with a cast iron shoe at its lower end Fig. 289 shows a type of cast iron shoe often provided.

Reinforcement in a pile

Main reinforcement. The main longitudinal reinforcement for a pile shall not be less than the following :

(*i*) 1·25 per cent of the gross cross-sectional area of the pile, for piles of length up to 30 times their least width.

(*ii*) 1·50 per cent of the gross cross-sectional area of the pile, for piles of length between 30 and 40 times their least width

(*iii*) 2·00 per cent for piles of length over 40 times their least width.

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Lateral reinforcement. Lateral reinforcement in the form of links shall be provided. The diameter of this reinforcement shall not be less than 5 *mm*. In the body of the pile the reinforcement shall not be less than 0·2 per cent of the gross volume. The spacing of the lateral reinforcement shall not be greater than half the least width of the pile. From each end of the pile, for a distance of three times the least width the spacing of the lateral ties shall be closer so that the lateral reinforcement provided in these regions shall not be less than 0·6% of the gross volume. The transition between the close spacing at the ends and the spacing in the body of a pile shall be made gradually.

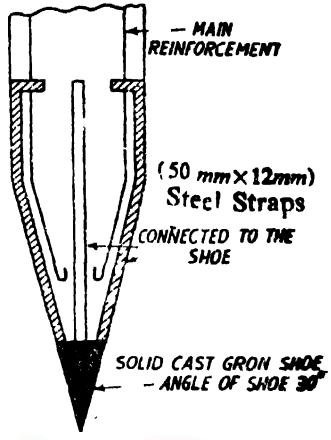

Fig. 289. Cast iron shoe for a pile.

In addition to the links, the longitudinal bars shall be held apart at intervals of at least 1·5 *metres* by spreaders made of 12 *mm* or 16 *mm* This is done to prevent inward bucking of the bars 6*mm* links should also be provided as ties to prevent outward displacement

If hard driving is expected, it is a visable to provide additional reinforcement in the top of a pile in the form of a helix extending a distance equal to three times the least width

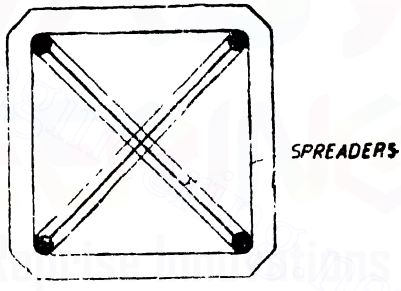

Fig. 290. Spreaders with 8 *mm*. dia. ties.

Shape of cross-section Reinforced concrete piles are generally square, rectangular or octagonal in section. Tee sections have been used for sheet piles. Usually the widths of the square piles are between 25 *cm*. and 45 *cm*. Precast piles of circular section are not found to have any advantage compared with octagonal piles. Further shuttering is not so simple for piles of circular section.

Where considerable transverse strength is required, piles of square and rectangular section are found most suitable These are therefore used in retaining walls also. For piles of cross-sections greater than 40 *cm*. octagonal bearing piles are generally preferred.

Size of a pile. The cross-section of a pile varies with its overall length The following table gives the usual cross-sections of a pile for various lengths.

Length of pile	*Cross section of pile*
Up to 10 *m*.	25 *cm*. × 25 *cm*.
Between 10 *m*. and 12 *m*.	30 *cm*. × 30 *cm*.

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Between 12 *m.* and 15 *m.*	35 *cm.* × 45 *cm.*
Between 15 *m.* and 18 *m.*	40 *cm.* × 40 *cm.*
Above 18 *m.*	45 *cm.* × 45 *cm.*

Cover. Cover to all reinforcement including binding wires shall not be less than 40 *mm.* If the pile is exposed to sea water or other corrosive effects, the cover to all steel shall not be less than 50 *mm.*

Stresses during handling. A pile is usually cast while in its horizontal position. When it therefore is lifted by a derrick it is subjected to bending stresses caused by its own weight. The maximum bending moments to which a pile may be subjected to while handling, depends upon the position and the number of the points of suspension. Piles less than 12 *metres* in length may be suspended at their middle points The maximum bending moment occurs in such a case at the middle point. A pile suspended at its middle point requires considerable amount of reinforcement to resist the induced bending stresses. Hence piles longer than 12 *metres* are suspended at two points during handling. Sometimes they are suspended at three points when the lengths of the pile are very large. It is very important for the designer to know before hand as to how the pile will be handled so that he can make provision for the requisite strength.

Pile suspended at one point. Suppose the pile is l units long and suspended at its middle point as shown in Fig. 291

Let the weight of the pile be w per unit run.

Fig. 291. One point suspension.

$$\therefore \text{ Max. B.M.} = \frac{wl}{2} \times \frac{l}{4} = \frac{wl^2}{8} \text{ at the point of suspension.}$$

(ii) Pile suspended at two points. Fig. 292 shows a pile lifted by suspending it at two points C and D. Let the length of the pile be l. Vertical component of the reaction at each point of suspension equals $\frac{wl}{2}$ where w is the weight per unit run of the pile. It is clear that the maximum hogging bending moment occurs at the points of suspension, while the maximum sagging bending moment occurs at the centre. The points of suspension are so selected that the hogging bending moment at the point of suspension equals the sagging bending moment at the centre.

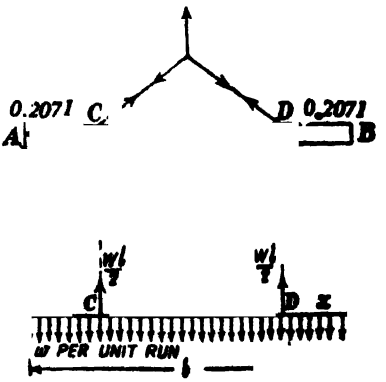

Fig. 292. Two points suspension.

In Fig. 292, let $AC = x$

$\therefore$ Hogging b.m. at C

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$$= \frac{wx^2}{2}$$

and the sagging b.m. at the centre

$$= \frac{wl}{2}\left(\frac{l}{2} - x\right) - \frac{wl^2}{8}$$

$$= \frac{wl}{2}\left(\frac{l}{4} - x\right)$$

Equating the two, we have

$$\frac{wx^2}{2} = \frac{wl}{2}\left(\frac{l}{4} - x\right)$$

$$\therefore \quad x^2 = \frac{l^2}{4} - lx$$

$$\therefore \quad x^2 + lx = \frac{l^2}{4}$$

Completing the square, we have

$$\left(x + \frac{l}{2}\right)^2 = \frac{l^2}{4} + \frac{l^2}{4} = \frac{l^2}{2}$$

$$\therefore \quad x + \frac{l}{2} = \frac{l}{\sqrt{2}}$$

$$\therefore \quad x = \frac{l}{\sqrt{2}} - \frac{l}{2} = \frac{l}{2}\left[\sqrt{2} - \right] = 0.207\,l$$

The b.m. at A or $C = \frac{wx^2}{2}$

$$= \frac{w(0.207\,l)^2}{2} = \frac{wl^2}{47}$$

Since hogging and sagging moments occur the same amount of steel will be provided near both the faces.

Pile suspended at three points. It can be shown proceeding as in points of suspension should be as shown in Fig 293, *i.e.*, one point of suspension is at the centre and the other two points of suspension are at a distance of 0·15 *l* from the respective ends. The maximum hogging and sagging bending moments will be $\frac{wl^2}{90}$.

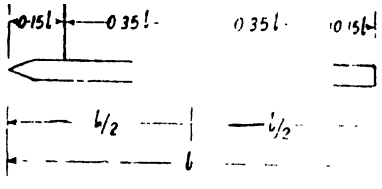

Fig. 293. Three points suspension.

Bending moment caused during erection by lifting one end of the pile, while the other end is resting on the ground. In this case the maximum bending moments will be at the point of supension and at another point between the lower end A and the point of suspension C.

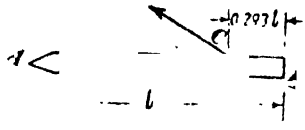

Fig 294

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Proceeding as in the previous cases, it can be shown that the point of suspension should be at a distance of 0·293 l from the head of the pile. The maximum hogging and sagging bending moments will in this case be $\frac{wl^2}{23}$.

Pile Formulae. Fig. 295 shows a pile driven to a depth of h. Let the angle of repose of the stratum into which the pile is driven be ϕ. Let the coefficient of friction between the concrete and the surrounding soil be μ. Let the safe bearing capacity of the soil on which the pile is resting be p.

Fig. 295

∴ **Total load on the pile**

= Capacity of the pile

= $p \times$ cross-sectional area of the pile + frictional resistance.

= $p \times$ sectional area.

$+ C_p \frac{wh^2}{2} \times \mu \times$ perimeter of pile section

where $C_p = \frac{1+\sin\phi}{1-\sin\phi}$ since the lateral pressure of the soil is passive.

In the above relation, μ can be taken from 0·3 to 0·35 and ϕ may be taken from 15° to 22° since the soil is likely to be wet.

Hence if p, ϕ and μ are known the length required for a pile to transmit a given load can be determined.

In case the frictional resistance is unreliable, then the piles are naturally driven upto hard stratum and the capacity of the pile will be the same as that of a column of the same section. Since it is difficult to exactly know the actual value of p, μ and ϕ the capacity computed is only approximate.

The capacity of a pile may be more accurately determined as follows :

Let W be the weight of the hammer in kg.

Let P be the weight of pile in kg.

H = Drop of the hammer

= fall of hammer (m)

s = Mean penetration per blow which is computed as the average of the penetrations in the last few blows. (m)

Fig. 295

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As the hammer drops through the distance of h *metres* under gravity it will attain a velocity of $\sqrt{2gh}$ *metre per sec.* just before hitting the pile. Immediately after the impact the pile hammer and the pile will attain a velocity of V *metre* per sec.

By the principle of conservation of momentum, we have,

Momentum before impact = Momentum after impact

$$\therefore \quad W\sqrt{2gH} = (W+P)V$$

$$\therefore \quad V = \frac{W}{W+P}\sqrt{2gH} \qquad \ldots(1)$$

Hence the two bodies, that is the pile hammer and the pile will have, together, a kinetic energy of $(W+P)\frac{V^2}{2g}$ *kg. m.* immediately after the impact. If the penetration per blow is s *metre* we have the total loss of energy

= Loss of kinetic energy + Loss of potential energy

$$= (W+P)\frac{V^2}{2g} + (W+P).$$

If R be the resistance offered by the soil in *kg.*, work done in overcoming the resistance $= Rs$. *kg. m.*

$$\therefore \quad Rs = (W+P)\frac{V^2}{2g} + (W+P)s$$

Substituting for V, we have,

$$R.s = \frac{(W+P)}{2g}\left[\frac{W}{W+P}\sqrt{2gH}\right]^2 + (W+P)s$$

$$\therefore \quad R\,s = \frac{W^2}{W+P}H + (W+P)s.$$

$$\therefore \quad R = \frac{W^2}{W+P}\cdot\frac{H}{s} + (W+P)\ kg.\ wt \qquad \ldots(2)$$

R is the total load the pile can carry. Hence the external load the pile can carry $= R' = R - P$

$$\therefore \quad R' = \frac{W^2}{W+P}\cdot\frac{H}{s} + W$$

In the above expression W is small in comparison with

$$\frac{W^2}{W+P}\cdot\frac{H}{s}$$

Hence $$R' = \frac{W^2}{W+P}\cdot\frac{H}{s}$$

or $$= \left(\frac{W}{1+\frac{P}{W}}\right)\frac{H}{s}$$

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A suitable factor of safety is used to obtain the safe load or safe capacity of the pile. The factor of safety varies from 4 to 6.

It is found when the average penetration is nearly 0·25 *cm.* the above formulae will give reliable results. But for smaller penetrations the capacity of the pile determined from the above formula is higher than its actual value.

Various formulae are used to estimate the bearing capacity of piles. Some of the commonly adopted formulae are given below :

(*ii*) **Static formula.** The ultimate bearing capacity R of a pile is given by

$$R = R_t + R_f$$

where, R_f = Tip resistance at ultimate load

R_f = Frictional resistance at ultimate load and is given by,

$$R_f = \mu \left[\frac{wH^2}{2} \frac{1+\sin\phi}{1-\sin\phi} \right] P$$

where, μ = Coefficient of friction between the pile and the soil (0·3 to 0·6)

w = Specific weight of the soil

P = Perimeter of the pile

H = Minimum length of pile required.

Fig. 297.

Design 70. (S.I.). *It is desired to drive a pile of 30 cm. diameter in a sandy clay formation to support a load of 200 kN. Find the length of pile required allowing a factor of safety of 4. Take* $\mu = 0{\cdot}35$, $\phi = 30°$ *and* $w = 18000$ *N/metre*^{3}.

Solution. Working load = 200 *kN*

Collapse load (ultimate load) = $200 \times 4 = 800$ *kN*

Tip reaction $R_t = \frac{1}{5}(800) = 160$ *kN*

$\therefore$ Frictional resistance = $R_f = 800 - 160 = 640$ *kN*

$= 640{,}000$ *N*

$$R_f = \mu \left[\frac{wH^2}{} \frac{1+\sin\phi}{1-\sin\phi} \right] P = 640{,}000$$

$$0{\cdot}35 \times 18000\, H^2 \left(\frac{1+\sin 30°}{1-\sin 30°} \right) \pi \times 0{\cdot}30 = 640{,}000$$

$$H = 8{\cdot}50 \text{ metre}$$

Design. 71. *Find the length of a 30 cm. diameter pile to support a working load of 20 t. The pile is to be driven in a sandy clay formation. Allow a factor of safety of 4. Take* $\mu = 0{\cdot}35$, $\phi = 30°$ *and* $w = 1800$ *kg./metre*^{3}.

Solution. Working load = 20 *t*

Ultimate load = $20 \times 4 = 80$ *t*

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Tip reaction $= R_t = \frac{80}{5} = 16\ t$

Frictional resistance $= R_f = 80 - 16 = 64\ t = 64000\ kg.$

$$R_f = p.\ \frac{wH^2}{2}\left[\begin{array}{l}1+\sin\phi\\1-\sin\phi\end{array}\right] P = 64000$$

$$0{\cdot}35 \times 1800\ \frac{H^2}{2}\ \frac{1+\sin 30^\circ}{1-\sin 30^\circ} \times \pi \times 0{\cdot}30 = 64{,}000$$

$$\therefore \qquad H = 8{\cdot}50\ metre.$$

(ii) Dynamic formulae

(a) *Engineering News Formula of Wellington.*

The allowable load Q_a for a pile is given by,

$$Q_a = \frac{Wh}{F(s+c)}$$

where W = Weight of the pile hammer.

h = Height of fall of hammer.

s = Average set or average penetration per blow. This is taken as the average penetration for the last 5 blows of a drop hammer, or the last 20 blows of a steam hammer.

c = Linear quantity = $\left\{\begin{array}{l}2{\cdot}50\ cm. \text{ for drop hummer and}\\0{\cdot}25\ cm. \text{ for single and double}\\\text{acting hammer.}\end{array}\right.$

F = Factor of safety = 6

Note : In the case of double acting hammer W is the sum of the weight of the hammer and the load due to steam pressure

The Engineering News formulae as applicable to the various types of hammers are given below :

When a drop hammer is used

$$Q_a = \frac{Wh}{6(s+2{\cdot}5)}$$

When a single acting steam hammer is used

$$Q_a = \frac{Wh}{6(s+0{\cdot}25)}$$

When a double acting steam hammer is used

$$Q_a = \frac{(W+ap)\,h}{6(s+0{\cdot}25)}$$

where, a = Effective piston area.

p = mean effective steam pressure.

Note. The quantities, 2·5 and 0·25 in the above formulae are linear quantities in *cm.*

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Design 72. (S.I). *A drop hammer weighing 20 kN is used to drive a wooden pile in a soil. If the hammer has a free fall of 1 metre and if the average penetration for the last five blows is 4·5 mm., calculate the load bearing capacity of the pile. Use Engineering News Formula.*

Solution. Since a drop hammer is used in driving the pile.

The safe load for the pile

$$= Q_u = \frac{Wh}{6\,(s+c)}$$

W = weight of hammer = 20 kN = 20,000 N

s = average set = 4·5 mm = 0·45 cm.

c = 2·5 cm. h = 1 m. = 100 cm.

$$Q_u = \frac{20{,}000 \times 100}{6\,(0{\cdot}45+2{\cdot}5)} = 112994 \ N$$

$$= 112{\cdot}994 \ kN$$

$$say \ 113 \ kN$$

(*b*) *Hiley's formula.* Hiley presented a formula for ultimate bearing capacity of a pile in his article "*A rational pile driving formula and its application in piling practice explained*" in the magazine *Engineering—May 29 and June 12, 1925.*

The ultimate bearing capacity Q_u is given by

$$Q_u = \frac{\eta_h \, Wh \, \eta_b}{s + \frac{c}{2}}$$

where, Q_u = Ultimate load (kg.).

W = Weight of the hammer (kg).

s = Penetration or set per blow for the last five blows.

c = Total temporary elastic compression of the pile, cap and the ground (cm.).

η_h = Efficiency of the hammer.

η_b = Efficiency of hammer blow.

= Ratio of energy just after impact to the striking energy of the hammer.

The efficiency of hammer blow should be taken as follows :

Let P = weight of the pile, anvil, helmet, follower *etc.*

e = coefficient of restitution (0 to 0·5)

When, $W > eP$, $\eta_b = \frac{W + e^2P}{W+P}$

When, $W < eP$, $\eta_b = \frac{W - e^2P}{W+P} = \left[\frac{W+eP}{W+P}\right]^2$

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The temporary elastic compression (c) consists of the following components :

(*i*) Elastic compression of the pile (c_1).

(*ii*) Elastic compression of the dolly and helmet (c_2).

and (*iii*) Elastic compression (quake) of the ground.

c_1, c_2 and c_3 are all proportional to the compressive stress in the pile induced by the resistance to driving.

Let A_p be the cross-sectional area of the pile. Hence the above quantities are proportional to $\frac{R}{A_p}$. In addition, c_1 will also depend on the length of the pile L (*metre*)

For R-C piles $c_1 = 0{\cdot}72 \frac{R}{A_p} L$ *cm.*

For timber piles $c_1 = 0{\cdot}96 \frac{R}{A_p} L$ *cm.*

R shall be taken in *tonnes*

A_p ,, ,, ,, ,, *sq. cm.*

L ,, ,, ,, ,, *metre*

c_1 ,, ,, ,, ,, *cm.*

Value of c_2 (elastic compression of dolly and helmet

R-C-pile with cushion *say* 25 *mm.* thick but no dolly or helmet — $c_2 = 1{\cdot}77 \frac{R}{A_p}$ *cm.*

R-C-pile width short dolly (up to 60 *cm.*) and helmet with cushion up to 75 *mm.* in thickness — $c_2 = 9 \frac{R}{A_p}$ *cm.*

Timber pile — $c_2 = 3{\cdot}54 \frac{R}{A_p}$ *cm.*

Effective fall of the hammer. The product $\eta_h h$ is called the effective fall of the hammer.

For a drop hammer with trigger release $\eta_h = 1$

For a drop hammer falling against friction $\eta_h = 0{\cdot}8$

For a single acting steam hammer $\eta_h = 0{\cdot}9$

When a double acting steam hammer is used, the quantity $W \eta_h h$ should be replaced by $WS + ASM$

where, W = Weight of the piston (*kg.*).

S = Stroke of piston (*cm.*).

A = Piston area acted on by steam ($cm.^2$).

M = Mean steam pressure in the cylinder ($kg./cm.^2$).

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Coefficient of restitution e. The coefficient of restitution e for various conditions met with are given below :

Condition met with	*Coefficient of restitution e*
For steel piston of double acting hammer, driving R-C-pile	0·5
For single acting or drop hammer striking directly on the head of R-C-pile	0·4
For single acting or drop hammer striking a well conditioned wood dolly or helmet on an R-C-pile	0·25
For single acting or drop hammer striking directly on the head of a timber pile	0·25
For a deteriorated head or dolly	$e=0$

Design 73. *A pile is likely to be subjected to a permanent load of 40 tonnes. It was proposed that a set corresponding to a resistance of 80 tonnes is to be obtained. The pile section is 30 cm. × 30 cm. and is 15 metres long. The pile is driven by a single acting steam hammer having a weight of 3 tonnes which falls a distance of 1 metre vertically. The weight of the helmet and dolly may be taken as 0·50 tonne. Determine the set required. Take coefficient of restitution $e=0·25$, and efficiency of the hammer $\eta_h=0·9$.*

Solution. In this case $R=80$ *tonnes*

$$A_p=30\times 30=900 \text{ cm.}^2$$

Let c = elastic compression of the pile, cap and ground

$$c=c_1+c_2+c_3$$

$$c_1=0·72\frac{R}{}L=0·72\times\frac{80}{900}\times 15=0·960 \text{ cm.}$$

$$c_2=9·01\frac{R}{A_p}=9·01\times\frac{80}{900}=0·800 \text{ cm.}$$

$$c_3=3·54\frac{R}{A_p}=3·54\times\frac{80}{900}=0·315 \text{ cm.}$$

$$c=c_1+c_2+c_3=2·075 \text{ cm.}$$

P = weight of the pile + weight of helmet and dolly

$$=\frac{0·30\times 0·30\times 15\times 2500}{1000}+0·50 \text{ tonne}$$

$$=3·875 \text{ tonne}$$

But weight of the hammer $= W = 3$ *tonnes.*

$$eP=0·25\times 3·875=0·97 \text{ tonne}$$

$$\therefore \quad W>eP$$

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∴ Efficiency of hammer blow

$$=\eta_h=\frac{W+e^2P}{W+P}=\frac{3+0{\cdot}25^2\times3{\cdot}857}{3+3{\cdot}875}=0{\cdot}47$$

∴ Ultimate bearing capacity or ultimate resistance

$$Q_u=\frac{\eta_h W h\eta_b}{s+\frac{c}{2}}=80$$

$$\frac{0{\cdot}9\times3\times100\times0{\cdot}47}{s+\frac{2{\cdot}075}{2}}=80$$

$$\therefore \quad s+1{\cdot}0375=1{\cdot}5862$$

$$\therefore \quad s=0{\cdot}5487 \text{ cm. say } 0{\cdot}55 \text{ cm.}$$

Hence, the set required is 0 55 cm. per blow.

(*c*) *Brix Formula*

According to this formula, the safe load Q_a per pile is given by

$$Q_a=\frac{W^2Ph}{F(W+P)^2s}$$

where, W = weight of hammer,

P = weight of pile,

h = height of hammer fall,

s = penetration per blow,

and F = Factor of safety (equal to 3).

In all major works where a large number of piles may have to be driven, it is preferable to conduct an actual load test and determine the capacity of the pile. This may be compared with the value obtained by penetration formulae, and then, the ratio of the bearing capacity of the pile determined by actual load tests to the bearing capacity of pile by the above formulae is determined. If this ratio is r, we can introduce r as a multiplying correcting coefficient, while using the penetration formulae, and thereby the load test need not be conducted for all other piles.

Testing a pile by load test. A platform is provided on the pile to be tested and the platform is loaded. The magnitude of the load is gradually increased, till the load for which the pile is to be designed is reached. The whole system is left in this position for 24 hours so as to assume the full settlement. The load on the pile is now gradually increased by 50% more. The system is now left fer 24 hours and the settlement is noted. The pile is now slowly and uniformly unloaded and the consequent rebound is noted. The net settlement which is equal to the total settlement minus the rebound should not be greater than 0·25 *mm.* per *tonne* of the test load.

Design 74. (S I). ***The foundation for a structure is to consist of 16 piles to carry a total load of 8000 kN. The piles are spaced at***

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1·25 metres centres. They are driven through made up ground into a hard stratum which is available at a depth of 6 metres. Design one of the piles. Use M 20 grade concrete mix.

Solution.

Safe stresses adopted

c *(direct)* $= 5\ N/mm^2 = 500\ N/cm^2$

c *(bending)* $= 7\ N/mm^2 = 700\ N/cm^2$

Safe compressive stress in steel

$$= 130\ N/mm^2 = 13000\ N/cm^2$$

Safe tensile stress in steel

$$140\ N/mm^2 = 14000\ N/cm^2$$

modular ratio $m = \dfrac{280}{3 \times 7} = 13$

Load on each pile $\dfrac{8000}{16} = 500$ kN

Let the piles be 6·5 *metres* long

Let the piles be 25 *cm.* × 25 *cm.*

$$\frac{l}{D} = \frac{650}{25} = 26$$

This is greater than 12. Hence the pile will be treated as a long column.

Reduction coefficient $C = 1.25 - \dfrac{l}{48D}$

$$= 1.25 - \frac{26}{48} = 0.708$$

Safe compressive stress in concrete

$$0.708 \times 500 = 354\ N/cm^2$$

Safe compressive stress in steel

$$= 0.708 \times 13000 = 9204\ N/cm^2$$

Safe load on the pile = safe load on concrete + safe load on steel

$$500{,}000 = 354(625 - A_t) + 9204\ A_t\ Newton$$

$$A_t = 31.50\ cm^2$$

The main reinforcement for a pile whose length is not greater than 30 times the least width shall not be less than 1·5% of the gross area of section. In our case the minimum reinforcement in steel

$$\frac{1.25}{100} \times 25 \times 25 = 7.81\ cm^2$$

Hence the actual steel required $21.50\ cm^2$

Provide 8 bars of 25 mm. ϕ (39.28 cm.²)

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Pitch of lateral ties

Let 8 mm. diameter ties be provided

In the body of the pile, the ties shall have a volume equal to 0·2% of the volume of pile.

Let p = pitch of ties in *cm*.

Volume of pile per pitch length = $625\ p\ cm.^3$

Clear cover to main bar = 4 *cm*.

Cover to centre of tie = 4 − 0·4 = 3·6 *cm*.

Volume of one tie

$= 0{\cdot}5(4 \times 17{\cdot}8) = 35{\cdot}6\ cm.^3$

$35{\cdot}6 = \frac{0{\cdot}2}{100} \times 625\ p$

$28{\cdot}48\ cm.$ *say* $28\ cm.$

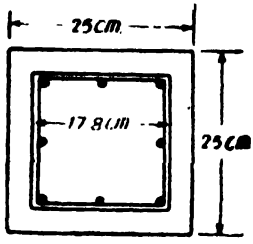

Fig. 298

Maximum permissible spacing

= half the least width

$= \frac{25}{2} = 12{\cdot}5\ cm.$

Hence, provide 8 mm. ϕ ties @ 12 cm. c/c.

Lateral ties near the ends

From each end of the pile, for a distance of three times the least width the spacing of the lateral ties shall be such that the volume of the ties will be 0·6% the gross volume.

In our case, three times the least width

$= 3 \times 25 = 75\ cm.$

Hence, for 75 *cm*. length from each end, the volume of lateral ties will be atleast 0·6% of the volume of pile.

$$\therefore \quad 0{\cdot}5(4 \times 17{\cdot}8) = \frac{0{\cdot}6}{100} \times 625\ p$$

$$\therefore \quad p = 9{\cdot}49\ cm.\ \textit{say}\ 9\ cm.$$

In addition to the links the longitudinal bars shall be held apart at intervals of atleast 1·50 *metres* by spreaders of 12 *mm*. diameter to prevent inward buckling. 8 *mm*. diameter ties shall also be provided.

The pile shall be provided with holes at 0·293 $l = 0{\cdot}293 \times 6{\cdot}5 = 1{\cdot}90$ *metre* from the upper end for hoisting and at 0·207 $l = 0{\cdot}207 \times 6{\cdot}5 = 1{\cdot}35$ *metre* from each end for stalking.

Check for handling stresses

Weight of the pile per *metre* run

$= 0{\cdot}25 \times 0{\cdot}25 \times 25000\ N/metre$

$= 1562{\cdot}5\ N/metre$

B.M. during hoisting $= \frac{wl^2}{23}$

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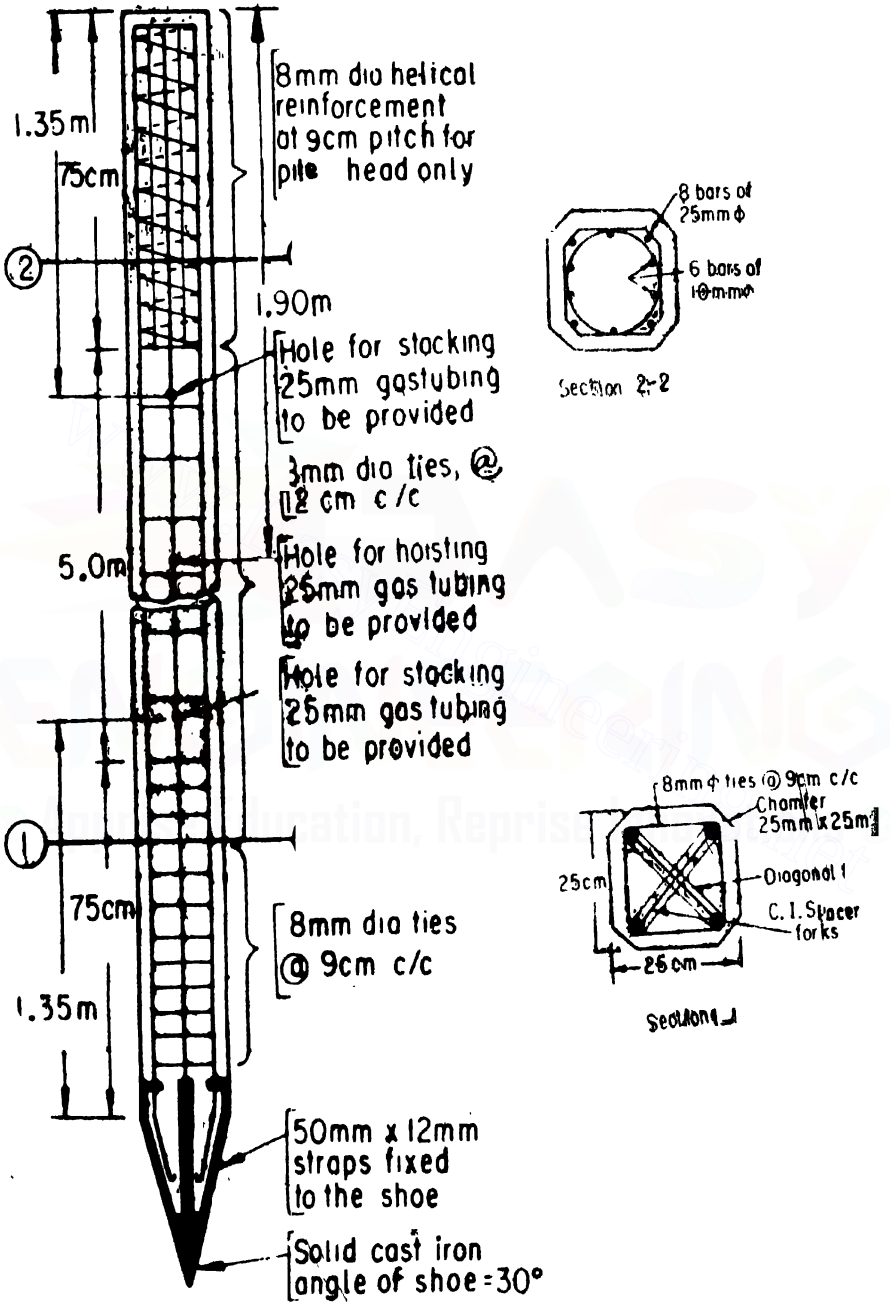

Fig. 299

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$$\frac{1562.5 \times 6.5^2}{23} \times 100 \text{ N cm.}$$

$$287020 \text{ N cm}$$

Depth of neutral axis

For this estimation the bars E and F in the middle may be ignored and the section can be analysed as doubly reinforced section

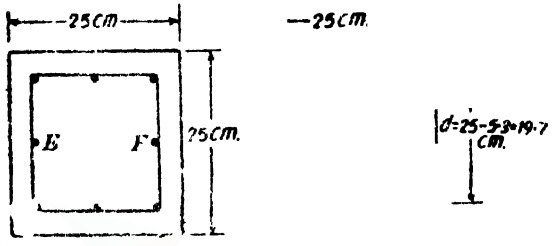

Fig. 300

Taking moments about the neutral axis,

$$\frac{5n^2}{2} + (1.5 \times 13 - 1)4.91 \times 3(n - 5.3) = 13 \times 4.91 \times 3 \times (19.7 - 3)$$

$$\therefore \quad n^2 + 37.12\, n - 411.33 = 0$$

$$\therefore \quad n = 9.04 \text{ cm.}$$

Equating the moment of resistance to the maximum bending moment

$$25 \times 9.04 \times \frac{c}{2}\left(19.7 - \frac{9.04}{3}\right) + (1.5 \times 13 - 1)(4.91 \times 3)\left(\frac{9.04 - 5.3}{9.04}\right) c\,(19.7 - 5.3)$$

$$= 287.20 \text{ N cm}$$

Solving we get $c = 81.8 \text{ N/cm}^2 = 0.818 \text{ N/mm}^2$

Design 75 *The foundation for a structure is to consist of 16 piles to carry a total load of 800 t. The piles are spaced at 1.25 metres centres. They are driven through made up ground into a hard stratum which is available at a depth of 6 metres. Design one of the piles. Use M 200 grade concrete mix*

Solution. *Safe stresses adopted*

c (*direct*) = 50 *kg./cm.*^{2}

c (*bending*) = 70 *kg./cm.*^{2}

Safe compressive stress in steel = 1300 *kg./cm.*^{2}

Safe tensile stress in steel = 1400 *kg./cm.*^{2}

Modular ratio $= m = \frac{2800}{3 \times 7} = 13$

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Load on each pile $= \frac{800}{16} = 50\ t$

Let the piles be 6·50 *metres* long

Let the piles be 25 *cm.* × 25 *cm.*

$$\frac{l}{D} = \frac{650}{25} = 26$$

This is greater than 12. Hence the pile will be treated as a long column.

Reduction coefficient $C_r = 1{\cdot}25 - \frac{l}{48D}$

$$= 1{\cdot}25 - \frac{26}{48} = 0{\cdot}708$$

Safe compressive stress in concrete $= 0{\cdot}708 \times 50 = 35{\cdot}4\ kg./cm.^2$

Safe compressive stress in steel $= 0{\cdot}708 \times 1300 = 920{\cdot}4\ kg./cm.^2$

Safe load on the pile = Safe load on concrete + Safe load on steel

$$50{,}000 = 35{\cdot}4\,(625 - A_t) + 920{\cdot}4\,A_t$$

$$\therefore \quad A_t = 31{\cdot}50\ cm.^2$$

The main reinforcement for a pile whose length is not greater than 30 times the least width shall not be less than 1·25% of the gross area of section. In our case the minimum requirement of steel

$$= \frac{1{\cdot}25}{100} \times 25 \times 25 = 7{\cdot}81\ cm^2$$

Hence, the actual steel required $= 31{\cdot}50\ cm.^2$

Provide 8 bars of 25 mm. ϕ (39·28 cm.²)

Pitch of lateral ties

Let 8 mm. diameter ties be provided

In the body of the pile the ties shall have a volume equal to 0·2% of the volume of pile.

Let p = pitch of ties in *cm.*

Volume of pile per pitch length $= 625\ p\ cm.^3$

Clear cover to main bars $= 4\ cm.$

Cover to centre of tie $= 4 - 0{\cdot}4 = 3{\cdot}6\ cm.$

Volume of one tie $= 0{\cdot}5\,(4 \times 17{\cdot}8) = 35{\cdot}6\ cm^3$

$$\therefore \quad 35{\cdot}6 = \frac{0{\cdot}2}{100} \times 625\ p$$

$$\therefore \quad p = 28{\cdot}48\ cm.\ say\ 28\ cm.$$

Maximum permissible pitch = half the least width

$$= \frac{25}{2} = 12{\cdot}50\ cm.$$

Hence, provide 8 mm. ϕ ties @ 12 cm. c/c.

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Lateral ties near the ends

From each end of the pile, for a distance of three times the least width the spacing of the lateral ties shall be such that the volume of the ties will be 0·6% of the gross volume.

In our case, three times the least width

$$= 3 \times 25 = 75 \text{ cm.}$$

Hence, for 75 *cm.* length from each end. the volume of lateral ties will be at least 0·6% of the volume of pile.

$$\therefore \quad 0{\cdot}5\,(4 \times 17{\cdot}8) = \frac{0{\cdot}6}{100} \times 625\, p$$

$$\therefore \quad p = 9{\cdot}49 \text{ cm. say } 9 \text{ cm.}$$

In addition to the links the longitudinal bars shall be held apart at intervals of atleast 1·50 *metres* by spreaders of 12 *mm.* diameter to prevent inward buckling 8 *mm.* diameter ties shall also be provided.

The pile shall be provided with holes at 0·293 $l = 0{\cdot}293 \times 6{\cdot}5 = 1{\cdot}90$ *metre* from the upper end for hoisting and at 0·207 $l = 0{\cdot}207 \times 6{\cdot}5 = 1{\cdot}35$ *m.* from each end for hoisting.

Check for handling stresses

Weight of the pile per *metre* run

$$= 0{\cdot}25 \times 0{\cdot}25 \times 2500 = 156{\cdot}25 \text{ kg./metre}$$

$$\text{B.M. during hoisting} = \frac{wl^2}{23} = \frac{156{\cdot}25 \times 6{\cdot}5^2}{23} \times 100 \text{ kg. cm.}$$

$$= 28700 \text{ kg. cm.}$$

Depth of neutral axis

For the estimation the bars in the middle parallel near the neutral axis can be ignored. Taking moments about the neutral axis,

$$\frac{25\, n^2}{2} + (1{\cdot}5 \times 13 - 1)(4{\cdot}91 \times 3)(n - 5{\cdot}3) = 13 \times 4{\cdot}91 \times 3(19{\cdot}7 - 3)$$

$$n^2 + 37{\cdot}12\, n - 417{\cdot}33 = 0$$

$$n = 9{\cdot}04 \text{ cm.}$$

Equating the moment of resistance to the maximum bending moment,

$$25 \times 9{\cdot}04 \times \frac{c}{2}\left(19{\cdot}7 - \frac{9{\cdot}04}{3}\right) + (1{\cdot}5 \times 13 - 1)(4{\cdot}91 \times 3) \times \left(\frac{9{\cdot}04 - 5{\cdot}3}{9{\cdot}04}\right) c\,(19{\cdot}7 - 3) = 28700 \text{ kg. cm.}$$

Solving we get $c = 8{\cdot}18$ *kg /cm.*^{2}

Lengthening and Re-driving Piles. Sometimes even after driving a pile into the ground so that its top is at ground level, the specified or permissible set might have been exceeded. In such cases it will

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be necessary to lengthen the pile and redrive it. In such situations it is usual to chip off the concrete for a sufficient length at the top so that the bars are exposed. Additional bars with binding links are then fixed in continuation with the exposed bars.

Each butt joint should be covered by a piece of bar of the same diameter and properly secured by wiring to the exposed and additional bars. Now the shuttering is erected around the bars and the extension is concreted. The length of pile chipped off of concrete should be at least 40 times the diameter or the least lateral dimension. The splice bar should be twice this length. Alternatively the jointing of the longitudinal reinforcement can be done by butt welding. The links which shall be similar to those in the original pile shall be provided at a closer spacing. At the top of the extended pile the spacing of the links should be close.

It is important that the extension shall be in line with the original pile to ensure that there is no eccentricity when the driving is renewed. To avoid considerable delay while the extension is curing, it is advisable to use high-alumina cement concrete which needs a very short period to gain strength so that the pile can be redriven in a few days. Otherwise the driving work has to be discontinued for several weeks to complete the curing period.

Pile Caps. When two or more piles are to be provided under a column or pier it is necessary to provide a slab to distribute the load to the individual piles. This slab is called the pile cap. A pile cap will consist of a rigid deep slab well bonded with longitudinal reinforcement of the piles.

The pile caps should be designed primarily from punching shear considerations, for the punching shear around the heads of the piles and around the column base. They should also be designed for bending moment due to the transference of loads from the column to the individual piles. The overall depth of the pile cap shall be such that it provides adequate bond length for the pile reinforcement and the column dowell reinforcements

Superimposed loads are taken to be dispersed through a pile cap at 45° to the main tension steel. The critical sections for shear should be those sections where the lines of pressure cut the tensile steel. Since the practice is not to use shear reinforcements in pile caps, the shear resistance of the concrete at the critical section must be equal to or greater than the maximum permissible shear strength at the critical section.

For purposes of design, the pile cap may be treated as a system of beams even though they take the form of a slab of uniform thickness covering all the area occupied by the piles. The width of the beam may be taken as the width of the pile supporting it.

Fig. 301 shows an arrangement of a pile cap provided for three piles which are placed on the corners of an equilateral triangle of side L. The column is located at the centroid of this triangle.

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For purposes of design the load W on the column is assumed to be applied on a beam DC of length $\frac{L\sqrt{3}}{2}$ at a point G at a distance of $\frac{L}{2\sqrt{3}}$ from the end D. The vertical reactions for this beam will be $\frac{W}{3}$ at C and $\frac{2W}{3}$ at D. The maximum bending moment for the beam $DC = \frac{W}{3} \cdot \frac{L}{\sqrt{3}} = \frac{WL}{3\sqrt{3}}$.

The beam DC has one support at C over a pile and the other support on the middle point of the beam AB. The beam AB has a span L and is loaded at its middle point with a load of $\frac{2W}{3}$. Each

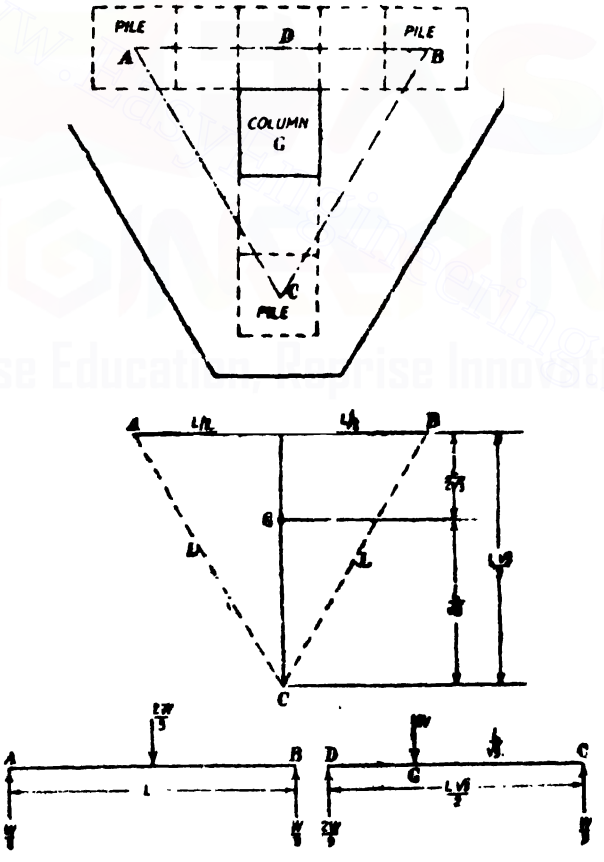

Fig. 301

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vertical reaction for this beam is $\frac{W}{3}$. The maximum bending moment for this beam will be $\frac{W}{3} \quad \frac{L}{2} \quad \frac{WL}{6}$. The reinforcement for these two beams can now be easily computed.

Secondary reinforcements. These are provided to prevent the piles from splaying outwards from a pile cap. This reinforcement shall be provided to the following requirements :

(*i*) This reinforcement shall be provided at the bottom of the pile cap running round the longitudinal reinforcements projecting from the piles into the pile cap.

(*ii*) There should be a change in direction of the secondary reinforcement at the head of every pile.

The amount of secondary reinforcement changing its direction at the head of each pile shall be not less than 20% of the main tensile reinforcement and it shall be well bonded.

Design 76. (S.I.). *A column 30 cm. × 30 cm in section stands on a pile cap supported on three piles. The column is situated at the centroid of the pile group. The total load transferred to the column is 600 kN. The piles are 1·20 metres centre to centre. Design the pile cap. Use M 15 concrete.*

Solution. Fig. 302 shows the plan of the column and the three piles and the imaginary beams *AB* and *CD* having a width of 30 *cm. i.e.*, the same as the corresponding lateral dimension of the pile.

Overall depth of pile cap

(*a*) *Punching shear consideration.* Let the permissible punching shear stress be 1 $N/mm.^2 = 100\ N/cm.^2$

Let the overall depth of the footing be *D cm.* Equating the punching resistance to the punching load, we have,

$$4 \times 30D \times 100 = 600 \times 1000$$

$$\therefore \qquad D = 50\ cm.$$

(*b*) *B.M. consideration.* Consider the beam *DC*. Maximum live load bending moment for this beam

$$= 400 \times 0{\cdot}2\sqrt{3} = 80\sqrt{3}\ kNm.$$

$$= 138{\cdot}56\ kNm.$$

B.M. at G due to self-weight of pile cap

The dead load consists of the weight of the beam plus the weight of part of the slab For design purposes the self-weight may be calculated as the weight of the slab whose width equals three times the width of the beam. The depth of the pile cap from bending moment consideration is generally found to be more than 15 *cm*

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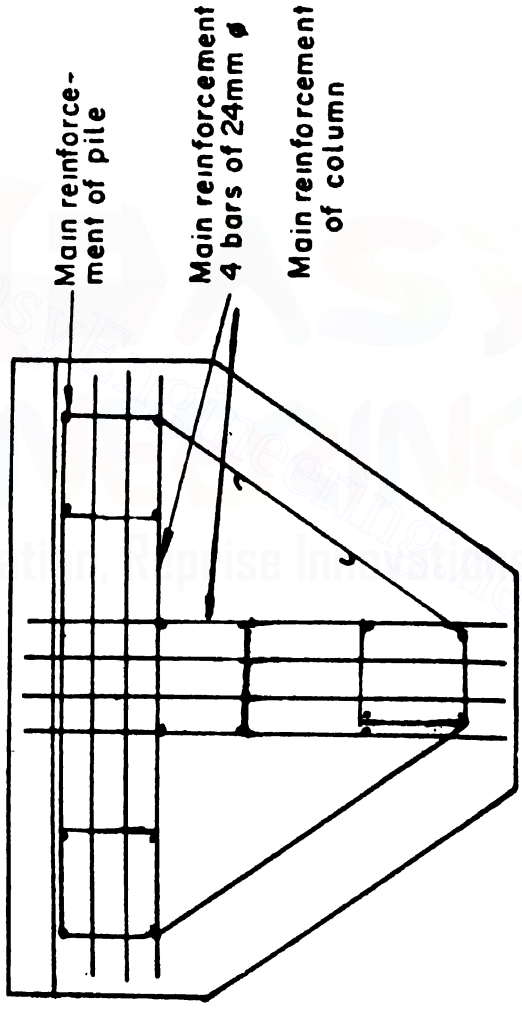

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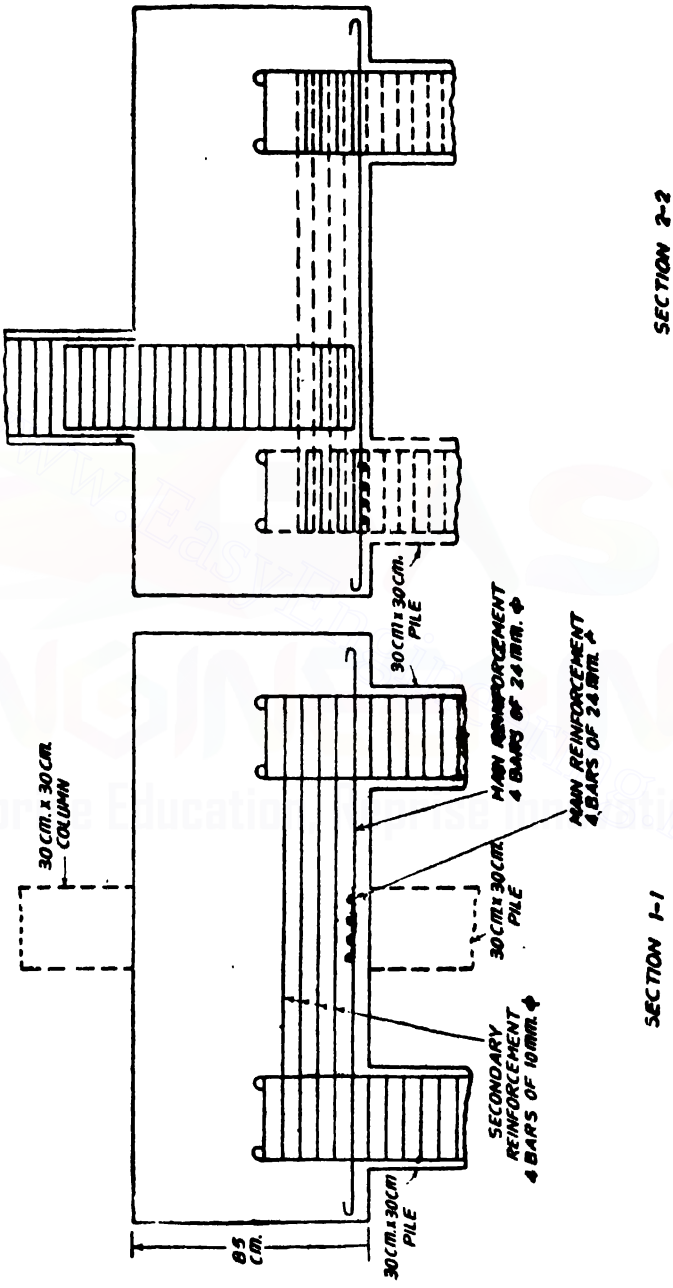

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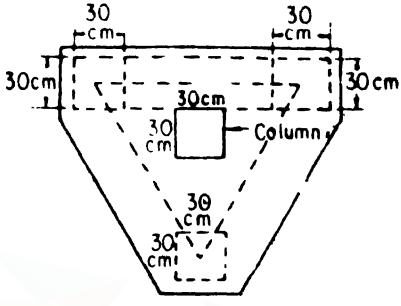
Let us therefore assume the pile cap to be 80 *cm.* deep for estimating the self-weight of the pile cap.

$$\text{Dead load of the beam} = \frac{3 \times 30 \times 80}{100 \times 100} \times 25000 \ N/metre$$

$$= 18000 \ N/metre$$

$$= 18 \ kN/metre$$



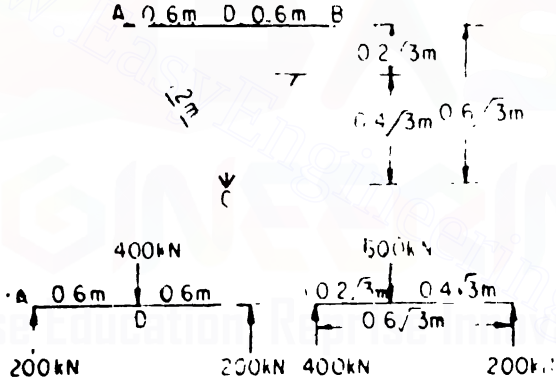


Fig. 302

B.M. at G due to self-weight of the pile cap

$$= \frac{18 \times 0.6\sqrt{3}}{2} \times 0.2\sqrt{3} - \frac{18}{2} (0.2\sqrt{3})^2 \ kNm$$

$$= 2.16 \ kNm.$$

$\therefore$ Total B.M. $= 138.56 + 2.16 = 140.72 \ kNm.$

Now consider the beam AB

Live load moment $= 200 \times 0.6 = 120 \ kNm.$

Dead load moment $= 18 \times \frac{1.2^2}{8} = 3.24 \ kNm.$

Dead load moment due to dead load transferred by the beam DC to the beam AB

$$= 18 \times \left(\frac{0.6\sqrt{3}}{2}\right) \times \frac{1.2}{4} = 2.80 \ kNm.$$

Total bending moment $= 120 + 3.24 + 2.80 = 126.04 \ kNm.$

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Equating the moment of resistance to the maximum bending moment for the beam CD,

$$85 \times 30\, d^2 = 140{\cdot}72 \times 1000 \times 100$$

$$d = 74{\cdot}30 \text{ cm. say } 75 \text{ cm.}$$

Since the beam DC has been taken as supported at one end on a pile and at the other end over the beam AB, the reinforcement for the beam DC must be above the level of the reinforcement for the beam AB.

$$A_s \text{ for beam } DC = \frac{140{\cdot}72 \times 1000 \times 100}{14000 \times 0{\cdot}87 \times 75} = 15{\cdot}40 \text{ cm.}^2$$

Provide 4 bars of 24 mm. ϕ (18·10) cm.²

If the same size of bar be provided for the beam AB also, the effective depth of the beam AB

$$= 75 + 2{\cdot}4 = 77{\cdot}40 \text{ cm.}$$

$$A_s \text{ for beam } AB = \frac{126{\cdot}04 \times 1000 \times 100}{14000 \times 0{\cdot}87 \times 77{\cdot}4} = 13{\cdot}4 \text{ cm.}$$

Provide 4 bars of 24 mm. ϕ.

Secondary reinforcement. This reinforcement shall be provided round each pile head. This shall be not less than 20% of the maximum steel provided for a beam.

$$= 0{\cdot}2 \times 15{\cdot}4 = 3{\cdot}08 \text{ cm.}^2$$

Hence provide four bars of 10 mm ϕ.

Design 77. *A column 30 cm. × 30 cm. in section stands on a pile cap supported on three piles. The column is situated at the centroid of the pile group The total load transferred to the column is 60 tonnes The piles are 1·20 metres centre to centre Design the pile cap.*

Fig. 303 shows the plan of the column and the three piles and the imaginary beams AB and CD having a width of 30 *cm. i.e.*, the same as the corresponding lateral dimension of the pile.

Overall Depth of Pile Cap

(*a*) *Punching shear consideration*

Let the permissible punching stress be 10 *kg./cm.*². Let the overall depth of footing be D *cm* Equating the punching resistance to the punching load, we have,

$$4 \times 30 \times D \times 10 = 60{,}000 \text{ kg.}$$

$$\therefore \quad D = 50 \text{ cm.}$$

(*b*) *B M. consideration*

Consider Beam DC. Maximum live load bending moment for this beam $= 40 \times 0{\cdot}2 \times 3 = 8 \times 3$ *tonne metre*

$= 13{\cdot}86$ *tonne metre.*

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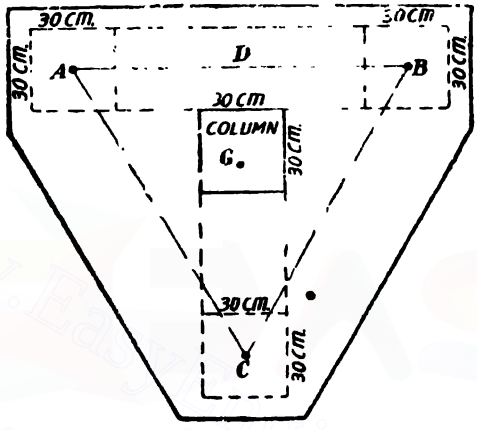
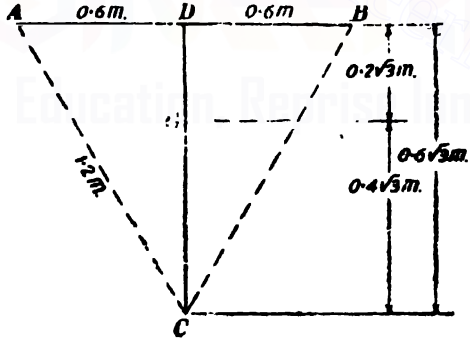
B.M. at G due to self-weight of pile cap

The dead load consists of the weight of the beam plus the weight of part of the slab. For design purpose the self-weight may be calculated as the weight of the slab whose width equals three times the width of the beam. The depth of the pile cap from bending moment consideration is likely to be more than 75 *cm.* usually. Let us therefore assume the pile cap to be 80 *cm.* deep for estimating the self-weight of the pile cap.





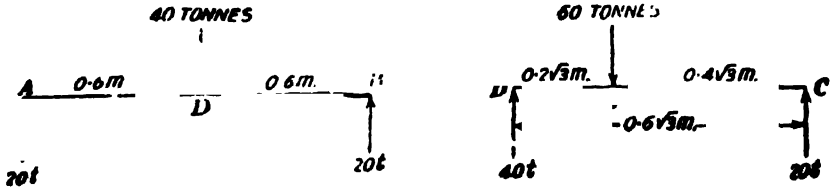


Fig. 303

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Dead weight on beam $= \frac{3 \times 30 \times 80}{100 \times 100} = 2500$ *kg. per metre*

$= 1800$ *kg./m.*

$= 1{\cdot}80$ *tonne/metre*

$\therefore$ B.M. at G due to self-weight of the pile cap

$$= \frac{1{\cdot}80 \times 0{\cdot}6\sqrt{3}}{2} \times 0{\cdot}2\sqrt{3} - \frac{1{\cdot}80}{2}(0{\cdot}2\sqrt{3})^2 \text{ tonne metre}$$

$= 0{\cdot}22$ *tonne metre*

$\therefore$ Total bending moment

$= 13{\cdot}86 + 0{\cdot}22$ *tonne metre*

$= 14{\cdot}08$ *tonne metre*

Now consider beam AB

Live load moment $= 20 \times 0{\cdot}6 = 12$ *tonne metre*

Dead load moment $= \frac{1{\cdot}80 \times 1{\cdot}2^2}{8}$ *tonne metre*

$= 0{\cdot}32$ *tonne metre.*

Dead load moment due to dead load transferred by the beam DC to the beam AB

$$= 1{\cdot}80 \times \left(\frac{0{\cdot}6\sqrt{3}}{2}\right) \times \frac{1{\cdot}2}{4} \text{ tonne metre}$$

$= 0{\cdot}28$ *tonne metres*

$\therefore$ Total bending moment

$= 12 + 0{\cdot}32 + 0{\cdot}28$ *tonne metre*

$= 12{\cdot}60$ *tonne metre*

Equating the moment of resistance to the maximum bending moment for the beam CD, we have,

$$8{\cdot}5 \times 30d^2 = 14{\cdot}08 \times 1000 \times 100 \text{ kg. cm.}$$

$\therefore$ $d = 74{\cdot}3$ *cm.* say 75 *cm.*

Since the beam DC has been taken as supported at one end on a pile and at the other end over the beam AB, the reinforcement for the beam DC must be above the level of the reinforcement for the beam AB

A_s for beam DC $= \frac{14{\cdot}08 \times 1000 \times 100}{1400 \times 0{\cdot}87 \times 75}$ *cm.*^{2}

$= 15{\cdot}40$ *cm*^{2}.

Provide 4 bars of 24 mm. diameter (18·10 *cm.*^{2}). If the same size of bar be provided for the beam AB also, the effective depth of the beam AB

$= 75 + 2{\cdot}4 = 77{\cdot}4$ *cm.*

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$$A_s \text{ for beam } AB = \frac{12{\cdot}60 \times 1000 \times 100}{1400 \times 0{\cdot}87 \times 77{\cdot}4} \ cm.^2$$

$$= 13{\cdot}40 \ cm.^2$$

Provide 4 bars of 24 mm. diameter.

Secondary reinforcement. This reinforcement shall be provided round each pile head. This shall be not less than 20% of the maximum steel provided for a beam

$$= 0{\cdot}2 \times 15{\cdot}4 = 3{\cdot}08 \ cm.^2$$

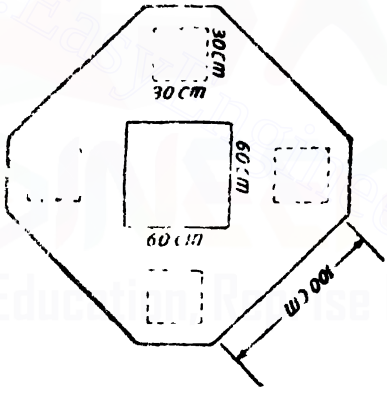
Hence provide four bars of 10 mm ϕ.

Design 78. (S I.). *A column carrying a load of 2500 kN has to be supported by four piles each of size 30 cm. × 30 cm. The piles are spaced at 100 cm. centres as shown in Fig. 304. The column size is 60 cm. × 60 cm. Design the pile cap. Use M 15 concrete.*

Solution. Load on the column = 2500 *kN*

Approximate weight of the pile cap

= 5% of column load



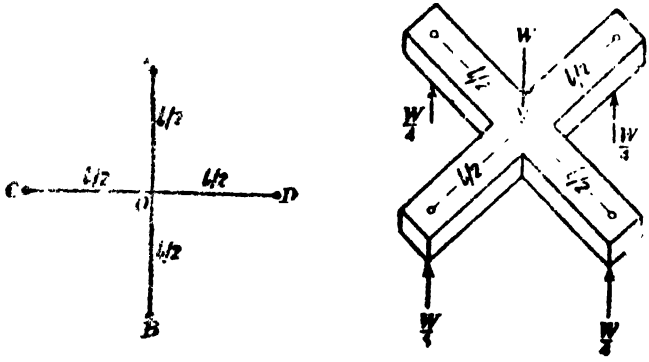


Fig. 304

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$$= \frac{2}{100} \times 2500 = 125 \text{ kN}$$

Total load on the piles $= W = 2625$ kN

Load per pile $= \frac{W}{4} = \frac{2625}{4} = 656{\cdot}25$ kN

The pile cap may be taken to provide two beams like AB and CD, together supporting the total load W.

For purposes of design the maximum bending moment per beam may be taken equal to

$$\frac{W}{4} \cdot \frac{l}{2} = \frac{WL}{8}$$

In our case $l = 100\sqrt{2} = 141{\cdot}4$ cm.

Maximum bending moment

$$= \frac{WL}{8} = \frac{(2625 \times 1000)141{\cdot}4}{8} \text{ N cm}$$

$$= 46396875 \text{ N cm.}$$

The width of the beam will be assumed to be equal to the corresponding width of the column *i.e.* equal to 60 cm. Equating the moment of resistance to the bending moment,

$$85 \times 60\, d^2 = 46396875$$

$$d = 95{\cdot}4 \text{ cm.}$$

If 22 mm. diameter bars be used at a clear cover of 6 cm. (Note there are two layers of bars).

Overall depth required

$$= 95{\cdot}4 + 1{\cdot}1 + 2{\cdot}2 + 6 = 95{\cdot}4 + 9{\cdot}3$$

$$= 104{\cdot}7 \text{ cm. say } 105 \text{ cm.}$$

Actual effective depth $= 105 - 9{\cdot}3 = 95{\cdot}7$ cm.

$$\therefore \quad A_s = \frac{46396875}{14000 \times 0{\cdot}87 \times 95{\cdot}7} = 39{\cdot}80 \text{ cm.}^2$$

Provide 12 bars of 22 mm. ϕ (45·60 cm.²)

Check for punching shear stress

Punching shear stress due to column load

$$= \frac{2500 \times 100}{4 \times 60 \times 105} = 99{\cdot}2 \text{ N/cm.}^2$$

$$= 0{\cdot}992 \text{ N/mm.}^2 < 1 \text{ N/mm.}^2$$

Punch shear stress due to pile reaction

$$= \frac{656{\cdot}25 \times 1000}{4 \times 30 \times 105} = 52 \text{ N/cm}^2$$

$$= 0{\cdot}52 \text{ N/mm.}^2 < 1 \text{ N/mm.}^2$$

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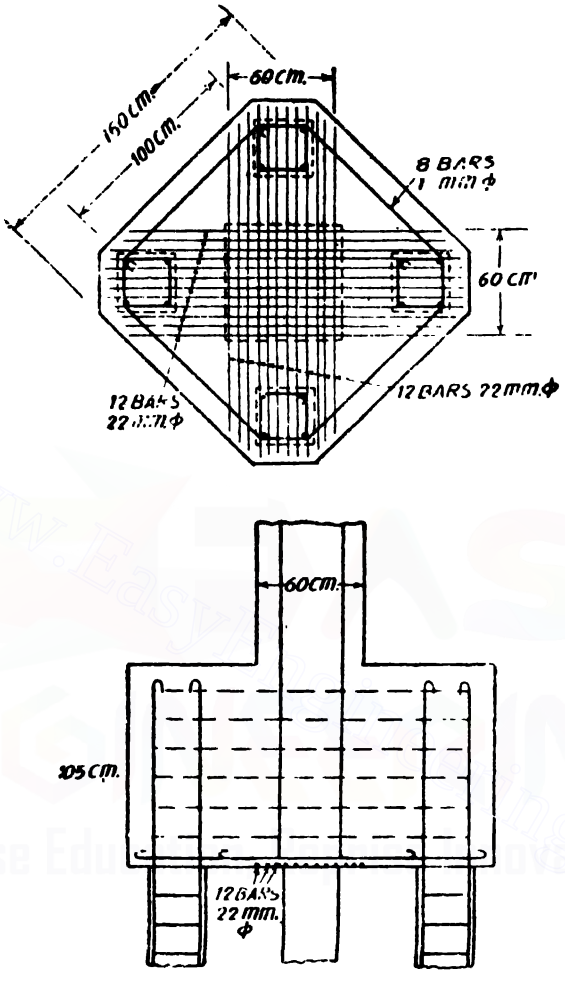

Fig. 305

Design 79. *A column 60 cm. × 60 cm. in section carries a load of 360,000 kg. The column is to be supported on a pile cap which has to stand on six piles arranged as shown in Fig. 306. Design the pile cap. Use M 150 concrete.*

Solution. Depth of pile cap from punching shear consideration

$$= \frac{360{,}000}{4 \times 60 \times 10} = 150 \text{ cm.}$$

Depth from B M. consideration

Load on column = 360,000 kg.
Weight of pile cap = 5% of column load = 18,000 kg.
Total = 378,000 kg.

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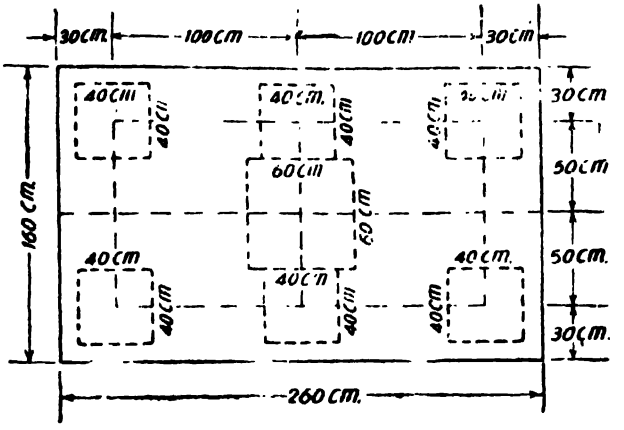

Fig. 306

$\therefore$ Load per pile $= \frac{378{,}000}{6} = 63{,}000\ kg.$

Maximum B.M. for the pile cap

$= 2 \times 63{,}000 \times (100 - 30)\ kg.\ cm.$

$= 8{,}820{,}000\ kg.\ cm.$

Equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}5 \times 150 \times d^2 = 8{,}820{,}000$$

$\therefore$ $d = 83{\cdot}2\ cm.$

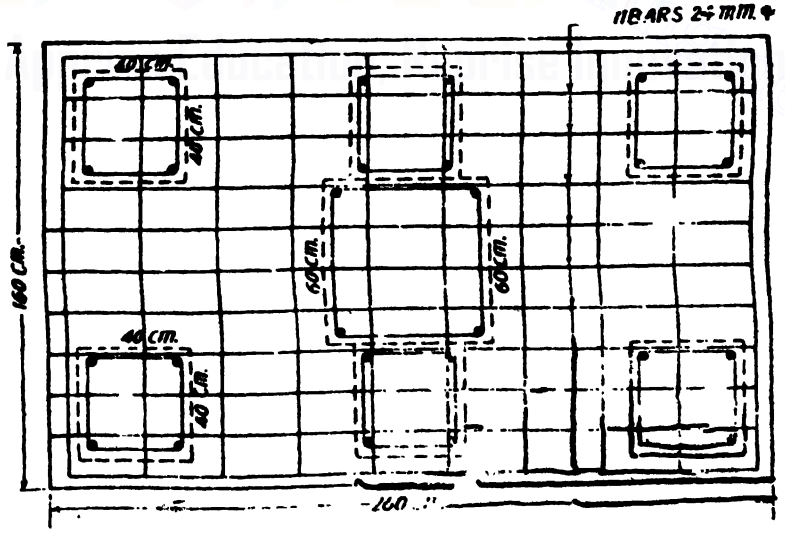

Fig. 307

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Hence provide 150 *cm.* overall as determined from punching shear stress consideration.

Providing 8 *cm.* effective cover the effective depth

$$= 150 - 8 = 142 \text{ cm.}$$

$$A_t = \frac{8,820,000}{1400 \times 0{\cdot}87 \times 142} \text{ cm.}^2 = 50 \text{ cm.}^2$$

Provide 11 bars of 24 mm. diameter.

Provide also nominal transverse reinforcement of 10 bars of 20 mm. ϕ.

Provide also secondary reinforcement binding the reinforcement of the piles

Cast-in-situ Piles. These are piles which are cast in the position in which they are expected to occupy in the completed structure. Cast-in-situ piles are all patented. A few important types of these piles are explained below.

Simplex Piles The pile is made by first driving a hollow steel tube 40 *cm.* to 50 *cm* in diameter This steel tube is fitted with a detachable cast iron conical driving point. The pipe is then partially filled with concrete Then a plunger of slightly lesser diameter is placed on the concrete and the concrete is pressed down and during this process the pipe is slowly withdrawn partially. Thus the pipe leaves part of the concrete with the driving point below it. The above process is repeated each time, by removing the plunger and

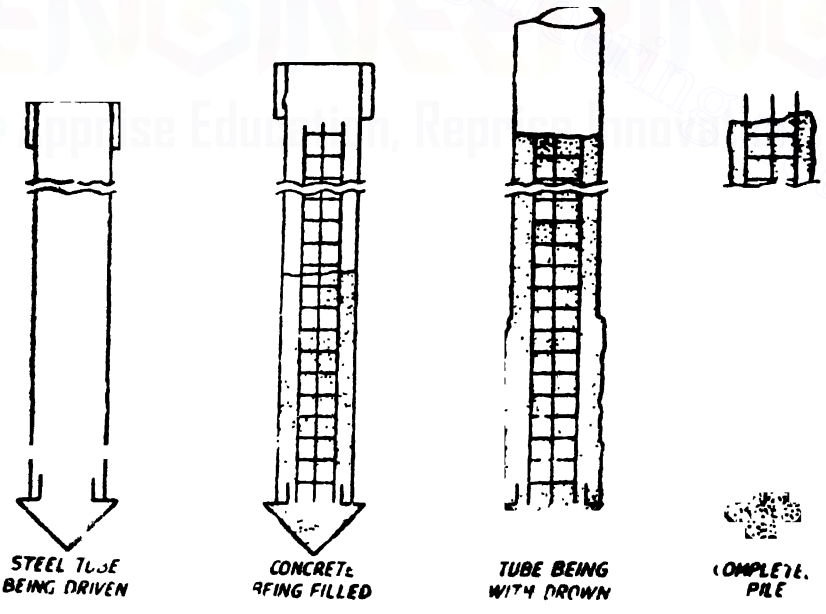

Fig. 308. **Simplex standard pile.**

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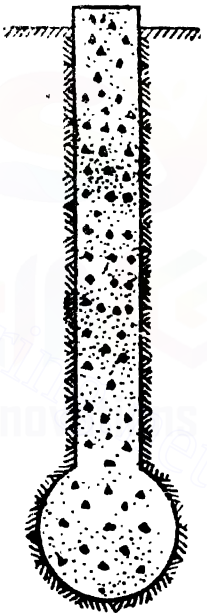
pouring a little more concrete, then inserting the plunger and slowly withdrawing the steel pipe. The process is continued until the pile is built up to the required height.

Sometimes the cast-in-situ pile may be reinforced. In such a case, the steel tube with its cast iron shoe is first driven to the required depth. The hammer and the drive cap are now removed. A cage of reinforcement is inserted into the tube. The tube is filled with concrete to the required level. The steel tube is slowly withdrawn. The concrete gets forced downwards and outwards by its own weight.

The Pedestal Pile. Fig 309 shows a pedestal pile. It consists of a vertical shaft of plain concrete with a bulbed enlargement called the pedestal. The pedestal assists in increasing the bearing area. This pile is built as follows.

A steel casing of about 40 *cm.* diameter provided with a solid plunger fitted to it is driven into the soil by a pile driven to the required depth. The plunger is then withdrawn. The casing is now partly filled with concrete. Now the plunger and the hammer are dropped over the concrete deposited. This will force the concrete sidewards. by repeating the process a bulb is formed at the bottom. The rest of the pile is now filled and the steel casing is gradually withdrawn. In order that the pile may not be lifted up during the process of the withdrawal of the steel casing, the plunger and the hammer will rest on the concrete as the casing is slowly withdrawn.



The Pedestal Pile. Fig. 309.

Pressure piles A boring is first made and the hole is lined with steel tubes. The steel tubes are driven in the ground either by their own weight or by tapping lightly. After the tube is sunk to the required depth, some quantity of concrete is lowered into the tube. The concrete is consolidated by dropping a ram on it. The reinforcement cage is lowered into the tube. This is followed by the first batch of concrete. The steel lining is slowly withdrawn and concrete is consolidated by admitting compressed air to the tubes. Sometimes the consolidation is also done by vibrating. Some more concrete is again added and the steel tubes are further pulled up and the process is continued and thus the pile is completed to the desired finished level. During the withdrawal of the casing the high pressure of the compressed air will force the concrete into soil crevices at the bottom and a firm base is automatically formed.

Merits and demerits of cast-in-situ piles. The cast-in-situ piles have the advantage that they can be built rapidly. Neither a big

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storage place nor a very expensive equipment is needed for this pile. Hence this is a special advantage when piles are to be driven in built-up localities. The length of the pile can always be regulated and hence it is unnecessary to predetermine it by actual borings as in the case of the pre-cast piles. The cast-in-situ pile has the following disadvantages. Its shape cannot be inspected after its erection. It requires a properly regulated driving, otherwise the setting of the concrete may be seriously affected by the shocks and vibrations effected by driving adjacent piles Special care should also be taken to ensure that no voids are formed in the body of the pile. It is particularly important to note that when the casing is gradually withdrawn, precaution should be taken to prevent lifting of the concrete which has been placed earlier along with the casing. If concrete gets lifted, the cross-sectional area of the pile will be reduced. It is also possible that the pile may be broken into sections.

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13
Retaining Walls

Retaining walls are walls meant to support soils or such materials. Hence in order to design a retaining wall it is necessary to reasonably determine the pressures exerted by and acting on plane faces of soil. A number of theories have been evolved on the determination of pressure exerted by soil. It is proposed to briefly discuss the usually followed theories of earth pressure. It will be assumed that the earth that exerts pressure is dry and free from moisture and cohesion. The earth material will be considered as a granular mass of particles. Any resistance to shear failure is entirely offered as a frictional resistance.

§144. Rankine's Theory of Earth Pressure

According to this theory the resultant pressure on any plane surface within an earth mass cannot be inclined to the normal to the plane, at an angle greater than the angle of repose of the soil mass. The angle of repose of a soil is the maximum angle which the exterior face of a granular mass can make with the horizontal.

Angle of Internal friction. This is the angle the tangent of which is equal to the ratio of the maximum resistance to sliding on any internal plane to the normal pressure acting on the plane. For sandy soils this angle can be considered as equal to the angle of repose of the soil.

§145. Stress Analysis of Soils

Mohr's circle of stress. It is necessary to be familiar this graphical construction which enables to determine the state of stress at any point in a body.

Consider the equilibrium of any element of a body subjected to principal stresses p_1 and p_2 (*see* Fig. 310).

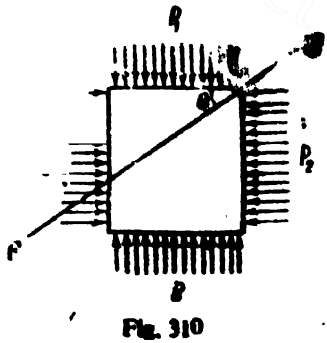

Fig. 310

The normal and tangential stresses on any plane EF inclined at an angle θ to the principal plane carrying the principal stress p_1 are given by

$$p_n = \frac{p_1+p_2}{2} + \frac{p_1-p_2}{2}\cos 2\theta$$

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$$p_t = \frac{p_1 - p_2}{2} \sin 2\theta$$

Along an axis OX, distances OA and OB representing the principal stresses p_1 and p_2 are set off. A circle is drawn on AB as diameter. This circle is called *Mohr's circle*

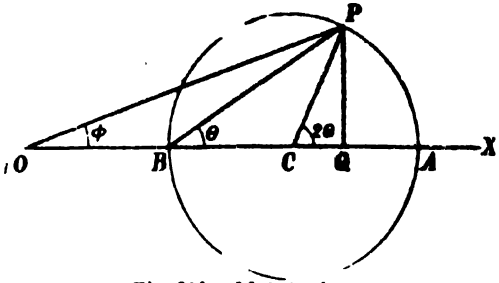

Fig. 311. Mohr's circle.

Let C be the centre of the Mohr's circle. Draw the radius CP making an angle 2θ with the axis OX. Draw PQ perpendicular to OX.

OQ represents the normal stress on the plane EF and QP represents the tangential stress on the plane EF. This can be proved easily.

$$PQ = CP \sin 2\theta$$

But $$CP = \frac{BA}{2} = \frac{p_1 - p_2}{2}$$

$$PQ = \frac{p_1 - p_2}{2} \sin 2\theta = p_t$$

$$OQ = OC + CQ = OC + CP \cos 2\theta$$

But $$OC = \frac{p_1 + p_2}{2}$$

$$OQ = \frac{p_1 + p_2}{2} + \frac{p_1 - p_2}{2} \cos 2\theta = p_n.$$

Hence OP represents the resultant stress on the plane EF.

The angle POB is the angle between the resultant stress and the normal to the plane EF. This angle is called the *obliquity*.

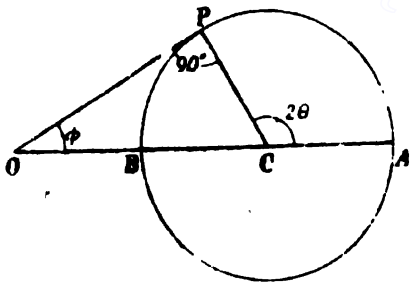

Fig. 312. Mohr's circle.

By studying the Mohr's , we find that there a plane on which the resultant stress is inclined to the normal to the maximum. Corresponding to the plane on which the obliquity is maximum the Mohr's diagram is shown in Fig. 312.

If ϕ = obliquity corresponding to the above condition, we have

$$\sin \phi = \frac{PC}{OC} = \frac{(p_1 - p_2)/2}{(p_1 + p_2)/2}$$

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Coefficients Used in Earth Pressure Computations

ϕ in degrees	$\frac{1-\sin\phi}{1+\sin\phi}$	$\frac{1+\sin\phi}{1-\sin\phi}$	$\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2$	$\left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2$
14	0·61	1·64	0·37	2·69
15	0·59	1·70	0·35	2·88
16	0·56	1·77	0·31	3·13
17	0·55	1·82	0·30	3·33
18	0·53	1·89	0·28	3·57
19	0·51	1·96	0·26	3·84
20	0·49	2·04	0·24	4·22
21	0·47	2·13	0·22	4·54
22	0·45	2·22	0·20	4·93
23	0·44	2·27	0·19	5·15
24	0·42	2·38	0·18	5·66
25	0·41	2·46	0·16	6·07
26	0·39	2·56	0·15	6·56
27	0·37	2·70	0·14	7·29
28	0·36	2·78	0·13	7·73
29	0·35	2·88	0·12	8·30
30	0·33	3·00	0·11	9·00
31	0·32	3·12	0·10	9·76
32	0·30	3·26	0·09	10·59
33	0·29	3·40	0·09	11·50
34	0·28	3·53	0·08	12·51
35	0·27	3·69	0·07	13·62
36	0·26	3·87	0·07	14·84
37	0·25	4·02	0·06	16·18
38	0·24	4·20	0·06	17·68

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ϕ in degrees	$\frac{1-\sin\phi}{1+\sin\phi}$	$\frac{1+\sin\phi}{1-\sin\phi}$	$\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2$	$\left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2$
39	0·22	4·43	0·05	19·64
40	0·22	4·60	0·05	21·16
41	0·21	4·81	0·04	23·14
42	0·20	5·04	0·04	25·40
43	0·19	5·29	0 04	27·98
44	0 18	5·55	0 03	30·84
45	0·17	5·83	0·03	33 94
46	0·16	6·12	0·01	37·45
47	0·15	6·43	0·02	41·34
48	0·15	6·78	0·02	45·97
49	0·14	7·16	0 02	51·27
50	0·13	7·54	0·02	56·85

Values of ϕ for cohesionless materials

Material	ϕ (*degrees*)
Sandy gravel	35—45
Compact sand	35—40
Loose sand	30—35
Shale filling	30—35
Rock filling	35—45
Ashes	35—45
Broken brick	35—45

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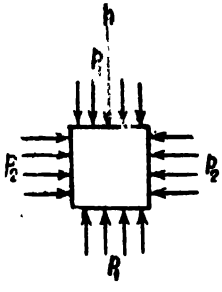

Fig. 313

$$\sin\phi = \frac{p_1 - p_2}{p_1 + p_2}$$

or $$\frac{p_1}{p_2} = \frac{1-\sin\phi}{1+\sin\phi}$$

In earth pressure problems this greatest obliquity reaches the angle of repose of the soil.

Consider an elemental block of earth at a depth of h from the earth surface (Fig. 313). Let p_1 be the vertical intensity of pressure at the depth h. We have $p_1 = wh$ where w is the weight per unit volume of the earth material. If p_2 is the lateral intensity of pressure we have

$$\frac{p_2}{p_1} = \frac{1-\sin\phi}{1+\sin\phi}$$

or $$p_2 = p_1 \frac{(1-\sin\phi)}{(1+\sin\phi)}$$

or $$p_2 = wh \frac{(1-\sin\phi)}{(1+\sin\phi)}$$

or $$p_2 = c_p\, wh \text{ where } c_p = \frac{1-\sin\phi}{1+\sin\phi}$$

Total pressure on a retaining wall

Case 1. *Wall with vertical face on the earth side and the retained earth level.*

Consider a retaining wall retaining h units of height of earth The pressure exerted by the earth on the wall per unit run of the wall is given by the area of the pressure triangle

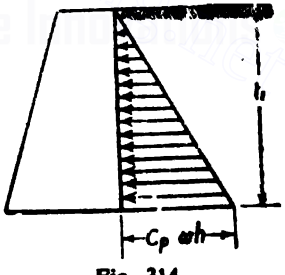

Fig. 314

∴ Total pressure on the retaining wall per unit run $= P = C_p \frac{wh^2}{2}$

or $$P = \frac{wh^2}{2} \frac{1-\sin\phi}{1+\sin\phi}$$

The table on page 515 gives the values of the quantity

$$C_p = \frac{1-\sin\phi}{1+\sin\phi}$$

for various values of ϕ.

Case 2. *Surcharged retaining walls with vertical face on the earth side.* Fig. 315 shows a retaining wall retaining earth surcharged

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at an angle α with the horizontal.

Consider an elemental parallelopiped *EFGH* as shown in Fig. 315 and situated at a depth h below the earth's free surface. The faces *EH* and *FG* are vertical, while the faces *EF* and *GH* are taken parallel to the free surface.

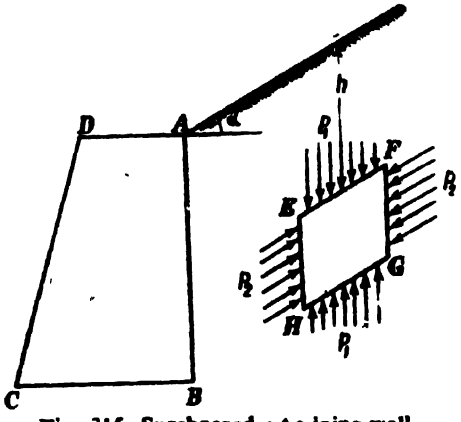

Fig. 315. Surcharged retaining wall.

On the faces *EF* and *HG* the resultant stresses will be $p_1 = w/ \cos \alpha$. Let p_2 be the resultant pressure on the faces *FG* and *EH*.

Hence the planes *EF* and *FG* are such that the resultant stresses on them are inclined at the same angle α with their respective normals

Let P_a and P_v be the principal stresses in terms of which p_1 and p_2 can be expressed.

Fig. 316 shows the Mohr's circle corresponding to the principal stresses P_a and P_v.

In the figure $OA = P_a$

$OB = P_v$

The line OP_2P_1 is shown at an angle α with OA. Hence OP_1 and OP_2 represent the resultant stresses p_1 and p_2 on the planes on which the obliquity is α.

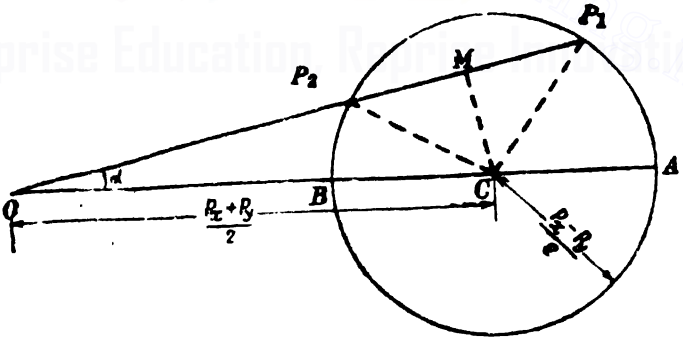

Fig. 316. Mohr's circle corresponding to the principal stresses p_a and p_v.

Draw *CM* perpendicular to P_1P_2.

We have

$$p_1 = OP_1 = OM + MP_1$$

But $$MP_1 = MP_2 = \sqrt{P_2C^2 - MC^2}$$

$$= \sqrt{\left(\frac{P_a - P_v}{2}\right)^2 - \left(\frac{P_a + P_v}{2}\right)^2 \sin^2}$$

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But $\frac{p_a - p_v}{p_a + p_v} = \sin \phi$

$\therefore \quad p_a - p_v = (p_a + p_v) \sin \phi$

$\therefore \quad MP_1 = MP_2 = \sqrt{\left(\frac{p_a + p_v}{2}\right)^2 \sin^2 \phi - \left(\frac{p_a + p_v}{2}\right)^2 \sin^2 \alpha}$

$\therefore \quad MP_1 = MP_2 = \left(\frac{p_a + p_v}{2}\right) \sqrt{\sin^2 \phi - \sin^2 \alpha}$

$\therefore \quad MP_1 = MP_2 = \frac{p_a + p_v}{2}\left(\sqrt{\cos^2 \alpha - \cos^2 \phi}\right)$

$p_1 = OM + MP_1$

$= \left(\frac{p_a + p_v}{2}\right) \cos \alpha + \left(\frac{p_a + p_v}{2}\right)\left(\sqrt{\cos^2 \alpha - \cos^2 \phi}\right)$

$\therefore \quad p_1 = \frac{p_a + p_v}{2}\left(\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}\right)$

Similarly $p_2 = OM - MP_2$

$p_2 = \frac{p_a + p_v}{2}\left(\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}\right)$

$\therefore \quad \frac{p_2}{p_1} = \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}}$

$\therefore \quad p_2 = p_1 \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}}$

But $p_1 = wh \cos \alpha$

$\therefore \quad p_2 = wh \cos \alpha \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}}$

Hence the total earth pressure on a retaining wall per uuit run for a depth h from the free surface

$$= P = \frac{wh^2}{2} \cos \alpha \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}}$$

P is taken to act at $\frac{h}{3}$ from the base of the wall and parallel to the free surface.

Thus $P = C_p \frac{wh^2}{2}$, where $C_p = \cos \alpha \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}}$

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The table below gives the values of C_p for various values of α and ϕ.

ϕ degrees	Values of C_p						
	Slope with Horizontal						
	1 to 1	1 to 1½	1 to 2	1 to 2½	1 to 3	1 to 5	Level
	Corresponding surcharge angle α						
	45°	34° 40′	26° 40′	21° 50′	18° 30′	14° 0′	0°
55	0·18	0·13	0·12	0·11	0·11	0·10	0·10
50	0·29	0·18	0·16	0·15	0·14	0·14	0·13
45	..	0·26	0·22	0·20	0·19	0·18	0·17
40	...	0·36	0·29	0·26	0·24	0·23	0·22
35	...	0·58	0·38	0·33	0·31	0·29	0·27
30	...	—	0·54	0·44	0·40	0·37	0·33
25	...	—	...	0·60	0·52	0·46	0·40
20	...	—	..	...	0·72	0·58	0·49

Case 3. *Retaining wall with a sloping back.*

Fig. 317 shows a retaining wall which is inclined to the vertical on the earth side. In this case, the total horizontal lateral pressure remains at $\frac{wh^2}{2}\frac{1-\sin\phi}{1+\sin\phi}$.

But the resultant pressure on the face AB will be the resultant of the lateral pressure on the face BC and the weight of earth ABC.

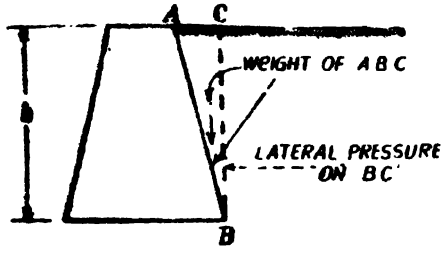

Fig. 317. Wall with a sloping back.

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4. *Retained earth level, Earth subjected to super load.*

Fig. 318 shows a retaining wall retaining a superloaded soil. Let the superload intensity on the retained earth be q per unit area.

In such a case the superimposed distributed load is converted into equivalent additional height of earth $h_e = \frac{q}{w}$ where w is the weight per unit volume of the soil.

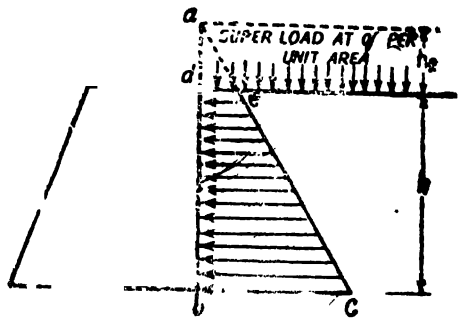

Fig. 318. Wall retaining superloaded earth

The pressure diagram abc is drawn, the pressure intensity at the bottom

$$= bc = w(h+h_e)\frac{1-\sin\phi}{1+\sin\phi}$$

The effective lateral loading on the wall is given by the area *decb* of the stress diagram. The ordinate *de* of the pressure diagram

$$= wh_e\frac{1-\sin\phi}{1+\sin\phi}$$

Let $de = p_2$ and $bc = p_1$

Total horizontal pressure per unit length of the wall

$$= P = \frac{p_1+p_2}{2}h$$

This acts at a height $\bar{h}$ above the base

where $$\bar{h} = \frac{p_1+2p_2}{p_1+p_2}\frac{h}{3}.$$

§146. Rebhann Haseler's Graphical Construction

The cases discussed above may all be dealt with by a single graphical construction. In this method it is also possible to include the friction of the earth on the wall. Let ϕ' be the angle of friction between earth and masonry.

Let AB be the sloping side of the retaining wall on the earth side. Draw BC at an angle ϕ with the horizontal to meet the free surface of the earth at C. Draw the line AD at an angle $(\phi+\phi')$ with AB, to meet BC at D. Draw a semicircle with BC as diameter. Draw the line DE at right angles to BC to meet the semicircle at E. BF is cut off on BC equal to BE. Draw FG parallel to AD, to meet the free earth surface at G. Cut off FH on FB and equal to FG. Join GH.

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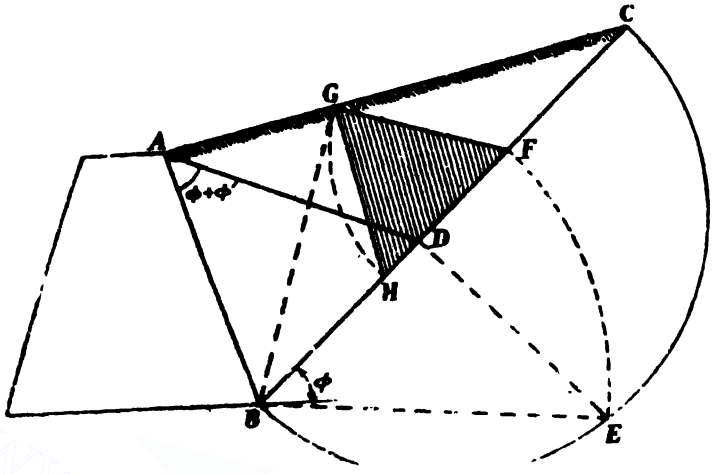

Fig. 319. Rebhann-Haseler's graphical construction.

The total earth pressure on the face AB

$= w \times$ area of $\triangle FGH$

The plane of rupture is given by BG.

§147. Minimum Depth of Foundation

Consider a column carrying a certain load which is transmitted to the soil. Let W be the total load transmitted to the soil.

Let the plan area of the foundation be A.

Hence the vertical intensity of pressure on an elemental cube under the foundation $\Big] = p_1 = \dfrac{W}{A}$

This is now accompanied by a horizontal pressure intensity of

$$p_2 = p_1 \frac{(1-\sin\phi)}{(1+\sin\phi)}$$

Let us now consider another elemental cube which is just clear of the foundation. The horizontal pressure p_2 acting on this cube should be accompanied by vertical pressure intensity p_3 for its equilibrium, so that

$$p_3 \frac{(1-\sin\phi)}{(1+\sin\phi)} = p_1 \left[\frac{1-\sin\phi}{1+\sin\phi}\right]^2$$

But p_3 can be provided by the weight of the earth immediately above the cube.

Hence $p_3 = wh$

$$wh = p_1 \left[\frac{1-\sin\phi}{1+\sin\phi}\right]^2$$

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or $$h=\frac{p_1}{w}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]^2$$

or $$\frac{W}{Aw}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]^2$$

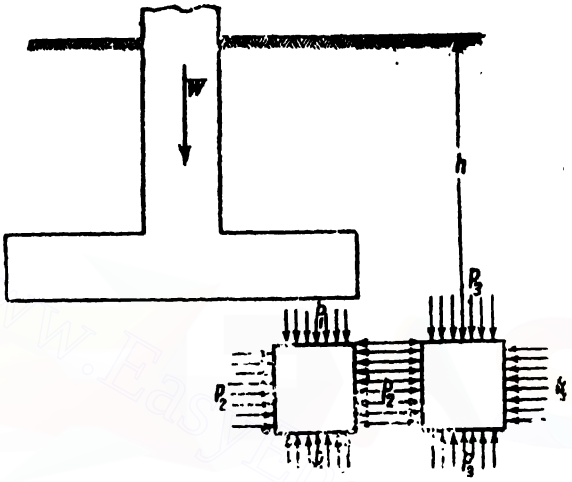

Fig 320

§148. Stability of Retaining Walls

Fig 321 shows a retaining wall retaining earth. Consider a unit length of the wall. Let R be the resultant of the weight of the wall and the earth pressure P.

This resultant force meets the base at an eccentricity of e. The distance e is the distance between the middle point of the base and the point of intersection of the resultant on the base. This resultant will produce a non-uniform distribution of pressure at the base. If the eccentricity is less than $\frac{b}{6}$ where b is the width of the base, the pressures remain wholly compressive and are given by,

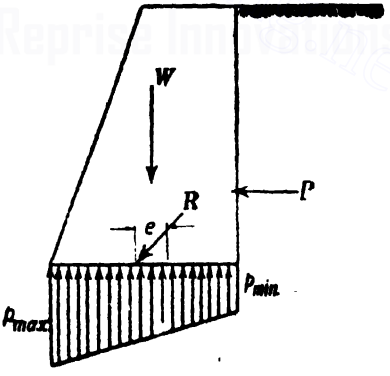

Fig. 321

$$p_{max}=\frac{W}{b}\left[1+\frac{6e}{b}\right]$$

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and $p_{min} = \frac{W}{b}\left[1 - \frac{6e}{b}\right]$

In order that a wall may be stable the following conditions should be satisfied :

(*i*) All the components of the wall must be adequately strong to resist the bending moments and shear forces.

(*ii*) The wall shall not overturn. This of course is fulfilled if the resultant does not strike outside the base.

(*iii*) The maximum pressure at the base shall not exceed the safe bearing capacity of the soil.

(*iv*) The wall shall not be pushed out by lateral pressure of the earth. Hence there shall be enough frictional resistance between the base and the soil opposing the lateral earth pressure If W is the total vertical load transferred to the soil the greatest available frictional resistance equals μW where μ is the coefficient of friction between the wall and the soil on which it rests. Hence for the wall to remain safe against sliding.

$$\mu W > P$$

where P = Horizontal earth pressure.

The factor of safety against sliding equals $\frac{\mu W}{P}$. It is usual to design retaining walls so as to provide a factor of safety of at least 1·5 against sliding.

The case when the eccentricity e is greater than $\frac{b}{6}$.

Studying the equation

$$p = \frac{W}{b}\left[1 \pm \frac{6e}{b}\right]$$

We find that when $e > \frac{b}{6}$, p_{min} works out to be negative indicating that tensile stresses are developed at the inner edge of the base. But since the wall is simply resting on the soil, the only type of stress between the soil and the base should be compressive. Tensile stresses are not possible since obviously the soil cannot pull down the base. Hence, in such cases the formula $p = \frac{W}{p}\left[1 \pm \frac{6e}{b}\right]$ should not be used since the distribution of pressure will not follow the above law.

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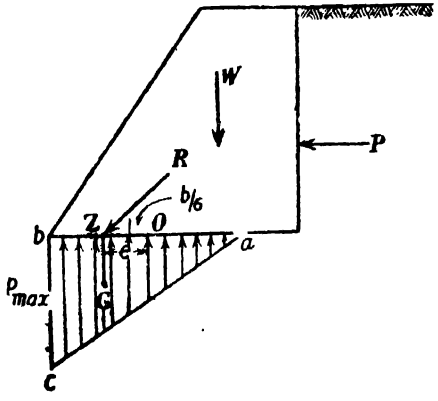

Fig. 322. Pressure distribution at the base when $e < \frac{b}{6}$

The only possible stresses being compressive at the base, the distribution of pressure will be as shown in Fig. 322.

Let the resultant force meet the base at Z. Let O be the middle point of the base. Then the eccentricity of the resultant will be $e = OZ$. The shape of the pressure diagram will be such that the centroid G of the pressure diagram and the point Z shall remain on the same vertical

Hence the base length of the stress diagram

$$= ba = 3\,bZ$$

p_{max} can be determined by equating the total upward pressure to the total downward load

$$\therefore \quad \tfrac{1}{2} p_{max} \times ba = W$$

$$p_{max} = \frac{2W}{ba} = \frac{2W}{3bZ}$$

If p_{max} is less than the safe bearing capacity of the soil, the design will be safe.

§149. Stresses on Horizontal Sections of the Body of the Wall

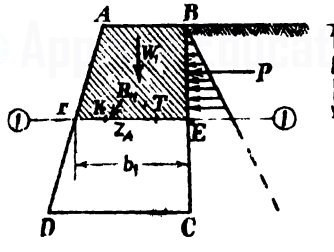

Fig. 323

Consider for unit run of the wall, a horizontal section 1–1 at a depth h_1 from the free earth surface. Let W_1 be the total vertical load acting on the plane (1)–(1). Let P_1 = total Horizontal Pressure acting above (1)–(1).

Let R_1 be the resultant of W_1 and P_1.

Let Z_1 be the point where the resultant meets the sectional face EF.

Let b_1 be the breadth at the section (1)–(1). If e_1 is the eccentricity of R_1 on the section 1–1, the maximum and minimum pressures at the section 1–1 are given by

$$p = \frac{W_1}{b_1}\left[1 \pm \frac{6e_1}{b_1}\right]$$

The maximum value of p determined with the positive sign shall not be greater than the permissible compressive stress of the wall material.

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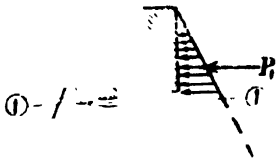

Fig. 324. Failure of masonry wall due to tensile stresses caused on the horizontal section 1 - 1.

If $e_1 < \frac{b_1}{6}$ the minimum pressure will remain compressive. If $e_1 > \frac{b_1}{6}$ tensile stresses are caused in the wall material. If the retaining wall is of masonry, the design shall not allow any tensile stresses to be caused in masonry. Hence b_1 should be considerably large so that e_1 shall not be greater than $\frac{b_1}{6}$. But if the retaining wall is an R.C. retaining wall, b_1 need not be so large and a thinner wall will do, since by providing reinforcement the tension developed may be resisted. Hence an R.C. wall will require an appreciably thinner section as compared to a masonry wall. Masonry has little tensile strength. But reinforced concrete can be designed to resist tension.

Instead of providing a masonry wall which can crack due to the tensile stress caused when $e_1 > \frac{b}{6}$ (see Fig. 324) it may be better to use an R.C. retaining wall.

§159. Reinforced Concrete Retaining Walls

Cantilever Retaining Walls. A cantilever retaining wall consists of a vertical cantilevering slab called the stem and a base slab (Fig 325). This type of wall may by used to retain earth up to 6 *metres*. For greater heights a counterfort retaining wall should be used. The base slab consists of :

(*i*) The toe slab and (*ii*) Heel slab. The heel slab is that part of the base slab, under the retained earth. The toe slab is the projection of the base slab on the front side of the retaining wall.

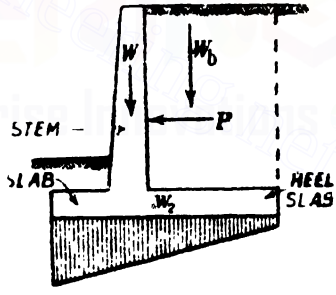

Fig. 325. Cantilever retaining wall.

Wall Proportions. The following wall proportions are usually followed :

Top width of stem = 20 *cms.*
Bottom width of stem = To be computed
Width of base slab = b

$b = 0{\cdot}5\ H$ to $0{\cdot}6\ H$ for walls without surcharge
$b = 0{\cdot}7\ H$ for surcharged walls

Toe projection $= \frac{H}{6}$ or $\frac{b}{3}$

H = Overall height of wall.

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The bottom width of the stem should be determined from bending moment considerations. Usually the thickness of the base slab is made equal to the bottom thickness of the stem.

Method of design. (*a*) A tentative cross-section should be first assumed.

(*b*) For one *metre* run of the wall the maximum bending moment at the bottom of the stem is determined. By equating the bending moment to the moment of resistance of $Q\,bd^2$ the effective depth is determined. The cover to the reinforcement in the stem is usually 3 to 4 *cm*. Hence assuming the diameter of the reinforcement the overall width of the stem at the bottom is determined. The base slab thickness is made equal to the width of the stem at the bottom.

(*c*) The stability of the structure is studied. The maximum and minimum pressures at the base are worked out. The maximum pressure at the base shall not exceed the safe bearing capacity of the soil.

(*d*) The reinforcement required for the stem is computed. This is given by

$$A_s\ (sq.\ cm.) = \frac{\text{Max. B.M. in } Newton\ cm.}{14000 \times 0.87\,d} \quad \text{or} \quad \frac{\text{Max. B.M. } in\ kg.\ cm.}{1400 \times 0.87\,d}$$

(*e*) Calculate the maximum bending moment for the toe slab. The toe slab is designed as a cantilever and acted upon by upward soil reaction. The depth provided is checked for the bending moment. A cover of 5 *cm*. to 6 *cm*. may be provided to reinforcement in the base slab.

(*f*) Calculate the maximum bending moment for the heel slab. The heel slab is also designed as a cantilever subjected to downward loads consisting of its own weight, the weight of the soil above it and super load and any loading due to the surcharge, and upward soil reactions. The reinforcement required is now determined.

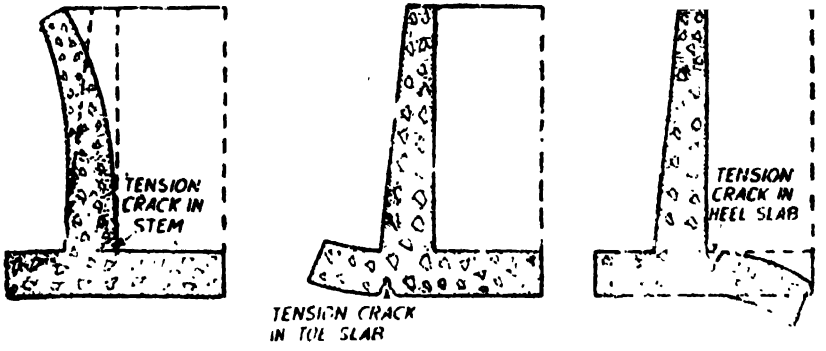

Fig. 326

(*g*) The tendency for the wall to slide forward due to the lateral earth pressure should be investigated. A factor of safety of

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at least 1·5 shall be provided against sliding. If the factor of safety is insufficient a key (also called rib or cut off wall) should be designed to prevent the lateral movement of the structure. (See Fig. 328).

Figs. 326 and 327 show how the various components of a cantilevering wall can fail.

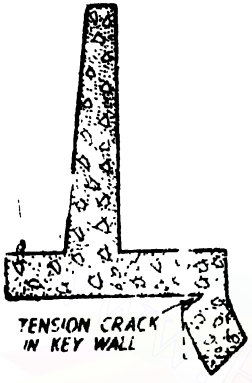

Fig. 327

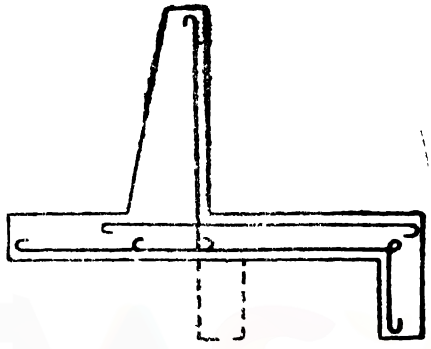

Fig. 328

The above figures must obviously give an idea as to where the reinforcements are required. For the stem, reinforcement is required near the earth side. For the toe slab the reinforcement is required at the bottom of the slab. For the heel slab the reinforcement is required at the top of the slab. For the key the reinforcement is required on that side where convexity of bending will occur (see Figs. 327 and 328). Fig. 328 shows a section of an R.C. retaining wall showing places where the main reinforcements are required.

Usually the requirement of reinforcement for the key will be small and hence alternate bars of toe slab may be continued and bent down to form the reinforcement for the key. The key may alternatively be provided under the stem so that alternate bars of the stem reinforcement may serve as reinforcement for the key.

Shear reinforcement. Shear reinforcement is not normally used in retaining walls. The resistance of the concrete itself must be sufficient to resist the shearing forces imposed.

Distribution steel. Horizontal distribution steel should be provided in both the stem and the base slab. This reinforcement shall not be less than 0·15% of the gross area of concrete.

Expansion and contraction joints. Keyed expansion and contraction joints should be provided at 30 *m* intervals.

Design 80. (S.I.) ***A cantilever type retaining wall has a 5·5 metre tall stem. It retains earth level with its top. The soil weighs 19000 newtons per cubic metre and has an angle of repose of 30°. The bearing capacity of the soil is 200 kilonewtons per square metre. Design the wall assuming suitable working stresses. Use M 15 concrete.***

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Solution.

Top width. This may be kept at 20 *cm*.

Bottom thickness This will be determined from bending moment consideration.

Height of stem $= h = 5{\cdot}5$ *metre*.

Maximum bending moment per *metre* length

$$= c_p \frac{wh^3}{6} = \frac{1}{3} \times 19000 \times \frac{5{\cdot}5^3}{6} \ Nm.$$

$$= 175620 \ Nm. = 17{,}562{,}000 \ Ncm.$$

Adopting $c = 5 \ N/mm.^2 = 500 \ N/cm.^2$,

$t = 140 \ N/mm.^2 = 14000 \ N/cm.^2$ and $m = 18$

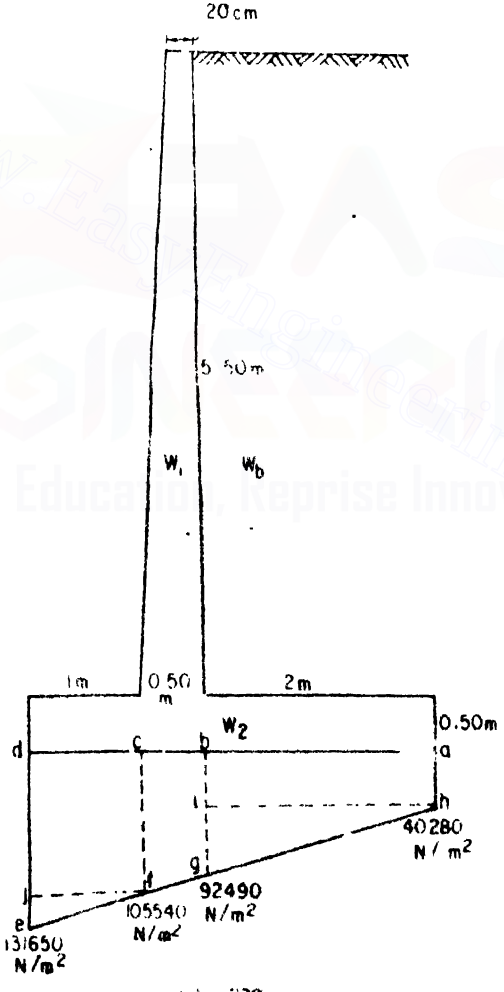

Fig. 329

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and equating the moment of resistance to the bending moment, we have,

$$85\ bd^2 = 85 \times 100\ d^2 = 17{,}562{,}000$$

$$d = \sqrt{\frac{17562000}{85 \times 100}} = 45{\cdot}5\ cm.$$

Providing an effective cover of 4 *cm.* the overall thickness

$$= 45{\cdot}5 + 4 = 49{\cdot}5\ cm.,\ say\ 50\ cm.$$

The base slab also will be made 50 cm. thick.

∴ Total height of wall

$$= 5{\cdot}50 + 0{\cdot}50 = 6\ metres.$$

Width of base slab $= b = 0{\cdot}5\ H$ to $0{\cdot}6\ H$

$$0{\cdot}5\ H = 0{\cdot}5 \times 6 = 3\ metres$$

$$0{\cdot}6\ H = 0{\cdot}6 \times 6 = 3{\cdot}6\ metres.$$

Let us provide a base width of 3·50 metres.

Toe projection. This may be made about one-third of the base width

$$= \frac{3{\cdot}5}{3} = 1{\cdot}17\ metre$$

Let us provide a toe projection of 1 metre.

The proposed overall dimensions of the retaining wall are shown in Fig. 329.

Stability Calculations for One Metre Run of Wall

Load due to	*Magnitude of load (Newtons)*	*Distance from a (metres)*	*Moment about a Newton metre)*
W_1 { $0{\cdot}20 \times 5{\cdot}50 \times 25000$	27500	2·1	57750
W_1 { $\frac{1}{2} \times 0{\cdot}30 \times 5{\cdot}50 \times 25000$	20625	2·3	47438
W_2 $3{\cdot}50 \times 0{\cdot}50 \times 25000$	43750	1·75	76563
W_3 $2 \times 5{\cdot}5 \times 19000$	209000	1	209000
Moment of lateral pressure $= c_p \frac{wH^3}{6} = \frac{1}{3} \times \frac{19000 \times 6^3}{6}$			228000
Total	300875		618751

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∴ Distance of the point of application of the resultant force from the end a $= Z = \frac{618751}{300875} = 2{\cdot}06$ *metre*

∴ Eccentricity $= e = Z - \frac{b}{2}$

$= 2{\cdot}06 - 1{\cdot}75 = 0{\cdot}31$ *metre*

But, $\frac{b}{6} = \frac{3{\cdot}5}{6} = 0{\cdot}58$ *metre*

∴ Extreme pressure intensity on the base

$$p = \frac{W}{b}\left[1 \pm \frac{6e}{b}\right]$$

$$= \frac{300875}{3{\cdot}5}\left[1 \pm \frac{6 \times 0{\cdot}31}{3{\cdot}5}\right] N/metre^2$$

$p_{max} = 131650$ *N/metre*^{2}

$= 131{\cdot}65$ *kilo newtons/metre*^{2}

and $p_{min} = 40280$ *N/metre*^{2}

$= 40{\cdot}28$ *kilo newtons/metre*^{2}

Safe bearing capacity of the soil

$= 200$ *kilo newtons/metre*^{2}

Design of the stem

Maximum bending moment for the stem per *metre* length

$= 17562000$ *N cm.*

∴ $A_t = \frac{17562000}{14000 \times 0{\cdot}87 \times 46} = 31{\cdot}34$ *cm.*^{2}

∴ Spacing of 20 *mm.* diameter bars

$$= \frac{3{\cdot}14 \times 100}{31{\cdot}34} = 10{\cdot}02 \text{ cm.}$$

Provide 20 mm. ϕ bars @ 10 cm. centres

Distribution steel. This will be provided at 0·15 percent of gross area $= \frac{0{\cdot}15}{100} = (100 \times 50) = 7{\cdot}50$ *cm.*^{2}

∴ Spacing of 10 *mm* ϕ bars

$$= \frac{0{\cdot}79 \times 100}{7{\cdot}5} = 10{\cdot}53 \text{ cm.}$$

say 10 cm. centres

If the distribution steel is provided near both the faces, then the spacing of 10 mm. ϕ bars will be at 20 cm. centres near both the faces.

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Design of the toe slab

B.M. calculations for the toe slab per *metre* run of the wall are shown in the table on page 533.

Hence, for one *metre* strip of the toe slab, the maximum bending moment $= 55223\ N\ metre$

$= 5522300\ N\ cm.$

$$\therefore \quad A_t = \frac{5522300}{14000 \times 0{\cdot}87 \times 44} = 10{\cdot}30\ cm.^2$$

$\therefore$ Spacing of 16 *mm* ϕ bars

$$= \frac{2{\cdot}01 \times 100}{10{\cdot}30} = 19\ cm.$$

Provide 16 mm. ϕ bars @ 18 cm. centres.

Design of the heel slab

B.M. calculations for the heel slab per *metre* run of the wall are shown in the table on page 533.

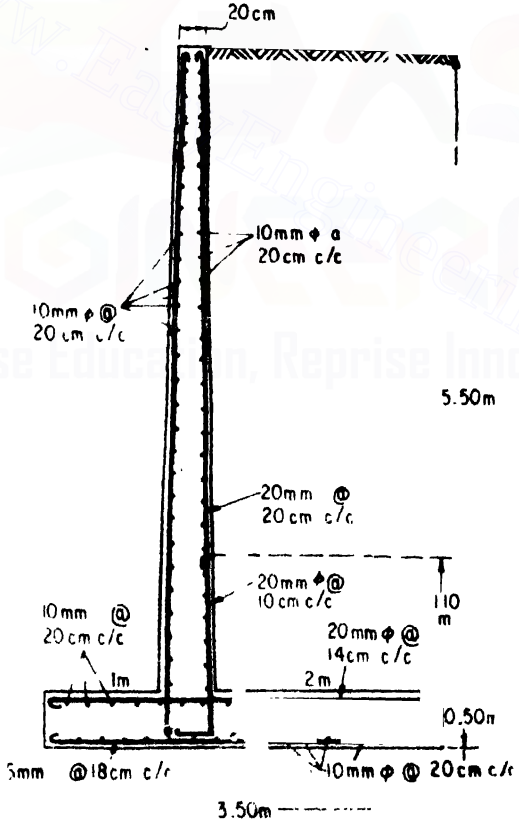

Fig 331

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Load due to	*Magnitude of the load (Newtons)*	*Distance from c (metre)*	*Moment about c (Newton metre)*
Upward pressure, *cdjf* 105540×1	105540	0·5	52770
Upward pressure, *efj* $\frac{1}{2} \times 1 \times 26110$	13055	$\frac{2}{3}$	8703
			61473
Deduct for self-weight of toe slab $1 \times 1 \times 0.5 \times 25000$	12500	0·5	6250
B.M. for toe slab			55223

Load due to	*Magnitude of load (Newtons)*	*Distance from b (metres)*	*Moment about b (Newton metre)*
Weight of backing $W_1 2 \times 5{\cdot}50 \times 19000$	209000	1	209000
Weight of heel slab $2 \times 0{\cdot}50 \times 25000$	25000	1	25000
			234000
Deduct for upward pressure,			
abih 40280×2	80560	1	80560
ghl $\frac{1}{2} \times 2 \times 52170$	52170	$\frac{2}{3}$	34780
Total deduction			115340
B.M. for Heel Slab			118660

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Hence for one *metre* strip of heel slab, the maximum bending moment

$$= 1186\,60 \text{ } N \text{ } metre = 11866000 \text{ } N \text{ } cm.$$

$$A_s = \frac{11866000}{14000 \times 0{\cdot}87 \times 41} \text{ } cm.^2 = 22{\cdot}14 \text{ } cm.^2$$

Spacing of 20 *mm.* ϕ bars

$$= \frac{3{\cdot}14 \times 100}{22{\cdot}14} = 14 \text{ } cm. \text{ } centres.$$

Provide *20 mm.* ϕ *bars @ 14 cm. centres*

Check for sliding

Total horizontal earth pressure per *metre* length of wall

$$P = c_p \frac{wh^2}{2}$$

$$= \frac{1}{3} \times 19000 \times \frac{6^2}{2} \text{ } Newtons$$

$$= 114000 \text{ } Newtons$$

Assuming $\mu = 0{\cdot}6$,

Limiting friction $= \mu W$

$$= 0{\cdot}6 \times 300875 \text{ } kg. = 180525 \text{ } kg.$$

Factor of safety against sliding

$$= \frac{\mu W}{P} = \frac{180525}{114000} = 1{\cdot}58$$

Design 81. (M.K.S). *The stem of a cantilever type retaining wall is 5 metres tall The wall retains soil level with its top. The soil weighs 1900 kg. per cubic metre and has an angle of repose of 30°. The bearing capacity of the soil is 20 tonnes per square metre. Design the wall assuming suitable working stresses.*

Solution.

Top width of stem. This may be kept equal to 20 *cm.*

Bottom thickness of stem This will be determined from bending moment consideration.

Height of stem $= 5$ *metres*

Consider one *metre* run of the wall.

Maximum bending moment

$$= c_p \frac{wh^3}{6}$$

$$= \frac{1}{3} \times 1900 \times \frac{5^3}{6} \text{ } kg. \text{ } m.$$

$$= 13194 \text{ } kg. \text{ } m. = 1319400 \text{ } kg. \text{ } cm.$$

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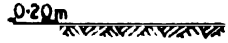
Adopting, $c=50$ *kg./cm.*^{2} $t=1400$ *kg./cm.*^{2} and $m=18$ and



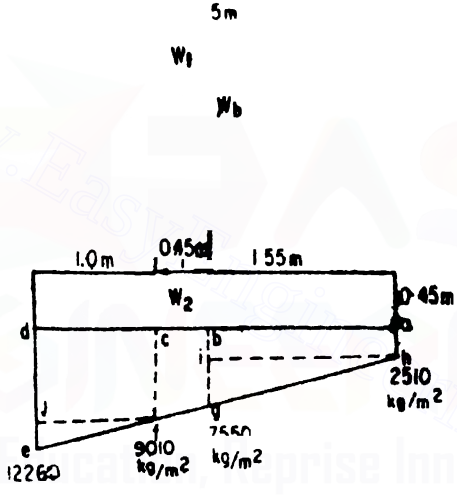


Fig. 331

equating the moment of resistance to the bending moment, we have

$$8{\cdot}5\,bd^2=8{\cdot}5\times100\,d^2=1319400$$

$$\therefore \qquad d=\sqrt{\frac{1319400}{8{\cdot}5\times100}}=39{\cdot}40\ cm.$$

Providing an effective cover of 4 *cm.* overall thickness required

$$=39{\cdot}40+4=43\ 4\ cm$$

Provide a thickness of 45 cm.

The base slab also will be made 45 *cm.* thick

Total height of wall $=H=5+0{\cdot}45=5\ 45$ *metre*

Width of base slab $b=0{\cdot}5\ H$ to $0{\cdot}6\ H$

$0{\cdot}5\ H=0{\cdot}5\times5{\cdot}45=2{\cdot}73\ m.$

$0{\cdot}6\ H=0{\cdot}6\times5{\cdot}45=3{\cdot}27\ m.$

Provide a base width of 3 ***metres.***

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Toe projection. This may be about one-third the base width

$$= \frac{3}{3} = 1 \text{ metre.}$$

Stability Calculations for one Metre Run of Wall

Load due to	Magnitude of load kg	Distance from a (metres)	Moment about a (kg. m.)
W_1 { $0{\cdot}20\times5\times2500$	2500	1·65	4125
W_1 { $\frac{1}{2}\times0{\cdot}25\times5\times2500$	1563	1·83	2860
W_2 $3\times0{\cdot}45\times2500$	3375	1·50	5063
W_3 $1{\cdot}55\times5\times1900$	14725	0·775	11412
Moment of lateral pressure $c_p \frac{wH^3}{6} = \frac{1}{3}\times1900\times\frac{5{\cdot}45^3}{6}$			17087
Total	22163		40547

∴ Distance of the point of application of the resultant from the end a

$$= Z = \frac{40547}{22163} = 1{\cdot}83 \text{ metre.}$$

∴ Eccentricity $e = Z - \frac{}{2}$

$$= 1{\cdot}83 - 1{\cdot}50 = 0{\cdot}33 \text{ metre}$$

But, $\frac{b}{6} = \frac{3}{6} = 0{\cdot}50$ metre $\quad e < \frac{b}{6}$

∴ Extreme pressure intensity on the base

$$= p = \frac{W}{b}\left[1 \pm \frac{6e}{b}\right]$$

$$\frac{22163}{3}\left[1 \pm \frac{6\times0{\cdot}33}{3}\right] \text{ kg./m.}$$

∴ $p_{max} = 12260$ kg /m^2 and $p_{min} = 2510$ kg /m.^{2}

Safe bearing capacity of the soil

$$= 20 \text{ t/metre}^2 = 20{,}000 \text{ kg./metre}^2.$$

Design of the stem

Maximum bending moment = 1319400 kg. cm.

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$$A_t = \frac{1319400}{1400 \times 0{\cdot}87 \times 41} = 26{\cdot}42\ cm.^2$$

Spacing of 20 *mm.* ϕ bars $= \dfrac{3{\cdot}14 \times 100}{26{\cdot}42} = 11{\cdot}9\ cm.$

Provide 20 mm. ϕ *@ 11 cm. centres.*

Distribution steel. This will be provided at 0·15% of gross area

$$= \frac{0{\cdot}15}{100}(100 \times 45) = 6{\cdot}75\ cm.^2$$

Spacing of 10 *mm.* ϕ bars

$$= \frac{0{\cdot}79 \times 100}{6{\cdot}75} = 11{\cdot}7\ cm.\ \text{say } 11\ cm.$$

If the distribution steel is provided near both the faces, then the spacing of 10 mm. ϕ *bars will be at 22 cm. centres.*

Design of the toe slab. Bending moment calculations for the toe slab for one *metre* length of the wall are shown in the table below :

Load due to	*Magnitude of load (kg.)*	*Distance from c (metre)*	*Moment about c (kg. m)*
Upward pressure			
cdjf 9·10 × 1	9010	0·5	4505
Upward pressure			
cfl ½ × 1 × 3250	1625	⅔	1083
			5588
Deduct for self-weight of toe slab			
1 × 0·45 × 2500	1125	0·5	562
B.M. for toe slab			5026

Hence, for one *metre* strip of the toe slab, the maximum bending moment

$$= 5026\ kg.\ m. = 502600\ kg.\ cm.$$

$$A_t = \frac{502600}{1400 \times 0{\cdot}87 \times 39} = 10{\cdot}58\ cm.^2$$

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∴ Spacing of 16 *mm.* ϕ bars

$$= \frac{2{\cdot}01 \times 100}{10{\cdot}58} = 18{\cdot}9 \text{ cm.}$$

Provide 16 mm. ϕ *@ 18 cm. centres.*

Design of the heel slab Bending moment calculations for the heel slab for one *metre* length of the wall are shown in the table below.

Load due to	*Magnitude of load (kg)*	*Distance from b (metre)*	*Moment about b (kg. m.)*
Weight of backing			
Wb $1{\cdot}55 \times 5 \times 1900$	14725	0·775	11412
Weight of the heel slab			
$1{\cdot}55 \times 0{\cdot}45 \times 2500$	1744	0·775	1352
			12761
Deduct for upward pressure			
$abth$ $2510 \times 1{\cdot}55$	3890	0·775	3015
ghl $\frac{1}{2} \times 1{\cdot}55 \times 5040$	3906	$\frac{1{\cdot}55}{3}$	2018
Total deduction			5033
B.M. for heel slab			7731

Hence, for one *metre* strip of the heel slab, the maximum bending moment,

$$= 7731 \text{ kg. m.} = 773100 \text{ kg. cm.}$$

$$\therefore \quad A_s = \frac{773100}{1400 \times 0{\cdot}87 \times 39} = 16{\cdot}28 \text{ cm.}^2$$

∴ Spacing of 20 *mm.* ϕ bars

$$= \frac{3{\cdot}14 \times 100}{16{\cdot}28} = 19 \text{ cm. centres}$$

Provide 20 mm. ϕ ***bars @ 19 cm. centres.***

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Check for sliding

Total horizontal earth pressure per *metre* length of wall

$$= P = c_p \frac{wH^2}{2}$$

$$= \frac{1}{3} \times 1900 \times \frac{5{\cdot}45^2}{2} \text{ kg.} = 9406 \text{ kg.}$$

Assuming $\mu = 0{\cdot}65$

Limiting friction $= \mu W = 0{\cdot}65 \times 22163 = 14406$ *kg.*

∴ **Factor of safety against sliding**

$$= \frac{\mu W}{P} = \frac{14406}{9406} = 1{\cdot}53$$

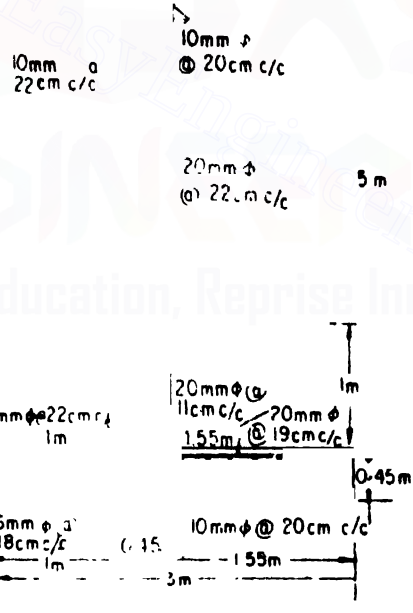

Fig. 332

Design 82. (S.I.). ***Design a retaining wall for a road to the following requirements :***

(i) Height of wall from the bottom of the base slab to the top of the stem } ***= 4·50 metre***

(ii) Superimposed load due to traffic = 10000 N/metre²

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(*iii*) *Angle of repose* = 30°

(*iv*) *Safe bearing capacity of the soil* = 200 *kN/metre*^{2}

(*v*) *Weight of soil* = 18000 *N/metre*^{3}

(*vi*) *Coefficient of friction between the concrete and the ground* = 0·5

(*vii*) *Top surface of the backing is level*

(*viii*) *A parapet wall one metre high shall also be provided.*

Solution *Top width of stem* = 20 *cm.*

Bottom width of the base slab. This is taken between 0·5 H and 0·6 H.

In our case, 0·5 H = 0·5 × 4·5 = 2·25 *metre*

0·6 H = 0·6 × 4·5 = 2·70 *metre*

Let us provide a base width of 2·50 *metre*

Toe projection. This is taken nearly at one-third the base width.

In our case let us provide a toe projection of

$$\frac{2{\cdot}50}{3} = 0{\cdot}83 \text{ metre say } 0{\cdot}80 \text{ metre}$$

There is a super load due to traffic of 10,000 *N/metre*^{2}. The equivalent additional height of soil corresponding to the above super load equals

$$\frac{10{,}000}{18000} = 0{\cdot}55 \text{ metre.}$$

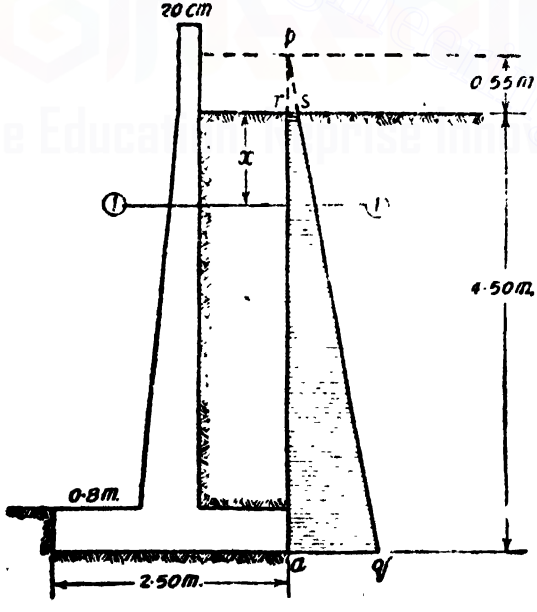

Fig. 333

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The lateral pressure on the wall is given by the area *rsqa* of the pressure diagram.

But in order to simplify the calculations we shall include the small area *prs* also.

Consider a section at a depth of *x metres* from the soil surface.

Consider one *metre* run of the wall.

B.M. at the section 1—1

$$= M = \frac{w(x+0.55)^3}{6} \frac{1-\sin\phi}{1+\sin\phi}$$

$$= \frac{18000(x+0.55)^3}{6} \times \frac{1-\sin 30°}{1+\sin 30°}$$

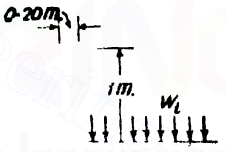
$$= 1000(x+0.55)^3 \ Nm.$$

Making an allowance of 30 *cm* for the base slab maximum bending moment for the stem

$$= 1000(4.50-0.30+0.55)^3 \ Nm.$$

$$= 107170 \ Nm. = 10717000 \ N\ cm.$$

Adopting $c = 5\ N/mm.^2 = 500\ N/cm.^2$; $t = 140\ N/mm.^2 = 14000\ N/cm.^2$ and $m = 18$, and equating the moment of resistance to the maximum bending moment, we have,



$$85\ bd^2 = 85 \times 100\ d^2 = 10717000$$

$$\therefore \quad d = 35.50\ cm.$$

Providing an effective cover of 4 *cm.* total thickness required

$$= 35.5+4 = 39.50\ cm.\ say\ 40\ cm.$$

The thickness of the base slab als will be made 40 *cm.*

We have thus provisionally determined the overall sizes of the various components of the retaining wall.

Let *z metres* be the distance of the point of application of the resultant force at the base from the heel end *a*. We have,

$$z = \frac{\text{Total moment about } a}{\text{Total vertical load}}.$$

The total moment about *a* and the total vertical load for one *metre* run of the wall may be determined as shown in table below.

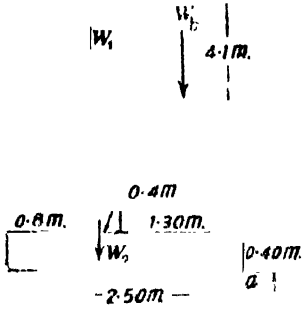


Fig. 334

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Stability Calculations for One Metre Run of the Wall

Load due to	*Magnitude of load (N)*	*Distance from a (m)*	*Moment about a (Nm)*
W_1 $5{\cdot}10\times0{\cdot}20\times25000$	25500	1·40	35700
$\frac{4{\cdot}10\times0{\cdot}20}{2}\times2500$	10250	1·57	16092
W_2 $2{\cdot}50\times0{\cdot}40\times25000$	25000	1·25	31250
W_3 $1{\cdot}30\times4{\cdot}10\times18000$	95940	0·65	62361
W_4 $10000\times1{\cdot}30$	13000	0·65	8450
Moment of lateral pressure $1000(4{\cdot}0+0{\cdot}55)^3$			128788
Total	169690		282641

$$z=\frac{282641}{169690}=1{\cdot}67 \text{ metre.}$$

Eccentricity $= e = z-\frac{b}{2}=1{\cdot}67-1{\cdot}25=0{\cdot}42$ *metre.*

But, $\frac{b}{6}=\frac{2{\cdot}50}{6}=0{\cdot}42$ *metre* $\therefore e=\frac{b}{6}$

Extreme pressure intensity

$$=\frac{W}{b}\left(1\pm\frac{6e}{b}\right)=\frac{W}{b}(1\pm1)$$

$$=\frac{169690}{2{\cdot}50}(1\pm1)$$

$\therefore$ $p_{max}=135750$ *N/metre*^{2} and $p_{min}=0$

Bearing capacity of the soil

$=200$ *kN/metre*$^2=200{,}000$ *N/metre*^{2}

Design of the stem

Actual height of stem

$=4{\cdot}10$ *metre*

$\therefore$ **Actual bending moment for the stem**

$=1000(4{\cdot}10+0{\cdot}55)^3$ *Nm.*

$=100545$ *Nm.*

$=10054500$ *N cm.*

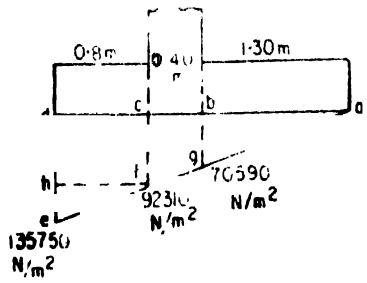

Fig. 335

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Effective depth $= 40 - 4 = 36$ *cm.*

$$\therefore \quad A_t = \frac{10054500}{14000 \times 0{\cdot}87 \times 36} = 22{\cdot}93 \text{ cm.}^2$$

Spacing of 20 *mm.* ϕ bars

$$= \frac{3{\cdot}14 \times 100}{22{\cdot}93} = 13{\cdot}70 \text{ cm.}$$

Provide 20 mm. ϕ *bars @ 13 cm. c/c.*

Distribution steel $= \frac{0{\cdot}15}{100} \times 40 \times 100 = 6 \text{ cm.}^2$

Spacing of 8 *mm.* ϕ bars

$$= \frac{0{\cdot}5 \times 100}{6} = 8{\cdot}3 \text{ cm. say } 8 \text{ cm.}$$

Since the distribution bars will be provided near both the faces, the spacing of 8 mm. ϕ *bars near each face will be 16 cm. c/c.*

The bending moment being less at higher levels the main reinforcement may be decreased at higher levels.

At any depth x *metres* below the soil surface,

$$A_t \propto M \propto (x + 0{\cdot}55)^3$$

If A_{t1} and A_{t2} are the requirements of the reinforcements at sections x_1 *metre* and x_2 *metre* from the top, we have,

$$\frac{A_{t1}}{A_{t2}} = \frac{(x_1 + 0{\cdot}55)^3}{(x_2 + 0{\cdot}55)^3}$$

If x_1 be the depth at which alternate bars may be curtailed, we have,

$$\frac{A_{t1}}{A_{t2}} = \frac{1}{2} = \frac{(x_1 + 0{\cdot}55)^3}{(4{\cdot}1 + 0{\cdot}55)^3}$$

$$\therefore \quad x_1 = 3{\cdot}10 \text{ metre from the top.}$$

Hence alternate bars of the stem can be curtailed at $4{\cdot}10 - 3{\cdot}10 = 1$ *metre* above the base slab.

Design of the toe slab

This is subjected to an upward load due to soil reaction.

Maximum bending moment for the toe slab

= Moment of the soil reaction acting on the toe slab − moment of dead load of the toe slab.

The B.M. calculations are made in the table on page 544.

$$\therefore \quad A_t = \frac{35548 \times 100}{14000 \times 0{\cdot}87 \times 34} = 8{\cdot}58 \text{ cm.}^2$$

Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{8{\cdot}58} = 13{\cdot}17 \text{ cm.}$$

Provide 12 mm. ϕ *bars @ 12 cm. c/c.*

Design of the heel slab

The loading on the heel slab consists of the following :

(*a*) Self-weight of the slab. (*b*) Weight of soil on the heel slab.
(*c*) Traffic load. (*d*) Upward soil reaction.

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Load due to	*Magnitude of load (N)*	*Distance from c (m)*	*Moment about c (Nm)*
Upward soil pressure			
cdhf 92310 × 0·8	73848	0·4	29539
efh 43440 × $\frac{0·8}{2}$	17376	$\frac{2}{3}$ × 0·8 = 0·53	9209
			38748
Deduct for self-weight of toe slab			
0·8 × 0·4 × 25000	8000	0·4	3200
B.M. for toe slab			35548

The B.M. calculations for heel slab are given in the table below :

Load due to	*Magnitude of load (N)*	*Distance from b (m)*	*Moment about b (Nm)*
Backing 1·30 × 4·10 × 18000	95940	0·65	62361
Traffic load 10000 × 1·30	13000	0·65	8450
Weight of heel slab			
1·30 × 0·40 × 25000	13000	0·65	8450
			79261
Deduct for upward pressure *a g*			
$\frac{1·3}{2}$ × 70590	4·883	$\frac{1·3}{3}$	19883
B.M. for Heel Slab			59378

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$$A_s = \frac{59378 \times 100}{14000 \times 0{\cdot}87 \times 34} = 14{\cdot}33 \text{ cm.}^2$$

Spacing of 20 *mm.* ϕ bars

$$= \frac{3{\cdot}14 \times 100}{14{\cdot}33} = 21{\cdot}9 \text{ cm.}$$

Provide 20 mm ϕ *bars @ 20 c/c*

Check for sliding

Total horizontal earth pressure

$$= P = \frac{18000\,(4{\cdot}50+0{\cdot}55)^2}{2} \cdot \frac{1-\sin 30^\circ}{1+\sin 30^\circ} \text{ N (approximately)}$$

$$= 76507 \text{ N}$$

Maximum friction $= \mu W = 0{\cdot}5 \times 169690 = 84845$ N

Factor of safety against sliding

$$= \frac{\mu W}{P} = \frac{84845}{76507} = 1{\cdot}10$$

But it is necessary to provide a factor of safety of 1·5 against sliding.

Hence, the wall should be safe for a lateral pressure of 1·5 P

1·5 $P = 1{\cdot}5 \times 76507$ $= 114760$ N

Available friction $= 84845$ N

$\therefore$ Unbalanced force $= 29915$ N

Safe bearing pressure for lateral loading may be taken at 0·7 times the safe bearing pressure for vertical loads. But safe bearing pressure for vertical loads

$= 200$ *kN/metre*^{2}

$= 200{,}000$ *N/metre*^{2}

Safe horizontal pressure

$= 0{\cdot}7 \times 200{,}000$

$= 140{,}000$ *N/metre*^{2}

Let the height of the key be y *metre*

$\therefore$ $140{,}000\, y = 29915$

$\therefore$ $y = 0{\cdot}21$ *metre*

It is usual to provide a key of height atleast 30 *cm.*

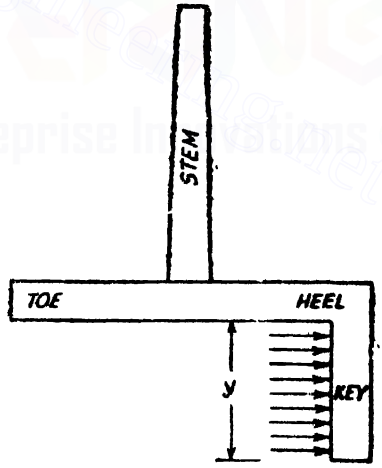

Fig. 336

$$\text{Max. B.M.} = 29915 \times \frac{30}{2} \text{ m.} = 448725 \text{ N cm.}$$

Equating the moment of resistance to the bending moment.

$$85 \times 100\, d^2 = 448725$$

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$\therefore$ $d = 7{\cdot}27$ *cm.*

A key should be atleast 20 *cm.* thick.

Providing a key of thickness 20 *cm.*, effective depth

$= 20 - 6 = 14$ *cm.*

$$A_t = \frac{448725}{14000 \times 0{\cdot}87 \times 14} = 2{\cdot}63 \text{ cm.}^2$$

This is a very small quantity. Let alternate bars of the toe slab be extended and bent down to [illegible] the key reinforcement.

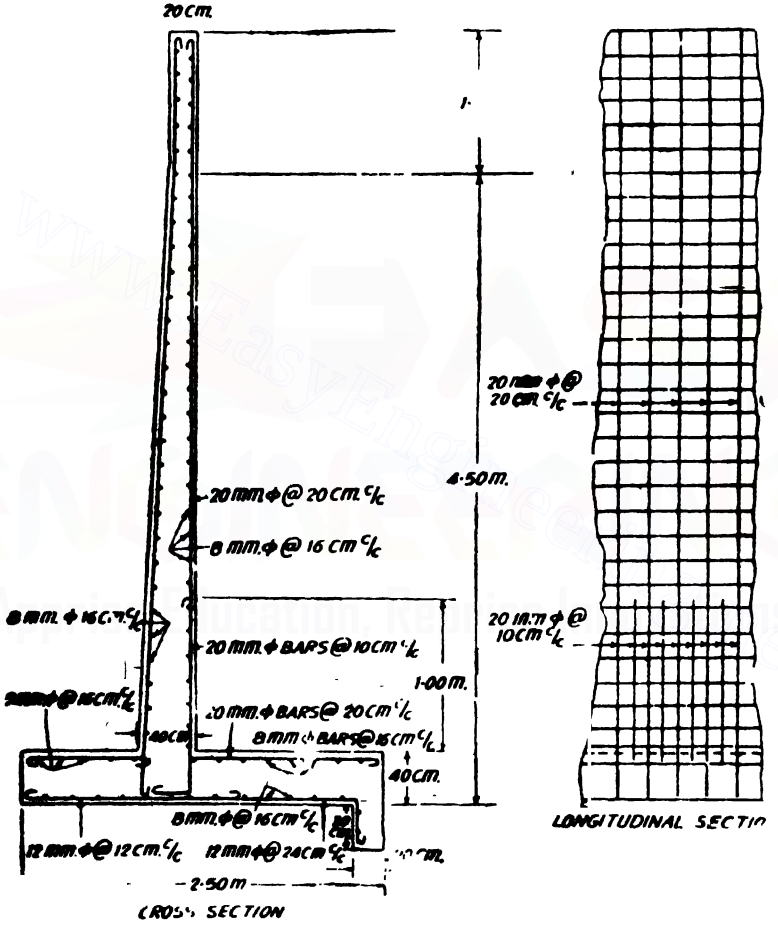

Fig. 337

Design 83. *Design a retaining wall for a road to the following requirements :*

(*i*) *Height of wall from the bottom of the base slab to the top of the stem* } = 4·50 *metres*

(*ii*) *Superimposed load due to traffic* = 1000 *kg./metre*^{2}

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(iii) *Angle of repose* = 30°

(iv) *Safe bearing capacity of the soil* = 20 *tonnes/metre*²

(v) *Weight of soil* = 1800 *kg./metre*³

(vi) *Coefficient of friction between concrete and the ground* = 0·5

(vii) *Top surface of the backing .. level*

(viii) *A parapet wall one metre high shall also be provided.*

Solution. Top width of stem = 20 *cm.*

Bottom width of the base slab. This is taken between 0·5 *H* and 0·6 *H*.

In our case, 0·5 *H* = 0·5 × 4·5 = 2·25 *metre*

0·6 *H* = 0·6 × 4·5 = 2·70 *metre.*

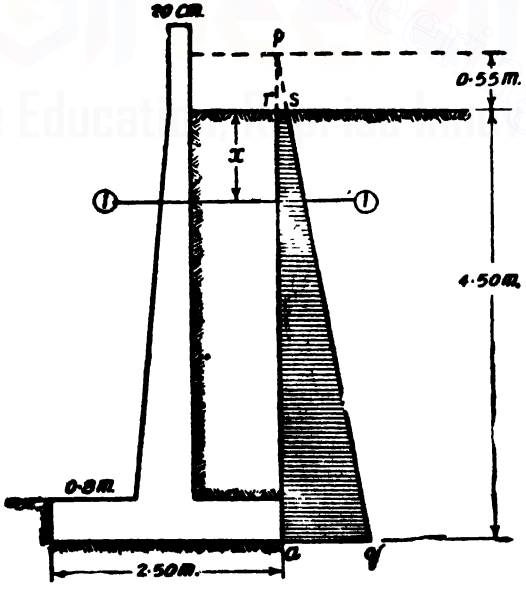
Let us provide a base width of 2·50 *metre.*

Toe projection. This is taken nearly at one-third the base width. In our case let us provide a toe projection of

$$\frac{2{\cdot}5}{3} = 0{\cdot}83 \text{ metre say } 0{\cdot}8 \text{ metre.}$$

There is a super load due to traffic of 1000 *kg./metre*². The equivalent additional height of soil corresponding to the above super load equals

$$\frac{1000}{1800} = 0{\cdot}55 \text{ metre.}$$



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The lateral pressure on the wall is given by the area *rsqa* of the pressure diagram (Fig. 338).

But in order to simplify the calculations we shall include the small area *prs* also.

Consider a section at a depth of *x metres* from the soil surface.

Consider one *metre* run of the wall.

B.M. at the section 1—1

$$= M = \frac{w(x+0{\cdot}55)^3}{6} \frac{1-\sin\phi}{1+\sin\phi}$$

$$= \frac{1800(x+0{\cdot}55)^3}{6} \frac{1-\sin 30^\circ}{1+\sin 30^\circ} \; kg.\, m$$

$$= 100(x+0{\cdot}55)^3 \; kg.\, m.$$

Assuming the base slab to be 30 *cm.* thick.

Maximum B.M. for the stem

$$= 100(4{\cdot}5-0{\cdot}30+0{\cdot}55)^3 \; kg.\, m.$$

$$= 10720 \; kg.\, m.$$

Adopting $c=50$ *kg./cm.*^{2}, $t=1400$ *kg/cm.*^{2}, $m=18$ and equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}50 \times 100\, d^2 = 10720 \times 100$$

$$\therefore \quad d = 35{\cdot}50 \; cm.$$

Providing an effective cover of 4 *cm.*, the overall depth required

$$= 35{\cdot}5+4 = 39{\cdot}5 \; cm.$$

Let us provide 40 *cm.* overall.

The base slab also will be made 40 *cm.* thick.

We have thus provisionally determined the overall sizes of the various components of the structure. With these dimensions let us check for the stability of the structure.

The relevent calculations are made in the table below:

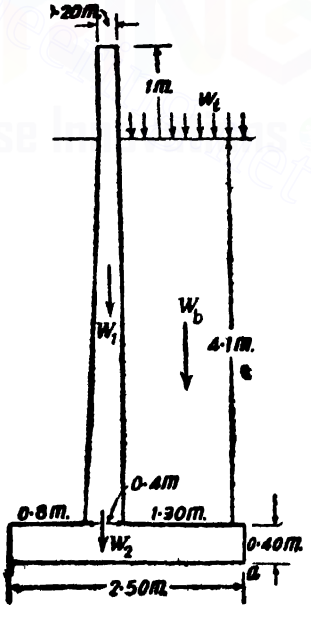

Fig. 339

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Stability Calculations for One Metre Run of the Wall

Load due to	*Magnitude of load (kg.)*	*Distance from a (m.)*	*Moment about a (kg. m.)*
W_1 $5{\cdot}1\times0{\cdot}2\times2500$	2550	1·4	3570
$\frac{1\times0{\cdot}2}{2}\times2500$	1025	1·57	1609
W_2 $2{\cdot}5\times0{\cdot}4\times2500$	2500	1·25	3125
W_b $1{\cdot}3\times4{\cdot}1\times1800$	9594	0·65	6236
W_t $1000\times1{\cdot}3$	1300	0·65	845
Moment of lateral pressure about a			
$100(4{\cdot}5+0{\cdot}55)^2$			12880
Total	16969		28265

$$z=\frac{28265}{16969}=1{\cdot}67\ m.$$

$$\text{Eccentricity}=e=z-\frac{b}{2}=1{\cdot}67-1{\cdot}25=0{\cdot}42\ \textit{metre}.$$

But, $$\frac{b}{6}=\frac{2{\cdot}50}{6}=0{\cdot}42\ \textit{metre} \quad \therefore\ e=\frac{b}{6}$$

$\therefore$ Extreme pressure intensity

$$=\frac{W}{b}\left(1\pm\frac{6e}{b}\right)=\frac{W}{b}(1\pm1)$$

$$\frac{16969}{2{\cdot}5}(1\pm1)$$

$$\therefore \qquad p_{max}=13575\ kg./m^2 \text{ and } p_{min}=0$$

The bearing capacity of the soil being 20 $t/metre^2$ the maximum pressure is within the safe limit.

Design of the stem

Actual height of stem $=4{\cdot}1\ m.$

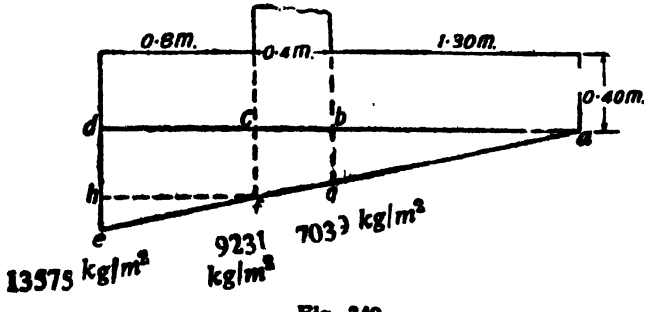

Fig. 340

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$\therefore$ Actual bending moment for the stem
$= 100(4{\cdot}1+0{\cdot}55)^2$ *kg. m.* $= 10060$ *kg. m.*

Effective depth $= 40-4 = 36$ *cm.*

$$\therefore \quad A_t = \frac{10060 \times 100}{1400 \times 0{\cdot}87 \times 36} = 22{\cdot}9 \text{ cm.}^2$$

$\therefore$ Spacing of 20 *mm.* ϕ bars

$$= \frac{3{\cdot}14 \times 100}{22{\cdot}9} \text{ cm.}^2$$

$= 13{\cdot}7$ *cm.* say 13 *cms. c/c.*

Distribution steel $= \frac{0{\cdot}15}{100} \times 40 \times 100 \text{ cm.}^2$

$= 6 \text{ cm.}^2$

$\therefore$ Spacing of 8 *mm.* ϕ bars

$$= \frac{0{\cdot}5 \times 100}{6} \text{ cm.}^2 = 8{\cdot}3 \text{ cm. say } 8 \text{ cm.}$$

Since the distribution bars will be provided near both the faces, the spacing of the bars will be 16 cm. c/c near each face.

The bending moment being less at higher levels the main reinforcement may be decreased at higher levels. We have, at any depth x below the soil surface

$$A_t \propto M \propto (x+0{\cdot}55)^3$$

If A_{t1} and A_{t2} are the requirements of the reinforcements at sections x_1 *m.* and x_2 *m.* from the top, we have,

$$\frac{A_{t1}}{A_{t2}} = \frac{(x_1+0{\cdot}55)^3}{(x_2+0{\cdot}55)^3}$$

If x_1 be the depth at which alternate bars of the stem can be curtailed, we have,

$$\frac{A_{t1}}{A_{t2}} = \frac{(x_1+0{\cdot}55)^3}{(4{\cdot}1+0{\cdot}55)^3}$$

$\therefore \quad x_1 = 3{\cdot}10$ *metre* from the top.

Hence alternate bars of the stem may be curtailed at

$4{\cdot}10-3{\cdot}10 = 1$ *m.* above the base slab.

Design of the toe slab

This is subjected to an upward load due to soil reaction. The maximum bending moment for the toe slab

= Moment of the soil reaction acting on the toe slab
− moment of self-weight of the toe slab.

The B.M. calculations for the toe slab are given in the table on page 551.

$$\therefore \quad A_t = \frac{3555 \times 100}{1400 \times 0{\cdot}87 \times 34} = 8{\cdot}60 \text{ cm.}^2$$

Provide 12 mm. bars @ 12 cm c/c.

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Load due to	Magnitude of load (kg.)	Distance from C (m.)	Moment about C (kg. m.)
Upward pressure			
cdhf 9231 × 0·8	7385	0·4	2954
efh 4344 × $\frac{0·8}{2}$	1738	$\frac{2}{3}$ × 0·8 = 0·53	921
			3875
Deduct for self-weight of toe slab			
0·8 × 0·4 × 2500	800	0·4	320
Actual B.M. for Toe Slab			3555

Load due to	Magnitude of load (kg)	Distance from b (m.)	Moment about b (kg. m.)
Backing	9594	0·65	6236
Traffic load	1300	0·65	845
Weight of heel slab			
1·30 × 0·4 × 2500	1300	0·65	845
Total			7926
Deduct for upward pressure *abg*			
$\frac{1·3}{2}$ × 7059	4588	$\frac{1·3}{3}$	1988
B M. for Heel Slab			5938

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Design of the heel slab

The loading on the heel slab consists of the following :

(*a*) Self-weight of the slab. (*b*) Weight of soil on the heel slab.

(*c*) Traffic load. (*d*) Upward soil reaction.

The bending moment due to the above loading is calculated as shown in the table shown on page 551.

$$A_t = \frac{5938 \times 100}{1400 \times 0{\cdot}87 \times 34} \ cm.^2$$

$$= 14{\cdot}33 \ cm.^2$$

$\therefore$ Spacing of 20 *mm.* diameter bars

$$= \frac{3{\cdot}14 \times 100}{14{\cdot}33} \ cm.$$

$$= 21 \ cm. \ say \ 20 \ cm.$$

But we find that the spacing of the 20 *mm* diameter bars for the stem is 13 *cms. c/c.* In order to match the spacing of bars in the stem and heel slab, it may be convenient to provide in the stem a spacing of $\frac{20}{2} = 10$ *cm. c/c.*

Check for sliding

Total horizontal earth pressure

$$P = \frac{1800 \times (4{\cdot}5 + 0{\cdot}55)^2}{} \ \frac{1 - \sin 30}{1 + \sin 30} \ kg.$$

(approximately)

$$= 7650 \ kg.$$

Maximum available friction

$$= \mu W$$

$$= 0{\cdot}5 \times 16969 \ kg.$$

$$= 8485 \ kg.$$

Factor of safety against sliding

$$= \frac{8485}{7650} = 1{\cdot}10$$

But it is absolutely necessary to provide a factor of safety of at least 1·5 against sliding.

Hence, the wall should be safe for a lateral pressure of 1·5 *P*

$$= 1{\cdot}5 \times 7650 = 11475 \ kg.$$

Available friction $= 8485 \ kg.$

$\therefore$ Unbalanced force $= 2990 \ kg.$

Safe bearing pressure for lateral loads may be taken at 0·7 times the safe bearing pressure for vertical loads.

But safe bearing pressure for vertical loads = 20 *tonnes/metre*^{2}

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$\therefore$ Safe horizontal pressure
$= 0{\cdot}7 \times 20 = 14$ *tonnes/metre*^{2}
$= 14000$ *kg./metre*^{2}.

Let the height of the key be *y metre*.

$\therefore \quad 14000 \times y = 2990$

$\therefore \quad y = \dfrac{2990}{14000}$ *m.*

$= 0{\cdot}21$ *m.*

It is usual to provide a key of height at least 30 *cm.* Hence we will provide a key of height 30 *cm.*

$\therefore$ Max B.M.

$= 2990 \times \dfrac{30}{2}$ *kg. cm.*

$= 44850$ *kg. cm.*

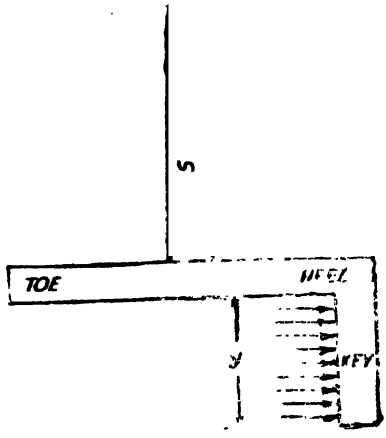
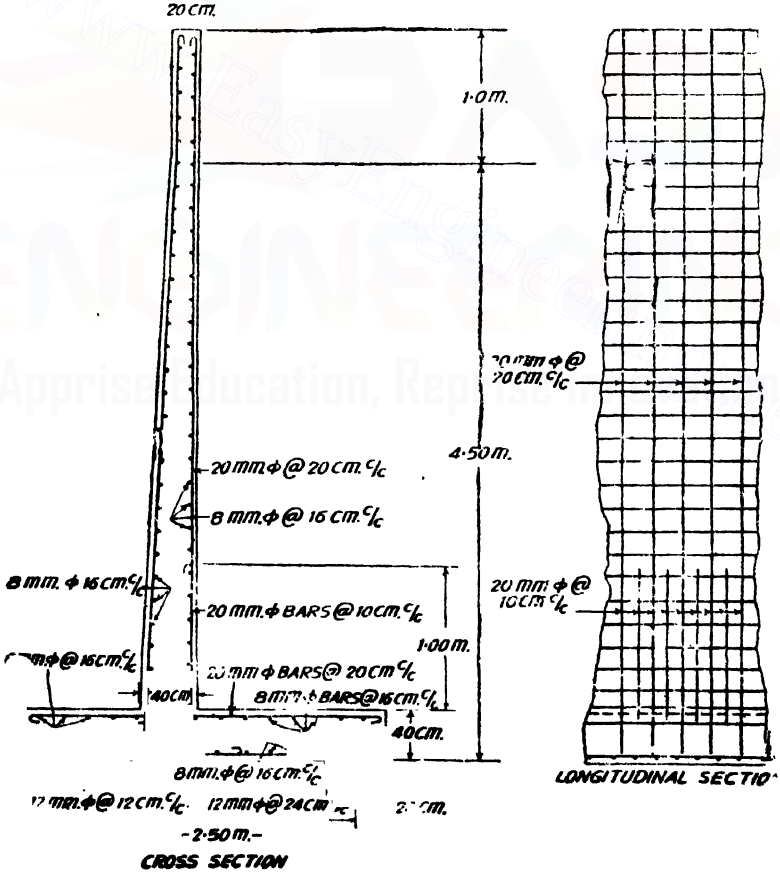


Fig. 341



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Equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}50 \times 100\, d^2 = 44850 \quad \ldots \quad d = 7{\cdot}27 \text{ cm.}$$

A key should be atleast 20 *cm.* in thickness.

Effective depth $= 20 - 6 = 14$ *cm.*

$$A_s = \frac{44850}{1400 \times 0{\cdot}87 \times 14} \text{ cm.}^2$$

$$= 2{\cdot}74 \text{ cm.}^2$$

Thus, we find that the key requires only a nominal amount of reinforcement. Alternate bars of the toe reinforcement may be extended and bent down to form the reinforcement for the key.

Design 84 (S I.). *A reinforced concrete cantilever type retaining wall has a 6 metre tall stem retaining earth level with its top. The base of the retaining wall is 3 metres wide with a toe projection of 0·75 metre. The soil weighs 16000 N per cubic metre and has an angle of repose of 30°. Design the wall. Take* $\mu = 0{\cdot}70$, *Use M 15 concrete mix.*

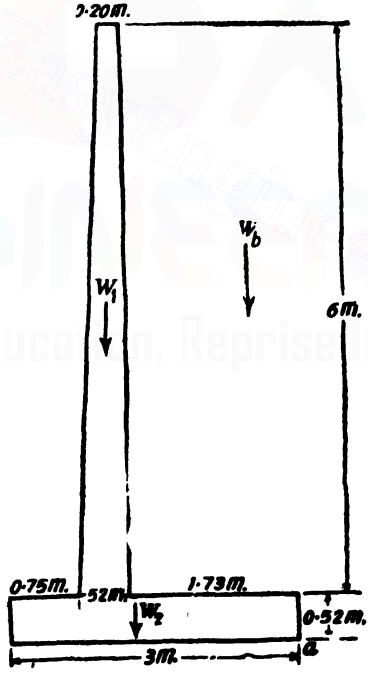

Fig. 343

Solution. Consider one *metre* run of the wall.

Maximum bending moment at the bottom of the stem

$$= \frac{wh^3}{6} \frac{1 - \sin \phi}{1 + \sin \phi}$$

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$$= \frac{16000 \times 6^3}{6} \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = 192000 \text{ Nm.}$$

Adopting $c = 5$ N/mm.^{2} $= 500$ N/cm.^{2}; $t = 140$ N/mm.^{2} $= 14000$ N/cm.^{2} and $m = 18$, and equating the moment of resistance to the bending moment, we have

$$85\, bd^2 = 85 \times 100\, d^2 = 192000 \times 100$$

$$d = 47{\cdot}50 \text{ cm.}$$

Providing an effective cover of 4 cm., the overall depth required

$$= 47{\cdot}5 + 4 = 51{\cdot}50 \text{ cm. say } 52 \text{ cm.}$$

Let the thickness of the base slab also be made 52 cm.

Let the top width of the stem be 20 cm.

Stability calculations

Consider one *metre* run of the wall. Loads on the walls and their moments about the heel end *a* are determined in the table below.

Stability Calculations for one metre Run of the Wall

Load due to	*Magnitude of load (N)*	*Distance from a (m.)*	*Moment about a (Nm.)*
W_1 $0{\cdot}20 \times 6{\cdot}0 \times 25000$	30000	1·83	54900
$\frac{0{\cdot}32 \times 6}{2} \times 25000$	24000	2·04	48900
W_2 $3 \times 0{\cdot}52 \times 25000$	39000	1·50	58500
W_b $1{\cdot}73 \times 6 \times 16000$	166080	0·865	143660
Moment of lateral earth pressure $16000 \times \frac{6{\cdot}52^2}{6} \times \frac{1}{3}$			246370
Total	259080		552390

Distance of the point of application of the resultant force from *a*

$$= z = \frac{552390}{259080} = 2{\cdot}13 \text{ metre}$$

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Eccentricity $e = z - \frac{b}{2} = 2{\cdot}13 - 1{\cdot}50 = 0{\cdot}63$ *metre*

But, $\frac{b}{6} = \frac{3}{6} = 0{\cdot}50$ *metre*

$\therefore \quad e > \frac{b}{6}$

Hence the pressure distribution at the base is *not* given by

$$\frac{W}{b}\left[1 \pm \frac{6e}{b}\right]$$

The actual pressure distribution for the base will follow the pattern shown in Fig. 344.

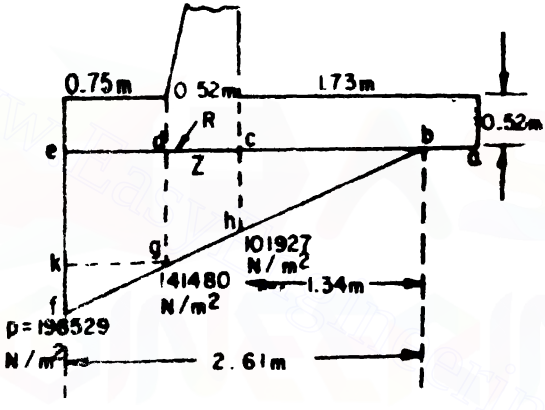

Fig. 344

Let Z be the point where the resultant R meets the base.

We have, $aZ = 2{\cdot}13$ *m*

$\therefore \quad eZ = 3 - 2{\cdot}13 = 0{\cdot}87$ *m.*

Width of the pressure triangle

$$= eb = 0{\cdot}87 \times 3 = 2{\cdot}61 \text{ } m.$$

Let the maximum pressure be p *N/metre*^{2}

Equating the upward and downward forces, we have,

$$\frac{1}{2} p \times 2{\cdot}61 = 259080$$

$\therefore \quad p = 198529$ *N/m.*^{2}

Design of the stem

Maximum bending moment $= 192000 \times 100$ *N cm.*

$$A_t = \frac{192000 \times 100}{14000 \times 0{\cdot}87 \times 48} = 32{\cdot}84 \text{ } cm.^2$$

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Spacing of 22 *mm.* ϕ bars

$$= \frac{3{\cdot}80 \times 100}{32{\cdot}84} = 11{\cdot}57 \text{ cm.}$$

Provide 22 mm ϕ *bars @ 11 cm. c/c.*

Distribution steel $= \frac{0{\cdot}15}{100} \times 52 \times 100 = 7{\cdot}80 \text{ cm.}^2$

Spacing of 8 *mm.* ϕ bars

$$= \frac{0{\cdot}5 \times 100}{7{\cdot}8} = 6{\cdot}4 \text{ cm. say } 6 \text{ cm.}$$

Since the distribution bars will be provided near both the faces the spacing of the bars will be 12 cm. c/c near each face.

Design of the toe slab

The bending moment for a 1 *metre* wide strip of the toe slab is calculated in the following table :

Load due to	*Magnitude of the load (N)*	*Distance from d (m.)*	*Moment about d (Nm.)*
Upward pressure *edgk*			
$141480 \times 0{\cdot}75$	106110	0·375	39791
Upward pressure *fgk*			
$0{\cdot}75 \times \frac{57049}{2}$	21393	0·50	10697
			50488
Deduct for self-weight of toe slab			
$0{\cdot}75 \times 0{\cdot}52 \times 25000$	9750	0·375	3656
B.M. for toe slab			46832

Let an effective cover of 6 *cm.* be provided to the reinforcement.

$\therefore$ Effective depth $= 52 - 6 = 46$ *cm.*

$\therefore$ $$A_s = \frac{46832 \times 100}{14000 \times 0{\cdot}87 \times 46} = 8{\cdot}35 \text{ cm.}^2$$

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Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{8{\cdot}35} = 13{\cdot}5 \text{ cm}$$

Provide 12 mm. ϕ *bars @ 12 cm. c/c.*

Design of the heel slab. The bending moment for a *metre* strip of the heel slab is calculated in the following table :

Load due to	*Magnitude of load (N)*	*Distance from c (m.)*	*Moment about c (Nm.)*
Weight of backing $1{\cdot}73 \times 6 \times 16000$	166080	0·865	143660
Weight of heel slab $1{\cdot}73 \times 0{\cdot}2 \times 25000$	22490	0·865	19454
			163114
Deduct for upward pressure *bch* $1{\cdot}34 \times \frac{101927}{2}$	68291	$\frac{1{\cdot}34}{3}$	30503
∴ Maximum bending moment for heel slab			132611

$$A_s = \frac{132611 \times 100}{14000 \times 0{\cdot}87 \times 46} = 23{\cdot}67 \text{ cm.}^2$$

Spacing of 22 *mm.* ϕ bars

$$= \frac{3{\cdot}80 \times 100}{23{\cdot}67} = 16 \text{ cm.}$$

Provide 22 mm. ϕ *bars @ 15 cm. c/c.*

Check for sliding. Total horizontal soil pressure

$$= P = 16000 \times \frac{6{\cdot}52^2}{2} \times \frac{1}{3} = 113361 \text{ N}$$

Maximum limiting friction

$$= \mu W = 0{\cdot}7 \times 259080 = 181356 \text{ N}$$

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∴ Factor of safety against sliding

$$= \frac{\mu W}{P} = \frac{181356}{13361} = 1{\cdot}60 \quad (\textit{greater than } 1{\cdot}5)$$

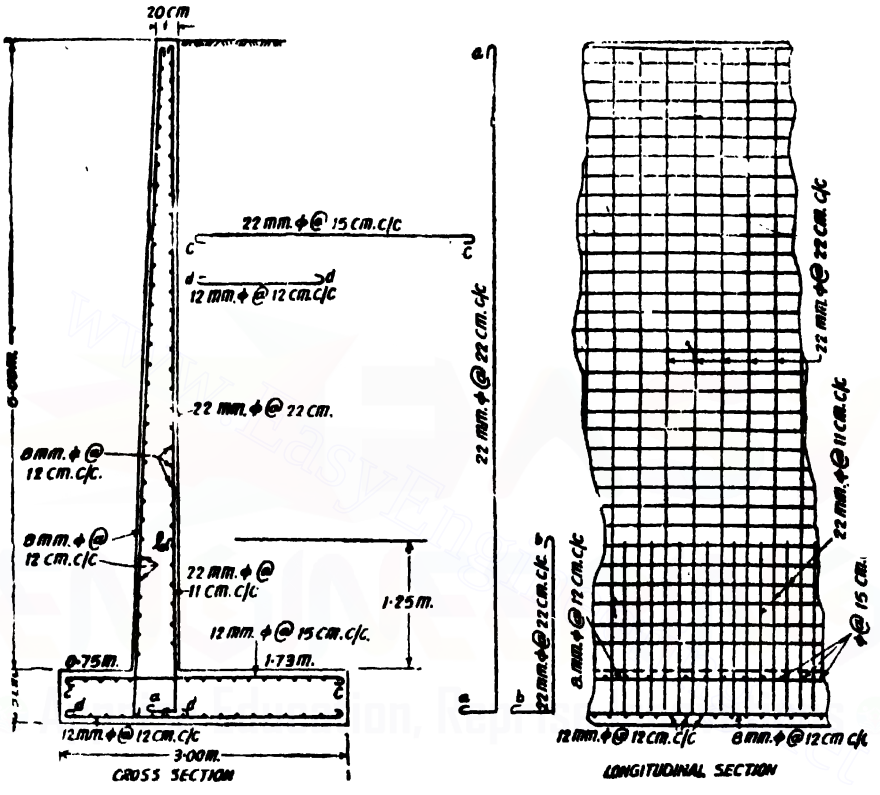

Fig. 345

§151. Surcharged Cantilever Retaining Wall

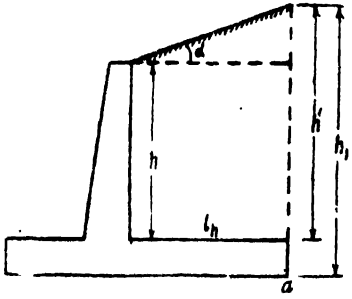

Fig. 346

Overall dimensions. The base width of such a wall may be taken at $0{\cdot}7^1H$ where H is the total height of the wall. Toe projection may be taken at one-third of the base width.

Earth pressure on the stem for unit run of the wall

$$= P = C_p \frac{wh^2}{2}$$

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where $C_p = \cos\alpha \cdot \dfrac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}$

P acts at $\frac{h}{3}$ above the base slab in a direction parallel to the earth surface.

Max. B.M. for the stem $= P \cos\alpha \cdot \frac{h}{3}$.

Hence the thickness of the stem may be determined to suit the above bending moment.

Stability calculation. As usual the maximum and minimum pressures at the base are determined. The total lateral earth pressure to be taken in this computation shall be at a vertical section through the edge a of the slab. If the vertical height of the earth's surface above a be h_1,

then $$P = C_p \frac{wh_1^2}{2}$$

P can be resolved into its vertical component P_v whose line of action passes through a and the horizontal component P_h acting at $\frac{h_1}{3}$ above a.

After determining the maximum and minimum pressures at the base, the toe can be designed as usual for the upward pressure on it.

But while designing the heel slab the following points should be considered.

The various loads to be considered on the heel are :

(*i*) Weight of the backing

(*ii*) Self weight of heel slab

(*iii*) *Vertical component of lateral earth pressure*

(*iv*) Upward pressure on the base.

The vertical component of lateral Earth Pressure on the Heel. Consider unit area of the heel slab.

Load on this unit area $= C_p\, wh \tan\alpha$ acting parallel to retained earth surface.

Hence the vertical component of this pressure

$$= C_p\, wh \tan\alpha \sin\alpha$$

Hence this will be an additional loading on the heel slab which is a trapezoidal load varying from $C_p\, wh \sin\alpha \tan\alpha$ at the end of the heel near the stem to $C_p\, wh' \sin\alpha \tan\alpha$ at the end of the heel remote from the stem.

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$\therefore$ B.M. due to this load

$$= \frac{wlh^2}{6}[h+2h']C_p \sin \alpha \tan \alpha$$

where l_h is the heel projection.

(Students may make a special note of this moment. It is observed that often this moment is ignored and the bending moment for the heel slab is calculated incorrectly).

Design 85 (S.I). *Design a cantilever type retaining wall to retain sand for 4 metres above the ground. The sand fill slopes at the rate of 1 vertical to 2 horizontal. The weight of sand is 18000 N/metre^3. the angle of repose being 30°. The safe bearing capacity of the soil is 200 kN/metre^2 at 1·25 metre below the ground level. Use M 15 concrete mix.*

Solution Total height of wall $= H = 4 + 1{\cdot}25 = 5{\cdot}25$ *metre*

Base width $= 0{\cdot}7\ H = 0{\cdot}7 \times 5{\cdot}25 = 3{\cdot}675$ *metre*

Provide a base width of 4 metre.

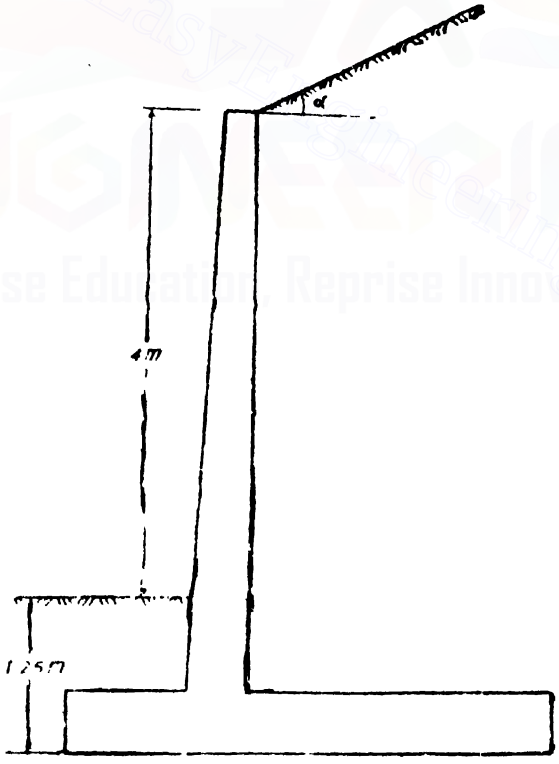

Fig. 347

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Toe projection = About one-third the base width = $\frac{4}{3} = 1{\cdot}33$ *m*

Provide a toe projection of 1 *metre*.

Consider one *metre* run of the wall.

Making an allowance of 30 *cm.* for the thickness of the base slab,

Height of the stem = 5·25 − 0·30 = 4·95 *metre*

Maximum bending moment for the stem

$$= M = C_p \frac{wh^3}{6} \cos \alpha$$

where $$C_p = \cos \alpha \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}}$$

In our case, $\tan \alpha = \frac{1}{2}$ $\therefore$ $\alpha = 26° 34'$

$$\cos \alpha = \frac{2}{\sqrt{5}} = 0{\cdot}8944$$

$$\cos^2 \alpha = \frac{4}{5} = 0{\cdot}80, \cos \phi = \cos 30° = \frac{\sqrt{3}}{2}$$

$$\cos^2 \phi = \frac{3}{4} = 0{\cdot}75$$

$$\therefore \quad C_p = 0{\cdot}8944 \frac{0{\cdot}8944 - \sqrt{0{\cdot}8000 - 0{\cdot}7500}}{0{\cdot}8944 + \sqrt{0{\cdot}8000 - 0{\cdot}7500}} = 0{\cdot}5368$$

$\therefore$ Maximum bending moment for the stem

$$= 0{\cdot}5368 \times 18000 \times \frac{4{\cdot}95^3}{6} \times 0{\cdot}8944 = 174695 \ Nm.$$

Adopting $c = 5$ *N/mm.*^{2} = 500 *N/cm.*^{2}, $t = 140$ *N/mm.*^{2} = 14000 *N/cm.*^{2}, and $m = 18$, and equating the moment of resistance to the maximum bending moment, we have,

$$85\, bd^2 = 85 \times 100\, d^2 = 174695 \times 100$$

$\therefore$ $d = 45{\cdot}3$ *cm.*

Providing an effective cover of 4 *cm.* to the reinforcement, in the stem, overall thickness = 45·3 + 4 = 49·3 *cm. say 50 cm.*

The base slab also will be made 50 *cm.* thick.

The top width of the stem may be made equal to 20 *cm.*

The overall dimensions of the wall are shown in Fig. 348.

Height of stem = $h = 4{\cdot}75$ *metre.*

Height of soil above the heel slab near the heel end

$$= h' = 4{\cdot}75 + \frac{2{\cdot}50}{2} = 6 \ metre.$$

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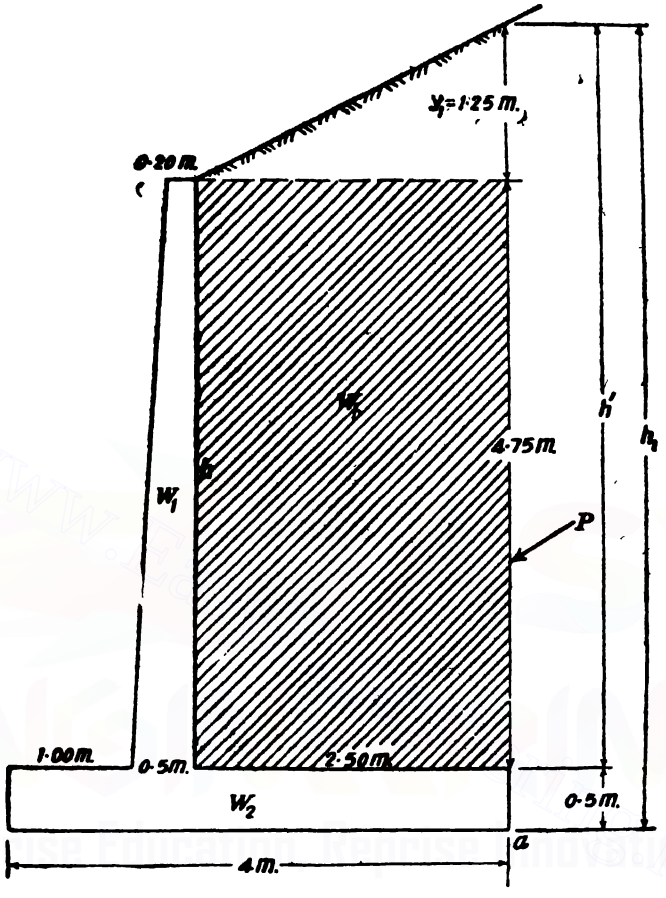

Fig. 348

Height of soil above the heel edge $a = h_1 = 6 + 0{\cdot}5 = 6{\cdot}50$ *metre.*

Total lateral pressure on the whole structure for one *metre* run of the wall

$$= P = C_a \frac{wh_1^2}{2} = \frac{0{\cdot}5368 \times 18000 \times 6{\cdot}5^2}{2} N$$

$$= 204118 \ N$$

Horizontal component of P

$$= P_h = P \cos \alpha = 204118 \times 0{\cdot}8944 = 182564 \ N$$

Vertical component of P

$$= P_v = P \sin \alpha = 204118 \times \frac{1}{\sqrt{5}} = 91282 \ N$$

P acts at $\frac{h_1}{3} = \frac{6{\cdot}50}{3}$ *metre* above *a*.

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Stability Calculations for one metre Run of the Wall

Load due to	*Magnitude of load (N)*	*Distance from a (m)*	*Moment about a (Nm)*
W_1 $0{\cdot}20\times4{\cdot}75\times25000$	23750	2·60	61750
$0{\cdot}30\times\frac{4{\cdot}75}{2}\times25000$	17813	2·80	49875
W_2 $4\times0{\cdot}5\times25000$	50000	2	100000
W_3 $2{\cdot}5G\times4{\cdot}75\times18000$	213750	1·25	267188
$\frac{2{\cdot}50\times1{\cdot}25}{2}\times18000$	28125	$\frac{2{\cdot}50}{3}$	23438
P_s	91282	0	0
Moment of P_h $182564\times\frac{6{\cdot}5}{3}$			395555
Total	424720		897806

$$z=\frac{897806}{424720}=2{\cdot}11 \text{ metre}$$

Eccentricity $$e=z-\frac{b}{2}=2{\cdot}11-2{\cdot}00=0{\cdot}11 \text{ metre}$$

But $$\frac{b}{6}=\frac{4}{6}=0{\cdot}67 \text{ metre}$$

$$e<\frac{b}{6}$$

∴ The extreme pressure intensity on the base

$$=\frac{W}{b}\left[1\pm\frac{6e}{b}\right]$$

$$=\frac{424720}{4}\left[1\pm\frac{6\times0{\cdot}11}{4}\right]$$

∴ $p_{max}=123700\ N/metre^2$ and

$p_{min}=88660\ N/metre^2$

Safe bearing capacity of the soil

$$=200\ kN/m^2=200{,}000\ N/metre^2$$

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Design of the stem

Max. B.M. = 174695 *Nm.*

(Note. Actually the B.M. for the stem is slightly less than this figure. The above value was arrived at taking the height of the stem as 4·95 *metre*.)

$$\therefore \quad A_s = \frac{174695 \times 100}{14000 \times 0{\cdot}87 \times 46} = 31{\cdot}20 \ cm.^2$$

Spacing of 20 *mm.* ϕ bars

$$= \frac{3{\cdot}14 \times 100}{31{\cdot}2} = 10{\cdot}06 \ cm.$$

Provide 20 mm ϕ *bars @ 10 cm. c/c.*

Distribution steel $= \frac{0{\cdot}15}{100} \times 50 \times 100 = 7{\cdot}50 \ cm.^2$

Spacing of 8 *mm.* ϕ bars

$$\frac{0{\cdot}5 \times 100}{7{\cdot}50} = 6{\cdot}67 \ cm. \ say \ 6{\cdot}50 \ cm.$$

Since the distribution steel will be provided near both the faces, the spacing of the bars will be at 13 cm. c/c near each face.

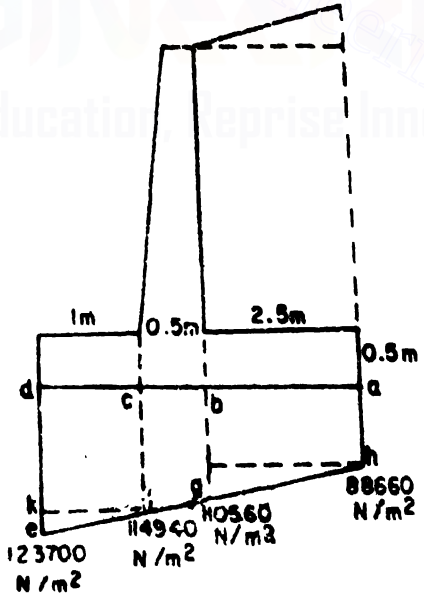

Fig. 349

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Design of the toe slab

The B.M. calculations for one metre strip of the toe slab are shown in the table below :

Load due to	*Magnitude of the load (N)*	*Distance from c (m)*	*Moment about c (Nm)*
Upward pressure *cdkf* 114940×1	114940	0·5	57470
Upward pressure *fke* $\frac{8760}{2} \times 1$	4380	$\frac{2}{3}$	2920
			60390
Deduct for self-weight of toe slab $1 \times 0{\cdot}5 \times 25000$	12500	0·5	6250
B.M. for toe slab			54140

$$A_s = \frac{54140 \times 100}{140000 \times 0{\cdot}87 \times 44} = 10{\cdot}1 \ cm.^2$$

Spacing of 16 *mm.* ϕ bars

$$= \frac{2{\cdot}01 \times 100}{10{\cdot}1} = 19{\cdot}9 \ cm.^2$$

Provide 16 mm. ϕ *bars @ 19 cm. c/c.*

Design of heel slab

Note. The soil resting on the heel slab due to its surcharge will exert additional pressure due to the vertical component of the lateral pressure. The additional moment

$$= \frac{wh^2}{6}[h+2h'] \ C_v \sin\alpha \tan\alpha \quad \text{(See page 561)}$$

$$= \frac{18000 \times 2{\cdot}50^2}{6}[4{\cdot}75+12] \times 0{\cdot}5368 \times \frac{1}{\sqrt{5}} \times \frac{1}{2} \ Nm.$$

$$= 37697 \ Nm.$$

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B.M. calculations for a one *metre* wide strip of the heel slab are shown in the table below :

Load due to	*Magnitude of the load (N)*	*Distance from b (m)*	*Moment about b (Nm)*
Weight of backing			
$2{\cdot}50 \times 4{\cdot}75 \times 18000$	213750	1·25	267188
$\frac{2{\cdot}50 \times 1{\cdot}25}{2} \times 18000$	28125	$\frac{2}{3} \times 2{\cdot}5$	46875
Weight of heel slab			
$2{\cdot}5 \times 0{\cdot}5 \times 2500$	31250	1·25	39063
Moment due to vertical component of lateral pressure			37697
			390823
Deduct for upward pressure			
abih $88660 \times 2{\cdot}5$	221650	1·25	277063
igh $21900 \times \frac{2{\cdot}5}{2}$	27375	$\frac{2{\cdot}50}{3}$	22813
Total deduction			299876
B.M. for heel slab			90947

$$A_s = \frac{90947 \times 100}{14000 \times 0{\cdot}87 \times 44} \text{ cm.}^2 = 17 \text{ cm.}^2$$

Provide 20 mm. ϕ @ 17 cm. c/c.

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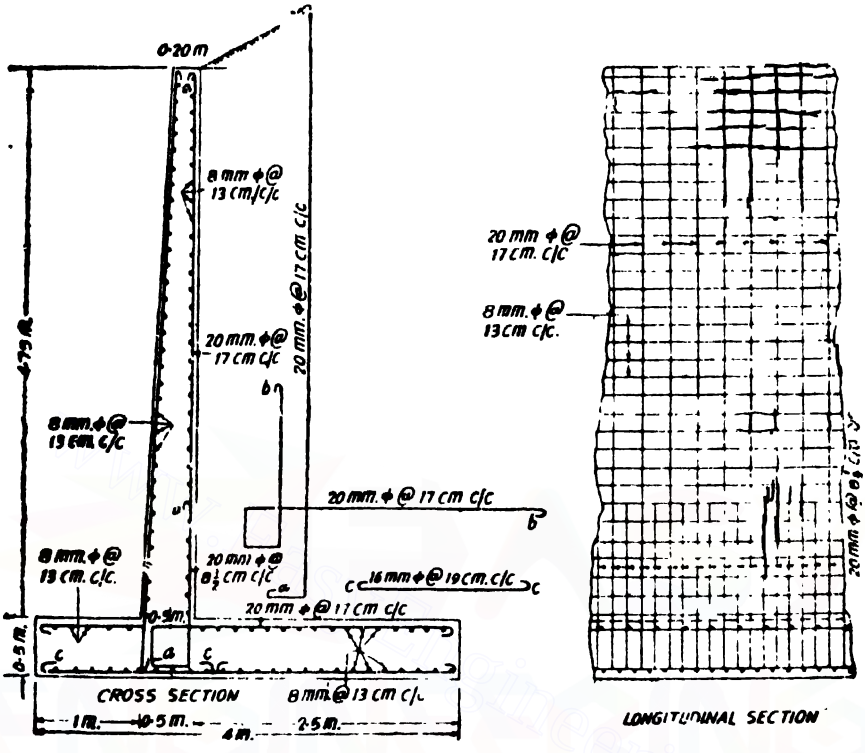

Fig. 350

Design 86. *Design a cantilever type retaining wall to retain sand for 4 metres above ground. The sand fill slopes at the rate of 1 vertical to 2 horizontal. The weight of sand is 1800 kg/metre³, the angle of repose being 30°. The safe bearing capacity of soil is 20 tonnes/metre² at 1·25 metres below ground level. Use M 150 concrete.*

Solution.

Total height of the wall

$$H = 4 + 1{\cdot}25 = 5{\cdot}25 \text{ m.}$$

$$\text{Base width} = 0{\cdot}7H = 0{\cdot}7 \times 5{\cdot}25 = 3{\cdot}675 \text{ m}$$

say 4 *m.*

Toe projection = about one-third base width = 1·33 *m.*

Let us provide a toe projection of 1 *m.*

Consider one *metre* run of the wall

Assuming the base slab to be 30 *cm.* thick,

Total lateral pressure on stem

$$= P = C_p \frac{wh^2}{2}$$

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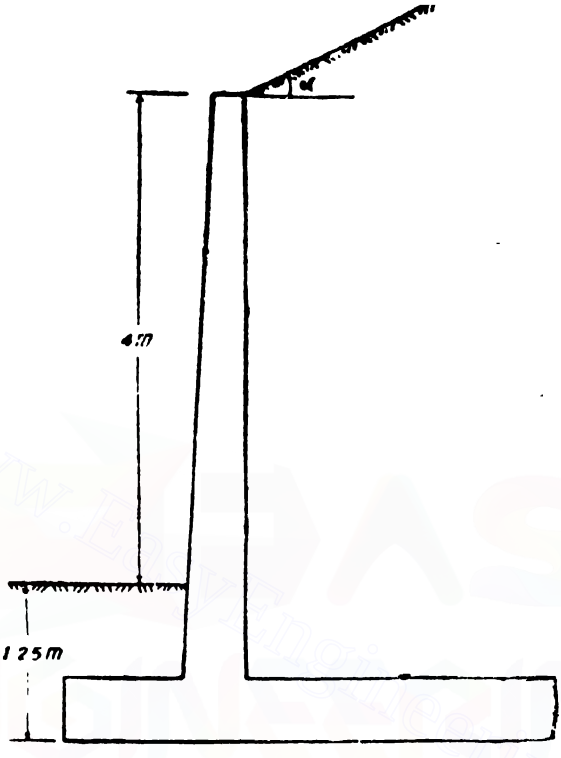

Fig. 351

where $$C_p = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}$$

In our case $\tan\alpha = \frac{1}{2}$

$\therefore$ $\alpha = 26^\circ\ 34'$

$\therefore$ $\cos\alpha = \frac{2}{\sqrt{5}} = 0.8944,$

$\cos^2\alpha = \frac{4}{5} = 0.8$

$\cos\phi = \cos 30^\circ = 0.866,\ \cos^2\phi = 0.75$

$\therefore$ $$C_p = 0.8944 \frac{0.8944 - \sqrt{0.8000 - 0.7500}}{0.8944 + \sqrt{0.8000 - 0.7500}}$$

$\therefore$ $C_p = 0.5368$

$\therefore$ Total lateral pressure on the stem

$$P = 0.5368 \times 1800 \times \frac{(5.25 - 0.30)^2}{2}\ kg$$

$= 11840\ kg.$

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Horizontal component of this pressure

$$= P \cos \alpha$$
$$= 11840 \times 0{\cdot}8944 = 10590 \text{ kg.}$$

$\therefore$ Maximum B.M. for the stem

$$= 10590 \times \frac{(5{\cdot}25 - 0{\cdot}30)}{3} \text{ kg. m}$$
$$= 17470 \text{ kg. m.}$$

Adopting $c = 50$ kg/cm.^{2} $t = 1400$ kg./cm.^{2} $m = 18$ and equating the moment of resistance to the maximum bending moment, we have

$$8{\cdot}50 \times 100\, d^2 = 17470 \times 100$$
$$\therefore \quad d = 45{\cdot}3 \text{ cm.}$$

Providing an effective cover of 4 cm. to the reinforcement in stem the overall thickness $= 45{\cdot}3 + 4 = 49{\cdot}3$ cm. say 50 cm.

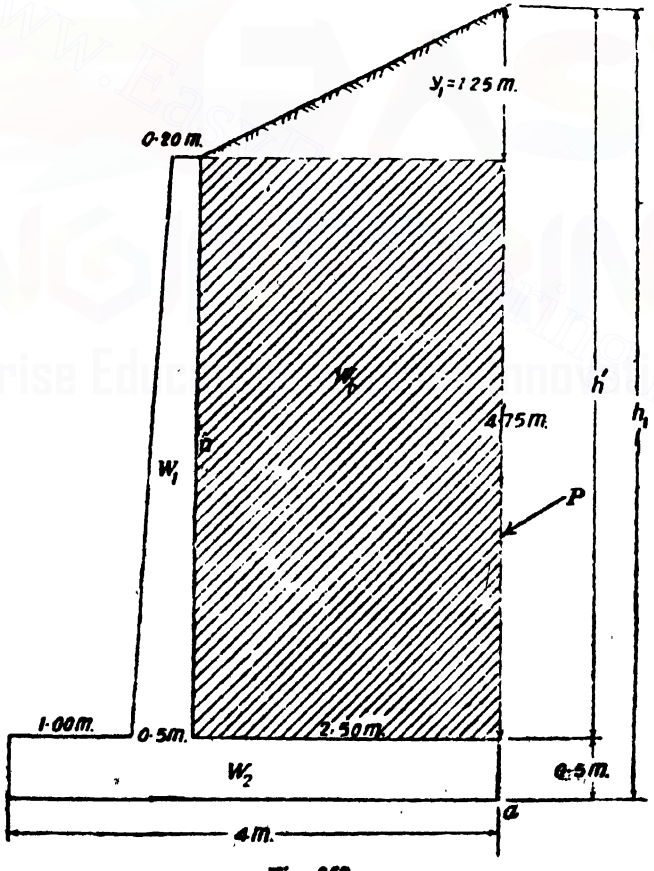

Fig. 352

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Stability Calculations for one metre Run of the Wall

Loads due to	*Magnitude of load (kg.)*	*Distance from a (m.)*	*Moment about a (kg.m.)*
W_1			
$0{\cdot}2\times4{\cdot}75\times2500$	2375	2·60	6175
$\frac{0{\cdot}3\times4{\cdot}75}{2}\times2500$	1781	2·80	4987
W_2			
$4\times0{\cdot}5\times2500$	5000	2·00	10000
W_3			
$2{\cdot}5\times4{\cdot}75\times1800$	21375	1·25	26719
$\frac{2{\cdot}5\times1{\cdot}25}{2}\times1800$	2813	$\frac{2{\cdot}50}{3}$	2344
P_v	9125	0	0
Moment of P_h			
$18250\times\frac{6{\cdot}5}{3}$			39542
Total	42469		89767

The base slab also may be made 50 *cm.* thick.
The top width of the stem may be made 20 *cm.*
The overall dimensions estimated have been shown in Fig. 352.
Height of soil above heel slab near the stem $=h=4{\cdot}75$ *m.*
Height of soil above heel slab near the heel end

$$=h'=4{\cdot}75+1{\cdot}25=6 \text{ m.}$$

$$h_1=6+0{\cdot}5=6{\cdot}5 \text{ m.}$$

Total lateral earth pressure on the whole structure for one *metre* run of the wall

$$=P=C_p\frac{wh_1^2}{2}$$

$$=\frac{0{\cdot}5368\times1800\times6{\cdot}5^2}{2} \text{ kg.}$$

$$=20410 \text{ kg.}$$

Horizontal component of P

$$=P_h=P\cos\alpha$$

$$=20410\times0{\cdot}8944 \text{ kg.}$$

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$= 18250\ kg.$

Vertical component of P

$= P_v = P \sin \alpha$

$= 20410 \times \frac{1}{\sqrt{5}}\ kg.$

$= 9125\ kg.$

P acts at $\frac{h_1}{3} = \frac{6.5}{3} = m$ above a

$\therefore \quad z = \frac{89767}{42103} = 2.11$

$\therefore$ eccentricity $= e = z - \frac{b}{2}$

$= 2.11 - 2.00 = 0.11\ m.$

But $\frac{b}{6} = \frac{4}{6} = 0.67\ m.$

$\therefore \quad e < \frac{b}{6}$

$\therefore$ The maximum and minimum pressures at the base are given by

$$p = \frac{W}{b}\left[1 \pm \frac{6e}{b}\right]$$

$$= \frac{42469}{4}\left[1 \pm \frac{6 \times 0.11}{4}\right] kg./m^2$$

$\therefore \quad p_{max} = 12370\ kg./m.^2$

and $\quad p_{min} = 8866\ kg./m.^2$

Design of Stem

Max. B.M. $= 17470\ kg.\ m.$

(**Note.** Actually the B M. for the stem is slightly less than this figure.

The above value was arrived at takir the height of stem as 4 95 $m.$ while estimating the thickness of stem at the bottom).

$\therefore \quad A_t = \frac{17470 \times 100}{1400 \times 0.87 \times 46}\ cm.$

$= 31.20\ cm.^2$

$\therefore$ Spacing of 20 $mm.$ diameter bars

$$\frac{3.14 \times 100}{31.2}\ cm.$$

$= 10.07\ cm.$ *say 10 cm. c/c.*

Distribution steel

$= \frac{0.15}{100} \times 50 \times 100\ cm.^2$

$= 7.50\ cm.^2$

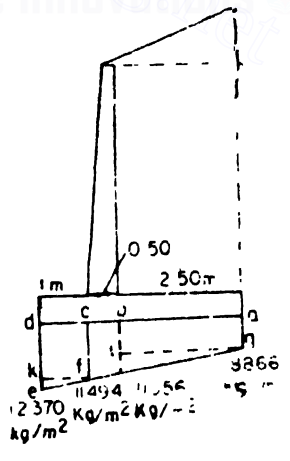

Fig. 353

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$\therefore$ Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{7{\cdot}5} \text{ cm.}$$

$$= 6{\cdot}67 \text{ cm. say } 6{\cdot}50 \text{ cm.}$$

Since the distribution steel will be provided near both the faces the spacing of the bars will be at 13 cm. c/c.

Design of the toe slab

The B.M. calculations for the toe slab are shown in table below :

Load due to	*Magnitude of load (kg.)*	*Distance from c (m.)*	*Moment about c (kg. m.)*
Upward pressure *cdkf* $= 11494 \times 1$	11494	$\frac{1}{2}$	5747
Upward pressure *fke* $\frac{876}{2} \times 1$	438	$\frac{2}{3}$	292
			6039
Deduct for self wt. $1 \times 0{\cdot}5 \times 2500$	1·50	$\frac{1}{2}$	625
$\therefore$ B.M. for toe slab			5414

Let an effective cover of 6 *cm.* be provided to the reinforcement of toe slab

$$\therefore \quad A_s = \frac{5414 \times 100}{1400 \times 0{\cdot}87 \times 44} \text{ cm.}^2$$

$$= 10{\cdot}1 \text{ cm.}^2$$

$\therefore$ Spacing of 16 *mm.* diameter bars

$$= \frac{2{\cdot}01 \times 100}{10{\cdot}1} \text{ cm.}$$

$$= 19 \text{ cm. c/c.}$$

Design of heel slab

Note. The soil resting on the heel slab due to its surcharge will exert additional pressure due to the vertical component of the lateral pressure. Hence there will be an additional moment due to the vertical component of the lateral pressure. This additional moment

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Load due to	*Magnitude of load (kg.)*	*Distance from b (m.)*	*Moment about b (kg. m.)*
Wt. of backing $2{\cdot}5\times4{\cdot}75\times1800$	21375	1·25	26719
$\frac{2{\cdot}5\times1{\cdot}25}{2}\times1800$	2813	$\frac{2}{3}\times2{\cdot}5$	4688
Wt. of heel slab $2{\cdot}5\times0{\cdot}5\times2500$	3125	1·25	3906
Moment due to vertical component of lateral pressure			3770
			39083
Deduct for upward pressure *abih* $8866\times2{\cdot}5$	22165	1·25	27706
tgh $2190\times\frac{2{\cdot}5}{2}$	2737·5	$\frac{2{\cdot}50}{3}$	2281
Total deduction			29987
∴ B.M. for heel slab			9096

$$=\frac{wh^2}{6}\,[h+2h']\,C_v \sin\alpha \tan\alpha$$

[See page 561]

In our case $h=4{\cdot}75$ *m.* and $h'=6$ *m.*

∴ Moment due to vertical component of lateral pressure

$$=\frac{1800\times2{\cdot}50^2}{6}\,[4{\cdot}75+12]\times0{\cdot}5{\cdot}68\,\frac{1}{\sqrt{5}}\times\frac{1}{2}\ kg.\ m.$$

$$=3770\ kg.\ m.$$

The B.M. calculations for the heel slab are shown in the table above.

∴ A_s for heel slab $=\frac{9096\times100}{1400\times0{\cdot}87\times4}\ cm.^2=17\ cm.^2$

Provide 20 mm. ϕ bars @ 17 cm. c/c.

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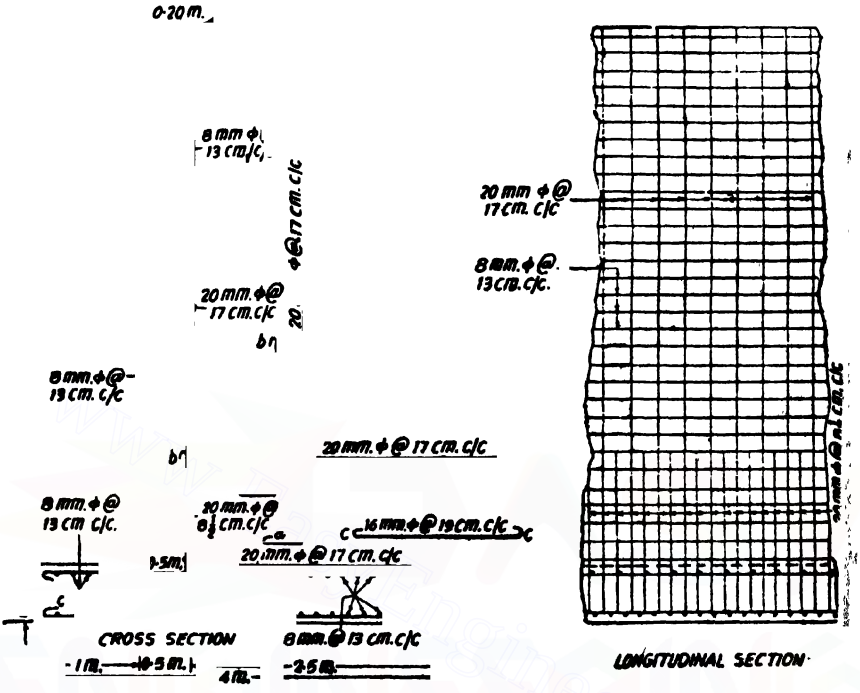

Fig. 354

§152. Cantilever Retaining Wall Without a Heel Projection

Sometimes due to certain boundary limitations a wall may have to be provided without a heel projection. In this case the dead load of the wall should be considerably large since no assistance is available due to the weight of the soil. The following points may be noted in this type of design :

(*i*) The thickness of the stem may be kept the same throughout the height to provide adequate dead weight.

(*ii*) The base slab may be made about 10 *cm.* thicker than the stem.

(*iii*) The width of the base slab may be kept about 0·7 times the total height of the wall.

(*iv*) It may most probably require a key to be provided to prevent the wall from sliding.

Design 87. *Design a cantilever retaining wall to the following requirements* :

(*a*) *Overall height of wall* = *5 m.*

(*b*) *Weight of soil* = *16000 N/cm.*

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(c) *Top surface of backing is level.*

(d) *Angle of repose of the soil* $= 30°$

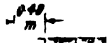
(e) *Foundation shall not project on the retained side.*

(f) *Safe bearing capacity of the soil* $= 200\ kN/metre^2$

(g) *Coefficient of friction* $\mu = 0{\cdot}45$.

Use M 15 concrete.

Solution.



Overall height of wall $= 5\ m$.

Base width $= b = 0{\cdot}7\ H = 0{\cdot}7 \times 5$
$= 3{\cdot}50\ m$.

Making an allowance of 50 *cm.* for the thickness of the base slab.

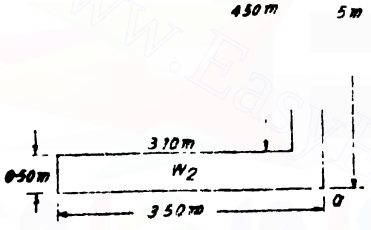


Fig. 355

Maximum bending moment for stem per *metre* run

$$M = C_p \frac{wh^3}{6}$$

$$= \frac{1}{3} \times 16000 \frac{(5-0{\cdot}5)^3}{6} \times 100\ N\ cm.$$

$$= 8100000\ N\ cm.$$

Equating the moment of resistance to the bending moment,

$85\ bd^2 = 85 \times 100\ d^2 = 8100000$

$\therefore \quad d = 30{\cdot}90\ cm.$

Effective cover $= 4\ cm.$

Total thickness required

$$= 30{\cdot}9 + 4 = 34{\cdot}9\ cm.$$

Provide a thickness of 40 *cm.* for the stem.

Let the base slab be 50 *cm.* thick.

Stability calculations

The relevant calculations for stability are made in the table below for one *metre* run of the wall.

$$z = \frac{196674}{88750} = 2{\cdot}22\ m.$$

Eccentricity $\quad e = z - \frac{b}{2} = 2{\cdot}22 - 1{\cdot}75 = 0{\cdot}47\ m.$

But $\quad \frac{b}{6} \quad \frac{3{\cdot}5}{6} = 0{\cdot}58\ m.$

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Stability Calculations for one metre Run of the Wall

Load due to	*Magnitude of the load (N)*	*Distance from a (m.)*	*Moment about a (Nm.)*
W_1 0·40×4·50×25000	45000	0·2	9,000
W_2 3·50×0·50×25000	43750	1·75	76563
Moment of lateral pressure $-C_p\frac{w\ s^2}{6}=\frac{1}{3}\times16000\times\frac{5^2}{6}$			111111
Total	88750		196674

$\therefore \quad e < \frac{b}{6}$

$\therefore$ Extreme pressure intensity on the base

$$= \frac{W}{b}\left[1 \pm \frac{6e}{b}\right] = \frac{88750}{3{\cdot}50}\left[1 \pm \frac{6\times0{\cdot}47}{3{\cdot}50}\right]$$

$$p_{max} = 45788 \ N/metre^2$$

and $$p_{min} = 4927 \ N/metre^2$$

Fig. 356 shows the pressure distribution on the base.

Design of the stem

Maximum bending moment per *metre* run

$= 8100000 \ N\ cm.$

Thickness of the stem

$= 40 \ cm.$

Effective depth

$= 40 - 4 = 36 \ cm.$

$$A_s = \frac{8100000}{14000\times0{\cdot}87\times36}$$

$18{\cdot}47 \ cm.^2$

Spacing of 18 *mm.* diameter bars

$$= \frac{2{\cdot}54\times100}{18{\cdot}47}$$

$= 13{\cdot}75 \ cm.$

Provide 18 mm. ϕ bars 13 cm. c/c.

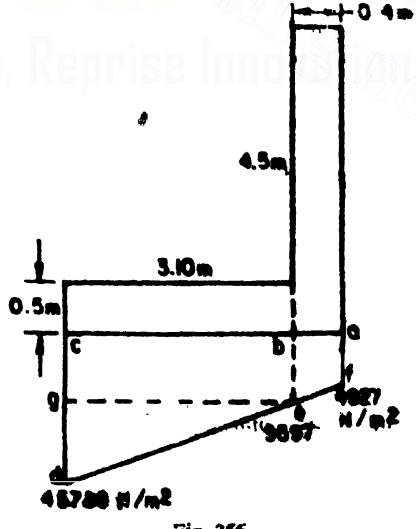

Fig. 356

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$$= \frac{0.15}{100} \times 40 \times 100 = 6 \text{ cm.}^2$$

Spacing of 8 mm. ϕ bars

$$= \frac{0.5 \times 100}{6} = 8 \text{ cm.}$$

Provide 8 mm. ϕ bars @ 16 cm. c/c near each face.

Design of the toe slab The bending moment for a one *metre* wide strip of the heel slab is calculated as shown in the table below

Load due to	*Magnitude of load (N)*	*Distance from a (m.)*	*Moment about a (Nm.)*
Upward pressure *base* 9397×3.10	29751	1.55	46114
due $\frac{1}{2} \times 3.10 \times 36191$	56096	$\frac{2}{3} \times 3.1$	115932
			162046
Deduct for self-weight of toe slab			
$0.5 \times 3.10 \times 25000$	38750	1.55	60063
	B.M. for Toe Slab		101983

depth available

$$= 50 - 6 = 44 \text{ cm.}$$

$$\therefore \quad A_s = \frac{101983 \times 100}{14000 \times 0.87 \times 44} = 19.03 \text{ cm.}^2$$

Spacing of 18 mm. ϕ bars

$$= \frac{2.54 \times 100}{19.03} = 13.3 \text{ cm.}$$

Provide 18 mm. ϕ bars @ 13 cm. c/c.

Distribution steel $= \frac{0.15}{100} \times 50 \times 100 = 7.50 \text{ cm.}^2$

Provide 8 mm ϕ bars @ 12 cm c/c near each face.

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Check for sliding

Total horizontal soil pressure per *metre* run of the wall

$$P = \frac{wH^2}{2} = \frac{1}{3} \times 16000 \times \frac{5^2}{2} N = 66667 \text{ N}.$$

Limiting frictional resistance

$$= \mu W = 0{\cdot}45 \times 88750 = 39938 \text{ N}.$$

Since this friction is inadequate we should provide a key.

Design of the key

Providing a factor of safety of 1·5 against sliding, the wall should be safe when the horizontal soil pressure is 1·5 *P*

$1{\cdot}5\,P = 1{\cdot}5 \times 66667 = 100000\ N$
Limiting friction $= 39938\ N$

$\therefore$ Unbalanced force $= 60062\ N$

This force has to be resisted by the key.

Bearing strength of soil for horizontal loading

$= 0{\cdot}7 \times$ Bearing capacity for vertical loading
$= 0{\cdot}7 \times 200 - 140\ kN/metre^2$
$= 140{,}000\ N/metre^2$

Let the height of the key be *y metres*

$\therefore \quad 140{,}000 \times y \times 1 = 60062 \quad \therefore \quad y = 0{\cdot}43$ *metre*

Provide a key of height 45 cm.

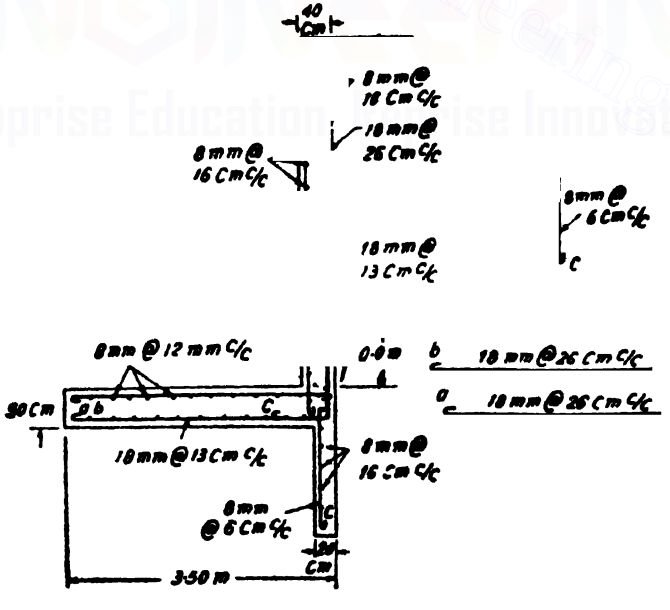

Fig. 357

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B.M. for the key $= 60062 \times \frac{45}{2}$ *N cm.*

$= 1351395$ *N cm.*

Equating the moment of resistance to the bending moment,

$85 \times 100\ d^2 = 1351395$

$\therefore \quad d = 12{\cdot}60$ *cm.*

Provide a 20 cm. thick key.

Effective depth available

$= 20 - 6 = 14$ *cm.*

$\therefore \quad A_s = \frac{1351395}{14000 \times 0{\cdot}87 \times 14} = 7{\cdot}90\ cm^2.$

Spacing of 8 *mm.* ϕ bars

$= \frac{0{\cdot}5 \times 100}{7{\cdot}90} = 6$ *cm.*

§153. Cantilever Retaining Wall Without a Toe Slab

This is another extreme case which arises due to boundary restriction. This type of wall may be designed as in the case of ordinary cantilever retaining wall. The base slab may be made about 10 *cm.* thicker than the stem. Width of the base slab may be made 0·5 to 0·6 times the overall height of wall.

Design 88. *Design a cantilever retaining wall to the following requirements :*

(*a*) *Overall height of wall* $= 5$ *m.*
(*b*) *Weight of soil* $= 16000$ *N/metre*^{3}
(*c*) *Top surface of backing is level*
(*d*) *Angle of repose of the soil* $= 30°$
(*e*) *Foundation shall not project on the front side.*
(*f*) *Safe bearing capacity of the soil* $= 200$ *kN/metre*^{2}.

Overall height of wall $= 5$ *m*

Base width $= 0{\cdot}5$ to $0{\cdot}6\ H$

$0{\cdot}5\ H = 0{\cdot}5 \times 5 = 2{\cdot}5$ *m.*

$0{\cdot}6\ H = 0{\cdot}6 \times 5 = 3$ *m.*

Provide a base width of 2·75 *m.*

Making an allowance of 50 *cm.* for the base slab, maximum bending moment for the stem per *metre* run

$= M = C_p \frac{wh^3}{6}$

$= \frac{1}{3}(16000)\ (5 - 0{\cdot}5)^3 \times 100$ *N cm.*

$= 8100000$ *N cm.*

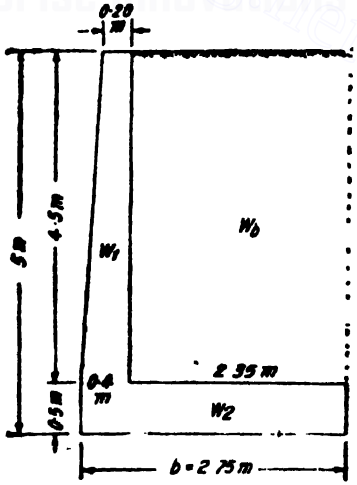

Fig. 358

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Equating the moment of resistance to the bending moment

$$85 \times 100\, d^2 = 8100000$$

$$\therefore \quad d = 30{\cdot}9 \text{ cm.}$$

Providing an effective cover of 4 *cm.*, overall thickness of the stem at the bottom $= 30{\cdot}9 + 4 = 34{\cdot}9$ *cm.*

Provide a thickness of 40 *cm.* at the bottom of the stem. Thickness at the top of the stem may be made equal to 20 *cm.*

The base slab will be 50 *cm.* thick.

Stability Calculations for one metre Run of the Wall

Load due to	*Magnitude of load (N)*	*Distance from a (m.)*	*Moment about a (Nm).*
W_1 $0{\cdot}2 \times 4{\cdot}5 \times 25000$	22500	2·45	55125
$\frac{1}{2} \times 0{\cdot}2 \times 4{\cdot}5 \times 25000$	11250	2·62	29475
W_2 $2{\cdot}75 \times 0{\cdot}5 \times 25000$	34375	1·375	47265
W_b $2{\cdot}35 \times 4{\cdot}5 \times 16000$	169200	1·175	198810
Moment of lateral pressure $C_p \frac{wH^3}{6} = \frac{1}{3} \times 16000 \times \frac{5^3}{6}$			111111
Total	237325		441787

$$\therefore \quad z = \frac{441787}{237325} = 1{\cdot}86 \text{ m.}$$

$$e = z - \frac{b}{2} = 1{\cdot}86 - \frac{2{\cdot}75}{2} = 0{\cdot}48 \text{ m.}$$

But $$\frac{b}{6} = \frac{2{\cdot}75}{6} = 0{\cdot}46 \text{ m.}$$

Since $e > \frac{b}{6}$, the pressure distribution will follow the geometry hown in Fig. 359.

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Let p = maximum upward pressure intensity. Equating the upward force to the downward force

$\frac{1}{2}(2{\cdot}67)\,p = 237325$

$\therefore \quad p = 177772\ N/m^2.$

This is less than the bearing capacity of the soil (200 $kN/metre^2$)

Design of the stem

B.M. for the stem

$= 8100000\ N\ cm.$

$$A_s = \frac{8100000}{14000 \times 0{\cdot}87 \times 36}$$

$= 18{\cdot}47\ cm.^2$

Spacing of 16 *mm.* bars

$$= \frac{2{\cdot}01 \times 100}{18{\cdot}47} = 10{\cdot}9\ cm.$$

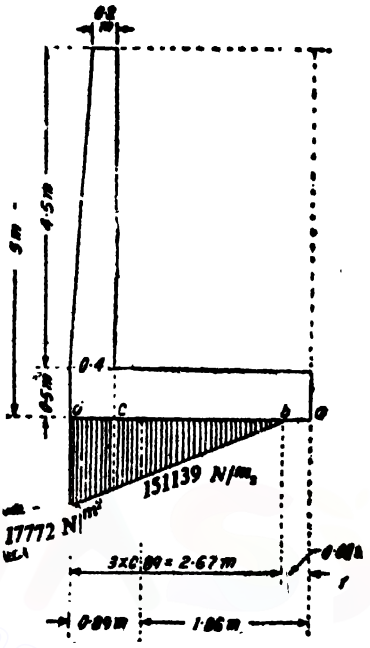

Fig. 359

Provide 16 mm. bars @ 10 cm c/c

Design of the heel slab

The bending moment calculations are shown in the table

Load due to	*Magnitude of load (N.)*	*Distance from c (m.)*	*Moment about c (N. m.)*
$2{\cdot}35 \times 4{\cdot}50 \times 16000$	169200	1·375	232650
Dead load of heel slab $2{\cdot}35 \times 0{\cdot}5 \times 25000$	29375	1·375	40390
			273040
Deduct for upward pressure $\frac{1}{2} \times 2{\cdot}27 \times 151139$	171543	$\frac{2{\cdot}27}{3}$	129800
B.M. for heel slab			143240

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Equating the M.R. to the B.M.

$$85 \times 100\, d^2 = 143240 \times 100$$

$$\therefore \quad d = 41{\cdot}1 \text{ cm.}$$

Available effective depth

$$= 50 - 6 = 44 \text{ cm.}$$

$$\therefore \quad A_s = \frac{143240 \times 100}{14000 \times 0{\cdot}87 \times 44} = 26{\cdot}7 \text{ cm.}^2$$

Spacing of 16 mm. bars

$$= \frac{2{\cdot}01 \times 100}{26{\cdot}7} = 7{\cdot}5 \text{ cm. c/c.}$$

Distribution steel

For stem : $\frac{0{\cdot}50}{100} \times 40 \times 100 = 6 \text{ cm.}^2$

Provide 8 mm. @ 16 cm. c/c near each face.

For heel slab : $\frac{0{\cdot}15}{100} \times 50 \times 100 = 7{\cdot}5 \text{ cm.}^2$

Provide 8 mm. @ 12 cm. c/c near each face.

Check for sliding

Horizontal soil pressure

$$P = c_p \frac{wH^2}{2} = \frac{1}{3} \times 16000 \times \frac{5^2}{2}$$

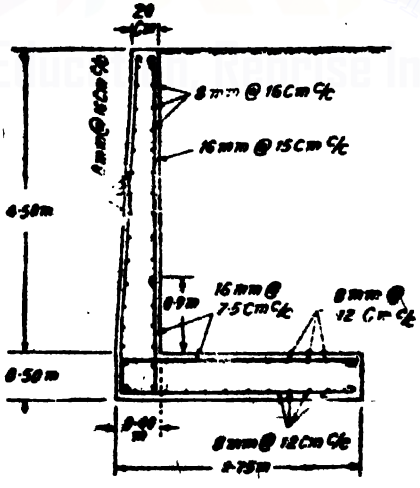

Fig. 360

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$= 66667\ N/metre^2$

Assuming $\mu = 0{\cdot}5$

Max. limiting friction

$$= \mu W = 0{\cdot}5 \times 237325 = 118662\ N$$

$\therefore$ Factor of safety against sliding

$$= \frac{\mu W}{P} = \frac{118662}{66667} = 1{\cdot}78\ (greater\ than\ 1{\cdot}5)$$

§154. Counterfort Retaining Walls

Retaining walls over 5·5 *metres* in height are usually made of the counterfort type. The various components of such a wall are shown in Fig. 361.

(*i*) **Upright slab.** The upright slab will be designed as a continuous slab spanning on the counterforts and subjected to lateral earth pressure. Let the lateral, horizontal pressure intensity at the bottom of the upright slab be *p per sq. metre*. Consider the bottom 1 *metre* deep strip of the upright slab. If the spacing of counterforts be *l metres* centre to centre then the maximum bending moment for the upright slab $= \pm \frac{pl^2}{12}$. The thickness required to

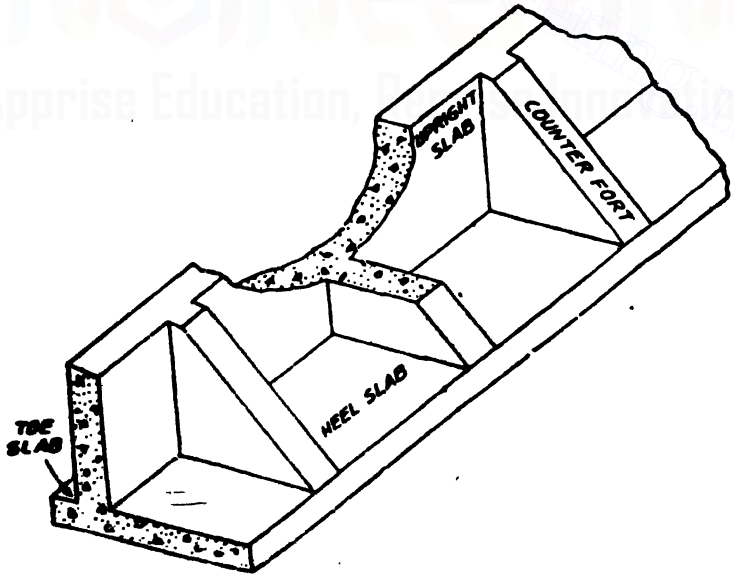

Fig. 361. The counterfort wall.

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suit this bending moment may now be computed. The slab is usually built of the same thickness. The main reinforcement requirement may now be calculated. This steel runs horizontally, its requirement being away from the earth side at sections mid-way between the cou .terforts and near the earth side at the sections on the counterforts.

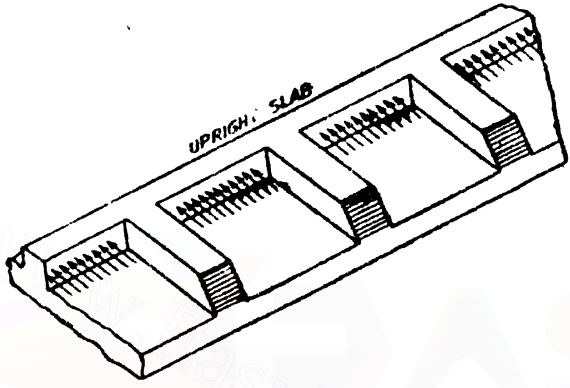

Fig. 362 Upright s ab subjected to lateral loading.

The slab shall also be provided with distribution steel at not less than 0·15% of the gross area of the section. The distribution steel is placed vertically near both the faces, since the upright slab is considerably thick. These bars should form a mesh with the horizontal bars. Hence these bars will have to be supported by additional horizontal bars at certain places where main horizontal bars are not forming a mesh with them.

(*ii*) **The base slab.** The width of the base slab may be made 0·6 *H* to 0·7 *H* where *H* is the overall height of the retaining wall. The base slab consists of the toe slab and the heel slab. The toe projection is usually one-fourth of the total width of the base slab.

Heel slab. The heel slab should be designed as a continuous horizontal slab with the counterforts as the supports. The slab is designed as a continuous slab consisting of cont'nuous strips parallel to the wall.

Each strip is uniformly loaded ; but the loading on the various strips varies from a maximum at the heel edge to a minimum near the wall.

The loading on a strip of heel slab will consist of the following :

(*a*) **Dead load of the strip**

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(*b*) Weight of earth above the strip

(*c*) Vertical component of lateral pressure in the case of earth surcharged at an angle. If the surcharge angle is α, then the intensity of vertical component of lateral pressure

$$= C_p wh' \sin \alpha \tan \alpha$$

where

$$C_p = \cos \alpha \cdot \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}}$$

h' = height of earth above the strip

ϕ = angle of repose.

(*d*) Superload intensity acting on the retained soil if any

(*e*) Upward soil pressure.

It will be seen that the net load on the heel slab will be a downward load. If the net load be Q per unit area near the heel end, then consider a one *metre* wide strip near the heel end. The maximum bending moment for the strip $= \pm \frac{Ql^2}{12}$. The moment will be a sagging moment at sections midway between the counterforts and will be a hogging moment at the sections over the supports.

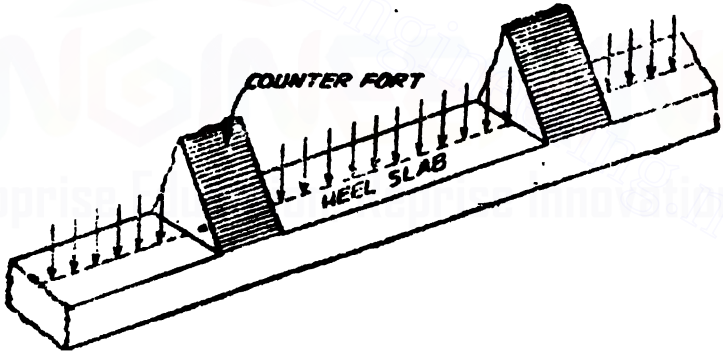

Fig. 36.1

Thickness of the base slab. The author suggests that this may be taken not less than the following, in order it may not be found unsafe from B.M. and S.F. considerations.

$$D = 4{\cdot}17\, l \sqrt{H}$$

$$D = 2\, l H$$

where

D = thickness of the base slab in *cm.*

l = spacing of counterforts in *metres*

H = overall height of wall in *metres.*

If the soil is surcharged at angle α, increase H by 0·7 *m.*

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If the soil is surcharged increase H by $\frac{\text{surcharge load intensity}}{\text{wt. per unit volume of the soil}}$

Counterforts are spaced from 3 *metres* to 3·50 *metres*. This spacing may also be taken from one third the height of the wall to half the height of the wall.

The spacing may also be computed as the spacing for which the maximum bending moment for the upright slab requires an overall thickness of 30 *cm*.

Let the spacing be l *metres*. Let the height of upright slab be h *metre*.

In M.K.S. units, to satisfy the above condition,

$$l = \frac{454 \cdot 8}{\sqrt{wh}} \text{ metre for } M \text{ 150 concrete.}$$

In S.I. units,

$$l = \frac{1438 \cdot 25}{\sqrt{wh}} \text{ metre for } M \text{ 15 concrete.}$$

Toe slab. The design of the toe slab depends upon whether the toe slab is allowed to remain a cantilever, or it is made to act as a continuous slab by providing front counterforts. When the front counterforts are not provided the toe slab should be designed as a cantilever slab subjected to upward soil reaction. But if a front counterfort be provided then, the toe slab shall be designed as a continuous slab with the front counterforts as the supports. In such a case at section midway between the front counterforts the bending moment for the toe slab will be of a hogging type, while at the section on the supports, the bending moment for the toe slab will be of the sagging type.

(*iii*) **Counterforts.** As mentioned already the retaining wall may have main counterforts or main counterforts and front counterforts.

Main counterforts. These are designed as vertical cantilevers held in position by the base slab. The loading on these counterforts is due to the lateral earth pressure acting on the upright slab.

Let h be the height of cantilever above the base.

l = spacing of counterforts

α = surcharge angle

Total horizontal force transferred to one counterfort

$$P_h = C_p \frac{wh^2}{2} l \cos \alpha \text{ acting at a height of}$$

$\frac{h}{3}$ above the base

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$\therefore$ Max. bending moment for the counterfort

$$= M = P_h \cdot \frac{h}{3}$$

$$\therefore \qquad M = C_p \frac{wh^3}{6} l \cos \alpha$$

The reinforcement required to resist this bending moment can be easily calculated.

$$A_t = \frac{M}{a\,t} \sec \beta$$

where β = inclination of the reinforcement with the normal to the horizontal section of counterfort (*i.e.*, inclination of the reinforcement with the vertical).

Curtailment of steel

We know $A_t \propto \frac{M}{a} \propto \frac{M}{d} \propto \frac{M}{h}$ approximately

$$\therefore \qquad A_t \propto \frac{h^3}{h}$$

$$\therefore \qquad A_t \propto h^2$$

If A_{t1} and A_{t2} are the areas of steel required at depths h_1 and h_2 we have $\frac{A_{t1}}{A_{t2}} = \frac{h_1^2}{h_2^2}$

But A_t is proportional to the number of bars.

Let n be the number of bars at the depth h

n_1 ,, ,, ,, ,, depth h_1

n_2 ,, ,, ,, ,, depth h_2

n_3 ,, ,, ,, ,, depth h_3

Then we have

$$\frac{n_1}{n} = \frac{h_1^2}{h^2} \qquad \ldots(1)$$

$$\frac{n_2}{n} = \frac{h_2^2}{h^2} \qquad \ldots(2)$$

$$\frac{n_3}{n} = \frac{h_3^2}{h^2} \qquad \ldots(3)$$

and so on.

Hence at what depth a certain number of bars can be curtailed, can be determined.

Front counterforts. These are designed as horizontal cantilevers. The loading on these will be due to the upward soil reaction on the toe slab. It is quite likely that the front counterfort will be subjected to considerable shear force. Hence shear stirrups of two or four legs be provided.

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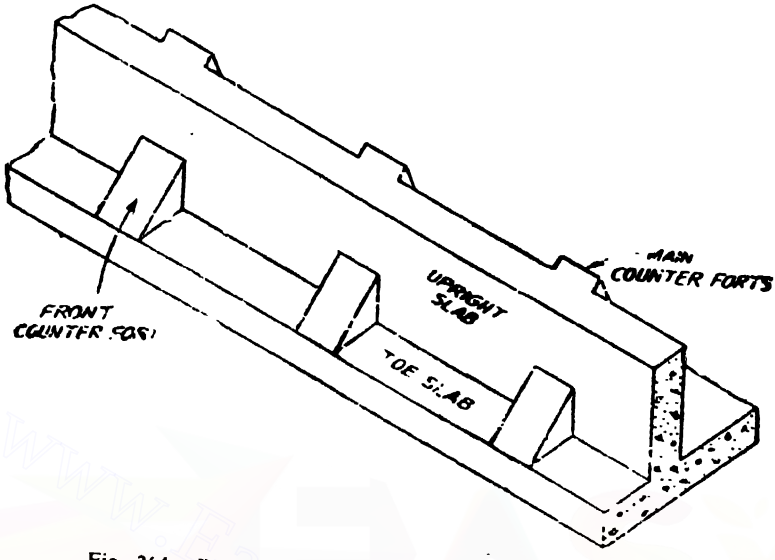
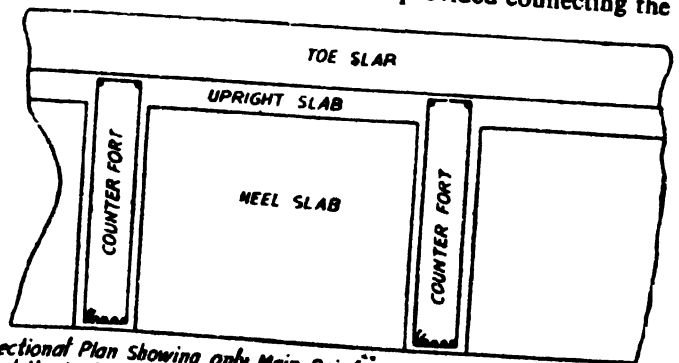


Fig. 364. Retaining wall withfront andmain counterfort.

The main reinforcement of the main counterfort and also that of the front counterfort should be embedded into the base slab for sufficient length to develop the necessary bond strength. The bars of the main counterfort should be securely anchored at the bottom by bending them back into the base slab.

In the case of a wall provided with main as well as front counterforts the critical section for the max. bending moment for the main counterforts shall be taken at a level corresponding to the top level of the front counterfort.

Horizontal ties connecting the main counterforts and upright slab. Horizontal links of two legs are provided connecting the main



Sectional Plan Showing only Main Reinforcement of the Counter Fort and the Horizontal Links, Connecting the Counter Fort and the upright Slab

Fig. 365

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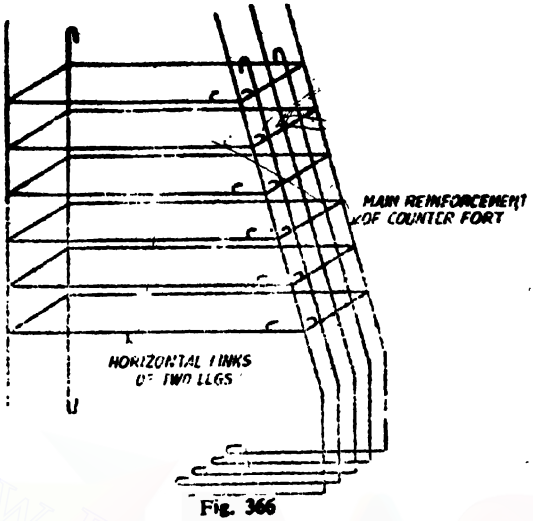

Fig. 366

counterfort and the upright slab to tie the wall to the counterfort and also to resist diagonal tension in the counterfort. These links must be looped around the main reinforcement of the counterfort. Fig. 364 shows the horizontal links and Figs. 365 and 366 show two alternative ways in which the horizontal links may be provided.

Vertical ties connecting the counterforts and the Heel slab. We know that the heel slab will transfer its load to the counterforts which are supporting them. In order that the heel slab may transfer its load to the counterfort, it is necessary to provide vertical ties which are in the form of vertical links of two legs.

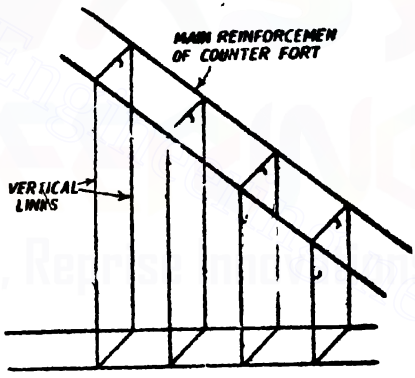

Fig. 367. Vertical links connecting the counterfort and the Heel slab.

Design 89 (S.I.). *Design a counterfort type retaining wall to th following particulars :*

(i) *Height of wall above G.L.* = 5·50 *metre*

(ii) *Safe bearing capacity of the soil* = 160 *kN/metre*^{2}

(iii) *Angle of repose* = 30°

(iv) *Weight of soil* = 16000 *N/metre*^{3}

(v) *Spacing of counterforts* = 3 *metres centres*

(vi) *Weight of R.C.C.* = 25000 *N/metre*^{3}

(vii) *Use M 15 concrete.*

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Solution. Depth of foundation

$$= \frac{p}{w}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]^2$$

$$= \frac{160000}{16000}\left(\frac{1}{3}\right)^2 = 1{\cdot}11\ m.$$

Let us provide a foundation depth of 1·20 *metre*.

Overall height of wall

$$= 5{\cdot}50 + 1{\cdot}20 = 6{\cdot}70\ metre$$

Thickness of the base slab

This shall be nearly

(*i*) $4{\cdot}17\ l\sqrt{H}\ cm. = 4{\cdot}17 \times 3\ \sqrt{6{\cdot}7} = 32{\cdot}38\ cm.$

(*ii*) $2\ l\ H\ cm. = 2 \times 3 \times 6{\cdot}7 = 40{\cdot}20\ cm.$

Let us provide 40 *cm.* overall for the base slab

∴ Actual height of the upright slab

$$= 6{\cdot}70 - 0{\cdot}40 = 6{\cdot}30\ metre$$

Pressure intensity at the bottom of the upright slab

$$= wh\ \frac{1-\sin\phi}{1+\sin\phi} = 16000 \times 6{\cdot}3 \times \frac{1}{3}$$

$$= 33600\ N/metre^2$$

Maximum bending moment for the upright slab for the bottom 1 *metre* strip

$$= M = 33600 \times \frac{3^2}{12} \times 100\ N\ cm.$$

$$= 2{,}520{,}000\ N\ cm.$$

Adopting $c = 5\ N/mm.^2 = 500\ N/cm.^2$; $t = 140\ N/mm.^2 = 14000\ N/cm.^2$; and $m = 18$, and equating the moment of resistance to the bending moment

$$85\ bd^2 = 85 \times 100\ d^2 = 2520000$$

∴ $d = 17\ cm.$

Effective cover $= 4\ cm.$

∴ Overall depth required

$$= 17 + 4 = 21\ cm.$$

The thickness of the upright slab will be kept uniformly at 21 *cms.*

Base width. This will be kept from 0·6 *H* to 0·7 *H*

$$0{\cdot}6 \times 6{\cdot}7 = 4{\cdot}02\ m.$$

$$0{\cdot}7 \times 6{\cdot}7 = 4{\cdot}69\ m.$$

Let us provide a base width of 4·50 *metre*

Toe projection. This will be kept at about one-fourth of the width of base

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In our case the toe projection

$$= \frac{4{\cdot}5}{4} = 1{\cdot}1 \text{ metre}$$

Let us provide a toe projection of 1 *metre*.

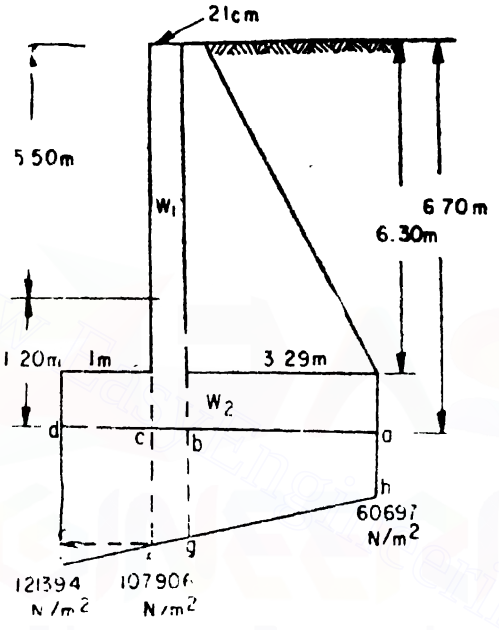

Fig. 368

Stability calculations. Stability calculations for 1 *metre* run of the wall are shown in the table on page 593.

∴ Distance of the point of application of the resultant from *a*

$$= z = \frac{1026420}{409707} = 2{\cdot}50 \text{ m}.$$

∴ Eccentricity $= e = z - \frac{b}{2} = 2{\cdot}50 - \frac{4{\cdot}50}{2} = 0{\cdot}25 \text{ m}.$

But, $\frac{b}{6} = \frac{4{\cdot}5}{6} = 0{\cdot}75 \text{ m}$ $\qquad e < \frac{b}{6}$

Extreme pressure intensity on the base

$$= \frac{W}{b}\left[1 \pm \frac{6e}{b}\right] = \frac{409707}{4{\cdot}5}\left[1 \pm \frac{6 \times 0{\cdot}25}{4{\cdot}5}\right]$$

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Stability Calculations for 1 Metre Run of Wall

Load due to	*Magnitude of the load (N)*	*Distance from a (m.)*	*Moment about a (Nm.)*
W_1 0·21×6·30×25000	33075	3·395	112290
W_2 4·50×0·4×25000	45000	2·25	101250
W_3 3·29×6·30×16000	331632	1·645	545535
Moment of earth pressure $C_p\frac{wH^3}{6}=\frac{1}{3}\times\frac{16000\times6.7^3}{6}$			267345
Total	409707		1026420

$\therefore$ $p_{max}=121394\ N/metre^2$

and $p_{min}=60697\ N/metre^2$.

Design of the upright slab

Maximum bending moment for a 1 *metre* deep strip

$=2520000\ N\ cm.$

Overall thickness $=21\ cm.$

Effective depth $d=21-4=17\ cm.$

$$A_t=\frac{2520000}{14000\times0.87\times17}=12.20\ cm.^2$$

Spacing of 16 *mm.* ϕ bars

$$=\frac{2.01\times100}{12.2}=16.47\ cm.$$

Provide 16 mm. ϕ bars @ 16 cm. c/c.

Distribution steel per *metre* run

$$\frac{0.15}{100}\times\times21\times100=3.15\ cm.^2$$

Spacing of 8 *mm* ϕ bars

$$\frac{0.5\times100}{3.15}=15\ cm.$$

If the distribution steel is provided near both the faces, then the spacing of 8 mm ϕ bars will be 30 cm. centres near each face.

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Design of the toe slab

R.M. calculation for 1 *metre* wide cantilevering strip of the toe slab are shown in the table below :

Load due to	*Magnitude of load* (*N*)	*Distance from c* (*m.*)	*Moment about c* (*Nm.*)
Upward pressure *cdlf* 107906×1	107906	0·5	53953
Upward pressure *efl* ½×1×13488	6744	⅔	4496
			58449
Deduct for self-weight of toe slab 1×0·4×25000	10000	0·5	5000
	B.M. for toe slab		53449

Hence maximum bending moment for the toe slab per *metre* width

$$= 53449 \ Nm.$$

Effective depth available

$$= d = 40 - 6 = 34 \ cm.$$

$$A_s = \frac{53449 \times 100}{14000 \times 0{\cdot}87 \times 34} = 12{\cdot}90 \ cm.^2$$

Spacing of 16 *mm.* ϕ bars

$$= \frac{2{\cdot}01 \times 100}{12{\cdot}90} = 15{\cdot}6 \ cm.$$

Provide 16 mm. ϕ *bars @ 15 cm. c/c.*

Distribution steel $= \frac{0{\cdot}15}{100} \times 100 \times 40 = 6 \ cm.^2$

Spacing of 8 *mm.* ϕ bars

$$= \frac{0{\cdot}15 \times 100}{6} = 8 \ cm.$$

If the distribution steel is provided near both the faces, then the spacing of 8 mm. ϕ *bars will be at 16 cm. centres near each face.*

Design of the heel slab

Consider a one *metre* wide strip of the heel slab near the heel end *a*.

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Weight of the strip $= 16000 \times 6{\cdot}3 = 100800\ N/metre^2$
D.L. of the strip $= 1 \times 0{\cdot}4 \times 25000 = 10000\ N/metre^2$

$110800\ N/metre^2$
Deduct for upward pressure $60697\ N/metre^2$

Net downward loading $50103\ N/metre^2$

Spacing of counterforts $= 3$ *metre*

$\therefore$ Maximum bending moment

$$= \frac{50103 \times 3^2}{12} \times 100\ N\ cm.$$

$$= 3757725\ N\ cm.$$

$$\therefore \quad A_t = \frac{3757725}{14000 \times 0{\cdot}87 \times 34} = 9\ cm.^2$$

Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{9} = 12{\cdot}5\ cm.$$

Provide 12 mm ϕ *@ 12 cm. c/c.*

Design of counterforts

It is proposed to provide for the counterfort a top width of *cm.* The thickness of the counterfort also may be made equal *2 cm.*

Total lateral earth pressure transferred to one counterfort ove the level of the base slab

$$= C_p \frac{wh^2}{2} \cdot l$$

$$= \frac{1}{3} \times \frac{16000 \times 6{\cdot}3^2}{2} \times 3\ N.$$

$$= 317520\ N$$

$\therefore$ Maximum bending moment for the counterfort

$$= 317520 \times \frac{6{\cdot}3}{3}\ Nm.$$

$$= 666792\ Nm$$

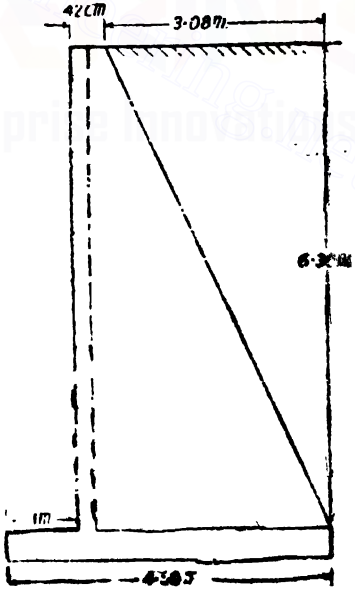

Fig. 369

Overall width of counterfort at the base

$= 3{\cdot}50\ metre = 350\ cm.$

Effective depth available

$= 350 - 6 = 344\ cm.$

Let the inclination of the main steel with the vertical be β

$$\tan \beta = \frac{3{\cdot}03}{6{\cdot}3} \quad \therefore \beta = 26^\circ 3'$$

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$$\therefore \quad A_s = \frac{666792 \times 100}{14000 \times 0{\cdot}87 \times 344} \text{ sec. } 26^\circ\ 3'$$

$$= 18 \text{ cm.}^2$$

Provide 8 bars of 18 mm. ϕ (20·36 cm.^{2})

Curtailment of bars

Let h_1 be the depth at which 2 bars can be curtailed. then

$$\frac{8-2}{8} = \frac{h_1^2}{6{\cdot}3^2} \quad \therefore \quad h_1 = 5{\cdot}50 \text{ metre from the top.}$$

Let h_2 be the depth at which 2 more bars can be curtailed. Then

$$\frac{8-4}{8} = \frac{h_2^2}{6{\cdot}3^2} \quad \therefore \quad h_2 = 4{\cdot}40 \text{ metre from the top.}$$

Let h_3 be the depth at which 2 more bars can be curtailed, then,

$$\frac{8-6}{8} = \frac{h_3^2}{6{\cdot}3^2} \quad \therefore \quad h_3 = 3{\cdot}10 \text{ metre from the top.}$$

Connection between the counterfort and the upright slab

Consider the bottom 1 *metre* deep strip of the upright slab. The pressure on this strip

$$= 33600 \text{ N/metre}^2$$

Lateral earth pressure transferred to the counterfort for 1 *metre* height

$$= 33600\ (3-0{\cdot}42) = 86688 \text{ N}$$

$\therefore$ Steel required per *metre* height

$$= \frac{86688}{14000} = 6{\cdot}2 \text{ cm.}^2$$

$\therefore$ Spacing of 8 *mm* ϕ bars

$$= \frac{0{\cdot}5 \times 100}{6{\cdot}2} = 8 \text{ cm.}$$

Since this reinforcement is provided in the form of horizontal links of two legs each, the spacing of such links will be at 16 cm. c/c.

Connection between the counterfort and the heel slab

Tension transferred in one *metre* width of the counterfort near the heel end

$$= 50103\ (3-0{\cdot}42) = 129266 \text{ N}$$

$\therefore$ Steel required in 1 *metre* width

$$= \frac{129266}{14000} = 9{\cdot}23 \text{ cm.}^2$$

Provide 8 mm. ϕ two legged links @ 10 cm. c/c.

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Design 90. Counterfort Retaining Wall

Design a counterfort type retaining wall to the following particulars :

(i) *Height of wall above G.L.* $= 5{\cdot}5$ *metre*

(ii) *Safe bearing capacity of soil* $= 16$ *tonnes/sq. metre*

(iii) *Angle of repose* $= 30°$

(iv) *Weight of soil* $= 1600$ *kg./metre*^{3}

(v) *Spacing of counterforts* $= 3$ *metres centres*

(vi) *Concrete weighs* 2500 *kg./m*^{3}

(vii) *Adopt the following stresses* $c = 50$ *kg./cm.*^{2}

$t = 1400$ *kg./cm.*^{2} *and* $m = 18$.

Solution. Depth of foundation

$$= \frac{p}{w}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]^2$$

$$= \frac{16 \times 1000}{1600} \cdot \frac{1}{9} = 1{\cdot}11 \text{ metres}$$

Let us provide a foundation depth of $1{\cdot}20$ *metres*

$\therefore$ Overall height of the wall

$$= 5{\cdot}50 + 1{\cdot}20 = 6{\cdot}70 \text{ metres.}$$

Thickness of the base slab

This shall be nearly

(i) $4{\cdot}17\, l\sqrt{H}$ *cms.* $= 4{\cdot}17 \times 3\sqrt{6{\cdot}7} = 32{\cdot}38$ *cm.*

(ii) $2lH$ *cms.* $= 2 \times 3 \times 6{\cdot}7 = 40{\cdot}2$ *cm.*

Let us provide 40 *cm.* overall for the base slab

$\therefore$ Actual height of the upright slab

$$= 6{\cdot}70 - 0{\cdot}40 = 6{\cdot}30 \text{ metres.}$$

Pressure intensity at the bottom of the upright slab

$$= wh\,\frac{1-\sin\phi}{1+\sin\phi}$$

$$= 1600 \times 6{\cdot}3 \times \tfrac{1}{3} = 3360 \text{ kg./m}^2$$

$\therefore$ Max. B.M. for the upright slab for the bottom 1 *metre* height

$$= \frac{3360 \times 3^2}{12} \times 100 \text{ kg. cm.}$$

$$= 252{,}000 \text{ kg. cms.}$$

Adopting $c = 50$ *kg./cm.*^{2}, $t = 1400$ *kg./cm.*^{2}, $m = 18$

We have $a = 0{\cdot}87\, d$

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and $Q = 8{\cdot}50$

$\therefore$ The effective depth requirement is given by

$$Q\,bd^2 = M$$

$\therefore$ $8{\cdot}50 \times 100\,d^2 = 252{,}000$

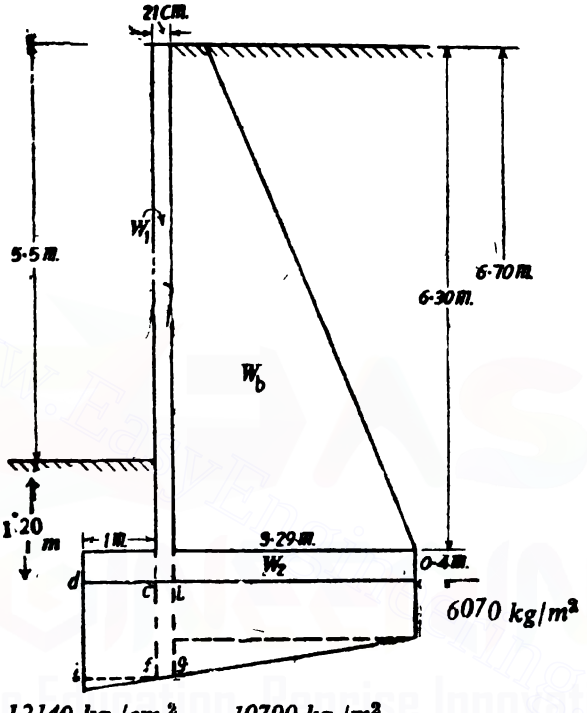

Fig. 370

$d = 17$ *cms.*

Assuming an effective cover of 4 *cms.*

Overall depth $= 17 + 4 = 21$ *cms.*

The thickness of the upright slab will be kept uniformly at 21 *cms.*

Base Width

This will be kept from $0{\cdot}6\,H$ to $0{\cdot}7\,H$

In our case $0{\cdot}6\,H = 0{\cdot}6 \times 6{\cdot}7 = 4{\cdot}02$ *metres*

$0{\cdot}7\,H = 0{\cdot}7 \times 6{\cdot}7 = 4{\cdot}69$ *metres*

Let us provide a base width of 4·50 *metres.*

Toe Projection. This will be kept at about one-fourth of the width of the base.

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In our case the toe projection

$$=\frac{4\cdot5}{5}=1\cdot1 \text{ metres}$$

Let us provide a toe projection of *1 metre*

Fig. 370 shows the overall dimensions of the wall.

Stability calculations for *1 metre* run of the wall. See table below :

Load due to	*Magnitude of the load (kg)*	*Distance from a (m.)*	*Moment about a (kg. m.)*
W_1 0·21×6·3×2500	3307	3·395	11229
W_2 4·50×0·4×2500	4500	2·25	10125
W_3 3·29×6·30×1600	33163	1·645	54553
Moment of earth Pressure $C_p \frac{wH^3}{6}$			
$\frac{1}{3}\ \frac{1600\times6\cdot7^3}{6}$			26734
Total	40970		102641

Distance of the point of application of the resultant from

$$=z=\frac{102641}{40970}=2\cdot50 \text{ m.}$$

$\therefore$ Eccentricity $e=z-\frac{b}{2}=2\cdot50-\frac{4\cdot50}{2}=0\cdot25$ m.

$$\frac{b}{6}=\frac{4\cdot5}{5}=0\cdot75 \text{ metre}$$

$\therefore$ $e<\frac{b}{6}$

$\therefore$ The maximum and minimum pressures at the base are given by

$$p_{max}=\frac{40970}{4\cdot5}\left[1+\frac{6\times0\cdot25}{4\cdot5}\right] \text{kg./m}^2.$$

$$=12140 \text{ kg./m.}^2$$

$$=12\cdot14 \text{ tonnes/m.}^2$$

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But the safe bearing capacity of the soil

$$= 16 \text{ tonnes/m.}^2$$

$$p_{min} = \frac{40970}{4{\cdot}5}\left[1 - \frac{6 \times 0{\cdot}25}{4{\cdot}5}\right] kg./m.^2$$

$$= 6070 \text{ kg./m}^2.$$

Design of the upright slab

Max. B.M. per *metre* run of the wall

$$= 252{,}000 \text{ kg. cm.}$$

Overall thickness $= 21$ *cms.*

Effective depth $= 21 - 4 = 17$ *cms.*

$$\therefore \quad A_t = \frac{252000}{1400 \times 0{\cdot}870 \times 17} = 12{\cdot}2 \text{ cm.}^2$$

$\therefore$ Spacing of 16 *mm.* dia. bars

$$= \frac{2{\cdot}01 \times 100}{12{\cdot}2} = 16{\cdot}47 \text{ cm.}$$

Hence provide 16 mm. dia. bars at 16 cm. centres.

Distribution steel per *metre* run

$$= \frac{0{\cdot}15}{100} \times 21 \times 100 = 3{\cdot}15 \text{ cm.}^2$$

$\therefore$ Spacing of 8 *mm.* bars

$$= \frac{0{\cdot}50 \times 100}{3{\cdot}15} = 15 \text{ cms.}$$

Hence if the distribution steel is provided near both faces, then the spacing of 8 mm. bars will be at 30 cms. centres near both faces.

Design of toe slab

Consider one *metre* run of the wall.

See page 601 for B.M. calculations.

Hence the max. bending moment for the toe slab per *metre* width

$$= 5345 \times 100 \text{ kg. cm.}$$

$\therefore$ Effective depth available

$$= 40 - 6 = 34 \text{ cms.}$$

$$\therefore \quad A_t = \frac{534500}{1400 \times 0{\cdot}870 \times 34} \text{ cms.}^2 = 12{\cdot}9 \text{ cm.}^2$$

$\therefore$ Spacing of 16 *mm.* dia. bars

$$= \frac{2{\cdot}01 \times 100}{12{\cdot}9} = 15{\cdot}6 \text{ cms.}$$

Hence provide 16 mm. dia. bars at 15 cms. centres.

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Load due to	*Magnitude of load c (kg.)*	*Distance from c (metre)*	*Moment about c (kg. m.)*
Upward pressure *cdif*			
10790×1	10790	0·5	5395
Upward pressure *efi*			
$\frac{1}{2} \times 1 \times 1350$	675	$\frac{2}{3}$	450
			5845
Deduct for self-weight of toe slab			
$1 \times 0{\cdot}40 \times 2500$	1000	0·5	500
Maximum Bending Moment for toe			5345

Distribution steel $= \frac{0{\cdot}15}{100} \times 100 \times 40 = 6$ *cm.*^{2}

$\therefore$ Spacing of 8 *m.m.* bars

$$= \frac{0{\cdot}50 \times 100}{6} = 8 \text{ cms.}$$

If the distribution steel is provided near both the faces, then the spacing of 8 mm. diameter bars will be at 16 cms. centres near both the faces.

Design of the Heel Slab

Consider a one *metre* wide strip of the heel slab near the heel end *a* [*see* Fig. 370].

Weight of soil on the strip $= 1600 \times 6{\cdot}3 = 10080$ *kg./m.*^{2}

Self-weight of strip $= 1 \times 0{\cdot}4 \times 2500 = 1000$ *kg./m.*^{2}

Total 11080 *kg./m.*^{2}

Deduct for upward pressure 6070 *kg./m.*^{2}

$\therefore$ Net downward pressure 5010 *kg./m.*^{2}

Spacing of counterforts $= 3{\cdot}00$ *metres*

Max. B.M. $= \frac{5010 \times 3^2}{12} \times 100$ *kg. cm.*

$= 375{,}750$ *kg. cm.*

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$$\therefore \quad A_s = \frac{375750}{1400 \times 0{\cdot}870 \times 3} \ cm.^2$$

$$= 9 \ cms.^2$$

Provide 12 mm. dia. bars at 14 cms. centres.

Design of counterforts

It is proposed to provide for the counterfort a top width of 42 *cms.* The thickness of the counterfort also may be kept at 42 *cm.*

Total lateral earth pressure transferred to one counterfort above the level of the base slab

$$= C_p \frac{wh^2}{2} l$$

$$= \frac{1}{3} \times \frac{1600 \times 6{\cdot}3^2}{2} \times 3 \ kg.$$

$$= 31752 \ kg.$$

$\therefore$ Maximum B.M. for the counterfort

$$= 31752 \times \frac{6{\cdot}3}{3} \ kg.\ m.$$

$$= 66679{\cdot}2 \ kg.\ metres.$$

Overall width of the counterfort at the base

$$= 3{\cdot}50 \ metres = 350 \ cms.$$

$\therefore$ Eff. depth available

$$= 350 - 6 = 344 \ cms.$$

Inclination of the main steel with the vertical is given by

$$\tan \beta = \frac{3{\cdot}08}{6{\cdot}30}$$

$$\beta = 26^\circ\ 3'$$

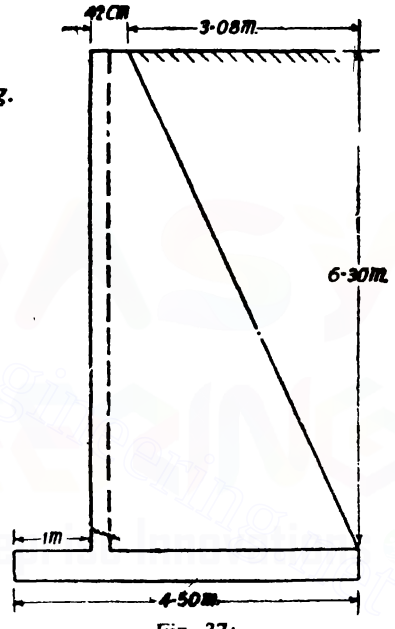

Fig. 37.1

$$A_s = \frac{6,667{\cdot}920}{1400 \times 0{\cdot}870 \times 344} \times \sec.\ 26^\circ\ 3'$$

$$= 18 \ cm.^2$$

Provide 8 bars of 18 mm. dia. (*Area provided* $= 20{\cdot}36 \ cm.^2$).

Curtailment of bars. Let h_1 be the depth at which 2 bars can be curtailed, then

$$\frac{8-2}{8} = \frac{h_1^2}{6{\cdot}3^2}$$

$\therefore$ $h_1 = 5{\cdot}5$ *metres* from the top.

Let h_2 be the depth at which 2 more bars can be curtailed, then

$$\frac{8-4}{8} = \frac{h_2^2}{6{\cdot}3^2}$$

$\therefore$ $h_2 = 4{\cdot}4$ *metres* from the top.

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Let h_3 be the depth at which 2 more bars can be curtailed, then

$$\frac{8-6}{8}=\frac{h_3^2}{6\cdot3^2}$$

$\therefore$ $h_3=3\cdot1$ *metres* from the top.

Connection between counterfort and the upright slab.

Consider the bottom 1 *metre* height of the upright slab. The pressure on this strip

$$=3360 \text{ kg./m.}^2$$

Lateral earth pressure transferred to the counterfort for 1 *metre* height

$$=3360\ (3-0\cdot42) \text{ kg.}=8670 \text{ kg.}$$

$\therefore$ Steel required per *metre* height

$$=\frac{8670}{1400}=6\cdot2 \text{ cm.}^2$$

Spacing of 8 *mm.* dia. bars

$$=\frac{0\cdot5\times100}{6\cdot2}=8 \text{ cms.}$$

Since this reinforcement is provided in the form of horizontal links of two legs each, the spacing of such links will be at 16 cms. centre.

Connection between the counterfort and the heel slab.

Tension transferred in 1 *metre* width of the counterfort near the heel end

$$=5010\ (3-0\cdot42) \text{ kg.}=12926 \text{ kg.}$$

$\therefore$ Steel required in 1 *metre* width

$$=\frac{12926}{1400}=9\cdot2 \text{ cm.}^2$$

Provide 8 mm. dia. two-legged links at 10 cms. centres.

Design 91 (S.I.). *Design a counterfort type retaining wall to the following particulars :*

(*i*) *Spacing of counterforts*	$=3$ *metres*
(*ii*) *Surcharge angle*	$=15°$
(*iii*) *Angle of repose*	$=35°$
(*iv*) *Weight of soil*	$=16000$ *N/metre*^{3}
(*v*) *Overall height of wall*	$=7$ *metres*
(*vi*) *Toe slab to be supported by front counterforts*	

Adopt the following working stresses :

$c=5$ *N/mm.*^{2}, $t=140$ *N/mm.*^{2}, $m=18$.

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Solution.

Lateral pressure intensity at any depth is given by

$p = C_p\, wh$ acting parallel to the top surface of the retained soil

where $$C_p = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}$$

In our case $\alpha = 15°$ and $\alpha = 35°$

$$\therefore \quad C_p = \cos 15° \frac{\cos 15° - \sqrt{\cos^2 15° - \cos^2 35°}}{\cos 15° + \sqrt{\cos^2 15° - \cos^2 35°}}$$

$$C_p = 0{\cdot}297.$$

Thickness of base slab

This may not be less than the following :

(*i*) $4{\cdot}17\, l \sqrt{H + 0{\cdot}7}$ *cm.* $= 4{\cdot}17 \times 3\sqrt{7 + 0{\cdot}7}$ *cm.* $= 34{\cdot}7$ *cm.*

(*ii*) $2\, l(H + 0{\cdot}7)$ *cm.* $= 2 \times 3 \times (7{\cdot}7)$ *cm.* $= 46{\cdot}2$ *cm.*

Let us provide a thickness of 50 *cm.*

Horizontal pressure intensity at a depth *h metres* is given by

$$p_h = C_p\, wh \cos\alpha$$

$$= 0{\cdot}297 \times 16000 \times h \cos 15°\ N/metre^2$$

$$= 4590\, h\ N/metre^2$$

Height of upright slab

$$= 7 - 0{\cdot}5 = 6{\cdot}5\ m.$$

Hence the horizontal pressure intensity on the upright slab at its bottom

$$= 4590 \times 6{\cdot}5 = 29835\ N/m.^2$$

Since the upright slab will be designed as a continuous slab, the maximum bending moment for this slab for a one *metre* deep strip

$$= \frac{29835 \times 3^2}{12} \times 100\ N\ cm.$$

$$= 2237625\ N\ cm.$$

Equating the moment of resistance to the maximum bending moment, we have,

$$Q\, bd^2 = 85 \times 100\, d^2 = 2237625$$

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$\therefore$ $d = 16$ *cm.*

Providing an effective cover of 4 *cm.*

Overall depth $= 16 + 4 = 20$ *cm.*

Width of the base $= 0{\cdot}6\ H$ to $0{\cdot}7\ H$.

In our case, $0{\cdot}6\ H = 0{\cdot}6 \times 7 = 4{\cdot}2$ *m.*

$0{\cdot}7\ H = 0{\cdot}7 \times 7 = 4{\cdot}9$ *m.*

Let us provide a base width of 4·5 *m.*

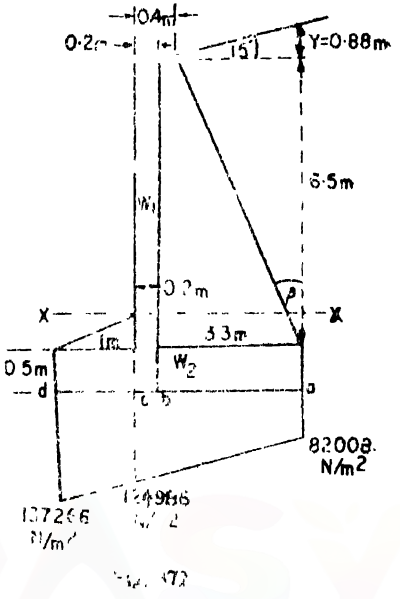
Toe projection = about $\frac{1}{4}$ base width

$$= \frac{4{\cdot}5}{4} = 1{\cdot}1 \text{ m.}$$

Let us provide a toe projection of 1 *m.*

$y = 3{\cdot}3 \tan 15° = 0{\cdot}88$ *m.*

Total lateral pressure on the wall per *metre* run



$$P = C_p\ wh_1$$

$$= 0{\cdot}297 \times 16000 \times \frac{7{\cdot}88}{2} \text{ N}$$

$$= 147536\ N$$

Horizontal component of P

$$= P_h = 147536 \cos 15° = 142508\ N$$

Vertical component of P

$$= P_v = 147536 \sin 15° = 38185\ N$$

$\therefore$ Distance of the point of application of the resultant from the end a

$$= z = \frac{1203222}{493367} = 2{\cdot}439 \text{ m}$$

Eccentricity $= e = z - \frac{b}{2} = 2{\cdot}439 - 2{\cdot}250 = 0{\cdot}189$ *m.*

$$\frac{b}{6} = \frac{4{\cdot}50}{6} = 0{\cdot}750 \text{ m.}$$

$\therefore$ The maximum and minimum pressures at the base are given by

$$p_{max} = \frac{493367}{4{\cdot}5}\left[1 + \frac{6 \times 0{\cdot}189}{4{\cdot}5}\right] N/metre^2$$

$$= 137256\ N/metre^2$$

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Stability Calculations for One Metre Run of the Wall

Load due to	*Magnitude of load (N)*	*Distance from a (m.)*	*Moment about a (N m.)*
W_1 $0{\cdot}2\times6{\cdot}50\times25000$	32500	3·40	110500
W_2 $4{\cdot}50\times0{\cdot}50\times25000$	56250	2·25	126563
Backing $3{\cdot}30\times6{\cdot}50\times16000$	343200	1·65	566280
$3{\cdot}30\times\frac{0{\cdot}88}{2}\times16000$	23232	$\frac{3{\cdot}30}{3}=1{\cdot}10$	25555
P_v	38185	0	0
Moment of P_h			
$142509\times\frac{7{\cdot}88}{3}$			374324
Total	493367		1203222

and $$p_{min}=\frac{493\cdot67}{4\cdot5}\left[1-\frac{6\times0\cdot189}{4\cdot5}\right]\ N/metre^2$$

$$=82008\ N/metre.^2$$

Design of the upright slab

Maximum bending moment

$$=2237625\ N\ cm.$$

$$\therefore\quad A_t=\frac{2237625}{14000\times0\cdot87\times16}=11\cdot48\ cm.^2$$

Spacing of 16 *mm.* ϕ bars

$$=\frac{2\cdot01\times100}{11\cdot48}=17\cdot5\ cm.$$

Provide 16 mm. ϕ *bars @ 17 cm. c/c*

Distribution steel $$=\frac{0\cdot15}{100}\times20\times100=3\ cm.^2$$

Spacing of 8 *mm.* ϕ bars

$$=\frac{0\cdot5\times100}{3}=13\ cm.$$

Since the distribution steel will be provided near both the faces, the spacing of these bars will be at 26 cm. c/c near each face

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Design of the toe slab

Since a front counterfort is provided, the toe slab should be designed as a continuous slab continuous over the front counterforts.

Loading on a one-*metre* wide strip of the toe slab will consist of the following :

(*i*) Upward pressure $= 137266$ *N/metre*^{2}

(*ii*) Dead load of slab $= 250 \times 50 = 12500$ *N/metre*^{2}

$\therefore$ Net upward pressure $= 124766$ *N/metre*^{2}

Maximum bending moment

$$= -\frac{124766 \times 3^2}{12} \times 100 \text{ } N \text{ } cm.$$

$$= 9357450 \text{ } N \text{ } cm.$$

Equating the moment of resistance to the maximum bending moment

We have $85 \, bd^2 = 85 \times 100 \, d^2 = 9357450$

$\therefore$ $d = 33.20$ *cm.*

If an effective cover of 6 *cm.* be provided to the reinforcement effective depth available $= 50 - 6 = 44$ *cm.*

$\therefore$ $$A_t = \frac{9357450}{1000 \times 0.87 \times 44} \text{ } cm.^2$$

$$= 17.46 \text{ } cm.^2$$

Spacing of 16 *mm.* ϕ bars

$$= \frac{2.01 \times 100}{17.46} = 11 \text{ } cm.$$

Design of the heel slab. Consider a one *metre* wide strip of the heel slab near the heel end. This strip is subjected to the following loads.

	N/metre^2
(*i*) Weight of the soil $= 16000 \, (6.50 + 0.88)$	$= 118080$
(*ii*) D.L. of base slab $= 0.5 \times 1 \times 25000$	$= 12500$
(*iii*) Vertical component of lateral pressure $= C_p \, wh' \sin \alpha \tan \alpha$	
$= 0.297 \times 16000 \, (6.5 + 0.88) \sin 15^\circ \tan 15^\circ$	$= 2430$
	133010
(*iv*) Deduct for upward pressure	$= 82008$
$\therefore$ Net downward load	$= 51002$ *N/metre*^{2}

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Maximum bending moment for this strip

$$= \frac{51002 \times 3^2}{12} \times 100 \ N\ cm.$$

$$= 3825150 \ N\ cm.$$

$$\therefore \quad A_s = \frac{3825150}{14000 \times 0{\cdot}87 \times 44} \ cm.^2$$

$$= 7{\cdot}14 \ cm.^2$$

$\therefore$ Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{7{\cdot}14} = 15{\cdot}8 \ cm.$$

Provide 12 mm. ϕ *bars @ 15 cm. c/c.*

Distribution steel $= \frac{0{\cdot}15}{100} \times 50 \times 100 = 7{\cdot}50 \ cm.^2$

Spacing of 8 *mm.* ϕ bars

$$= \frac{0{\cdot}5 \times 100}{7{\cdot}5} = 6{\cdot}7 \ cm.\ say\ 6{\cdot}5 \ cm.$$

Since the distribution steel will be provided near both the faces, the spacing of the bars near each face will be 13 cm. c/c.

Design of front counterfort

Total upward soil pressure transmitted to the front counterfort

$$\frac{(137266 + 124986)}{2} \times 3 \times 1 \ N$$

$$= 393378 \ N$$

This acts at a distance of

$$= \frac{124986 + 2 \times 137266}{124986 + 137266} \times \frac{1}{3} \ m.$$

$$= 0{\cdot}51 \ metre \ from \ c.$$

Moment of upward pressure

$$= 393378 \times 0{\cdot}51 \ N\ m.$$

$$= 200623 \ N\ m.$$

Moment due to dead load of slab

$$= 1 \times 0{\cdot}5 \times 3 \times 25000 \times \tfrac{1}{2} \ N\ m.$$

$$= 18750 \ Nm.$$

$\therefore$ Net bending moment for front counterfort

$$= 200623 - 18750 = 181873 \ Nm.$$

$$= 18187300 \ N\ cm.$$

Let the width of the counterfort be 45 *cm.*

Equating the moment of resistance to the bending moment,

We have, $85\ bd^2 = 85 \times 45\ d^2 = 18187300$

$\therefore$ $d = 68{\cdot}9 \ cm.$

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Effective cover

Cover to centre of toe slab bars = 6.0 *cm.*
Half diameter of toe slab bars = 0.8 *cm.*
Diameter of distribution bars = 0.8 *cm.*
Half diameter of counterfort bars (22 *mm*) = 1.1 *cm.*

8.7 *cm.*

∴ Overall depth required

$$= 68.9 + 8.7 = 77.6 \text{ cm}$$

Provide an overall depth of 80 cm.

Actual effective depth = 80 − 8.7 = 71.3 *cm.*

$$\therefore \quad A_s = \frac{18187300}{14000 \times 0.87 \times 71.3} = 21 \text{ cm}^2$$

Provide 6 bars of 22 mm. ϕ (22.81 cm.²)

Maximum shear force

$$= S = 393378 - 1 \times 0.5 \times 3 \times 25000$$
$$= 393378 - 37500 = 355878 \text{ N}$$

Nominal shear stress

$$= q_v = \frac{S}{bd} = \frac{355878}{45 \times 71.3} = 110.9 \text{ N/cm}^2$$
$$= 1.11 \text{ N/mm}^2$$

Percentage of steel provided

$$= \frac{22.81 \times 100}{45 \times 71.3} = 0.7\%$$

For 0.7% steel, $q_c = 0.33 \text{ N/mm}^2 = 33 \text{ N/cm}^2$.

∴ Shear resistance of concrete = $q_c bd$

$$= 33 \times 45 \times 71.3 = 105880 \text{ N}$$

∴ Shear to be resisted by stirrups = V_s = 355878 − 105880

$$= 249998 \text{ N}$$

∴ Spacing of 4-legged 10 *mm.* φ stirrups

$$= \frac{A_w f_w d}{V_s}$$

$$= \frac{4 \times 0.79 \times 14000 \times 71.3}{249998}$$

$$= 12.6 \text{ cm.}$$

Provide 4-legged 10 mm. ϕ stirrups @ 12 cm c/c

Design of main counterforts

The critical section for the main counterfort at a distance of

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bending moment should be determined is at the section XX (just at the level of the top of the front counterfort).

Width of the counterfort of the critical section XX,

$$=3{\cdot}50-\frac{3{\cdot}50-0{\cdot}40}{6{\cdot}5}\times 0{\cdot}3 \ m.$$

$$=3{\cdot}36 \ m.=336 \ cm.$$

Total horizontal pressure transferred to the counterfort above the critical section XX

$$=4590\times\frac{(6{\cdot}5-0{\cdot}3)^2}{2}\times 3 \ N$$

$$=264660 \ N$$

B.M. at the critical section

$$=264660\times\frac{6{\cdot}2}{3}\times 100 \ N \ cm.$$

$$=54{,}696{,}400 \ N \ cm.$$

Effective depth available $=336-6=330$ *cm.*

Let β be the inclination of the main reinforcement with the vertical

$$\tan\beta=\frac{3{\cdot}5-0{\cdot}4}{6{\cdot}5}=\frac{3{\cdot}1}{6{\cdot}5} \qquad \therefore \ \beta=25^\circ\ 30'$$

$$A_t=\frac{54696400\ \sec 25^\circ\ 30'}{14000\times 0{\cdot}87\times 330} \quad 15 \ cm.^2$$

Provide 5 bars of 20 mm. ϕ.

Out of these 5 bars, let two of them be curtailed at a depth of x *metres* from top

$$\therefore \qquad \frac{A_{t1}}{A_{t2}}=\frac{M_1}{M_2}=\frac{\frac{h_1^3}{a_1}}{\frac{h_2^3}{a_2}}=\frac{h_1^2}{h_2^2}$$

$$\frac{5-2}{5}=\frac{x^2}{(6{\cdot}5-0{\cdot}3)^2} \qquad \therefore \ x=4{\cdot}8 \ \textit{metres} \text{ from the top.}$$

Hence two of the five bars may be curtailed at $6{\cdot}2-4{\cdot}8=1{\cdot}4$ *m.* above the base slab.

Connection between the upright slab and the counterfort

Horizontal soil pressure intensity at the level of the critical section XX

$$=4590\times 6{\cdot}2=28458 \ N/metre^2$$

(Width of counterfort=45 *cm*)

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∴ Tension transmitted to the counterfort per *metre* height at this level $= 28458(3 - 0{\cdot}45) = 72568\ N$

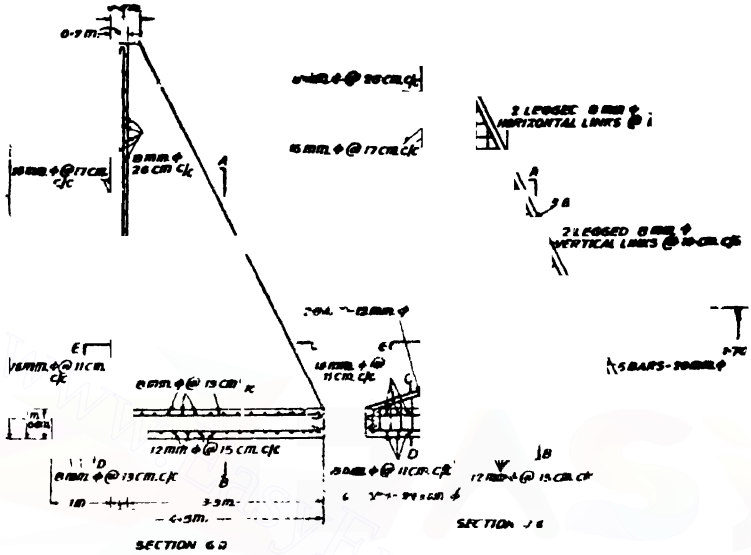

Fig. 373 (*a*)

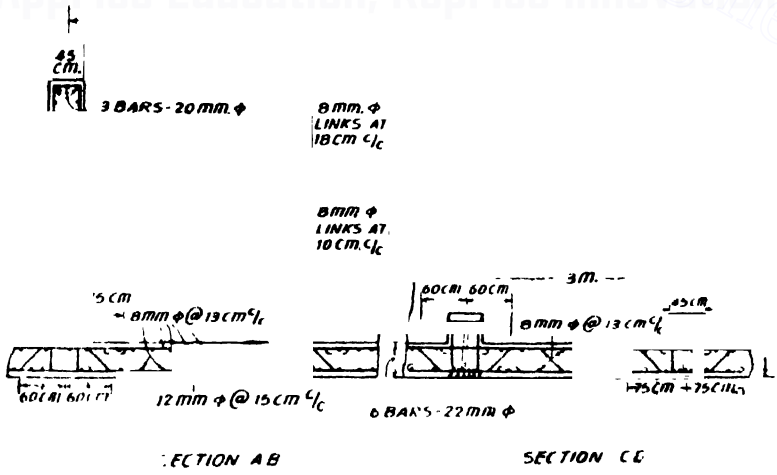

Fig. 373 (*b*)

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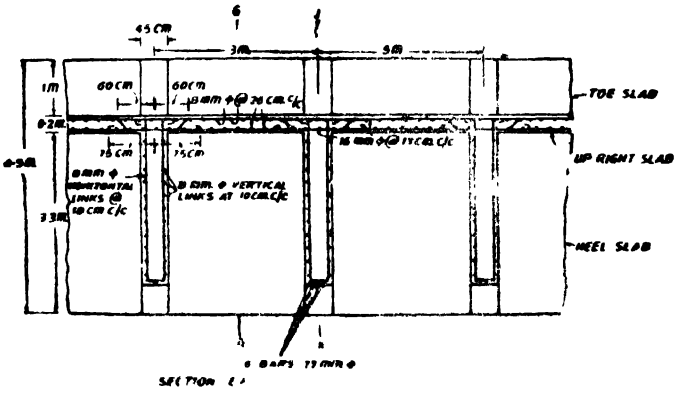

Fig. 373 (c)

∴ Horizontal steel required per *metre* height

$$= \frac{72568}{14000} = 5{\cdot}18 \text{ cm}^2$$

Spacing of 8 *mm.* ϕ bars

$$= \frac{0{\cdot}5 \times 100}{5{\cdot}18} = 9{\cdot}6 \text{ cm., say } 9 \text{ cm}$$

Since this reinforcement is provided in the form of horizontal links of two legs, the spacing of the links will be 10 cm. c/c.

Connection between the base slab and the counterfort

Consider a one *metre* wide strip of the heel slab near the heel end.

Net load on the strip = 51002 *N/metre*^{2}

Tension transmitted to the counterfort in one *metre* width

$$= 51002\ (3 - 0{\cdot}45) = 130055 \text{ N}$$

∴ Steel required per *metre* width

$$= \frac{130055}{14000} = 9{\cdot}29 \text{ cm.}^2$$

Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{9{\cdot}29} = 5{\cdot}3 \text{ cm. say } 5 \text{ cm.}$$

Since this reinforcement will be provided in the form of vertical links of two legs, the spacing of the links will be 10 cm. c/c.

Design 92. *Design a counterfort retaining wall to the following particulars :*

(*i*) *Spacing of counterforts* = *3 metres*

(*ii*) *Surcharge angle* = *15°*

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(*iii*) *Angle of repose* $= 35°$

(*iv*) *Weight of soil* $= 1600$ *kg/metre*^{3}

(*v*) *Overall height of wall* $= 7$ *metres*

(*vi*) *Toe slab to be supported by front counterforts*

Adopt the following working stresses :

$c = 50$ *kg./cm.*^{2}, $t = 1400$ *kg /cm.*^{2}, $m = 18$.

Solution.

Lateral pressure intensity at any depth is given by

$p = C_p wh$ acting parallel to the top surface of the retained soil

where $$C_p = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}$$

In our case $\alpha = 15°$ and $\phi = 35°$

$$\therefore \quad C_p = \cos 15° \frac{\cos 15° - \sqrt{\cos^2 15° - \cos^2 35°}}{\cos 15° + \sqrt{\cos^2 15° - \cos^2 35°}}$$

$$C_p = 0{\cdot}297.$$

Thickness of base slab

This may not be less than the following :

(*i*) $4{\cdot}17\, l\sqrt{H} + 0{\cdot}7$ *cm.* $= 4{\cdot}17 \times 3\sqrt{7} + 0{\cdot}7$ *cm.* $= 34{\cdot}7$ *cm.*

(*ii*) $2\, l(H + 0{\cdot}7)$ *cm.* $= 2 \times 3 \times (7{\cdot}7)$ *cm.* $= 46{\cdot}2$ *cm.*

Let us provide a thickness of 50 *cm*

Horizontal pressure intensity at a depth h *metres* is given by

$$p_h = C_p wh \cos\alpha$$

$$= 0{\cdot}297 \times 1600 \times h \times \cos 15° \text{ kg./metre}^2$$

$$= 459\, h \text{ kg./m}^2$$

Height of upright slab

$$= 7 - 0{\cdot}5 = 6{\cdot}5 \text{ m.}$$

Hence the lateral pressure intensity on the upright slab at its bottom

$$= 459 \times 6{\cdot}5 \text{ kg./m.}^2$$

$$= 2984 \text{ kg./m}^2.$$

Since the upright slab will be designed as a continuous slab, the maximum bending moment for this slab for a one *metre* deep strip

$$= \frac{2984 \times 3^2}{12} \times 100 \text{ kg. cm.}$$

$$= 223800 \text{ kg. cm.}$$

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Equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}50 \times 100\, d^2 = 223800$$

$$\therefore \quad d = 16 \text{ cm.}$$

Providing an effective cover of 4 *cm*.

Overall depth $= 16 + 4 = 20$ *cm*.

Width of the base $= 0{\cdot}6\ H$ to $0{\cdot}7\ H$

In our case, $0{\cdot}6\ H = 0{\cdot}6 \times 7 = 4{\cdot}2$ *m*.

$0{\cdot}7\ H = 0{\cdot}7 \times 7 = 4{\cdot}9$ *m*.

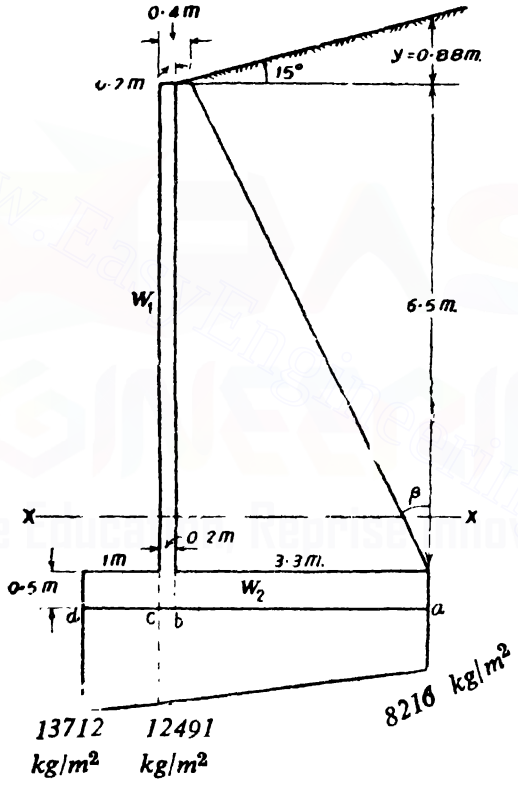

Fig. 374

Let us provide a base width of 4·5 *m*.

Toe projection $=$ about $\frac{1}{4}$ base width

$$= \frac{4{\cdot}5}{4} = 1{\cdot}1 \text{ m.}$$

Let us provide a toe projection of 1 *m*.

$$y = 3{\cdot}3 \tan 15° = 0{\cdot}88 \text{ m.}$$

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Total lateral pressure on the wall per *metre* run

$$P = C_p \frac{wh_1^2}{2}$$

$$= 0{\cdot}297 \times 1600 \times \frac{7{\cdot}88^2}{2} \text{ kg.}$$

$$= 14750 \text{ kg.}$$

Horizontal component of P

$$= P_h = 14750 \cos 15° \text{ kg.}$$

$$= 14250 \text{ kg.}$$

Vertical component of P

$$= P_v = 14750 \sin 15° \text{ kg.}$$

$$= 3820 \text{ kg.}$$

Stability Calculations for one metre run of the Wall

Load due to	*Magnitude of load (kg.)*	*Distance from a (m.)*	*Moment about a (kg. m.)*
W_1			
$0{\cdot}2 \times 6{\cdot}5 \times 2500$	3250	3·4	11050
W_2			
$4{\cdot}5 \times 0{\cdot}5 \times 2500$	5625	2·25	12656
Backing			
$3{\cdot} \times 6{\cdot}5 \times 600$	34320	1·65	56628
$3{\cdot}3 \times \frac{0{\cdot}88}{2} \times 1600$	2323	$\frac{3{\cdot}3}{3} = 1{\cdot}1$	2555
P_e	3820	0	0
Moment of P_h			
$14250 \times \frac{7{\cdot}88}{3}$			37430
Total	49338		120319

∴ Distance of the point of application of the resultant from the heel end a

$$= z = \frac{120319}{49338} \text{ metre}$$

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$= 2{\cdot}438$ *metre*

$\therefore$ Eccentricity $e = z - \frac{b}{2}$

$= 2{\cdot}438 - 2{\cdot}250 = 0{\cdot}188$ *m*

But, $\frac{b}{6} = \frac{4{\cdot}5}{6} = 0{\cdot}75$ *m.*

$\therefore$ $e < \frac{b}{6}$

$\therefore$ The maximum and minimum pressures at the base are given by

$$p_{max} = \frac{49338}{4{\cdot}5}\left[1 + \frac{6 \times 0{\cdot}188}{4{\cdot}5}\right] kg./m.^2$$

$= 13712$ *kg./m.*^{2}

and $p_{min} = \frac{49338}{4{\cdot}5}\left[1 - \frac{6 \times 0{\cdot}188}{4{\cdot}5}\right] kg./m.^2$

$= 8216$ *kg./m.*^{2}

Design of upright slab

Maximum B.M. $= 223800$ *kg. cm.*

Overall depth $= 20$ *cm.*

Effective cover to reinforcement

$= 4$ *cm.*

$\therefore$ Effective depth $= 20 - 4 = 16$ *cm.*

$\therefore$ $A_s = \frac{223800}{1400 \times 0{\cdot}87 \times 16}$ *cm.*^{2}

$= 11{\cdot}48$ *cm.*^{2}

Spacing of 16 *mm.* diameter bars

$= \frac{2{\cdot}01 \times 100}{11{\cdot}48}$ *cm.*

$= 17{\cdot}5$ *cm. say* 17 *cm. c/c*

Distribution steel $= \frac{0{\cdot}15}{100} \times 20 \times 100$ *cm.*^{2}

$= 3$ *cm.*^{2}

$\therefore$ Spacing of 8 *mm.* diameter bars

$= \frac{0{\cdot}5 \times 100}{3} = 13$ *cm.*

Since the distribution steel will be provided near both the faces, the spacing of these bars will be at 26 cm. c/c near each face.

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Design of the toe slab. Since a front counterfort is provided, the toe slab should be designed as a continuous slab continuous over the front counterforts.

Loading on a one-*metre* strip of the toe slab will consist of the following :

(*i*) Upward pressure $= 13712\ kg./m.^2$

(*ii*) Self-weight of slab 25×50 $= 1250\ kg./m.^2$

$\therefore$ Net upward pressure $= 12462\ kg./m.^2$

$\therefore$ Max. B.M. $= \frac{12462 \times 3^2}{12} \times 100\ kg.\ cm.$

$= 934650\ kg.\ cm.$

Equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}50 \times 100\ d^2 = 934650$$

$$\therefore \quad d = 33{\cdot}2\ cm.$$

If an effective cover of 6 *cm.* be provided to the reinforcement effective depth available $= 50 - 6 = 44\ cm.$

$$\therefore \quad A_t = \frac{934650}{1400 \times 0{\cdot}87 \times 44}\ cm^2$$

$$= 17{\cdot}40\ cm^2$$

$\therefore$ Spacing of 16 *mm.* diameter bars

$$= \frac{2{\cdot}01 \times 100}{17{\cdot}40}\ cm. = 11\ cm.$$

Design of the heel slab. Consider a one *metre* wide strip of the heel slab near the heel end. This strip is subjected to the following loads :

	$kg./m.^2$
(*i*) Weight of soil $= 1600 \times (6{\cdot}5 + 0{\cdot}88)$	$= 11808$
(*ii*) Self weight of base slab $= 0{\cdot}5 \times 1 \times 2500$	$= 1250$
(*iii*) Vertical component of lateral pressure $= C_p\, wh' \sin \alpha \tan \alpha$ $= 0{\cdot}297 \times 1600\ (6{\cdot}5 + 0{\cdot}88) \sin 15^\circ \tan 15^\circ$	$= 243$
	13301
(*iv*) Upward pressure	$= 8216$
$\therefore$ Net downward load	$= 5085\ kg./m.^2$

Maximum bending moment for this strip

$$\frac{5085 \times 3^2}{12} \times 100\ kg.\ cm.$$

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$$= 381{,}375 \text{ kg. cm.}$$

$$A_t = \frac{381{,}375}{1400 \times 0{\cdot}87 \times 44} \text{ cm.}^2$$

$$= 7{\cdot}12 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{7{\cdot}12} \text{ cm.}$$

$$= 15{\cdot}8 \text{ cm. say } 15 \text{ cm. c/c.}$$

Distribution steel $\frac{0{\cdot}15}{100} \times 50 \times 100 \text{ cm.}^2$

$$= 7{\cdot}50 \text{ cm.}^2$$

Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{7{\cdot}5} \text{ cm.}$$

$$= 6{\cdot}7 \text{ cm. say } 6{\cdot}5 \text{ cm.}$$

Since the distribution steel will be provided near both the faces, the spacing of the bars near each face will be 13 cm. c/c.

Design of front counterforts. Total upward soil pressure transferred to the front counterfort

$$= \frac{13712 + 12491}{2} \times 3 \times 1 \text{ kg.}$$

$$= 39304 \text{ kg.}$$

This acts at a distance of

$$= \frac{12491 + 2 \times 13712}{12491 + 13712} \times \frac{1}{3} \text{ m.}$$

$$= 0{\cdot}51 \text{ m. from } c$$

$\therefore$ Moment of upward pressure

$$= 39304 \times 0{\cdot}51 \text{ kg. m.}$$

$$= 20045 \text{ kg. m.}$$

Moment due to self-weight of slab

$$= 1 \times 0{\cdot}5 \times 3 \times 2500 \times \frac{1}{2} \text{ kg. m.}$$

$$= 1875 \text{ kg. m.}$$

$\therefore$ Net B.M. for front counterfort

$$= 20045 - 1875 \text{ kg. m.}$$

$$= 18170 \text{ kg. m.}$$

$$= 1{,}817{,}000 \text{ kg. cm.}$$

Let the width of the counterfort be 45 *cm.*

Equating the moment of resistance to the maximum bending moment, we have

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$$8{\cdot}5 \times 45\, d^2 = 1{,}817{,}000$$
$$d = 68{\cdot}9 \text{ cm.}$$

Effective cover

Cover to the centre of toe slab bars	$= 6{\cdot}0$ cm.
Half diameter of toe slab bars	$= 0{\cdot}8$ cm.
Dia. of distribution steel	$= 0{\cdot}8$ cm.
Half dia. of counterfort bars (22 mm. dia.)	$= 1{\cdot}1$ cm.
	$8{\cdot}7$ cm.

$\therefore$ Overall depth required
$$= 68{\cdot}9 + 8{\cdot}7 \text{ cm.}$$
$$= 77{\cdot}6 \text{ cm.} \quad \text{say } 80 \text{ cm.}$$

Hence actual effective depth
$$= 80 - 8{\cdot}7 \text{ cm.}$$
$$= 71{\cdot}3 \text{ cm.}$$

$\therefore$
$$A_t = \frac{1{,}817{,}000}{1400 \times 0{\cdot}87 \times 71{\cdot}3} \text{ cm.}^2$$
$$= 21 \text{ cm}^2$$

Let us provide 6 bars of 22 mm. diameter ($22{\cdot}81$ cm.^{2})

Maximum shear force
$$= 39304 - 1 \times 0{\cdot}50 \times 3 \times 2500 \text{ kg.}$$
$$= 35554 \text{ kg.}$$

Nominal shear stress
$$q_v = \frac{S}{bd} = \frac{35554}{45 \times 71{\cdot}3} \text{ kg./cm.}^2$$
$$= 11{\cdot}08 \text{ kg./cm.}^2$$

Percentage of steel provided
$$= \frac{22{\cdot}81 \times 100}{45 \times 71{\cdot}3} = 0{\cdot}7$$

For $0{\cdot}7\%$ steel $\quad q_c = 3{\cdot}30$ kg./cm.^{2}

Shear resistance of concrete
$$q_c bd = 3{\cdot}3 \times 45 \times 71{\cdot}3 = 10588 \text{ kg.}$$

$\therefore$ Shear to be resisted by stirrups
$$= V_s = 35554 - 10588 = 24966 \text{ kg.}$$

$\therefore$ Spacing of 4-legged 10 mm. ϕ stirrups
$$= \frac{A_v f_v d}{V_s} = \frac{4 \times 0{\cdot}79 \times 1400 \times 71{\cdot}3}{24966} = 12 \text{ cm.}$$

Design of main counterforts

The critical section for the main counterfort at which the bend-

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ing moment should be determined is at the section *XX* (just at the top of the front counterfort). See Fig. 374.

Width of the counterfort at the critical section *XX*.

$$= 3{\cdot}50 - \frac{(3{\cdot}50 - 0{\cdot}40)}{6{\cdot}5} \times 0{\cdot}3 \, m.$$

$$= 3{\cdot}36 \, m.$$

$$= 336 \, cm.$$

Total horizontal pressure transferred to the counterfort above the critical section *XX*

$$= 459 \times \frac{(6{\cdot}5 - 0{\cdot}3^2)}{2} \times 3 \, kg.$$

$$= 26460 \, kg.$$

$\therefore$ B.M. at the critical section

$$= 26460 \times \frac{6{\cdot}2}{2} \times 100 \, kg. \, cm.$$

$$= 5{,}470{,}000 \, kg. \, cm$$

Effective depth available

$$= 336 - 6 = 330 \, cm.$$

Let β be the inclination of the main reinforcement with the vertical

$$\therefore \quad \tan \beta = \frac{3{\cdot}5 - 0{\cdot}4}{6{\cdot}5} = \frac{3{\cdot}1}{6{\cdot}5}$$

$$\therefore \quad \beta = 25^\circ \, 30'$$

$$\therefore \quad A_t = \frac{5{,}470{,}000 \sec 25^\circ \, 30'}{1400 \times 0{\cdot}87 \times 330} \, cm.^2$$

$$= 15 \, cm^2.$$

Let us provide 5 bars of 20 mm diameter. Out of these 5 bars, let two of them be curtailed at a depth of *x metres* from top

$$\therefore \quad \frac{A_{t1}}{A_{t2}} = \frac{M_1}{M_2} = \frac{\frac{a_1 h_1^3}{a_2}}{h_2^3} = \frac{h_1^2}{h_2^2}$$

$$\frac{5-2}{5} = \frac{x^2}{(6{\cdot}5 - 0{\cdot}3)^2}$$

$$\therefore \quad x = 4{\cdot}8 \, metres \text{ from the top}$$

Hence two of the 5 bars may be curtailed at $6{\cdot}2 - 4{\cdot}8$

$$= 1{\cdot}4 \, m. \text{ above the section } XX$$

or $1{\cdot}4 + 0{\cdot}3 = 1{\cdot}7$ *metres* above the base slab.

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Connection between the upright slab and the counterfort

Horizontal soil pressure at the level of critical section XX

$$= 459 \times 6{\cdot}2 \ kg./metre^2$$

$$= 2846 \ kg./metre^2$$

(width of counterfort = 45 *cm*).

$\therefore$ Tension transferred to the counterfort per *metre* height at this level

$$= 2846\,[3 - 0{\cdot}45] \ kg.$$

$$= 7256 \ kg.$$

$\therefore$ Steel required per *metre* height

$$= \frac{7256}{1400} \ cm.^2$$

$$= 5{\cdot}18 \ cm.^2$$

$\therefore$ Spacing of 8 *mm*. diameter bars

$$= \frac{0{\cdot}5 \times 100}{5{\cdot}18} \ cm.$$

$$= 9{\cdot}6 \ cm. \text{ say } 9 \ cm$$

Since the reinforcement is provided in the form of horizontal links of two legs the spacing of the links will be at 18 cm. centres.

Connection between the base slab and the counterfort. Consider a one *metre* wide strip of the heel slab near the heel end.

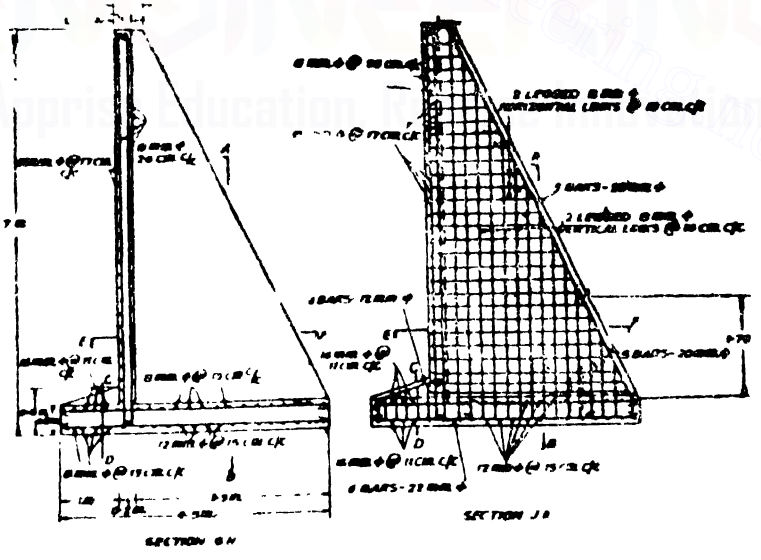

Fig. 375 (*a*)

Net load on the strip = 5085 *kg /m.*

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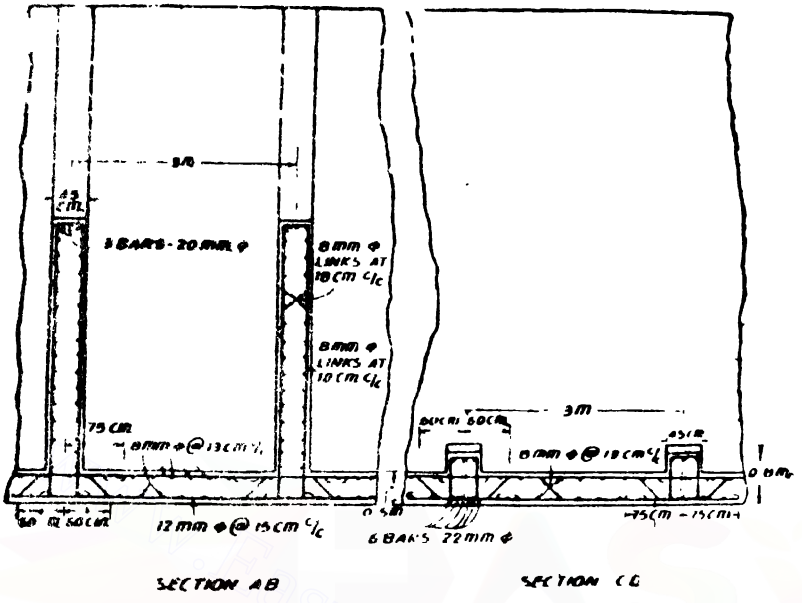

Fig. 375 (*b*)

∴ Tension transferred to the counterfort in one *metre* width $= 5085\ (3 - 0{\cdot}45)\ kg.$

$= 12967\ kg.$

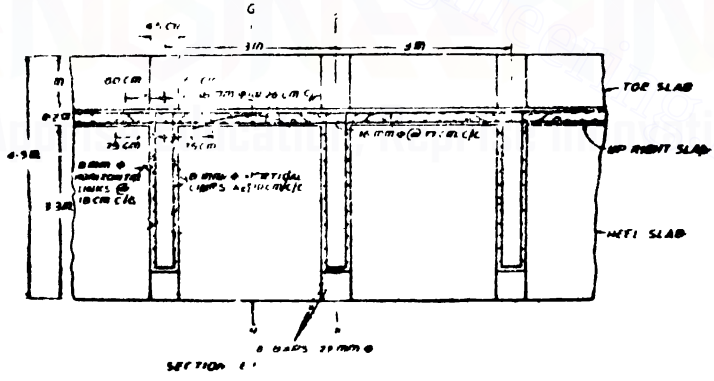

Fig. 375 (*c*)

∴ Steel required per *metre* width

$$= \frac{12967}{1400}\ cm.^2$$

$$= 9{\cdot}26\ cm.^2$$

∴ Spacing of 8 *m.m.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{9{\cdot}26}\ cm. = 5{\cdot}4\ cm.\ \text{say}\ 5\ cm.$$

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Since this reinforcement will be provided in the form of vertical links of two legs, the spacing of the links

$$= 10 \text{ cm. c/c.}$$

§155. Buttressed Retaining Walls

Sometimes buttresses are used instead of counterforts to strengthen the upright slab of the retaining wall. Fig. 376 shows a general layout of buttressed wall. Such a wall is designed similar to the counterfort wall but the following points must be noted :

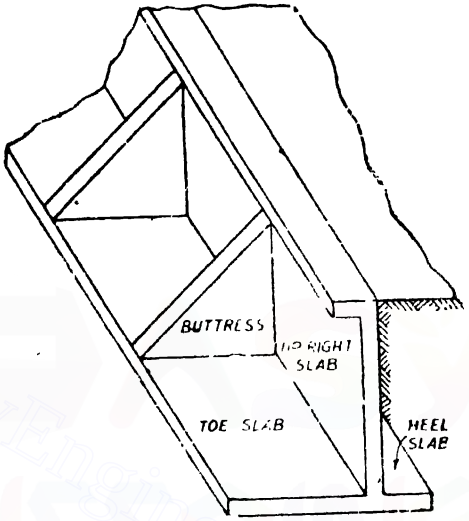

Fig. 376. Buttressed Wall.

(*i*) The buttresses should be designed as vertical cantilevers, but cantilevering from the toe slab.

(*ii*) Heavy reinforcement in the form of vertical links or U-stirrups should be provided connecting the buttresses and the toe slab But only nominal horizontal reinforcement connecting the buttresses and the upright slab will be provided.

(*iii*) The toe slab should be designed as a continuous slab spanning between the buttresses.

(*iv*) The heel slab should be designed as a cantilever as the heel slab of a cantilever retaining wall.

Wall with tie rods. If the height to which the soil is to be retained is large it may be convenient to anchor the wall back to the earth itself by means of tie rods. The tie rods may be encased in concrete The tie rods pass through the centroid of the lateral earth pressure Hence, if the height of the stem be h, the tie rods may be provided at a height of $\frac{h}{3}$ above the base slab. One end of the tie rod is properly secured or fixed to the stem while its other end is embedded into a concrete block. The lateral movement of the block is resisted by friction between the block and the soil above and below it.

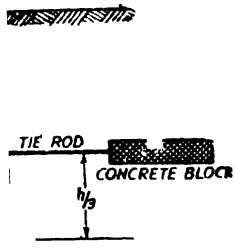

Fig. 377

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If the tie rods be provided at a spacing of x *metres* the tension in each tie rod.

$$= T = C_p \frac{wh^2}{2} x \text{ kg.}$$

Hence the sectional area of each rod

$$= \frac{T}{1400} \text{ cm}^2.$$

Size of the concrete block. Let the area of the block be A *metres*^{2}. Hence effective area of the block in contact with the soil equals $2A$ *metre*^{2} (since the depth of the block is small only the area of the top and bottom faces of the block are considered).

Suppose the block be y *metres* below the retained surface (y will be $\frac{2}{3}h$).

Vertical pressure intensity on the block

$$= w\ y \text{ kg./m}^2.$$

$\therefore$ Frictional resistance

$$= 2 \times w\ y\ A \times \mu$$

Equating the frictional resistance to the tension in the rod, we have,

$$C_p \frac{wh^2}{2} x = wy\ A\ .\ \mu$$

From the above relation the area A of the block can be determined.

Sometimes, at the end of the tie rod a vertical steel plate is provided. This adds to the resistance to horizontal movement considerably.

Connection between the tie rods and the stem. Corresponding to the level of ties a horizontal beam may be cast monolithically with the stem. The stem can now be designed as cantilevering on either side of the horizontal beam. The beam can be designed as a continuous beam, the ties serving as the supports.

Alternatively vertical beams (cantilevers) are built monolithically with the stem and to these beams the horizontal ties may be connected. In such a case, the stem should be designed as a continuous slab spanning horizontally over the vertical beams. The vertical beams serve as counterforts. The vertical beam should be designed as two cantilevers cantilevering on either side of the point at which the tie rod is connected to it.

Sometimes retaining walls may be provided on both sides of a high embankment (Fig. 378).

In such cases the tie rods will simply connect the two retaining walls. The lateral pressure on one wall provides the necessary anchoring force.

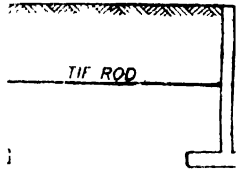

Fig. 378

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In this connection Taylor Thompson and Smulski in their work *Concrete, Plain and Reinforced Vol I* have mentioned about an interesting wall designed by Gustave Lindenthal for *New York connecting rail road*. In this case, there are two walls 20 *metres* high enclosing a rail road fill which is nearly 18 *metres* wide. The rail road fill carries four lanes of traffic (subjected to £ 60 loading and designed for 10% impact). The longitudinal walls are connected by transverse walls at intervals of 16 *metres*. The lateral earth pressure acting on the longitudinal walls is resisted by tie rods of 60 *mm* diameter spaced at intervals of 3 *metres* apart vertically and horizontally. The ends of the tie rods are threaded and nuts are provided over plates. The pressure is transferred to the concrete by longitudinal as well as vertical channels placed near the outside face of the wall. The bolts at the ends of the tie rods bear against the anchorage channels. The tie rods are surrounded by concrete to form 20 *cm* square pillars. This was done only for protection against rusting.

1156. Sheet Piled Retaining Walls

Concrete piles may be driven as a row to retain an earth backing. In these cases the piles are made side by side so as to minimize their number. The piles may be driven closely alongside one another. In order to keep the piles in line either *tongue and groove* arrangement may be made or any other form of key may be provided. Piled retaining walls may be built in many ways. The piles may be driven one by the side of the other continuously. They may also be driven at intervals the spaces between them being filled with slabs spanning horizontally, (See Fig. 379). The slabs support the soil and transfer the pressure to the piles. The slabs should be taken for sufficient depth below

Fig 379

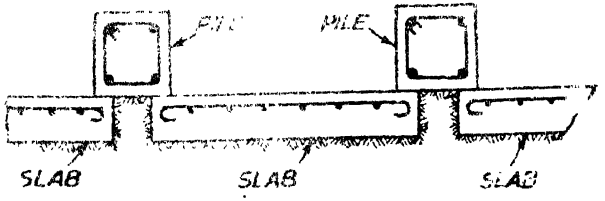

Fig. 380

the lower ground level so as to prevent the movement of the retained earth under them.

When the piles are driven continuously their tops will automatically form the top of the retaining wall. They are generally surmounted by a continuous capping.

Sometimes it is found economical to design the piles having

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a T-shape which increases their width without considerable increase of weight or area.

Though it is usual to form the point of a sheet pile with a rake so that it is induced to close up to its neighbouring pile, generally a little opening exists between adjacent piles. Often this is an advantage since it prevents the accumulation of water behind the piles.

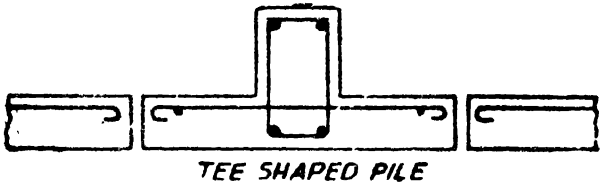

Fig. 381

But, in case it is necessary to provide watertight piling and perfectly make it earthtight, grooves may be provided on both sides of each pile, and after the piles are driven, the tube or the cavity formed can be cleared out and filled with cement grout or fine concrete.

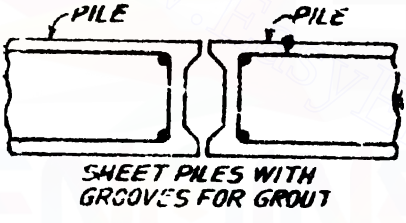

Fig. 382

Anchorage of walls. It is necessary to provide means to prevent the forward movement of piled retaining walls. This is done by providing *anchor-ties* at intervals near the tops of the piles. The anchor-ties are secured to a horizontal *walling* which connects all the piles. The walling transmits the thrust of the earth to the ties. The walling is designed as a beam. Generally the lower ends of the piles are automatically retained by driving them for sufficient depth into the ground. However, if found necessary, the lower ends of the piles may also be secured by a second series of anchor ties. The anchor tie may consist of one or more steel bars of requisite sectional area. In order to protect them from corrosion particularly in damp porous soils, it is advisable to surround the ties with dense concrete. Alternatively, the ties may be wrapped with canvas soaked in bitumen. If this is not possible, the ties may atleast be coated with hot tar.

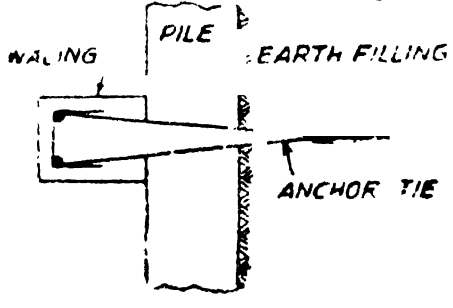

Fig. 383

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14
Stair Cases

Stairs consist of steps arranged in a series for purpose of giving access to different floors of a building. Since a stair is often the only means of communication between the various floors of a building, the location of the stair requires good and careful consideration. In a residential house the stair case may be provided near the main entrance. In a public building, the stairs must be from the main entrance itself and located centrally, to provide quick accessibility to the principal apartments. All stair cases should be adequately lighted and properly ventilated.

§157. Technical Terms used in Connection with Stair Case

(*i*) **Tread.** Tread means the horizontal upper portion of a step.

(*ii*) **Riser.** This is the vertical portion of a step.

Rise means the vertical distance between the horizontal surfaces of two consecutive steps.

(*iii*) **Winders.** These are steps used for changing the direction of the stair. These are usually triangular in plan.

(*iv*) **Landing.** This is a horizontal platform provided at the head of a series of steps. A landing extending right across a stair case is called a *half space landing*. If a landing extends only half across a stair case, it is called *quarter space landing*.

(*v*) **Nosing.** This is the outer projecting edge of a tread.

(*vi*) **Flight.** This consists of a series of steps provided between landings.

(*vii*) **Line of Nosing.** This is straight line touching the nosings of the various steps and parallel to the slope of the stair.

Classification of stairs

(*i*) **Straight Stairs.** This consists of steps leading in the same direction. This is provided in a long narrow stair case. This often consists of one flight ; but in some circumstances it may consist of two flights with an intermediate landing.

(*ii*) **Dog legged stair.** In this type the succeeding flights rise in opposite directions. The two flights, in plan, are not separated by a well. A landing is provided corresponding to the level at which the direction of flight changes.

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(*iii*) **Open newel stair.** In this type a rectangular well or opening is left between forward and backward flights. This type of stairs is a very comfortable form. The well assists as a means for good lighting. The forward and the backward flights may be connected by a landing platform. But if space allows, a short flight is also introduced connecting the forward and the backward flights with a quarter space landing on each side of the short flight.

(*iv*) **Geometrical stair.** In this type of stairs the well between the forward and the backward flights is curved. Winders are therefore used and these winders have a certain width even at the inner edge. This is helpful while walking over the stair even near the inner edge.

Fig. 384 shows the above types of stairs.

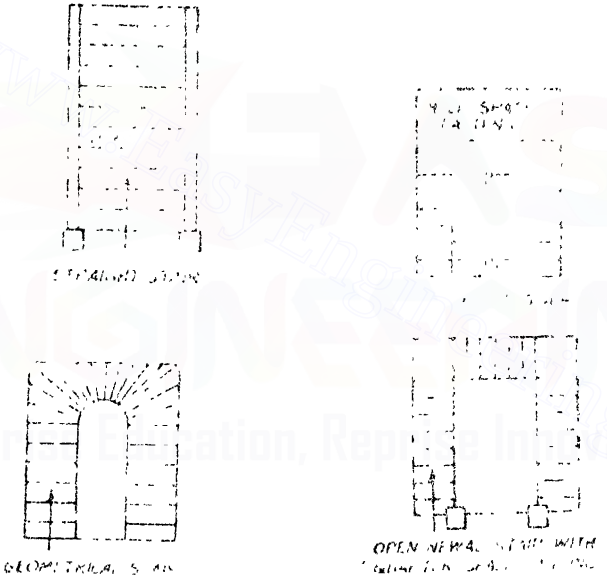

Fig. 384

Tread and rise determination. Let the tread be T *cms.* and the rise be R *cms*, T and R may be chosen to satisfy the following relations :

(*i*) $TR = 400$ to 410

(*ii*) $2R + T = 60$

In dwelling houses the tread may be 25 *cm* and the rise may be 16 *cm*. In public buildings the tread may be 27 *cm* to 30 *cm*. and the rise may be 15 *cm*. In factories a tread of 25 *cm* and a rise of 19 *cm* may be adopted. In cinema theatres the tread may be 27 *cm*. and the rise may be 15 *cm*.

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The minimum width of a stair shall be 85 *cm*. In residential buildings the minimum width is preferably 105 *cm*.

Head room over a stair The head room over a stair shall be at least 2·10 *metres*.

Number of steps in a flight The number of steps at one stretch in a flight may not preferably be more than 12. If more are provided than the above limit, it becomes difficult and tiresome to ascend and descend the stairs.

Live loads on stairs The live loads to be considered on stairs have been stipulated by *I S 875 (Revised) Code of Practice for Structural Safety of Buildings : Loading Standards*

As per this code the live load on stairs, landings and balconies may be taken at 3000 *N per metre*^{2} (300 *kg per sq metre*) in buildings, where there are no possibilities of overcrowding. But in the case of public buildings and warehouses, where overcrowding is likely, the live load may be taken at 5000 *N per metre*^{2} (500 *kg. per sq. metre*)

§158. Design of Stairs

Based on the direction along which a stair slab spans, the stairs may be classified into the following two types :

(*i*) Stairs spanning horizontally,

(*ii*) Stairs spanning longitudinally.

§159 Stairs Spanning Horizontally

These stairs are supported at each side by walls, stringer beams, or at one side by wall and at the other side by a beam. When the span of the stair does not exceed 1·50 *metres* one bar of 10 *mm* dia. per step will be found sufficient. A distribution steel of 6 *mm* dia. bars at 30 *cms* centres may be provided.

Each step will be designed as spanning horizontally. For purposes of design each step will be taken as equivalent to a rectangular beam of width *b* and effective depth *D*/2. For specifica-

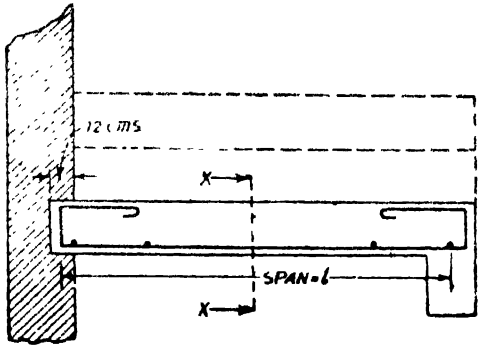

Fig. 385. Stairs spanning horizontally.

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tions of b and D see Fig. 386. A waist of about 8 *cm.* may be provided.

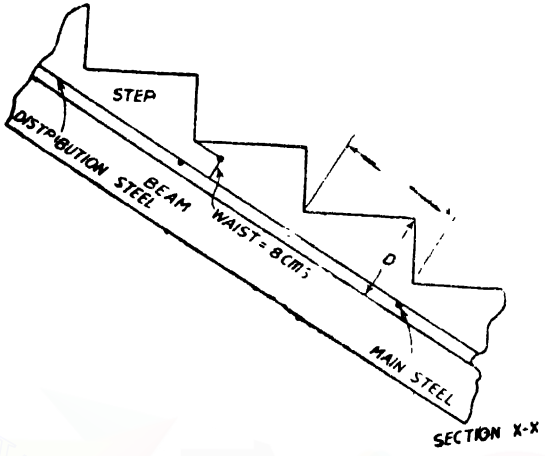

Fig. 386

Each step may be designed as an equivalent rectangular beam having a breadth b and effective depth $d = D/2$

Design 93 (S I). *A flight of stairs to be provided in an office building is to be supported by a stringer beam on one edge and a brick wall on the other. The effective horizontal span of the stairs may be taken as 1·50 metre. The risers are 15 cm. and the treads are 27 cm. Design the steps allowing a live load of 3000 N/metres². Use M 15 concrete.*

Solution. Fig. 387 shows the steps.

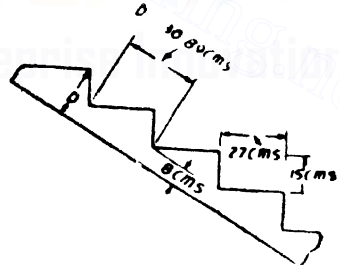

Fig. 387

Let the thickness of the waist be 8 *cm.*

$$b = \sqrt{27^2 + 15^2}$$

$$= 30{\cdot}89 \text{ cm.}$$

$$D = 8 + \frac{27 \times 15}{30{\cdot}89} = 21{\cdot}11 \text{ cm.}$$

Load per metre run of step

D.L. of step $= \frac{1}{2} \times 0{\cdot}27 \times 0{\cdot}15 \times 25000 =$ 506 *N/metre*

D.L. of waist $= 0{\cdot}3089 \times 0{\cdot}08 \times 25000 =$ 618 *N/metre*

13 *mm.* thick granolithic finish on tread

$0{\cdot}013 \times 0{\cdot}27 \times 24000 =$ 84 *N/metre*

Total dead load $= 1208$ *N/metre*

L.L. at 3000 *N/metre*² $= 0{\cdot}27 \times 3000 =$ 810 *N/metre*

Total load/*metre* run $= 2018$ *N/metre*

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Maximum bending moment

$$= \frac{wl^2}{8} = \frac{2018 \times 1{\cdot}5^2}{8} \times 100 \ N\ cm.$$

$$= 56756 \ N\ cm.$$

Equating the moment of resistance to the bending moment,

$$85\ bd^2 = 85 \times 30{\cdot}89\ d^2 = 56756$$

$$\therefore \quad d = 4{\cdot}65\ cm.$$

But effective depth available

$$= \frac{D}{2} = \frac{21{\cdot}11}{2} = 10{\cdot}56\ cm.$$

$$A_t = \frac{56756}{14000 \times 0{\cdot}87 \times 10{\cdot}56} = 0{\cdot}44\ cm.^2$$

Provide one bar of 8 mm. ϕ (0 50 cm.²)
Provide also a distribution steel of 6 mm. ϕ @ 30 cm. c/c.

Design 94. *A flight of stairs to be provided in an office building is to be supported by a stringer beam on one edge and a brick wall on the other. The effective horizontal span of the stairs may be taken as 1·50 metres. The risers are 15 cm and the treads are 27 cm. Design the steps allowing a live load of 300 kg./metre². Use M 150 concrete.*

Solution.

Fig. 388 shows the steps.

Let the thickness of the waist be 8 *cm.*

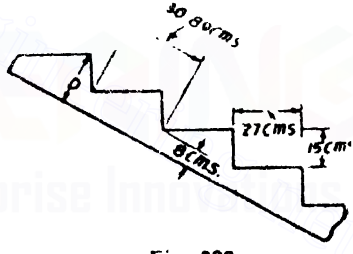

Fig. 388

$$b = \sqrt{27^2 + 15^2} = 30{\cdot}89\ cm.$$

$$D = 8 + \frac{27 \times 15}{30\ 89}\ cm. = 21{\cdot}11\ cm.$$

Load per metre run of step

Dead load of step

$$= \frac{1}{2} \times \frac{27}{100} \times \frac{15}{100} \times 1 \times 2500\ kg. = 50{\cdot}6\ kg/m.$$

Waist $\frac{30\ 89}{100} \times \frac{8}{100} \times 2500$ = 61·8 *kg./m.*

13 *mm* thick granolithic finish on tread $= \frac{1}{1000} \times \frac{27}{100} \times 2400$ = 8·4 *kg./m.*

Total dead load = 120 8 *kg /m.*

Live load at 300 *kg./metre²*

$$= \frac{27}{100} \times 1 \times 300 = 81\ kg./m.$$

Total load/*metre* run = 201·8 *kg./m. say* 202 *kg./m.*

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$\therefore$ **Maximum bending moment**

$$= \frac{wl^2}{8}$$

$$\frac{202 \times 1{\cdot}5^2}{8} \times 100 \text{ kg. cm.} = 5681 \text{ kg. cm.}$$

Adopting $c = 50$ *kg./cm.*^{2}, $t = 1400$ *kg./cm.*^{2} and $m. = 18$ and equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}5 \times 30{\cdot}89\, d^2 = 5681 \qquad \therefore \quad d = 4{\cdot}6 \text{ cm.}$$

But effective depth available

$$= \frac{D}{2} = \frac{21{\cdot}11}{2} = 10{\cdot}56 \text{ cm.}^2$$

$$\text{Area} = \frac{5681}{1400 \times 0{\cdot}87 \times 10{\cdot}56} \text{ cm.}$$

$$= 0{\cdot}44 \text{ cm.}^2$$

Provide one bar of 8 mm. diameter (0·5 cm.^{2}*)*

Provide also a distribution steel of 6 mm. diameter bars at 30 cm. centres

Stairs spanning horizontally, with a newel wall provided between the two flights Fig. 389 shows a usual arrangement of stairs spanning horizontally. Each stair is supported at one end by a side wall and at the other end by the newel wall into which it is chased for 12 *cm.* The stairs should be designed as in the previous case. A small beam may also be provided to span from end wall to the newel wall. The landing slab can now be designed as a continuous slab supported on the side walls and the intermediate beam.

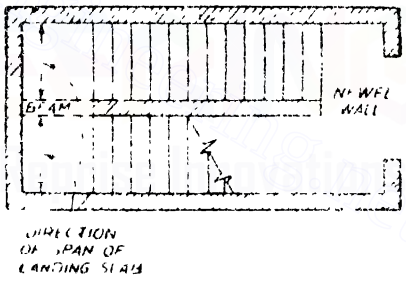

Fig. 389

§160. Stairs Spanning Longitudinally

Stairs of this category span between supports at the bottom and top of the flight and remain unsupported at the sides. In such a case the bending moment per unit width may be taken as $\frac{wl^2}{10}$ where w is the load per unit horizontal area and l is the effective horizontal span. Sometimes beams are avoided to support the stairs and the flight will be cast along with the landings which may be supported on walls. In such a case the effective horizontal span may be taken as the horizontal distance between the centres of bearings. The maximum bending moment may be taken as $\frac{wl^2}{8}$.

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In this connection I.S. : 456 (Revised) Code of Practice for Plane and Reinforced Concrete has stipulated the following :

Effective span of stairs. *The effective span of stairs without stringer beams shall be taken as the following horizontal distances.*

(a) Where supported at top and bottom risers by beams spanning parallel to the risers : Distance between centres of beams

(b) Where spanning on the edge of a landing slab which spans parallel with the risers : (see Fig. 390) a distance equal to the going of the stairs plus at each end either half the width of the landing or one metre which [illegible]

(c) Where the landing slab spans in the same direction as the stairs, they [illegible] as acting together to form a single slab and the span determined in the distance [illegible] of the supporting beams or walls, the going being measured [illegible]

[illegible]	[illegible]	Spans in metres
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

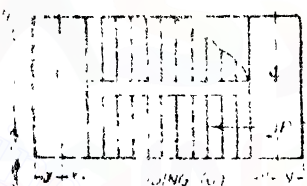

Fig. 390

Distribution of loading on stairs In the case of stairs with open wells where spans partly crossing at right angles occur, the load on area common to any two such spans may be taken as one half in each direction as shown in Fig. 391.

Where flights or landings are embedded into walls for a length of not less than 110 *mm* and are designed to span in the direction of the flight, a 150 *mm* strip may be deducted from the loaded area, and the effective breadth of the section increased by 75 *mm*. for purposes of design. See Fig. 391 (*a*).

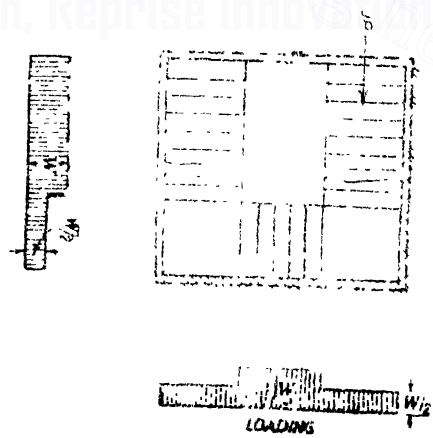

Fig. 391

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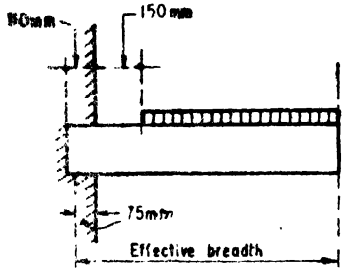

Fig. 391 (a)

$$\frac{\sqrt{R^2+T^2}}{T}$$

Depth of section. The depth of section shall be taken as the minimum thickness perpendicular to the soffit of the stair case.

Estimation of weight

Let the waist be 15 *cm.* thick measured at right angles to the slope. Its weight per *sq. m.* on slope

$$= 250 \times 15 = 3750 \ N/metre^2$$

To obtain the corresponding load per *sq. metre* of horizontal area, this value should be multiplied by

$$\frac{\sqrt{R^2+T^2}}{T}$$

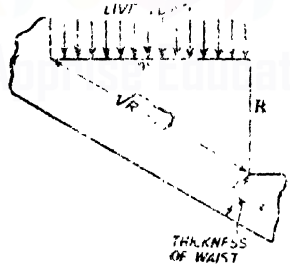

Fig. 392

where R = Rise and T = Tread

Suppose $R = 15$ *cm.*

and $T = 27$ *cm.*

Then $$\frac{\sqrt{R^2+T^2}}{T} = \frac{\sqrt{15^2+27^2}}{27}$$

$$= 1{\cdot}14$$

$\therefore$ Weight on plan = weight on slope $\times$ 1·14.

The weight of the steps per *sq. metre* of horizontal area can be estimated by treating all the steps as equivalent to a horizontal slab of thickness $\frac{R}{2}$.

Design 95 (S.I.) *The main stair of an office building has to be located in a stair measuring 3·5 m. × 5·5 m. The vertical distance*

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between the floors is 3·75 m. Design the stairs. Allow a L.L. of 3000 N/metre². Use M 15 concrete.

Solution. It is proposed to provide two flights for the stairway.

Hence, the height of each flight $= \frac{3\cdot75}{2}$ *m.*

$= 1\cdot875$ *metres*

$= 187\cdot5$ *cm.*

Assuming 15 *cms.* risers,

number of risers required $= \frac{187\cdot5}{15} = 12$

Hence the actual rise of each riser $= \frac{187\cdot5}{12} = 15\cdot62$ *cm.*

Number of treads in each flight = number of risers − 1

$= 12 - 1 = 11$ treads.

Let the width of the stairs be 160 *cms.*

Let the tread of the steps be 27 *cms.*

Fig. 393 shows the arrangement of the stairs in plan.

Design of the flight AB. Let the bearing for the flight be 15 *cms.*

Effective horizontal span

$= 2\cdot97 + 1\cdot60 + \frac{0\cdot15}{2} = 4\cdot645$ *metres*

Let the thickness of the waist be 22 *cms.* (This can be assumed at 4 to 5 *cm.* per *metre* run of horizontal span).

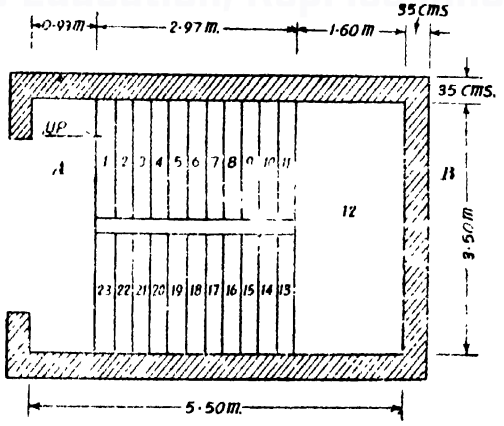

Fig. 393

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Loads

D.L. of 22 *cm.* waist $= 250 \times 22 = 5500$ *N/metre*^{2}

Ceiling finish (1·25 *cm*) $= 240 \times 1{\cdot}25 = 300$ *N/metre*^{2}

5800 *N/metre*^{2}

Corresponding load per *sq metre* on plan

$$= \frac{\sqrt{R^2 + T^2}}{T} \times 5800 = \frac{\sqrt{15{\cdot}62^2 + 27^2}}{27} \times 5800 = 6700 \text{ N/metre}^2$$

Hence, the actual load per *sq metre* of plan area will consist of the following :

Waist and ceiling finish	$= 6700$ *N/metre*^{2}
DL of steps $\left(\frac{15{\cdot}62}{2}$ *cm.* average$\right)$	
$= 7{\cdot}81 \times 250$	$= 1952$ *N/metre*^{2}
Top finish (1·25 *cm*) $= 1{\cdot}25 \times 240$	$= 300$ *N/metre*^{2}
Live Load	$= 3000$ *N/metre*^{2}
Total	$= 11952$ *N/metre*^{2}

Maximum bending moment per *metre* width of stairs

$$= \frac{11952 \times 4{\cdot}645^2}{8} \times 100 \text{ N cm}$$

$$= 32234{\cdot}0 \text{ N cm.}$$

Adopting $c = 5$ *N/mm.*$^2 = 500$ *N/cm*^{2}, $t = 140$ *N/mm.*$^2 = 14000$ *N/cm*^{2}; $m = 18$; and equating the moment of resistance to the bending moment, we have,

$$85\, bd^2 = 85 \times 100\, d^2 = 3223460$$

$$\therefore \quad d = 19{\cdot}5 \text{ c.m.}$$

Providing 14 *mm.* bars at a clear cover of 15 *mm.*

Effective cover $= 15 + 7 = 22$ *mm.* $= 2{\cdot}2$ *cm.*

Effective depth $= 22 - 2{\cdot}2 = 19{\cdot}8$ *cm.*

$$\therefore \quad A_t = \frac{3223460}{14000 \times 0{\cdot}87 \times 19{\cdot}8} = 13{\cdot}37 \text{ cm.}^2$$

Spacing of 14 *mm.* ϕ bars

$$= \frac{1{\cdot}54 \times 100}{13{\cdot}37} = 11{\cdot}5 \text{ cm.}$$

Provide 14 mm. ϕ @ 11 cm. c/c.

Destribution steel $= \frac{0{\cdot}15}{100} \times 22 \times 100 = 3{\cdot}3$ *cm.*^{2}

Provide 8 mm. ϕ @ 11 cm

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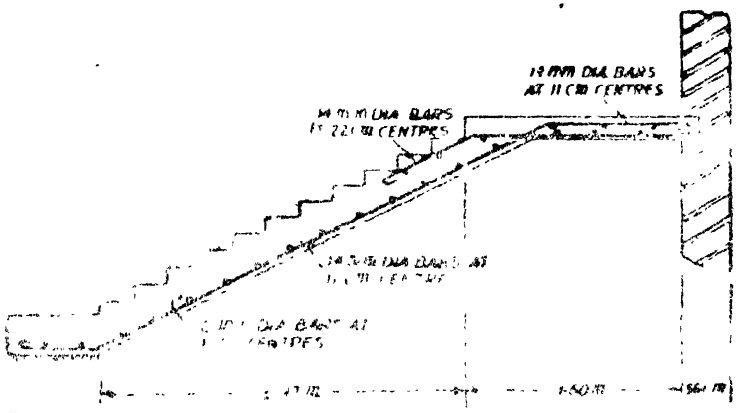

Fig. 394

[illegible] stair of an office building has to be located [illegible] m. × 5·5 m. The vertical distance between the [illegible] Design the stairs, allowing a live load of 300 [illegible] concrete.

[illegible] to provide two flights for the stair was

[illegible] each flight

$$= \frac{[illegible]}{2} = 1{\cdot}875 \text{ metre} = 187{\cdot}5 \text{ cm}$$

[illegible] risers, number of risers required

$$= \frac{187{\cdot}5}{15} = 12$$

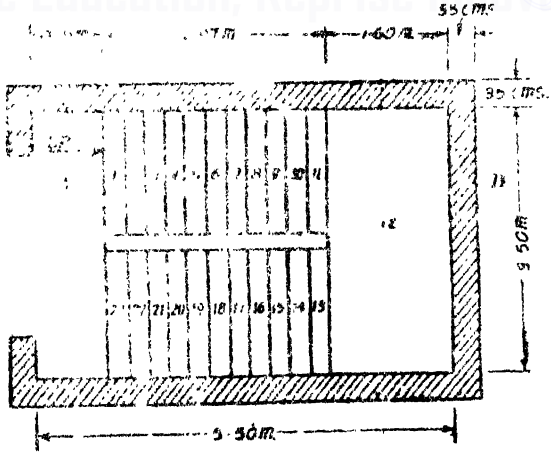

Fig. 395

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Hence the actual rise of each riser $= \frac{187{\cdot}5}{12} = 15{\cdot}62$ *cms.*

Number of treads in each flight = number of risers − 1
$= 12 - 1 = 11$ treads.

Let the width of the stair be 160 *cms.*

Let the tread of the steps be 27 *cms.*

Fig. 395 shows the arrangement of the stairs in plan.

Design of the flight AB. Let the bearing for the flight be 15 *cms.*

Effective horizontal span

$$= 2{\cdot}97 + 1{\cdot}60 + \frac{0{\cdot}15}{2} = 4{\cdot}645 \text{ metres}$$

Let the thickness of the waist be 22 *cms.* (This can be assumed at 4 to 5 *cm.* per *metre* run of horizontal span).

Loads. Dead load of 22 *cms.* waist $= 25 \times 22 = 550$ *kg./metre*^{2}

Ceiling finish (1·25 *cm*) thick $= 24 \times 1{\cdot}25 = 30$ *kg./metre*^{2}

$= 580$ *kg./metre*^{2}

Corresponding load per *sq. metre* on plan

$$= \frac{\sqrt{R^2+T^2}}{T} \times 557 = \frac{\sqrt{15{\cdot}6^2+27^2}}{27} \times 580 \text{ kg./metre}^2$$

$= 670$ *kg /m*^{2}.

Hence the actual load per *sq. metre* of plan area will consist of he following :

Waist and ceiling finish $= 670$ *kg /m.*^{2}

Dead load of steps $\left(\frac{15{\cdot}62}{2} \text{ cm. average}\right)$

$= 7{\cdot}81 \times 25 = 195$ *kg /m.*^{2}

Top finish (1·25 *cm.* thick)

$= 1{\cdot}25 \times 24 = 30$ *kg./m.*^{2}

Live Load $= 300$ *kg /m*^{2}

$= 1195$ *kg./m.*^{2}

Max. bending moment per *metre* width of stairs

$$= \frac{1195 \times (4{\cdot}645)^2}{8} \times 100 \text{ kg. cm.}$$

$= 322290$ *kg. cm.*

Adopting $c = 50$ *kg./cm.*^{2} $t = 1400$ *kg /cm.*^{2}

and $m = 18$, we have $a = 0{\cdot}87\,d$

and $Q = 8{\cdot}50$

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$\therefore \quad 8{\cdot}50 \times 100\, d^2 = 322{,}290$

$d = 19{\cdot}5$ *cms.*

Providing 14 *mm.* ϕ bars at a clear cover of 15 *mm.*

Effective cover $= 15 + 7 = 22$ *mm.* $= 2{\cdot}2$ *cm.*

$\therefore$ Effective depth $= 22 - 2{\cdot}2 = 19{\cdot}8$ *cm.*

$$\therefore \quad A_s = \frac{322290}{1400 \times 0{\cdot}87 \times 19{\cdot}8} \text{ cm.}^2$$

$$= 13{\cdot}36 \text{ cm.}^2$$

Spacing of 14 *mm.* dia. bars (area $1{\cdot}54$ *cm.*^{2})

$$= \frac{1{\cdot}54 \times 100}{13{\cdot}36} = 11{\cdot}5 \text{ cms}$$

Provide 14 mm. ϕ *@ 11 cm. c/c.*

Distribution steel $= \frac{0{\cdot}15}{100} \times 22 \times 100 = 3{\cdot}3$ *cm.*^{2}

Provide 8 mm. ϕ *@ 11 cm c/c.*

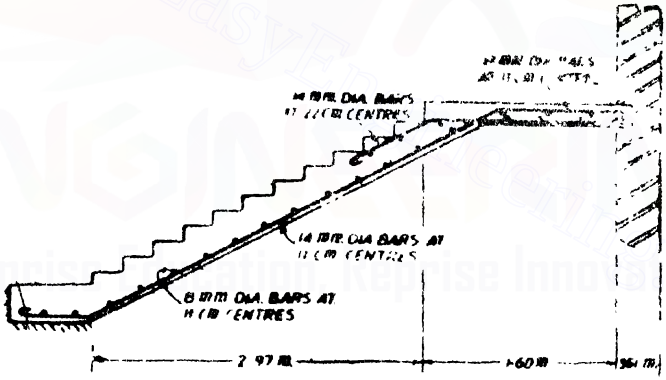

Fig. 396

Design 97 (S.I.). *Fig. 397 shows an arrangement for a stair case to be provided for an office building. Design the stair case. The risers are 15 cm. and the treads are 25 cm. The walls are 40 cm. thick and the stair slab has full bearing on the masonry walls. The supporting beam is 40 cm. wide. Use M 15 concrete.*

Solution. Let us provide a waist of 20 *cm.*

Loads

DL of waist	$= 20 \times 250$	$= 5000$ *N/metre*^{2}
Ceiling finish		$= 300$ *N/metre*^{2}
		$= 5300$ *N/metre*^{2}

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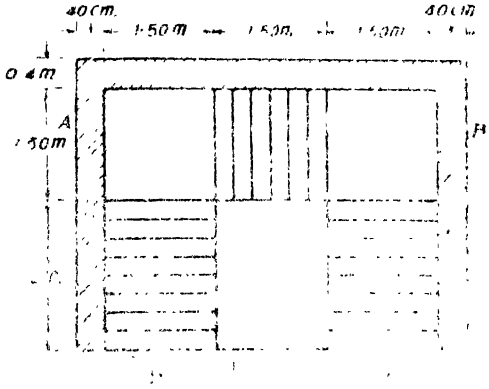

[illegible]

$= 5$ [illegible] $N/metre^2$

[illegible] $\left[\frac{[illegible] + 25^2}{25}\right]$ $N/metre^2$

[illegible] $N/metre^2$

Load per sq. metre [illegible]

Waist and cell [illegible]	$= 6130\ N/metre^2$
DL of [illegible] $\left(\frac{[illegible]}{2}\right.$ [illegible] $\left.\right) = [illegible]5 \times 250$	$= 187[illegible]\ N/metre^2$
Top limit	$= [illegible]00\ N/metre^2$
Live Load	$= 3000\ N/metre^2$
Total	$= 1135[illegible]\ N/metre^2$

Since the landing slab is a two way slab, the load on the landing slab may be taken as

$$\frac{11355}{[illegible]} = 5677.5\ N/metre^2$$

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Flight AB

Effective span = centre to centre distance between the supporting walls.

$$= 1{\cdot}70 + 1{\cdot}50 + 1{\cdot}70 = 4{\cdot}90 \text{ metre}$$

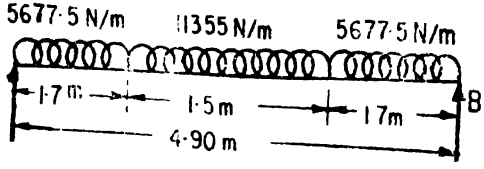

Fig 398

Consider a one *metre* wide strip of the flight.

Reaction at each support

$$\frac{11355 \times 1{\cdot}5 + 5677{\cdot}5 \times 3{\cdot}4}{2} = 18168 \text{ N.}$$

Maximum bending moment

$$= \frac{18168 \times 4{\cdot}9}{2} - (5677{\cdot}5 \times 1{\cdot}7 \times 1{\cdot}6) - \left(11355 \times \frac{0{\cdot}75^2}{2}\right)$$

$$= 44511{\cdot}6 - 15442{\cdot}8 - 3193{\cdot}6 \text{ N m.}$$

$$= 25875{\cdot}2 \text{ N m.}$$

Equating the moment of resistance to the maximum bending moment,

$$85\ bd^2 = 85 \times 100\ d^2 = 25875{\cdot}2 \times 100$$

$$\therefore \quad d = 17{\cdot}4 \text{ cm.}$$

Providing a clear cover of 15 *mm.* and using 12 *mm.* ϕ bars,

Effective cover $= 15 + 6 = 21$ *mm.* $= 2{\cdot}1$ *cm.*

Effective depth available

$$= 20 - 2{\cdot}1 = 17{\cdot}9 \text{ cm.}$$

$$\therefore \quad A_t = \frac{25875{\cdot}2 \times 100}{14000 \times 0{\cdot}87 \times 17{\cdot}9} = 11{\cdot}87 \text{ cm.}^2$$

Provide 12 mm. ϕ @ 10 cm. c/c.

Flight DA

Effective span = Centre of beam to centre of bearing on wall

$$= 2 + 1{\cdot}5 + 0{\cdot}20 + 0{\cdot}20 = 3{\cdot}90 \text{ metre}$$

Consider a one *metre* wide strip of the flight. Fig. 399 shows the loading on the strip.

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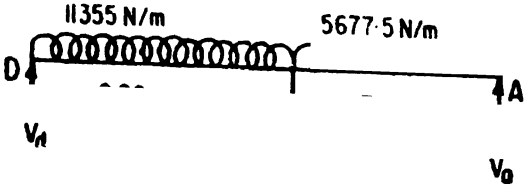

[illegible] 399

Taking moments about D,

$$V_a \times 3{\cdot}9 = (11355 \times 2{\cdot}2 \times 1{\cdot}1) + 5677{\cdot}5 \times 1{\cdot}7\ (2{\cdot}2 + 0{\cdot}85)$$

$$V_a = 14594{\cdot}09\ N.$$

$$V_d = (11355 \times 2{\cdot}2) + (5677{\cdot}5 \times 1{\cdot}7) - 14594{\cdot}09$$

$$= 20038{\cdot}66\ N$$

Let the shear force be zero at a distance of x *metres* from D. For this condition,

$$20038{\cdot}66 - 11355\ x = 0$$

$$x = \frac{20038{\cdot}66}{11355} = 1{\cdot}76\ m.$$

Maximum bending moment

$$= 20038{\cdot}66 \times 1{\cdot}76 - 11355 \times \frac{1{\cdot}76^2}{2}\ Nm.$$

$$= 17681{\cdot}40\ Nm.$$

$$= 1768140\ N\ cm.$$

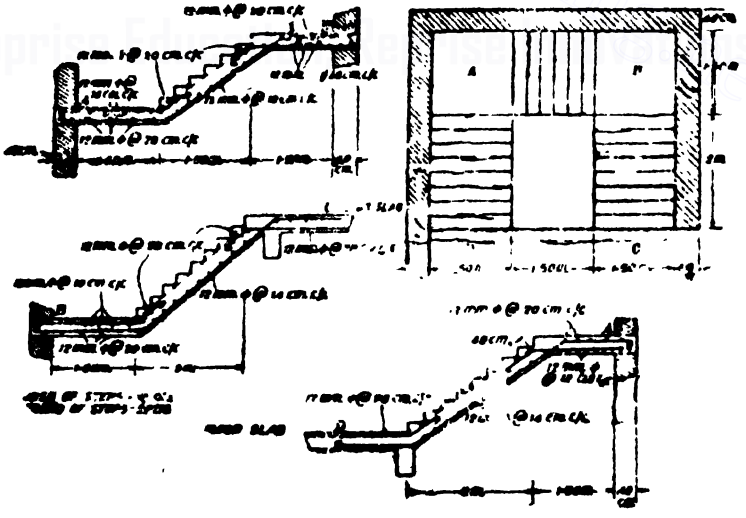

Fig. 4.[illegible]

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$$A_t = \frac{1768140}{14000 \times 0{\cdot}87 \times 17{\cdot}9} = 8{\cdot}10 \text{ cm.}^2$$

Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{8{\cdot}10} = 14 \text{ cm.}$$

Provide 12 mm. ϕ *@ 14 cm. c/c.*

Fig. 400 shows details of reinforcements.

Design 98. *Fig. 401 shows an arrangement for a stair case to be provided for an office building. Design the stair case. The risers are 15 cm. and the treads are 25 cm. The walls are 40 cm. thick and the stair slab has full bearing on the masonry walls. The supporting beam is 40 cm. wide.*

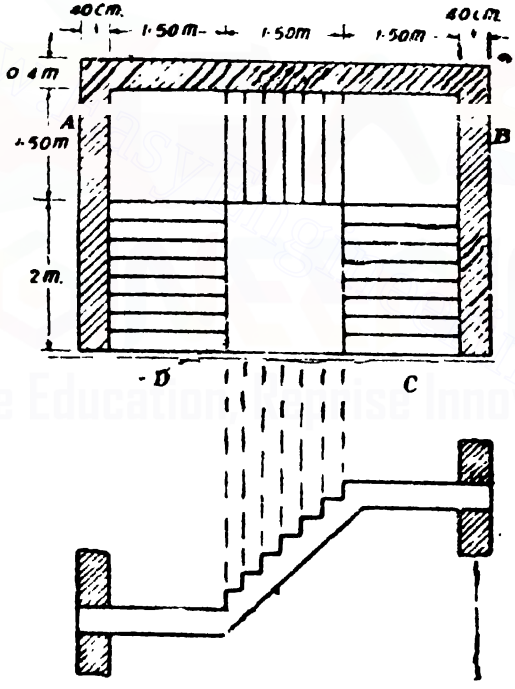

Fig. 401

Solution. Let us provide a waist of 20 *cm.*

Loads. Dead load of waist $= 20 \times 25$ $= 500$ *kg./m.*^{2}
Ceiling finish $= 30$ *kg./m.*^{2}
530 *kg./m.*^{2}

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Corresponding load per *sq. metre* on plan

$$=530\left\{\frac{\sqrt{R^2+T^2}}{T}\right\}\ kg./m.^2=530\left\{\frac{\sqrt{15^2+25^2}}{25}\right\}kg./m.^2$$

$$=618\ kg./m.^2$$

Load per sq. metre on plan

Waist and ceiling finish	$= 618\ kg./m.^2$
D.L. of steps $\left(\frac{15}{2}\ cm.\ \text{average}\right)$	$= 187\ kg./m.^2$
Top finish	$= 30\ kg./m.^2$
Live load	$= 300\ kg./m.^2$
Total	$= 1135\ kg./m.^2$

Since the landing slab is a two way slab, the load on the landing slab may be taken at $567{\cdot}5\ kg/m.^2$

Flight AB

Effective span = Centre to centre distance between the supporting walls

$=1{\cdot}70+1{\cdot}50+1{\cdot}70\ m$

$=4{\cdot}9\ m.$

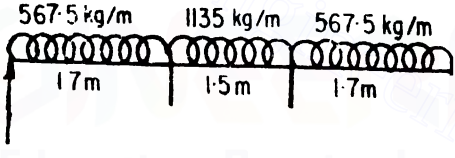

Fig. 402

Consider a one *metre* wide strip of the flight.

Each vertical reaction

$$=\frac{1135\times1{\cdot}5+567{\cdot}5\times3{\cdot}4}{2}\ kg.$$

$$=1816\ kg.$$

$\therefore$ Maximum bending moment

$$=\frac{1816\times4{\cdot}9}{2}-(567{\cdot}5\times1{\cdot}7\times1{\cdot}6)-\left(1135\times\frac{0{\cdot}75^2}{2}\right)kg.\ m.$$

$$=2586{\cdot}4\ kg.\ m.=258640\ kg.\ cm.$$

Adopting $c=50\ kg./cm.^2$, $t=1400\ kg./cm.^2$ and $m=18$ and equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}5\times100\ d^2=258640$$

$$\therefore \quad d=17{\cdot}4\ cm.$$

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Providing a clear cover of 15 *mm.* and using 12 *mm.* ϕ bars.

Effective cover $= 15+6=21$ *mm.* $= 2{\cdot}10$ *cm.*

$\therefore$ Effective depth available $= 20-2{\cdot}1 = 17{\cdot}9$ *cm.*

$$\therefore \quad A_t = \frac{258640}{1400\times 0{\cdot}87\times 17{\cdot}9}\ cm.^2 = 11{\cdot}87\ cm.^2$$

Provide 12 mm. ϕ *@ 10 cm. c/c*

Flight DA. Effective span = Centre of beam to centre of bearing on wall

$$= 2+1{\cdot}5+0{\cdot}20+0{\cdot}2\ m.$$
$$= 3{\cdot}9\ m.$$

Consider a one *metre* wide strip of the flight. Fig 403 shows the loading on the strip.

Taking moments about *D*, we have,

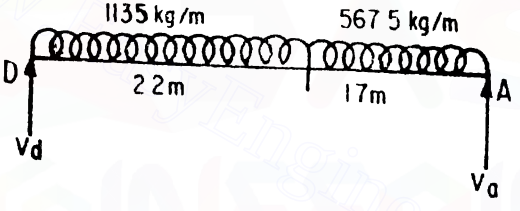

Fig. 403

$$V_a\times 3{\cdot}9 = (1135\times 2{\cdot}2\times 1{\cdot}1)+567{\cdot}5\times 1{\cdot}7\ (2{\cdot}2+0{\cdot}85)$$

$\therefore \quad V_a = 1458{\cdot}76\ kg.$

$\therefore \quad V_d = (1135\times 2{\cdot}2)+(567{\cdot}5\times 1{\cdot}7)-1458{\cdot}76$

$= 2002{\cdot}99\ kg.$

Let the shear force be zero at distance of x *metres* from *D*. For this condition.

$$1135x = 2002{\cdot}99$$

$\therefore \quad x = 1{\cdot}76\ m.$

$\therefore$ B.M. at $x = 1{\cdot}76\ m.$

$$2002{\cdot}99\times 1{\cdot}76-1135\times\frac{1{\cdot}76^2}{2}\ kg.\ m.$$

$$= 1767\ kg.\ m.$$

$$\therefore \quad \frac{1767\times 100}{1400\times 0{\cdot}87\times 18}\ cm.^4$$

$$= 8{\cdot}10\ cm.^2$$

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$\therefore$ Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{8{\cdot}1} \text{ cm.} = 14 \text{ cm.}$$

Fig. 400 shows details of reinforcements.

Design 99. *Fig. 404 shows the general layout for a cinema balcony. Design the balcony. Allow a live load of 450 kg./m².* *The tread and rise of the steps are respectively 80 cm. and 12 cm. The fulcrum girder is 50 cm. × 100 cm. Take weight of R.C.C. = 2400 kg./m³. Use M 150 concrete.*

Solution.

Design of the decking slab. The decking slab is supported on the rakers and is designed as a continuous slab.

Loads

Dead load of waist slab (10 *cm.* thick)

$$= 24 \times 10 \frac{\sqrt{R^2+T^2}}{T}$$

$$= 240 \frac{\sqrt{0{\cdot}12^2+0{\cdot}80^2}}{0{\cdot}8} = 243 \text{ kg./m.}^2$$

Dead load of steps	$= \frac{1}{2}(12)\ 24$	$= 144$ kg./m.²
Live load		$= 450$ kg./m.²
	Total	$= 837$ kg./m.²

Consider 1 *m* wide strip of the slab

Maximum bending moment

$$= M = \frac{wl^2}{10} = 837 \times \frac{2{\cdot}5^2}{10} \times 100 \text{ kg. cm.}$$

$$= 52310 \text{ kg. cm.}$$

Equating the M R. to the B.M.

$$8{\cdot}5 \times 100d^2 = 52310 \qquad \therefore \quad d = 7{\cdot}84 \text{ cm}$$

Effective depth available

$$= 10 - 2 = 8 \text{ cm.}$$

$$\therefore \qquad A_s = \frac{52310}{1400 \times 0{\cdot}87 \times 8} = 5{\cdot}37 \text{ cm.}^2$$

Spacing of 10 *mm.* bars

$$= \frac{0{\cdot}79 \times 100}{5{\cdot}37} = 14{\cdot}7 \text{ cm.}$$

Provide 10 mm. ϕ bars @ 14 cm. centres.

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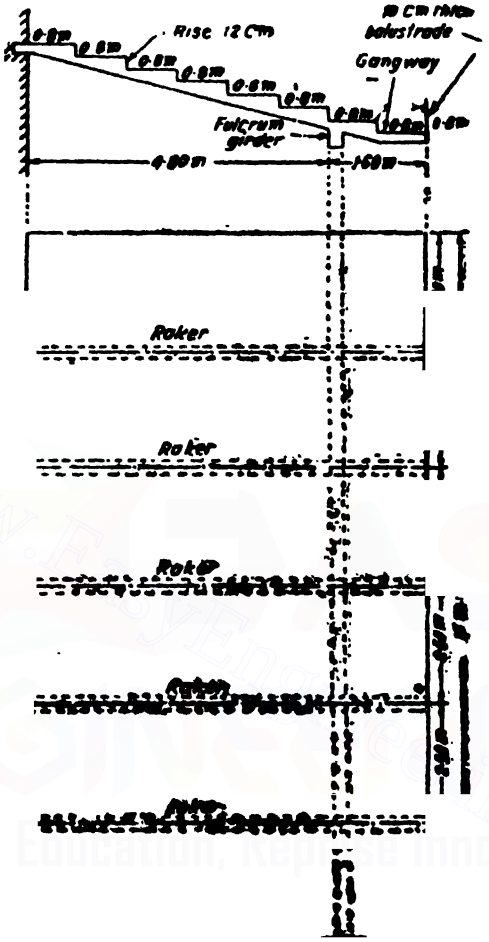

Fig. 404

Design of rakers

Consider one intermediate raker. The raker is a T-beam.

Let the depth of the rib be 30 *cm*. Let the width of the rib be also 30 *cm*.

Load from decking slab = $837 \times 2{\cdot}5$ = 2093 *kg./m.*

Dead load of rib = $0{\cdot}3 \times 0{\cdot}3 \times 2400$ = 216 *kg./m.*

= 2009 *kg./m.*

The loading on the gangway will also be taken as 2309 *kg./m.*

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Load due to 10 *cm.* balustrade $=0{\cdot}10\times0{\cdot}80\times2{\cdot}5\times2400$ $=480$ *kg.*

This load acts as a concentrated load at the free end of the raker.

Provide a bearing of 50 *cm.* for the raker. Since the width of the fulcrum girder is also 50 *cm.*, the effective span of the raker beam between the fulcrum girder and the supporting wall

$$=4{\cdot}80+0{\cdot}50=5{\cdot}30\ m.$$

Fig. 405 shows the raker beam and the loading on it.

Let V_a and V_b be the reactions at the supports A and B. Taking moments about A,

$$V_b\times5{\cdot}30=2309\times\frac{6{\cdot}65^2}{2}+480\times6{\cdot}65$$

$$\therefore\quad V_b=10235\ kg.$$

$$\therefore\quad V_a=(2309\times6{\cdot}65)+480-10235=5599{\cdot}85\ kg.$$

Fig. 423 shows the S.F. diagram for the raker beam.

Let the section of zero shear be x *metre* from A.

Equating the shear force to zero

$$5599{\cdot}85-2309\,x=0\quad\therefore\quad x=2{\cdot}43\ m.$$

B.M. calculations

$$M_a=M_c=0$$

$$M_b=-2309\times\frac{1{\cdot}35^2}{2}-480\times1{\cdot}35$$

$$=-2752\ kg.\ m.$$

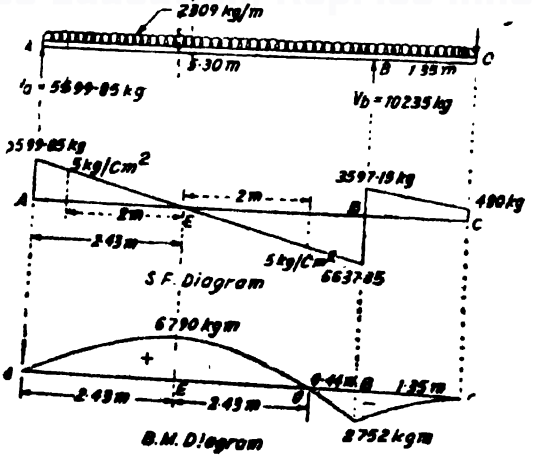

Fig. 405

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B.M. at 2·43 *m.* from *A*

$$= 5599{\cdot}85 \times 2\,43 - 2309 \times \frac{2{\cdot}43^2}{2}$$

$$= +6790\ kg.\ m.$$

Fig. 405 shows the B.M. diagram for the raker beam.

In the region of sagging bending moment, the beam will be considered as a T-beam. In the region of hogging bending moment, the beam will be considered as a rectangular beam.

Consider the region of hogging bending moment.

Maximum hogging bending moment

$$= 2752\ kg.\ m.$$

$$= 275{,}200\ kg.\ cm.$$

Equating the moment of resistance to the bending moment,

$$8{\cdot}50 \times 30\ d^2 = 275200 \quad \therefore\ d = 32{\cdot}85\ cm.$$

Overall depth of the beam

$$= 30 + 10 = 40\ cm.$$

Available effective depth (with an effective cover of 6 *cm.*)

$$= 40 - 6 = 34\ cm.$$

$$\therefore \quad A_t = \frac{275{,}200}{1400 \times 0{\cdot}87 \times 34} = 6{\cdot}64\ cm.^2$$

Provide 3 bars of 20 *mm.* diameter ($3 \times 3{\cdot}14 = 9{\cdot}42\ cm.^2$)

Maximum shear force near the support *B*

$$= 6637{\cdot}85\ kg.$$

$$\text{Nominal shear stress} = q_v = \frac{S}{bd} = \frac{6637{\cdot}85}{30 \times 34}\ kg./cm.^2$$

$$= 6{\cdot}5\ kg./cm.^2$$

Percentage of tensile steel

$$= \frac{9{\cdot}42}{30 \times 34} \times 100 = 0{\cdot}92\%$$

For 0·92% steel $q_c = 3{\cdot}6\ kg./cm.^2$

Shear resistance of concrete $= q_c\ bd$

$$= 3{\cdot}6 \times 30 \times 34 = 3672\ kg.$$

$\therefore$ Shear to be resisted by stirrups

$$= V_s = 6637{\cdot}85 - 3672 = 2965{\cdot}85\ kg.$$

Spacing of 4-legged 10 *mm.* ϕ stirrups

$$= p = \frac{A_v\ f_v\ d}{V_s} \quad \frac{(4 \times 0{\cdot}79)\ 1400 \times 34}{2965{\cdot}85} = 50\ cm.$$

Provide a spacing of 30 cm. c/c.

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Now let us consider the region of sagging bending moment. As mentioned earlier the region of sagging bending moment will be designed as a T-beam.

Maximum sagging bending moment

$$= 6790 \text{ kg. m.}$$
$$= 679{,}000 \text{ kg. cm.}$$

Flange width of the T-beam

Flange width $B = \frac{l}{6} + b_w + 6\,d_s$

$$\frac{530}{6} + 30 + (6 \times 10) = 178{\cdot}3 \text{ cm. say } 178 \text{ cm.}$$

Effective depth $= d = 40 - 6 = 34$ cm.

Approximate lever arm $= a = d - \frac{d_s}{2}$

$$= 34 - \frac{10}{2} = 29 \text{ cm.}$$

$\therefore$ Approximate steel required

$$= \frac{679{,}000}{1400 \times 29} = 16{\cdot}72 \text{ cm.}^2$$

Provide 6 bars of 20 mm. diameter.(18·84 cm.²)

Position of Neutral Axis

Assuming the neutral axis to lie within the flange and taking moments about the neutral axis,

$$\frac{178 \times n^2}{2} = 18 \times 18{\cdot}84\,(34 - n)$$

Solving, we get $n = 9{\cdot}64$ cm.

Actual lever arm $= a = d - \frac{n}{3}$

$$= 34 - \frac{9{\cdot}64}{3} = 30{\cdot}79 \text{ cm.}$$

Actual stress in steel $= \frac{679000}{18{\cdot}84 \times 30{\cdot}79}$ kg./cm.²

$$= 1170 \text{ kg./cm.}^2$$

Corresponding compressive stress in concrete

$$= c = \frac{t}{m} \cdot \frac{n}{d-n} = \frac{1170}{18} \cdot \frac{9{\cdot}64}{34 - 9{\cdot}64}$$
$$= 25{\cdot}7 \text{ kg./cm.}^2$$

Shear analysis

S.F. at A and at the point of contraflexure

$$= 5599{\cdot}85 \text{ kg.}$$

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∴ Nominal shear stress

$$= \frac{S}{bd} = \frac{5599{\cdot}85}{30 \times 34} = 5{\cdot}49 \text{ kg./cm.}^2$$

Percentage of tensile steel

$$= \frac{4 \times 3{\cdot}14}{30 \times 34} \times 100 = 1{\cdot}23\%$$

For 1·23% steel $q_c = 4$ *kg./cm.*²

Shear resistance of concrete

$$= q_c\, bd = 4 \times 30 \times 34 = 4080 \text{ kg.}$$

∴ Shear to be resisted by stirrups

$$= V_s = 5599{\cdot}85 - 4080 = 1519{\cdot}85 \text{ kg.}$$

Spacing of 4-legged 10 *mm.* ϕ stirrups

$$= \frac{4 \times 0{\cdot}79 \times 1400 \times 34}{1519{\cdot}85} = 99 \text{ cm.}$$

Provide 4-legged 10 mm. ϕ *@ 20 cm. c/c.*

Design of the fulcrum girder

The fulcrum girder supports the raker beams. Hence the loading on the raker beam consists of (*i*) a number of concentrated loads transmitted by the raker beams and (*ii*) its own weight.

Concentrated load transmitted by each raker beam = 10235 *kg.*

Depth of the fulcrum girder = 100 *cm.*

∴ Dead load of the fulcrum girder

$$= 0{\cdot}5 \times 1 \times 2400 = 1200 \text{ kg./m.}$$

Providing a bearing of 60 *cm.* for the fulcrum girder effective span of the girder = 15 + 0·60 = 15·60 *m.*

Total load on the fulcrum girder

$$= (5 \times 10235) + 1200 \times 15{\cdot}6 = 69895 \text{ kg.}$$

∴ Reaction at each support

$$= \frac{69895}{2} = 34947{\cdot}5 \text{ kg.}$$

Fig. 406 shows the S F. and B.M. diagram for the fulcrum girder.

Max. B M. at the centre

$$= 34947{\cdot}5 \times 7{\cdot}80 - 1200 \times \frac{7{\cdot}8^2}{2} - 10235 \times 5 - 10235 \times 2{\cdot}5$$

$$= 199{,}320 \text{ kg. m.}$$

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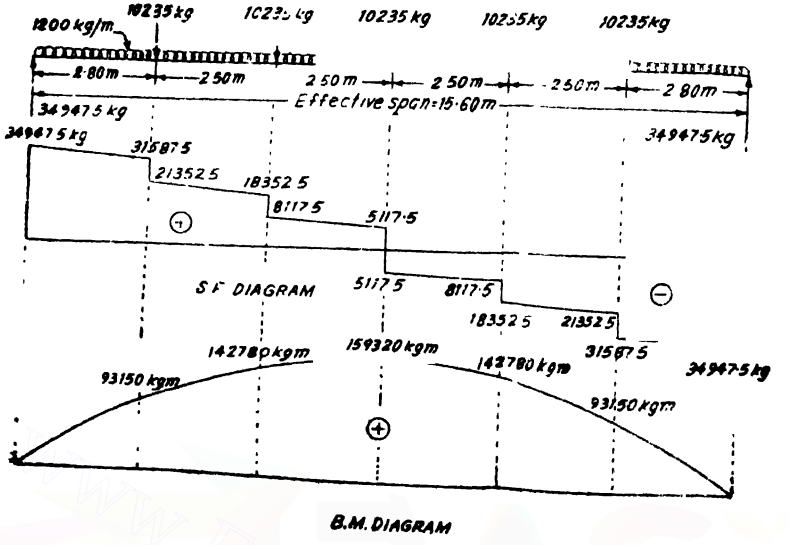

Fig. 405

Equating the M.R. to the maximum B M.

$$8{\cdot}50 \times 50\, d^2 = 159320 \times 100 \qquad \therefore \quad d = 194 \text{ cm.}$$

The overall depth of the fulcrum girder is only 100 *cm.*

The fulcrum girder will therefore be designed as a doubly reinforced beam.

Providing an effective cover of 8·5 *cm.* to the compression as well as tension reinforcement.

Distance between the centres of the top and bottom steel

$$= 100 - 2 \times 8{\cdot}5 = 83 \text{ cm.}$$

$$\therefore \qquad A_c = A_t = \frac{159320 \times 100}{1400 \times 83} = 137{\cdot}1 \text{ cm.}^2$$

Providing 40 *mm.* ϕ bars number of bars at top or bottom

$$\text{Number of bars} = \frac{137{\cdot}1}{12{\cdot}57} = 11 \text{ bars}$$

Let us provide 12 bars of 40 *mm.* diameter arranged in two layers with 6 bars per layer.

Check for shear

Maximum S.F. $= S = 34947{\cdot}5$ *kg.*

$$\therefore \quad \text{Shear stress} = \frac{S}{ab} = \frac{34947{\cdot}5}{83 \times 50} = 8{\cdot}42 \text{ kg./cm.}^2$$

Spacing of 4-legged 12 *mm.* diameter stirrups

$$= \frac{A_{wtwa}}{S} = \frac{4 \times 1{\cdot}13 \times 14000 \times 83}{34947{\cdot}5} = 15 \text{ cm.}$$

$\therefore$ *Provide 4-legged 12 mm. diameter stirrups at 15 cm. c/c.*

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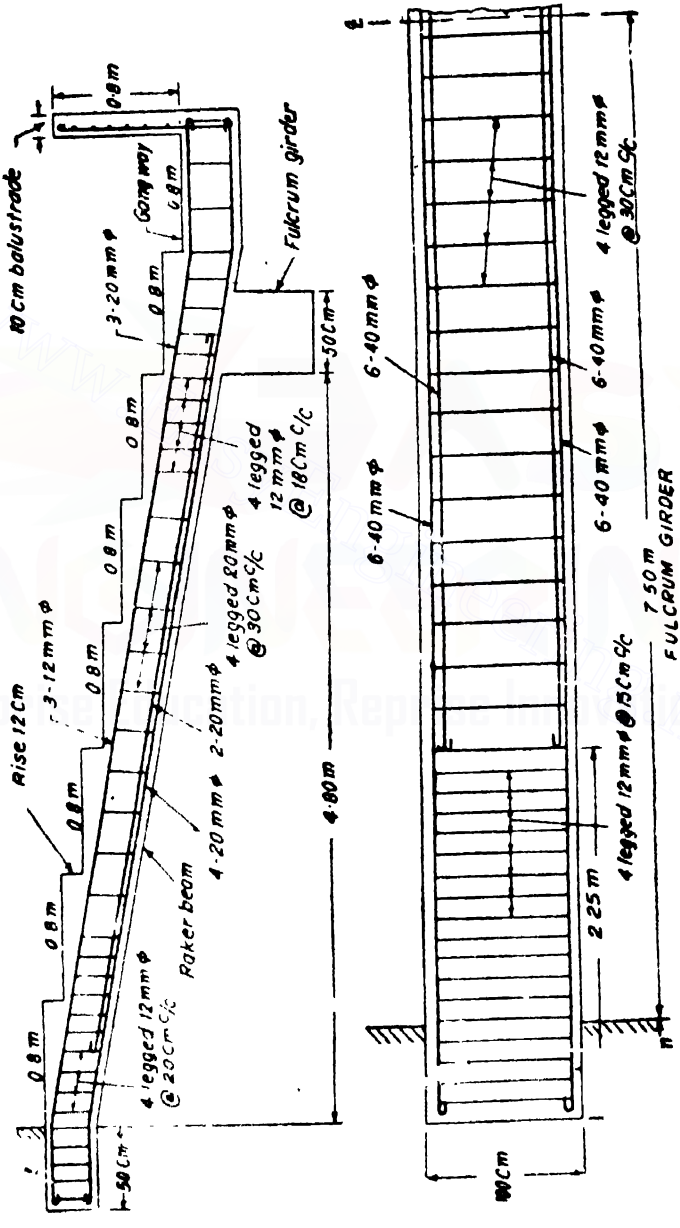

Fig. 407

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These stirrups will be provided for a distance of 2·50 *m.* from each support. In the remaining region provide nominal 4-legged 12 *mm.* diameter stirrups at a spacing of 30 *cm.* centres.

Curtailment of steel

Perimeter of bottom bars required from local bond stress consideration, near the supports

$$= \frac{S}{a s_b} = \frac{34947{\cdot}5}{83 \times 10} = 42{\cdot}10 \text{ cm.}$$

Perimeter of 1 bar $= \pi \times 4 = 12{\cdot}56$ *cm.*

$$\therefore \text{ Number of bars} = \frac{42{\cdot}10}{12{\cdot}56} = 4$$

Let the lower 6 bars of the bottom bars be present for this whole span. The other 6 bars of the upper layer will be curtailed at $\frac{1}{7}$ of the span *i.e.* at $\frac{1}{7}(15{\cdot}6) = 2{\cdot}23$ *m.* say 2·25 *m.* from the support. Similarly the bars may be curtailed in the compression zone also.

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15

Design of Water Tanks

A reinforced concrete tank is a very useful structure which is meant for the storage of water, for swimming baths, sewage sedimentation and for such similar purposes. The usual types of water tanks are the following :

(*i*) *Tanks situated on the ground.*

(*ii*) *Tanks situated under ground.*

(*iii*) *Tanks situated above ground level.*

The tanks may be either open or roofed over and they may be either circular or rectangular in plan.

Concrete used has to be watertight. This can be achieved by proper mixing and placing and curing of concrete. Well graded cement in richer proportion shall be used. The mix usually adopted is *M* 20 in S.I. designs and *M* 200 in M.K.S. designs (nearly 1 : 1½ : 3). The impermeability can be appreciably increased by asphalt lining or puddle backing.

§161. Stresses in Steel

In order to reduce any possible tensile cracking of concrete of the tank wall, it is usual to assume a working stress of 100 $N/mm.^2$ (1000 kg/cm^2) in steel reinforcement situated near the water face. The stress in the steel reinforcement situated away from the water face will be taken at 125 $N/mm.^2$ (1250 $kg./cm^2$.)

§162. Basis of Design

There are two important factors to be noted in the design of an R.C. tank. It is not enough if the structure is designed so as to have a desired strength. What is required in addition is *watertightness.*

It is necessary to prevent shrinkage cracks in the tank walls. This is possible by adopting a distribution reinforcement amounting to at least 0·3 percent of the gross area of the wall. Further, slabs should be concreted in lengths not greater than 7·5 *metres.* It is also necessary to provide contraction joints between adjacent slab units. In order to accommodate any additional thermal displacements, expansion joints shall also be provided at intervals of 30 *m.* For floors it is necessary that the ground should be first covered with a 8 *cm.* thick layer of plain concrete.

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Figs. 408 to 411 show the common forms of joints used in tanks.

163. Joints in Tanks

Rigid Joints These are joints provided to serve a continuity between a first cast concrete and a second cast concrete. When a good bond is required, it is usual that the whole face of the joint is completely removed for a depth of 3 *mm.* by jetting or bush hammering and a layer of 1½ : 1 sand cement mortar is spread on the face which is cast first immediately before the new concrete is placed. A typical detail of such a joint is shown in Fig. 408. A galvanised steel-water bar is also cast into the concrete work to prevent the passage of water across the line of the joint.

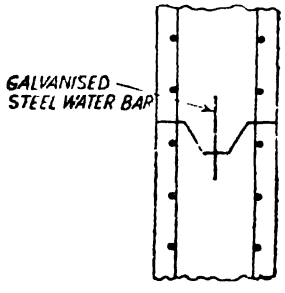

Fig. 408. Rigid Joint.

Flexible Joint. For larger structures flexible joints are preferred.

These joints are meant to provide a free movement. The joint faces should not be allowed to remain rough. They shall be finished evenly. The second-cast concrete should be worked into a layer of 1½ : 1 sand cement mortar which should be spread on to the first cast concrete face.

Fig. 409 (*a*) shows a flexible horizontal construction joint in the wall of a structure. Such joints are required for the sake of convenience in setting the form work and also to afford proper access for compaction of concreting.

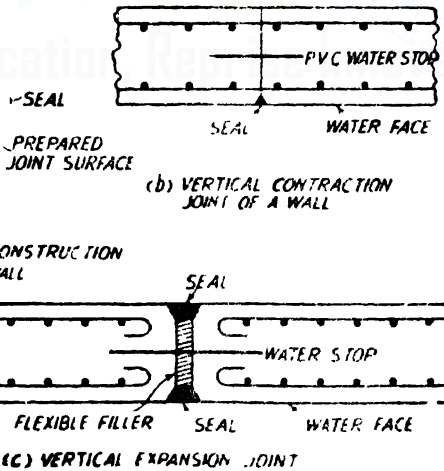

Fig. 409. Wall joints.

Fig. 409 (*b*) shows a vertical contraction joint in a wall. Such joints are to be provided at intervals of 7·5 *metres* at places of zero

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shear. The first placed concrete surface should be left smooth and unkeyed so that the joint is allowed to open as the concrete on either side cools or shrinks. Thus, all movements are localised or collected at pre-determined positions. A poly-vinyl chloride (PVC) water stop is found satisfactory. It is also necessary to provide a triangular fillet seal on the water face

Fig 409 (*c*) shows vertical expansion joint in a wall. Such joints are necessary at intervals of 30 *metres* at a place of zero shear. A flexible filler of 2 *cm.* thickness is provided in the joint. This allows the joint to partially close when the concrete on either side should expand. A suitable filler consists of cellular cane fibres

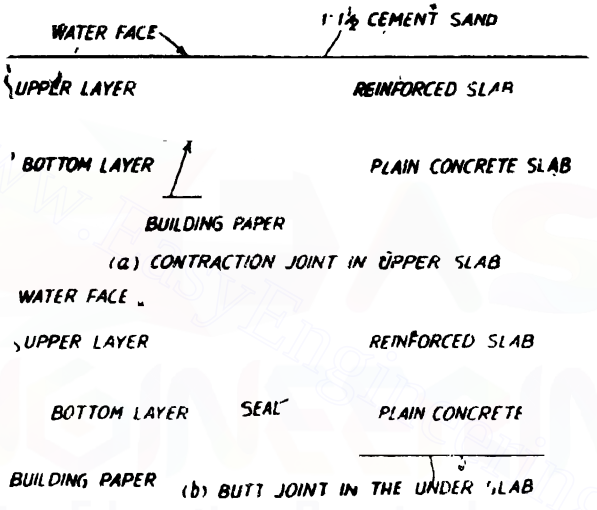

Fig. 410. Tank floor joints.

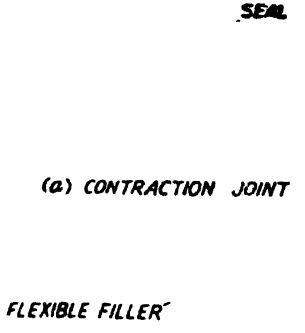

Fig. 411. Roof slab joints.

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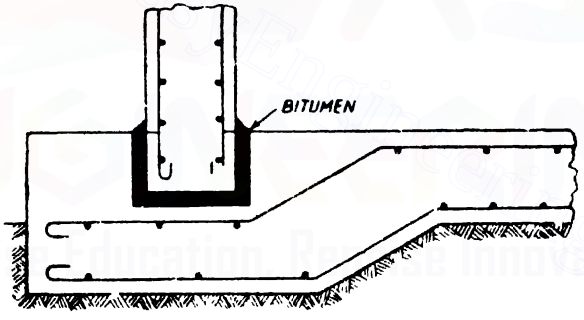
impregnated with bitumen The joint is sealed at both faces The seal on the water face is meant to provide watertightness while the seal on the outer face is meant to protect the flexible filler.

Bottom slabs of tanks. These slabs are bettor cast in two separate layers The joints of the layers are staggered Figs. 410 (*a*) and (*b*) show the various arrangements. The joints of the upper layer are contraction joints. A gap is left between adjacent panels. This gap should be filled only after seven days are over since concreting of the last adjacent slab. So far as the joints of the bottom layer are concerned they form butt joints The slab panels are cast alternately in chess-board fashion The slab in the upper layer is reinforced with at least 0 3 percent reinforcement.

Roof slab joints Contraction joints are provided at intervals of 7 5 *metres* at position of zero shear. The panels which are formed by the joints are cast alternately in a chessboard fashion. Expansion joints are provided at intervals of 30 *metres*. *See* Fig. 411.

Joints between wall and floor. Sometimes the design of the tank might allow the tank walls to be unrestrained at the base. Fig. 412 shows a usual arrangement for the joint.



CONNECTION BETWEEN VERTICAL WALL AND BASE OF TANK

Fig. 412

TANKS SITUATED ON GROUND LEVEL

§164. Circular Tanks

Circular tanks on ground may be designed either with flexible connection of the wall with the base or with a rigid connection of the wall with the base. In the former case the expansion or contraction of the side walls are possible. In the latter case the walls are monolithic with the base.

§165. Circular Tanks with a flexible joint between the walls and the base

The walls of such a tank will be designed as vertical cylinders subjected to water pressure. It is usual to apply the theory of thin cylinders in the analysis of the walls.

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The intensity of water pressure at any depth h is equal to wh units per unit area.

The corresponding hoop tension per *metre* height at this level is given by

$$T = \frac{wh \times D}{2}$$

where D = Internal diameter of the tank

The area of reinforcement to be provided should be calculated to resist the above tension. The permissible stress in steel will be taken as 100 $N/mm.^2$ (1000 $kg/cm.^2$)

For 1 *metre* height

$$A_s = \frac{whD}{2 \text{ (safe stress in steel)}} \ cm.^2$$

w = specific weight of water in $N/metre^3$ ($kg./metre^3$)

= 9810 $N/metre^3$ or 1000 $kg./metre^3$

h = depth in *metres*

D = Internal diameter in *metres*.

This hoop reinforcement may be provided preferably near both faces with a cover of 25 *mm*. However, the amount of steel provided shall not be less than 0·3% of the gross area.

The spacings of hoop steel may be increased at higher levels.

Thickness of wall. Even though the reinforcement has been provided so as to safely resist the hoop tension, the concrete *has not been prevented* to resist a part of the tension. Hence cracks are likely to be formed and the watertightness of the tank may be affected. To take this factor into account the tensile stress on the equivalent area of concrete is limited to a certain value. On the assumption that the concrete has not yet cracked, the tensile stress in concrete is given by,

$$c = \frac{T}{bt + (m-1)A_s}$$

where T = Hoop load on b units of height of wall

t = Thickness of tank wall

m = Modular ratio

In our case since T has been calculated per unit height of the wall ($b = 100\ cm$)

$$\therefore \quad c = \frac{T}{100t + (m-1)A_s} \ N/cm.^2 \ (kg./cm.^2)$$

(t should be taken in *cms.*)

The above stress may be limited to 120 $N/cm.^2$ (12 $kg./cm.^2$)

Hence the thickness of the wall may be such that the above stress on the equivalent area of concrete is not exceeded. It is con-

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sidered that walls of thicknesses less than 12 *cm.* are not advantageous since the cost of fixing reinforcement, pouring and tamping concrete will only further increase the cost of the structure. It is even preferred to adopt a thickness of at least 15 *cm.* Hence to sum up, the following rules may be adopted in finding the thickness of the walls.

The thickness of the wall should not be less than the following :

(*i*) 15 *cm.*

(*ii*) 3 *cm.* per *metre* depth + 5 *cm.*

(*iii*) Thickness required to limit the tensile stress in concrete to 120 $N/cm.^2$ (12 $kg./cm.^2$)

§166. Vertical Distribution Steel

A minimum percentage of 0·3% of the gross area of the wall will be provided. These bars will serve to hold not only the hoop bars but also the various layers of concrete.

§167. Bottom Slab

This may consist of a 15 *cm.* to 20 *cm.* thick slab with a reinforcement forming a top mesh and a bottom mesh. Usually a mesh with 10 *mm.* dia. bars at 25 *cms.* centres will be sufficient.

§168. Design Requirements

It is necessary that the reinforced concrete member for liquid retaining structures should be impervious. For reinforced concrete to be impervious the following two conditions must be satisfied :

(*a*) The concrete used must be of uniform well graded mix of low water-cement ratio and it should be fully compacted and should be free from all defects such as segregation and honeycombing.

(*b*) The concrete must be free from cracks.

Cracks in reinforced concrete. Reinforced concrete may develop various types of cracks. The cracks are generally of the following types :

(*a*) Very fine well distributed cracks in the tension zone of a concrete member.

(*b*) Excessive cracks in the tension zone of a concrete member caused by overloading it.

(*c*) Cracks caused by expansion and contraction of restrained members.

(*d*) Cracks caused by differential expansion in thick members due to the heat of hydration.

(*e*) Cracks caused by shrinkage of concrete.

(*f*) Cracks caused by settlement of a structure.

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Avoiding normal tensile cracks. The very fine cracks which occur in the tension zone of a member designed by the normal methods must be avoided in impervious reinforced concrete by designing it in two ways :

(*a*) By designing against cracking having regard to the tensile strength of concrete.

(*b*) By designing for structural strength ignoring the tensile strength of concrete.

It is worthwhile to note the following points concerning this aspect of design :

(*i*) The safety factor against cracking is less than the safety factor required for structural safety.

(*ii*) Since the safety factor in design against cracking is relatively small, the possibility of tensile cracks occurring must be envisaged. Hence to restrict the width and depth of such potential cracks a low permissible tensile stress in steel is adopted in the structural design.

Inspite of this general rule, there are many instances where designers design members for structural strength alone when the thickness is 25 *cm.* or more and if the liquid retained is in contact with the compression face only.

In addition to these steps taken to prevent tensile cracks the creep of concrete may turn out to be a practical advantage to prevent cracking by applying the working load slowly. This is often possible in the case of a tank or reservoir by slow filling.

Avoidance of other cracks

Cracks caused due to expansion and contraction due to shrinkage can be avoided by the proper use of movement joints in reinforced concrete. Shrinkage cracks may also be avoided by slow drying out of concrete.

Cracks caused by differential expansion in thick members due to the heat of hydration are not likely to occur in members less than 45 *cm.* thick.

Cracks caused by settlement can be minimized or avoided by careful site selection and good foundation design and construction.

§16. IS Design Requirements (IS 3370)

The following are the general design requirements recommended by the I.S. code :

(*i*) Plain concrete members of reinforced concrete liquid retaining structures may be designed against structural failure by allowing tension in plain concrete as per the permissible limits for tension in bending (IS 456). This introduces the necessary provision to prevent failure due to cracking. However, nominal reinforcement shall be provided for plain concrete structural members complying with the requirements of IS : 456.

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(i) Permissible stresses in concrete

In all calculations pertaining to the resistance of members to cracking, the permissible stresses in tension (direct as well as due to bending, shall conform to the values specified in the table below. The permissible stresses in tension due to bending are applicable to the face of the member in contact with the liquid. In the case of members of thickness less than 225 *mm.* and in contact with liquid on one side, the permissible stresses in bending specified in the table apply also to the face remote from the liquid.

Permissible stresses in concrete in computations pertaining to resistance to cracking

S.I. UNITS			*M.K.S. UNITS*		
Grade of concrete	*Permissible tensile stress N/cm.²*		*Grade of concrete*	*Permissible stress in kg./cm²*	
	Direct	*Bending*		*Direct*	*Bending*
M 15	110	150	M 150	11	15
M 20	120	170	M 200	12	17
M 25	130	180	M 250	13	18
M 30	150	200	M 300	15	20
M 35	160	220	M 350	16	22
M 40	170	240	M 400	17	24

(ii) Permissible streses in steel

(a) Cracking stress consideration. When steel and concrete are assumed to act together while determining the tensile stress in concrete to avoid cracks, it is necessary that the tensile stress in steel is so that the permissible tensile stress in concrete is not exceeded. From this consideration the tensile stress shall not exceed the product of the modular ratio and the corresponding allowable tensile stress in concrete.

(b) Strength consideration. The permissible stress in steel in strength calculations shall be taken as given in the stable below.

(iv) Stresses due to drying shrinkage or temperature change

Stresses due to drying shrinkage or temperature change may be ignored provided that :

(a) The permissible stresses specified in the above table are not otherwise exceeded.

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(*b*) Adequate precautions are taken to prevent cracking of concrete at the time of construction and until the reservoir is put to use.

(*c*) Recommendations regarding joints explained in the earlier articles for suitable sliding layer beneath the reservoir are complied with or the reservoir is meant to be used for storing water or aqueous liquids at or near ambient temperature and that the conditions are such that the concrete is never liable to dry out.

In special cases shrinkage stresses may however have to be calculated. A shrinkage coefficient of 300×10^{-6} may be assumed. When shrinkage stresses are allowed the permissible tensile stress in concrete may be increased by 33⅓ per cent.

Permissible Stress in Steel (strength consideration)

Nature of tensile stress	*Permissible tensile stress*	
	*N/cm.*²	*kg/cm.*²
Tensile stress due to direct tension	10,000	1000
Tensile stress due to bending in steel near the liquid face or away from the liquid face when the thickness of the member is less than 225 *mm.*	10,000	1000
Tensile stress in steel provided away from the liquid face in members of thickness greater than 225 *mm.*	12,500	1250
Tensile stress in shear reinforcements :		
When the thickness of members is less than 225 *mm.*	10,000	1000
When the thickness of members is greater than 225 *mm.*	12,500	1250
Compressive stress in steel in R.C.C. columns subjected to direct load.	12,500	125

Note 1. The limitations of stress for liquid retaining faces shall also apply to :

(1) Other faces within 225 *mm.* of the liquid retaining face.

(2) Outside or external faces of structures away from the liquid but provided in water-logged soils up to the level of the highest sub-soil water level.

Note 2. The permissible stress 10,000 *N/cm.*² (1000 *kg./cm.*²) in the table above may be increased to 11250 *N/cm.*² (1125 *kg/cm.*²) for plain mild steel bars when the cross reinforcement is spot welded to the main reinforcement.

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Design 100 (S.I.). *Design a circular tank to the following particulars :*

(i) Diameter of tank = 3·50 metre.

(ii) Depth of water = 3 metre.

(iii) The tank rests on ground.

(iv) The walls and base slab are not monolithic with each other.

(v) Specific weight of water = 9810 N/metre³.

Solution. Consider the bottom 1 *metre* height of wall. Pressure intensity corresponding to the centre of the bottom one *metre* height of wall,

$$= p = wh = 9810 \times 2{\cdot}5 = 24525 \text{ N per metre}^2$$

$$\text{Hoop tension} = T = \frac{pD}{2} = \frac{24525 \times 3{\cdot}5}{2} = 42919 \text{ N}$$

Steel required for 1 *metre* height

$$= \frac{42919}{10000} = 4{\cdot}3 \text{ cm.}^2$$

Spacing of 10 *mm.* ϕ bars

$$= \frac{0{\cdot}79 \times 100}{4{\cdot}3} = 18{\cdot}4 \text{ cm.}$$

Provide 10 mm. ϕ *@ 18 cm. c/c (4·4 cm.²)*

If the steel is provided near both the faces, the spacing of bars will be at 36 *cm. c/c.*

Thickness of wall. This shall not be less than the following :

(*i*) 15 *cm*

(*ii*) 3 *cm.* per *metre* depth + 5 *cm.* = (3 × 3) + 5 = 14 *cm.*

(*iii*) Thickness required to limit the tensile stress in steel to 120 *N/cm*² ($m = 13$)

$$120 = \frac{42919}{100t + 12 \times 4{\cdot}4} \qquad \therefore \quad t = 3{\cdot}04 \text{ cm.}$$

Hence let us provide an overall thickness of 15 *cm.*

Vertical Reinforcement (distribution steel)

This shall be at least 0·3% of the gross area,

$$= \frac{0{\cdot}3}{100} (15 \times 100) = 4{\cdot}5 \text{ cm.}^2 \text{ per metre run}$$

Spacing of 10 *mm.* ϕ bars

$$= \frac{0{\cdot}79 \times 100}{4{\cdot}5} = 17{\cdot}5 \text{ cm.}$$

say 10 mm. ϕ *@ 17 cm. c/c.*

If the reinforcement is provided near both the faces, the spacing of these 10 mm. ϕ *vertical bars will be @ 34 cm. c/c.*

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Base slab. This will be a 15 *cm.* thick slab with a top mesh and a bottom mesh of steel with 10 *mm.* ϕ bars at 24 *cm. c/c.* Fig. 413 shows the details of reinforcement.

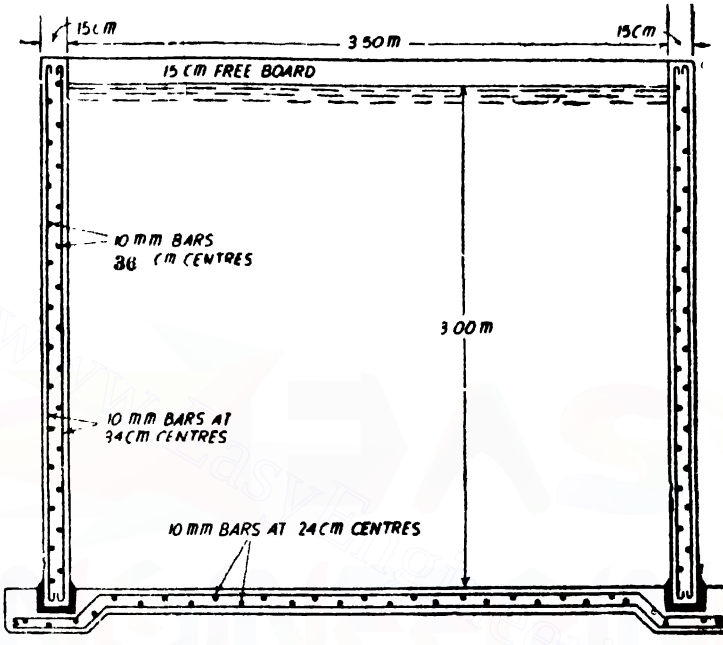

Fig. 413. Circular tank with a flexible connection of the wall with the base

Design 102. *Design a circular tank to the following requirements :*

(i) Diameter of tank = 3·50 metres.

(ii) Depth of water = 3·00 metres.

(iii) The tank rests on ground.

(iv) The walls and the base slab are not monolithic.

Solution. Consider the bottom 1 *metre* height of wall Pressure intensity corresponding to centre of the bottom one *metre* height of wall,

$$= 1000 \times 2{\cdot}5 = 2500 \ kg./m.^2$$

$\therefore$ Hoop tension $= \dfrac{2500 \times 3{\cdot}5}{2} \ kg.$

$$= 4375 \ kg$$

$\therefore$ Steel for 1 *metre* height

$$= \frac{4375}{1000} = 4{\cdot}375 \ cm.^2$$

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$\therefore$ Spacing of 10 *mm.* bars

$$= \frac{0{\cdot}79 \times 100}{4{\cdot}37} = 18{\cdot}1 \text{ cm.}$$

say 10 mm. bars at 18 cm. centres ($4{\cdot}4$ *cm.*^{2})

If these bars are provided at both faces the spacing would be at 36 *cm* centres.

Thickness of wall. This shall not be less than the following.

(*i*) 15 *cm.*

(*ii*) 3 *cm.* per *metre* depth $+5$ *cm.* $= 3 \times 3 + 5 = 14$ *cm.*

(*iii*) Thickness required to limit the tensile stress in concrete to 12 *kg./cm*^{2} $m = 13$)

$$12 = \frac{4375}{100t + 12 \times 4{\cdot}4}$$

$$\therefore \quad t = 3{\cdot}12 \text{ cm.}$$

Hence let us prove an overall thickness of 15 *cm.*

Vertical reinforcement

This shall be at least 0·3% of the gross area

$$= \frac{0{\cdot}3}{100} \times 15 \times 100 = 4{\cdot}5 \text{ sq. cm. per metre run}$$

Spacing of 10 *mm.* ϕ bars

$$= \frac{0{\cdot}79 \times 100}{4{\cdot}5} = 17{\cdot}5 \text{ cm. say 17 cm. c/c.}$$

If the reinforcement is provided near both the faces, the spacing of these 10 mm. ϕ *vertical bars will be at 34 cm. c/c.*

Bottom slab. This will be a 15 *cm.* thick slab with a top mesh and a bottom mesh of steel with 10 *mm* ϕ bars at 24 *cm. c/c.*

Fig. 413 shows the details of reinforcement.

§170. Circular Tanks with the wall restrained at the base

In this case the wall will resist the water pressure partly by hoop action and partly by cantilever action. Deformation observations reveal that for a certain height from the base there will be predominant cantilever action, and that at higher levels there will be predominant hoop action. (*See* Fig. 414).

The tanks of this category are analysed by the following methods :

(*i*) ***Dr. Reissner's Method***

(*ii*) ***Carpenter's Simplification of Dr. Reissner's Method.***

(The simplified method by Carpenter may be found more easy to analyse tanks.)

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(*i*) **Dr. Reissner's Method.** According to Reissner the load distribution curves for tanks with equal values of a constant K will be similar ; where $K = \dfrac{12\,H^4}{\left(\dfrac{D}{2}\right)^2 t^2}$

where H = Height of the tank wall.

D = Internal diameter of the tank.

t = Thickness of the wall.

The cantilever bending moment M_r for the various values of K are given in Table 1.

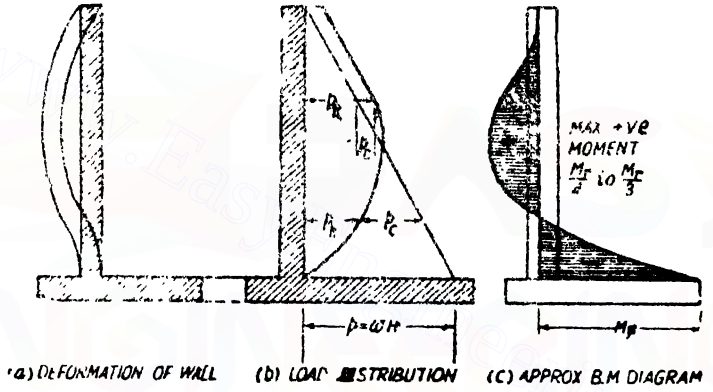

Fig. 414. Tank wall restraint at the base.

Table 1. Values of Restraint Moment M_r.

($p = wH$)

K	*Rectangular wall section*	*Triangular wall section*
0	0·167 pH^2	0·167 pH^2
10	0·110 pH^2	0·140 pH^2
100	0·0582 pH^2	0·0707 pH^2
1000	0·024 pH^2	0·026 pH^2
10000	0·0085 pH^2	0·009 pH^2
∞	0	0

The position and magnitude of maximum ring tension can be obtained from the table given below :

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Table 2. Ring Tension in Tank Wall

($p = wH$)

K	Rectangular wall section		Triangular wall section	
	Max. Tension	Height from base	Max. Tension	Height from base
0	0	—	0	—
10	$0.18\ p\left(\frac{D}{2}\right)$	$1.0\ H$	$0.09\ p\left(\frac{D}{2}\right)$	$0.65\ H$
100	$0.27\ p\left(\frac{D}{2}\right)$	$1.0\ H$	$0.31\ p\left(\frac{D}{2}\right)$	$0.58\ H$
1000	$0.47\ p\left(\frac{D}{2}\right)$	$0.47\ H$	$0.52\ p\left(\frac{D}{2}\right)$	$0.44\ H$
10000	$0.67\ p\left(\frac{D}{2}\right)$	$0.31\ H$	$0.70\ p\left(\frac{D}{2}\right)$	$0.30\ H$
∞	$1.0\ p\left(\frac{D}{2}\right)$	0	$1.0\ p\left(\frac{D}{2}\right)$	0

Design 103 (S.I.). *A circular water tank has an internal diameter of 12 metres, the maximum depth of water being 4 metres. The walls of the tank are restrained at the base. The tank rests on ground. Design the tank.*

Solution. $H = 4$ *metres*

$D = 12$ *metres*

Thickness of wall. This shall be not less than

(*i*) 15 *cms.*

(*ii*) 3 *cm.* per *metre* depth + 5 *cm.* $= 3 \times 4 + 5 = 17$ *cm.*

Hence let us provide a thickness of 17 *cm.*

$$K = \frac{12\,H^4}{\left(\frac{D}{2}\right)^2 t^2} = \frac{12 \times 4^4}{6^2 \times (0.17)^2} = 2953$$

From table 1, we find

For $K = 1000$ $\quad M = 0.024\ pH^2$

$K = 10{,}000$ $\quad M = 0.0085\ pH^2$

(Adopting logarithmic interpolation),

For $K = 2953$, $\quad M = \alpha\ pH^2$

where, $\alpha = 0.0085 + (0.024 - 0.0085)\ \frac{\log 10{,}000 - \log 2953}{\log 10{,}000 - \log 1000}$

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$$= 0{\cdot}0085 + (0{\cdot}024 - 0{\cdot}0085) \times 0{\cdot}5298$$
$$= 0{\cdot}0167$$
$$M = 0{\cdot}0167\, pH^2$$
$$M = 0{\cdot}0167\, (9810 \times 4)\, 4^2\ Nm.$$
$$= 10485\ Nm. = 1048500\ N\ cm.$$

Adopting $c = 700\ N\ cm.^2$, $t = 10{,}000\ N/cm.^2$ and $m = 13$

$n_1 = 0{\cdot}476\ d$, $a = 0{\cdot}84\ d$, $Q = 140{\cdot}3$

Equating the moment of resistance to the bending moment, we have,

$$140{\cdot}3 \times 100\, d^2 = 1048500$$

$$\therefore \quad d = 8{\cdot}64\ cm.$$

But effective depth available to the centre of 12 *mm.* ϕ bars

$$= 17 - 2{\cdot}5 - 0{\cdot}6 = 13{\cdot}90\ cm.$$

$$A_t = \frac{1048500}{10000 \times 0{\cdot}84 \times 13{\cdot}90} = 9\ cm.^2$$

Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{9} = 12{\cdot}5\ cm.$$

Provide 12 mm. ϕ *@ 12 cm. c/c.*

Hoop tension. From table 2, we have

For $K = 1000$, max. hoop tension

$$T = 0{\cdot}47\, p\left(\frac{D}{2}\right)$$

at $0{\cdot}47\ H$ from the base

For $K = 10000$,

$$T = 0{\cdot}67\, p\left(\frac{D}{2}\right)$$

at $0{\cdot}31\ H$ from the base.

Following a logarithmic interpolation,

max. hoop tension for $K = 2953$ will be equal to $\beta p\left(\frac{D}{2}\right)$

where

$$\beta = 0{\cdot}47 + (0{\cdot}67 - 0{\cdot}47)\, \frac{\log 2953 - \log 1000}{\log 10000 - \log 1000}$$
$$= 0{\cdot}47 + 0{\cdot}20 \times 0{\cdot}4702$$
$$= 0{\cdot}47 + 0{\cdot}094 = 0{\cdot}564$$

$\therefore$ **Max. hoop tension**

$$= 0{\cdot}564\, p\left(\frac{D}{2}\right)$$

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$$= 0{\cdot}564\ (9810 \times 4)\ \frac{12}{2}\ N$$

$$= 132788\ N$$

This occurs at γH from the base

where $\gamma = 0{\cdot}47 - (0{\cdot}47 - 0{\cdot}31) \times 0{\cdot}4702$

$= 0{\cdot}40$

∴ Maximum hoop tension will occur at $0{\cdot}4\ H$ from the base, *i.e.* at a height of $0{\cdot}4 \times 4 = 1{\cdot}6$ *metre* from the base

Steel for hoop tension $= \frac{132788}{10{,}000} = 13{\cdot}28\ cm.^2$

Spacing of 12 *mm.* ϕ bars $= \frac{1{\cdot}13 \times 100}{13{\cdot}28} = 8{\cdot}5$ *cm. say 8 cm. c/c.*

Since this steel is provided in the form of rings near both the faces the spacing of the rings will be @ 16 cm. c/c near each face. Fig. 433 shows the details of reinforcement.

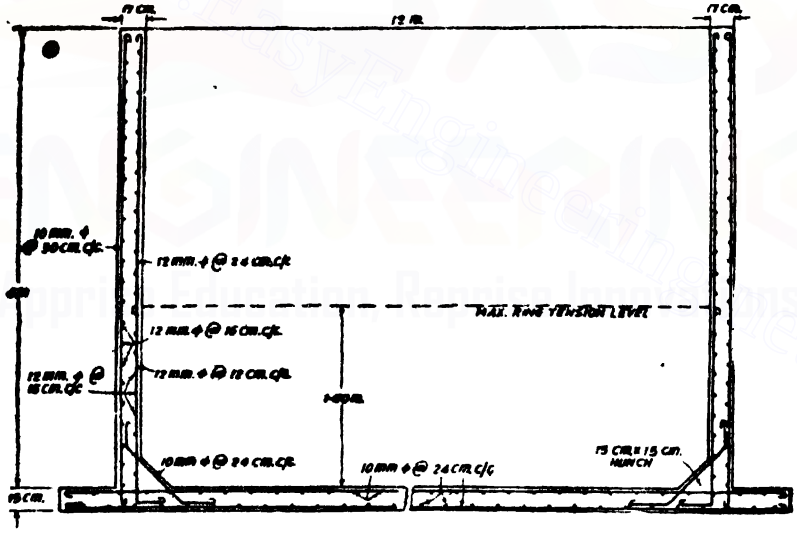

Fig. 415.

Check for tensile stress in concrete

Tensile stress $= \frac{132788}{100 \times 17 + (13 - 1)13{\cdot}54}$

$= 71\ N/cm.^2$ (less than $120\ N/cm.^2$)

Base slab This slab may be 15 *cm.* thick with a top mesh and a bottom mesh of steel with 10 *mm.* ϕ bars @ 24 *cm. c/c.* It is also used to provide 15 *cm.* × 15 *cm.* haunches at the junction of

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wall and base. A haunch reinforcement of 10 *mm* ϕ @ 24 *cm. c/c* may be provided.

Design 101. *A circular water tank has an internal diameter of 12 metres, the maximum depth of water being 4 metres. The walls of the tank are restrained at the base. The tank rests on ground Design the tank.*

Solution. $H=4$ *metres*, $D=12$ *metres.*

Thickness of wall. This shall be not less than

(*i*) 15 *cm.*

(*ii*) 3 *cm.* per *metre* depth $+5$ *cm.* $=3\times4+5=17$ *cm.*

Hence let us provide a thickness of 17 *cm.*

$$K=\frac{12H^4}{\left(\frac{D}{2}\right)^2 t^2}=\frac{12\times4^4}{6^2\times(0{\cdot}17)^2}=2953$$

From table 1, we find

For $K=1000$ $\quad M=0{\cdot}024\ pH^2$

$K=10{,}000$ $\quad M=0{\cdot}0085\ pH^2$

(Adopting logarithmic interpolation),

For $K=2953$, $\quad M=\alpha pH^2$

where, $\alpha=0{\cdot}0085+(0{\cdot}024-0{\cdot}0085)\dfrac{\log 10{,}000-\log 2953}{\log 10{,}000-\log 1000}$

$=0{\cdot}0085+(0{\cdot}024-0{\cdot}0085)\times0{\cdot}5298$

$=0{\cdot}0167$

$M=0{\cdot}0167\ pH^2$

$=0{\cdot}0167(1000\times4)4^2$ *kg. m.*

$=1069$ *kg. m.*

Adopting $\quad c=70$ *kg./cm.*^{2} $\quad t=1000$ *kg./cm.*^{2}

$m=\dfrac{2800}{3c}$

$=13$

$n=0{\cdot}476\ d$, $\quad a=0{\cdot}84\ d$, $\quad Q=14{\cdot}03$

Equating the moment of resistance to the maximum bending moment, we have,

$14{\cdot}03\times100\ d^2=1069\times100$

$\therefore \quad d=8{\cdot}73$ *cm.*

But available effective depth to the centre of 12 *mm.* dia. bars

$=17-2{\cdot}5-0{\cdot}6=13{\cdot}9$ *cm.*

$$A_t=\frac{1069\times100}{1000\times0{\cdot}84\times13{\cdot}9}\ \text{cm.}^2=9{\cdot}16\ \text{cm.}^2$$

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$\therefore$ Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{9{\cdot}16} \ cm$$

$$= 12{\cdot}3 \ cm. \qquad \text{say } 12 \ cm. \ c/c.$$

Hoop tension. From table 2, we have

For $K = 1000$, max. hoop tension

$$T = 0{\cdot}47 \ p \left(\frac{D}{2} \right)$$

at $0{\cdot}47 \ H$ from the base

For $K = 10000$,

$$T = 0{\cdot}67 \ p \left(\frac{D}{2} \right)$$

at $0{\cdot}31 \ H$ from the base.

Following a logarithmic interpolation,

Max. hoop tension for $K = 2953$ will be equal to $\beta p \left(\frac{D}{2} \right)$

where

$$\beta = 0{\cdot}47 + (0{\cdot}67 - (0{\cdot}47) \frac{\log 2953 - \log 1000}{\log 10000 - \log 1000}$$

$$= 0{\cdot}47 + 0{\cdot}20 \times 0{\cdot}4702$$

$$= 0{\cdot}47 + 4{\cdot}094 = 0 \ 564$$

$\therefore$ Max. hoop tension $= 0{\cdot}564 \ p \left(\frac{D}{2} \right)$

$$= 0{\cdot}564 \times (1000 \times 4) \frac{12}{2} \ kg.$$

$$= 13536 \ kg.$$

This occurs at γH from the base

where $\gamma = 0{\cdot}47 - (0{\cdot}47 - 0{\cdot}31) \times 0{\cdot}4702$

$$= 0{\cdot}40$$

$\therefore$ Maximum hoop tension will occur at $0{\cdot}4 \ H$ from the base *i.e.* at a height of $0{\cdot}4 \times 4 = 1{\cdot}6$ *metre* from the base.

Steel for hoop tension

$$= \frac{13536}{1000} \ cm^2.$$

$$= 13{\cdot}54 \ cm^2$$

$\therefore$ Spacing of 12 *mm.* bars $= \frac{1{\cdot}13 \times 100}{13{\cdot}54} \ cm.$

$$= 8{\cdot}35 \ cm. \quad \textit{say } 8 \ cm. \ c/c.$$

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Since this steel is provided in the form of rings near both the faces, the spacing of the rings near each face will be @ 16 *cm. c/c.*

Check for tensile stress in concrete

$$\text{Tensile stress} = \frac{13536}{100 \times 17 + (13-1)13\,54}$$

$$= 7{\cdot}3 \text{ kg./cm.}^2 \text{ (less than 12 kg./cm.}^2\text{)}$$

Base slab. This slab may be 15 *cm.* thick with a top mesh and a bottom mesh of steel with *10 mm.* ϕ @ *24 cm. c/c.* It is also usual to provide *15 cm.* × *15 cm* haunches at the junction of wall and base slab. A haunch reinforcement of *10 mm.* ϕ *bars* @ *24 cm. c/c* may be provided.

Circular tanks with the wall restrained at the base—Carpenter's simplification of Dr. Reissner's Method.

As stated earlier, the design of tanks with Carpenter's simplification will be found comparatively easier than the previous method.

The maximum cantilever bending moment and the position and amount of maximum ring tension are given by the following relations :

Let H = Height of wall

D = Internal Diam. of Tank

t = Thickness of Tank wall.

(*i*) Position of maximum ring tension = KH from the base where K = a coefficient depending on the values of $\frac{H}{D}$ and $\frac{H}{t}$.

Table 3. Coefficients F and K.

FACTORS		*F*				*K*			
H/t :		10	20	30	40	10	20	30	40
Values of H/D	0·2	0·046	0·025	0·017	0·015	—	0·50	0·45	0·40
	0·3	0·032	0·019	0·014	0·010	0·55	0·43	0·38	0·33
	0·4	0·024	0·014	0·010	0·007	0·50	0·39	0·35	0·30
	0·5	0·020	0·012	0·009	0·006	0·45	0·37	0·32	0·27
	1·0	0·012	0·006	0·5	0·003	0·37	0·28	0·24	0·21
	2·0	0·006	0·003	0·002	0·002	0·30	0·22	0·19	0·16
	4·0	0·004	0·002	0·002	0·001	0·27	0·20	0·17	0·14

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Refer to table 3 find K.

(*ii*) Max. hoop tension

= Hoop tension at KH *metres* above base

$$= w(H-KH)\frac{D}{2} = \frac{w}{2}(1-K)HD \quad ..$$

(*iii*) Max. cantilever bending moment

$$= FwH^3$$

where F=a coefficient depending on the values of $\frac{H}{D}$ and $\frac{H}{t}$.

Refer to table 3 and find F.

The following examples show the use of the following table.

Design 104 (S.I.). *A circular water tank has an internal diameter of 12 metres with 4 metres depth of water. The walls of the tank are restrained at the base. The tank rests on the ground. Design the tank with carpenter's simplifications.*

Solution. Thickness of wall. This shall be not less than.

(*i*) 15 *cm.*

(*ii*) 3 *cm. per metre depth* + 5 *cm.* = 12 + 5 = 17 *cm.*

Let us adopt 17 *cm* overall thickness.

Thus, in this case, $H = 4$ *m.*

$D = 12$ *m.*

$t = 0{\cdot}17$ *m*

$$\therefore \quad \frac{H}{D} = \frac{4}{12} = 0{\cdot}33 \quad \text{and} \quad \frac{H}{t} = \frac{4}{0{\cdot}17} = 23{\cdot}5$$

Referring to table 3, we have

For $\frac{H}{D} = 0{\cdot}3$ and $\frac{H}{t} = 20$, $F = 0{\cdot}019$ and $K = 0{\cdot}43$

and For $\frac{H}{D} = 0{\cdot}3$ and $\frac{H}{t} = 30$, $F = 0{\cdot}014$ and $K = 0{\cdot}38$

∴ By linear interpolation,

For $\frac{H}{D} = 0{\cdot}3$ and $\frac{H}{t} = 23{\cdot}5$, $F = 0{\cdot}017$ and $K = 0{\cdot}41$ ...(1

Again

For $\frac{H}{D} = 0{\cdot}4$ and $\frac{H}{t} = 20$, $F = 0{\cdot}011$ and $K = 0{\cdot}39$

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and for $\frac{H}{D}=0{\cdot}4$ and $\frac{H}{t}=30$, $F=0{\cdot}010$ and $K=0{\cdot}35$

$\therefore$ By linear interpolation

For $\frac{H}{D}=0{\cdot}4$ and $\frac{H}{t}=23{\cdot}5$, $F=0{\cdot}013$ and $K=0{\cdot}38$ ...(2)

Hence from relations (1) and (2), we have by further linear interpolation,

For $\frac{H}{D}=0{\cdot}33$ and $\frac{H}{t}=23{\cdot}5$, $F=0{\cdot}016$ and $K=0{\cdot}40$

$\therefore$ Position of maximum ring tension

$$=KH$$

$$=0{\cdot}40\times 4=1{\cdot}6 \text{ metres from the base}$$

or $4-1{\cdot}6=2{\cdot}4$ *metres* from the top.

Pressure intensity at this level

$$=9810\times 2{\cdot}4=23544 \ N/metre^2$$

Maximum hoop tension

$$=23544\times\frac{12}{2}=141264 \ N$$

Safe stress in steel $=10{,}000 \ N/cm.^2$

$$\therefore \quad A_s=\frac{141264}{10{,}000}=14{\cdot}12 \ cm.^2$$

Spacing of 12 *mm.* ϕ bars

$$=\frac{1{\cdot}13\times 100}{14{\cdot}12}=8 \ cm. \ c/c.$$

Since the hoop stress is provided near both the faces, *provide 12 mm.* ϕ *@ 16 cm. c/c near each face* ($14{\cdot}12 \ cm^2$.)

Check for tensile stress in concrete

Tensile stress in concrete

$$=\frac{141264}{(17\times 100)+(13-1)14{\cdot}12} \ N/cm.^2$$

$$=76 \ N/cm.^2 \ (less \ than \ 120 \ N/cm.^2)$$

Maximum bending moment

$$=F\,wH^3$$

$$=0{\cdot}016\times 9810\times 4^3=10045 \ Nm.$$

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Taking $c=700\ N/cm.^2$, $t=10{,}000\ N/cm.^2$, $m=13$, $a=0{\cdot}84$ and $Q=140{\cdot}3$ and equating the moment of resistance to the bending moment

$$140{\cdot}3\times 100\ d^2=1004500$$

$$\therefore \quad d=8{\cdot}5\ cm.$$

But effective depth available (overall depth − clear cover to hoop bar − diameter of hoop bar − half diameter of vertical bar).

$$=17-2{\cdot}5-1{\cdot}2-0{\cdot}6=12{\cdot}7\ cm.$$

$$\therefore \quad A_t=\frac{1004500}{10000\times 0{\cdot}84\times 12{\cdot}7}=9{\cdot}4\ cm.^2$$

Spacing of 12 *mm.* $\phi=\dfrac{1{\cdot}13\times 100}{9{\cdot}4}=12{\cdot}02\ cm.$

Provide 12 mm. ϕ @ 12 cm. c/c.

Distribution steel $=\dfrac{0{\cdot}3}{100}\times 17\times 100=5{\cdot}10\ cm.^2$

Spacing of 12 *mm* ϕ bars

$$=\frac{1{\cdot}13\times 100}{5{\cdot}1}=22\ cm.$$

Spacing of bars near each face = 44 *cm. c/c.*

Fig. 415 shows the details of reinforcement.

We find that there is no appreciable difference in results between Reissner's method and Carpenter's method.

The bottom slab will be 15 *cm.* thick with a top mesh and a bottom mesh of *10 mm. ϕ @ 24 cm. c/c.*

Design 105. *Design a circular tank 12 metre diameter with 4 metre depth of water. The walls of the tank are monolithic with the base. The tank rests on the ground. Design the tank with Carpenter's simplification.*

Solution.

Thickness of wall. This shall be not less than,

(*i*) 15 *cm.*

(*ii*) *3 cm. per metre depth* + 5 *cm.* = 12 + 5 = 17 *cm.*

Let us adopt 17 *cm.* overall thickness

Thus, in this case, $H=4\ m.$

$$D=12\ m.$$

$$t=0{\cdot}17\ m.$$

$$\therefore \frac{H}{D}=\frac{4}{12}=0{\cdot}33 \text{ and } \frac{H}{t}=\frac{4}{0{\cdot}17}=23{\cdot}5$$

Referring to table 3, we have,

for $\dfrac{H}{D}=0{\cdot}3$ and $\dfrac{H}{t}=20$, $F=0{\cdot}019$ and $K=0{\cdot}43$

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and For $\frac{H}{D}=0{\cdot}3$ and $\frac{H}{t}=30$, $F=0{\cdot}014$ and $K=0{\cdot}38$

$\therefore$ By linear interpolation,

For $\frac{H}{D}=0{\cdot}3$ and $\frac{H}{t}=23{\cdot}5$, $F=0{\cdot}017$ and $K=0{\cdot}41$ ...(1)

Again,

For $\frac{H}{D}=0{\cdot}4$ and $\frac{H}{t}=20$, $F=0{\cdot}014$ and $K=0{\cdot}39$

and For $\frac{H}{D}=0{\cdot}4$ and $\frac{H}{t}=30$, $F=0{\cdot}010$ and $K=0{\cdot}35$

$\therefore$ By linear interpolation

For $\frac{H}{D}=0{\cdot}4$ and $\frac{H}{t}=23{\cdot}5$, $F=0{\cdot}013$ and $K=0{\cdot}38$ ...(2)

Hence from relations (1) and (2), we have by further linear interpolation,

For $\frac{H}{D}=0{\cdot}33$ and $\frac{H}{t}=23{\cdot}5$, $F=0{\cdot}016$ and $K=0{\cdot}40$

$\therefore$ Position of maximum ring tension

$=KH$

$=0{\cdot}40\times4=1{\cdot}6$ *metres* from the base

or $4-1{\cdot}6=2{\cdot}4$ *metres* from the top

Pressure intensity at this level

$=1000\times2{\cdot}4=2400$ *kg./m.*^{2}

Maximum hoop tension

$=2400\times\frac{12}{2}=14400$ *kg.*

With stress in steel at 1000 *kg/cm.*^{2}, we have steel for hoop tension

$=\frac{14400}{1000}=14{\cdot}4$ *cm.*^{2}

$\therefore$ Spacing of 12 *mm.* bars

$=\frac{1{\cdot}13\times100}{14{\cdot}4}=8$ *cms.*

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Since this ring steel will be placed near both the faces, the spacing on each side $= 16$ *cms.*

Let us provide 12 mm. bars at 16 cms. centres near both faces (14·12 cm.²)

Check for the stress on the equivalent area of concrete (tensile stress in concrete should not exceed 12 *kg./cm.²* on the equivalent area).

$$\therefore \text{ Tensile stress } = \frac{14400}{17 \times 100 + (13-1) \times 14{\cdot}12} = 7{\cdot}56 \text{ kg./cm.}^2 < 12 \text{ kg/cm.}^2$$

Maximum bending moment

$$= F\, w\, H^3$$
$$= 0{\cdot}016 \times 1000 \times 4^3 = 1024 \text{ kg. m.}$$

With $c = 70$ *kg./cm.²* and $t = 1000$ *kg./cm.²*,

$$m = \frac{2800}{3c} = 13, \qquad a = 0{\cdot}84\, d$$

and $Q = 14{\cdot}03$

$$\therefore \quad 14{\cdot}03 \times 100 \times d^2 = 1024 \times 100$$
$$d = 8{\cdot}5 \text{ cm.}$$

But effective depth provided

$$= 17 \text{ cm.} - 2{\cdot}5 \text{ cm.} - 0{\cdot}6 \text{ cm.} = 13{\cdot}9 \text{ cm.}$$

(overall depth − clear cover − ½ dia. of 12 *mm.* bar)

$\therefore$ Steel requirement

$$= A_t = \frac{1024 \times 100}{1000 \times 0{\cdot}84 \times 13{\cdot}9} = 8{\cdot}77 \text{ cm.}^2$$

$\therefore$ Spacing of 12 *mm.* bars

$$= \frac{1{\cdot}13 \times 100}{8{\cdot}77} = 12{\cdot}8 \text{ cms.}$$

Let us provide 12 mm. bars at 12 cms. centres.

Distribution steel

$$= \frac{0{\cdot}3}{100} \times 17 \times 100 = 5{\cdot}1 \text{ cm.}^2$$

Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{5{\cdot}1} = 22 \text{ cm.}$$

$\therefore$ Spacing of bars near each face $= 44$ *cm. c/c.*

Fig. 415 shows the details of reinforcement.

The bottom slab will be 15 *cm.* thick with a top mesh and a bottom mesh of *10 mm.* ϕ @ *24 cm. c/c.*

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§171. The I.S. Code Method

The bending moments, and hoop tension and shear at base for the tank wall of circular tank may be determined by using appropriate coefficients given by the I.S. code. These coefficients depend on the $\frac{H^2}{Dt}$ ratio.

Hoop tension per *metre* height = coefficient $\times (wH) \frac{D}{2}$ *kg /m.*

B.M. per *metre* run = coefficient $\times wH^3$ *kg. m./m.*

The S.F. at the base of the wall = coefficient $\times wH^2$ *kg.*

Cylindrical Tank (Fixed at Base and Free at Top)

Coefficient for Shear Force

$\frac{H^2}{Dt}$	*Coefficient*	$\frac{H^2}{Dt}$	*Coefficient*	$\frac{H^2}{Dt}$	*Coefficient*
0·4	+0·436	3	+0·262	10	+0·158
0·8	+0·374	4	+0·236	12	+0·145
1·2	+0·339	5	+0·213	14	+0·135
1·6	+0·337	6	+0·197	16	+0·127
2·0	+0·299	8	+0·174	20	+0·114

Note : Positive sign indicates inward acting shear.

Design 106 (S I.). *Design a circular tank 12 m. diameter and 4 metre high. The tank rests on firm ground. The walls of the tank are restrained at the base*

Solution. $H = 4$ *metres,* $D = 12$ *metres.*

Thickness of Wall. This is taken as the greater of the following :

(*i*) *15 cm.*

(*ii*) *3 cm. per metre depth* $+ 5$ *cm* $= (3 \times 4) + 5 = 17$ *cm.*

Provide a thickness of 17 *cm.*

$$\frac{H^2}{Dt} = \frac{4 \times 4}{12 \times 0{\cdot}17} = 7{\cdot}8$$

Max. coefficient for hoop tension for $\frac{H^2}{Dt} = 6$,

is 0·514 at 0·6 H depth

Max. coefficient for hoop tension, for $\frac{H^2}{Dt} = 8$,

is 0·575 at 0·6 H depth.

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Cylindrical Tank (Fixed at Base and Free at Top) Coefficient for Tension in the Wall

$\frac{H^2}{Dt}$	Coefficient at Point									
	0·0H	0·1H	0·2H	0·3H	0·4H	0·5H	0·6H	0·7H	0·8H	0·9H
0·4	+0·149	+0·134	+0·120	+0·101	+0·082	+0·066	+0·049	+0·029	+0·014	+0·004
0·8	+0·263	+0·239	+0·215	+0·190	+0·160	+0·130	+0·096	+0·063	+0·034	+0·010
1·2	+0·283	+0·271	+0·254	+0·234	+0·209	+0·180	+0·142	+0·099	+0·054	+0·016
1·6	+0·265	+0·268	+0·268	+0·266	+0·250	+0·226	+0·185	+0·134	+0·075	+0·023
2·0	+0·234	+0·251	+0·273	+0·285	+0·285	+0·274	+0·232	+0·172	+0·104	+0·031
3·0	+0·134	+0·203	+0·267	+0·322	+0·357	+0·362	+0·330	+0·262	+0·157	+0·052
4·0	+0·067	+0·164	+0·256	+0·339	+0·403	+0·429	+0·409	+0·334	+0·210	+0·073
5·0	+0·025	+0·137	+0·245	+0·346	+0·428	+0·477	+0·469	+0·398	+0·259	+0·092
6·0	+0·018	+0·119	+0·234	+0·344	+0·441	+0·504	+0·514	+0·447	+0·301	+0·112
8·0	−0·011	+0·104	+0·218	+0·335	+0·443	+0·534	+0·575	+0·530	+0·381	+0·151
10·0	−0·011	+0·098	+0·208	+0·323	+0·437	+0·542	+0·608	+0·589	+0·440	+0·179
12·0	−0·005	+0·097	+0·202	+0·312	+0·429	+0·543	+0·628	+0·633	+0·494	+0·211
14·0	−0·002	+0·098	+0·200	+0·306	+0·420	+0·539	+0·639	+0·666	+0·541	+0·241
16·0	−0·000	+0·099	+0·199	+0·304	+0·413	+0·531	+1·641	+0·687	+0·582	+0·265

Note : Positive sign indicates tension.

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Cylindrical Tank (Fixed at Base and Free at Top) Coefficient for Bending Moment in the Wall

$\frac{H^2}{Dt}$	Coefficient at Point									
	0·1H	0·2H	0·3H	0·4H	0·5H	0·6H	0·7H	0·8H	0·9H	1·0H
0·4	+0·0005	+0·0014	+0·0021	+0·0007	−0·0042	−0·0150	−0·0302	−0·0529	−0·0816	−0·1205
0·8	+0·0011	+0·0037	+0·0063	+0·0080	+0·0070	+0·0023	−0·0068	−0·0224	−0·0465	−0·0795
1·2	+0·0012	+0·0042	+0·0077	+0·0103	+0·0112	+0·0090	+0·0022	−0·0108	−0·0311	−0·0602
1·6	+0·0011	+0·0041	+0·0075	+0·0107	+0·0121	+0·0111	+0·0058	−0·0051	−0·0232	−0·0505
2·0	+0·0010	+0·0035	+0·0068	+0·0099	+0·0120	+0·0115	+0·0075	+0·0021	−0·0185	−0·0436
3·0	+0·0006	+0·0024	+0·0047	+0·0071	+0·0090	+0·0097	+0·0077	+0·0012	−0·0139	−0·0333
4·0	+0·0003	+0·0015	+0·0028	+0·0047	+0·0066	+0·0077	+0·0069	+0·0023	−0·0080	−0·0268
5·0	+0·0002	+0·0008	+0·0016	+0·0029	+0·0046	+0·0059	+0·0059	+0·0028	−0·0058	−0·0222
6·0	+0·0001	+0·0003	+0·0008	+0·0019	+0·0032	+0·0046	+0·0051	+0·0029	−0·0041	−0·0187
8·0	0	+0·0001	+0·0002	+0·0008	+0·0016	+0·0028	+0·0038	+0·0029	−0·0022	−0·0146
10·0	0	0	+0·0001	+0·0004	+0·0007	+0·0019	+0·0029	+0·0028	−0·0012	−0·0122
12·0	0	−0·0001	+0·0001	+0·0002	+0·0003	+0·0013	+0·0023	+0·0026	−0·0005	−0·0104
14·0	0	0	0	0	+0·0001	+0·0008	+0·0019	+0·0023	−0·0001	−0·0090
16·0	0	0	−0·0001	−0·0001	−0·0001	+0·0004	+0·0013	+0·0019	−0·0001	−0·0079

Note : Positive sign indicates tension near the outer face.

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∴ Coefficient for hoop tension, for $\frac{H^2}{Dt} = 7{\cdot}8$,

is equal to $0{\cdot}514 + \frac{(0{\cdot}575 - 0{\cdot}514)}{2}$ (1·8)

$= 0{\cdot}514 + 0{\cdot}055 = 0{\cdot}569$

∴ Max. hoop tension

$= 0{\cdot}569\ wH\ \frac{D}{2}$

$= 0{\cdot}569 \times 9810 \times 4 \times \frac{12}{2} N$

$= 133965\ N$

Hoop steel per *metre* height

$= \frac{133965}{10{,}000} = 13{\cdot}4\ cm.^2$

Spacing of 12 *mm.* ϕ bars

$= \frac{1{\cdot}13 \times 100}{13{\cdot}4} = 8{\cdot}4\ cm.$

Provide 12 mm. ϕ *@ 16 cm. c/c. near each face*

Vertical distribution steel

$= \frac{0{\cdot}3}{100} \times 17 \times 100 = 5{\cdot}1\ cm.^2$

Spacing of 8 *mm.* ϕ bars

$= \frac{0{\cdot}5 \times 100}{5{\cdot}1} = 9{\cdot}8\ cm.$

Provide 8 mm. ϕ *@ 18 cm. c/c near each face.*

Max. B.M. coefficient for $\frac{H^2}{Dt} = 7{\cdot}8$,

is equal to $-\left[0{\cdot}0187 - \frac{(0{\cdot}0187 - 0{\cdot}0146}{2} \times 1{\cdot}8\right]$

$= -0{\cdot}0150$

Max. cantilever moment

$= 0{\cdot}015\ wH^3$

$= 0{\cdot}015 \times 9810 \times 4^3 \times 100\ N\ cm.$

$= 941760\ N\ cm.$

Effective depth to the centre of 8 *mm.* ϕ vertical bars

$= 17 - (2{\cdot}5 + 1{\cdot}2 + 0{\cdot}4) = 12{\cdot}90\ cm.$

Vertical steel for B.M.

$= \frac{941760}{10000 \times 0{\cdot}84 \times 12{\cdot}9} = 8{\cdot}7\ cm.^2$

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Spacing of 8 *mm.* ϕ bars

$$= \frac{0.5 \times 100}{8.7} = 5.7 \text{ cm.}$$

Provide 8 *mm.* ϕ @ 4·5 *cm. c/c.*

Fig 416 shows the details of reinforcement.

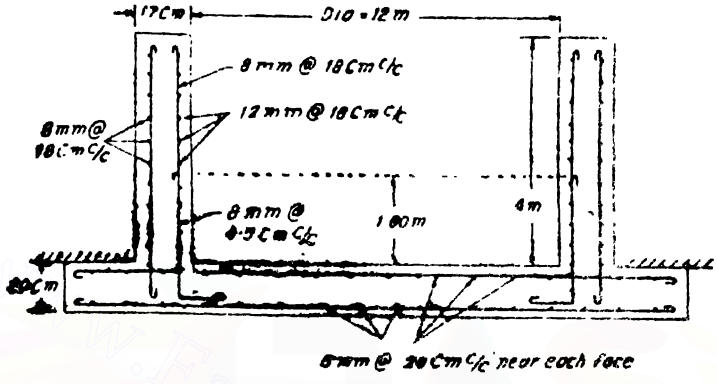

Fig 416

Design 107. *Design a circular tank 12 metre diameter and 4 metre high The tank rests on firm ground. The walls of the tank are restrained at the base.*

Solution. $H = 4$ *metres*, $D = 12$ *metres.*

Thickness of wall This is taken as the greater of the following :

(*i*) *15 cm.*

(*ii*) *3 cm. per metre depth* + *5 cm.* $= (3 \times 4) + 5 = 17$ *cm.*

Provide a thickness of 17 cm. $\therefore t = 0.17$ *m.*

$$\therefore \quad \frac{H^2}{Dt} = \frac{4 \times 4}{12 \times 0.17} = 7.8$$

Max. coefficient for hoop tension for $\frac{H^2}{Dt} = 6$

is 0 514 at 0·6 H depth

Max. coefficient for hoop tension for $\frac{H^2}{Dt} = 8$

is 0·575 at 0·6 H depth

$\therefore$ Coefficient for hoop tension for $\frac{H^2}{Dt} = 7.8$

is equal to

$$= 0.514 + \frac{(0.575 - 0.514)}{2} \quad 1.8$$

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$$= 0{\cdot}514 + 0{\cdot}055 = 0{\cdot}569$$

$\therefore$ Max. hoop tension $= 0{\cdot}569 \times wH\,\frac{D}{2}$

$$= 0{\cdot}569 \times 1000 \times 4 \times \frac{12}{2} = 13656\ kg.$$

Hoop steel per *metre* height

$$= \frac{13656}{1000} = 13{\cdot}6\ cm.^2$$

Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{13{\cdot}6} = 8{\cdot}3\ cm.$$

Provide 12 mm. bars @ 16. cm. c/c near each face.

Vertical distribution steel

$$= \frac{0{\cdot}3}{100} \times 17 \times 100 = 5{\cdot}1\ cm.$$

Spacing of 8 *mm.* bars $= \frac{0{\cdot}5 \times 100}{5{\cdot}1} = 9\,8\ cm.$

Provide 8 mm. bars @ 18 cm. c/c near each face.

Max. B.M. B.M. coefficient for $\frac{H^2}{Dt} = 7{\cdot}8$

is equal to

$$= -\left[\,0\,0187 - \frac{(0{\cdot}0187 - 0\,0146)}{2}\,1\,8\,\right]$$

$$= -0{\cdot}0150$$

$\therefore$ Maximum cantilever moment

$$= -0{\cdot}0150\ wH^3$$

$$= -0{\cdot}0150 \times 1000 \times 4^3 \times 100\ kg.\ cm.$$

$$= -96000\ kg.\ cm.$$

Effective depth to the centre of vertical 8 *mm.* bars

$$= 17 - (2{\cdot}5 + 1{\cdot}2 + 0\,4) = 12{\cdot}9\ cm.$$

$\therefore$ Vertical steel for B.M.

$$= \frac{96000}{1000 \times 0{\cdot}84 \times 12{\cdot}9} = 8{\cdot}8\ cm.^2$$

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$\therefore$ Spacing of 8 *mm.* bars

$$= \frac{0{\cdot}5 \times 100}{8\,8} = 5{\cdot}6 \text{ cm. c/c.}$$

Provide 8 *mm.* at 4·5 *cm. c/c.*

Fig. 416 shows the details of reinforcement.

§172. Approximate Design of Circular Tanks with Walls Restrained at the Base

When Reissner's tables or Carpenter's tables are not available, an approximate design of the tank can be made as follows.

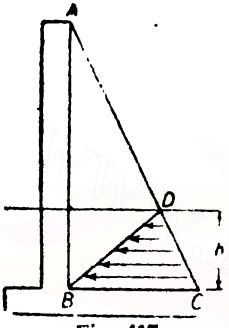

Fig. 417

It will be assumed that the cantilevering effect of the wall will be present for a height approximately equal to one-fourth of the height of the tank. This height shall be taken for design purposes at least at 1 *metre* (*see* Fig. 417). The pressure variation is given by the pressure diagram *ABC* with the ordinate *BC* representing the maximum water pressure of *wH*. Draw a horizontal line intersecting the pressure diagram at *D* at a height of $h' = \frac{H}{4}$ or 1 *metre* from the base whichever is greater. For the bottom h' units height the tank wall will be designed as a cantilever subjected to a triangular load given by the area *DBC* of the pressure diagram. Hence the bottom h' units height of the wall will be treated as a vertical cantilever carrying a load varying from zero at the top of the cantilever to *wH* at the bottom (*see* Fig 418). Vertical steel to resist the maximum bending moment can now be computed.

Fig. 418

The hoop tension will be calculated at the level *D*.

Maximum hoop tension per unit height at this level

$$= \frac{w(H-h)D}{2}$$

Steel for this hoop tension can be computed and may be provided near both faces. The hoop steel may be provided at a uniform spacing for a height h' from the base. At higher levels the spacing of the hoop reinforcements can be proportionately increased. This is illustrated in the following design.

Design 108 (S I.) *Design a clarifier tank of diameter 30 metres. The walls stand 5 metres above the floor. The tank rests on ground.*

Solution. It will be assumed that the walls cantilever up from

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the floor for the bottom $\frac{H}{4}$ or 1 *metre* whichever is greater. In our case the cantilevering height will be $\frac{5}{4} = 1{\cdot}25$ *metre*, from the base.

Maximum ring tension at this level per *metre* height

$$= 9810(5 - 1{\cdot}25) \times \frac{30}{2} \ N$$

$$= 551813 \ N$$

Steel for ring tension $= \frac{551813}{10{,}000} = 55{\cdot}18 \ cm.^2$

Spacing of 22 *mm.* ϕ bars

$$= \frac{3{\cdot}80 \times 100}{55{\cdot}18} = 6{\cdot}9 \ cm. \ say, \ 6 \ 5 \ cm.$$

Since the reinforcement is provided near both the faces, the spacing of the bars near each face will be 13 cm. c/c.

∴ Steel provided per *metre* height

$$= \frac{3{\cdot}8 \times 100}{6{\cdot}5} = 58{\cdot}50 \ cm.^2$$

Thickness of wall This shall be not less than the following :

(i) *15 cm.*

(ii) *3 cm. per metre depth* $+ 5 \ cm = (3 \times 5) + 5 = 20 \ cm.$

(iii) *Requirement to limit the tensile stress on the equivalent concrete area to 120 N/cm.*^{2}

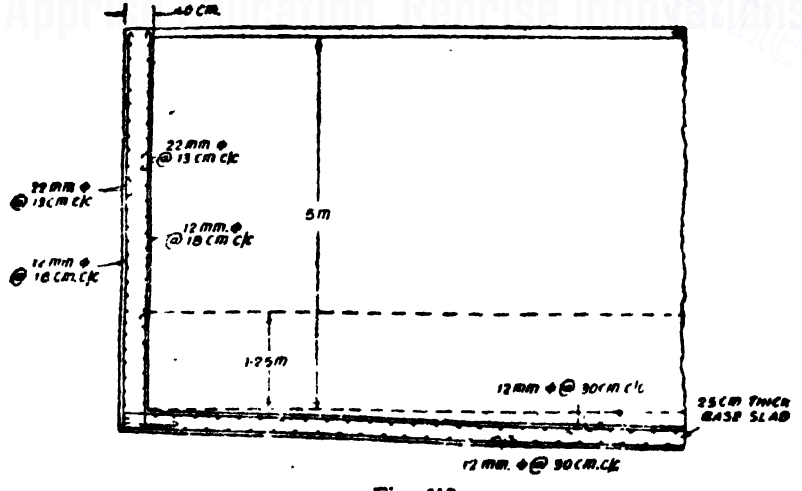

Fig. 419

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$$\therefore \quad 120 = \frac{551813}{100t + (13-1)58\,5}$$

$$\therefore \quad t = 38{\cdot}96 \text{ cm.}$$

Provide a thickness of 40 cm.

$$\text{Distribution steel} = \frac{0{\cdot}3}{100} \times 40 \times 100 \text{ cm.}^2 = 12 \text{ cm.}^2$$

$\therefore$ Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{12} \text{ cm.} = 9{\cdot}4 \text{ cm. say } 9 \text{ cm.}$$

Since this steel is provided near both the faces the spacing of 12 mm. diameter bars near each face will be 18 cm. c/c

Design of the bottom 1·25 m. cantilevering part

Water pressure at the bottom

$$= 9810 \times 5 = 49050 \text{ N/metre}^2$$

Maximum cantilevering bending moment

$$= \frac{1}{2} \times 49050 \times 1{\cdot}25 \times \frac{1{\cdot}25}{3} \text{ Nm}$$

$$= 12{\cdot}773 \text{ Nm}$$

If 12 *mm.* ϕ bars be provided at a clear cover of 25 *mm.*, the effective depth available

$$= 40 - 2{\cdot}50 - 2{\cdot}20 - 0{\cdot}6 = 34{\cdot}7 \text{ cm.}$$

$$\therefore \quad A_t = \frac{1277300}{10000 \times 0{\cdot}84 \times 34{\cdot}7} = 4{\cdot}38 \text{ cm.}^2$$

Available yertical steel near water e as part of distribution steel

$$= \frac{12}{2} = 6 \text{ cm.}^2$$

The base slab may be made 25 *cm.* thick. Nominal reinforcement of a top mesh and a bottom mesh of steel with 12 *mm.* ϕ @ 30 *cm. c/c.* may be provided.

Design 109. *Design a clarifier tank of diameter 30 meters. The walls stand 5 metres above the floor. The tank rests on ground.*

Solution. It will be assumed that the walls cantilever up from the floor for the bottom $\frac{H}{4}$ or 1 *metre* whichever is greater. In our case the cantilevering height will be $5/4 = 1{\cdot}25$ *metre*, from the base.

Maximum ring tension at this level per metre height

$$= 1000(5 - 1{\cdot}25) \times \frac{30}{2} \text{ kg} \quad 56250 \text{ kg}$$

$\therefore$ Steel for ring tension

$$= \frac{56250}{1000} \text{ cm}^2 = 56{\cdot}25 \text{ cm}^2$$

If 22 *mm* diameter bars be used the spacing of the bars will be

$$= \frac{3{\cdot}80 \times 100}{56{\cdot}25} \text{ cm}$$

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$= 6{\cdot}7$ *cm.* say *6·5 cm.*

Since the reinforcement is placed near both faces the spacing of the bars near each face will be 13 cm. centres.

∴ Steel provided per *metre* height

$$= \frac{3{\cdot}8 \times 100}{6{\cdot}5} = 58{\cdot}5 \text{ cm.}^2$$

Thickness of wall This shall not be less than the following :

(*i*) *15 cm.*

(*ii*) *3 cm. per metre depth* $+ 5$ *cm.* $= (3 \times 5) + 5 = 20$ *cm.*

(*iii*) *Requirement to limit the tensile stress on th equivalent concrete area to 12 kg./cm.*^{2}

$$12 = \frac{6250}{100t + (13-1)58{\cdot}4} = 40 \text{ cm.}$$

Hence provide a thickness of 40 *cm.*

∴ Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{12} \text{ cm.} = 9{\cdot}4 \text{ cm. say } 9 \text{ cm.}$$

Since this steel is provided near both the faces the spacing of 12 mm. diameter bars near each face will be 18 cm. c/c.

Design of the bottom 1·25 m. cantilevering part

Water pressure at the bottom

$$= 1000 \times 5 = 5000 \text{ kg./metre}^2$$

Maximum cantilevering bending moment

$$= \frac{1}{2} \times 5000 \times 1{\cdot}25 \times \frac{1{\cdot}25}{3} \text{ kg. m.} = 1302 \text{ kg. m.}$$

If 12 *mm.* diameter bars be provided at a clear cover of 25 *mm.*, the effective depth available

$$= 40 - 2{\cdot}5 - 2{\cdot}2 - 0{\cdot}6 = 34{\cdot}7 \text{ cm.}$$

$$\therefore \quad A_s = \frac{1302 \times 100}{1000 \times 0{\cdot}84 \times 34{\cdot}7} \text{ cm.}^2 = 4{\cdot}4 \text{ cm.}^2$$

The vertical distribution bars holding the rings near the water face will also serve to resist the cantilever bending moment. The floor may be made 25 *cm.* thick. Nominal reinforcement of a top and bottom mesh with 12 *mm. diameter bars at 30. cm. c/c* may be provided.

§173. Rectangular Tank Resting on Ground

Rectangular tanks are useful for small capacities. For large capacities circular tanks are preferred. Circular tanks, by virtue of their having the least perimeter for a given capacity, require the minimum amount of materials. But often this economy is overlooked, from the point of view of the costly formwork they require.

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The vertical walls of rectangular tank are subjected to bending in two directions The walls are liable to be bent as horizontal strips supported at the corners. They are also liable to be bent as vertical cantilevers, cantilevering for a certain height from the base. It is difficult to exactly estimate as to for what height there may be cantilevering effects and from what level onwards the strips span horizontally The following practice may be adopted in this regard.

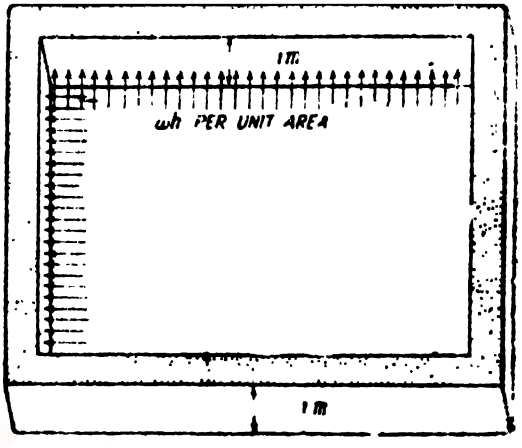

Fig. 420. A continuous horizontal strip at h units below the water surface.

Let the centre line dimensions of the tank be $L \times l$ where L is the longer plan dimension and l is the shorter plan dimension.

If $\frac{L}{l}$ is less than 2, the walls are designed as continuous horizontal slabs and subjected to a water pressure of wh per unit area at depth h from the water surface. One such continuous strip is shown in Fig. 420.

The continuous strip of unit height, that is, 1 *metre* can be analysed by moment distribution or any other method. Often the bending moments are taken at the following values.

B.M. at the centre of span

$$= \frac{whl^2}{16}$$

producing tension away from the water side

B.M. at the ends of the span

$$= -\frac{whl^2}{12}$$

producing tension near the water side

If the ratio $\frac{L}{l} = 2$ or greater than 2, the long walls are considered as vertical cantilevers, cantilevering for the whole height from the base. The short walls are considered as spanning between the

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long walls and with ends taken as fixed.

Cantilever effect of short wall. Though the short wall in principle spans horizontally between the long walls, it is possible that the short wall may have a cantilevering effect for a certain height from the base. It is usual to consider that for the bottom 1 *metre* the short wall will have a cantilevering effect. The water pressure taken for design, on the cantilevering part, is given by *BDC* of the pressure diagram (*see* Fig. 411).

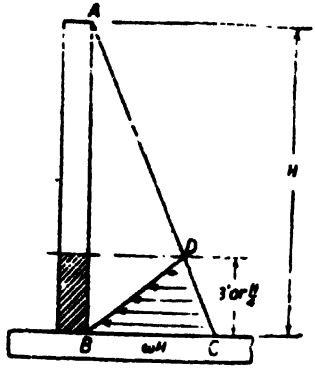

Fig. 421

Pull on the long and short walls

Pull on the long wall. Since the short wall spans between the long walls at higher levels (*i.e.* above the level *D*), the water pressure on the short wall will be transferred to the supporting long walls as tension. Suppose we consider that the bottom 1 *metre* of short wall will act as a cantilever, consider at a level of 1 *metre* from the base. Pressure intensity at this level $= w\ (H-1)$ $N/metre^2$ ($kg./metre^2$). On a 1 *metre* deep strip of short wall spanning between the long walls, the water pressure force will be $w(H-1)\times 1\times l$. Hence, the tension per *metre* height of the long wall at a level of 1 *metre* above the base will be $\frac{w\,(H-1)\,l}{}$

Fig. 422. Pull in long walls.

Pull on the short wall. In fact, if the long walls will behave actually as cantilevers cantilevering from the base slab, no part of the water pressure on the long wall can produce any tension in the short wall. But it is found that a part of the water pressure acting on the long wall for a certain distance from the corners will produce a tension in the short wall.

For design purpose this length is taken as 1 *metre*. (*See* Fig. 423).

This tension is computed at a height of 1 *metre* above the base.

The tension in the short wall at 1 *metre* above the base is therefore equal to $w(H-1)\times 1$ per *metre* height.

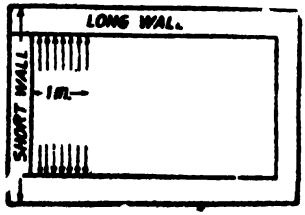

Fig. 423

The short wall being subjected to moment and a pull, it should be designed for the combined effect. So

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far as the long wall is concerned, the steel for pull is horizontal bu the steel requirement for the bending moment is vertical. In this case, however, the steel requirement for each of the effects may be separately computed and provided.

The base slab will be 15 *cm.* to 20 *cm.* thick reinforced with a top mesh and a bottom mesh with 10 *mm.* bars at 24 *cm.* centres.

§174. Analysis of a Tank Wall Section Subjected to Bending Momen and Pull

The following approximate method is adopted in the analysis of tank wall subjected to a bending moment accompanied by a pull. Fig. 424 shows a wall section subjected to an axial pull T and a bending moment M. Let the nature of the bending moment be such that tension is induced on the right side of the section (*see* Fig. 424). Note the direction of the arrow indicating the type of moment. Let two equal and opposite forces T be imagined in the reinforcements The moments and the various forces may therefore be considered as equivalent to a resultant moment $(M-Tx)$ and a pull T in the reinforcement

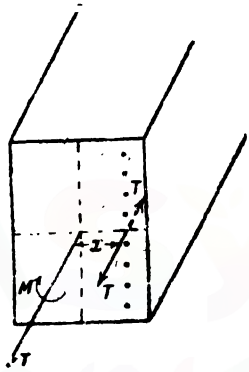

Fig. 424. Section subjected to bendi. g moment and pull.

The steel for this resultant bending moment and the pull may now be separately calculated. Steel for the resultant B.M

$$\frac{M-Tx}{\text{Safe stress in steel} \times \text{lever arm}} = A_{s1}$$

Safe stress in steel will be taken at 10,000 *N/cm.* or 1,000 *kg./cm.*^{2} near the water face and 12,500 *N/cm.*^{2} or 1,250 *kg./cm.*^{2}, away from the water face.

$$\text{Steel for pull} = \frac{T}{\text{Safe stress in steel}} = A_{s2}$$

$\therefore$ Total steel requirement $= A_s = A_{s1} + A_{s2}$

Design 110 (S.I.) *An open square tank 5 m. × 5 m. × 3 m. deep rests on firm ground. Design the tank.*

Solution. Thickness of wall. This will be not less than,

(*i*) *15 cm.*

(*ii*) *3 cm. per metre depth* + *5 cm.* = (3 × 3) + 5 = 14 *cm.*

(*iii*) *6 cm. per metre length of side* = 6 × 5 = 30 *cm.*

Provide a thickness of 30 *cm.*

Effective span of the slab = 5 + 0·30 = 5·30 *metre*

Consider a level 1 *metre* above the floor.

Water pressure intensity at this level

= 9810 × 2 = 19620 *N/metre*^{2}

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Bending moment per *metre* height at this level at corners

$$= \frac{19620 \times 5.3^2}{12} \text{ Nm.} = 45927 \text{ Nm.}$$

$$= 4592700 \text{ N cm.}$$

Bending moment at mid span

$$= \frac{19620 \times 5.3^2}{16} \text{ Nm.} = 34445 \text{ Nm.}$$

$$= 3444500 \text{ N cm.}$$

Pull on the wall per *metre* height at this level

$$= T = \frac{19620 \times 5}{2} \text{ N} = 49050 \text{ N}$$

Design of the corner section. Providing a cover of 25 *mm.* and using 18 *mm.* ϕ bars,

effective depth $= 30 - 3.4 = 26.6$ *cm.*

Effective bending moment at corners

$$= M - Tx$$

$$= 4592700 - 49050\ (26.6 - 15) \text{ N cm.}$$

$$= 4023720 \text{ N cm.}$$

Since this bending moment produces tension near the water face the stresses and the design coefficients to be adopted are,

$t = 10{,}000$ *N/cm.*^{2}, $c = 700$ *N/cm*^{2}, $m = 13$, $Q = 140.3$, $a = 0.84\ d$

$\therefore$ Steel for bending moment

$$= A_{t1} = \frac{4023720}{10000 \times 0.84 \times 26.6} \text{ cm.}^2$$

$$= 18 \text{ cm.}^2$$

Steel for pull $= A_{t2} = \dfrac{49050}{10000} = 4.90 \text{ cm.}^2$

Total steel required $= A_t = A_{t1} + A_{t2} = 18 + 4.90 = 22.90 \text{ cm.}^2$

Spacing of 18 *mm.* bars

$$= \frac{2.54 \times 100}{22.90} = 11 \text{ cm.}$$

Provide 18 mm. ϕ *@ 10 cm. c/c.*

Design of the mid span section

Bending moment $= 3444500$ *N cm*

Pull $= 49050$ *N*

$\therefore$ Effective bending moment

$$= M - Tx$$

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$$= 344450 - 49050\ (26{\cdot}6 - 15)\ N\ cm.$$

$$= 2875520\ N\ cm.$$

Since this bending moment produces tension away from the water face, the stresses and the design coefficients to be adopted are,

$t = 12500\ N/cm.^2$, $c = 700\ N/cm.^2$, $m = 13$ and $a = 0{\cdot}86\ d$

$$\text{Steel for B M.} = A_{s1} = \frac{2875520}{12500 \times 0{\cdot}86 \times 26{\cdot}6} = 10{\cdot}06\ cm.^2$$

$$\text{Steel for pull} = A_{s2} = \frac{49050}{12500} = 3{\cdot}92\ cm.^2$$

Total steel required

$$= A_s = A_{s1} + A_{s2} = 10{\cdot}06 + 3{\cdot}92 = 13{\cdot}98\ cm.^2$$

Provide 18 mm. ϕ @ 17 cm. c/c.

Design of the bottom 1 metre height of wall

This will be designed as a cantilever 1 *metre* high and subjected to a triangular load ranging from zero at the top of the cantilever to $9810 \times 3 = 29430\ N/metre^2$ at the bottom.

∴ Maximum pressure force on the cantilever (for 1 *metre* width)

$$= \tfrac{1}{2} \times 29430 \times 1 = 14715\ N$$ acting at $\tfrac{1}{3}$ *metre* from the base.

∴ B.M. for the cantilever

$$= 14715 \times \tfrac{1}{3} = 4905\ Nm. = 490500\ N\ cm.$$

Since this bending moment produces tension near the water face,

$$\text{Steel for B.M.} = \frac{490500}{10000 \times 0{\cdot}84 \times 25{\cdot}2} = 2{\cdot}32\ cm.^2$$

But 0·3% of gross area

$$= \frac{0{\cdot}3}{100} \times 3 \times 100 = 9\ cm.^2$$

Spacing of 10 *mm.* ϕ bars

$$= \frac{0{\cdot}79 \times 100}{9} = 8{\cdot}8\ cm.$$

say, 10 mm. ϕ @ 8 cm. c/c.

Provide 10 *mm.* ϕ vertical bars @ 16 *cm* c/c near each face. These vertical bars near the water face are enough to resist cantilever moment.

Base slab. The base slab will be 24 *cm.* thick with a top mesh and a bottom mesh of reinforcement of 10 *mm. bars at 20 cm. centres.*

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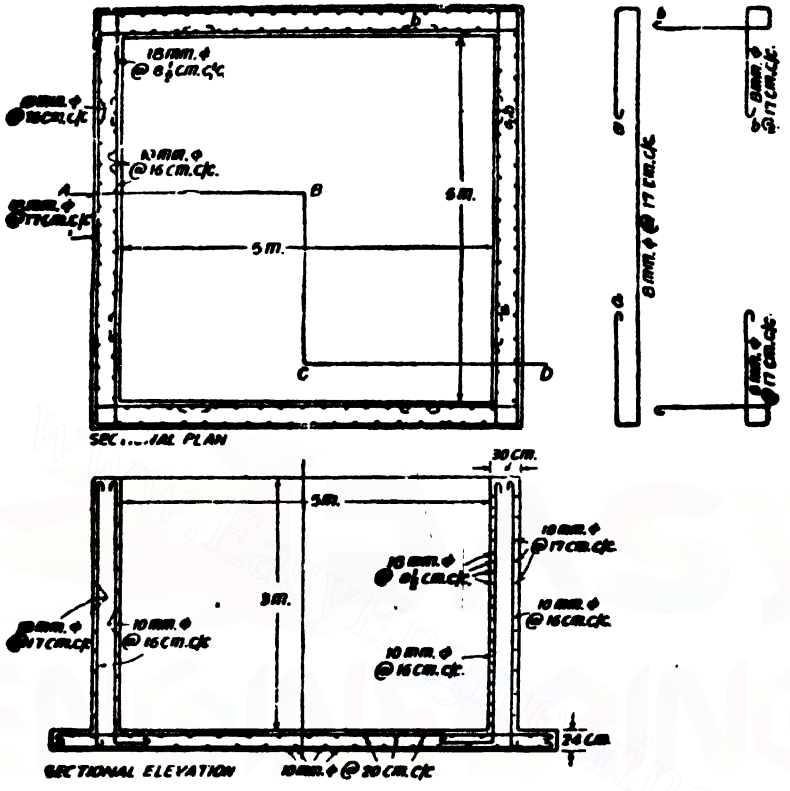

Fig. 425

Design 111. *An open square water tank 5 m. × 5 m. × 3 m. deep rests on firm ground. Design the tank.*

Solution. Design of walls. Thickness of the wall shall not be less than

(*i*) 15 *cms.*

(*ii*) 3 *cm.* per *metre* depth + 5 *cm.*

$= 9 + 5 = 14$ *cm.*

(*iii*) 6 *cm.* per *metre* length of side

$= 6 \times 5 = 30$ *cm.*

Let us adopt 30 *cm.* of overall thickness

∴ Effective span of the wall slab

$= 5 + 0{\cdot}30$ *m.* $= 5{\cdot}30$ *m.*

Consider a level 1 *m.* above the floor.

Water pressure at this level

$= 1000 \times 2$ *kg./sq. m.* $= 2000$ *kg./sq. m.*

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Bending moment per *metre* height at this level at corners

$$= \frac{2000 \times 5{\cdot}3^2}{12} \text{ kg. m.}$$

$$= 4681 \text{ kg. m.} = 468100 \text{ kg. cm.}$$

Bending moment at mid span

$$= \frac{2000 \times 5{\cdot}3^2}{16} \text{ kg. m.}$$

$$= 3511 \text{ kg. m.} = 351100 \text{ kg. cm.}$$

Pull on the wall per *metre* height at this level

$$= \frac{2000 \times 5}{2} \text{ kg.} = 5000 \text{ kg.}$$

Design of corner section. Assume a cover of 25 *mm.* and with 18 *mm.* dia. bars, the effective cover $= 30 - 3{\cdot}4 = 26{\cdot}6$ *cm.*

$\therefore$ Effective b.m. at corners

$$= M - Tx$$

$$= 468100 - 5000 \times (26{\cdot}6 - 15) \text{ kg. cm.}$$

$$= 410100 \text{ kg. cm.}$$

Since this bending moment produces tension near the cater face the stresses and the design coefficients to be adopted are.

$$t = 1000 \text{ kg./cm.}^2,\ c = 70 \text{ kg./cm.}^2,\ m = 13$$

$$Q = 14{\cdot}03,\ a = 0{\cdot}84\ d.$$

$\therefore$ Steel for bending moment

$$= \frac{410100}{1000 \times 0{\cdot}84 \times 26{\cdot}6} \text{ cm.}^2 = 18{\cdot}35 \text{ cm.}^2$$

$$\text{Steel for pull} = \frac{5000}{1000} \text{ cm.}^2 = 5{\cdot}00 \text{ cm.}^2$$

$\therefore$ Total steel $= 18{\cdot}35 + 5{\cdot}00 \text{ cm.}^2 = 23{\cdot}35 \text{ cm.}^2$

$\therefore$ Spacing of 18 *mm.* bars

$$= \frac{2{\cdot}54 \times 100}{23{\cdot}35} = 10{\cdot}87 \text{ cms.}$$

say *18 mm. bars at 10 cms. centres.*

Design of the mid-span section

Bending moment $= 351100$ *kg. cm.*

Pull $= 5000$ *kg.*

$\therefore$ Effective bending moment

$$= M - Tx$$

$$= 351100 - 5000 \times (26{\cdot}6 - 15) = 293100 \text{ kg. cm.}$$

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Since this bending moment produces tension away from the water face, the stresses and the design coefficients to be adopted are,

$$t = 1250 \text{ kg./cm.}^2,\ c = 70 \text{ kg /cm.}^2,\ m = 13,\ a = 0{\cdot}86\ d$$

$$\text{Steel for B.M.} = \frac{293100}{1250 \times 0{\cdot}86 \times 26{\cdot}6} = 10{\cdot}25 \text{ cm.}^2$$

$$\text{Steel for pull} = \frac{5000}{1250} = 4{\cdot}00 \text{ cm.}^2$$

$\therefore$ Total steel required $= 10{\cdot}25 + 4{\cdot}00 = 14{\cdot}25 \text{ cm.}^2$

$\therefore$ Spacing of 18 mm. dia bars $= \frac{2{\cdot}54 \times 100}{14{\cdot}25} = 17{\cdot}82$ cms.

say *18 mm. dia. at 17 cms. centres.*

Design of the bottom 1 metre height of wall. This will be designed as a cantilever 1 *metre* high and subjected to a triangular load ranging from zero at the top of the cantilever to $1000 \times 3 = 3000$ *kg./metre*^{2}, at the bottom.

$\therefore$ Max. pressure on the cantilever (for 1 *m.* width)

$$= \tfrac{1}{2} \times 3000 \times 1 = 1500 \text{ kg.}$$

acting at $\frac{1}{3}$ *metre* from the base.

$\therefore$ B.M. for the cantilever

$$= 1500 \times \tfrac{1}{3} = 500 \text{ kg. m.} = 50{,}000 \text{ kg. cm.}$$

Since this bending moment produces tension near the water face

$$\text{Steel for b.m.} = \frac{50{,}000}{1000 \times 0{\cdot}84 \times 26{\cdot}6} \text{ cm.}^2 = 2{\cdot}24 \text{ cm.}^2$$

But 0·3% of gross area

$$= \frac{0{\cdot}3}{100} \times 3 \times 100 = 9 \text{ cm.}^2$$

$\therefore$ Spacing of 10 *mm.* bar

$$= \frac{0{\cdot}79 \times 100}{9} = 8{\cdot}8 \text{ cms.}$$

say *10 mm. bars at 8 cm. centres.*

Since even half the above amount is sufficient to resist the cantilever bending moment, the above steel will be provided near both the faces. *Hence steel near each face will be 10 mm bars at 15 cms centres*

Base slab. The base slab will be 24 *cm.* thick with a top mesh and a bottom mesh of reinforcement of *10 mm. bars at 20 cm. centres.*

Design 112 (S I.) *A reinforcement concrete tank is 6 metres × 3 metres with a maximum depth of 2·50 metres of water. The tank rests on ground. 15 cm × 15 cm splays are provided at the junction of walls and base slab. Design the tank*

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Solution. $L = 6$ *metres*, $l = 3$ *metres*

$$\frac{L}{l} = 2$$

Since this ratio equals 2, the long walls will be designed as vertical cantilevers and short walls as spanning horizontally between the long walls.

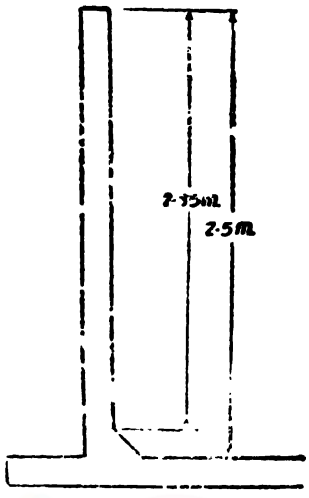

Fig. 426

Design of long walls

These are designed as vertical cantilevers. Since splays are provided it is enough if the cantilevering effect is considered above the top of the splay, *i.e.*, the effective height of the cantilevering long wall will be

$$2{\cdot}50 - 0{\cdot}15 = 2{\cdot}35 \text{ metre}$$

.. Maximum bending moment per *metre* width of the long wall

$$= \frac{wH^3}{6} = \frac{9810 \times 2{\cdot}35^3}{6} \text{ Nm.}$$

$$= 21219 \text{ Nm.} = 2121900 \text{ N cm.}$$

This bending moment produces tension near the water face.

Adopting $c = 700$ *N/cm.*^{2}, $t = 10{,}000$ *N/cm.*^{2} and $m = 13$ and equating the moment of resistance to the bending moment, we have

$$140{\cdot}3\, bd^2 = 140{\cdot}3 \times 100\, d^2 = 2121900$$

$$\therefore \quad d = 12{\cdot}30 \text{ cm.}$$

With a clear cover of 25 *mm.* and providing 16 *mm.* ϕ bars, the overall depth required

$$= 12{\cdot}30 + 0{\cdot}80 + 2{\cdot}50 = 15{\cdot}6 \text{ cm.}$$

Provide a thickness of 16 *cm.*

$\therefore$ Actual effective depth

$$= 16 - 3{\cdot}3 = 12{\cdot}7 \text{ cm}$$

$$\therefore \quad A_s = \frac{2121900}{10000 \times 0{\cdot}84 \times 12{\cdot}7} = 19{\cdot}9 \text{ cm.}^2$$

Spacing of 16 *mm.* ϕ bars

$$= \frac{2{\cdot}01 \times 100}{19{\cdot}9} = 10 \text{ cm. c/c.}$$

Pull on the long wall. The water pressure on the short wall will be transferred to the long wall as tension. Since the short wall will cantilever from the base slab for the bottom 1 *metre* of

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its height, the pull transferred to the long wall shall also be calculated at a level, one *metre* above the haunch.

∴ Pull on the long wall per *metre* height at the above level

$$= \frac{9810\,(2{\cdot}35-1)\times 3}{2} N = 19865\ N$$

Steel required for this pull

$$= \frac{19865}{10000} = 2\ cm.^2$$

But 0·3% of gross area

$$= \frac{0{\cdot}3}{100} \times 16 \times 100 = 4{\cdot}80\ cm.^2$$

∴ Spacing of 10 *mm.* ϕ bars

$$= \frac{0{\cdot}79 \times 100}{4{\cdot}8} = 16{\cdot}48\ cm.$$ say ... cm.

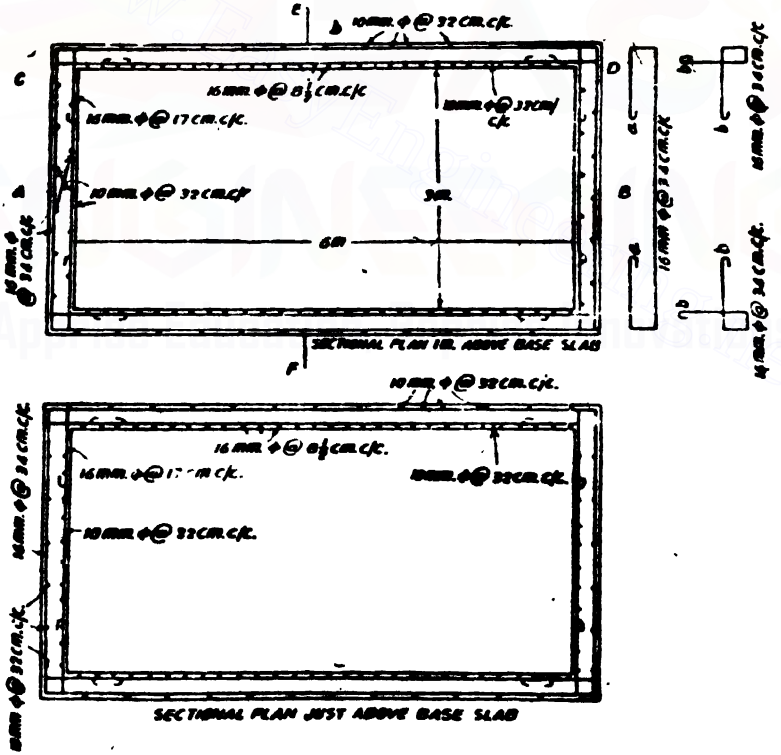

Fig. 427

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Since this steel is provided near both the faces, the spacing of the 10 mm. ϕ bars will be 32 cm c/c near each face.

Design of short walls. Consider a level one *metre* above the top of the haunch.

The pressure intensity at this level
$= 9810\ (2{\cdot}35 - 1)$
$= 13243{\cdot}5\ N/metre^2$

Effective span of the horizontally spanning slab
$= 3 + 0{\cdot}16 = 3{\cdot}16$ *metre*

Corner section B.M. at corner section (*i.e.* ends)

$$= -\frac{pl^2}{12} = -\frac{13243{\cdot}5 \times 3{\cdot}16^2}{12}\ Nm. = 11020\ Nm.$$
$= 1102000\ N\ cm.$

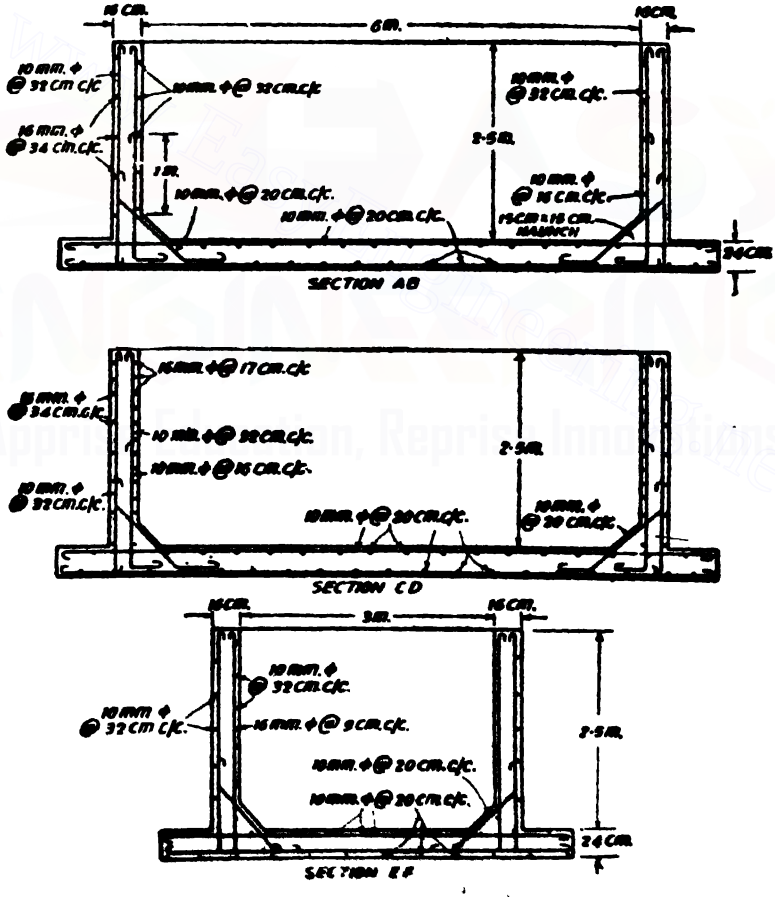

Fig. 428

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Water pressure per *metre* height at this level, on one *metre* length of the long wall will be transferred to the short wall.

∴ Tension transferred per *metre* height of short wall

$$= 13243{\cdot}5 \times 1 = 13243{\cdot}5\ N$$

∴ Resultant bending moment

$$= M - Tx$$
$$= 1102000 - 13243{\cdot}5\ (12{\cdot}7 - 8)$$
$$= 1039760\ N\ cm.$$

This bending moment produces tension near the water face. Hence steel for bending moment

$$= A_{s1} = \frac{1039760}{10{,}000 \times 0{\cdot}84 \times 12{\cdot}7} = 9{\cdot}75\ cm.^2$$

Steel for pull $= A_{s2} = \dfrac{13243{\cdot}5}{10{,}000} = 1{\cdot}32\ cm.^2$

∴ Total steel required

$$= A_s = 9{\cdot}75 + 1{\cdot}32 = 11{\cdot}07\ cm.^2$$

Provide 16 mm. ϕ @ 17 cm. c/c

Mid span section. Bending moment at mid span

$$= \frac{pl^2}{24} = \frac{13243{\cdot}5 \times 3{\cdot}16^2}{24}\ Nm.$$

$$= 5510\ Nm. = 551000\ N\ cm.$$

Pull $= 13243{\cdot}5\ N$

Resultant B.M. $= M - Tx$

$$= 551000 - 13243{\cdot}5\ (12{\cdot}7 - 8)\ N\ cm.$$
$$= 488755\ N\ cm.$$

This bending moment produces tension away from the water face.

∴ Steel for B.M. $= A_{s1} = \dfrac{488755}{12500 \times 0{\cdot}86 \times 12{\cdot}7} = 3{\cdot}6\ cm.^2$

Steel for pull $= A_{s2} = \dfrac{13243{\cdot}5}{12500} = 1{\cdot}1\ cm.^2$

Total steel required $= A_{s1} + A_{s2} = 3{\cdot}6 + 1{\cdot}1 = 4{\cdot}7\ cm.^2$

Spacing of 16 *mm.* ϕ bars

$$= \frac{2{\cdot}01 \times 100}{4{\cdot}7} = 42{\cdot}8\ cm.$$

It is convenient to provide 16 mm ϕ bars @ 34 cm. c/c which is just double the spacing of the bars provided near the corners.

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Design of the bottom one metre cantilevering part of the short wall.

Maximum bending moment for cantilevering effect

$$= \frac{9810}{2} \times 2{\cdot}35 \times 1 \times \tfrac{1}{3} \; Nm.$$

$$= 3842{\cdot}25 \; Nm.$$

$$= 384225 \; N \; cm.$$

Effective depth to the centre of 10 *mm.* ϕ vertical bars

$$= 16 - 2{\cdot}5 - 1{\cdot}6 - 0{\cdot}5 = 11{\cdot}4 \; cm.$$

$$\therefore \qquad A_t = \frac{384225}{10{,}000 \times 0{\cdot}84 \times 11{\cdot}4} = 4{\cdot}01 \; cm.^2$$

But 0·3% gross area $= \frac{0{\cdot}3}{100} \times 16 \times 100 = 4{\cdot}8 \; cm.^2$

Spacing of 10 *mm.* ϕ bars

$$= \frac{0{\cdot}79 \times 100}{4{\cdot}8} = 16{\cdot}46 \; cm. \; say \; 16 \; cm. \; c/c.$$

If the steel be provided near both the faces, the spacing will be at 32 *cm. c/c.* But the requirement for cantilevering effect itself is 4·09 cm^2. *We will provide a spacing of 32 cm. c/c away from the water face and a spacing of 16 cm. c/c near the water face.* Of these vertical bars near the water face alternate bars will be curtailed at a height of one *metre* above the haunch. Figs. 427 and 428 show the details of reinforcement.

Design 113. *A reinforced concrete tank is 6 m. × 3 m. with a maximum depth of 2·5 m. of water. The tank rests on ground. 15 cm. × 15 cm. splays are provided at the junction of walls and base slab. Design the tank.*

Solution. $L = 6$ *metres*

$l = 3$ *metres*

$$\therefore \qquad \frac{L}{l} = \frac{6}{3} = 2.$$

Since this ratio equals 2 the long walls will be designed as vertical cantilevers and short wall as spanning horizontally between long walls.

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Design of long walls. These are designed as vertical cantilevers. Since splays are provided it is enough if the cantilevering effect is considered above the top of the splay, *i.e.*, the effective height of the cantilevering long wall will be $2{\cdot}50 - 0{\cdot}15 = 2{\cdot}35$ *metres*.

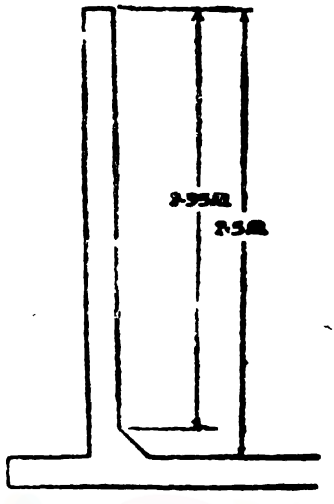

Fig. 4.9

$\therefore$ Maximum bending moment per *metre* width of the long wall

$$= \frac{wH^3}{6}$$

$$= \frac{1000 \times 2{\cdot}35^3}{6} \text{ kg. m.}$$

$$= 2163 \text{ kg. m.}$$

$$= 216300 \text{ kg. cm.}$$

This bending moment produces tension near the water face.

Adopting $c = 70$ *kg. cm.*^{2}

$t = 1000$ *kg./cm.*^{2}

$m = 13$.

Equating the moment of resistance to the maximum bending moment, we have,

$$14{\cdot}03 \times 100d^2 = 216300$$

$$d = 12{\cdot}42 \text{ cm.}$$

With a clear cover of 25 *mm.* and providing 16 *mm.* diameter bars, the overall depth required

$$= 12{\cdot}42 + 0{\cdot}80 + 2{\cdot}50 \text{ cm.} = 15{\cdot}72 \text{ cm.}$$

Hence let us provide 16 *cm.* overall.

$\therefore$ Actual effective depth

$$= 16 - 3{\cdot}3 = 12{\cdot}7 \text{ cm.}$$

$\therefore$ $$A_s = \frac{216320}{1000 \times 0{\cdot}84 \times 12{\cdot}7} \text{ cm.}^2 = 20{\cdot}28 \text{ cm}^2$$

$\therefore$ Spacing of 16 *mm.* diameter bars

$$= \frac{2{\cdot}01 \times 100}{20{\cdot}28} \text{ cm.}$$

$$= 9{\cdot}9 \text{ cm. } \textit{say } 9 \text{ cm c/c.}$$

Pull on the long wall. The water pressure on the short wall will be transferred to the long wall as tension Since the short wall will cantilever from the base slab for the bottom one *metre* of its height, the pull transferred to the long wall shall also be calculated at a level, one *metre* above the haunch

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∴ Pull on the long wall per *metre* height at the above level

$$= \frac{1000(2{\cdot}35-1)\times 3}{2} \text{ kg.} = 2025 \text{ kg.}$$

∴ Steel required for this pull

$$= \frac{2025}{1000} = 2{\cdot}02 \text{ cm}^2.$$

But 0·3% of gross area

$$= \frac{0{\cdot}3}{100} \times 16 \times 100 \text{ cm}^2. = 4{\cdot}8 \text{ cm}^2.$$

∴ Spacing of 10 *mm.* diameter bars

$$= \frac{0{\cdot}79 \times 100}{4{\cdot}8} \text{ cm.}$$

$$= 16{\cdot}46 \text{ cm. say } 16 \text{ cm.}$$

Since the steel is provided near both the faces the spacing of the 10 mm. diameter bars will be at 32 cm. c/c near each face.

Design of short walls. Consider a level one *metre* above the top of the haunch.

The pressure intensity at this level

$$= 1000(2{\cdot}35-1)$$

$$= 1350 \text{ kg./metre}^2$$

Effective span of horizontally spanning slab

= clear span + thickness of wall

$= 3 + 0{\cdot}16 \text{ m.} = 3{\cdot}16 \text{ m.}$

Corner Section. B.M. at corner section (*i.e.* ends)

$$= \frac{pl^2}{12}$$

$$= \frac{1350 \times 3{\cdot}16^2}{12} \text{ kg. m.}$$

$$= 1123 \text{ kg. m.} = 112{,}300 \text{ kg. cm.}$$

Water pressure per *metre* height at the level on one *metre* length of the long wall will be transferred to short wall.

∴ Tension transferred per *metre* height of short wall

$$= 1350 \times 1 = 1350 \text{ kg.}$$

∴ Resultant bending moment

$$= M - Tx$$

$$= 112300 - 1350(12{\cdot}7-8) \text{ kg. cm.}$$

$$= 105955 \text{ kg. cm.}$$

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This bending moment produces tension near the water face. Hence steel for bending moment

$$= \frac{105955}{1000 \times 0.84 \times 12.7} \ cm.^2 = 9.94 \ cm.^2$$

Steel for pull

$$\frac{1350}{1000} = 1.35 \ cm.^2$$

∴ Total steel per *metre* height

$$= 11.29 \ cm.^2$$

∴ Spacing of 16 *mm.* diameter bars

$$= \frac{2.01 \times 100}{11.29} \ cm.$$

$$= 17.8 \ cm. \text{ say } 17 \ cm. \ c/c.$$

Mid span section. Bending moment at mid span

$$M = \frac{pl^2}{24} = \frac{1350 \times 3.16^2}{24} \ kg. \ m.$$

$$= 561.5 \ kg. \ m. = 56150 \ kg. \ cm.$$

Pull $= T = 1350 \ kg$

Resultant bending moment

$$= M - Tx$$

$$= 56150 - 1350(12.7 - 8) \ kg. \ cm.$$

$$= 49805 \ kg. \ cm$$

∴ Steel for bending moment

$$= A_{s1} = \frac{49805}{1250 \times 0.86 \times 12.7} \ cm.^2$$

$$= 3.65 \ cm.^2$$

Steel for pull $= A_{s2} = \frac{1350}{1250} = 1.08 \ cm.^2$

∴ Total steel required per *metre* height

$$= 3.65 + 1.08 = 4.73 \ cm.^2$$

∴ Spacing of 16 *mm.* diameter bars

$$= \frac{2.01 \times 100}{4.75} \ cm. = 42.5 \ cm.$$

It is convenient to provide 16 mm. diameter bars at 34 cm. c/c which is just double the spacing of bars provided near the corners.

Design for the bottom one metre cantilevering effect of the short wall

Maximum bending moment for the cantilevering effect

$$= \frac{1000}{2} \times 2.35 \times 1 \times \tfrac{1}{3} \ kg. \ m.$$

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$= 392$ *kg. m.* $= 39200$ *kg. cm.*

Effective depth to the centre of 10 *mm.* diameter vertical bars

$= 16 - 2{\cdot}5 - 1{\cdot}6 - 0{\cdot}5$ *cm*

$= 11{\cdot}4$ *cm.*

$$\therefore \quad A_t = \frac{39200}{1000 \times 0{\cdot}84 \times 11{\cdot}4} \text{ cm.}^2 = 4{\cdot}09 \text{ cm.}^2$$

But 0·3% of gross area

$$= \frac{0{\cdot}3}{100} \times 16 \times 100 \text{ cm.}^2 = 4{\cdot}8 \text{ cm.}^2$$

∴ Spacing of 10 *mm.* diameter bars

$$= \frac{0{\cdot}79 \times 100}{4{\cdot}8} = 16{\cdot}46 \text{ cm.}$$

say *16 cm. c/c.*

If the steel be provided near both the faces the spacing will be at 32 *cm. c/c.* But the requirement of cantilever steel itself is 4·09 cm^2. *We will provide a spacing of 32 cm. c/c away from the water face and a spacing of 16 cm. near the water face.* Of these vertical bars near the water face alternate bars will be curtailed one *metre* above the haunch.

OVERHEAD TANKS

(Tanks situated above ground level)

§175. Rectangular Tanks Supported on Masonry Walls all Round

The design of such a tank consists of :

(*i*) Design of vertical walls, and

(*ii*) Design of base slab.

So far as the vertical walls are concerned, they are designed as in the case of tanks resting on ground level.

Base slab. The base slab will be designed as a one-way or a two-way slab depending on the lengths of the longer plan dimension L and the shorter dimension l

If $\frac{L}{l} = 2$ or greater than 2, the slab is designed as a one-way slab.

If $\frac{L}{l} < 2$ the slab will be designed as a two-way slab.

While designing the base slab, the effect of a possible transference of tension due to the pressure on the vertical walls should also be included.

Base slab as a one-way slab. As stated earlier this case will

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arise when $\frac{L}{l}=2$ or greater than 2. This slab will be spanning along the shorter dimension l.

§176. Bending Moment for the Base Slab

The bending moments are to be computed at the end and at mid span.

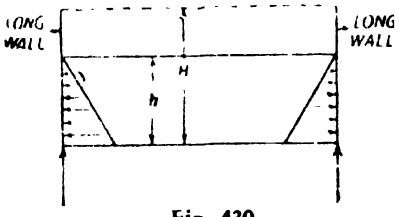

Fig. 430

At ends the bending moment will be a hogging moment caused by water pressure on the long wall. This hogging moment will be $\frac{wH^3}{6}$ for a strip of unit width.

This hogging bending moment will be a maximum when the tank is full. This will also be accompanied by a pull equal to $\frac{wH^2}{2}$.

The bending moment at mid span will consist of the sagging moment due to weight of water and weight of base slab and a hogging moment due to water pressure on the side walls. The net sagging bending moment per *metre* width when the depth of water is h is given by

$$M=\frac{whl^2}{8}-\frac{wh^3}{6}+w_d\cdot\frac{l^2}{8}$$

where w_d = wt. of the slab per *metre*^{2}

For M to be a maximum,

$$\frac{dM}{dh}=0$$

$$\therefore \quad \frac{dM}{dh}=\frac{wl^2}{8}-\frac{3wh^2}{6}=0$$

$$\therefore \quad h^2=\frac{l^2}{4}$$

$$\therefore \quad h=\frac{l}{2}$$

$$\therefore \quad M_{max}=\frac{wl^2}{8}\left(\frac{l}{2}\right)-\frac{w}{6}\left(\frac{l}{2}\right)^3+\frac{w_dl^2}{8}$$

$$M_{max}=\frac{wl^3}{24}+\frac{w_dl^2}{8}$$

The pull corresponding to this condition

$$=T=\frac{wh^2}{2}=\frac{w}{2}\left(\frac{l}{2}\right)^2=\frac{wl^2}{8}$$

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There may be a possibility that the height of the tank itself may be less than $\frac{l}{2}$. In such a case with the full tank condition the maximum sagging bending moment at the centre should be determined and the pull should be computed with the tank full.

When the height of the tank is considerably greater than $\frac{l}{2}$, it is possible that when the tank is full, there may be a hogging bending moment at mid span. In such a case, the mid span section should also be designed for this hogging bending moment accompanied by the pull corresponding to the full tank condition.

Design 114 (S I). *A reinforced concrete water tank is 6 metres × 3 metres with a maximum depth of 2·50 metres. 15 cm. × 15 cm. splays are provided at the junction of walls and base slab. The tank is supported on brick masonry walls all round. Design the tank.*

Solution.

Design of the walls. These are designed as in the case of the tank resting on ground. See designs 112 and 113.

Design of the base slab The thickness of the base slab may be made 4 *cm* greater than the thickness of the walls.

In our case, the walls are 16 *cm.* thick and hence the base slab will be 16 + 4 = 20 *cm.* thick.

Midspan section of the base slab Maximum sagging bending moment at the centre due to weight of water and water pressure on the long walls

$$= \frac{wl^3}{24} = \frac{9810 \times 3{\cdot}16^3}{24} \ Nm$$

$$= 12897{\cdot}9 \ Nm. = 1289790 \ N \ cm.$$

Sagging moment due to weight of base slab

$$= (250 \times 20) \frac{3{\cdot}16^2}{8} = 6241 \ Nm.$$

$$= 624100 \ N \ cm.$$

Maximum sagging moment

$$= 1289790 + 624{\cdot}0 = 1913890 \ N \ cm.$$

This is accompanied by a pull of $\frac{wl}{8}$

$$= \frac{9810 \times 3{\cdot}16^2}{8} = 12245 \ N$$

If 16 *mm.* ϕ bars be used at a clear cover of 25 *mm.*, the effective depth

$$= 20 - 2{\cdot}5 - 0{\cdot}8 = 16{\cdot}7 \ cm.$$

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Resultant bending moment

$$= M - Tx = 1913890 - 12245\ (16{\cdot}7 - 10)\ N\ cm.$$

$$= 1913890 - 82042 = 1831848\ N\ cm.$$

This bending moment produces tension away from the water face.

Steel for bending moment

$$= A_{s1} = \frac{1831848}{12500 \times 0{\cdot}86 \times 16{\cdot}7} = 10{\cdot}20\ cm.^2$$

Steel for pull $= A_{s2} = \dfrac{12245}{12500} = 0{\cdot}98\ cm.^2$

Total steel required

$$= A_s = A_{s1} + A_{s2} = 10{\cdot}20 + 0{\cdot}98 = 11{\cdot}18\ cm.^2$$

Spacing of 16 *mm.* ϕ bars

$$= \frac{2{\cdot}01 \times 100}{11{\cdot}18} = 17{\cdot}9\ cm.$$

Provide 16 mm. ϕ @ 17 cm. c/c.

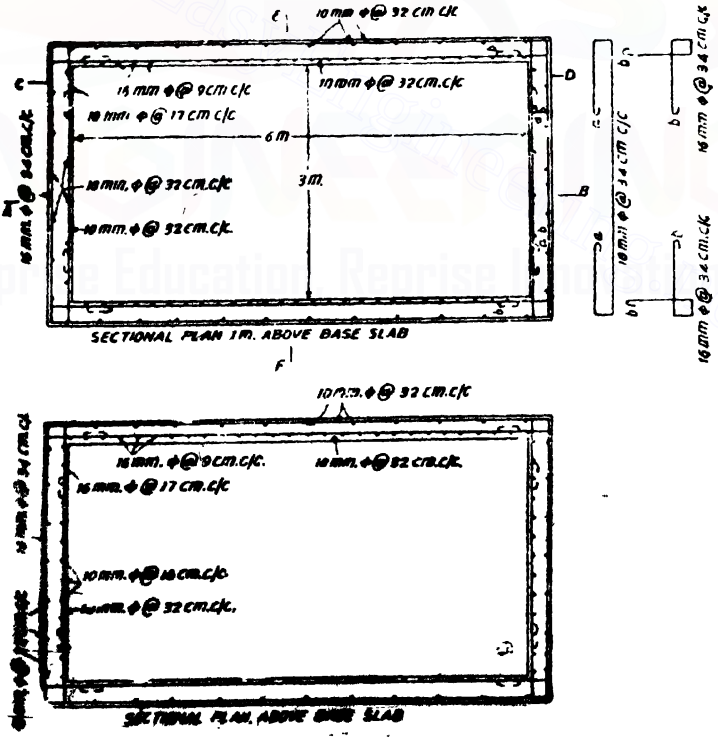

Fig. 431

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Distribution steel $= \frac{0{\cdot}3}{100} \times 20 \times 100 = 6$ *cm.*²

∴ **Spacing of 10 *mm.* ϕ bars**

$$= \frac{0{\cdot}79 \times 100}{6} = 13 \text{ cm. c/c}$$

Hence provide 10 mm. ϕ @ 26 cm. c/c. near each face.

Check for a possibility of a hogging moment at mid span. Let us calculate the bending moment at the mid span when the tank is full. If even, when the tank is full the bending moment at mid span is a sagging moment, it should be concluded that there is no possibility of occurrence of a hogging moment at mid span.

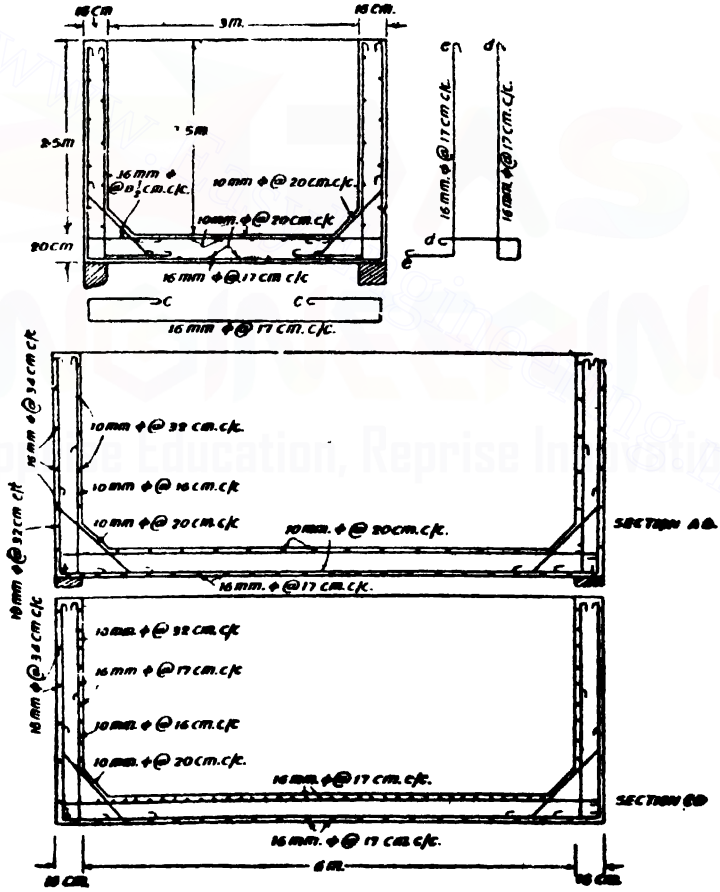

Fig. 432

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When the tank is full, the bending moment at the centre

$$= \left[\frac{9810 \times 2{\cdot}5 \times 3{\cdot}16^2}{8} + \frac{250 \times 20 \times 3{\cdot}16^2}{8} - 9810 \times \frac{2{\cdot}5^3}{6} \right] Nm.$$

$$= +11306\ Nm.$$

This is a sagging moment. Hence, there is no possibility of occurrence of any hogging moment at the mid span section.

End section. Maximum hogging moment (at full tank condition).

$$M = -\frac{wH^3}{6} = -\frac{9810 \times 2{\cdot}5^3}{6}\ Nm.$$

$$= -25547\ Nm. = 2554700\ N\ cm.$$

This is accompanied by a pull of T

$$= \frac{wH^2}{2}$$

$$T = \frac{9810 \times 2{\cdot}5^2}{2}\ N = 30656\ N$$

Resultant moment $= M - Tx$

$$= 2554700 - 30656\ (16{\cdot}7 - 10)\ N\ cm.$$

$$= 2349300\ N\ cm.$$

This bending moment produces tension near the water face

∴ Steel for bending moment

$$= A_{s1} = \frac{2349300}{10{,}000 \times 0{\cdot}84 \times 16{\cdot}7} = 16{\cdot}7\ cm.^2$$

Steel for pull $= A_{s2} = \frac{30656}{10{,}000} = 3{\cdot}1\ cm.^2$

Total steel required $= A_s = A_{s1} + A_{s2}$

$$= 16{\cdot}7 + 3{\cdot}1 = 19{\cdot}8\ cm.^2$$

Spacing of 16 *mm.* ϕ bars

$$= \frac{2{\cdot}01 \times 100}{19{\cdot}8} = 10{\cdot}1\ cm.$$

Provide 16 mm. ϕ @ 10 cm. c/c.

Figs. 431 and 432 show the details of reinforcement.

Design 115. *A reinforced concrete water tank is 6 metres × 3 metres with a maximum depth of 2·5 metres. 15 cm. × 15 cm. splays are provided at the junction of the walls and base slab. The tank is supported on brick masonry walls all round. Design the tank.*

Solution. Design of vertical walls

These are designed as in the case of the tank resting on ground. See designs 112 and 113.

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Design of the base slab

The thickness of the base slab may be assumed about 4 *cm.* greater than the thickness of the vertical walls.

In our case the vertical walls are 16 *cm.* thick and hence the base slab will be 16+4 = 20 *cm.* thick.

Mid span section of the base slab

Max sagging moment at the centre due to weight of water and water pressure on long walls

$$= \frac{wl^3}{24} = \frac{1000 \times 3{\cdot}16^3}{24} \text{ kg. m.}$$

$$= 1315 \text{ kg. m.} = 131500 \text{ kg. cm.}$$

Sagging moment due to self-weight

Self-weight of the slab

$$= 25 \times 20 = 500 \text{ kg./m.}^2$$

Sagging moment due to self weight of base slab

$$= \frac{500 \times 3{\cdot}16^2}{8} \text{ kg. m.}$$

$$= 624{\cdot}1 \text{ kg. m.} = 62410 \text{ kg. cm.}$$

∴ Maximum sagging moment

$$= 131500 + 62410 \text{ kg. cm.}$$

$$= 193{,}910 \text{ kg. cm.}$$

This is accompanied by a pull of $\frac{wl^2}{8}$

$$= \frac{1000 \times 3{\cdot}16^2}{8} \text{ kg.} = 1248 \text{ kg.}$$

If 16 *mm.* diameter bars be used at a clear cover of 25 *mm.* the effective depth

$$= 20 - 2{\cdot}5 - 0{\cdot}8 \text{ cm.} = 16{\cdot}7 \text{ cm.}$$

∴ Resultant bending moment

$$= M - Tx$$

$$= 193{,}910 - 1248(16{\cdot}7 - 10) \text{ kg. cm.}$$

$$= 185550 \text{ kg. cm.}$$

This bending moment produces tension away from the water face

Steel for bending moment

$$= \frac{185550}{1250 \times 0{\cdot}86 \times 16{\cdot}7} \text{ cm.}^2$$

$$= 10{\cdot}3 \text{ cm.}^2$$

Steel for pull $= \frac{1248}{1250} = 1{\cdot}00 \text{ cm.}^2$

∴ Total steel $= 10{\cdot}30 + 1{\cdot}00 = 11{\cdot}3 \text{ cm.}^2$

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∴ Spacing of 16 *mm.* diameter bars

$$= \frac{2{\cdot}01 \times 100}{11{\cdot}3} \text{ cm.}$$

$$= 17{\cdot}8 \text{ cm. say } 17 \text{ cm. c/c.}$$

Distribution steel $= \frac{0{\cdot}3}{100} \times 20 \times 100 = 6 \text{ cm.}^2$

∴ Spacing of 10 *mm.* dia. bars

$$= \frac{0{\cdot}79 \times 100}{6} \text{ cm.} = 13 \text{ cm. c/c.}$$

Hence provide 10 mm dia. bars at 26 cm. c/c near each face.

Check for possibility of a hogging moment at mid span

Let us calculate the bending moment at mid span when the tank is full. If even when the tank is full the bending moment at mid span is of the sagging type it should be concluded that there is no possibility of occurrence of a hogging moment at mid span.

In our case, when the tank is full, the bending moment at the centre will be

$$\left[\frac{1000 \times 2{\cdot}5 \times 3{\cdot}16^2}{8} + \frac{500 \times 3{\cdot}16^2}{8} - 1000 \times \frac{2{\cdot}5^3}{6}\right] \text{kg. m.}$$

$$= +1140 \text{ kg. m.}$$

This is sagging moment. Hence there is no possibility of any hogging bending moment to occur at the mid span section.

End Section. Maximum hogging moment

$$= \frac{wH^3}{6} = \frac{1000 \times 2{\cdot}5^3}{6} \text{ kg. m.}$$

$$= 2603 \text{ kg. m.} = 260300 \text{ kg. cm.}$$

This is accompanied by a pull of $\frac{wH^2}{2}$

$$= \frac{1000 \times 2{\cdot}5^2}{2} \text{ kg.} = 3125 \text{ kg.}$$

Resultant moment $= M - Tx$

$$= 260300 - 3125(16{\cdot}7 - 10) \text{ kg. cm.}$$

$$= 239{,}360 \text{ kg. cm.}$$

This bending moment produces tension near the water face.

∴ Steel for bending moment

$$= \frac{239360}{1000 \times 0{\cdot}84 \times 16{\cdot}7} \text{ cm.}^2 = 17{\cdot}06 \text{ cm.}^2$$

∴ Steel for pull $= \frac{3125}{1000} = 3{\cdot}12 \text{ cm.}^2$

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$\therefore$ Total steel required

$$= 17{\cdot}06 + 3{\cdot}12 \text{ cm.}^2 = 20{\cdot}18 \text{ cm.}^2$$

$\therefore$ Spacing of 16 *mm.* diameter bars

$$= \frac{2{\cdot}01 \times 100}{20{\cdot}18} = 10 \text{ cm.}$$

§177. Square Tank Supported on Masonry Walls

Walls. These are designed as in the case of the tank resting on ground.

Base slab

The base slab will be designed as a slab spanning in two directions.

Mid span section Let the depth of water be h *metres*. Consider a strip of unit width spanning in a principal direction.

Sagging moment at the mid span due to weight of water and pressure on the vertical walls

$$= M = \left(\frac{wh}{2}\right)\frac{l^2}{8} - \frac{wl}{6}$$

For M to be a maximum,

$$\frac{dM}{dh} = 0$$

$\therefore$

$$\frac{dM}{dh} = \frac{wl^2}{16} - \frac{wh^2}{2} = 0$$

$\therefore$

$$h^2 = \frac{l^2}{8}$$

$\therefore$

$$h = \frac{l}{2\sqrt{2}}$$

$\therefore$ Max. sagging moment due to weight of water and lateral pressure

$$= \left(\frac{w}{2} \cdot \frac{l}{2\sqrt{2}}\right)\frac{l^2}{8} - \frac{w}{6}\left(\frac{l}{2\sqrt{2}}\right)^3 = \frac{wl^3}{48\sqrt{2}}$$

Let w_s be the weight of the base slab per unit area.

$\therefore$ B.M. at the centre due to self-weight of base slab

$$= \frac{w_s}{2} \cdot \frac{l^2}{8} = \frac{w_s l^2}{16}$$

$\therefore$ Maximum sagging moment at mid span

$$= \frac{wl^3}{48\sqrt{2}} + \frac{w_s l^2}{16}$$

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This bending moment is accompanied by a pull of $\frac{wh^2}{2}$

$$= \frac{w}{2}\left[\frac{l}{2\sqrt{2}}\right]^2 = \frac{wl^2}{16}.$$

End section. It is usual to design the end section for maximum bending moment of $\frac{wH^3}{6}$, accompanied by a pull of $\frac{wH^2}{2}$ where H is the maximum depth of water in the tank.

Thickness of base slab

This may be kept equal to the thickness of the vertical walls.

§178. Wind Load Analysis of Columns and Bracings of a Water Tower

The wind load on the tank walls and the supporting structure should also be included in the design of the supporting columns and the braces. The exact sway analysis for the wind load effect is very lengthy and for all design purposes the approximate method outlined below is found to be sufficiently accurate. The method is based on approximate estimation of the points of contraflexure in the columns and the braces.

The following cases usually occur in practice.

Case 1. *Tank supported on four identical columns hinged at the base.* Fig. 433 shows four columns each of height h supporting a tank. Let P be the total wind load on the tank face. Since the four columns have the same stiffness the lateral load P will be resisted equally by the horizontal thrusts. Hence each horizontal thrust $= \frac{P}{4}$. Considering now any one column, the bending moment at any section at a hight y above the hinge due to the horizontal thrust $= Py$.

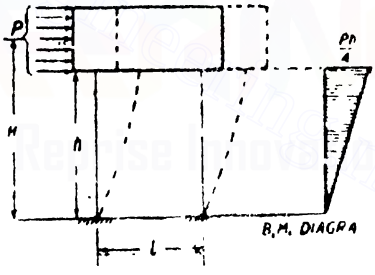

Fig. 433

Hence the bending moment for the column varies from O at the hinge to $\frac{Ph}{4}$ at the top.

The vertical reactions induced by wind load can be easily determined as follows :

Let V be the upward reaction at each of the leeward columns and let V be the downward reaction at each of the windward columns. Let l be the distance between the leeward and the windward columns.

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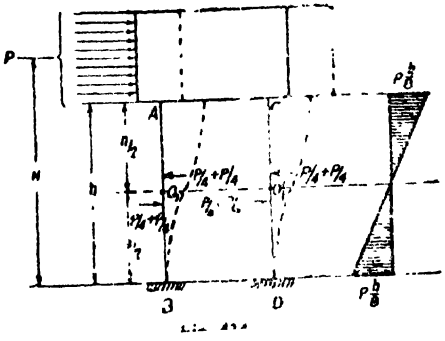
Taking moments about the windward columns, we have,

$$2Vl = PH$$

$$\therefore \quad V = \frac{PH}{2l}$$

Hence the effect of a wind load is to increase the compressive load by the above extent in the leaward columns and to produce a tension of the above extent in the windward columns.



Case 2. *Tank supported on four identical columns with their lower end fixed to the base.*

Assume that points of contraflexure occur at the middle points of the columns. These are shown in Fig. 43.4 O_1, O_1' represent the points of contraflexure. Introduce horizontal forces of magnitude $\frac{P}{4}$ just above the levels of the points of contraflexure in a direction opposing the direction of the wind load. Introduce also horizontal forces $\frac{P}{4}$ just below the points of contraflexure in the direction of the wind load.

$$\text{B.M. at the top of each column} = \frac{P}{4} \times \frac{h}{2} = \frac{Ph}{8}$$

$$\text{B.M. at the bottom of the column} = \frac{P}{4} \times \frac{h}{2} = \frac{Ph}{8}$$

Assuming the columns to be hinged at the levels of the points of contraflexure the vertical reactions are determined by taking moments about the points of contraflexure O_1.

$$\therefore \quad 2Vl = P\left(H - \frac{h}{2}\right)$$

$$\therefore \quad V = \frac{P\left(H - \frac{h}{2}\right)}{2l}$$

There will be a compression to the above extent in the leeward columns and a tension to the same extent in the windward columns.

Hence the bottom of the column should be designed due to the following forces and moments :

(*i*) ¼ Dead load of the tank,

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(*ii*) Self-weight of the column

(*iii*) A compression of $\frac{P\left(H-\frac{h}{2}\right)}{2l}$

(*iv*) A B.M. of $\frac{Ph}{8}$.

Case 3. *Tank supported on four identical columns with their lower ends fixed and braced at intermediate levels.*

Fig. 435 shows a water tank supported on four columns braced at intermediate levels

It will be assumed that the points of contraflexure will occur at the middle points of the various panels. Introduce horizontal forces $\frac{P}{4}$ just above and below the levels of the points of contraflexure of the columns.

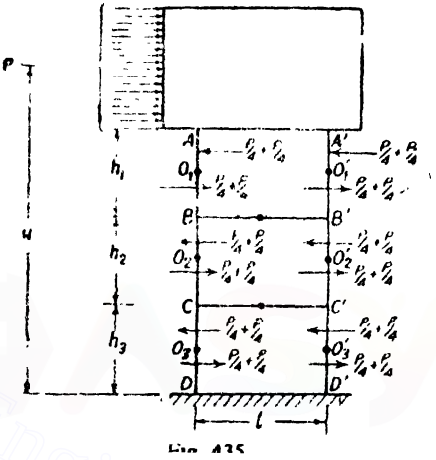

Fig. 435

The b.m. at the various conspicuous points are given by :

B.M. at A and A' $= \frac{P}{4} \times \frac{h_1}{2} = \frac{Ph_1}{8}$

B.M. just above B and $B' = \frac{Ph_1}{8}$

B.M. just below B and $B' = \frac{Ph_2}{8}$

B.M. just above C and $C' = \frac{Ph_2}{8}$

B.M. just below C and $C' = \frac{Ph_3}{8}$

B.M. at D and $D' = \frac{Ph_3}{8}$

Let the vertical force in each of the columns at the levels O_1 and $O_1' = V_1$

Taking moments about O_1 we have

$$2V_1 \times l = P\left[H-\left(h_3+h_2+\frac{h_3}{2}\right)\right]$$

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$$V_1 = \frac{P\left[H-\left(h_3+h_2+\frac{h_1}{2}\right)\right]}{2l}$$

This will be a compressive force in the leeward columns and a tensile force in the windward columns.

Let the vertical force in the columns at the levels of O_2 and O_2' be V_2

Taking moments about O_2 we have

$$2V_2 l = P\left[H-\left(h_3+\frac{h_2}{2}\right)\right]$$

$$\therefore \quad V_2 = \frac{P\left[H-\left(h_3+\frac{h_2}{2}\right)\right]}{2l}$$

This will be a compressive force in the leeward columns and a tensile force in the windward columns.

Let the vertical force in the columns at the levels of O_3 and O_3' be V_3.

Taking moments about O_3, we have,

$$2V_3 l = P\left[H-\frac{h_3}{2}\right]$$

$$\therefore \quad V_3 = \frac{P\left(H-\frac{h_3}{2}\right)}{2l}$$

This will be a compressive force on the leeward columns and a tensile force in the windward columns.

Braces. Brace BB'

Consider the equilibrium of the part $O_1BO_2O_4$ (see Fig. 436).

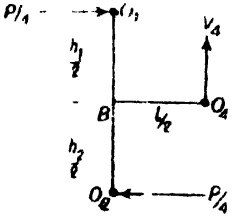

Fig. 436

The vertical reaction V_4 at the point of contraflexure O_4 can be easily determined by taking moments about O_2.

$$\therefore \quad \frac{P}{4}\left(\frac{h_1+h_2}{2}\right) = V_4 \cdot \frac{l}{2}$$

$$\therefore \quad V_4 = \frac{P}{4}\,\frac{(h_1+h_2)}{l}$$

$\therefore$ **B.M. at the end of the brace**

$$= V_4 \cdot \frac{l}{2}$$

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$$= \frac{P(h_1+h_2)}{4l} \cdot \frac{l}{2}$$

$$= \frac{P(h_1+h_2)}{8}$$

= sum of the moments for the columns just above and below B.

The shear in the brace BB' will be V_4

$$= \frac{P(h_1+h_2)}{4l}$$

Similarly,

B.M. at the ends of brace CC'

$$= \frac{P}{8}(h_2+h_3)$$

S.F. in the brace $CC' = \frac{P(h_2+h_3)}{4l}$

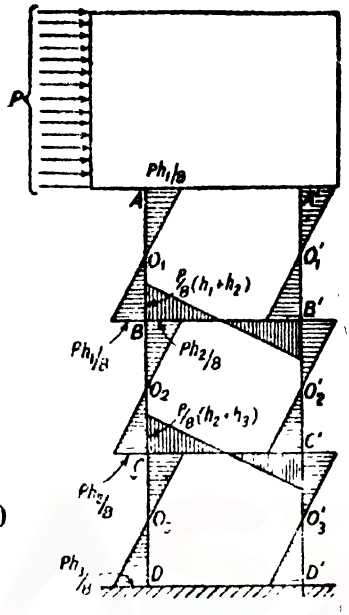

Fig. 437

Fig. 437 shows the B.M diagram for the frame

§179. Design of a Square Tank Supported on Columns and Beams

The various components to be designed are

(*i*) the tank walls (*ii*) the bottom slab

(*iii*) The supporting columns and braces.

Tank walls. These are designed exactly as in the case of the tank resting on the ground. (*See* Design 110).

Bottom slab. The bottom slab should be designed as a two-way slab. Water load per *square metre* in each way

$$= \frac{wH}{2}$$

Let the dead load on the slab be w_d per unit area.

The water pressure on the tank walls will induce a hogging moment. For design purposes, in the design of the bottom slab full cantilever effect of the tank walls is usually allowed. Hogging bending moment caused by water pressure on the tank walls

$$= \frac{wh^3}{6}$$

where the depth of water is h units.

Mid span section Sagging b.m. at mid span ignoring self wt. of base slab

$$= M = \left(\frac{wh}{2}\right)\frac{l^2}{8} - \frac{wh^3}{6}$$

for any depth of water above the base slab.

$$M = \frac{whl^2}{16} - \frac{wh^3}{6}$$

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For M to be maximum,

$$\frac{dM}{dh}=0$$

$$\therefore \quad \frac{dM}{dh}=\frac{wl^2}{16}-\frac{wh^2}{2}=0$$

$$\therefore \quad h^2=\frac{l^2}{8}$$

$$\therefore \quad h=\frac{l}{2\sqrt{2}}$$

$\therefore$ Maximum sagging B.M. at mid span

$$-M_{max}=\frac{wl^2}{16}\left(\frac{l}{2\sqrt{2}}\right)-\frac{w}{6}\left(\frac{l}{2\sqrt{2}}\right)^3$$

$$=\frac{wl^3}{32\sqrt{2}}-\frac{wl^3}{96\sqrt{2}}=\frac{wl^3}{48\sqrt{2}}$$

B.M. due to dead load $=\frac{w_d l^2}{16}$ (allowing two-way action of slab)

The pull corresponding to the head of h units

$$=\frac{wh^2}{2}$$

$$=\frac{w}{2}\left(\frac{l}{2\sqrt{2}}\right)^2=\frac{w}{2}\cdot\frac{l^2}{8}$$

$$=\frac{wl^2}{16}$$

The overall thickness of the bottom slab may be kept equal to the thickness of the walls.

End section. The maximum hogging bending moment at the end section will occur when the tank is filled completely. The bending moment at the end section corresponding to this condition will be

$$\frac{wH^3}{6}$$

This is accompanied by a pull of

$$\frac{wH^2}{2}.$$

Beams supporting the tanks. When supported on columns, the beams should be designed for partially fixed end conditions. It is usual to design for a B.M. of $\frac{ql^2}{12}$ for the ends and mid section where q is the load per *metre* run on the beam. The overall depth of the beam may be kept at 15 *cm.* to 16 *cm.* per *metre* run of the span. The section may be doubly reinforced if found necessary. The additional strength due to the composite action of the tank walls with the bottom slab is usually ignored.

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Columns. These are to be designed for the direct load transferred to them due to the weight of the tank and its contents, beams and self-weight of columns, and braces and also for the wind pressures. Braces should be provided at 3 *metres* to 4·5 *metres* intervals.

Design 116 (S I) *An open square tank 5 m. × 5 m × 3 m. deep and supported 6 metres above the ground level on beams and columns. Design the tank and the supporting beams and columns, and the foundation.*

Solution.

Tank walls. These are designed as in the case of the tank resting on ground. See design 110

Bottom slab. Overall thickness = 30 *cm.*

Mid section of the slab

Consider a 1 *metre* wide strip of the slab.

Effective span = Distance between centres of walls

$$= 5 + 0{\cdot}3 = 5{\cdot}30 \ m.$$

Maximum sagging moment due to weight of water and pressure on walls

$$= \frac{wl^3}{48\sqrt{2}} = \frac{9810\,(5{\cdot}3)^3}{48\sqrt{2}} \ Nm.$$

$$= 21520 \ Nm.$$

D.L. of base slab = $2{\cdot}0 \times 30 = 7{\cdot}00 \ N/metre^2$

Since the slab is a two way slab, the load distribution each way

$$= \frac{7500}{2} = 3750 \ N/metre^2$$

∴ Sagging moment due to dead load of slab

$$= \frac{3750 \times 5{\cdot}30^2}{8} \ Nm. = 13170 \ Nm.$$

Total sagging moment

$$= M = 21520 + 13170 = 34690 \ Nm.$$

$$= 3469000 \ cm.$$

$$\text{Pull} = T = \frac{wl^2}{16} = \frac{9810 \times 5{\cdot}3^2}{16} = 17220 \ N.$$

Providing 20 *mm.* ϕ bars at a clear cover of 25 *mm.*

Effective depth $= d = 30 - 2{\cdot}5 - 2 - 1 = 24{\cdot}50$ *cm.*

Resultant B.M. $= M - Tx$

$$= 3469000 - 17220(24{\cdot}5 - 15) \ Nm.$$

$$= 3305410 \ N \ cm.$$

This B.M. produces tension away from the water face. Hence adopt $c = 700 \ N/cm.^2$; $t = 12500 \ N/cm.^2$ and $a = 0{\cdot}86 \ d$.

$$\text{Steel for B.M.} = A_{s1} = \frac{3305410}{12500 \times 0{\cdot}86 \times 24{\cdot}5} = 12{\cdot}60 \ cm.^2$$

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Steel for pull $= A_{t2} = \frac{17220}{12500} = 1{\cdot}40$ *cm.*^{2}

Total steel required $= A_t = A_{t1} + A_{t2} = 12{\cdot}60 + 1{\cdot}40 = 14$ *cm.*^{2}

Provide 20 mm. ϕ *@ 21 cm. c/c.*

End section

Maximum bending moment

$$= M = \frac{wH^3}{6}$$

$$= \frac{9810 \times 3^3}{6} = 44145 \text{ Nm.}$$

$$= 4414500 \text{ N cm.}$$

$$\text{Pull} = T = \frac{wH^2}{2} = \frac{9810 \times 3^2}{2} = 44145 \text{ N}$$

Resultant bending moment

$$= M - Tx$$
$$= 4414500 - 44145\,(24{\cdot}5 - 15)$$
$$= 3995120 \text{ N cm.}$$

This bending moment produces tension near water face. Hence adopt $c = 700$ *N/cm.*^{2}, $t = 10{,}000$ *N/cm.*^{2} and $a = 0{\cdot}84\,d$.

$\therefore$ Steel for B.M $= A_{t1} = \frac{3995120}{10{,}000 \times 0{\cdot}84 \times 24{\cdot}5} = 19{\cdot}4$ *cm.*^{2}

Steel for pull $= A_{t2} = \frac{44145}{10{,}000} = 4{\cdot}4$ *cm.*^{2}

Total steel required $= A_{t1} + A_{t2} = 19{\cdot}4 + 4{\cdot}4 = 23{\cdot}8$ *cm.*^{2}

Spacing of 20 *mm* ϕ bars

$$= \frac{3{\cdot}14 \times 100}{23{\cdot}8} = 13{\cdot}2 \text{ cm. say } 13 \text{ cm. c/c}$$

Design of beams

Width of beam $= 30$ *cm.*

Overall depth of beam : 15 *cm.* per *metre* of span

$$= 15 \times 5{\cdot}3 = 79{\cdot}5 \text{ cm. say } 80 \text{ cm.}$$

Load per metre run

(*i*) Weight of wall $= 3{\cdot}30 \times 0{\cdot}3 \times 1 \times 25000$ *N* $= 24750$ *N/m.*

(*ii*) D.L. of slab (two-way distribution) $= 9375$ *N/m.*

$= 3750 \times \frac{5}{4}$

(*iii*) Weight of water (two-way distribution)

$$= \frac{9810 \times 3}{2} \times \frac{5}{2} = 36790 \text{ N/m.}$$

(*iv*) Weight of beam + 30 *cm.* slab

$$= \left[\frac{80 + 30}{100}\right] \times 0{\cdot}3 \times 25000 = 8250 \text{ N/m.}$$

Total load per *metre* run $= 79165$ *N/m.*

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Maximum bending moment

$$= \frac{Ql^2}{12} = \frac{79165 \times 5{\cdot}3^2}{12} \ Nm.$$

$$= 185310 \ Nm.$$

$$= 18531000 \ Ncm.$$

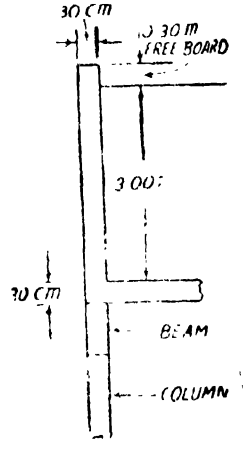

Fig 438

Now onwards let the concrete used be *M* 15.

Taking $c = 500 \ N/cm.^2$, $t = 14000 \ N/cm.^2$, $m = 18$, $a = 0{\cdot}87$ d and $Q = 85$ and equating the moment of resistance to the bending moment,

$$85 \ bd^2 = 85 \times 30 \ d^2 = 18531000$$

$$\therefore \quad d = 85{\cdot}2 \ cm.$$

With 25 *mm.* cover and 24 *mm.* ϕ bars

Effective cover $= 3{\cdot}7 \ cm.$

Overall depth required $= 85{\cdot}2 + 3{\cdot}7 = 88{\cdot}9 \ cm.$

Provide an overall depth of 90 *cm.*

Since the previously assumed overall depth is 80 *cm.*, increase in depth $= 10 \ cm.$

Additional weight per *metre run*

$$= 0{\cdot}10 \times 0{\cdot}30 \times 25000 = 750 \ N/m.$$

Corresponding increase in B.M.

$$= \frac{750 \times 5{\cdot}30^2}{12} = 1756 \ N \ m.$$

$$= 175600 \ N \ m.$$

$\therefore$ Total B.M. $= 18531000 + 175600 = 18706600 \ N \ cm.$

Equating M.R. to B.M.,

$$85 \ bd^2 = 85 \times 30 \ d^2 = 18706600$$

$$\therefore \quad d = 85{\cdot}6 \ cm.$$

Effective depth available

$$= 90 - 3{\cdot}7 = 86{\cdot}3 \ cm.$$

$$\therefore \quad A_t = \frac{18706600}{14000 \times 0{\cdot}87 \times 86{\cdot}3} = 17{\cdot}79 \ cm.^2$$

Provide 4 bars of 24 mm. ϕ (18·08 cm.^{2})

Shear analysis

Maximum shear force

$$= \frac{(79165 + 750) \times 5{\cdot}3}{2} = 211775 \ N$$

Nominal shear stress $= q_v = \frac{S}{bd} = \frac{211775}{30 \times 86{\cdot}3} = 82 \ N/cm.^2$

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$$= 0{\cdot}82 \ N/mm.^2$$

Percentage of steel $= \dfrac{18{\cdot}08}{30 \times 86{\cdot}3} \times 100 = 0{\cdot}7\%$

For 0·7% steel, $q_c = 0{\cdot}33 \ N/mm.^2 = 33 \ N/cm.^2$

$\therefore$ Shear resistance of concrete $= q_c \ bd$

$$= 33 \times 30 \times 86{\cdot}3 = 85437 \ N$$

$\therefore$ Shear to be resisted by stirrups

$$= V_s = 211775 - 85437 = 126338 \ N$$

Spacing of 4-legged 8 *mm.* ϕ stirrups

$$= \frac{A_w \ f_w \ d}{V_s} = \frac{(4 \times 0{\cdot}5) \ 13000 \times 86{\cdot}3}{126338}$$

$$= 19 \ cm.$$

Let the section where the nominal shear stress is 0·33 $N/mm.^2$ be at x *metres* from the centre.

$$\frac{x}{\frac{5{\cdot}3}{2}} = \frac{0{\cdot}33}{0{\cdot}82} \quad \therefore \ x = 1{\cdot}10 \ m.$$

Design of columns

Load on one column $= (79165 + 750) \times 5{\cdot}3 = 423550 \ N.$

Area of tank exposed to wind pressure

$$= 5{\cdot}6 \times \begin{bmatrix} 3{\cdot}00 + 0{\cdot}3 \ m. \text{ free board} \\ + 0{\cdot}3 \ m \text{ thickness of slab} \\ + 0{\cdot}9 \ m. \text{ depth of beam} \end{bmatrix} = 5{\cdot}6 \times 4{\cdot}5 \ metre^2 = 25{\cdot}20 \ metre^2$$

Wind load at 1500 $N/metre^2 = 25{\cdot}20 \times 1500 = 37800 \ N$

Wind load on one pair of columns

$$= \frac{37800}{2} = 18900 \ N$$

Assume that the points of contraflexure are at the middle points of the respective panels. See Fig. 439. Taking moments about the lower points of contraflexure C, we have,

Direct load on column due to wind

$$= \frac{18900 \times 6{\cdot}75}{5{\cdot}3} = 24070 \ N$$

Column section at B

Load from the tank and its contents	$= 423550 \ N$
D.L. of column (30 *cm.* × 30 *cm*) $= 0{\cdot}3 \times 0{\cdot}3 \times 6 \times 25000$	$= 13500 \ N$
Weight of one brace $= (30 \ cm. \times 30 \ cm) = 0{\cdot}3 \times 0{\cdot}3 \times 5 \times 25000$	$= 11250 \ N$
Direct load due to wind	$= 24070 \ N$
Total	$= 472370 \ N$

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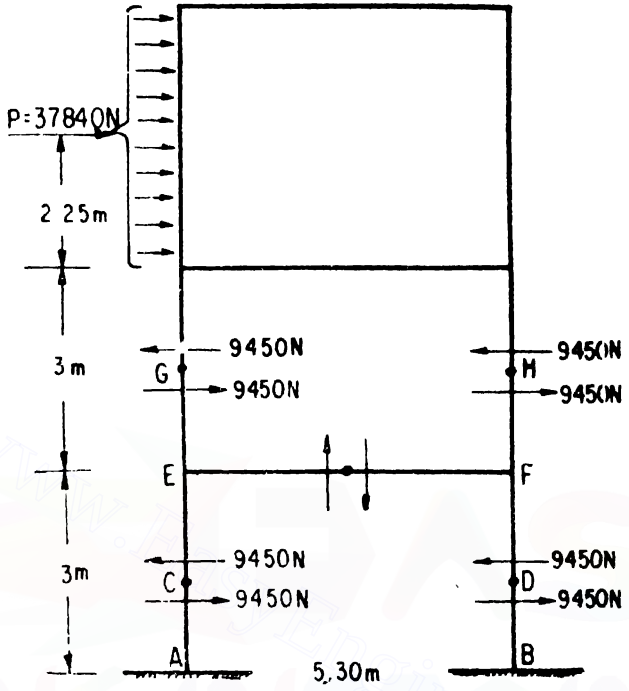

Fig. 4.39

B.M. at $B = 9450 \times 1{\cdot}5 = 14175\ Nm.$

Let 8 bars of 24 $mm.$ ϕ be provided.

Area of 8 bars $= 8 \times 4{\cdot}52 = 36{\cdot}16\ cm^2$

Equivalent area of concrete

Equivalent concrete area

$$= 30^2 + (18-1)\ 36{\cdot}16$$

$$= 1514{\cdot}7\ cm.^2$$

Moment of inertia about the centroidal axis

$$= I = \frac{30 \times 30^3}{12} + (18-1)\ 27{\cdot}12 \times 9{\cdot}8^2\ cm.^4$$

$$= 111778\ cm.^4$$

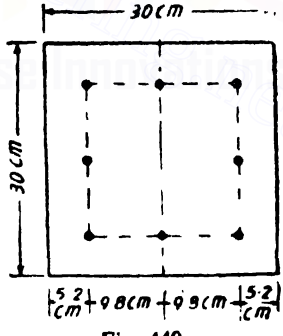

Fig. 4.40

Extreme stresses on the section

$$= -\frac{472370}{1514{\cdot}7} \pm \frac{14175 \times 100}{111778} \times 15\ N/cm.^2$$

$$= 311{\cdot}8 \pm 190{\cdot}2\ N/cm.^2$$

$$= 502\ N/cm.^2 \text{ and } 121{\cdot}6\ N/cm.^2$$

$$= 5{\cdot}02\ N/mm.^2 \quad \text{and} \quad 1{\cdot}216\ N/mm.^2$$

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Design of the brace

The brace will be designed as a doubly reinforced beam section, by steel beam theory.

Maximum bending moment for the brace

$$=(9450 \times 1{\cdot}5)+(9450 \times 1{\cdot}5)\ Nm.$$

$$=28350\ Nm.$$

Providing an effective cover of 4 *cm.* to top and bottom bars, distance between centres of top and bottom bars $=30-8=22$ *cm.*

$$\therefore \quad A_c=A_t=\frac{28350 \times 100}{14000 \times 22}$$

$$=9{\cdot}20\ cm^2$$

Provide 4 bars of 18 mm. ϕ (10·18 cm.²)

at top as well as at bottom.

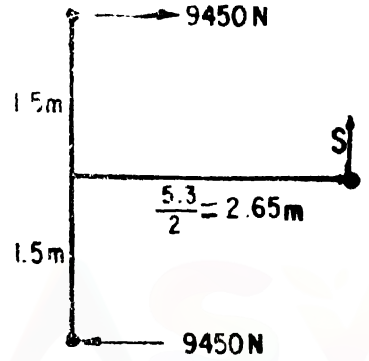

Fig. 441

Shear force

$$=S=\frac{B.M \text{ for brace}}{\text{half span of brace}}$$

$$=\frac{28350}{2{\cdot}65}=10698\ N$$

$$\text{Nominal shear stress}=q_v=\frac{10698}{30 \times 26}=13\ N/cm^2$$

$$=0{\cdot}13\ N/mm^2$$

Percentage of tensile steel

$$=\frac{10{\cdot}18}{30 \times 26} \times 100=1{\cdot}3\%$$

For 1·3% steel $q_c=0{\cdot}41\ N/mm^2$ $\therefore$ $q_v<q_c$

Provide 2-legged 10 mm. ϕ stirrups @ 22 cm. c/c.

Foundation

Total load at the base of column $=472370\ N$

Approximate weight of footing (10%) $=47240\ N$

Total $=519610\ N$

Moment $=14175\ Nm.$

Eccentricity $e=\frac{14175}{519610}=0{\cdot}03\ m.$

The eccentricity is very small.

Approximate area required for the footing

$$=\frac{\text{Total load}}{\text{Safe bearing capacity of soil}}$$

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Let us assume that the bearing capacity of the soil

$$= 200 \ kN/metre^2$$

$\therefore$ Area of foundation

$$= \frac{519610}{200 \times 1000} = 2{\cdot}598 \ metre^2$$

$\therefore$ Side of foundation

$$= \sqrt{2\,598} = 1{\cdot}61 \ m.$$

Let us provide 1·75 *metre* × 1·75 *metre*

Extreme pressure intensity at the base

$$= \frac{W}{A}\left\{1 \pm \frac{6e}{b}\right\}$$

$$= \frac{519610}{1{\cdot}75^2}\left\{1 \pm \frac{6 \times 0{\cdot}03}{1{\cdot}75}\right\} N/metre^2$$

$$p_{max} = 187140 \ N/metre^2$$

$$p_{min} = 152190 \ N/metre^2$$

Depth from B.M. consideration

Total load on cantilevering part

$$= \frac{172660 + 187140}{2} \times 0{\cdot}725 \times 1{\cdot}75 \ N$$

$$= 228248 \ N$$

This load acts at a distance of

$$\frac{172660 + 2 \times 187140}{172660 + 187140} \times \frac{0{\cdot}725}{3} \ m. \text{ from the column face}$$

$$= 0{\cdot}367 \ m. \text{ from the column face}$$

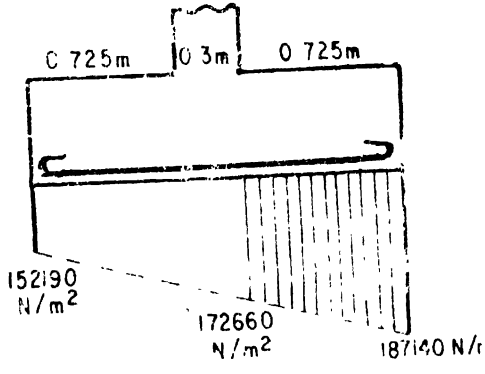

Fig. 442

$\therefore$ Max. B.M. $= 228248 \times 0{\cdot}367 = 83767 \ Nm.$

$= 8376700 \ N \ cm.$

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Equating the moment of resistance to the maximum bending moment,

$$85 \times 175\, d^2 = 8376700$$

$$\therefore \quad d = 23{\cdot}7 \text{ cm.}$$

Providing an effective cover of 6 *cm.*

Overall depth required

$$= 23{\cdot}7 + 6 = 29{\cdot}7 \text{ cm. say } 30 \text{ cm.}$$

Depth from punching shear consideration

Punching shear $= 472370$ *N (approximately)*

Let the overall depth be D *cm.*

Safe punching shear stress

$$= 1 \text{ N/mm.}^2 = 100 \text{ N/cm.}^2$$

Equating the punching resistance to the punching load.

$$4 \times 30 \times D \times 100 = 472370$$

$$\therefore \quad D = 39{\cdot}36 \text{ cm.}$$

Provide an overall depth of 40 *cm.*

Actual effective depth

$$= 40 - 6 = 34 \text{ cm.}$$

$$\therefore \quad A_t = \frac{8376700}{14000 \times 0{\cdot}87 \times 34} = 20{\cdot}22 \text{ cm.}^2$$

Provide 8 bars of 18 mm. ϕ (20·36 cm²) in each principal direction.

Design 117. *An open square tank is 5 m. × 5 m. × 3 m. deep and supported 6 metres above the ground level on beams and columns. Design the tank and the supporting beams and columns and the foundation.*

Solution Tank walls. These are designed as in the case of the tank supported on ground level. Refer to Design 111.

Bottom slab

Overall thickness $= 30$ *cm.*

Mid span section Consider a 1 *metre* wide strip of the slab

Effective span = Distance between centre to centre of walls

$= 5{\cdot}00 + 0{\cdot}30$ *metres*

$= 5{\cdot}30$ *metres*

Sagging b m $= \frac{wl^3}{48\sqrt{2}}$

$= \frac{1000 \times (5{\cdot}30)^3}{48\sqrt{2}}$ *kg. m.*

$= 2194$ *kg. m.* $= 219400$ *kg. cm.*

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Dead load of 30 *cm.* thick slab *sq. metre*

$$= 25\ D\ kg./m.^2 = 25 \times 30 = 750\ kg/m.^2$$

Since the slab is a two-way slab, the load distribution in each way will be $\frac{750}{2} = 375\ kg./m.^2$

$\therefore$ Sagging b.m. due to self weight

$$= \frac{375 \times 5\,3^2}{8}\ kg.\ m.$$

$$= 1316\ kg.\ m. = 131600\ kg.\ cm.$$

$\therefore$ Total sagging b m.

$$= 219400 + 131600 = 351000\ kg.\ cm.$$

Pull $$= \frac{wl^3}{16}$$

$$= \frac{1000 \times 5\,3^3}{16}\ kg. = 1756\ kg.$$

With 20 *mm.* bars at a clear cover of 25 *mm.*

Effective depth $= d = 30 - 2{\cdot}5 - 2 - 1 = 24{\cdot}5\ cm.$

$\therefore$ Resultant b.m. $= 345800 - 1756(24{\cdot}5 - 15{\cdot}00)\ kg.\ cm.$

$$= 334318\ kg\ cm.$$

This b m. produces tension away from the water face. Hence adopt $c = 70\ kg./cm.^2$, $t = 1250\ kg./cm.^2$ and $a = 0{\cdot}86\ d$.

$\therefore$ Steel for b.m. $= \frac{334318}{1250 \times 0\,86 \times 24{\cdot}5}\ cm.^2 = 12{\cdot}69\ cm.^2$

Steel for pull $= \frac{1756}{1250} = 1{\cdot}40\ cm.^2$

$\therefore$ Total steel $= 12{\cdot}6 + 1{\cdot}40\ cm.^2 = 4{\cdot}09\ cm.^2$

Provide 20 mm. ϕ bars @ 21 cm. c/c.

End section of the slab

Per *metre* width of the slab

Maximum b.m. $= \frac{wH^3}{6}$

$$= \frac{1000 \times 3^3}{6} = 4500\ kg.\ m.$$

$$= 450{,}000\ kg.\ cm.$$

Pull $= \frac{wH^2}{2} = \frac{1000 \times 3^2}{2} = 4500\ kg.$

Resultant b.m. $= 450{,}000 - 4500(24{\cdot}50 - 15{\cdot}00)\ kg.\ cm.$

$$= 407250\ kg.\ cm.$$

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This b m. produces tension near the water face.

Hence, adopt $c = 70\ kg./cm.^2$, $t = 1000\ k.g/cm.^2$, $Q = 14{\cdot}03$ and $a = 0{\cdot}84\ d$.

$$\text{Steel for B.M.} = A_{t1} = \frac{407250}{1000 \times 0{\cdot}84 \times 24{\cdot}5} = 19{\cdot}79\ cm.^2$$

$$\text{Steel for pull} = A_{t2} = \frac{4500}{1000} = 4{\cdot}50\ cm.^2$$

Total steel required

$$= A_t = A_{t1} + A_{t2} = 19{\cdot}79 + 4{\cdot}50 = 24{\cdot}29\ cm.^2$$

Spacing of 20 *mm.* ϕ bars

$$= \frac{3{\cdot}14 \times 100}{24{\cdot}29} = 12{\cdot}8\ cm.\ say\ 12\ cm.$$

Check for effective depth

$$14{\cdot}03 \times 100\ d^2 = 407250$$

$$d = 17\ cm.$$

But effective depth available $= 24{\cdot}50\ cm.$

Design of beams

Width of beam $= 30\ cm.$

Assume a depth of 80 *cm.*

Effective span $= 5{\cdot}3$ *metres*

Load per metre run of the beam (*See Fig 443*)

(*i*) Weight of wall $= 3{\cdot}30 \times 0{\cdot}3 \times 1 \times 2500$

$= 2475\ kg.$

(*ii*) Dead load of the slab (two-way distribution)

$$= 375 \times \frac{5}{2}$$

$$= 938\ kg.$$

(*iii*) Weight of water (two-way distribution)

$$= \frac{1000 \times 3}{2} \times \left(\frac{5}{2}\right)$$

$$= 3750\ kg.$$

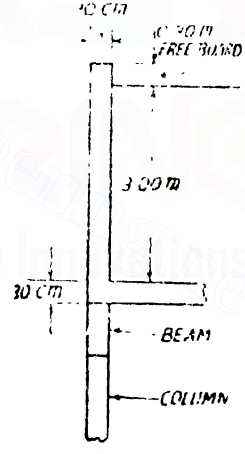

Fig. 443

(*iv*) Weight of 80 *cm.* beam + 30 *cm.* slab

$$= 1{\cdot}10 \times 0{\cdot}30 \times 2500$$

$$= 825\ kg.$$

Total load per *metre* run

$$= 2475 + 938 + 3750 + 825 = 7988\ kg./m.$$

$\therefore$ Max. B.M. with fixity at the end

$$= \frac{7988 \times 5{\cdot}3^2}{12}\ kg\ m.$$

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$= 18699$ *kg. m.* $= 1{,}869{,}900$ *kg. cm*

Now onwards let the usual stresses be adopted.

With $c = 50$ *kg./cm.*^{2}, $t = 1400$ *kg./cm.*^{2}, $m = 18$, $a = 0{\cdot}8\ d$ and $Q = 8{\cdot}50$.

$\therefore \quad 8{\cdot}50 \times 30 \times d^2 = 1869900$

$\therefore \quad d = 85{\cdot}6$ *cm.*

With 25 *mm.* cover and 24 *mm.* dia. bars

Effective cover $= 3{\cdot}70$ *cm.*

Overall depth required

$= 85{\cdot}6 + 3{\cdot}7 = 89{\cdot}3$ *cm.*

Provide an overall depth of 90 *cm.*

Since the previously assumed overall depth is 80 *cm*, increase in depth $= 10$ *cm*

Additional weight per *metre* run

$= 0{\cdot}10 \times 0{\cdot}30 \times 2500 = 75$ *kg /m.*

Corresponding increase in bending moment

$$= \frac{75 \times 5{\cdot}3^2}{12} = 176 \text{ kg. m.} = 17600 \text{ kg. cm}$$

Total bending moment

$= 1869900 + 17600 = 1887500$ *kg cm.*

Equating M.R. to B.M.,

$8{\cdot}5 \times 30\ d^2 = 1887500 \qquad \therefore \quad d = 86{\cdot}03$ *cm.*

Effective depth available

$= 90 - 3{\cdot}7 = 86{\cdot}30$ *cm.*

$$\therefore \qquad A_t = \frac{1887500}{1400 \times 0{\cdot}87 \times 86{\cdot}3} = 17{\cdot}96 \text{ cm}^2$$

Provide 4 bars of 24 mm. ϕ (18·08 cm.^{2}*)*

Shear analysis

Maximum S.F. $= S = \dfrac{(7988 + 75) \times 5{\cdot}3}{2} = 21367$ *kg.*

Nominal shear stress

$$= q_v = \frac{S}{bd} = \frac{21367}{30 \times 86{\cdot}3} = 8{\cdot}25 \text{ kg./cm.}^2$$

Percentage of steel $= \dfrac{18{\cdot}08}{30 \times 86{\cdot}3} \times 100 = 0{\cdot}7\%$

For 0·7% steel $\quad q_c = 3{\cdot}3$ *kg./cm*^{2}

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$\therefore$ Shear resistance of concrete

$$= q_c\, bd = 3{\cdot}3 \times 30 \times 86{\cdot}3 = 8544\ kg.$$

$\therefore$ Shear to be resisted by stirrups

$$= V_s = 21367 - 8544 = 12823\ kg.$$

Spacing of 4-legged 8 $mm.$ ϕ stirrups

$$= \frac{A_{sv} f_s d}{V_s}$$

$$= \frac{(4 \times 0{\cdot}5) 1400 \times 86{\cdot}3}{12823} = 18\ cm\ /cc$$

Let the section where the nominal shear stress is 3·3 $kg/cm.^2$ be at x *metres* from the centre.

$$\frac{x \times 6{\cdot}3}{2} = 8{\cdot}25$$

$\therefore$ $x = 1{\cdot}1\ m.$

Design of columns

Load on one column $= (7988 + 75) \times 5{\cdot}3 = 42734\ kg$

Area of tank exposed to wind pressure

$$= 5{\cdot}6 \times \left\{ \begin{matrix} (3{\cdot}0 + 0{\cdot}3\ m.\ \text{free board}) \\ +0{\cdot}3\ m.\ \text{thickness of slab} \\ +0{\cdot}9\ m.\ \text{depth of beam} \end{matrix} \right\} = 5{\cdot}6 \times 4{\cdot}5\ metre^2 = 25{\cdot}20\ metre^2$$

Wind load at 150 $kg/metre^2$

$$= 25{\cdot}2 \times 150\ kg. = 3780\ kg$$

$\therefore$ Wind load on one pair of columns

$$= \frac{3780}{2} = 1890\ kg.$$

Assume that the points of contraflexure are at the middle points of the respective panels.

Taking moments about the point of contraflexure C, we have,

Direct load on column due to wind

$$= \frac{1890 \times 6{\cdot}75}{5{\cdot}3}\ kg.$$

$$= 2407\ kg.$$

Column Section at B

Load from the tank and its contents	$= 42734\ kg.$
D.L. of column (30 $cm.$ × 30 $cm.$) $0{\cdot}3 \times 0{\cdot}3 \times 6 \times 2500$	$= 1350\ kg.$
Weight of one brace (30 $cm.$ × 30 $cm.$) $0{\cdot}3 \times 0{\cdot}3 \times 5 \times 2500$	$= 1125\ kg.$
Direct load due to wind	$= 2407\ kg.$
	Total $= 47616\ kg.$

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B.M. at B $= 945 \times 1{\cdot}5$ *kg. m.*

$= 1418$ *kg. m.*

Let eight bars of 24 *mm.* diameter be used.

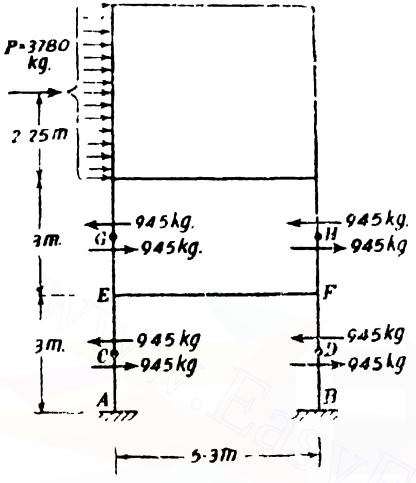

Fig. 44

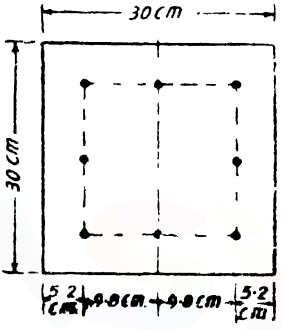

Fig. 445

Area of eight bars $= 30{\cdot}20$ *cm*^{2}

Equivalent area of concrete

$= (30)^2 + (18-1)36{\cdot}2$ *cm*$^2 = 1515$ *cm.*^{2}

Moment of inertia about the centroidal axis

$$= \frac{30 \times (30)^3}{12} + (18-1)27{\cdot}12 \times (9{\cdot}8)^2$$

$= 109170$ *cm.*^{4}

$\therefore$ Extreme stresses on the section

$$= \frac{47616}{1515} \pm \frac{1418 \times 100}{109170} \times 15 \text{ kg/cm.}^2$$

$= 31{\cdot}4 \pm 19{\cdot}5$ *kg/cm.*^{2}

$\therefore$ Max compressive stress

$= 31{\cdot}4 + 19{\cdot}5$ *kg/cm.*^{2}

$= 50{\cdot}9$ *kg/cm.*^{2}

Design of the brace

Maximum bending moment for the brace

$= 945 \times 1{\cdot}5 + 945 \times 1{\cdot}5$ *kg. m.*

$= 2835$ *kg. m.*

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The brace will be designed as a doubly reinforced beam section on the steel beam theory.

Let the effective cover to the reinforcement be 4 *cm.*

∴ Distance between the centres of top and bottom steel

$$= 30 - 8 = 22 \text{ cm.}$$

$$\therefore \quad A_c = A_t = \frac{2835 \times 100}{1400 \times 22} \text{ cm}^2$$

$$= 9{\cdot}21 \text{ cm}^2$$

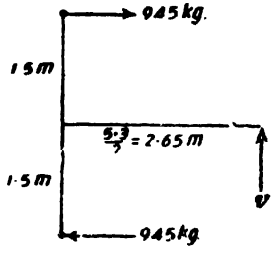

Fig. 446

Provide 4 bars of 18 mm. diameter (10 18 cm.²)

Shear

$$\text{Shear force} \quad = v = \frac{\text{B.M. for brace}}{\text{half span of brace}}$$

$$= \frac{2835}{2{\cdot}65} \text{ kg.} = 1070 \text{ kg.}$$

$$\text{Shear stress} \quad = \frac{1070}{22 \times 30} \text{ kg./cm.}^2 = 1{\cdot}62 \text{ kg./cm.}^2$$

Provide two-legged 10 mm. diameter stirrups at a spacing of 22 cm. centres.

Foundation

Total load at the base of column	47616 *kg.*
Approximate weight of footing (10%)	4762 *kg.*
Total	52378 *kg.*

Moment $= 1418 \text{ kg. m.}$

$$\text{Eccentricity} \quad = e = \frac{1418}{52378} = 0{\cdot}03 \text{ m.}$$

The eccentricity is very small

Approximate area of footing

$$= \frac{\text{Total load}}{\text{bearing capacity of soil}}$$

Let us assume that the bearing capacity of the soil

$$= 20 \text{ t/metre}^2$$

∴ Area of foundation

$$= \frac{52378}{20000} \text{ metre}^2$$

$$= 2{\cdot}62 \text{ metre}^2$$

Side of foundation $\sqrt{2{\cdot}62} \text{ m} = 1{\cdot}62 \text{ m.}$

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Let us provide 1·75 *metre* × 1·75 *metre*.

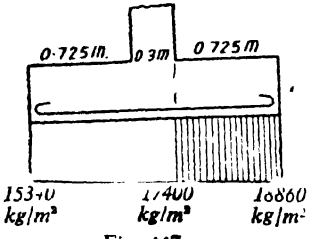

Fig. 447

Pressure at base

$$= \frac{W}{A}\left\{1 \pm \frac{6e}{b}\right\}$$

$$= \frac{52378}{1{\cdot}75^2}\left\{1 \pm \frac{6 \times 0{\cdot}03}{1{\cdot}75}\right\} kg./m.^2$$

$$p_{max} = 18860 \ kg./m.^2$$

$$p_{min} = 15340 \ kg./m.^2$$

Total load on cantilevering part

$$= \frac{17400 + 18860}{2} \times 0{\cdot}725 \times 1{\cdot}75 \ kg.$$

$$= 23002 \ kg.$$

This load acts at a distance of

$$\frac{17400 + 2 \times 18860}{17400 + 18860} \times \frac{0{\cdot}725}{3} \ m \text{ from the column face}$$

$$= 0{\cdot}366 \ m. \text{ from the column face}$$

$\therefore$ Max. B.M. $= 23002 \times 0{\cdot}366 \times 100 \ kg.\ cm.$

$$= 841870 \ kg.\ cm.$$

Equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}5 \times 175 \ d^2 = 841870$$

$$\therefore \quad d = 23{\cdot}8 \ cm.$$

Providing an effective cover of 6 *cm*.

Overall depth required

$$= 23{\cdot}8 + 6 = 29{\cdot}8 \ cm. \text{ say } 30 \ cm.$$

Punching shear $= 47616 \ kg.$ (*approximately*)

Let D be the overall depth.

Equating the punching resistance to the punching load,

$$4 \times 30 \times D \times 10 = 47616$$

$$\therefore \quad D = 39{\cdot}68 \ cm$$

Provide an overall depth of 40 cm.

Actual effective depth

$$= d = 40 - 6 = 34 \ cm.$$

$$\therefore \quad A_t = \frac{841870}{1400 \times 0{\cdot}87 \times 34} = 20{\cdot}32 \ cm.^2$$

Provide 8 bars of 18 mm. ϕ (20·36 cm.²) in each principal direction.

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§180. Aqueducts

When an irrigation channel has to cross over a drainage, an aqueduct is provided. An aqueduct consists of a horizontal slab supported on two side walls. The side walls act as beams spanning across the drainage.

The beams and the slab are cast monolithically. The slab will be designed as spanning between the beams. The effect of water pressure on the side walls should also be considered. The sagging bending moment at the mid span of the slab will be a maximum when the depth of the water is one-half the span of the slab. But often the depth of the aqueduct itself may be less than one-

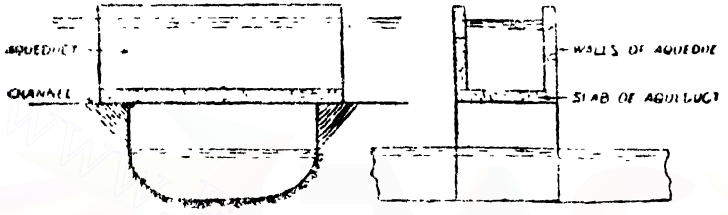

Fig. 448

half the span of the slab. In such cases the maximum bending moment at the mid span will occur when the aqueduct is full. If the depth of the aqueduct is greater than half the span of the bottom slab, there is a possibility of a hogging bending moment to occur at the mid span when the aqueduct is full. Where applicable, this condition should also be investigated. In each case the pull transferred to the bottom slab due to water pressure on the side walls should also be included. The end section of the bottom slab will be subjected to a hogging moment. This bending moment will be a maximum when the aqueduct is full.

Design 118 (S I). *Design an aqueduct for a 3 metres wide irrigation channel to cross a drainage having a water way of 6 metres. The depth of water in the aqueduct is 1 metre.*

Solution. Base slab.

Let the thickness of the base slab be 25 *cm*. provide a free board of 15 *cm*

Loads

D.L. of slab : $250 \times 25 = 6250$ *N/metre*^{2}

Weight of water : $9810 \times 1{\cdot}15 = 11280$ *N/metre*^{2}

17530 *N/metre*^{2}

Maximum sagging moment

Let the depth of water be *h metre*

Sagging moment due to weight of water

$$9810\, h \times \frac{3{\cdot}3^2}{8} = 12354\, h \text{ Nm.}$$

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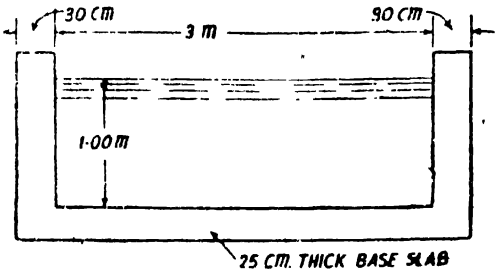

Fig. 449

Sagging moment due to weight of slab

$$= 6250 \times \frac{3\,3^2}{8} = 8508 \; Nm.$$

Water pressure on side walls for 1 *metre* run

$$= \frac{9810\,h^2}{2} = 4905\,h^2 \; N$$

This acts at $\frac{h}{3} + \frac{0{\cdot}25}{2}$ *m*.

$$= \left(\frac{h}{3} + 0{\cdot}125\right) \textit{metre}$$ above the centre line of the bottom slab.

Hogging moment due to lateral water pressure

$$= 4905\,h^2\left(\frac{h}{3} + 0{\cdot}125\right) Nm.$$

Net B.M. at the centre of the bottom slab

$$= M_c = 13354\,h + 8508 - 4905\left(\frac{h^3}{3} + 0{\cdot}125\,h^2\right)$$

For M_c to be maximum,

$$\frac{dM_c}{dh} = 0$$

$$\frac{dM_c}{dh} = 13354 - 4905\,(h^2 + 0{\cdot}25\,h) = 0$$

$\therefore \; h^2 + 0{\cdot}25\,h - 2{\cdot}722 = 0$

$\therefore \qquad h = 1{\cdot}53$ *metre*

But, the depth available is only 1·15 *metre*

Hence the maximum sagging moment will occur only when the aqueduct is full.

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$\therefore$ Maximum sagging moment

$$= M = 13354 \times 1{\cdot}15 + 8508 - 4905 \left(\frac{1{\cdot}15^3}{3} + 0{\cdot}125 \times 1{\cdot}15^2 \right) Nm.$$

$$= 20570\ Nm. = 2057000\ N\ cm.$$

Corresponding pull

$$= T = \frac{9810 \times 1{\cdot}15^2}{2}\ N$$

$$= 6487\ N$$

Providing an effective cover of 4 *cm.*

effective depth $= d = 25 - 4 = 21$ *cm.*,

Resultant B.M. $= M - Tx$

$$= 2057000 - 6487\ (21 - 12{\cdot}5)\ N\ cm.$$

$$= 1995370\ N\ cm.$$

Steel for B.M $= A_{t1} = \frac{1995370}{12500 \times 0{\cdot}86 \times 21} = 8{\cdot}84\ cm.^2$

Steel for pull $= A_{t2} = \frac{6487}{12500} = 0{\cdot}52\ cm^2$

Total steel required

$$= A_t = A_{t1} + A_{t2} = 8{\cdot}84 + 0{\cdot}52 = 9{\cdot}36\ cm.^2$$

Spacing of 12 *mm.* ϕ @ bars

$$= \frac{1{\cdot}13 \times 100}{9{\cdot}36} = 12{\cdot}1\ cm.$$

Provide 12 mm. ϕ *bars @ 12 cm. c/c.*

End section. Maximum hogging moment

$$= \frac{9810\ h^2}{2} \left(\frac{h}{3} + 0{\cdot}125 \right) Nm.$$

$$= 4905 \times 1{\cdot}15^2 \left(\frac{1{\cdot}15}{3} + 0{\cdot}125 \right) Nm.$$

$$= 3298\ Nm.$$

$$= 329800\ N\ cm.$$

$$\text{Pull} = \frac{9810 \times 1{\cdot}15^2}{2} = 6487\ N$$

Resultant B.M. $= M - Tx$

$$= 329800 - 6487\ (21 - 12{\cdot}5)\ N\ cm.$$

$$= 268170\ N\ cm.$$

This bending moment produces tension near the water face.

Hence adopt $c = 700\ N/cm.^2$

$t = 10{,}000\ N/cm^2$

$a = 0{\cdot}84\ d$

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$\therefore$ Steel for B.M $= A_{t1} = \dfrac{268170}{10{,}000 \times 0{\cdot}84 \times 21} = 1{\cdot}52\ cm.^2$

Steel for pull $= A_{t2} = \dfrac{6487}{10000} = 0{\cdot}65\ cm.^2$

Total steel required $= A_{t1} + A_{t2} = 1{\cdot}52 + 0{\cdot}65 = 2{\cdot}17\ cm.^2$

Let alternate bars of the bottom bars be extended as to serve as reinforcement at top.

Distribution steel

$$= \frac{0{\cdot}3}{100} \times 25 \times 100\ cm.^2 = 7{\cdot}5\ cm.^2$$

Spacing of 10 *mm*. ϕ bars

$$= \frac{0{\cdot}79 \times 100}{7{\cdot}5} = 10{\cdot}5\ cm.\ say\ 10\ cm.$$

If this steel be provided near both the faces, the spacing of the distribution bars will be @ 20 cm. c/c near each face.

Beams (Side walls)

Load per metre run of the beam

Weight of water $= 9810 \times 1{\cdot}15 \times \dfrac{3}{2} = 16922\ N/m.$

D.L. of the slab $= 3 \times 0{\cdot}25 \times \dfrac{25000}{2} = 9375\ N/m$

D.L. of the beam $= 0{\cdot}3 \times 1{\cdot}15 \times 25000 = 8625\ N/m.$

Total $34922\ N/m.$

Effective span $=$ clear span $+$ bearing
$= 6{\cdot}00 + 0{\cdot}6\ m.$
$= 6{\cdot}60\ m.$

$\therefore$ Max. B.M. $= \dfrac{34922 \times 6{\cdot}6^2}{8}\ Nm.$

$= 190150\ Nm.$
$= 19015000\ N\ cm.$

Providing an effective cover of 8 *cm.*
Effective depth $= 140 - 8 = 132\ cm.$

$\therefore$ $A_t = \dfrac{19015000}{14000 \times 0{\cdot}87 \times 132} = 11{\cdot}83\ cm.^2$

Provide fagr bars of 20 mm. ϕ (12·57 cm.^{2})

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Maximum S.F.

$$-S = 34922 \times \frac{6{\cdot}6}{2} \ N$$

$$= 115243 \ N$$

Nominal shear stress

$$= q_v = \frac{S}{bd} = \frac{115243}{30 \times 132} \ N/cm.^2$$

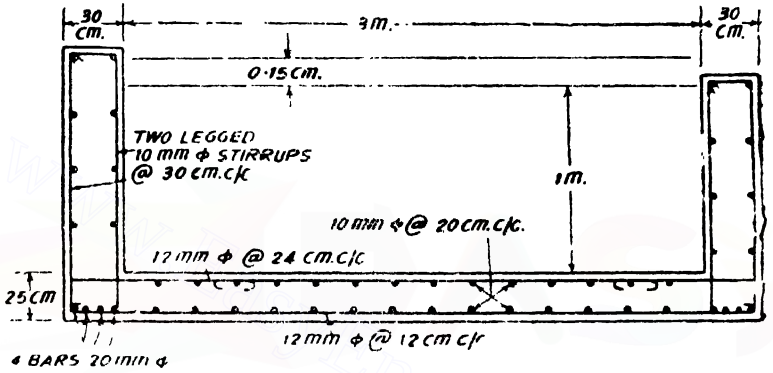

Fig. 450

$$= 29 \ N/cm.^2$$

$$= 0{\cdot}29 \ N/mm.^2$$

Percentage of steel $= \frac{12{\cdot}57}{30 \times 132} \times 100\%$

$= 0{\cdot}32\%$

For 0·32% steel $q_c = 0{\cdot}25 \ N/mm.^2$ (This is nearly equal to q_v)

Provide two-legged 10 mm. ϕ stirrups @ 30 cm. c/c.

Design 119. *Design an aqueduct for a 3 metres wide irrigation channel to cross a drainage having a waterway of 6 metres. The depth of water in the aqueduct is one metre.*

Solution Let us provide a free board 15 *cm.* so that the inside depth of the aqueduct is 1·15 *m.*

Assuming that the supporting beams (side walls) are 30 *cm.* thick the effective span of the bottom slab = 3·30 *metres.*

Base slab

With a 25 *cm.* thick base slab the load per *square metre* will be as follows :

Weight of slab	$= 25 \times 25$	$= 625 \ kg./m.^2$
Weight of water	$= 1000 \times 1{\cdot}15$	$= 1150 \ kg./m.^2$
	Total	$= 1775 \ kg./m.^2$

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Maximum sagging bending moment

Let the depth of water be h *metres*

Sagging moment due to weight of water

$$= 1000\, h \times \frac{3{\cdot}3^2}{8} = 1361\, h \text{ kg. m.}$$

Sagging moment due to self-weight of base slab

$$= 625 \times \frac{3\,3^2}{8} = 850 \text{ kg. m.}$$

Water pressure on side walls for one *metre* length

$$= \frac{1000\, h^2}{2} = 500\, h^2 \text{ kg.}$$

This acts at $\frac{h}{3} + \frac{0{\cdot}25}{2}$ *m.*

$$= \left(\frac{h}{3} + 0{\cdot}125\right) \textit{ metres} \text{ above the centre line of the bottom slab.}$$

Hogging moment due to lateral water pressure

$$= 500\, h^2 \left(\frac{h}{3} + 0{\cdot}125\right) \text{ kg. m.}$$

$\therefore$ Net B.M. at the centre of the bottom slab

$$= M_c = 1361\, h + 850 - 500\left(\frac{h^3}{3} + 0{\cdot}125\, h^2\right)$$

For M_c to be a maximum,

$$\frac{dM_c}{dh} = 0$$

$$\therefore \quad \frac{dM_c}{dh} = 1361 - 500\,(h^2 + 0{\cdot}25\, h) = 0$$

or $h^2 + 0{\cdot}25\, h - 2{\cdot}722 = 0$

$$\therefore \quad h = 1{\cdot}53 \textit{ metres}$$

But the depth available is only 1·15 *metres.*

Hence the maximum possible sagging bending moment will occur only when the aqueduct is full.

$\therefore$ Maximum sagging moment

$$= M_{max}$$

$$= 1361 \times 1{\cdot}15 + 850 - 500\left(\frac{1{\cdot}15^3}{3} + 0{\cdot}125 \times 1{\cdot}15^2\right) \text{ kg. m.}$$

$$= 2079 \text{ kg. m.}$$

Co rresponding pull $= \frac{1000 \times 1{\cdot}15^2}{2}$ kg.

$$= 661{\cdot}25 \text{ kg.}$$

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Let the effective cover to steel reinforcement be 4 *cm.*

$\therefore$ Effective depth $= 25 - 4 = 21$ *cm.*

Resultant bending moment

$$= 2079 - \frac{661{\cdot}25}{100}(21 - 12{\cdot}5) \text{ kg. m.}$$

$$= 2023 \text{ kg. m.}$$

Adopting $c = 70$ *kg/cm.*^{2}

$t = 1250$ *kg./cm.*^{2}

$m = 13$

Steel for bending moment

$$= \frac{2023 \times 100}{1250 \times 0{\cdot}86 \times 21} \text{ cm.}^2$$

$$= 8{\cdot}96 \text{ cm.}^2$$

Steel for pull $= \frac{661{\cdot}25}{1250} = 0{\cdot}53$ *cm.*^{2}

$\therefore$ Total steel required

$$= 8{\cdot}96 + 0{\cdot}53 = 9{\cdot}49 \text{ cm.}^2$$

$\therefore$ Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{9{\cdot}49} \text{ cm.}$$

$$= 12 \text{ cm.}$$

End section. Maximum hogging moment

$$= 500\, h^2\left(\frac{h}{3} + 0{\cdot}125\right) \text{ kg. m.}$$

$$= 500 \times 1{\cdot}15^2\left(\frac{1{\cdot}15}{3} + 0{\cdot}125\right) \text{ kg. m.}$$

$$= 335{\cdot}9 \text{ kg. m.}$$

Pull $= 661{\cdot}25$ *kg.*

Resultant moment $= 335{\cdot}9 - \frac{661{\cdot}25}{100}(21 - 12{\cdot}5)$ *kg. m.*

$$= 273 \text{ kg. m.}$$

This moment produces tension near the water face

Hence adopt $c = 70$ *kg./cm.*^{2}

$t = 1000$ *kg./cm.*^{2}

$a = 0{\cdot}84\, d$

$\therefore$ Steel for b.m. $= \frac{273 \times 100}{1000 \times 0{\cdot}84 \times 21}$ *cm.*$^2 = 1{\cdot}55$ *cm.*^{2}

Steel for pull $= \frac{661{\cdot}25}{1000} = 0{\cdot}66$ *cm.*^{2}

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∴ Total steel requirement

$$= 1{\cdot}55 + 0{\cdot}66 = 2{\cdot}21 \text{ cm.}^2$$

Let alternate bars of the bottom bars be extended to serve as steel at the top.

Distribution steel

$$= \frac{0{\cdot}3}{100} \times 25 \times 100 \text{ cm.}^2$$

$$= 7{\cdot}5 \text{ cm.}^2$$

∴ Spacing of 10 *mm.* dia. bars

$$= \frac{0{\cdot}79 \times 100}{7{\cdot}5} \text{ cm.} = 10 \text{ cm. c/c.}$$

If this steel be provided near both the faces the spacing of the distribution bars will be at 20 cm. c/c near each face.

Beams (side walls). Load per *metre* run of the beam :

Weight of water $= 1000 \times 1{\cdot}15 \times \frac{3}{2} = 1725$ *kg./m.*

Dead load of the slab $= 2 \times 0{\cdot}25 \times \frac{2500}{2} = 938$ *kg./m.*

Dead load of the beam $= 0{\cdot}3 \times 1{\cdot}15 \times 2500 = 863$ *kg./m.*

Total $= 3526$ *kg./m.*

Effective span = clear span + bearing
= 6·00 *m.* + 0·6 *m.*
= 6·6 *m.*

∴ Max. bending moment

$$= \frac{3526 \times 6{\cdot}6^2}{8} \text{ kg. m.}$$

$$= 19199 \text{ kg. m.}$$

Providing an effective cover of 8 *cm.*

Effective depth $= 140 - 8 = 132$ *cm.*

$$\therefore \quad A_s = \frac{19199 \times 100}{1400 \times 0{\cdot}87 \times 132} \text{ cm.}^2$$

$$= 11{\cdot}94 \text{ cm.}^2$$

Provide four bars of 20 mm. diameter (12·57 cm.²)

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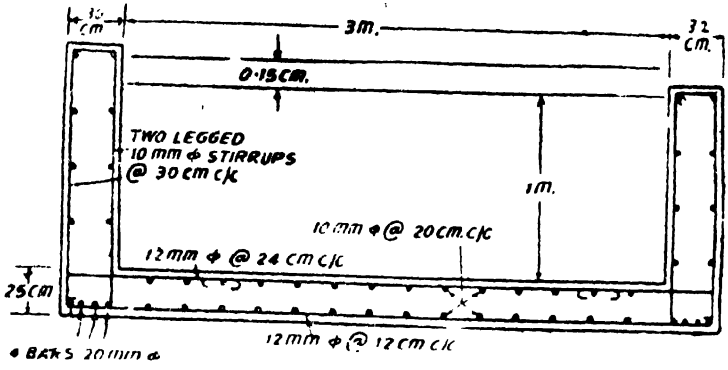

Fi . 451

Maximum S F.

$$= S = 3526 \times \frac{6{\cdot}6}{2} = 11636 \text{ kg.}$$

$$\text{Nominal shear stress} = q_v = \frac{S}{bd} = \frac{11636}{30 \times 132} = 2{\cdot}9 \text{ kg./cm.}^2$$

$$\text{Percentage of steel} = \frac{12{\cdot}57}{30 \times 132} \times 100\% = 0{\cdot}32\%$$

For 0·32% steel, $q_c = 2\ 50$ kg./cm.^{2}

(This is nearly equal to q_c)

Provide two-legged 10 mm. ϕ stirrups @ 30 cm. c/c.

§181. Domes

A dome consists of a shell which is generated by the revolution of a geometrical curve about an axis. If a segment of a circle is revolved about the vertical diameter, we get a spherical dome. A conical dome is obtained by revolving a triangle round a central pivot. Domes are used to roof circular areas. They are used in buildings and water tanks. The domes provided for water tanks have a rise equal to $\frac{1}{4}$ to $\frac{1}{8}$ of the span. For the usual cases we come across the thickness of the dome slab is found to be 10 *cm.* to 15 *cm.*

Stresses in a spherical dome. Two types of stresses are caused in a dome slab. They are : (*i*) Meridional thrust, (*ii*) Hoop stress.

Meridional thrust. For purposes of analysis let us consider the dome as formed by a series of horizontal rings of decreasing diameters placed one above the other (*see* Fig. 452). Hence if a load is applied on the dome it gets resisted by these horizontal rings. There will thus be a thrust of one ring on the other. This thrust is called the meridional thrust.

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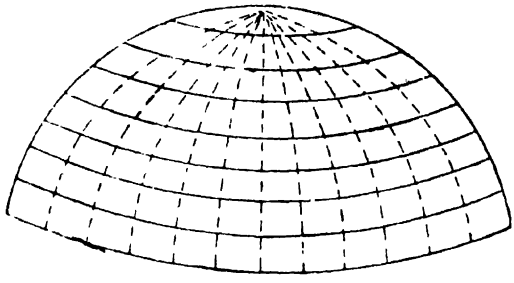

Fig. 452 Dome formed by placing a series of concentric rings of decreasing diameters, one above the other.

Consider one such ring *DEFG*. Let the radius of this ring be r and let the radius of the sphere be R. Let this ring be corres-

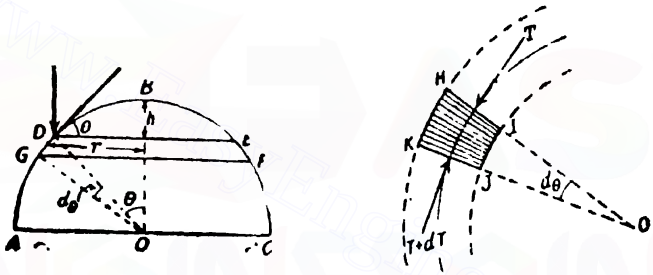

Fig. 453

ponding to the radius vector at angle θ with the vertical. Consider unit length of this ring. The meridional thrust on this ring can now be determined as follows.

Case 1. *Uniformly distributed load of w per unit area of the dome surface.*

Area of the dome surface above ring *DEFG*

$$= 2\pi Rh$$
$$= 2\pi R[R - R\cos\theta]$$
$$= 2\pi R^2(1 - \cos\theta)$$

∴ Load above the ring *DEFG*

$$= 2\pi R^2(1 - \cos\theta)\, w.$$

This load is now transferred as a thrust on the circumference of the ring *DEFG*. Let the thrust per unit run on this ring be T.

∴ Total vertical component of the thrust on the ring

$$= T \sin\theta \times 2\pi r$$
$$= 2\pi R^2 (1 - \cos\theta)\, w$$

But $r = R \sin\theta$

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$$\therefore T \sin\theta \cdot 2\pi R \sin\theta = 2\pi R^2 (1-\cos\theta) w$$

$$\therefore \quad T = \frac{wR(1-\cos\theta)}{\sin^2\theta}$$

$$= \frac{wR(1-\cos\theta)}{1-\cos^2\theta}$$

$$\therefore \quad T = \frac{wR}{1+\cos\theta}.$$

Hoop stress. Let T be the thrust per unit run on the ring *DEFG*.

The horizontal component of this thrust will produce a hoop tension of

$$H = T\cos\theta \cdot R\sin\theta$$

$$\therefore \quad H = \frac{wR^2}{1+\cos\theta} \sin\theta\cos\theta \qquad \ldots(1)$$

Similarly the thrust on this ring just on its lower face will produce a hoop compression of $H+dH$.

Hence the net hoop compression on the ring

$$= dH$$

Area of the section of the ring

$$= Rd\theta\, t$$

$$\therefore \quad \text{Hoop stress} = f = \frac{dH}{Rd\theta\, t}$$

$$\therefore \quad f = \frac{1}{Rt} \cdot \frac{dH}{d\theta} \qquad \ldots(2)$$

But fro)

$$H = wR^2 \frac{\sin\theta\cos\theta}{1+\cos\theta}$$

$$\therefore \quad \frac{dH}{d\theta} = wR^2 \cdot \frac{(1+\cos\theta)(-\sin^2\theta+\cos^2\theta)+\sin^2\theta\cos\theta}{(1+\cos\theta)^2)}$$

$$= wR^2 \cdot \frac{(1+\cos\theta)(\cos^2\theta-\sin^2\theta)+\cos\theta(1-\cos^2\theta)}{(1+\cos\theta)^2}$$

$$= wR^2 \times \frac{\cos^2\theta-\sin^2\theta+\cos\theta-\cos^2\theta}{1+\cos\theta}$$

$$\frac{dH}{d\theta} = wR^2 \frac{\cos\theta-\sin^2\theta}{1+\cos\theta}$$

$$= wR^2 \frac{\cos\theta-(1-\cos^2\theta)}{1+\cos\theta}$$

$$= wR^2 \frac{\cos^2\theta+\cos\theta-1}{1+\cos\theta}$$

$$\frac{dH}{d\theta} = wR^2\left[\cos\theta - \frac{1}{1+\cos\theta}\right]$$

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$$\therefore \quad \text{the hoop stress } f = \frac{1}{Rt}\,\frac{dH}{d\theta}$$

$$\therefore \qquad f = \frac{wR}{t}\left[\cos\ \theta - \frac{1}{1+\cos\theta}\right].$$

If the above expression simplifies to a positive quantity the hoop stress is compressive. If the above expression simplifies to a negative quantity the hoop stress is tensile.

It is possible for a certain value of θ the hoop stress is zero. This is obtained from the relation,

$$f = \frac{wR}{t}\left[\cos\theta - \frac{1}{1+\cos\theta}\right] = 0$$

$$\therefore \qquad \cos\theta = \frac{1}{1+\cos\theta}$$

$$\therefore \quad \cos^2\theta + \cos\theta - 1 = 0$$

Solving as a quadratic in $\cos\theta$, we have

$$\cos\theta = \frac{-1+\sqrt{1+4}}{2}$$

$$= \frac{-1+\sqrt{5}}{2}$$

$$= 0{\cdot}618$$

$$\therefore \qquad \theta = 51^\circ\ 51'.$$

Case 2. *Concentrated load on the crown of the dome.*

If now there is a concentrated load W on the dome, considering the ring $CDEF$ the load transferred per unit run of the circumference of the ring $= \frac{W}{2\pi R \cdot \sin\theta}$. If the thrust per unit run on this ring be T we have, resolving vertically,

$$T \sin\theta = \frac{W}{2\pi R \sin\theta}$$

$$T = \frac{W}{2\pi R \sin^2\theta}$$

The horizontal component of this thrust will produce a hoop tension of

$$H = T\cos\theta\ R\sin\theta$$

$$= \frac{W}{2\pi}\cot\theta.$$

Similarly the thrust on the lower face of the ring $CDEF$ will produce a hoop compression of $H + dH$.

$$\therefore \quad \text{Net hoop load} = dH$$

$$\therefore \quad \text{Hoop stress} = \frac{dH}{R\,d\theta\,t} = \frac{1}{Rt}\,\frac{dH}{d\theta}$$

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But $$H=\frac{W}{2\pi}\cot\theta$$

$$\therefore \quad \frac{dH}{d\theta}=\frac{W}{2\pi}\operatorname{cosec}^2\theta=-\frac{W}{2\pi\sin^2\theta}$$

$$\therefore \text{ Hoop stress } =-\frac{W}{2\pi Rt\sin^2\theta}.$$

The negative sign indicates that the hoop stress induced is a tensile stress. We find from the above expressions for thrust and hoop stress, at $\theta=0$ that is, at the crown of the dome,

$$T=\frac{W}{2\pi R\sin^2(O)}=\infty$$

and $$f=-\frac{W}{2\pi Rt\sin^2(O)}=\infty.$$

Hence strictly a truly point load should not be applied on the crown of the dome. In practice it is usual the load at the crown should be distributed over a sufficient area so as not to cause excessive stresses at the crown. In order to reach this aim, sometimes an opening is provided at the crown.

Summary of formulae

Concentrated load on the dome $= W$

Distributed load on the dome $= w$ per unit area

$$\text{Meridional thrust} = T=\frac{wR}{1+\cos\theta}+\frac{W}{2\pi R\sin^2\theta}$$

$$\text{Hoop stress } =f=\frac{wR}{t}\left[\cos\theta-\frac{1}{1+\cos\theta}\right]-\frac{W}{2\pi Rt\sin^2\theta}.$$

The meridional tnrust is always a compressive force. The hoop stress will be compressive if f is positive and tensile if f is negative.

Design of a dome. From load considerations the thickness requirement for a dome is very small. The thickness practically adopted is never less than 10 *cm*. The domes provided for water tanks are usually circular with a rise equal to $\frac{1}{4}$ to $\frac{1}{8}$ of the span. Often the stresses in domes are compressive and in dome constructions the compressive stresses adopted are from $\frac{1}{3}$ to $\frac{2}{3}$ of the direct stresses. If the direct compressive stress be taken at 400 N/cm^2 or 40 kg/cm^2 the compressive stress in concrete in dome construction may be taken at 80 N/cm^2 or 8 $kg./cm^2$ to 160 N/cm^2 or 16 kg/cm^2. The safe compressive stress in steel shall be taken at 10,000 N/cm^2 or 1000 $kg./cm^2$.

Ring beam for a dome. Very often the domes constructed are segmental and not hemispherical. Hence the meridional thrust at the base of the dome will be at an inclination with the horizontal. The horizontal component of the meridional thrust at the base will

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induce an outward push of the cylindrical wall carrying the dome. In order to prevent this, a ring beam is provided at the base of the dome. The horizontal component of the meridional thrust will be resisted by the ring beam by hoop action. Often the ring beam at the base is made by thickening the edges and providing adequate hoop steel to resist the hoop tension. Sometimes no thickness of the edges may become necessary and the requisite amount of hoop steel will be sufficient. Often a ring beam section may be based on an architectural consideration. A minimum of 0·3% of gross area shall be provided as the reinforcement in each principal direction for the dome section.

Design 120 (S I.). *A spherical dome of a water tank of span 6 metres has a rise of 1·20 metres It carries an all inclusive distributed load of 6000 N/metre² and a lantern load of 8000 N at the crown. Design the dome.*

Solution. Let the radius of the dome be R *metres*

$\therefore \quad 1{\cdot}2\,(2R-1{\cdot}2) = 3^2$

$\therefore \quad R = 4{\cdot}35$ *metres*

$$\sin\theta = \frac{3}{4{\cdot}35} \quad \therefore \quad \theta = 43^\circ\,36'$$

$\therefore \quad \sin\theta = 0{\cdot}6896$ and $\cos\theta = 0{\cdot}7242$.

Meridional thrust per metre run at springing level

$$= T = \frac{wR}{1+\cos\theta} + \frac{W}{2\pi R \sin^2\theta}$$

$$= \frac{6000 \times 4{\cdot}35}{1{\cdot}7242} + \frac{8000}{2\pi \times 4{\cdot}35 (0{\cdot}6896)^2}$$

$$= 15753 \text{ N.}$$

Let the shell be 10 *cm.* thick

Compressive stress due to meridional thrust

$$= \frac{15753}{100 \times 10} = 15{\cdot}7 \text{ N/cm.}^2$$

$$= 0{\cdot}157 \text{ N/mm.}^2$$

Hoop stress

$$= f = \frac{wR}{t}\left[\cos\theta - \frac{1}{1+\cos\theta}\right] - \frac{W}{2\pi Rt \sin^2\theta}$$

$$= \frac{6000 \times 4{\cdot}35}{0{\cdot}15}\left[0{\cdot}7242 - \frac{1}{1{\cdot}7242}\right] - \frac{8000}{2\pi \times 4{\cdot}35 \times 0{\cdot}15 \times (0{\cdot}6896)^2}$$

$$= 25094 - 4103 = 20991 \text{ N/metre}^2$$

$$= 2{\cdot}09 \text{ N/cm.}^2$$

$$= 0{\cdot}0209 \text{ N/mm}^2$$

Similarly these stresses may be calculated at other levels also.

These stresses are very low and hence a minimum of 0·3% steel will be provided, in each principal direction.

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Minimum steel requirement

$$= \frac{0.3}{100} \times 100 \times 10 = 3 \ cm.^2$$

Spacing of 10 *mm.* ϕ bars

$$= \frac{0.79 \times 100}{3} = 26 \ cm.$$

Ring beam

Horizontal component of meridional thrust

$$= T \cos \theta = 15753 \times 0.7242 = 11408 \ N.$$

Hoop tension in the ring beam

$$= T \cos \theta . r = 11408 \times 3 = 34224 \ N.$$

Safe stress in steel $= 10{,}000 \ N/cm.^2$

$$\therefore \quad A_t = \frac{34224}{10{,}000} = 3.42 \ cm.^2$$

Provide 4 bars of 14 mm. ϕ.

Size of the ring beam. The tensile stress on the equivalent concrete area shall not exceed 120 $N/cm.^2$

Let A be the minimum area of the ring beam section.

$\therefore$ Equivalent concrete area $= A + (13 - 1)\ 6.16 = A + 73.92 \ cm.^2$

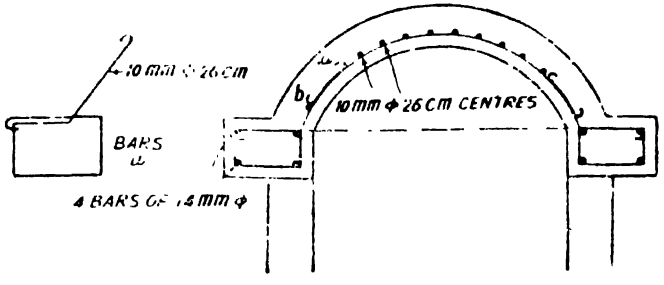
Limiting the tensile stress in concrete to 120 $N/cm.^2$

We have,

$$\frac{34224}{A + 73.92} = 120$$

$$\therefore \quad A = 211.3 \ cm.^2$$

Let us provide 20 *cm.* $\times$ 15 *cm.* section.



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Design 121. *A spherical dome of span 6 metres has a rise of 1·20 metres. It carries an all inclusive distributed load of 650 kg. per sq. metre and a lantern load of 800 kg. at the crown. Design the dome.*

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Solution. Let the radius of the dome be R *metres*

$\therefore \quad 1{\cdot}2(2R-1{\cdot}2)=3^2$

$\therefore \quad R=4{\cdot}35$ *metres*

$\therefore \quad \sin\theta=\frac{3}{4{\cdot}37}$

$\therefore \quad \theta=43°36'$

$\therefore \quad \sin\theta=0{\cdot}6896$

and $\quad \cos\theta=0{\cdot}7242$

$\therefore$ Meridional thrust per *metre* run

$$-T=-\frac{wR}{1+\cos\theta}+\frac{W}{2\pi R\sin^2\theta}$$

$$=-\left[\frac{650\times4{\cdot}35}{1{\cdot}7242}+\frac{800}{2\pi\times4{\cdot}35(0{\cdot}6896)^2}\right] kg.$$

$$=1700\ kg.$$

Let the shell be 10 *cm.* thick.

$\therefore$ Compressive stress due to meridional thrust

$$=\frac{1700}{100\times10}=1{\cdot}70\ kg./cm.^2$$

Hoop stress

$$f=\frac{wR}{t}\left[\cos\theta-\frac{1}{1+\cos\theta}\right]-\frac{W}{2\pi Rt\sin^2\theta}$$

$\therefore$ f (*kg.* per *sq. m.*)

$$=\frac{650\times4{\cdot}35}{0{\cdot}10}\left[0{\cdot}7242-\frac{1}{1{\cdot}7242}\right]-\frac{800}{2\pi\times4{\cdot}35\times0{\cdot}15(0{\cdot}6896)^2}$$

$$=3459\ kg./sq.\ m.=0{\cdot}3459\ kg./cm.^2$$

Similarly these stresses may be calculated at other levels also.

These stresses are extremely low stresses and hence a minimum of 0·3% of gross area of the section will be provided as the reinforcement along the principal directions.

0·3% of gross area for 1 *metre* length

$$=\frac{0{\cdot}3}{100}\times100\times10=3\ cm.^2$$

$\therefore$ Spacing of 10 *mm.* bars

$$=\frac{0{\cdot}79\times100}{3}=26\ cms.$$

Hence provide 10 mm. bars at 26 cms. centres, along each of the principal directions.

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Ring beam

Horizontal component of meridional thrust

$$= T \cos \theta = 1700 \cos 43^\circ 36'$$

$$= 1700 \times 0{\cdot}7242 = 1230 \text{ kg./m.}$$

$\therefore$ Hoop tension in the ring beam $= 1230\ r = 1230 \times 3 = 3690$ kg.

Safe stress in steel $= 1000$ kg./cm.^{2}

$$\therefore \quad A_t = \frac{3690}{1000} = 3{\cdot}7 \text{ cm.}^2$$

.. *Provide 4 bars of 14 mm. diameter.*

Size of the ring beam The tensile stress on the equivalent area of concrete on the ring beam section shall not exceed 12 kg./cm.^{2}

Let A be the area of the ring beam section. Equivalent concrete area $= A + (13 - 1) \times 6{\cdot}16 = (A + 70{\cdot}92)$ cm.^{2} (using M 200 mix)

$$\therefore \quad \frac{3690}{A + 73{\cdot}92} = 12$$

$$\therefore \quad A = 234 \text{ cm.}^2$$

Let us provide 20 cm. × 15 cm. section.

Fig. 454 shows details of reinforcement.

The ring beam reinforcement should be bound by ties of *10 mm. dia. bars at 26 cm. centres.* These are lapped with the dome reinforcement.

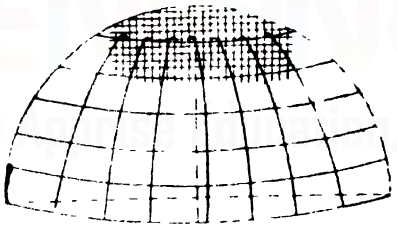

Fig. 455

If the bars running along the meridian are allowed to be continuous over the crown, there will be a congestion of reinforcement at the crown. It is, therefore, preferable to stop the meridian reinforcement near the crown and a separate mesh be provided near the zone of the crown. This is shown in Fig. 455.

§182. Openings

If an opening is provided in a dome, sufficient trimming re-

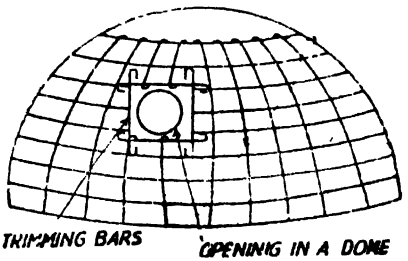

Fig. 456

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inforcement should be provided all round the opening. The reinforcement reaching the opening should be well anchored to the trimming reinforcement. *See* Fig. 456.

§183. Conical Domes

The thrust and hoop stress in a conical dome can be determined on lines similar to those of the spherical dome.

Stresses due to distributed load. Let the distributed load on the dome be w per unit surface area. Fig. 457 shows a conical dome with a semi apex angle θ.

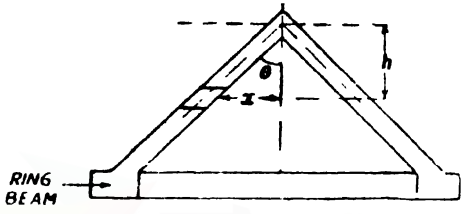

Fig. 457

The dome can be visualised as consisting of a series of rings placed one above the other.

Consider one such ring at a depth h from the apex and of height dh.

Load on the ring $= w \pi x h \sec \theta$

$= w \pi h \tan \theta \, h \sec \theta$

$= w \pi h^2 \sec \theta \tan \theta$

Let the thrust on the ring be T per unit run.

But vertical component of the thrust on the ring

= vertical load on the ring

$= T \times 2\pi h \tan \theta \cos \theta = w\pi h^2 \sec \theta \tan \theta$

$$\therefore \quad T = \frac{w h \sec^2 \theta}{2}.$$

Hoop stress. Hoop tension caused by the load on the ring

$= H = T \sin \theta \, . \, h \tan \theta$

$$= \frac{\pi h^2}{2} \sec.^2 \theta \, . \sin \theta \tan \theta$$

Let t be thickness of the dome slab Due to the thrust immediately below the ring there will be a hoop compression of $H + dH$. Hence the net hoop compression in the ring $= dH$.

Area of cross-section of the ring

$= d h \sec \theta \, t$

$\therefore$ Hoop stress on the ring section $= f$

$$= \frac{dH}{d h \sec \theta \, t}$$

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$$\therefore \quad f = \frac{1}{\sec\theta \, t} \cdot \frac{dH}{dh}$$

But $$H = \frac{wh^2}{2} \sec^2\theta \cdot \sin\theta \cdot \tan\theta$$

$$\therefore \quad \frac{dH}{dh} = \frac{2wh}{2} \sec^2\theta \cdot \sin\theta \cdot \tan\theta$$

$$= wh \sec^2\theta \sin\theta \cdot \tan\theta$$

$$\therefore \quad f = \frac{wh \sec^2\theta \cdot \sin^2\theta \cdot \tan\theta}{\sec\theta \cdot t}$$

$$\therefore \quad f = \frac{wh \tan^2\theta}{t}.$$

Concentrated load on the apex. In this case the thrust per unit length on a ring of height dh at a depth h from the apex is given by

$$2\pi h \tan\theta \cos\theta = W$$

$$\therefore \quad T = \frac{W \sec\theta}{2\pi h \tan\theta}.$$

Hoop stress due to a concentrated load W on the apex. Consider an elemental ring of depth of dh at a depth of h from the apex.

The thrust on the top of this ring $= \frac{W \sec\theta}{2\pi h \tan\theta}$. Hoop tension caused due to the horizontal component of the thrust

$$= \frac{W \sec\theta}{2\pi h \tan\theta} \sin\theta \cdot h \tan\theta$$

$$H = \frac{W}{2\pi} \tan\theta.$$

The thrust on the bottom of the ring will also produce a hoop compression equal to H itself

Hence net hoop load on the section = 0.

Summary of formulae for conical domes. Concentrated load on the apex of the dome $= W$.

Distributed load per unit area of the conical surface $= w$

$$\text{Meridional thrust} = T = \frac{wh}{2} \sec^2\theta + \frac{W \sec\theta}{2\pi h \tan\theta} \text{ (compressive)}$$

$$\text{Hoop stress} = \frac{wh}{t} \tan^2\theta \text{ (compressive)}.$$

Ring beam at the base. As for a spherical dome, a conical dome also requires a ring beam to resist by hoop action the horizontal component of thrust at the base of the dome.

Design 122. *A conical dome has a base diameter of 9·00 metres. ri buted load of 4900 N per sq metre and a concentra- The height of the dome is 4 50 metres.*

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Solution. Assume thickness of 10 *cms.* for the dome slab. The meridional thrust per *metre* run at the base $= T$

$$= \frac{wh}{2} \sec^2\theta + \frac{W \sec\theta}{2\pi h \tan\theta}$$

$$\frac{4900 \times 4{\cdot}5}{2} \times 2 + \frac{9000\sqrt{2}}{2\pi \times 4{\cdot}5} N$$

$$= 22050 + 450\ N$$

$$= 22500\ N$$

$\therefore$ Compressive stress due to meridional thrust

$$= \frac{22500}{10 \times 100}\ N/cm.^2 = 22{\cdot}5\ N/cm.^2$$

Hoop stress $= \frac{wh}{t} \tan^2\theta$

$$= \frac{4900 \times 4{\cdot}5}{0{\cdot}10} = 220500\ N/m.^2$$

$$= \frac{220500}{100 \times 100} = 22{\cdot}05\ N/cm.^2$$

The stresses being considerably low, only nominal steel will be provided at 0·3% of the gross area of the section.

$\therefore$ $A_s = \frac{0{\cdot}3}{100} \times 100 \times 10$

$= 3\ cm.^2$ per *metre* run.

$\therefore$ Spacing of 8 *mm.* bars

$$= \frac{0{\cdot}5 \times 100}{3} = 16\ cms.$$

Hence, provide 8 mm. bars at 16 cm. centres along the meridians and the latitudes.

Ring beam. Horizontal component [of meridional thrust at the base

$= 22500 \cos 45° = 15920\ N$

$\therefore$ Hoop tension in the ring beam

$= 15920 \times R\ N$

$= 15920 \times 4{\cdot}5\ N = 71640\ N$

Allowing a stress of 10000 $N/cm.^2$ in the reinforcement

$$A_s = \frac{71640}{10000} = 7{\cdot}16\ cm.^2$$

$\therefore$ *Provide 4 bars of 16 mm. dia. bars (8·04 cm.²)*

Let the area of the ring section be $A\ cm.^2$

$\therefore$ Equivalent area of concrete

$= A + 12 \times 8{\cdot}04\ cm.^2$ $(m = 13)$

$= A + 96{\cdot}48\ cm.^2$

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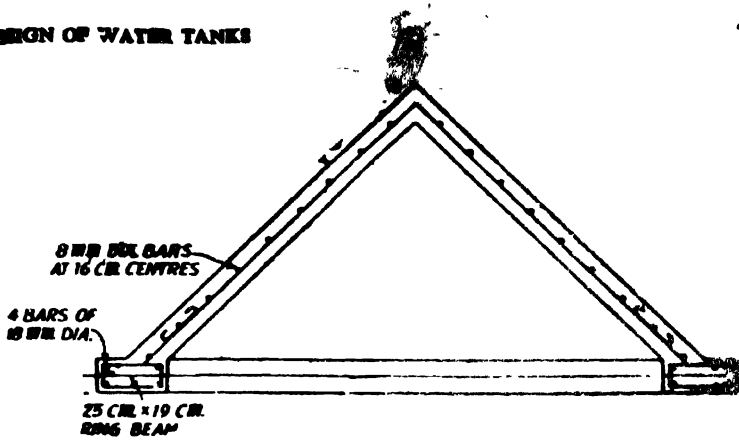

Fig. 458. Conical dome.

Allowing the tensile stress on the equivalent area of concrete at 120 $N/cm.^2$, we have

$$120 = \frac{71640}{A + 96{\cdot}48}$$

$$= A - 500 \ cm.^2$$

Adopt a section 25 *cm.* × 20 *cm.*

See Fig. 458 for detail of reinforcement of this dome.

Suspended bottoms of Tanks. Sometimes the bottoms of tanks are not made horizontal but are made hemispherical or conical. In such a case the floor slab will be part of a surface of revolution. Such slabs are analysed by assuming them to be inextensible surfaces, which do not offer any sensible resistance to flexure. Fig. 459 shows a tank bottom. In its general occurrence, the surface is doubly curved. Let r_1 and r_2 be the radii of curvature in two principal directions.

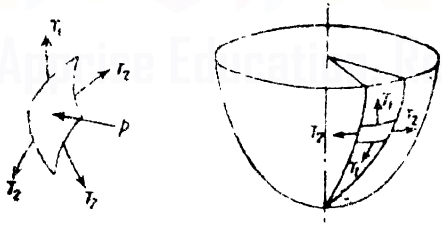

Fig. 459. Suspended bottom of a tank.

If the forces acting on a small element of the slab be resolved normal to the surface of the elemental surface, we have

$$\frac{T_1}{r_1} + \frac{T_2}{r_2} = p$$

where p = water pressure intensity on the element.

T_1 and T_2 = the tension per unit run along the respective principal directions.

Further the sum of the vertical components of the meridional tensions must be equal to the resultant vertical pressure on the por-

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tion of the surface together with the weight on the shell just below a horizontal plane through the section considered.

Hemispherical bottom

Let us apply the above principle to the hemispherical bottom. Let t = thickness of the shell.

Since in this case $r_1 = r_2 = R$ = Radius of the sphere

and $T_1 = f_1 t$

$T_2 = f_2 t$

and $p = wh$

Substituting in the equation,

$$\therefore \quad p = \frac{T_1}{r_1} + \frac{T_2}{r_2}$$

We have

$$\frac{f_1 t}{R} + \frac{f_2 t}{R} = wh$$

$$\therefore \quad \frac{f_1 + f_2}{R} = \frac{wh}{t}$$

Further,
$$f_1 = \frac{wR}{t}\left[\frac{H-R}{2} + \frac{R}{3}\,\frac{1-\sin^3\theta}{\cos^2\theta}\right]$$

where H = Depth of water in the tank

θ = Angle between a horizontal radius of the hemisphere and a radius drawn to the point considered

Conical Bottom

In this case $\frac{1}{r_1} = 0$

But $p = \frac{T_1}{r_1} + \frac{T_2}{r_2}$

$$\therefore \quad p = \frac{T_2}{r_2}$$

$$= \frac{f_2 t}{r_2}$$

$$\therefore \quad \frac{p}{t} = \frac{f_2}{r_2}$$

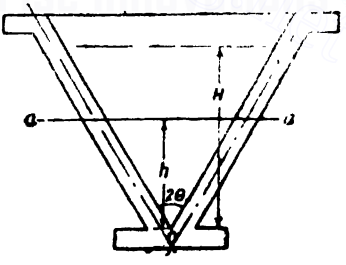

Fig. 460. Conical bottom.

Consider a level aa at a height of h above the apex of the cone.

Here $r_2 = \frac{h \tan\theta}{\cos\theta}$ and $p = w(H-h)$

$$f_2 = \frac{p}{t} r_2 = \frac{w(H-h)\,h \tan\theta}{t \cos\theta}$$

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Considering the equilibrium of the portion of the cone held below the level *aa*, we have

$$f_1 = \frac{wh \tan \theta}{6\, t \cos \theta}(3H - 2h)$$

For the particular case when $2\theta = 60°$

$$f_1 = 0{\cdot}667 \frac{w}{t} h(3H - 2h)$$

$$\therefore \quad f_2 = 0{\cdot}667 \frac{w}{t} h(H - h)$$

Design 123 (S.I.). *Design a circular tank of diameter 5·75 m. with a dome of rise 90 cm. The cylindrical part has a height of 3 metres. The tank is supported on brick masonry walls all round. Allow a live load of 1350 N/metre² on the dome surface.*

Solution. Let R be the radius of the dome

$$0{\cdot}9(2R - 0{\cdot}9) = (2{\cdot}875)^2$$

$$\therefore \quad R = 5{\cdot}04 \text{ m.}$$

Provide a thickness of 15 *cm.* for the dome.

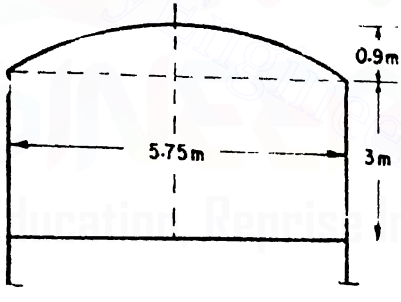

Fig. 461

Wt. of dome $= 250 \times 15 = 3750$ N/m.²

Live load $= 1350$ N/m.²

Total $= 5100$ N/m.²

$$\sin \theta = \frac{2{\cdot}875}{5{\cdot}04} \qquad \therefore \quad \theta = 34° 47'$$

$$\cos \theta = \cos 34° 47' = 0{\cdot}8213$$

Hoop stress at springing level

$$f = \frac{wR}{t}\left[\cos \theta - \frac{1}{1 + \cos \theta}\right]$$

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$$= \frac{5100}{0{\cdot}15}\left[0{\cdot}8213 - \frac{1}{1{\cdot}8213}\right] N/metre^2$$

$$= 46650\ N/metre^2 = 4{\cdot}665\ N/cm.^2$$

Hoop stress at the crown

$$f = \frac{wR}{t}\left[1 - \frac{1}{1+1}\right] = \frac{wR}{2t}$$

$$= \frac{5100 \times 5{\cdot}04}{2 \times 0{\cdot}15} = 85680\ N/metre^2$$

$$= 8{\cdot}568\ N/cm.^2$$

Meridional thrust at springing level per metre run

$$= T = \frac{wR}{1 + \cos\theta} = \frac{5100 \times 5{\cdot}04}{1{\cdot}8213} = 14110\ N/metre$$

Meridional stress $= \dfrac{14110}{100 \times 15} = 9{\cdot}4\ N/cm.^2$

These stresses are very small. **Provide nominal meridional and hoop steel.** ***Provide 8 mm. ϕ @ 11 cm. c/c both ways.***

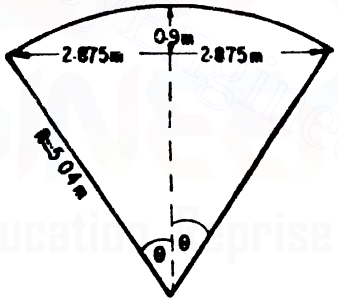

Fig. 462

Ring beam supporting the dome

Meridional thrust per *metre* run at the springing level

$$= T = 14110\ N/metre$$

Hoop tension in the ring beam

$$= 14110 \cos\theta \times r$$

$$= 14110 \times 0{\cdot}8213 \times 2{\cdot}875\ N = 33320\ N$$

$\therefore$ **Hoop steel** $= \dfrac{33320}{10{,}000} = 3{\cdot}332\ cm.^2$

Provide 4 bars of 12 mm. ϕ (4·52 cm.^{2})

Provide also 8 mm. ϕ bars @ 11 cm. c/c as stirrups which will also be embedded in the dome to lap with the dome reinforcement.

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Let the ring beam section be 20 *cm.* × 15 *cm.*

Equivalent concrete area

$$= (20 \times 15) + (13 - 1)\ 4{\cdot}52\ cm.^2$$
$$= 354{\cdot}24\ cm.^2$$

Tensile stress on the equivalent concrete area

$$= \frac{14110}{354{\cdot}24} = 39{\cdot}8\ N/cm^2$$

Cylindrical wall

Height of wall $= H = 3$ *metres*

Diameter $= D = 5{\cdot}75$ *metre*

Thickness of wall $= t = 15$ *cm.* $= 0{\cdot}15$ *metre*

$$\therefore \quad \frac{H^2}{Dt} = \frac{3 \times 3}{5{\cdot}75 \times 0{\cdot}15} = 10{\cdot}4 \text{ } \textit{say } 10.$$

Referring to I.S. tables (page 612), corresponding to $\frac{H^2}{Dt} = 10$, coefficient for maximum hoop tension $= 0{\cdot}608$

$\therefore$ Maximum hoop tension

$$= 0{\cdot}608\ wHr = 0{\cdot}608 \times 9810 \times 3 \times 2{\cdot}875\ N$$
$$= 51444\ N$$

The maximum hoop tension occurs at a depth of

$$0{\cdot}6\ H = 0{\cdot}6 \times 3 = 1{\cdot}80\ \textit{metre}$$ from the top.

Steel for hoop tension

$$= \frac{51444}{10{,}000} = 5{\cdot}14\ cm.^2$$

Spacing of 8 *mm.* ϕ bars

$$= \frac{0{\cdot}5 \times 100}{5{\cdot}14} = 9{\cdot}7\ cm.\ \textit{say } 9\ cm.\ c/c.$$

Provide 8 mm. ϕ *bars @ 18 cm. c/c near each face.*

Coefficient for cantilever moment $= 0{\cdot}0122$

$\therefore$ Maximum cantilever moment $= 0{\cdot}0122\ wH^2$

$$= 0{\cdot}0122 \times 9810 \times 3^2 \times 100\ N\,cm$$
$$= 323140\ N\ cm.$$

producing tension *near* the water face.

Coefficient for positive moment $= 0{\cdot}0029$

$\therefore$ Maximum positive moment $= 0{\cdot}0029\ wH^2$

$$= 0{\cdot}0029 \times 9810 \times 3^2 \times 100\ N\,cm.$$
$$= 76810\ N\ cm.$$

producing tension *away* from the water face.

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The maximum positive moment occurs at 0·7 H from the top

$= 0{\cdot}7 \times 3 = 2{\cdot}1$ *m.* from the top.

Coefficient for S.F. at base $= 0{\cdot}158$

$\therefore$ S.F. at base $= S = 0{\cdot}158\ wH^2$

$= 0{\cdot}158 \times 9810 \times 3^2\ N$

$= 13950\ N$

Providing a clear cover of 25 *mm.* ϕ and using 8 *mm.* ϕ bars, effective depth

$= 15 - 3{\cdot}3 = 11{\cdot}7$ *cm.*

Steel for cantilever moment

$$= \frac{323140}{10{,}000 \times 0{\cdot}84 \times 11{\cdot}7}\ cm.^2$$

$= 3{\cdot}40\ cm.^2$

Provide 8 mm. ϕ bars @ 14 cm. c/c.

Steel for positive moment

$$= \frac{76810}{12500 \times 0{\cdot}86 \times 11{\cdot}7}\ cm.^2$$

$= 0{\cdot}63\ cm.^2$ (*very small*)

Nominal shear stress $= \frac{S}{bd} = \frac{13950}{100 \times 11{\cdot}7} = 12{\cdot}3\ N/cm.^2$

$= 0{\cdot}123\ N/mm.^2$

Design of the bottom slab

Provide a thickness of 20 *cm.*

Loads

D.L. of slab $= 250 \times 20 = 5000\ N/metre^2$

Weight of water $= 9810 \times 3 = 29430\ N/metre^2$

Total $= 34430\ N/metre^2$

Assume fixity at the ends

Maximum circumferential moment $= \frac{Qr^2}{16}$

Maximum positive radial moment $= \frac{Qr^2}{16}$

Maximum negative radial moment $= \frac{2}{16}\ Qr^2 = \frac{Qr^2}{8}$

Maximum B.M. $= \frac{34430 \times 2{\cdot}875^2}{8} \times 100\ N\ cm.$

$= 3557320\ N\ cm.$

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Equating the M.R. to the B.M.

$$140{\cdot}3 \times 100\, d^2 = 3557320$$

$$\therefore \quad d = 15{\cdot}92\ cm.$$

Providing an effective cover of 4 *cm.* effective depth available

$$= 20 - 4 = 16\ cm.$$

$$\therefore \quad A_s = \frac{3557320}{10{,}000 \times 0{\cdot}84 \times 16} = 26{\cdot}5\ cm.^2$$

Spacing of 16 *mm.* ϕ bars

$$= \frac{2{\cdot}01 \times 100}{26{\cdot}5} = 7{\cdot}58\ cm.$$

Provide 16 mm. ϕ *bars @ 7 50 cm. c/c.*

This is the radial steel at top and is provided for a distance 0·2 diameter

$$= 0{\cdot}2 \times 5{\cdot}75 = 1\ 15\ m.$$ from the edge.

Maximum positive radial moment

$$= \frac{3557320}{2} = 1778660\ N\ cm.$$

Maximum circumferential moment is also equal to 1778660 *N cm.*

$$A_s = \frac{1778660}{12500 \times 0{\cdot}86 \times 16} = 10{\cdot}34\ cm.^2$$

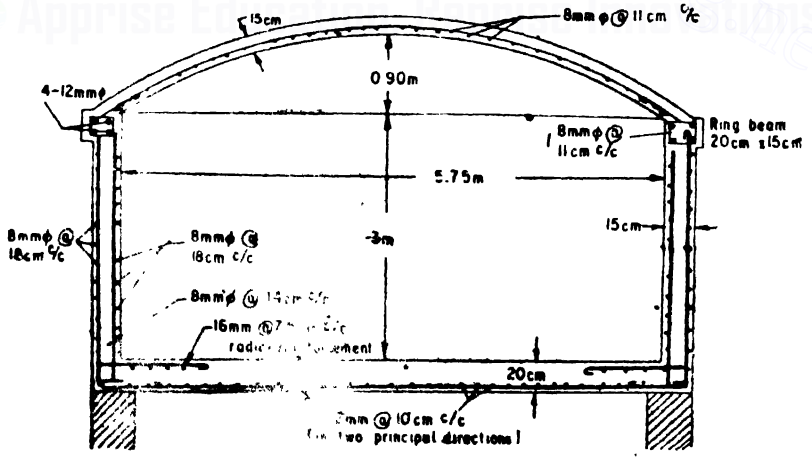

Fig. 463

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Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{10{\cdot}34} = 10{\cdot}9 \text{ cm.}$$

Provide 12 mm. ϕ bars @ 10 cm. c/c in each principal direction.

Fig. 463 shows the details of reinforcement.

Design 124. *Design a circular tank of diameter 5·75 m. with a dome of rise 90 cm. The cylindrical part has a height of 3 metres. The tank is supported on brick masonry walls all round. The live load on the dome is 135 kg./metre².*

Solution.

Design of the dome

Let R be the radius of the dome

$$0{\cdot}9\,(2R - 0{\cdot}9) = (2{\cdot}875)^2$$

$$\therefore \quad R = 5{\cdot}04 \text{ m.}$$

Provide a thickness of 15 *cm.* for the dome.

Loads **:**

Dead load of dome $= 25 \times 15$	$= 375$ *kg./metre²*
Live load	$= 135$ *kg /metre²*
Total	$= 510$ *kg./metre²*

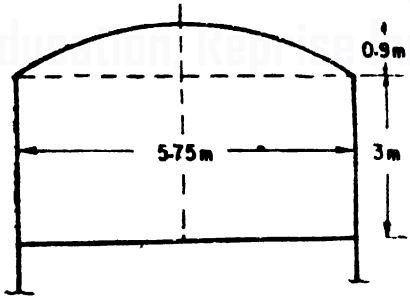

Fig. 464

$$\sin\theta = \frac{2{\cdot}875}{5{\cdot}04} \therefore \quad \theta = 34^\circ\ 47'$$

$$\cos\theta = \cos 34^\circ\ 47' = 0{\cdot}8213$$

Hoop stress at springing level

$$f = \frac{wR}{t}\left[\cos\theta - \frac{1}{1+\cos\theta}\right]$$

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$$= -\frac{510 \times 5{\cdot}04}{0{\cdot}15}\left[0{\cdot}8213 - \frac{1}{1{\cdot}8213}\right]$$

$$= -4665 \text{ kg./m.}^2 = -0{\cdot}4665 \text{ kg./cm.}^2$$

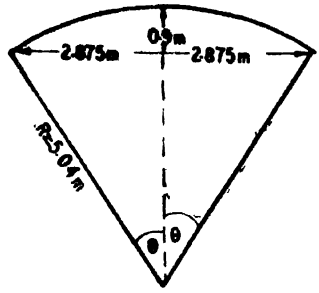

Fig. 465

Hoop stress at the crown, *i.e.*, **at** $\theta = 0$

$$f = \frac{wR}{t}\left[1 - \frac{1}{1+1}\right] = \frac{wR}{t} \cdot \frac{1}{2}$$

$$= \frac{510 \times 5{\cdot}04}{0{\cdot}15} \times \frac{1}{2} = 8568 \text{ kg./cm.}^2$$

$$= 0{\cdot}8568 \text{ kg./mm.}^2$$

Meridional thrust at springing level per *metre* **run**

$$= T = \frac{wR}{1+\cos\theta} = \frac{510 \times 5{\cdot}04}{1{\cdot}8213} = 1411 \text{ kg./m.}$$

$$\text{Meridional stress} = \frac{1411}{100 \times 15} = 0{\cdot}94 \text{ kg./cm.}^2$$

These stresses are very small. Provide nominal meridional and hoop steel. *Provide 8 mm.* @ *11 cm. c/c both ways.*

Ring beam supporting the dome

Meridional thrust per *metre* **run at the springing level**

$$= T = 1411 \text{ kg./m.}$$

Hoop tension in the ring beam section

$$= 1411 \cos\theta \times r$$

$$= 1411 \times 0{\cdot}8213 \times 2{\cdot}875 \text{ kg.}$$

$$= 3332 \text{ kg.}$$

$$\text{Hoop steel} = \frac{3332}{1000} = 3{\cdot}332 \text{ cm.}^2$$

Provide 4 bars of 12 mm. diameter **(4·25 cm.²)**

Provide also 8 mm. ϕ @ *11 cm. c/c* **(*stirrups lapping with the dome reinforcement*).**

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Let the ring beam section be 20 *cm.* × 15 *cm.*

∴ Equivalent concrete area

$$= (20 \times 15) + (13 - 1)\, 4{\cdot}52 \text{ cm.}^2$$

$$= 354{\cdot}24 \text{ cm.}^2$$

∴ Tensile stress on the equivalent concrete area

$$= \frac{1411}{354{\cdot}24} = 3{\cdot}98 \text{ kg./cm.}^2$$

Cylindrical wall

Height of wall $= H = 3$ *m.*

Diameter $= D = 5{\cdot}75$ *m.*

Thickness of wall $= t = 15$ *cm.* $= 0{\cdot}15$ *m.*

$$\therefore \quad \frac{H^2}{Dt} = \frac{3 \times 3}{5{\cdot}75 \times 0{\cdot}15} = 10{\cdot}4 \text{ say } 10.$$

Referring to I.S tables corresponding to $\frac{H^2}{Dt} = 10$

Coefficient for maximum hoop tension $= 0{\cdot}608$

∴ Max. Hoop tension

$$= 0{\cdot}608\ wH\ r$$

$$= 0{\cdot}608 \times 1000 \times 3 \times 2{\cdot}875 \text{ kg.}$$

$$= 5244 \text{ kg.}$$

The maximum hoop tension occurs at *a* depth $0{\cdot}6\ H$

$$= 0{\cdot}6 \times 3 = 1{\cdot}8 \text{ m. from the top.}$$

Steel for hoop tension

$$= \frac{5244}{1000} = 5{\cdot}24 \text{ cm.}^2$$

Spacing of 8 *mm.* bars

$$= \frac{0{\cdot}5 \times 100}{5{\cdot}24} = 9 \text{ cm. c/c.}$$

Provide 8 mm. ϕ bars @ 18 cm. c/c near each face.

Coefficient for cantilever moment

$$= 0{\cdot}0122$$

∴ Maximum cantilever moment

$$= 0{\cdot}0122\ wH^3$$

$$= 0{\cdot}0122 \times 1000 \times 3^3 \times 100 \text{ kg. cm.}$$

$$= 32940 \text{ kg. cm}$$

producing tension near the water face.

Coefficient for positive moment

$$= 0{\cdot}0029$$

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∴ Maximum positive moment

$$= 0{\cdot}0029\ wH^3$$
$$= 0{\cdot}0029 \times 1000 \times 3^3 \times 100\ kg.\ cm.$$
$$= 7830\ kg.\ cm.$$

producing tension *away* from the water face.

The maximum positive moment occurs at $0{\cdot}7\ H$ from the top

i.e., $0{\cdot}7 \times 3 = 2{\cdot}1\ m.$ from the top

Coefficient for S.F. at base $= 0{\cdot}158$

∴ S.F. at base $= S = 0{\cdot}158\ wH^2$

$$= 0{\cdot}158 \times 1000 \times 3^2\ kg.$$
$$= 1422\ kg.$$

Providing a clear cover of 25 *mm.* and using 8 *mm.* ϕ bars

effective depth $= 15 - 3{\cdot}3 = 11{\cdot}7\ cm.$

Steel for cantilever moment

$$= \frac{32940}{1000 \times 0.84 \times 11{\cdot}7}\ cm.^2$$
$$= 3{\cdot}47\ cm.^2$$

Provide 8 mm, ϕ bars @ 14 cm. c/c.

Steel for positive moment

$$= \frac{7830}{1000 \times 0{\cdot}84 \times 11{\cdot}7}$$
$$= 0{\cdot}82\ cm.^2\ (very\ small)$$

$$\text{Shear stress} = \frac{S}{ab} = \frac{1422}{0{\cdot}84 \times 11{\cdot}3 \times 100} = 1{\cdot}5\ kg./cm.^2$$

Design of the bottom slab

Provide a thickness of 20 *cm.* for the bottom slab.

Loads :

D.L. of the slab $= 25 \times 20$	$= 500\ kg./m.^2$
Weight of water $= 1000 \times 3$	$= 3000\ kg./m.^2$
Total	$3500\ kg./m.^2$

We will assume fixity at the ends.

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Max. circumferential moment

$$-\frac{Qr^2}{16}$$

Maximum positive Radial Moment

$$-\frac{Qr^2}{16}$$

Maximum negative Radial Moment

$$=\frac{2}{16}\ Qr^2=\frac{Qr^2}{8}$$

$\therefore$ Max. B.M $= -\dfrac{3500\times 2{\cdot}875^2}{8}\times 100$ *kg. cm.*

$=361620$ *kg. cm.*

producing tension *near* the water face

Equating the M.R. to the B.M.

$14{\cdot}03\times 100\ d^2=361620 \quad \therefore\ d=16$ *cm.*

Providing an effective cover of 4 *cm.*, effective depth available

$20-4=16$ *cm.*

$$\therefore \quad A_t=\frac{361620}{1000\times 0{\cdot}84\times 15}=26{\cdot}90\ cm.^2$$

$\therefore$ Spacing of 16 *mm.* ϕ bars

$$=\frac{2{\cdot}01\times 100}{26{\cdot}90}=7{\cdot}5\ cm.$$

Provide 16 mm. ϕ bars @ 7·5 cm. c/c.

This is the radial steel at top. The steel is provided for a distance of 0·2 diam. $=0{\cdot}2\times 5{\cdot}75=1{\cdot}15$ *m.* from the edge.

Maximum positive radial moment

$$=\frac{361620}{2}=180810\ kg.\ cm.$$

Maximum circumferential moment is also equal to 180810 *kg. cm.*

$$\therefore \quad A_t=\frac{180810}{1250\times 0{\cdot}86\times 16}=10{\cdot}51\ cm.^2$$

Spacing of 12 *mm.* ϕ ba

$$=\frac{1{\cdot}13\times 100}{10{\cdot}51}=10{\cdot}7\ cm.$$

Provide 12 mm. ϕ bars @ 10 cm. c/c in each way.

Circular tanks above ground level. Fig. 466 shows a usual type of tank often provided. It consists of a dome and a cylin-

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drical wall provided on a conical wall. The floor slab may be flat or domed. A ring beam will be required to support the dome. A ring beam is also provided at the junction of cylindrical and conical walls. The conical wall and the tank floor are supported on a ring girder. The ring girder will be supported on a number of columns.

If a domed floor is provided as shown in the figure, the ring girder supported on the columns will be relieved from the horizontal thrusts. This is possible because the horizontal thrusts of the conical slab and the domed floor will balance. Such a tank is called an **Intze Tank**.

Sometimes a vertical hollow shaft may be provided which may be supported on the domed floor.

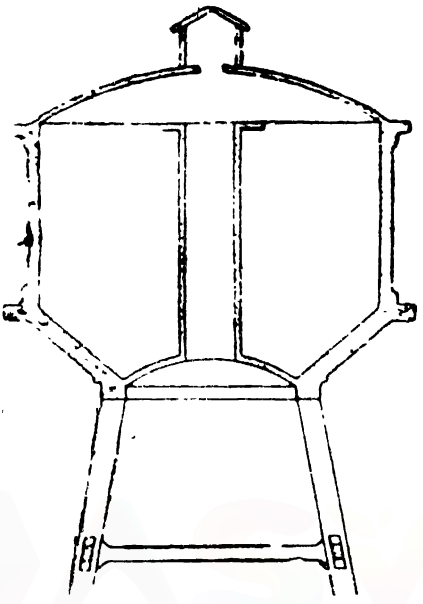

Fig. 466

The design of the tank will therefore involve the following :

(*i*) **The dome**. This is usually 10 *cm*. to 15 *cm*. thick with reinforcement along the meridians and lattitudes. The rise of the dome is usually $\frac{1}{5}$ of the span.

(*ii*) **Ring beam supporting the dome**. This ring beam is necessary to resist the horizontal component of the thrust of the dome. The ring beam will be designed for the hoop tension induced.

(*iii*) **Cylindrical walls**. This will be designed for hoop tension caused by horizontal water pressure.

(*iv*) **Ring beam at the junction of the cylindrical wall and the conical wall**. This ring beam is provided to resist the horizontal component of the reaction of the conical wall on the cylindrical wall. The ring beam will be designed for the induced hoop tension.

(*v*) **Conical slab** This will be designed for a hoop tension caused by water pressure. This slab will also be designed for a possibility of its spanning between the ring beam at its top and the ring girder at the bottom.

(*vi*) **Floor of the tank**. The floor may be a flat circular slab, or it may be domed. This slab is supported on the ring beam.

(*vii*) **The ring girder**. This will be designed to support the tank and its contents. The girder will be supported on columns. The girder should be designed for the bending moments and torsion.

(*viii*) **Columns**. These are to be designed for the total load

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transferred to them. The wind pressure on the tank also will have to be included. The columns shall be braced at intervals.

(*ix*) **Foundation.** A combined footing may be provided for all supporting columns. When this is done it is usual to make the foundation consist of a ring girder and a circular slab.

§184. Design of a Circular Beam (Ring Girder)

A ring girder is supported usually on four or more columns. When such a girder carries a distributed load, it is subjected to bending moments and torsion of the following magnitudes :

Total load carried by the ring girder $= W$, radius of girder $= r$

Number of columns	*Bending moment over support*	*Bending moment at centre of each span*	*Torsion*	*Angular distance for Max. torsion*
4	0·0342 Wr	0·0176 Wr	0·0053 Wr	15° 12′
6	0·0146 Wr	0·0075 Wr	0·0015 Wr	12° 44′
8	0·0083 Wr	0·00416 Wr	0·0006 Wr	9° 3′
10	0·0054 Wr	0·0023 Wr	0·0003 Wr	7° 39′
12	0·00375 Wr	0·0142 Wr	0·00017 Wr	6° 15′

The ring girder should be designed for (*i*) bending moments at supports and centre of each span, (*ii*) shear force near each support, (*iii*) torsion.

In this connection our code of practice (IS 456 clause 48) has stipulated the following :

Equivalent shear. The equivalent shear S_e shall be calculated from the formula

$$S_e = S + 1{\cdot}6\frac{T}{b}$$

where S_e = Equivalent shear,

S = Shear,

T = Torsional moment,

b = breadth of beam.

The equivalent nominal shear stress q_{ve} is given by,

$$q_{ve} = \frac{S_e}{bd}$$

The equivalent nominal shear stress shall not exceed q_c max specified earlier.

Let q_c = permissible nominal shear stress.

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If the equivalent nominal shear stress q_{ve} does not exceed q minimum shear reinforcement shall be provided. If q_{ve} exceeds q_c the reinforcement for the beam shall be determined as follows.

The reinforcement for torsion shall consist of longitudinal and transverse reinforcement.

Longitudinal reinforcement. The longitudinal reinforcement shall be designed to resist an equivalent bending moment M_{e1} given by,

$$M_{e1} = M + M_t$$

where, M = bending moment at the cross-section

and,
$$M_t = T\left[\frac{1+\frac{D}{b}}{1.7}\right]$$

where T is the torsional moment, D is the overall depth of the beam and b is the breadth of the beam.

If the numerical value of M_t exceeds the numerical value of the moment M, longitudinal reinforcement shall be provided in the flexural compression face, such that the beam can also withstand an equivalent moment $M_{e2} = M_t - M$, the moment M_{e2} being taken as acting in the opposite sense of the moment M.

Transverse reinforcement. Two-legged close hoops enclosing the corner longitudinal bars shall have an area of cross-section A_v given by,

$$A_v = \frac{Tp}{b_1 d_1 t_w} + \frac{Sp}{2.5 d_1 t_w},$$

but, the total transverse reinforcement shall not be less than

$$\frac{(q_{ve} - q_c) b v}{t_w}$$

where, T = Torsional moment,

S = Shear,

p = Spacing of the stirrup reinforcement,

b_1 = centre to centre distance between corner bars in the direction of the width,

d_1 = centre to centre distance between corner bars in the direction of the depth,

b = breadth of the member,

t_w = permissible tensile stress in shear reinforcement,

q_{ve} = equivalent shear stress,

q_c = shear strength of concrete.

Design 125 (S.I.). *Fig. 467 shows an arrangement for an overhead tank. Assuming suitable working stresses design the tank to the center-line dimensions shown in the figure. The equivalent uniformly distributed load on the dome may be taken as 6000 N/metre².*

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Solution. Let *M* 15 concrete be used for the tank.

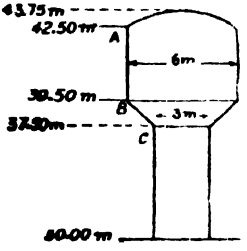

Fig. 467

For the sake of convenience the various loads for which the components of the tank should be designed are shown in Fig. 468.

(*i*) **Design of the dome**

Let the thickness of the dome be 15 *cm*.

Maximum thrust per *metre* run

$$= T_1 = \frac{wR}{1+\cos\theta}$$

In our case $w = 6000\ N/metre^2$

Rise of dome $= 1{\cdot}25\ m$.

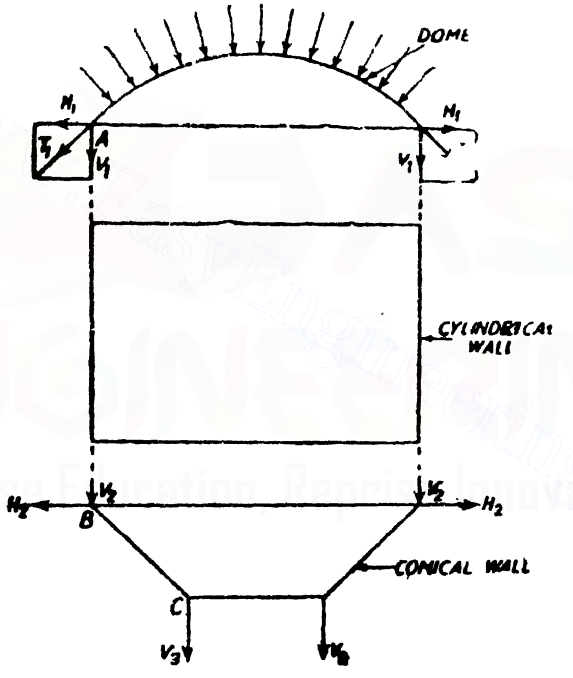

Fig. 468. Loads for which the components of the tanks are designed.

Let *R* be the radius of the dome

$$1{\cdot}25\,(2R - 1{\cdot}25) = 3^2$$

$$R = 4{\cdot}23\ m.$$

$$\sin\theta = \frac{3}{4{\cdot}23} = 0{\cdot}7092 \text{ and } \cos\theta = 0{\cdot}7049$$

$$\therefore \quad T_1 = \frac{6000 \times 4{\cdot}23}{1{\cdot}7049} = 14886{\cdot}5\ N/metre$$

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Intensity of stress due to the meridional thrust,

$$= \frac{14886{\cdot}5}{100 \times 15} = 9{\cdot}92 \ N/cm.^2$$

$$\text{Hoop stress} = f = \frac{wR}{t}\left[\cos\theta - \frac{1}{1+\cos\theta}\right]$$

$$= \frac{6000 \times 4{\cdot}23}{0{\cdot}15}\left[0{\cdot}7049 - \frac{1}{1{\cdot}7049}\right]$$

$$= 20026 \ N/metre^2$$

$$= 2 \ N/cm.^2$$

Similarly hoop stress may also be worked out at other levels. These stresses are low and hence a nominal reinforcement at 0·3% of gross area may be provided.

$\therefore$ Steel reinforcement per *metre*

$$= \frac{0{\cdot}3}{100} \times 100 \times 15 = 4{\cdot}50 \ cm^2$$

$\therefore$ spacing of 8 *mm.* ϕ bars

$$= \frac{0{\cdot}5 \times 100}{4{\cdot}5} = 11 \ cm. \ c/c.$$

(*ii*) Ring beam at A supporting the dome

Horizontal component of

$$T_1 = T_1 \cos\theta = 14886{\cdot}5 \times 0{\cdot}7049$$

$$= 10493 \ N/metre$$

$\therefore$ Hoop tension on the ring beam section

$$= 10493 \times \frac{6}{2} = 31479 \ N.$$

$\therefore$ Steel required $= \frac{31479}{10{,}000} = 3{\cdot}15 \ cm.^2$

Provide 4 bars of 10 mm. ϕ.

Size of the ring beam

Let the area of the section of the ring beam be $A \ cm.^2$

Equivalent area of concrete

$$= A + (m-1)\,A_s$$

$$= A + (13-1)\,3{\cdot}14 \ cm.^2$$

$$= A + 37{\cdot}68 \ cm.^2$$

Allowing a tensile stress of 120 $N/cm.^2$ on the equivalent concrete area,

$$\frac{31479}{A + 37{\cdot}68} = 120$$

$\therefore$ $A = 224{\cdot}6 \ cm.^2$

Provide 15 cm. × 15 cm. section.

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The reinforcement of the ring beam shall be wound by stirrups of 8 *mm*. ϕ bars @ 11 *cm. c/c.* These stirrups, after being wound round the reinforcement of the ring beam, should be embedded into the dome and lapped with the dome reinforcement.

(*iii*) **Cylindrical wall**

This will be designed for hoop tension.

Maximum water pressure intensity on the cylindrical wall

$$= 9810 \times 3 = 29430 \ N/metre^2$$

Maximum hoop tension per *metre* height

$$= 29430 \times \frac{6}{2} = 88290 \ N/metre$$

$$\therefore \qquad A_t = \frac{88290}{10{,}000} = 8{\cdot}83 \ cm.^2$$

Spacing of 8 *mm* ϕ bars

$$= \frac{0{\cdot}5 \times 100}{8{\cdot}8} = 5{\cdot}68 \ cm. \ say \ 5{\cdot}50 \ cm \ c/c.$$

Since the reinforcement will be provided near both the faces, the spacing near each face will be 11 cm. centres The reinforcement is in the form of rings.

Let the thickness of the wall be 15 *cm*

Tensile stress on the equivalent concrete area

$$= \frac{88290}{(15 \times 100) + (13-1)\dfrac{0{\cdot}5 \times 100}{5{\cdot}5}}$$

$$= 54{\cdot}9 \ N/cm^2$$

This is less than the permissible stress of 120 $N/cm.^2$

Check for compressive stress on the cylindrical wall

Vertical compressive of

$T_1 = V_1 = T_1 \sin\theta = 14886{\cdot}5 \times 0{\cdot}7092 = 10557 \ N$

Weight of wall per *metre*

$= 0{\cdot}15 \times 3 \times 25000 = 11250 \ N$

Weight of ring beam per *metre*

$= 0{\cdot}15 \times 0{\cdot}15 \times 25000 = 562 \ N$

Total vertical load per *metre* run $V_2 = 22369 \ N$

The component stress due to this vertical load

$$= \frac{22369}{15 \times 100} = 14{\cdot}91 \ N/cm.^2$$

This being low, only nominal vertical steel shall be provided at 0·3% of gross area, which in our case

$$= \frac{0{\cdot}3}{100} \times 100 \times 15 = 4{\cdot}5 \ cm.^2$$

Provide 8 mm. ϕ @ 22 cm. c/c near each face.

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(iv) Ring beam at B

Let T_2 be the thrust per *metre* run in the conical slab at the junction B. Resolving vertically, we have,

$$T_2 \sin \alpha = V_2$$

$$T_2 = \frac{V_2}{\sin \alpha}$$

$$\tan \alpha = \frac{3}{3/2} = \frac{4}{3}$$

$$\therefore \quad \sin \alpha = \frac{4}{5} \quad \text{and} \quad \cos \alpha = \frac{3}{5}$$

$$\therefore \quad T_2 = \frac{22369}{\left(\frac{4}{5}\right)} = 27956 \ N/m.$$

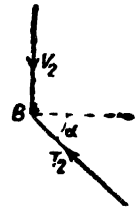

Fig. 469

Horizontal Component of T_2

$$= H_2 = T_2 \cos \alpha$$

$$= 27956 \times \frac{3}{5} = 16774 \ N/m.$$

This horizontal load H_2 will produce a hoop tension in the ring beam B.

Hoop tension due to H_2

$$= 16774 \times \frac{6}{2} = 50322 \ N$$

Let the ring beam be 20 *cm.* deep.

Water pressure on the ring beam

$$= 9810 \times 3 \times \frac{20}{100} = 5886 \ N/m.$$

Hoop tension due to water presure

$$= 5886 \times \frac{6}{2} = 17658 \ N$$

Total hoop tension $= 50322 + 17658 = 67980 \ N$

$$\therefore \quad A_s = \frac{67980}{20{,}000} = 6{\cdot}798 \ cm.^2$$

(Provide 4 bars of 16 mm. $\phi = 8{\cdot}04$ cm.^{2})

Size of the ring beam

Let the cross-sectional area of the ring be A cm.^{2}

Equivalent concrete area

$$= A + (m-1) A_s$$

$$= A + (13-1)\ 8{\cdot}04 \ cm.^2 = A + 96{\cdot}48 \ cm.^2$$

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Limiting the tensile stress on the equivalent concrete area to 120 $N/cm.^2$, we have

$$\frac{67980}{A+96{\cdot}48}=120 \quad \therefore \quad A=470{\cdot}02\ cm.^2$$

Provide a ring beam 24 *cm.* wide and 20 *cm.* deep.

(*iv*) Conical slab between the ring beam and the ring girder

The conical slab should be designed for (*a*) a hoop tension and (*b*) for a possibility of its spanning as a sloping slab from the ring beam at *B* to the ring girder at *C*.

Design for the hoop tension

The hoop tension of the whole conical slab section is given by,

Hoop load $= \frac{W_w+W_s}{2\pi}+\frac{W_w}{2\pi}\tan\alpha$

where W_w = Weight of water resting on the conical slab

W_s = weight of the conical slab

α = Inclination of the conical slab with the horizontal

Fig. 470

Area of the water section standing on the conical slab

$$=\frac{1{\cdot}5}{2}\left(3+5\right)=6\ sq.\ m.$$

$$\bar{x}=\frac{5+2\times3}{5+3}\times\frac{1{\cdot}5}{3}\ m.$$

$$=\frac{11}{8}\times\frac{1{\cdot}5}{3}=0{\cdot}688\ m.$$

Weight of water on the conical slab

$$=W_w=9810\,[6\times2\pi(1{\cdot}5+0{\cdot}688)]$$

$$=809190\ N$$

Weight of 15 *cm.* thick conical slab

$$=W_s=0{\cdot}15\times2{\cdot}5\left[2\pi\times\frac{3+1{\cdot}5}{2}\right]25000\ N$$

$$=132540\ N$$

Total hoop tension

$$=\frac{809190+132540}{2\pi}+\frac{809190}{2\pi}\times\frac{4}{3}=321600\ N$$

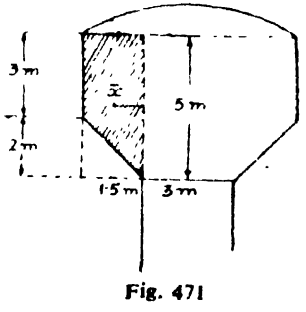

Fig. 471

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Total hoop steel

$$= \frac{321600}{10,000} = 32{\cdot}2 \text{ cm.}^2$$

Provide 16 bars of 18 mm. ϕ with 8 bars near each face.

Design for bending moment

Load per *metre* width of conical slab

$$= \frac{809190 + 132540}{2\pi \times \text{mean rudius}}$$

$$= \frac{941730}{2\pi \times 2{\cdot}25} = 66614 \text{ N}$$

Maximum bending moment for the conical slab

$$= \frac{66614 \times 1{\cdot}5}{8} \text{ Nm}$$

$$= 12490 \text{ Nm.} = 1249000 \text{ N cm.}$$

Axial compression

$$= T_2 = 27956 \text{ N}$$

Effective depth to the centre of 16 *mm.* ϕ bars with a clear cover of 25 *mm.* $= 15 - 2{\cdot}5 - 0{\cdot}8 = 11{\cdot}7$ *cm.*

Resultant B.M. $= M + Tx$

$$= 1249000 + 27956\ (11{\cdot}7 - 7{\cdot}5)$$

$$= 1366415 \text{ N cm.}$$

$$\therefore \quad A_t = \frac{1366415}{12500 \times 0{\cdot}86 \times 11{\cdot}7} \text{ cm.}^2$$

$$= 10{\cdot}86 \text{ cm.}^2$$

Spacing of 16 *mm.* ϕ bars

$$= \frac{2{\cdot}01 \times 100}{10\ 86} = 18{\cdot}5 \text{ cm.}$$

Provide 16 mm. ϕ bars @ 18 cm. c/c.

(v) Circular slab

This slab will be designed as supported all round the edge over the circular girder. If the load per *sq. metre* on the slab is q *N/metre*^{2}, the maximum bending moment at the centre will be $\frac{3}{16} qr^2$ per *metre* width of slab. An allowance may also be made for a hogging moment at the ends.

In our case, the loading on the circular slab will consist of the following :

Weight of water $= 9810 \times 5 = 49050$ *N/metre*^{2}
Weight of slab (20 *cm.*) $= 250 \times 20 = 5000$ *N/metre*^{2}

Total $= 54050$ *N/metre*^{2}

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Maximum sagging moment per *metre* width

$$= \frac{3}{16} \times 54050 \times 1{\cdot}5^2 \times 100 \; N\,cm.$$

$$= 2280230 \; N\,cm.$$

With $c = 700 \; N/cm.^2$, $t = 12500 \; N/cm.^2$, and $m = 13$, and $k = 126{\cdot}8$

equating M.R. to B.M.,

$$126{\cdot}8 \times 100 \; d^2 = 2280230$$

$$\therefore \quad d = 13{\cdot}41 \; cm.$$

If 16 *mm.* ϕ bars be provided at a clear cover of 25 *mm*, effective depth available $= 20 - 2{\cdot}5 - 1{\cdot}6 - 0{\cdot}8 = 15{\cdot}1 \; cm.$

$$\therefore \quad A_s = \frac{2280230}{12500 \times 0{\cdot}86 \times 15{\cdot}1} = 14{\cdot}04 \; cm.^2$$

Spacing of 16 *mm.* ϕ bars

$$= \frac{2{\cdot}01 \times 100}{14{\cdot}04} = 14{\cdot}3 \; cm.$$

Provide 16 mm. ϕ *bars @ 14 cm. c/c.*

(*vi*) Circular girder (Ring girder)

The loading on the circular girder consists of the following :

Weight of water	$= 809190 + 9810 \times \frac{\pi}{4}(3)^2 5 = 1155900 \; N$	
Dome and side walls	$= 22369 \times \pi \times 6$	$= 421650 \; N$
Ring beam at B	$= 0{\cdot}24 \times 0{\cdot}2 \times 25000 \times \pi \times 6$	$= 22620 \; N$
Conical slab		$= 132540 \; N$
Circular slab	$= \frac{\pi}{4}(3)^2 \times 0{\cdot}2 \times 25000$	$= 35340 \; N$
DL of ring girder (30 *cm.* × 50 *cm.*)	$= 0{\cdot}3 \times 0{\cdot}5 \times 25000 \times \pi \times 3$	$= 35340 \; N$
	Total	1803390 N

Let us provide 6 columns.

Maximum negative B.M.

$$= 0{\cdot}0148 \; Wr$$

$$= 0{\cdot}0148 \times 1803390 \times 1{\cdot}5 \; Nm.$$

$$= 40035 \; Nm.$$

Maximum positive B.M.

$$= 0{\cdot}0075 \; Wr$$

$$= 0{\cdot}0075 \times 1803390 \times 1{\cdot}5 \; Nm.$$

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Torsion $= 0{\cdot}0015\ Wr$
$= 0{\cdot}0015 \times 1803390 \times 1{\cdot}5\ Nm$
$= 4058\ Nm.$

S.F. at support $= \frac{W}{2} \cdot \frac{1}{\text{number of columns}}$
$= \frac{1803390}{2 \times 6} = 150282\ N$

Design at support section

$M = 40035\ Nm.$
$T = 4058\ Nm.$
$S = 150282\ N$

Equating M.R. to B.M.,

$85 \times 30\ d^2 = 4003500 \quad \therefore \quad d = 39{\cdot}5\ cm.$

Effective depth available

$= 50 - 6 = 44\ cm$

Equivalent S.F. $= S_e = S + 1{\cdot}6\ \frac{T}{b}$

$= 150282 + 1{\cdot}6 \times \frac{405800}{30}\ N$

$= 171925\ N$

Equivalent nominal shear stress

$= q_{ve} = \frac{S_e}{bd}$

$= \frac{171925}{30 \times 44} = 130\ N/cm.^2$

$= 1{\cdot}3\ N/mm.^2$

Longitudinal reinforcement

Equivalent B.M. $= M_{e1} = M + M_t$

where $M_t = f\ \frac{\left[1 + \frac{D}{b}\right]}{1{\cdot}7}$

$= 405800\ \frac{\left[1 + \frac{50}{30}\right]}{1{\cdot}7}$

$= 636550\ N\ cm.$

$\therefore \quad M_{e1} = M + M_t$
$= 4003500 + 636550 = 4640050\ N\ cm.$

$A_t = \frac{4640050}{12500 \times 0{\cdot}86 \times 44} = 9{\cdot}8\ cm.^2$

Provide 4 bars of 18 mm. ϕ (10·18 cm.²)

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Transverse reinforcement

Fig. 472 shows the section of the beam, showing the position of reinforcement.

Distance between centre of corner bars parallel to the width $\Big\}$ $= b_1 = 30 - 2 \times 4 = 22$ *cm.*

Distance between centres of corner bars parallel to the depth $\Big\}$ $= d_1 = 50 - 2 \times 4 = 42$ *cm.*

Area of section of stirrups

$$= A_w = \frac{Tp}{b_1 d_1 t_w} + \frac{Sp}{2{\cdot}5\, d_1 t_w}$$

Let 10 *mm.* ϕ two-legged stirrups be provided.

$$A_w = 2 \times 0{\cdot}79 = 1{\cdot}58 \text{ cm.}^2$$

$$1{\cdot}58 = \left[\frac{405800}{22 \times 42 \times 1400} + \frac{150282}{2{\cdot}5 \times 42 \times 14000} \right] p$$

$$1{\cdot}58 = [0{\cdot}0314 + 0{\cdot}1022]\, p$$

$$p = \frac{1{\cdot}58}{0{\cdot}1336} = 11{\cdot}8 \text{ cm.}$$

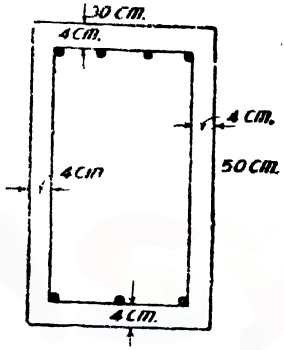

Fig 472

Provide 2-legged 10 mm. ϕ stirrups @ 10 cm. c/c.

Steel for sagging moment

$$\frac{2028800}{12500 \times 0{\cdot}86 \times 46} = 4{\cdot}10 \text{ cm.}^2$$

Provide 3 bars of 14 mm. ϕ (4·62 cm.²)

(vii) Columns

The columns should be designed for direct loads coming upon them and for the bending moments caused by wind load. We shall now discuss the wind load effect on the columns.

Let the wind load intensity on a flat vertical surface be $p = 1500$ *N/metre*²

Intensity of wind pressure on projected dome surface

$= 0{\cdot}5\, p = 0{\cdot}5 \times 1500 = 750$ *N/metre*²

Intensity of wind pressure on cylindrical and conical surfaces

$= 0{\cdot}75\, p = 0{\cdot}75 \times 1500$

$= 1125$ *N/metre*²

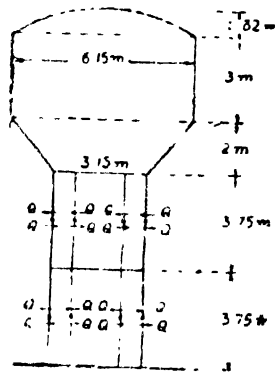

Fig. 473

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Fig. 473 shows the external dimensions of the tank.

Segmental area $= \frac{4h}{3}\sqrt{\frac{l^2}{4}+\frac{2h^2}{3}}$

$= \frac{4\times 1{\cdot}32}{3}\sqrt{\frac{6{\cdot}15^2}{4}+\frac{2\times 1{\cdot}32^2}{3}}$ *metre*^{2}

$= 5{\cdot}73$ *metre*^{2}

The centroid of the area is approximately at a height of $\frac{5}{12}h = \frac{5}{12}\times 1{\cdot}32 = 0{\cdot}55$ *m.* above the base of the segment.

The wind loads acting on this various components of the tank are given below :

Tank component	*Area* (*metre*^{2})	*Wind load* (*N*)
Dome	5·73	$5{\cdot}73\times 750 = 4300 = P_1$
Cylindrical wall	$6{\cdot}15\times 3 = 18{\cdot}45$	$18{\cdot}45\times 1125 = 20760 = P_2$
Conical wall	$\frac{6{\cdot}15+3{\cdot}15}{2}\times 2{\cdot}1 = 9{\cdot}30$	$9{\cdot}30\times 1125 = 10460 = P_3$
Total wind load		35520 *N*

Let the columns be braced at their mid-height. It will be assumed that points of contraflexure occur at the middle point of each vertical panel of each column.

Let the horizontal shear just above and below each point of contraflexure be *Q Newtons.*

See Fig. 473 $\therefore \quad 6Q = 35520\ N$

$\therefore \quad Q = 5920\ N$

B.M. at the top of the column, just above and below the brace and at the base of the column

$= 5920\times\frac{3{\cdot}75}{2} = 11100\ Nm.$

Column section at top

Load on one column transferred from ring girder

$= \frac{1803390}{6} = 300565\ N$

Let the columns be made 30 *cm.* in diameter.

Let us try 8 bars of 32 mm. ϕ (Area of 1 bar = 8·04 cm.^{2}*) provided at an effective cover of 5·5 cm.*

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Now onwards M15 concrete may be provided.

Hence adopt $c = 500\ N/cm.^2$, $t = 14000\ N/cm.^2$ and $m = 18$

Equivalent concrete area

$$A_e = \frac{\pi}{4}(30)^2 + (18-1)8{\cdot}04 \times 8\ cm.^2$$

$$= 1800\ cm.^2$$

Equivalent moment to inertia

$= I_e =$ Half the equivalent polar moment of inertia

$$= \frac{1}{2}\left[\frac{\pi \times 30^4}{32} + 17 \times 8{\cdot}04 \times 8 \times (15-5{\cdot}5)^2\right] cm.^4$$

$$= 89100\ cm.^4$$

Extreme stress intensity

$$= \frac{3{\cdot}0565}{1800} \pm \frac{11100 \times 100}{89100} \times 15\ N/cm.^2$$

$$= 166{\cdot}98 \pm 186{\cdot}86$$

$$= 353{\cdot}84\ N/cm.^2 \text{ or } -20{\cdot}02\ N/cm.^2$$

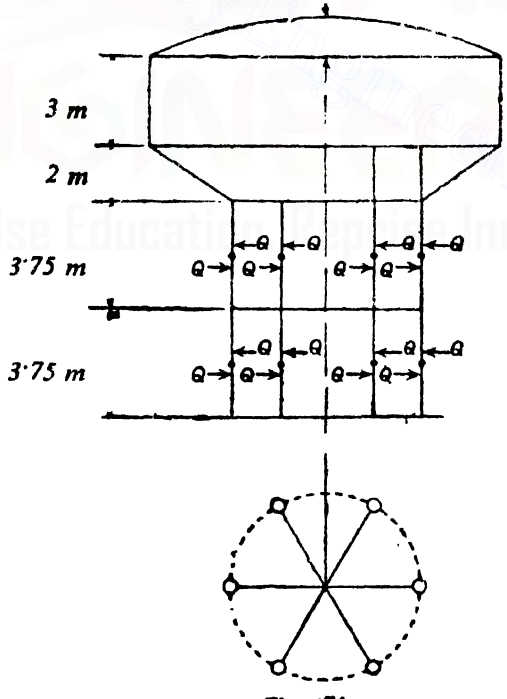

Fig. 474

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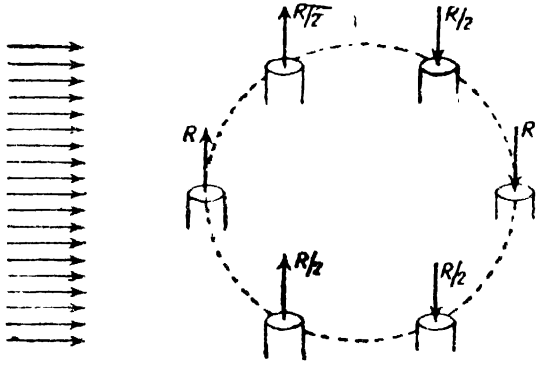

Fig. 475

Direct load transferred to the columns due to wind pressure at the level of the upper points of contraflexure.

Let the direct load transmitted to the column most distant from the centre of column be *R Newton*. Taking moments about the upper points of contraflexure.

$$4300\left[0{\cdot}55+3+2+\frac{3{\cdot}75}{2}\right]+20760\left[1{\cdot}5+2+\frac{3{\cdot}75}{2}\right]$$

$$+10460\left[\frac{3{\cdot}15+2\times6{\cdot}15}{3{\cdot}15+6{\cdot}15}\times\frac{2}{3}+\frac{3{\cdot}75}{2}\right]$$

$$=3R\times r=3R\times\frac{3}{2}$$

$$31928+111585+31197=174710=\frac{9R}{2}$$

$$\therefore \qquad R=38825\ N.$$

Column section just above the brace level

Load from tank $=300565\ N$

D.L. of column $=\frac{\pi}{4}(0{\cdot}3)^2\times3{\cdot}75\times25000=\ 6627\ N$

Direct load due to wind $=\ 38825\ N$

Total load $=346017\ N$

Extreme stress intensity

$$=\frac{346017}{1800}\pm\frac{11100\times100}{89100}\times15$$

$$=192{\cdot}23\pm186{\cdot}86\ N/cm.^2$$

$$=379{\cdot}09\ N/cm.^2 \text{ or } 5{\cdot}37\ N/cm.^2$$

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Column section just below the brace level

Load from above $= 346017\ N$

D.L. of one brace (30 *cm.* × 30 *cm.*).

$0{\cdot}3 \times 0{\cdot}3 \times 1{\cdot}5 \times 25000 = 3375\ N$

Total $= 349392\ N$

Extreme stress intensity

$$= -\frac{349392}{1800} \pm \frac{11100 \times 100}{89100} \times 15$$

$$= 194{\cdot}11 \pm 186{\cdot}86\ N/cm^2$$

$$= 380{\cdot}97\ N/cm.^2 \text{ or } 7{\cdot}25\ N/cm.^2$$

Direct load transferred to the column due to wind pressure at the level of the lower points of contraflexure.

Let the direct load in the column most distant from the centre of the column group be R' *Newton*. Taking moments about the lower points of contraflexure, we have,

$$4300\left[0{\cdot}55+5+3{\cdot}75+\frac{3{\cdot}75}{2}\right]+20760\left[1{\cdot}5+2+3{\cdot}75+\frac{3{\cdot}75}{2}\right]$$

$$+10460\left[1{\cdot}10+3{\cdot}75+\frac{3{\cdot}75}{2}\right]=3R'r=3R'\times\frac{3}{2}$$

$$\therefore \quad 48052{\cdot}5+189435+70343{\cdot}5=\frac{9R'}{2}$$

$$\frac{9R'}{2}=307831$$

$$\therefore \quad R'=68407\ N$$

Column section at the bottom

Load from tank $= 300565\ N$

D.L. column $6627 \times 2 = 13254\ N$

D.L. of brace $= 3375\ N$

Direct load due to wind $= 68407\ N$

Total $= 385601\ N$

Extreme stress intensity

$$= -\frac{385601}{1800} \pm \frac{11100 \times 100}{89100} \times 15$$

$$= 214{\cdot}22 \pm 186{\cdot}86\ N/cm.^2$$

$$= 401{\cdot}08\ N/cm.^2 \text{ or } 27{\cdot}36\ N/cm.^2$$

These stresses are within safe limits.

(*viii*) Braces

The brace is designed for a bending moment equal to sum of the moments in the column just above and below the brace level.

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$\therefore$ B.M. for the brace

$$= 11100 \times 2 = 22200 \ Nm.$$

Let the brace section be 30 *cm.* × 30 *cm.*

The section will be designed as a doubly reinforced beam section on steel beam theory. Let the effective cover to reinforcement be = 4 *cm.*

$\therefore$ Distance between centres of compressive and tensile steel

$$= 30 - 8 = 22 \ cm.^2$$

$\therefore$ $$A_c = A_t = \frac{22200 \times 100}{14000 \times 22} = 7{\cdot}21 \ cm.^2$$

Provide 4 bars of 16 mm. ϕ at the top and 4 similar bars at the bottom.

S.F. for the brace

$$= S = \frac{\text{B.M. for brace}}{\text{half span of brace}}$$

$$= \frac{22200}{\left(\frac{1{\cdot}5}{2}\right)} = 29600 \ N$$

Nominal shear stress

$$= q_v = \frac{S}{bd} = \frac{29600}{30 \times 26} = 38 \ N/cm.^2$$

$$= 0{\cdot}38 \ N/mm.^2$$

Percentage of tensile steel

$$= \frac{4 \times 2{\cdot}01}{30 \times 26} \times 100 = 0{\cdot}9\%$$

For 0·9% steel $q_c = 0{\cdot}36 \ N/mm.^2$

$\therefore$ q_v is almost equal to q_c

Hence provide nominal shear reinforcement.

Provide two-legged 8 mm. ϕ stirrups @ 22 cm. c/c.

(ix) Foundation

Weight of tank and girder	= 1803390 *N*
Weight of 6 column : 13254 × 6	= 79524 *N*
Weight of 6 braces : 3375 × 6	= 20250 *N*
	1903164 *N*
Weight of foundation (10%)	= 190316 *N*
	Total = 2093480 *N*
Safe bearing capacity of the soil	**= 120 *kN/metre*²**
	= 120,000 *N/metre*²

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∴ Area required for the foundation

$$= \frac{2093480}{120{,}000} = 17{\cdot}45 \ m.$$

Let D be the diameter of the base,

$$\frac{\pi D^2}{4} = 17{\cdot}45 \qquad \therefore \quad D = 4{\cdot}71 \ m.$$

Let us provide a diameter of 5·50 *metre* to make allowance for uneven stress distribution caused by wind loads.

Let the depth of the ring girder of the foundation be 60 *cm.*

Moment of wind pressure about the base

$$= 4300(0{\cdot}55 + 12{\cdot}50 + 0{\cdot}60) + 20760(1{\cdot}5 + 9{\cdot}5 + 0{\cdot}6) + 10460(1{\cdot}1 + 7{\cdot}5 + 0{\cdot}6) \ Nm$$
$$= 58695 + 240816 + 96232 = 395743 \ Nm.$$

Moment of inertia of a circle of 5·5 *metre* diameter

$$= \frac{\pi \times 5{\cdot}5^4}{64} = 44{\cdot}93 \ metre^4$$

Hence the pressure at the base is given by

$$p = \frac{2093480 \times 4}{\pi \times 5{\cdot}5^2} \pm \frac{395743}{44{\cdot}93} \times \frac{5{\cdot}5}{2}$$
$$= 88115 \pm 24222 \ N/metre^2$$

$$\therefore \quad p_{max} = 112337 \ N/metre^2$$

and $\quad p_{min} = 63893 \ N/metre^2$

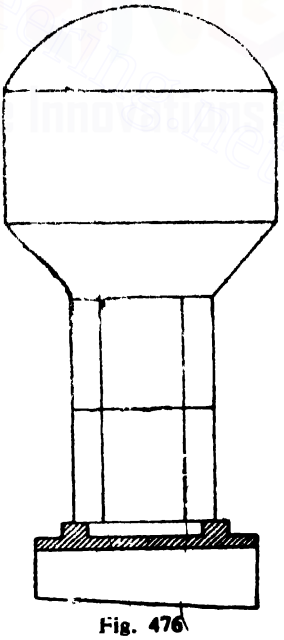

Fig. 476

Net upward pressure on the foundation can be determined by subtracting from the above the pressure due to dead load on the foundation.

Pressure due to D.L. of foundation

$$= \frac{1901{\cdot}6 \times 4}{\pi \times 5{\cdot}5^2} = 8010 \ N/metre^2$$

The maximum pressure

$$= 112337 - 8010 = 104327 \ N/metre^2$$

The minimum pressure

$$= 63893 - 8010 = 55883 \ N/metre^2$$

For design purposes, the upward equivalent uniform pressure may be taken as the average of the pressure at the centre and the maximum pressure.

Pressure at the centre

$$\frac{104327 + 55883}{2} = 80105 \ N/m.^2$$

∴ **Equivalent uniform pressure**

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$$= \frac{80105 + 104327}{2} = 92216 \ N/m.^2$$

Equivalent upward load on the bottom ring girder

$$= W = 92216 \times \frac{\pi \times 5{\cdot}5^2}{4} = 2190900 \ N$$

Design of the bottom ring girder

Radius to the centre line of the ring girder = 1·50 *metre*

∴ B.M. at support = 0·0148 *Wr* (*sagging*)

$= 0{\cdot}0148 \times 2190900 \times 1{\cdot}5 \ Nm.$

$= 48638 \ Nm. = 4863800 \ N_{\cdot} cm.$

B.M. at the centre of each span

$= 0{\cdot}0075 \ Wr$ (*hogging*)

$= 0{\cdot}0075 \times 2190900 \times 1{\cdot}5 \ Nm.$

$= 24648 \ Nm. = 2464800 \ N \ cm.$

Torsion $= 0{\cdot}0015 \ Wr$

$= 0{\cdot}0015 \times 2190900 \times 1{\cdot}5 \ N \ m.$

$= 4930 \ N \ m. = 493000 \ N \ cm.$

Design of the support section

$M = 4863800 \ N \ cm.$

$T = 493000 \ N \ cm.$

S.F. $= S = \frac{\text{Total load}}{2 \times \text{Number of columns}} = \frac{2190900}{2 \times 6}$

$= 182575 \ N$

Equating M.R. to B.M.

$= 85 \times 30 \ d^2 = 4863800$

$d = 43{\cdot}7 \ cm.$

Effective depth available

$= 60 - 8 = 52 \ cm.$

Equivalent S.F $= S_e = S + 1{\cdot}6 \ \frac{T}{b}$

$= 182575 + 1{\cdot}6 \times \frac{493000}{30} \ N$

$= 208870 \ N$

Equivalent nominal shear stress

$= q_{ve} = \frac{S_e}{bd} = \frac{208870}{30 \times 52} \ N/cm.^2$

$= 134 \ N/cm.^2 = 1{\cdot}34 \ N/mm.^2$

Longitudinal reinforcement

Equivalent B.M. $= M_{e1} = M + M_t$

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where, $$M_t = T\frac{\left[1+\frac{D}{b}\right]}{1{\cdot}7}$$

$$= 493000\frac{\left[1+\frac{60}{30}\right]}{1{\cdot}7} = 870000 \text{ N cm.}$$

$$\therefore \quad M_{e1} = M + M_t = 4863800 + 870000 \text{ N cm.}$$

$$= 5733800 \text{ N cm.}$$

$$\therefore \quad A_t = \frac{5733800}{14000 \times 0{\cdot}87 \times 52} = 9{\cdot}05 \text{ cm.}^2$$

Provide 4 bars of 18 mm. ϕ (10·16 cm.^{2})

Transverse reinforcement

Fig. 477 shows the section of the beam showing the position of reinforcement.

Distance between the centres of corner bars parallel to the width $= b_1 = 30 - 2 \times 4 = 22$ cm.

Distance between the centres of corner bars parallel to the depth $= d_1 = 60 - 2 \times 8 = 44$ cm.

Area of section of stirrups

$$= A_v = \frac{Tp}{b_1 d_1 t_w} + \frac{Sp}{2{\cdot}5\, d_1\, t_w}$$

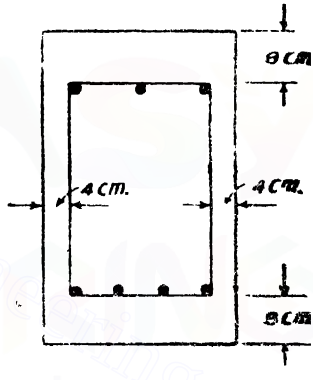

Fig. 477

Let 10 *mm.* ϕ two-legged stirrups be provided.

$$A_v = 2 \times 0{\cdot}79 = 1{\cdot}58 \text{ cm.}^2$$

$$\therefore \quad 1{\cdot}58 = \left[\frac{493000}{22 \times 44 \times 14000} + \frac{182575}{2{\cdot}5 \times 44 \times 14000}\right] p$$

$$1{\cdot}58 = [0{\cdot}0364 + 0{\cdot}1186]\, p$$

$$\therefore \quad p = \frac{1{\cdot}58}{0{\cdot}155} = 10{\cdot}2 \text{ cm.}$$

Provide 2-legged 10 mm ϕ stirrups @ 10 cm. c/c.

Steel for hogging moment

$$= \frac{2464800}{14000 \times 0{\cdot}87 \times 52} \text{ cm.}^2$$

$$= 3{\cdot}89 \text{ cm.}^2$$

Provide 3 bars of 14 mm. ϕ (4·62 cm.^{2})

Design 126. *Fig. 478 shows an arrangement for an overhead tank. Assuming suitable working stresses design the tank to the centre line*

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dimensions shown in the figure. The equivalent uniformly distributed load on the dome may be taken as 590 kg/metre² of dome surface. Weight of R.C.C. may be taken as 2300 kg/metre.³

Solution. Use *M* 200 concrete.

For the sake of convenience the various loads for which the components of the tank should be designed are shown in Fig. 478.

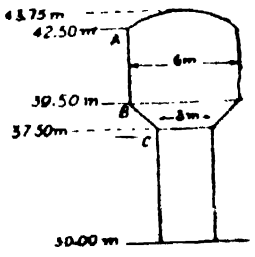

Fig. 478.

(i) Design of the dome

Let the thickness of the dome be 15 *cm*.

Maximum thrust per *metre* run

$$= T_1 = \frac{wR}{1+\cos\theta}$$

In our case

$w = 590$ *kg./m.²*

Rise of dome

$= 1{\cdot}25$ *m.*

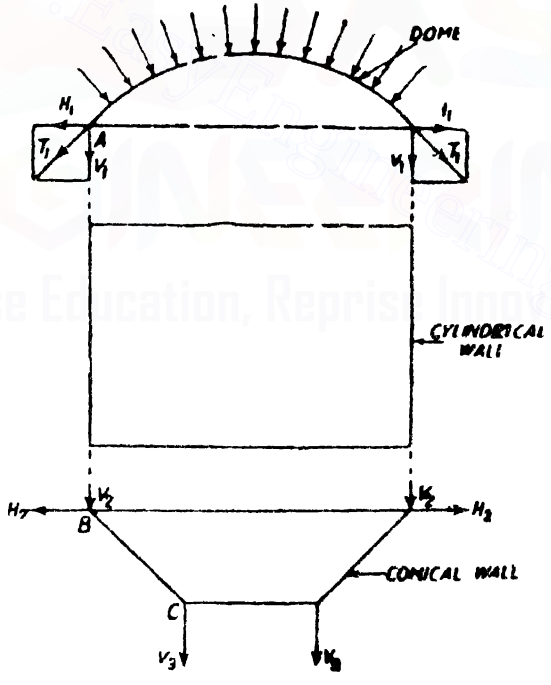

Fig. 479 Loads for which the components of the tanks are designed.

Let R be the radius of the dome

$$1{\cdot}25\,(2R - 1{\cdot}25) = 3^2$$

$$\therefore \quad R = 4{\cdot}23 \text{ m.}$$

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$$\therefore \quad \sin\theta = \frac{}{4{\cdot}23} = 0{\cdot}7092$$

and $\cos\theta = 0{\cdot}7049$

$$\therefore \quad T_1 = \frac{590 \times 4{\cdot}23}{1{\cdot}7049}\ kg./metre\ run$$

$$= 1464\ kg./m.$$

Intensity of stress due to the thrust

$$= \frac{1464}{100 \times 15} = 0{\cdot}98\ kg./cm.^2$$

Hoop stress $$= -\frac{wR}{t}\left[\cos\theta - \frac{1}{1+\cos\theta}\right]$$

$$= \frac{590 \times 4{\cdot}23}{0{\cdot}15}\left[0{\cdot}7049 - \frac{1}{1{\cdot}7049}\right] kg./m.^3$$

$$= 1968\ kg./m.^2$$

$$= \frac{1968}{100 \times 100}\ kg./cm.^2$$

$$= 0{\cdot}1968\ kg./cm^2$$

Similarly hoop stress may also be worked out at other levels.

These stresses are low and hence a nominal reinforcement at 0·3% of gross area may be provided.

$\therefore$ Steel reinforcement per *metre*

$$= \frac{0{\cdot}3}{100} \times 100 \times 15 = 4{\cdot}5\ cm.^2$$

$\therefore$ Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{4{\cdot}5} \quad 11\ \text{cm. c/c.}$$

(iii) Ring beam at A supporting the dome

Horizontal component of T_1

$$= T_1 \cos\theta$$

$$= 1464 \times 0{\cdot}7049\ kg./m.$$

$$= 1032\ kg./m.$$

$\therefore$ Hoop tension on the ring beam

$$= 1032 \times \frac{6}{2}\ kg. = 3096\ kg.$$

Steel required $$= \frac{3096}{1000}\ cm.^2 = 3{\cdot}096\ cm.^2$$

Provide 4 bars of 10 mm. diameter (3.14 cm.²)

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Size of the ring beam

Let the area of section of the ring beam be A *cm.*^{2}

Equivalent area of concrete

$$= A + (m-1)\, A_s$$
$$= A + (13-1)\, 3{\cdot}14 \text{ cm.}^2$$
$$= A + 37{\cdot}68 \text{ cm.}^2$$

Allowing the tensile stress on the equivalent area of concrete at 12 *kg./cm.*^{2},

We have, $$\frac{3096}{A+37{\cdot}6} = 12$$

$$\therefore \quad A = 220{\cdot}32 \text{ cm.}^2$$

Let us provide 15 cm. × 15 cm. section.

The reinforcement of the ring beam shall be wound by stirrups of *8 nm. diameter at 11 cm. centres.* These stirrups, after being wound round the reinforcement of the ring beam, should be embedded into the dome and lapped with the dome reinforcement.

(*iii*) Cylindrical wall

This will be designed for hoop tension. Maximum water pressure $= 1000 \times 3 = 3000$ *kg./m.*^{2}

$\therefore$ Max. hoop tension per *metre* height

$$= 3000 \times \frac{6}{2} = 9000 \text{ kg./m.}$$

$$\therefore \quad A_s = \frac{9000}{1000} = 9 \text{ cm.}^2$$

$\therefore$ Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{9} = 5{\cdot}5 \text{ cm.}$$

Since the reinforcement will be provided near both the faces the spacing near each face will be 11 cm. centres. The reinforcement is in the form of rings.

Let the thickness of the wall be 15 *cm.*

Tensile stress on the equivalent concrete area

$$\frac{9000}{15 \times 100 + (13-1)\, \frac{0{\cdot}5 \times 100}{5{\cdot}5}} \text{ kg./cm.}^2$$

$$= 5{\cdot}59 \text{ kg /cm.}^2$$

This is less than the permissible stress of 12 *kg./cm.*^{2}

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Check for the compressive stress on the cylindrical wall.

Vertical component of $T_1 = V_1 = T_1 \sin \theta$

$= 1464 \times 0{\cdot}7092$ $= 1039$ *kg.*

Weight of wall/*metre* $= 0{\cdot}15 \times 3 \times 2300$ $= 1035$ *kg.*

Weight of ring beam/*metre*

$= 0{\cdot}15 \times 0{\cdot}15 \times 2300$ $= 52$ *kg.*

Total vertical load/*metre* $V_2 = 2126$ *kg.*

The compressive stress due to this vertical load

$$= \frac{2126}{15 \times 100} = 1{\cdot}42 \text{ kg./cm.}^2$$

This being low, only nominal vertical steel shall be provided at 0·3% of gross area which in our case

$$= \frac{0{\cdot}3}{100} \times 100 \times 15$$

$$= 4{\cdot}5 \text{ cm.}^2$$

Provide 8 mm dia. bars at 22 cm c/c near each face.

(*v*) **Ring-Beam at B.** Let T_2 be the thrust *metre* run in the conical slab at the junction *B*. Resolving vertically, we have,

Fig. 480.

$$T_2 \sin \alpha = V_2$$

$$\therefore \quad T_2 = \frac{V_2}{\sin \alpha}$$

$$\tan \alpha = \frac{2}{3/2} = \frac{4}{3}$$

$$\therefore \quad \sin \alpha = \frac{4}{5}$$

and $$\cos \alpha = \frac{3}{5}$$

$$\therefore \quad T_2 = \frac{2126}{\left(\frac{4}{5}\right)} \text{ kg.} = 2658 \text{ kg./m.}$$

Horizontal component of T_2

$$= H_2 = T_2 \cos \alpha$$

$$= 2658 \times \frac{3}{5} \text{ kg.}$$

$$= 1595 \text{ kg./m.}$$

This horizontal load H_2 will produce a hoop tension in the ring beam *B*.

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Hoop tension due to H_2

$$= 1595 \times \frac{6}{2} \text{ kg.} = 4785 \text{ kg.}$$

Let the ring beam be 20 *cm.* deep.

Water pressure on the ring beam

$$= 1000 \times 3 \times \frac{20}{100} \text{ kg./m.} = 600 \text{ kg./m.}$$

Hoop tension due to this water pressure

$$= 600 \times \frac{6}{2} \text{ kg.} = 1800 \text{ kg.}$$

Total hoop tension $= 4785 + 1800$ *kg.*

$$= 6585 \text{ kg}$$

$$\therefore \quad A_t = \frac{6585}{1000} \text{ cm.}^2 = 6{\cdot}585 \text{ cm.}^2$$

Provide 4 bars of 16 mm. diameter (8·04 *cm.*^{2})

Size of the ring beam

Let the cross-sectional area of the ring beam be A *cm.*^{2}

Equivalent concrete area

$$= A + (m-1)\, A_t$$
$$= A + (13-1)\, 8{\cdot}04 \text{ cm.}^2$$
$$= A + 96{\cdot}48 \text{ cm.}^2$$

Limiting the tensile stress on the equivalent area of concrete to 12 *kg./cm.*^{2}, we have

$$\frac{6585}{A + 96{\cdot}48} = 12$$

$$\therefore \quad A = 452{\cdot}27 \text{ cm.}^2$$

Let the ring beam be 23 cm. wide and 20 cm. deep.

(*vi*) **Design of the conical slab**

The conical slab should be designed for (*a*) a hoop tension and (*b*) for a possibility of its spanning as a sloping slab from the ring beam at *B* to the ring girder at *C*.

Design for the hoop tension

The hoop tension on the whole conical slab section is given by,

$$\text{Hoop load} = \frac{W_w + W_s}{2\pi} + \frac{W_w}{2\pi} \tan \alpha$$

where W_w = Weight of water resting on the conical slab

W_s = Weight of the conical slab

α = Inclination of the conical slab with the horizontal

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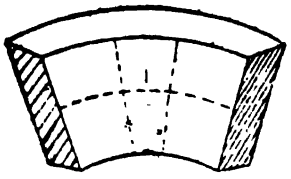

Fig. 481

Area of the water section standing on the conical slab

$$= \frac{15}{2}(3+5) = 6 \text{ sq. m.}$$

$$\bar{x} = \frac{5+2\times 3}{5+3} \times \frac{1.5}{3} \text{ m.}$$

$$= \frac{11}{8} \times \frac{1.5}{3} = 0.688 \text{ m.}$$

$\therefore$ Weight of water on the conical slab

$$= W_w = 1000\,[6 + 2\pi\,(1.5 + 0.688)]$$

$$= 82500 \text{ kg.}$$

Weight of 15 *cms.* thick conical slab

$$= W_s = 0.15 \times 2.5 \left[2\pi \times \frac{3+1.5}{2} \right] \times 2300 \text{ kg.}$$

$$= 12190 \text{ kg.}$$

$\therefore$ Total hoop tension

$$= \frac{82500 + 12190}{2\pi} + \frac{82500}{2\pi} \times \frac{4}{3}$$

$$= 32590 \text{ kg.}$$

$\therefore$ Total hoop steel

$$\frac{32590}{1000} = 32.5 \text{ cm.}^2$$

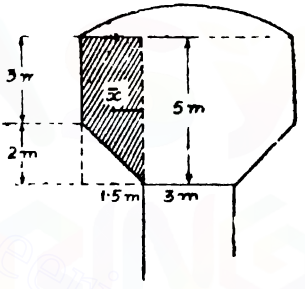

Fig. 482

Provide 16 bars of 18 mm. dia. (40·72 cm.²) in all, with 8 bars near each face.

Design for bending moment

Local per *metre* width of the conical slab

$$= \frac{82500 + 12190}{2\pi \times \text{mean radius}}$$

$$= \frac{94690}{2\pi \times 2.25} = 6700 \text{ kg.}$$

$\therefore$ Max. B.M. for the conical slab

$$= \frac{6700 \times 1.5}{8} \times 100 \text{ kg. cms.}$$

$$= 125600 \text{ kg. cms.}$$

Axial compression

$$= T_2 = 2658 \text{ kg.}$$

Effective depth to the centre of 16 *mm.* dia. bars with a clear cover of 25 *mm.* $= 15.00 - 2.50 - 0.80 = 11.7$ *cms.*

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∴ **Resultant bending moment**

$$= 125600 + 2658\,(11{\cdot}70 - 7{\cdot}50)\ kg.\ cm.$$

$$= 136760\ kg.\ cm.$$

$$\therefore \quad A_t = \frac{136760}{1250 \times 0{\cdot}86 \times 11{\cdot}7}\ cm.^2$$

$$= 10{\cdot}87\ cm.^2$$

∴ **Spacing of 16 *mm.* dia. bars**

$$= \frac{2{\cdot}01 \times 100}{10{\cdot}87}$$

$$= 18{\cdot}48\ cms.$$

Let us provide 16 mm. dia. bars at 18 cms. centres.

(*vi*) Design of the circular slab

This slab will be designed as supported all round the edge over the circular girder. If the load per *sq. metre* on the slab is *q kg.* per *sq. m.*, the maximum bending moment at the centre will be $\frac{3}{16}\,qr^2$ per *metre width* of the slab. An allowance will also be made for a possible hogging bending moment at the ends.

In our case the load on the circular slab will consist of the following :

Weight of water $= 1000 \times 5$	$= 5000\ kg./m.^2$
Weight of slab (20 *cm.* thick)	$= 460\ kg./m.^2$
	$5460\ kg./m.^2$

∴ Max. sagging b.m. per *metre* width

$$= \frac{3}{16} \times 5460 \times 1{\cdot}5^2 \times 100\ kg.\ cm.$$

$$= 230200\ kg.\ cms.$$

with $c = 70\ kg./cm.^2$, $t = 1250\ kg.\ cm.^2$ and $m = 13$, $Q = 12{\cdot}68$

$$\therefore \quad 12{\cdot}68 \times 100\ d^2 = 230200$$

$$d = 14\ cm.$$

If 16 *mm.* dia. bars be used at a clear cover of 25 *mm.* effective depth available

$$= 20 - 2{\cdot}50 - 1{\cdot}60 - 0{\cdot}80 = 15{\cdot}1\ cm.$$

$$\therefore \quad A_t = \frac{230200}{1250 \times 0{\cdot}86 \times 15{\cdot}1}\ cm.^2$$

$$= 14{\cdot}18\ cm.^2$$

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∴ Spacing of 16 *mm.* dia. bars

$$= \frac{2{\cdot}01 \times 100}{14{\cdot}18} = 14{\cdot}2 \text{ cm. say } 14 \text{ cm.}$$

Hence provide 16 mm. dia. bars at 14 cm. centres

(*vii*) Design of the circular girder

The total load on the circular girder will now be estimated.

Weight of water	= 117840 *kg.*
Dome and side walls	= 40070 *kg.*
Conical slab	= 12190 *kg.*
Circular slab	= 3250 *kg.*
Weight of circular girder (30 *cm.* × 50 *cm.*)	= 3250 *kg.*
	176600 *kg.*

Let us provide 6 columns

Maximum negative B.M.

$$= 0{\cdot}0148\ Wr$$
$$= 0{\cdot}0148 \times 176600 \times 1{\cdot}5 \text{ kg. m.}$$
$$= 3921 \text{ kg. m.}$$

Maximum positive B.M.

$$= 0{\cdot}0075\ Wr$$
$$= 0{\cdot}0075 \times 176600 \times 1{\cdot}5 \text{ kg. m.}$$
$$= 1987 \text{ kg. m.}$$

Torsion

$$= 0{\cdot}0015\ Wr$$
$$= 0{\cdot}0015 \times 176600 \times 1{\cdot}5 \text{ kg. m.}$$
$$= 398 \text{ kg. m.}$$

S.F. at support

$$= \frac{W}{2} \times \frac{1}{\text{number of columns}}$$

$$= \frac{176600}{2 \times 6} = 14720 \text{ kg.}$$

Design at support section

$$M = 392100 \text{ kg. cm.}$$
$$T = 39800 \text{ kg. cm.}$$
$$S = 14720 \text{ kg.}$$

Equating the M.R. to B.M.,

$$8{\cdot}5 \times 30\, d^2 = 392100$$

$$\therefore \quad d = 39{\cdot}21 \text{ cm.}$$

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Effective depth available

$$= 50 - 6 = 44 \text{ cm.}$$

Equivalent shear force

$$= S_e = S + 1{\cdot}6\frac{T}{b}$$

$$= 14720 + 1{\cdot}6 \times \frac{39800}{30} \text{ kg.}$$

$$= 16843 \text{ kg.}$$

Equivalent nominal shear stress

$$= q_{nom} = \frac{S_e}{bd}$$

$$\frac{16843}{30 \times 44} = 12 \text{ kg./cm.}^2$$

Longitudinal reinforcement

Equivalent B.M.

$$= M_{e1} = M + M_t$$

where

$$M_t = T\frac{\left[1 + \frac{D}{b}\right]}{1{\cdot}7}$$

$$= 39800\frac{\left[1 + \frac{50}{30}\right]}{1{\cdot}7} \text{ kg. cm.}$$

$$= 62430 \text{ kg. cm.}$$

$\therefore$

$$M_{e1} = M + M_t$$

$$= 392100 + 62430 = 454530 \text{ kg. cm.}$$

$$A_t = \frac{454530}{1250 \times 0{\cdot}86 \times 44} = 9{\cdot}61 \text{ cm.}^2$$

Provide 4 bars of 18 mm. ϕ (10·18 cm.^{2})

Transverse reinforcement

Fig. 483 shows the section of the beam, showing the position of reinforcement.

Distance between centres of corner bars parallel to the width $= b_1 = 30 - 2 \times 4 = 22$ cm.

Distance between centres of corner bars parallel to the depth $= d_1 = 50 - 2 \times 4 = 42$ cm.

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Area of section of stirrups

$$= A_w = \frac{Tp}{b_1 d_1 t_w} + \frac{Sp}{2{\cdot}5\, d_1 t_w}$$

Let 10 *mm.* ϕ two-legged stirrups be provided.

$$\therefore \quad A_w = 2 \times 0{\cdot}79 = 1{\cdot}58 \text{ cm.}$$

$$1{\cdot}58 = \left[\frac{39800}{22 \times 42 \times 1400} + \frac{14720}{2{\cdot}5 \times 42 \times 1400} \right] p$$

$$1{\cdot}58 = [0{\cdot}030 + 0{\cdot}100]\, p$$

$$\therefore \quad p = \frac{1{\cdot}58}{0{\cdot}13} = 12 \text{ cm.}$$

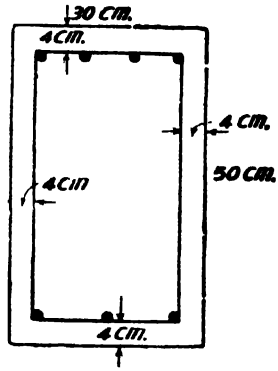

Fig. 483

Provide two-legged 10 mm. ϕ *stirrups @ 10 cm. c/c.*

Steel for sagging moment

$$= \frac{198700}{1250 \times 0{\cdot}86 \times 46} = 4{\cdot}02 \text{ cm.}^2$$

Provide 3 bars of 14 mm. ϕ *(4·62 cm.²)*

(*viii*) Design of columns.

The columns should be designed for direct loads coming upon them and for the bending moments caused by wind load. We shall now discuss the wind load effect on the columns.

Let the wind load intensity on a flat vertical surface be $p = 150$ *kg. per sq. metre.*

Intensity of wind pressure on projected dome surface

$$= 0{\cdot}5\, p = 75 \text{ kg. per sq. metre}$$

Intensity of wind pressure on cylindrical and conical surfaces

$$= 0{\cdot}75\, p = 113 \text{ kg./m.}^2$$

Fig. 484 shows the external dimensions of the tank.

Segmental area

$$= \frac{4h}{3} \sqrt{\frac{l^2}{4} + \frac{2h^2}{3}}$$

$$= \frac{4 \times 1{\cdot}32}{3} \sqrt{\frac{6{\cdot}15^2}{4} + \frac{2 \times 1{\cdot}32^2}{3}} \text{ sq. m.}$$

$$= 5{\cdot}73 \text{ sq. m.}$$

The centroid of this area will be at a height of $\frac{5}{12} h = \frac{5}{12} \times 1{\cdot}32 = 0{\cdot}55$ *m.* above the base.

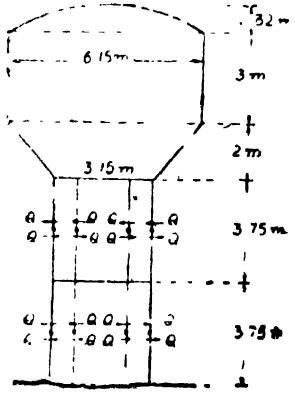

Fig. 484

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The wind loads acting on the various components of the tank are given below :

Tank component	*Area (sq. m.)*	*Wind load (kg)*
Dome	5·73	$5·73 \times 75 = 430 = P_1$
Cylindrical walls	$6·15 \times 3 = 18·45$	$18·45 \times 113 = 2085 = P_2$
Conical walls	$\frac{(6·15+3·15)}{2} \times 2 = 9·30$	$9·30 \times 113 = 1051 = P_3$
Total wind load		$= 3566$ *kg.*

Let the columns be braced at their mid-heights. It will be assumed that points of contraflexure occur at the middle point of each vertical panel of each column.

Let the horizontal shear just above and below the points of contraflexure be Q *kg.* *See* Fig. 484.

$$6Q = 3566 \text{ kg.}$$

$$\therefore \quad Q = \frac{1783}{3} \text{ kg.}$$

∴ B.M. at the top of the column, just above and below the braces and at the base of the column

$$= \frac{1783}{3} \times \frac{3·75}{2} = 1114 \text{ kg. m.}$$

Column section at the top

Load on one column transferred from ring girder

$$= \frac{176600}{6} = 29433 \text{ kg.}$$

Let us assume the columns to be of 30 *cm.* diameter. Let us try *8 bars of 32 mm. diameter* (*Area of one bar* $= 8·04$ *cm.*^{2}) *placed at an effective cover of 5·5 cms.*

Now onwards the usual working stresses may be adopted *i.e.*, $c = 50$ *kg./cm.*^{2}, $c = 1400$ *kg.cm.*^{2}, $m = 18$.

Equivalent concrete area

$$A_e = \frac{\pi}{4} \times 30^2 + (18-1) \times 8·04 \times 8$$

$$= 1800 \text{ cm.}^2$$

Equivalent Moment of Inertia

I_e = **Half the equivalent polar moment of inertia**

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$$= -\frac{1}{2}\left[\frac{\pi \times 30^4}{32} + 17 \times 8{\cdot}04 \times 8 \times (15 - 5{\cdot}5)^2\right] cm.^4$$

$$= -89100\ cm.^4$$

$$\therefore \quad \text{Resultant stress} = \frac{29433}{1800} \pm \frac{(1114 \times 100)}{89100} \times 15$$

$$= -16{\cdot}35 \pm 18{\cdot}75$$

$$= -35{\cdot}10 \text{ or } -2{\cdot}4\ kg./cm.^2$$

Direct load transferred to the columns due to wind pressure at the level of the upper points of contraflexure.

Let the direct load in the column most distant from the centre of the column system be R *kg*. Taking moments about the upper points of contraflexure, we get

$$430\left[0{\cdot}55 + 3 + 2 + \frac{3{\cdot}75}{2}\right] + 2085\left[1{\cdot}5 + 2 + \frac{3{\cdot}75}{2}\right]$$

$$+ 1051 \times \left[\frac{3{\cdot}15 + 2 \times 6{\cdot}15}{3{\cdot}15 + 6{\cdot}15} \cdot \frac{2}{3} + \frac{3{\cdot}75}{2}\right]$$

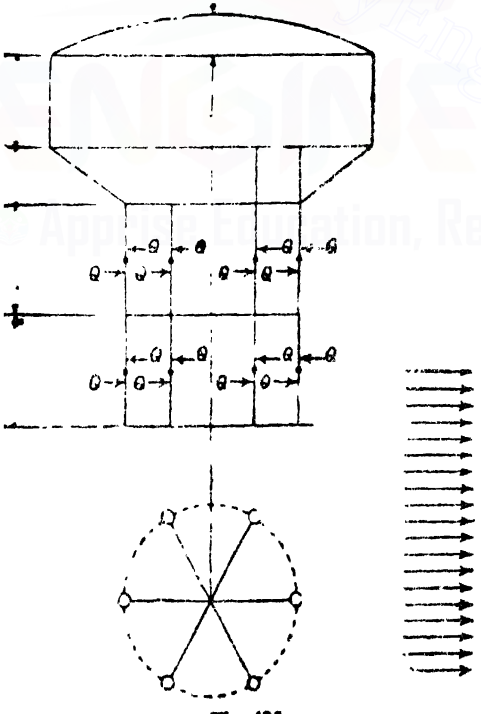

Fig. 485

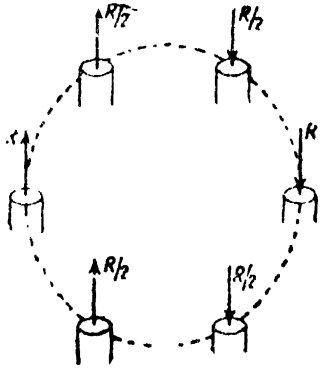

Fig. 486

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$$= 3R \times r$$

$$= 3R \times \frac{3}{2}$$

$$\therefore \quad R = 3899 \text{ kg.}$$

Column section just above the brace level.

Load from the tank = 29433 *kg.*

Wt. of column = 610 *kg.*

Direct load due to wind = 3899 *kg.*

Total load = 33942 *kg.*

$\therefore$ Stress in the section

$$= \frac{33942}{1000} \pm \frac{1114 \times 1000}{89100} \times 15$$

$$= 18{\cdot}86 \pm 18{\cdot}75 \text{ kg./cm.}^2$$

$$= 37{\cdot}61 \text{ or } 0{\cdot}11 \text{ kg./cm.}^2$$

Column section just below the brace level.

Load from above = 33942 *kg.*

Wt. of one brace (30 *cm.* × 30 *cm.*) = 310 *kg.*

= 34252 *kg.*

$\therefore$ Stress on the section

$$= \frac{34252}{1800} \pm \frac{1114 \times 100}{89100} \times 15$$

$$= 19{\cdot}02 \pm 18{\cdot}75 \text{ kg./cm.}^2$$

$$= 37{\cdot}77 \text{ kg./cm.}^2 \text{ or } 0{\cdot}27 \text{ kg./cm}^2.$$

... load transferred to the columns due to wind pressure, at the level of the lower points of contraflexure.

... the direct load in the column most distant from the centre of the column group be R' *kg.*

Taking moments about the lower points of contraflexure, we have

$$4[0{\cdot}55 + 5 + 3{\cdot}75 + \frac{3{\cdot}75}{2}] + 2085[1{\cdot}5 + 2 + 3{\cdot}75 + \frac{3{\cdot}75}{2}]$$

$$+ 1051[1{\cdot}10 + 3{\cdot}75 + \frac{3{\cdot}75}{2}]$$

$$= 3R'r$$

$$= 3R' \times \frac{3}{2}$$

$$\therefore \quad R' = 6870 \text{ kg.}$$

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Column section at the bottom

Load from tank $= 20433$ *kg.*
Wt. of column $= 1220$ *kg.*
Wt. of brace $= 310$ *kg.*
Direct load due to wind $= 6870$ *kg.*

$= 37833$ *kg.*

$\therefore$ Stress on the section

$$= \frac{37833}{1800} \pm \frac{1114 \times 100}{89100} \times 15 \text{ kg./cm.}^2$$

$$= 21{\cdot}01 \pm 18{\cdot}75 \text{ kg./cm.}^2$$

$$= 39{\cdot}76 \text{ or } 2{\cdot}26 \text{ kg./cm}^2.$$

These stresses are within safe limits.

Design of braces

A brace is designed for a bending moment equal to the sum of the moments in the column just above and below the brace level.

$\therefore$ B.M. for the brace $= 2 \times 1114 = 2228$ *g. metres*

Let the brace section be 30 *cm.* $\times$ 30 *cm*

The section will be designed as a doubly reinforced section on steel beam theory. But the effective cover to reinforcement be $= 4$ *cms.*

$\therefore$ Distance between the centres of compressive and tensile reinforcement $= 30 - 8 = 22$ *cms.*

Allowing a stress of 1400 *kg./cm.*^{2} in the reinforcement, we have, $A_c = A_t$

$$= \frac{222800}{1400 \times 22} = \text{cm.}^2 = 7{\cdot}24 \text{ cm}^2.$$

Provide 4 bars of 16 mm. dia. at the top and 4 similar bars at the bottom.

Shear force in the brace

$$= \frac{\text{B.M. for the brace}}{\text{Half the length of the brace}}$$

$$= \frac{2228}{\left(\frac{1{\cdot}5}{2}\right)} \text{ kg.}$$

$$= 2971 \text{ kg.}$$

Shear stress $= \frac{S}{ab}$

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$$= \frac{2971}{22 \times 30} \text{ kg./cm.}^2$$

$$= 4{\cdot}5 \text{ kg./cm}^2$$

This is less than 5 $kg./cm^2$. Provide therefore nominal two-legged stirrups of 8 *mm.* dia. at a spacing of 22 *cms.* centres.

Foundation

Weight of tank and girders		= 176600 *kg.*
Weight of six columns 1220 × 6		= 7320 *kg.*
Weight of braces 310 × 6		= 1860 *kg.*
		185780 *kg.*
Weight of foundation (10%)		= 18578 *kg.*
	Total	= 204358 *kg.*

Safe bearing capacity of the soil

$$= 12000 \text{ kg./sq. metre}$$

$\therefore$ Area required for the foundation

$$= \frac{204358}{12000}$$

$$= 17{\cdot}03 \text{ sq.m.}$$

Let D be the diameter of the base

$$\therefore \quad \frac{\pi D^2}{4} = 17{\cdot}03 \text{ sq.metre}$$

$$\therefore \quad D = 4{\cdot}65 \text{ metres.}$$

Let us provide a diameter of 5·5 *metres* to make allowance for uneven stress distribution caused by wind loads.

Let the depth of the ring girder of the foundation be 60 *cms.*

Moment of wind pressure about the base

$$= 430[0{\cdot}55 + 12{\cdot}50 + 0{\cdot}60] + 2085[1{\cdot}5 + 9{\cdot}5 + 0{\cdot}60] + 1051[1{\cdot}1 + 7{\cdot}5 + 0{\cdot}60] \text{ kg. m.}$$

$$= 39725 \text{ kg. m.}$$

Moment of inertia of a circle of 5·5 *metres* diameter

$$= \frac{\pi \times 5{\cdot}5^4}{64} = 44{\cdot}93 \text{ metres}^4$$

Hence the pressure at the base is given by

$$p = \frac{205358 \times 4}{\pi \times 5{\cdot}5^2} \pm \frac{39725}{44{\cdot}93} \times \frac{5{\cdot}5}{2} \text{ kg /m.}^2$$

$$= 8598 \pm 2432 \text{ kg./m.}^2$$

$p_{max.} = 11030 \text{ kg /m}^2$ and $p_{min.} = 6166 \text{ kg./m}^2$.

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Net upward pressure on the foundation can be determined by subtracting from the above, the stress due to self-weight of the foundation.

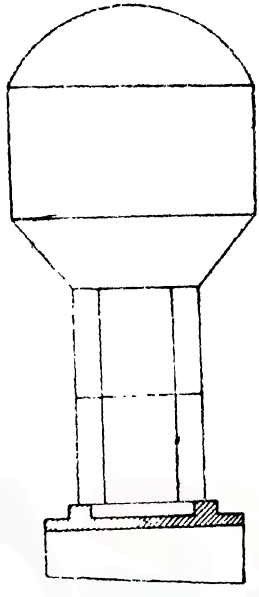

Fig. 487

Stress due to wt. of foundation

$$= \frac{18578 \times 4}{\pi(5{\cdot}5)^2} = 782 \ kg./m^2$$

Net max. pressure

$$= 11030 - 782 = 10248 \ kg./m.^2$$

Net min. pressure

$$= 6166 - 782 = 5384 \ kg./m^2.$$

For design purposes, the upward equivalent uniform pressure may be taken as the average of the pressure at the centre and the maximum pressure.

$\therefore$ Equivalent uniform upward pressure

$$= \frac{7816 + 10248}{2} = 9032 \ kg./m.^2$$

$\therefore$ Equivalent total upward load on the bottom ring girder

$$= W = 9032 \times \pi \times \frac{5{\cdot}5^2}{4} = 214600 \ kg.$$

Design of the ring girder

Radius to the centre line of the ring girder

$$= 1{\cdot}5 \ metres$$

$\therefore$ B.M. at the support

$$= 0{\cdot}0148 \ Wr \text{ (sagging)}$$
$$= 0{\cdot}0148 \times 214600 \times 1{\cdot}5 \ kg. \ m.$$
$$= 4764 \ kg. \ m. = 476400 \ kg. \ cm.$$

B.M. at the centre of each span

$$= 0 \ 0075 \ Wr \text{ (hogging)}$$
$$= 0{\cdot}0075 \times 214600 \times 1{\cdot}5 \ kg. \ m.$$
$$= 2414 \ kg. \ m.$$
$$= 241400 \ kg. \ cm.$$

Torsion

$$= 0{\cdot}0015 \ Wr$$
$$= 0{\cdot}0015 \times 214600 \times 1{\cdot}5 \ kg. \ m.$$
$$= 482{\cdot}8 \ kg. \ m. = 48280 \ kg. \ cm.$$

Design of the support section

$M = 476400 \ kg. \ cm.$, $T = 48280 \ kg. \ cm.$, $S = \dfrac{\text{Total load}}{\text{No. of columns}}$

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$$= \frac{214600}{6 \times 2} = 17883 \text{ kg.}$$

Equating M.R. to B.M.

$$8{\cdot}5 \times 30\ d^2 = 476400 \quad \therefore\ d = 43{\cdot}22 \text{ cm.}$$

Effective depth available

$$= 60 - 8 = 52 \text{ cm.}$$

Equivalent shear force $= S_e = S + 1{\cdot}6\,\frac{T}{b}$

$$= 17883 + 1{\cdot}6 \times \frac{48280}{30} \text{ kg.}$$

$$= 20458 \text{ kg.}$$

Equivalent nominal shear stress

$$q_{ve} = \frac{S_e}{bd} = \frac{20458}{30 \times 52} = 13{\cdot}1 \text{ kg./cm.}^2$$

Longitudinal reinforcement

Equivalent B.M. $\quad \therefore M_{e1} = M + M_t$

where,

$$M_t = T\,\frac{\left[1 + \frac{D}{b}\right]}{1{\cdot}7}$$

$$= 48280\,\frac{\left[1 + \frac{60}{30}\right]}{1{\cdot}7} = 85200 \text{ kg. cm.}$$

$$\therefore \quad M_{e1} = 476400 + 85200 = 561600 \text{ kg. cm.}$$

$$A_t = \frac{561600}{1400 \times 0{\cdot}87 \times 52} = 8{\cdot}87 \text{ cm.}^2$$

Provide 4 bars of 18 mm. ϕ (10·16 cm.^{2})

Fig. 488 shows the section of the beam showing the position of the reinforcement

Distance between the centres of corner bars parallel to the width. $\Big\}$ $= b_1 = 30 - 2 \times 4 = 22$ cm.

Distance between the centres of corner bars parallel to the depth. $\Big\}$ $= d_1 = 60 - 2 \times 8 = 44$ cm.

Area of section of stirrups

$$= A_v = \frac{Tp}{b_1\, d_1\, t_v} + \frac{Sp}{2{\cdot}5\, d_1\, t_v}$$

Let 10 *mm.* ϕ two-legged stirrups be provided,

$$A_v = 2 \times 0{\cdot}79 = 1{\cdot}58 \text{ cm.}^2$$

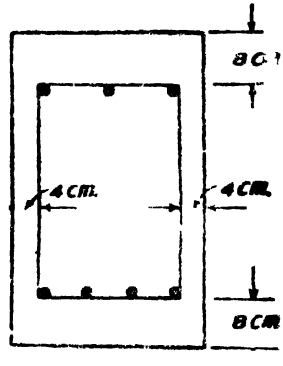

Fig. 488

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$$1{\cdot}58=\left[\frac{48280}{22\times44\times1400}+\frac{17883}{2{\cdot}5\times44\times1400}\right]p$$

$$1{\cdot}58=(0{\cdot}036+0{\cdot}116)\,p$$

$$\therefore \qquad p=\frac{1{\cdot}58}{0{\cdot}152}=10{\cdot}39\ cm.$$

Provide 2-legged 10 mm. ϕ stirrups @ 10 cm. c/c.

Steel for hogging moment

$$=\frac{241400}{1400\times0{\cdot}87\times52}\ cm.^2$$

$$=3{\cdot}81\ cm.^2$$

Provide 3 bars of 14 mm. ϕ (4·62 cm.^{2}).

Design 127 (S.I.). *Design an Intze tank to the dimensions shown in Fig. 489.*

Solution.

(i) Design of the Roof Dome

Provide a thickness of 15 *cm.* for the roof dome.

Loads

Dead Load $=15\times250=3750\ N/m.^2$

Live Load $=1500\ N/m.^2$

Total $=5250\ N/m.^2$

Let R be the radius of the dome

$$1{\cdot}6\,(2R-1{\cdot}6)=5\times5$$

$$\therefore \qquad R=8{\cdot}61\ m.$$

Let 2θ be the angle subtended by the dome at its centre

$$\sin\frac{d}{2R}$$

$$\sin\theta=\frac{10}{2\times8{\cdot}61}$$

$$\therefore \qquad \theta=35^\circ\ 30'.$$

$$\therefore \qquad \cos\theta=\cos 35^\circ\ 30'=0{\cdot}8141$$

Hoop stress at the level of the springing

$$=f=\frac{wR}{t}\left[\cos\theta-\frac{1}{1+\cos\theta}\right]$$

$$=\frac{5250\times8{\cdot}61}{0{\cdot}15}\left[0{\cdot}8141-\frac{1}{1{\cdot}8141}\right]N/m.^2$$

$$=79214\ N/m.^2$$

$$=7{\cdot}9214\ N/cm.^2$$

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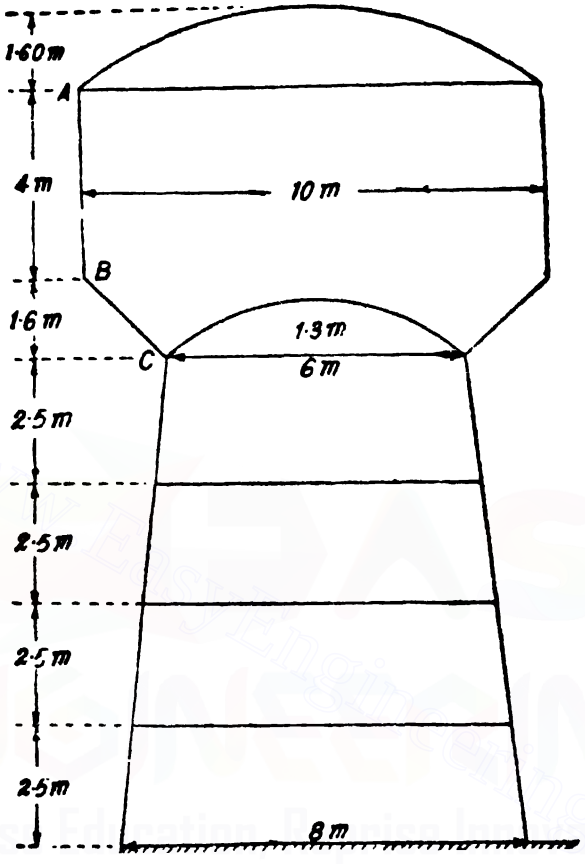

Fig. 489

Hoop stress at the crown

i.e. at $\theta = 0$

$$f = \frac{wR}{t}\left[1 - \frac{1}{2}\right]$$

$$= 0{\cdot}5 \quad \frac{wR}{t} = \frac{0{\cdot}5 \times 5250 \times 8{\cdot}61}{0{\cdot}15} \ N/m.^2$$

$$= 150675 \ N/m.^2 = 15{\cdot}07 \ N/cm^2$$

Meridional thrust at the level of the springing, per *metre* run

$$= T_1 = \frac{wR}{1 + \cos\theta} \qquad \frac{5250 \times 8{\cdot}61}{1{\cdot}8141} = 24917 \ N/m.$$

∴ **Meridional stress**

$$= \frac{24917}{15 \times 10} = 166 \ N/cm.^2$$

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These stresses are very small. Provide nominal reinforcement say 8 *mm.* ϕ @ 11 *cm* centres both ways.

(ii) **Ring beam at top.**

Horizontal component of T_1

$$= T_1 \cos\theta = 24917 \times 0.8141 = 20285 \text{ N/m.}$$

$\therefore$ Hoop tension in the ring beam

$$= 20285 \times \frac{10}{2} = 101425 \text{ N}$$

$\therefore$ Steel required for hoop tension

$$= \frac{101425}{10{,}000} = 10.14 \text{ cm}^2$$

Provide 4 bars of 18 mm. diameter (10.16 cm.²)

Size of the ring beam

Let the area of the ring beam section be A $cm.^2$

Equivalent concrete area $= A + (m-1)\, A_s$

$$= A + (13-1)10.16$$

$$= A + 121.92 \text{ cm}^2$$

Limiting the tensile stress on the equivalent concrete area to 120 N/cm^2.

$$\frac{101425}{A + 121.92} = 120$$

$\therefore$ $A = 723.3$ $cm.^2$

Provide 25 $cm \times 30$ *cm.* section

(iii) **Cylindrical wall**

Pressure intensity at the bottom of the cylindrical wall

$$= 9810 \times 4 = 39240 \text{ N/m.}^2$$

Consider the bottom 1 *m* strip of wall

$$\text{Hoop tension} = \frac{pd}{2} = 39240 \times \frac{10}{2} = 196200 \text{ N}$$

$$\therefore \quad A_t = \frac{196200}{10{,}000} = 19.6 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1.13 \times 100}{19.6} = 5.76 \text{ cm.}$$

Provide 12 mm. ϕ *bars @ 10 cm. centres near each face.*

Thickness of the wall may be kept at 20 *cm.*

$$\text{Distribution steel} = \frac{0.3}{100}(20 \times 100) = 6 \text{ cm.}^2$$

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Spacing of 8 *mm.* ϕ bars

$$\frac{0{\cdot}5\times 100}{6} = 8{\cdot}3 \text{ cm.}$$

Provide 8 mm. ϕ *@ 16 mm. centres near each face.*

Check for compressive stress at the bottom of the cylindrical wall.

Vertical component of T_1

$= V_1 = T_1 \sin\theta$

$24917 \sin 35° 30'$ $= 14470$ *N/m.*

Weight of wall

$= 0{\cdot}2 \times 4 \times 25000$ $= 20000$ *N/m.*

Weight of ring beam

$= 0{\cdot}25 \times 0{\cdot}30 \times 25000$ $= 1875$ *N/m.*

V_2 = Total vertical load per *metre* $= 36345$ *N/m.*

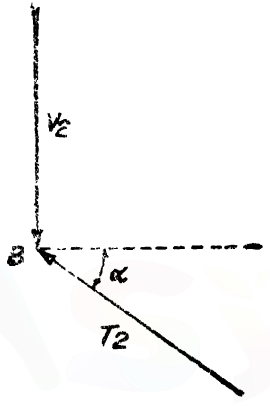

Fig. 490

$\therefore$ Compressive stress

$$= \frac{36345}{20\times 100} = 18{\cdot}20 \text{ N/cm.}^2$$

The stress being low provide nominal vertical steel at 0·30% gross area.

Vertical steel $\frac{0{\cdot}3}{100}(20\times 100) = 6 \text{ cm.}^2$

Spacing of 8 *mm.* ϕ bars

$$= \frac{0{\cdot}5\times 100}{6} \quad 8{\cdot}3 \text{ cm.}$$

Provide 8 mm. ϕ *bars @ 16 cm. c/c near each face.*

(*iv*) Ring beam at B

Let T_2 be the thrust per *metre* run exerted by the conical slab at the junction *B*.

Resolving vertically at *B*,

$$T_2 \sin\alpha = V_2$$

$$\tan\alpha = \frac{1{\cdot}6}{2} = 0{\cdot}8$$

Resolving horizontally at *B*,

$$H_2 = T_2 \cos\alpha = V_2 \cot\alpha$$

$$\frac{36345}{0{\cdot}8} = 45431 \text{ N/m.}$$

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This horizontal load H_2 will produce a hoop tension in the ring beam B.

Hoop tension due to H_2

$$= H_2 \frac{d}{2}$$

$$= 45431 \times \frac{10}{2} N$$

$$= 227155 N$$

Let the ring beam be 60 *cm.* deep

Water pressure on the ring beam

$$= 9810 \times 4 \times \frac{60}{100} N/m.$$

$$= 23544 N/m.$$

∴ Hoop tension due to water pressure

$$= 23544 \times \frac{10}{2} = 117720 N$$

Total hoop tension

$$= 227155 + 117720 = 344875 N$$

Steel for hoop tension

$$= \frac{344875}{10000} cm.^2 = 34{\cdot}5 cm.^2$$

Providing 20 *mm.* ϕ bars, number of bars required

$$\frac{34{\cdot}5}{3{\cdot}14} = 11 \text{ bars}$$

Provide 12 bars of 20 mm. diameter.

Steel provided $= 12 \times 3{\cdot}14 = 37{\cdot}68 cm.^2$

Let the area of the ring beam section be A

Equivalent concrete area

$$= A + (m-1) A_s$$
$$= A + (13-1)\, 37{\cdot}68$$
$$= A + 452{\cdot}16.$$

Limiting the tensile stress on the equivalent concrete area to 120 $N/cm.^2$

$$\frac{344875}{A + 452{\cdot}16} = 120$$

$$\therefore \quad A = 2422 cm.^2$$

Provide 40 *cm.* × 60 *cm.* section.

(v) Design of the conical slab

The conical slab should be designed for (*a*) a hoop tension and (*b*) for bending as it spans as a sloping slab from the ring beam at *B* to the ring girder at *C*.

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Design for hoop tension

The hoop tension on the conical slab is given by

$$\frac{W_w + W_s}{2\pi} + \frac{W_w}{2\pi} \tan \alpha$$

where W_w = Weight of water resting on the conical slab

W_s = Weight of the conical slab

α = Inclination of the conical slab with the horizontal.

Area of water section standing on the conical slab

$$= \frac{2}{2} [4 + 5{\cdot}6] = 9{\cdot}6 \text{ metre.}^2$$

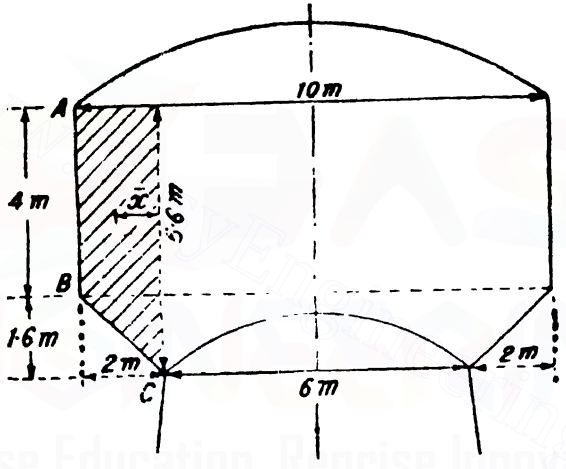

Fig. 491

$$\bar{x} = \frac{5{\cdot}6 + (2 \times 4)}{5{\cdot}6 + 4} \times \frac{2}{3} = 0{\cdot}94 \text{ m.}$$

∴ Weight of water resting on the conical slab

$$= W_w = 9810 \times 9{\cdot}6 \times 2\pi (3 + 0{\cdot}94) \text{ N}$$

$$= 2331400 \text{ N}$$

Length of the sloping slab

$$= \sqrt{2^2 + 1{\cdot}6^2} = 2{\cdot}561 \text{ metre}$$

Thickness of the sloping slab

$$= 20 \text{ cm.}$$

∴ Weight of the conical slab

$$W_s = 2{\cdot}561 \times 0{\cdot}20 \times 25000 \times 2\pi \left(\frac{5+3}{2} \right)$$

$$= 321825 \text{ N}$$

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$$\therefore \text{ Hoop tension } = \frac{2331400+321825}{2\pi} + \frac{2331400}{2\pi} \times \frac{1{\cdot}6}{2} N$$

$$= 719115 \ N$$

$\therefore$ Hoop steel on the entire section

$$= \frac{719115}{10000} = 71{\cdot}91 \ cm.^2$$

Provide 40 bars of 16 mm diameter.

These bars may be distributed near both the faces of the conical slab.

Design for bending moment

Load per *metre* width of the conical slab

$$= \frac{W_w + W_s}{2\pi \times \text{mean radius}}$$

$$= \frac{2331400+321825}{2\pi \times 4} N$$

$$= 105568 \ N$$

Maximum bending moment

$$= \frac{105568 \times 2}{8} \times 100 \ N \ cm.$$

$$= 2639200 \ N \ cm.$$

Axial compression

$$T_2 = \frac{V_2}{\sin \alpha}$$

$$= \frac{36345 \times 2{\cdot}561}{1{\cdot}6} = 58175 \ N$$

Providing 16 *mm.* diameter bars at a clear cover of 25 *mm.*

Effective depth $= 20 - 2{\cdot}5 - 0{\cdot}8 = 16{\cdot}70 \ cm.$

Distance between centre of section and centre of steel

$$= x = d - \frac{t}{2}$$

$$= 16{\cdot}70 - \frac{20}{2} = 6{\cdot}7 \ cm.$$

$\therefore$ Resultant bending moment

$$= M + T_2 x$$

$$= 2639200 + 58175 \times 6{\cdot}7 \ N \ cm.$$

$$= 3028972 \ N \ cm.$$

$$A_s = \frac{3028972}{12500 \times 0{\cdot}86 \times 10{\cdot}7} = 16{\cdot}87 \ cm.^2$$

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$\therefore$ Spacing of 16 *mm.* diameter bars

$$= \frac{2\cdot01 \times 100}{16\cdot87} = 11\cdot9 \text{ cm.}$$

Provide 16 mm. ϕ bars @ 11 cm. centre.

(vi) **The bottom dome**

Span of the dome $= 6$ *m*

Rise of the dome $= 1\cdot3$ *m*

Let R be the radius of the dome

$1\cdot3\ (2R - 1\cdot3) = 3 \times 3$

$\therefore \quad R = 4\cdot11$ *m.*

Let 2θ be the angle subtended by the dome its centre

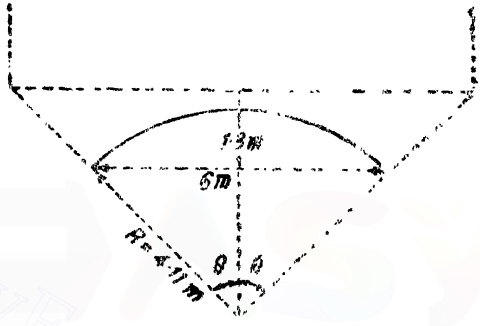

Fig. 492

$$\sin\theta = \frac{3}{4\cdot11} = 0\cdot73$$

$\theta = 46° 53'$

$\therefore \quad \cos\theta = 0\cdot68$

Thickness of dome

$= 20$ *cm.*

Loads

Dead load of the dome

$$= 250 \times 20 = 5000 \text{ N/m}^2$$

Weight of water resting on the dome

$$= 9810\left[\frac{\pi}{4} \times 6^2 \times 5\cdot6 - \frac{\pi \times 1\cdot3^2}{3}(3 \times 4\cdot11 - 1\cdot3)\right] N$$

$$= 1361820\ N$$

Area of dome surface

$$= 2\pi \times 4\cdot11 \times 1\cdot3 = 33\cdot57 \text{ metre}^2$$

Load intensity due to weight of water

$$= \frac{1361820}{33\cdot57} = 40566 \text{ N/metre}^2$$

Total load intensity on the dome

$$= w = 5000 + 40566 = 45566 \text{ N/metre}^2$$

Meridional thrust per *metre* run at the springing level

$$T' = \frac{wR}{1 + \cos\theta} = \frac{45566 \times 4\cdot11}{1\cdot68}$$

$$= 111474 \text{ N/m.}$$

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Meridional compressive stress

$$= -\frac{111474}{100 \times 20} = 55{\cdot}74 \ N/cm.^2$$

Hoop stress $$= \frac{wR}{t}\left[\cos\theta - \frac{1}{1+\cos\theta}\right]$$

Maximum hoop stress occurs at the crown, *i.e.*, at $\theta = 0$

$\therefore$ Maximum hoop stress

$$= \frac{45566 \times 4{\cdot}11}{0{\cdot}20}\left[1 - \frac{1}{2}\right]$$

$$= 468190 \ N/metre^2$$

$$= 46{\cdot}82 \ N/cm.^2$$

These stresses are low and hence provide nominal 0·3% steel.

Provide 8 mm. ϕ bars @ 8 cm. centres

(*vii*) Circular girder

The total load on the circular girder consists of the following :

Weight of water	$= 2331400 + 1361820$	$= 3693220 \ N$
Weight of top dome and side wall	$= 36345 \times 2\pi \times 5$	$= 1141815 \ N$
Weight of conical wall		$= 321825 \ N$
Weight of the lower dome	$= 5000 \times 33{\cdot}57$	$= 167850 \ N$
Weight of circular girder 40 cm. × 60 cm.	$= 0{\cdot}4 \times 0{\cdot}6 \times 25000 \times 2\pi \times 3$	$= 113100 \ N$
Totai load		$W = 5437810 \ N$

Let us provide 8 columns

Max. negative B.M. $= -0{\cdot}0083 \ Wr$

$$= -0{\cdot}0083 \times 5437810 \times 3 = -135400 \ Nm.$$

Max. positive B.M. $= -0{\cdot}00416 \ Wr$

$$= -0{\cdot}00416 \times 5437810 \times 3 = 67860 \ Nm.$$

Max. Torsion $= 0{\cdot}0006 \ Wr$

$$= 0{\cdot}0006 \times 5437810 \times 3 = 9790 \ Nm.$$

S.F. at support $$= -\frac{W}{2 \times \text{Number of columns}}$$

$$= \frac{5437810}{2 \times 8} = 339863 \ N$$

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Design at support section

Equating the moment of resistance to the B.M. at support

$$141{\cdot}1\ bd^2 = 13540000$$

$$d = \sqrt{\frac{13540000}{140{\cdot}3 \times 40}} = 49{\cdot}1\ cm.$$

Overall depth of beam $= 60\ cm.$

Actual effective depth $= 60 - 4 = 56\ cm.$

Equivalent S.F. $= S_e = S + 1{\cdot}6\ \frac{T}{b}$

$$= 339863 + 1{\cdot}6 \times \frac{979000}{40}\ N$$

$$= 379023\ N$$

Equivalent nominal shear stress

$$= q_{ve} = \frac{379023}{40 \times 56} = 169\ N/cm.^2$$

For M 15 concrete $q_{c\ max} = 1{\cdot}6\ N/mm.^2$

$$= 160\ N/cm.^2$$

Longitudinal reinforcement

Equivalent B.M. $= M_{e1} = M + M_t$

where, $$M_t = T\frac{\left[1 + \frac{D}{b}\right]}{1{\cdot}7} = \frac{979000}{1{\cdot}7}\left[1 + \frac{60}{40}\right] N\ cm.$$

$$= 1439705\ N\ cm.$$

$$\therefore \quad M_{e1} = 13540000 + 1439705 = 14979705\ N\ cm.$$

$$\therefore \quad A_t = \frac{14979705}{10{,}000 \times 0{\cdot}84 \times 56} = 31{\cdot}8\ cm.^2$$

Provide 10 bars of 20 mm. ϕ (31·4 cm.^{2}).

Transverse reinforcement

Fig. 493 shows the section of the beam

Distance between centres of corner bars parallel to the width

$$= b_1 = 40 - 2 \times 4 = 32\ cm.$$

Distance between centres of corner bars parallel to the depth

$$= d_1 = 60 - 8 = 52\ cm.$$

Area of section of stirrups

$$A_w = \frac{Tp}{b_1 d_1 f_w} + \frac{Sp}{2{\cdot}5\ d_1\ f_w}$$

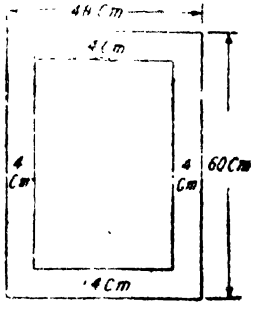

Fig. 493

Providing 4-legged 10 *mm.* ϕ *stirrups*

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$$A_w = 4 \times 0{\cdot}79 = 3{\cdot}16 \ cm.^2$$

$$3{\cdot}16 = \left[\frac{979000}{32 \times 52 \times 14000} + \frac{339863}{2{\cdot}5 \times 52 \times 14000}\right] p$$

$$3{\cdot}16 = (0{\cdot}042 + 0{\cdot}1867)\, p$$

$$\therefore \quad p = 13{\cdot}8 \ cm. \text{ say } 10 \ cm. \ c/c.$$

Steel for sagging moment

$$= \frac{6786000}{12500 \times 0{\cdot}86 \times 52} = 12{\cdot}14 \ cm^2$$

Provide 4 bars of 20 mm. ϕ.

Hoop stress

Let T_0 = thrust exerted by the conical slab on the girder

$\therefore$ $T_0 \sin \alpha \ 2\pi r = W_w + W_s + wt$ of cylindrical wall and upper dome

$$= 2331400 + 321825 + 1141815 \ N$$

$$= 3795040 \ N$$

$$\therefore \quad T_0 = \frac{3795040}{\sin 38^\circ 39' \times 2\pi \times 3} = 322360 \ N$$

Horizontal component of T_0

$$= 322360 \cos \alpha$$

$$= 322360 \cos 38^\circ 39' = 251755 \ N$$

Horizontal component of T'

$$= T' \cos \theta$$

$$= 111474 \cos 46^\circ 53' = 76190 \ N$$

$\therefore$ Net horizontal force

$$= 251755 - 76190 \ N$$

$$= 175565 \ N$$

$\therefore$ Hoop force $= 175565 \times 3 = 526695 \ N$

$\therefore$ Hoop compressive stress

$$= \frac{526695}{40 \times 60} = 219 \ N/cm.^2$$

(*viii*) **Columns.** The columns should be designed for direct loads coming upon them and for the bending moments caused by wind load.

Vertical load on one column at top

$$= \frac{5437810}{8} = 679726 \ N$$

Let α be the inclination of the column with the vertical

$$\tan \alpha = \frac{1}{10} \qquad \therefore \quad \sin \alpha = \frac{1}{\sqrt{101}}$$

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$$= 0{\cdot}0995 \text{ and } \cos\alpha = \frac{10}{\sqrt{101}} = 0{\cdot}9950$$

Actual length of the column

$$= 10 \sec\alpha$$

$$= \frac{10}{0{\cdot}9950} = 10{\cdot}05 \textit{ metre.}$$

Providing 40 *cm* diameter column

Weight of column $= \frac{\pi}{4}(0{\cdot}4)^2 \times 10{\cdot}05 \times 25000 = 31573\ N$

Total vertical load $= 679726 + 31573 = 711299\ N$

$\therefore$ Corresponding axial load

$$= \frac{711299}{0{\cdot}9950} = 714873\ N$$

Weight of water in the tank

$$= 3693220\ N$$

$\therefore$ Weight of water transmitted to one column

$$= \frac{3693220}{8} = 461652\ N$$

$\therefore$ Vertical load on one column when the tank is empty

$$= 711299 - 461652 = 249647\ N$$

Corresponding axial load

$$= \frac{249647}{0{\cdot}9950} = 250900\ N$$

Ignoring wind load effect if the steel requirement is A_s

Then $c(A - A_s) + tA_s = 714873\ N$

$$500\left[\frac{\pi}{4} \times 40^2 - A_s\right] + 13000\ A_s = 714873$$

$$A_s = 6{\cdot}92\ cm^2$$

Minimum requirement of steel

$$= 0{\cdot}8\% \text{ of gross area}$$

$$= \frac{0{\cdot}8}{100}\left(\frac{\pi}{4} \times 40^2\right) = 10{\cdot}10\ cm.^2$$

Provide 6 bar of 20 *mm.* diameter

$$= (18{\cdot}84)\ cm.^2$$

(More steel has been suggested since the column will also be subjected to bending moment caused by wind load.)

Analysis due to wind pressure

Assume a wind pressure of 1500 $N/metre^2$ and a reduction factor of 0 7.

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Wind force on the top dome and cylindrical walls

$$= \left(4 + \frac{1{\cdot}6}{2}\right) 10{\cdot}4 \times 1500 \times 0{\cdot}7 = 52420 \ N$$

acting at 14 *m*. above the base

Wind force on the circular wall

$$= \frac{10{\cdot}4 + 6{\cdot}4}{2} \times 1{\cdot}6 \times 1500 \times 0{\cdot}7 = 14110 \ N$$

acting at 10·8 *m*. above the base

Wind force on the circular girder

$$= 0\ 6 \times 6\ 4 \times 1500 \times 0{\cdot}7 = 4030 \ N$$

acting at 10 *m*. above the base

Wind force on column and braces

$$= 5 \times 0{\cdot}4 \times 10 \times 1500 \times 0{\cdot}7$$

$$+ 3 \times \left(\frac{6+8}{2}\right) \times 0{\cdot}30 \times 1500 \ N$$

$= 30450 \ N$ acting at 5 *m*. above the base

Total moment of wind pressure about the base

$$= (52420 \times 14) + (14110 \times 10{\cdot}8) + (4030 \times 10) + (30450 \times 5)$$

$= 1078820 \ Nm.$

Vertical load on any column

$$= \frac{M \cdot x}{\Sigma x^2}$$

$$\Sigma x^2 = (2 \times 4^2) + 4\left(\frac{4}{\sqrt{2}}\right)^2$$

$= 64 \ metre^2$

∴ **Maximum wind load force in the most leeward side and the most windward side**

$$= \frac{1078820 \times 4}{64} = 67430 \ N$$

Maximum wind load force in columns marked *z*

$$= \frac{1078820}{64} \times \frac{4}{\sqrt{2}} = 47690 \ N$$

Consider the windward column 1

Vertical load due to dead load and wind load

$$= 711299 + 67430 \ N$$

$$= 778729 \ N$$

Corresponding axial load

$$= \frac{778729}{0{\cdot}995} = 782642 \ N$$

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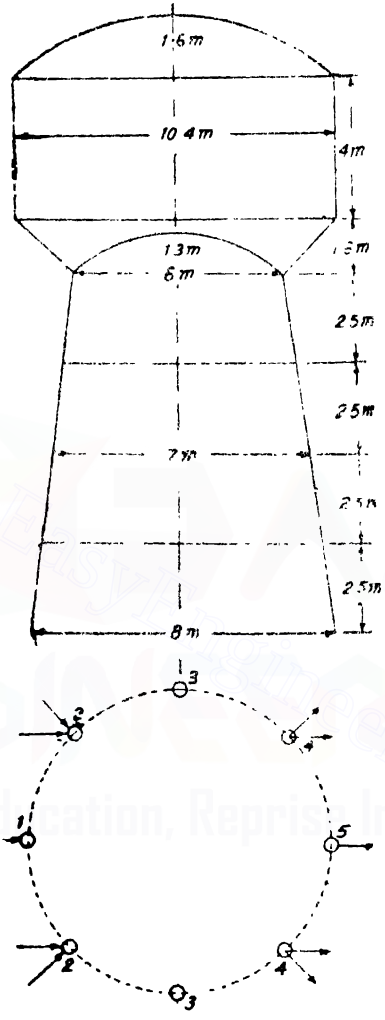

Fig. 494.

Since the columns are inclined the horizontal components of the axial forces caused by wind action reduce the horizontal shear in the column.

Horizontal component of the axial forces caused by wind action

$$= 2 \times 67430 \times 0{\cdot}0995 + 4 \times 47690 \times 0{\cdot}0995 \times \frac{1}{\sqrt{2}} N$$

$$= 26820\ N$$

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∴ Actual horizontal force at the base

$$=(52420+14110+4030+30450)-26820$$
$$=74190\ N$$

∴ Horizontal shear per column

$$=\frac{74190}{8}=9273{\cdot}75\ N$$

∴ Maximum bending moment for the column

$$=9273{\cdot}75\times\frac{2{\cdot}5}{2}=11592\ Nm.$$

Analysis for the column section

Direct load $=782642\ N$

Bending moment $=11592\ Nm$

Provide 40 *cm.* diameter column.

Provide 8 bars of 20 *mm.* diameter, at an effective cover of 5 *cm*

Area of steel A_s $=8\times3{\cdot}14=25{\cdot}12\ cm.^2$

Equivalent concrete area

$$=\frac{\pi}{4}(40)^2+(12\times25{\cdot}12)\ cm.^2$$
$$=1256{\cdot}64+301{\cdot}44$$
$$=1558\ cm.^2$$

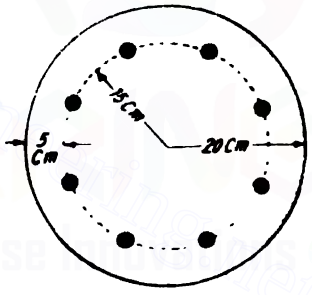

Fig. 495

Polar moment of inertia of the equivalent concrete section

$$=\frac{\pi\times40^4}{32}+12\times25{\cdot}12\times15^2$$
$$=251328+67824$$
$$=319152\ cm.^4$$

∴ Equivalent moment of inertia about a diameter

$$=\frac{319152}{2}=154576\ cm.^4$$

Direct stress in concrete

$$=\frac{782642}{1558}=502{\cdot}3\ N/cm.^2$$

Bending stress in concrete

$$=\frac{1159200}{159576}\times20=145{\cdot}3\ N/cm.^2$$

∴ Maximum stress $=502{\cdot}3+145{\cdot}3=647{\cdot}6\ N/cm.^2$

(ix) Design of Braces

Consider one of the braces, say the brace *BC*.

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For the condition of maximum bending moment for the brace *BC* wind pressure should act normal to an adjoining brace *AB*.

Moment in brace *BC* = twice the moment for the column × sec. 45°

$= 2 \times 11592 \times \sqrt{2}$ *Nm.*

$= 32782$ *Nm*

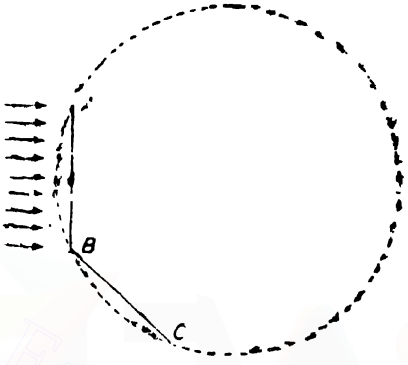

Fig. 496.

Providing 30 *cm* × 30 *cm.* section and designing as a doubly reinforced beam with equal steel at top and bottom

$$A_s = A_t = \frac{3278200}{14000 \times 22} \text{ cm.}^2 = 10{\cdot}64 \text{ cm.}^2$$

Provide 4 bars of 18 *mm.* diameter at top and equal amount of steel at bottom.

$$\text{S.F. for brace} = \frac{\text{B.M. for brace}}{\frac{1}{2} \text{ span of brace}}$$

$$\text{Span of brace} = 2 \times \frac{7{\cdot}5}{2} \sin 22^\circ 30' = 2{\cdot}87 \text{ m}$$

$$\therefore \text{ S.F. for brace} = \frac{32782}{\left(\frac{2 \times}{2}\right)} = 22844 \text{ N}$$

Nominal shear stress

$$q_v = \frac{22844}{bd} = \frac{22844}{30 \times 26} = 29 \text{ N/cm.}^2$$

Provide nominal stirrups *say 2-legged 10 mm ϕ stirrups @ 20 cm. c/c.*

(x) Design of foundation

Total load on the columns when the tank is full $= 714873 \times 8 = 5718984$ *N*

Approximate weight of foundation = (10% of column loads) 571898 *N*

6290882 *N*

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Assuming a safe bearing capacity of 200 $kN/metre^2$

Area of foundation $= \frac{6290882}{200000} = 31{\cdot}45\ metre^2$

Let us provide 10 *metre* outer diameter and 7 *metre* inside diameter, for the raft footing.

$\therefore$ Area of footing $= \frac{\pi}{4}\ [10^2 - 7^2] = 40\ 10\ metre^2$

$\therefore$ Net upward pressure intensity

$$= \frac{5718984}{40{\cdot}10}\ N/metre^2$$

$$= 142618\ N/metre^2$$

Design of the circular girder

Maximum negative B.M. at centre

$$= 0{\cdot}00416\,Wr$$

$$= 0{\cdot}00416 \times 6290882 \times 4\ Nm.$$

$$= 104680\ Nm$$

Maximum positive B.M. at support

$$= 0{\cdot}0083\ Wr.$$

$$= 0{\cdot}0083 \times 6290882 \times 4\ Nm.$$

$$= 208857\ Nm.$$

Maximum torsion $= 0{\cdot}0006\ Wr$

$$= 0{\cdot}0006 \times 6290882 \times 4\ Nm.$$

$$= 15098\ Nm.$$

Maximum S.F. at support

$$= \frac{6290882}{2 \times 8}\ N$$

$$= 393180\ N$$

Design at support section. Provide a girder section

$$40\ cm. \times 75\ cm.$$

Effective depth $= 75 - 6 = 69\ cm.$

Equivalent S.F. $= S_e = S + 1{\cdot}6\,\frac{T}{b}$

$$= 393180 + \frac{1{\cdot}6 \times 1509800}{40}\ N$$

$$= 453572\ N$$

Equivalent nominal shear stress

$$= q_{ve} = \frac{453572}{40 \times 69} = 164\ N/cm.^2$$

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For M 15 concrete,

$$\sigma_{cbc} = 1{\cdot}6\ N/mm.^2 = 160\ N/cm.^2$$

Longitudinal reinforcement

Equivalent B.M. $= M_{e1} = M + M_t$

where $$M_t = \frac{\left[1+\frac{D}{b}\right]}{1{\cdot}7}$$

$$= \frac{1509800}{1{\cdot}7}\left(1+\frac{75}{40}\right) N\ cm.$$

$$= 2553340\ N\ cm.$$

$$M_{e1} = 20885700 + 2553340 = 23439040\ N\ cm.$$

$$A_{st} = \frac{23439040}{14000 \times 0{\cdot}87 \times 69} = 27{\cdot}88\ cm.^2$$

Provide 10 bars of 20 mm. ϕ (31·4 cm.^{2})

Transverse reinforcement

Distance between centres of corner bars parallel to the width

$$= b_1 = 40 - 2 \times 4 = 32\ cm.$$

Distance between centres of corner bars parallel to the depth

$$= d_1 = 75 - 2 \times 6 = 63\ cm.$$

Area of section of stirrups

$$= A_v = \frac{Tp}{b_1 d_1 \sigma_{st}} + \frac{Sp}{2{\cdot}5\, d_1 \sigma_{st}}$$

Providing 4-legged 14 *mm* ϕ stirrups

$$A_v = 4 \times 0{\cdot}79 = 3{\cdot}16\ cm.^2$$

$$3{\cdot}16 = \left[\frac{1509800}{32 \times 63 \times 14000} + \frac{393180}{2{\cdot}5 \times 63 \times 14000}\right] p$$

$$3{\cdot}16 = (0{\cdot}0534 + 0{\cdot}178)\, p$$

$$\therefore p = 13{\cdot}7\ cm.$$ say *10 cm. c/c*

Steel for hogging moment

$$= \frac{19468000}{14000 \times 0{\cdot}87 \times 69} = 12{\cdot}40\ cm.^2$$

Provide 4 bars of 20 mm ϕ

Design of the bottom slab

Cantilever projection beyond the face of the beam

$$= \frac{1{\cdot}50 - 0{\cdot}40}{2} = 0{\cdot}55\ m.$$

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Maximum bending moment for a 1 *m.* wide strip

$$= 142618 \times \frac{0{\cdot}55^2}{2} \times 100 \ N\ cm.$$

$$= 2157100 \ N\ cm.$$

Equating the M.R. to the B M.

$$85 \times 100\, d^2 = 2157100$$

$$\therefore \quad d = \sqrt{\frac{2157100}{8500}} = 15{\cdot}93\ cm.$$

Let us provide an overall depth of 20 *cm.* and an effective depth of 16 *cm.*

$$\therefore \quad A_s = \frac{2157100}{14000 \times 0{\cdot}87 \times 16} = 11{\cdot}06\ cm.^2$$

Spacing of 16 *mm.* diameter bars

$$= \frac{2{\cdot}01 \times 100}{11{\cdot}06} = 18\ cm.\ centres.$$

Distribution steel $= \frac{0{\cdot}15}{100} \times 20 \times 100 = 3\ cm.^2$

Provide 8 mm. ϕ bars @ 16 cm. centres.

Check for sliding

Total load on the foundation when the tank is empty

$$= 6290882 - 3693220 = 2597662\ N$$

Horizontal force on the base

$$= 74190\ N$$

Assuming a coefficient of friction of 0·5

Factor of safety against sliding

$$= \frac{0{\cdot}5 \times 2597662}{74190} = 17$$

Design 128. *Design an Intze tank to the dimensions shown in Fig. 497*

Solution.

(*i*) Design of the Roof Dome.

Provide a thickness of 15 *cm.* for the roof dome.

Dead Load	$= 15 \times 24 = 360\ kg./m.^2$
Live Load	$= 150\ kg./m.^2$
Total	$= 510\ kg./m.^2$

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Let R be the radius of the dome

$$1{\cdot}6\,(2R-1{\cdot}6)=5\times 5$$

$$\therefore \quad R=8{\cdot}61 \text{ m}.$$

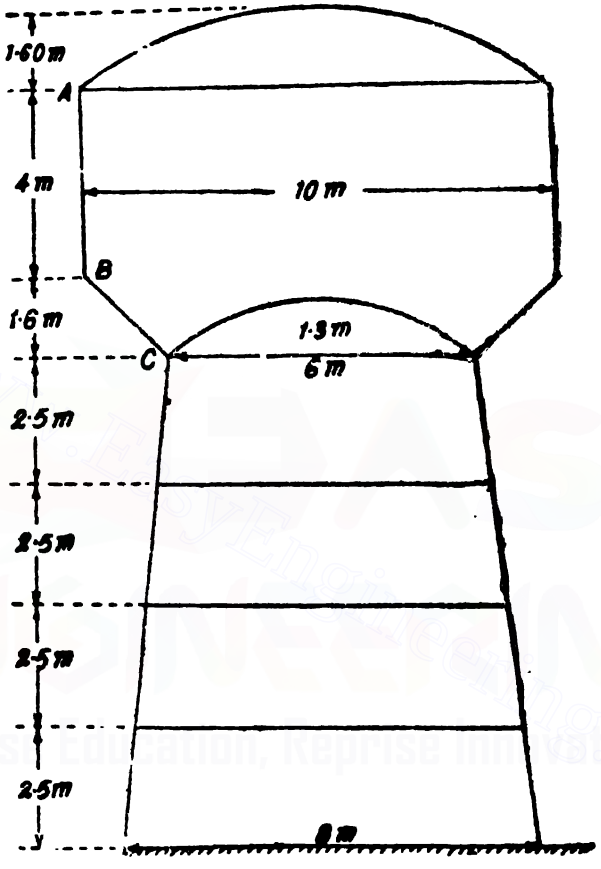

Fig. 497

Let 2θ be the angle subtended by the dome at its centre

$$\therefore \quad \sin\theta=\frac{d}{2R}$$

$$\sin\theta=\frac{10}{2\times 8{\cdot}61}$$

$$\therefore \quad \theta=35^\circ\ 30'$$

$$\therefore \quad \cos\theta=\cos 35^\circ\ 30'=0{\cdot}8141$$

Hoop stress at the level of the springing

$$-f=\frac{wR}{t}\left[\cos\theta-\frac{1}{1+\cos\theta}\right]$$

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$$= -\frac{510 \times 8{\cdot}61}{0{\cdot}15}\left[0{\cdot}8141 - \frac{1}{1{\cdot}8141}\right] kg./m.^2$$

$$= 7696\ kg./m.^2$$

$$= 0{\cdot}7696\ kg./cm.^2$$

Hoop stress at the crown

i.e. at $\theta = 0$

$$f = \frac{wR}{t}\left[1 - \frac{1}{2}\right]$$

$$= 0{\cdot}5\ \frac{wR}{t} = \frac{0{\cdot}5 \times 510 \times 8{\cdot}61}{0{\cdot}15}\ kg./m.^2$$

$$= 14636\ kg./m.^2 = 1{\cdot}46\ kg./cm.^2$$

per *metre* run

Meridional thrust at the level of the springing

$$= T_1 = \frac{wR}{1+\cos\theta} = \frac{510 \times 8{\cdot}61}{1{\cdot}8141} = 2420\ kg./m.$$

$\therefore$ Meridional stress $= \frac{2420}{15 \times 10} = 1{\cdot}61\ kg./cm^2.$

These stresses are very small. Provide nominal reinforcement say 8 *mm.* ϕ @ 11 *cm.* centres both ways.

(ii) **Ring beam at top**

Horizontal component of T_1

$$= T_1 \cos\theta = 2420 \times 0{\cdot}8141 = 1970\ kg./m$$

Hoop tension in the ring beam

$$= 1970 \times \frac{10}{2} = 9850\ kg.$$

$\therefore$ Steel required for hoop tension

$$= \frac{9850}{1000} = 9{\cdot}85\ cm.^2$$

Provide 4 bars of 18 mm. diameter (10·16 cm.²)

Size of the ring beam

Let the area of the ring beam section be $A\ cm.^2$

Equivalent concrete area $= A + (m-1)\ A_s$

$$= A + (13-1)\ 10{\cdot}16$$

$$= A + 121\ 92\ cm.^2$$

Limiting the tensile stress on the equivalent concrete area to 12 *kg./cm.*²

$$\frac{9850}{A + 121{\cdot}92} = 12$$

$$A = 699\ cm.^2$$

Provide 25 *cm* × 30 *cm* section.

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(iii) Cylindrical wall

Pressure intensity at the bottom of the cylindrical wall
$= 1000 \times 4 = 4000\ kg./m.^2$

Consider the bottom 1 m strip of wall

Hoop tensio $= \frac{pd}{2} = 4000 \times \frac{10}{2} = 20{,}000\ kg.$

$\therefore$ $A_s = \frac{20{,}000}{1000} = 20\ cm.^2$

Spacing of 12 *mm* diameter bars

$= \frac{1{\cdot}13 \times 100}{20} = 5{\cdot}65\ cm.$

Provide 12 mm. ϕ bars @ 10 cm centres near each face.

Thickness of the wall may be kept at 20 cm.

Distribution steel $= \frac{0{\cdot}3}{100}(20 \times 100) = 6\ cm.^2$

Spacing of 8 *mm* ϕ bars

$= \frac{0{\cdot}5 \times 100}{6} = 8{\cdot}3\ cm$

Provide 8 mm ϕ @ 16 cm centres near each face.

Check for compressive stress at the bottom of the cylindrical wall.

Vertical component of T_1

$= V_1 = T_1 \sin\theta$

$= 2420 \sin 35^\circ 30' = 1405\ kg/m.$

Weight of wall

$= 0{\cdot}2 \times 4 \times 2400 = 1920\ kg./m.$

Weight of ring beam

$0{\cdot}25 \times 0{\cdot}30 \times 2400 = 180\ kg/cm.$

V_2 = Total vertical load per *metre* $= 3505\ kg/m$

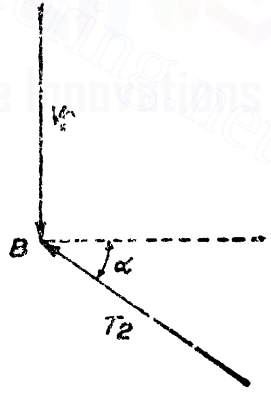

Fig. 498

$\therefore$ Compressive stress

$\frac{3505}{20 \times 100} = 1{\cdot}75\ kg/cm.^2$

The stress being low provide nominal vertical steel at 0·30% gross area.

$\therefore$ Vertical steel $= \frac{0{\cdot}3}{100}(20 \times 100) = 6\ cm.^2$

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Spacing of 8 *mm* ϕ bars

$$= \frac{0{\cdot}5 \times 100}{6} = 8{\cdot}3 \text{ cm.}$$

Provide 8 mm. ϕ bars @ 16 cm. c/c near each face.

(*iv*) **Ring beam at B.**

Let T_2 be the thrust per *metre* run exerted by the conical slab at the junction *B*.

Resolving vertically at *B*,

$$T_2 \sin \alpha = V_2$$

$$\tan \alpha = \frac{1{\cdot}6}{2} = 0{\cdot}8$$

Resolving horizontally at *B*,

$$H_2 = T_2 \cos \alpha = V_2 \cot \alpha$$

$$= \frac{3505}{0{\cdot}8} = 4381 \text{ kg./m.}$$

This horizontal load H_2 will produce a hoop tension in the ring beam *B*.

Hoop tension due to H_2

$$= H_2 \frac{d}{2}$$

$$= 4381 \times \frac{10}{2} \text{ kg.}$$

$$= 21905 \text{ kg.}$$

Let the ring beam be 60 *cm.* deep

Water pressure on the ring beam

$$= 1000 \times 4 \times \frac{60}{100} \text{ kg./m.}$$

$$= 2400 \text{ kg./m.}$$

$\therefore$ Hoop tension due to water pressure

$$= 2400 \times \frac{10}{2} = 12000 \text{ kg.}$$

$\therefore$ Total hoop tension

$$= 21905 + 12000 = 33905 \text{ kg.}$$

$\therefore$ Steel for hoop tension

$$= \frac{33905}{1000} = 33{\cdot}9 \text{ cm.}^2$$

Providing 20 *mm.* ϕ bars, number of bars required

$$= \frac{33{\cdot}9}{3{\cdot}14} = 11 \text{ bars}$$

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Provide 12 bars of 20 mm. diameter.

Steel provided $= 12 \times 3{\cdot}14 = 37{\cdot}68\ cm.^2$

Let the area of the ring beam section be A

Equivalent concrete area

$$= A + (m-1)\,A_s$$
$$= A + (13-1)\,37{\cdot}68$$
$$= A + 452{\cdot}16.$$

Limiting the tensile stress on the equivalent concrete area to 12 $kg./cm.^2$

$$\frac{33905}{A + 452\ 16} = 12$$

$$\therefore \quad A = 2374\ cm.^2$$

Provide 40 $cm.$ × 60 $cm.$ section.

(*v*) **Design of the conical slab.**

The conical slab should be designed for (*a*) a hoop tension and (*b*) for bending as it spans as a sloping slab from the ring beam at *B* to the ring girder at *C*.

Design for hoop tension

The hoop tension of the conical slab is given by

$$\frac{W_w + W_s}{2\pi} + \frac{W_w}{2\pi} \tan \alpha$$

where W_w = Weight of water resting on the conical slab

W_s = Weight of the conical slab

α = Inclination of the conical slab with the horizontal.

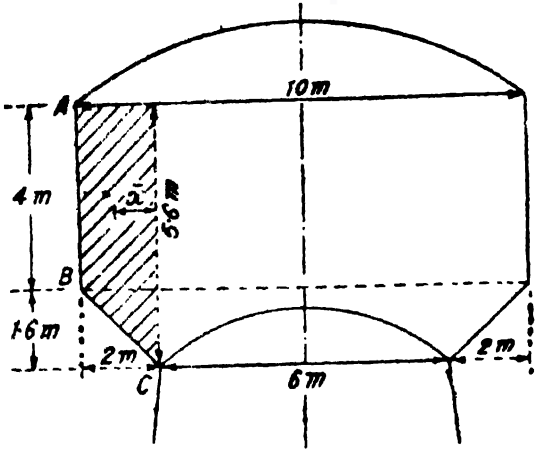

Fig. 499.

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Area of water section standing on the conical slab

$$= \frac{2}{2}\,[4+5{\cdot}6] = 9{\cdot}6 \text{ metre.}^2$$

$$\bar{x} = \frac{5{\cdot}6+(2\times 4)}{5{\cdot}6+4} \times \frac{2}{3} = 0{\cdot}94 \text{ m.}$$

∴ Weight of water resting on the conical slab

$$= W_w = 1000 \times 9{\cdot}6 \times \pi\,(3+0{\cdot}94) \text{ kg.}$$

$$= 237660 \text{ kg.}$$

Length of the sloping slab

$$= \sqrt{2^2+1{\cdot}6^2} = 2{\cdot}561 \text{ metre}$$

Thickness of the sloping slab

$$= 20 \text{ cm}$$

∴ Weight of the conical slab

$$W_s = 2{\cdot}561 \times 0{\cdot}20 \times 2400 \times 2\pi \left(3 + \frac{5{\cdot}6}{2}\right) \text{kg.}$$

$$= 30900 \text{ kg.}$$

∴ Hoop tension $= \dfrac{237660+30900}{2\pi} + \dfrac{237660}{2\pi} \times \dfrac{1{\cdot}6}{2}$

$$= 73000 \text{ kg.}$$

∴ Hoop steel on the entire section

$$= \frac{73000}{1000} = 73 \text{ cm}^2$$

Provide 40 bars of 16 mm diameter.

These bars may be distributed near both the faces of the conical slab.

Design for bending moment

Load per *metre* width of the conical slab

$$= \frac{W_w + W_s}{2\pi \times \text{mean radius}} = \frac{237660+30900}{2\pi \times 4} \text{ kg.}$$

$$= 10690 \text{ kg.}$$

Maximum bending moment

$$= \frac{10690 \times 2}{8} \times 100 \text{ kg cm.}$$

$$= 267250 \text{ kg cm.}$$

Axial compression

$$T_2 = \frac{V_2}{\sin\alpha}$$

$$= 3505 \times \frac{2{\cdot}561}{1{\cdot}6} = 5610 \text{ kg.}$$

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Providing 16 *mm.* diameter bars at a clear cover of 25 *mm.*

Effective depth $= 20 - 2{\cdot}5 - 0{\cdot}8 = 16{\cdot}70$ *cm.*

Distance between centre of section and centre of steel

$$= x = d - \frac{t}{2}$$

$$= 16{\cdot}70 - \frac{20}{2} = 6{\cdot}7 \text{ cm.}$$

$\therefore$ Resultant bending moment

$$= M + T_2 x$$
$$= 267250 + 5610 \times 6{\cdot}7 \text{ kg. cm.}$$
$$= 304840 \text{ kg. cm.}$$

$\therefore$ $$A_s = \frac{304840}{1250 \times 0{\cdot}84 \times 16{\cdot}7} = 17{\cdot}38 \text{ cm.}^2$$

$\therefore$ Spacing of 16 *mm.* diameter bars

$$= \frac{2{\cdot}01 \times 100}{17{\cdot}38} = 11{\cdot}5 \text{ cm.}$$

Provide 16 mm. ϕ bars @ 11 cm. centre

(*vi*) **The bottom dome**

Span of the dome $= 6$ *m.*

Rise of the dome $= 1{\cdot}3$ *m.*

Let R be the radius of the dome

$1{\cdot}3\ (2R - 1{\cdot}3) = 3 \times 3$

$\therefore$ $R = 4{\cdot}11$ *m*

Let 2θ be the angle subtended by the dome at its centre

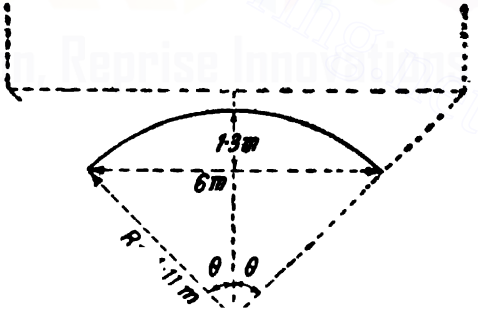

Fig. 500

$$\sin\theta = \frac{3}{4{\cdot}11} = 0{\cdot}73$$

$\theta = 46^\circ\ 53'$

$\therefore$ $\cos\theta = 0{\cdot}68$

Thickness of dome $= 20$ *cm*

Loads

Dead load of the dome

$$= 24 \times 20 = 480 \text{ kg./m.}^2$$

Weight of water resting on the dome

$$= 1 \left[\frac{\pi}{4} \times 6^2 \times 5{\cdot}6 - \frac{\pi \times 1{\cdot}3^2}{4} (3 \times 4{\cdot}11 - 1{\cdot}3) \right] \text{kg}$$

$$= 138820 \text{ kg.}$$

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Area of dome surface

$$= 2\pi \times 4{\cdot}11 \times 1{\cdot}3 = 33{\cdot}57 \text{ metre}^2$$

Load intensity due to weight of water

$$= \frac{138820}{33\,57} = 4135 \text{ kg./metre}^2$$

Total load intensity on the dome

$$= w = 480 + 4135 = 4615 \text{ kg./metre}^2$$

Meridional thrust per *metre* run at the springing level

$$T' = \frac{wR}{1+\cos\theta} = \frac{4615 \times 4{\cdot}11}{1\,68}$$

$$= 11290 \text{ kg./metre}$$

Meridional compressive stress

$$= \frac{11290}{100 \times 20} = 5{\cdot}645 \text{ kg./cm.}^2$$

Hoop stress $$= \frac{wR}{t}\left[\cos\theta - \frac{1}{1+\cos\theta}\right]$$

Maximum hoop stress occurs at the crown, *i.e.*, at $\theta = 0$

$\therefore$ Maximum hoop stress

$$= \frac{4615 \times 4{\cdot}11}{0{\cdot}20}\left[1 - \frac{1}{2}\right]$$

$$= 47420 \text{ kg./metre}^2$$

$$= 4{\cdot}74 \text{ kg./cm.}^2$$

These stresses are low and hence provide nominal 0·3% steel.

Provide 8 mm. ϕ bars @ 8 cm. centres

(*vii*) Circular girder

The total load on the circular girder consists of the following :

Weight of water	$= 237660 + 138820$	$= 376480$ kg.
Weight of top dome and side wall	$= 3505 \times 2\pi \times 5$	$= 110110$ kg.
Weight of conical wall		$= 30900$ kg.
Weight of the lower dome	$= 480 \times 33{\cdot}57$	$= 16110$ kg.
Weight of circular girder 40 cm × 60 cm	$= 0{\cdot}4 \times 0{\cdot}6 \times 2400 \times 2\pi \times 3$	$= 10860$ kg.
	Total load	$w = 544460$ kg.

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Let us provide 8 columns

Max. negative B.M. $=0{\cdot}0083\ Wr$

$=0{\cdot}0083 \times 544460 \times 3 = 13557\ kg.\ m.$

Max. positive B.M. $=0{\cdot}00416\ Wr$

$=0{\cdot}00416 \times 544460 \times 3 = 6795\ kg.\ m.$

Max. Torsion $=0{\cdot}0006\ Wr$

$=0{\cdot}0006 \times 544460 \times 3 = 980\ kg.\ m.$

S.F. at support $= \dfrac{W}{2 \times \text{Number of columns}}$

$= \dfrac{544460}{2 \times 8} = 34029\ kg.$

Design at support section

Equating the moment of resistance to the B.M. at support

$$14{\cdot}03\ bd^2 = 14{\cdot}03 \times 40\ d^2 = 1355700$$

$$\therefore \quad d = 49{\cdot}1\ cm.$$

Overall depth of beam

$$= 60\ cm.$$

Actual effective depth

$$= 60 - 4 = 56\ cm.$$

Equivalent S F. $= S_e = S + 1{\cdot}6\ \dfrac{T}{b}$

$$= 34029 + \frac{1{\cdot}6 \times 98000}{40} = 37949\ kg.$$

Equivalent nominal shear stress

$$= q_{ve} = \frac{37949}{40 \times 56} = 16{\cdot}9\ kg/cm.^2$$

For M 150 concrete

$$q_{c\,max} = 16\ kg./cm^2$$

Longitudinal reinforcement

Equivalent B.M. $= M_{e1} = M + M_t$

where $M_t = T\left[\dfrac{1 + \dfrac{D}{b}}{1{\cdot}7}\right] = \dfrac{98000}{1{\cdot}7}\left[1 + \dfrac{60}{40}\right]\ kg.\ cm.$

$= 144118\ kg.\ cm.$

$M_{e1} = 1355700 + 144118 = 1499818\ kg.\ cm.$

$$A_t = \frac{1499818}{1000 \times 6{\cdot}84 \times 56} = 31{\cdot}8\ cm.^2$$

Provide 10 bars of 20 mm. ϕ (31·4 cm²)

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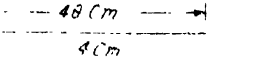
Transverse reinforcement

Distance between centres of corner bars parallel to the width
$$= b_1 = 40 - 2 \times 4 = 32 \text{ cm.}$$
Distance between centres of corner bars parallel to the depth
$$= d_1 = 60 - 2 \times 4 = 52 \text{ cm.}$$

Area of section of stirrups

$$A_w = \frac{fT}{b_1 d_1 t_w} + \frac{Sp}{2.5 d_1 t_w}$$



Providing 4-legged 10 mm. ϕ stirrups,

$$A_w = 4 \times 0.79 = 3.16 \text{ cm.}^2$$

$$3.16 = \left[\frac{98000}{32 \times 52 \times 1400} + \frac{24029}{2.5 \times 52 \times 1400}\right] p$$

$$3.16 = (0.042 + 0.167)\, p$$

$$\therefore \quad p = 15.8 \text{ c.m. say } 10 \text{ cm. c/c.}$$

Fig. 501.

Steel for sagging moment

$$= \frac{679500}{1250 \times 0.86 \times 52} = 12.15 \text{ cm.}^2$$

Provide 4 bars of 20 mm. ϕ.

Hoop stress

Let T_s = thrust exerted by the conical slab on the girder

$\therefore \quad T_s \sin \alpha \, 2\pi r = W_w + W_s + wt$ of cylindrical wall and upper dome

$$= 237660 + 30900 + 110110$$
$$= 378670 \text{ kg.}$$

$$\therefore \quad T_s = \frac{378670}{\sin 38^\circ 39' \times 2\pi \times 3} = 33142 \text{ kg.}$$

Horizontal component of T_s

$$= 32142 \cos \alpha$$
$$= 32142 \cos 38^\circ 39' = 25098 \text{ kg.}$$

Horizontal component of T'

$$= T' \cos \theta$$
$$= 11290 \cos 46^\circ 53' = 7717 \text{ kg.}$$

$\therefore$ **Net horizontal force**

$$= 17381 \text{ kg.}$$

$\therefore$ **Hoop force** $= 17381 \times 3 = 52143 \text{ kg.}$

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∴ Hoop compressive stress

$$= \frac{52143}{40 \times 60} = 21{\cdot}72 \text{ kg./cm}^2$$

(*viii*) **Columns.** The columns should be designed for direct loads coming upon them and for the bending moments caused by wind load.

Vertical load on one column at top

$$= \frac{54460}{8} = 68058 \text{ kg.}$$

Let α be the inclination of the column with the vertical

$$\tan \alpha = \frac{1}{10} \quad \therefore \quad \sin \alpha = \frac{1}{\sqrt{101}}$$

$$= 0{\cdot}0995 \text{ and } \cos \alpha = \frac{10}{\sqrt{101}} = 0{\cdot}9950$$

Actual length of the column

$$= 10 \sec \alpha$$

$$= \frac{10}{0{\cdot}9950} = 10{\cdot}05 \text{ metre.}$$

Providing 40 *cm.* diameter column

Weight of column $= \frac{\pi}{4} (0{\cdot}4)^2 \times 10{\cdot}05 \times 2400 = 3031$ kg.

Total vertical load $= 68058 + 3031 = 71089$ kg.

∴ Corresponding axial load

$$= \frac{71089}{0{\cdot}9950} = 71446 \text{ kg.}$$

Weight of water in the tank

$$= 376480 \text{ kg.}$$

∴ Weight of water transmitted to one column

$$= \frac{376480}{8} = 47060 \text{ kg.}$$

∴ Vertical load on column when the tank is empty

$$= 71089 - 47060 = 24029 \text{ kg.}$$

Corresponding axial load

$$= \frac{24029}{0{\cdot}9950} = 24150 \text{ kg.}$$

Ignoring wind load effect if the steel requirement is A_s

Then $c(A - A_s) + tA_s = 71446$ kg.

$$50\left[\frac{\pi}{4} + 40^2 - A_s\right] + 1300\ A_s = 71446$$

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$$A_t = 6{\cdot}626 \ cm.^2$$

Minimum requirement of steel

$$= 0{\cdot}8\% \text{ of gross area}$$

$$= \frac{0{\cdot}8}{100}\left(\frac{\pi}{4} \times 40^2\right) = 10{\cdot}10 \ cm.^2$$

Provide 6 bars of 20 mm. diameter

$$= (18{\cdot}84) \ cm.^2$$

(More steel has been suggested since the column will also be subjected to bending moment caused by wind load.)

Analysis due to wind pressure

Assume a wind pressure of 150 *kg./metre*^{2} and a reduction factor of 0·7.

Wind force on the top dome and cylindrical walls

$$= \left(4 + \frac{1{\cdot}6}{2}\right) 10{\cdot}4 \times 150 \times 0{\cdot}7 = 5242 \ kg.$$

acting at 14 *m* above the base.

Wind force on the circular wall

$$= \frac{10{\cdot}4 + 6{\cdot}4}{2} \times 1{\cdot}6 \times 150 \times 0{\cdot}7 = 1411 \ kg.$$

acting at 10·8 *m* above the base

Wind force on the circular girder

$$= 0{\cdot}6 \times 6{\cdot}4 \times 150 \times 0{\cdot}7 = 403 \ kg.$$

acting at 10 *m* above the base

Wind force on column and braces

$$= 5 \times 0{\cdot}4 \times 10 \times 150 \times 0{\cdot}7$$

$$+ 3 \times \left(\frac{6+8}{2}\right) \times 0{\cdot}30 \times 150 \ kg.$$

$$= 3045 \ kg.$$ acting at 5 *m.* above the base

Total moment of wind pressure about the base

$$= (5242 \times 14) + (1411 \times 10{\cdot}8) + (403 \times 10) + (3045 \times 5)$$

$$= 107882 \ kgm.$$

Vertical load on any column

$$\frac{M \cdot x}{\Sigma x^2}$$

$$\Sigma x^2 = (2 \times 4^2) + 4\left(\frac{4}{\sqrt{2}}\right)^2$$

$$= 64 \ metre^2$$

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∴ Maximum wind load force in the most leeward side and the most windward side

$$= -\frac{107882 \times 4}{64} = 6743 \text{ kg.}$$

Maximum wind load force in columns marked *z*

$$= -\frac{107882}{64} \times \frac{4}{\sqrt{2}} = 4769 \text{ kg.}$$

Consider, the windward column 1

Vertical load due to dead load and wind load

$$= 71089 + 6743 \text{ kg.}$$

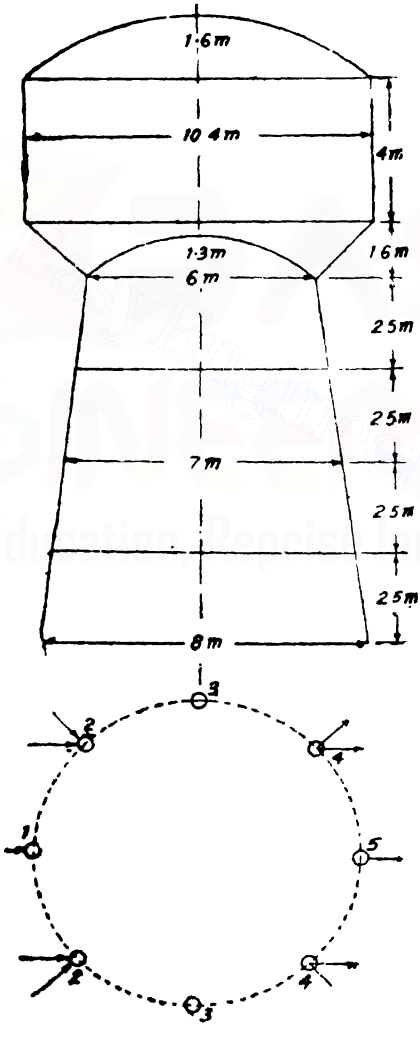

Fig. 502

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$= 77832\ kg.$

Corresponding axial load

$$= \frac{77832}{0\cdot995} = 78223\ kg.$$

Since the columns are inclined the horizontal components of the axial forces caused by wind action reduces the horizontal shear in the column.

Horizontal component of the axial forces caused by wind action

$$= -2 \times 6743 \times 0\cdot0995 + 4 \times 4769 \times 0\cdot0995 \times \frac{1}{\sqrt{2}}\ kg.$$

$$= -2682\ kg.$$

$\therefore$ Actual horizontal force at the base

$$= -(5242 + 1411 + 403 + 3045) - 2682$$

$$= 7419\ kg.$$

$\therefore$ Horizontal shear per column

$$\frac{7419}{8} = 927\ kg.$$

$\therefore$ Maximum bending moment for the column

$$= 927 \times \frac{2\cdot5}{2} = 1159\ kg.\ m.$$

Analysis of the column section

Direct load $= 78223\ kg.$

Bending moment $= 1159\ kg.\ m.$

Provide 40 *cm.* diameter column.

Provide 8 bars of 20 *mm.* diameter, at an effective cover of 5 *cm.*

Area of steel $A_s = 8 \times 3\cdot14 = 25\cdot12\ cm.^2$

Equivalent concrete area

$$= \frac{\pi}{4}(40)^2 + (12 \times 25\cdot12)\ cm.^2$$

$$= 1256\cdot64 + 301\cdot44$$

$$= 1558\ cm^2$$

Polar moment of inertia of the equivalent concrete section

$$= \frac{\pi \times 40^4}{32} + 12 \times 25\cdot12 \times 15^2$$

$$= 251328 + 67824$$

$$= 319152\ cm.^4$$

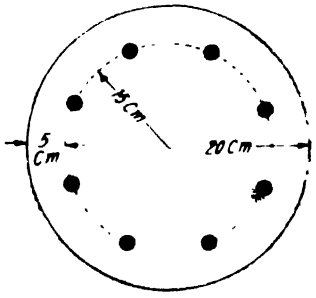

Fig. 503

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∴ Equivalent moment of inertia about a diameter

$$= \frac{319152}{2} = 154576 \ cm^4.$$

Direct stress in concrete

$$= \frac{782 \cdot 3}{1558} = 50 \cdot 21 \ kg/cm.^2$$

Bending stress in concrete

$$= \frac{115900}{159576} \times 20 = 14 \cdot 52 \ kg./cm.^2$$

∴ Maximum stress = 50·21 + 14·52 = 64·73 *kg./cm.*² **(Safe)**

(*ix*) Design of Braces

Consider one of the braces, say the brace *BC*.

For the condition of maximum bending moment for the brace *BC* wind pressure should act normal to an adjoining brace *AB*.

Moment in brace *BC*

= twice the moment for the column × sec. 45°

$= 2 \times 1159 \times \sqrt{2}$

$= 3278 \ kg. \ m.$

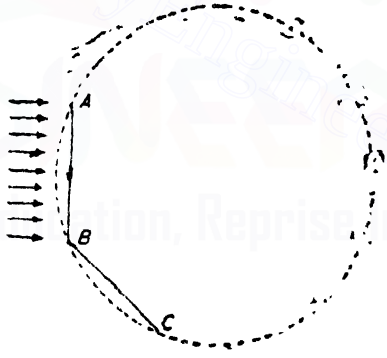

Fig. 504

Providing 30 *cm.* × 30 *cm.* section and designing as a doubly reinforced beam with equal steel at top and bottom

$$A_c = A_{t} = \frac{3278 \times 100}{1400 \times 22} = 10 \cdot 64 \ cm.^2$$

Provide 4 bars of 18 *mm.* diameter at top and an equal amount of steel at bottom.

S.F. for brace $= \dfrac{\text{B.M. for brace}}{\frac{1}{2} \text{ span of brace}}$

Span of brace $= 2 \times \dfrac{7 \cdot 5}{2} \sin 22° \ 30' = 2 \cdot 87 \ m.$

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$$\therefore \quad \text{S.F. for brace} = \frac{3278}{\left(\frac{2\cdot87}{2}\right)} = 2284 \ kg.$$

$$\therefore \quad \text{Shear stress} = \frac{S}{ab} = \frac{2284}{22 \times 30} = 3\cdot46 \ kg./cm.^2 \text{ (Safe).}$$

(x) Design of foundation

Total load on the columns when the tank is full $= 71446 \times 8 = 571568$ *kg.*

Approximate weight of foundation (10% of column loads) $= 57157$ *kg.*

628725 *kg.*

Assuming a safe bearing capacity of 20 *t/metre*^{2}

$$\text{Area of foundation} = \frac{628725}{20000} = 31\cdot44 \ metre^2$$

Let us provide 10 *metre* outer diameter and 7 *metre* inside diameter, for the raft footing.

$$\because \text{ Area of footing} = \frac{\pi}{4}\left[10^2 - 7^2\right] = 40\cdot10 \ metre^2$$

$$\therefore \quad \text{Net upward pressure intensity} = \frac{571568}{40\cdot10} \ kg./metre^2$$

$$= 14250 \ kg./metre^2$$

Design of the circular girder

Maximum negative B.M. at centre

$$= 0\cdot00416 \ Wr$$
$$= 0\cdot00416 \times 628725 \times 4 \ kg. \ m.$$
$$= 10462 \ kg. \ m.$$

Maximum positive B.M. at support

$$= 0\cdot0083 \times 628725 \times 4 \ kg. \ m.$$
$$= 20874 \ kg. \ m.$$

Maximum torsion $= 0\cdot0006 \times 628725 \times 4$
$= 1509 \ kg. \ m$

Maximum S.F. at support

$$= \frac{628725}{2 \times 8} = 39295 \ kg.$$

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Design at support section

Provide a girder section 40 *cm* × 75 *cm*.

Effective depth $= 75 - 6 = 69$ *cm*.

Effective S.F. $= S_e = S + 1{\cdot}6\frac{T}{b}$

$= 39295 + \frac{1{\cdot}6 \times 150900}{40}$ *kg*.

$= 45331$ *kg*.

Equivalent nominal shear stress

$$= q_{ve} = \frac{45331}{40 \times 69} = 16{\cdot}4 \text{ kg./cm.}^2$$

For *M* 150 concrete,

$$q_{max.} = 16 \text{ kg./cm}^2.$$

Longitudinal reinforcement

Equivalent B.M.

$$= M_{e1} = M + M_t$$

where $$M_t = T\frac{\left[1 + \frac{D}{b}\right]}{1{\cdot}7}$$

$$= \frac{150900}{1{\cdot}7}\left(1 + \frac{75}{40}\right) \text{kg. cm.} = 255200 \text{ kg. cm}$$

$$M_{e1} = 2087400 + 255200 = 2342600 \text{ kg. cm.}$$

$$\therefore \quad A_t = \frac{2342600}{1400 \times 0{\cdot}87 \times 69} = 27{\cdot}87 \text{ cm}^2.$$

Provide 10 bars of 20 mm. ϕ (31·4 cm.²)

Transverse reinforcement

Distance between centres of corner bars parallel to the width

$$= b_1 = 40 - 2 \times 4 = 32 \text{ cm.}$$

Distance between centres of corner bars parallel to the depth

$$= d_1 = 75 - 2 \times 6 = 63 \text{ cm.}$$

Area of section of stirrups

$$= A_v = \frac{Tp}{b_1 d_1 t_v} + \frac{Sp}{2{\cdot}5 d_1 t_v}$$

Providing 4-legged 10 *mm*. ϕ stirrups

$$A_v = 4 \times 0{\cdot}79 = 3{\cdot}16 \text{ cm.}^2$$

$$3{\cdot}16 = \left[\frac{150900}{32 \times 63 \times 1400} + \frac{39295}{2{\cdot}5 \times 63 \times 1400}\right]p$$

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$$3{\cdot}16=(0{\cdot}053+0{\cdot}178)p$$

$$\therefore \quad p=13{\cdot}7 \text{ cm. say } 10 \text{ cm. c/c.}$$

Steel for hogging moment

$$=\frac{1046200}{1400\times 0{\cdot}87\times 69} \text{ cm.}^2=12{\cdot}4 \text{ cm}^2.$$

Provide 4 bars @ 20 mm. ϕ.

Design of the bottom slab

Cantilever projection beyond the face of the beam

$$=\frac{1{\cdot}5-0{\cdot}40}{2}=0{\cdot}55 \text{ m.}$$

Maximum bending moment for a 1 *m.* wide strip

$$=14250\times\frac{0{\cdot}55^2}{2}\times 100 \text{ kg. cm.}$$

$$=215530 \text{ kg. cm.}$$

Equating the M.R. to the B.M.

$$8{\cdot}5\times 100\, d^2=215530$$

$$\therefore \quad d=\sqrt{\frac{215530}{850}}=15{\cdot}93 \text{ cm.}$$

Let us provide an overall depth of 20 *cm.* and an effective depth of 16 *cm.*

$$\therefore \quad A_t=\frac{215530}{1400\times 0{\cdot}87\times 16}=11{\cdot}06 \text{ cm.}^2$$

Spacing of 16 *mm.* diameter bars

$$=\frac{2{\cdot}01\times 100}{11{\cdot}06}=18 \text{ cm. centres}$$

Distribution steel $=\frac{0{\cdot}15}{100}\times 20\times 100=3 \text{ cm}^2.$

Provide 8 mm. ϕ bars @ 16 cm. centres.

Check for sliding

Total load on the foundation when the tank is empty

$$=628725-376480=252245 \text{ kg.}$$

Horizontal force on the base

$$=7419 \text{ kg.}$$

Assuming a coefficient of friction of 0·5

Factor of safety against sliding

$$=\frac{0{\cdot}5\times 252245}{7419}=17.$$

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Design 129. *Fig. 505 shows an arrangement in plan for an overhead tank. Ten columns are provided arranged in two rows with 5 columns in each row. The tank is to be supported 6 metres above ground. The tank shall also be roofed. Adopting suitable working stresses design the tank if the maximum depth of water is 2·5 metres. Take weight of R.C.C = 2400 kg /metre³*

Solution. The tank consists of the following components :

(*i*) Roof slab

(*ii*) Roof beams

(*iii*) Floor slab

(*iv*) Floor beams

(*v*) Vertical walls

(*vi*) Columns

(*vii*) Braces

(*viii*) Foundation.

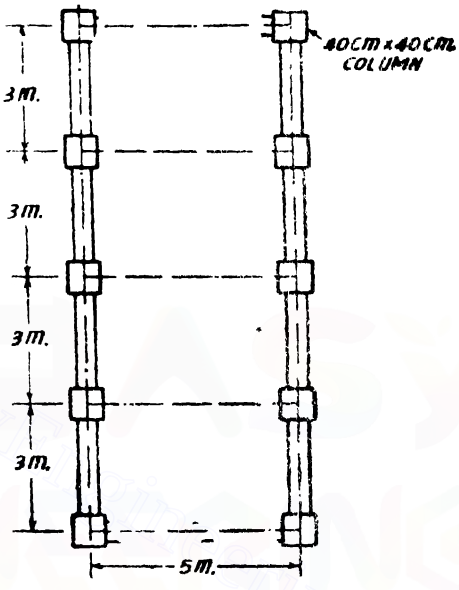

Fig. 505

(*i*) **Roof slab.** The roof slab may be designed as a continuous slab supported on roof beams.

Loads

D.L. of slab (10 *cm.*) = 240 *kg./m².*

Incidental live load = 150 *kg./m.²*

Total = 390 *kg.m.²*

Consider a one *metre* wide strip of the slab

$$\text{Max. B.M.} = \frac{wl^2}{10} = \frac{390 \times 3^2}{10} \text{ kg. m.}$$

$$= 351 \text{ kg./m.}$$

Adopting $c = 70$ *kg./cm.²*, $t = 1250$ *kg./cm.²*, $m = 13$ and equating the moment of resistance to the maximum bending moment, we have

$$12{\cdot}6 \times 100\, d^2 = 351 \times 100 \text{ kg. cm.}$$

$$d = 5{\cdot}3 \text{ cm.}$$

Effective depth available

$$= 10 - 2 = 8 \text{ cm.}$$

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$$\therefore \quad A_t = \frac{3510}{1250 \times 0{\cdot}86 \times 8} \text{ cm.}^2$$

$$= 4{\cdot}08 \text{ cm.}^2$$

$\therefore$ Spacing of 10 *mm.* diameter bars

$$= \frac{0{\cdot}79 \times 100}{4{\cdot}08} \text{ cm.}$$

$$= 19\,4 \text{ cm. say } 19 \text{ cm. centres.}$$

Distribution steel $= \frac{0{\cdot}3}{100} \times 10 \times 100 \text{ cm.}^2 = 3 \text{ cm.}^2$

Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{3} \text{ cm.} = 16 \text{ cm. centres.}$$

(ii) Roof beams

These will be designed as inverted *T*-beams so that there may be no projection of the beam within the tank.

Loads

Consider one *metre* run of the beam

Let the overall depth of beam be 40 *cm.*

Load from roof slab $= 390 \times 3 = 1170$ *kg./m.*

Weight of rib $= 0{\cdot}3 \times 0{\cdot}3 \times 2400 = 216$ *kg./m.*

Total $= 1386$ *kg./m.*

Max. B.M $= \frac{1386 \times 5^2}{8}$ *kg. m.* $= 4331$ *kg. m.*

Equating the moment of resistance to the max. bending moment

$$12{\cdot}6 \times 30\, d^2 = 4331 \times 100$$

$$d^2 = 33{\cdot}9 \text{ cm.}$$

Effective depth available

$$= 40 - 4 = 36 \text{ cm.}$$

$$\therefore \quad A_t = \frac{4331 \times 100}{1250 \times 0{\cdot}86 \times 36} \text{ cm.}^2 = 11{\cdot}2 \text{ cm.}^2$$

Provide four bars of 20 mm. diameter (12·57 cm.²).

Maximum shear force

$$= 1386 \times \frac{5}{2} \text{ kg.}$$

$$= 3465 \text{ kg.}$$

Nominal shear stress $= \frac{S}{bd}$

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$$= \frac{3465}{30 \times 36} \text{ kg./cm.}^2$$

$$= 3{\cdot}21 \text{ kg./cm.}^2$$

Provide two-legged 8 mm. diameter stirrups at 30 cm. centres.

(iii) Floor slab

The floor slab may be designed as a continuous slab supported on the floor beams.

Loads

D.L. of slab (18 *cm.*) = 432 *kg./m.*^{2}

Weight of water $= 1000 \times 2{\cdot}5 = 2500$ *kg./m.*^{2}

Total = 2932 *kg./m.*^{2}

Consider a one *metre* wide strip of the slab

Max. bending moment

$$= \frac{wl^2}{10}$$

$$= \frac{2932 \times 3^2}{10} \text{ kg. m.} = 2639 \text{ kg. m.}$$

Adopting $c = 70$ *kg./cm.*^{2}, $t = 1000$ *kg./cm.*^{2} and $m = 13$, and equating the moment of resistance to the max. bending moment, we have,

$$14{\cdot}11 \times 100\, d^2 = 2639 \times 100$$

$$d = 14{\cdot}2 \text{ cm.}$$

Effective depth available, providing 3 *cm.* effective cover

$$= 18 - 3 = 15 \text{ cm.}$$

$$A_t = \frac{2639 \times 100}{1000 \times 0{\cdot}84 \times 15} \text{ cm.}^2$$

$$= 20{\cdot}94 \text{ cm.}^2$$

Spacing of 18 *mm.* diameter bars

$$= \frac{2{\cdot}54 \times 100}{20{\cdot}93} \text{ cm.}$$

$$= 12{\cdot}13 \text{ cm. say } 12 \text{ cm. centres.}$$

Distribution steel $\frac{0{\cdot}3}{100} \times 18 \times 100$ *cm.*^{2}

$$= 5{\cdot}4 \text{ cm.}^2$$

Spacing of 10 *mm.* diameter bars

$$= \frac{0{\cdot}79 \times 100}{5{\cdot}4} \text{ cm.} = 14{\cdot}63 \text{ cm.}$$

say *14 cm. centres.*

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(*iv*) **Floor beams.** These will be designed as *T*-beams at mid span and rectangular beams at ends. Due to considerable restraint at the ends due to monolithic connection with the columns the beam may be designed for a bending moment of $\pm \frac{wl^2}{10}$.

Loads. Consider one *metre* run of the beam

Let the overall depth of the beam be 60 *cm.*

Load from the floor slab	$=2932\times 3$	$=8796$ *kg./m.*
Weight of rib	$=0{\cdot}3\times 0{\cdot}42\times 2400$	$=302$ *kg./m.*
	Total	$=9098$ *kg./m.*

Maximum bending moment

$$=\pm\frac{9098\times 5^2}{10}\ kg.\ m.$$

$$\pm 22750\ kg.\ m.$$

Mid Section of beam The mid section of the beam will be designed as a *T*-beam.

Approximate effective depth

$$=60-6=54\ cm.$$

Approximate lever arm

$$=d-\frac{d_s}{2}=54-9=45\ cm.$$

Approximate steel requirement

$$=\frac{22750\times 100}{1250\times 45}\ cm.^2=40{\cdot}44\ cm.^2$$

Let us try 8 bars of 24 mm. diameter (36·19 cm.²).

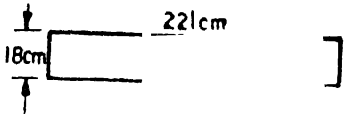
Width of flange. This will be taken as follows :

$$B=\frac{l}{6}+b_r+6\ d_s$$

$$=\frac{500}{6}+30+(6\times 18)$$

$$=221{\cdot}3\ cm.$$

say 221 *cm.*



Clear cover to beam bars $=25$ *mm.*

Clear distance between the two layers of bars

$=12$ *mm.*

$\therefore$ Actual effective depth

$=60-2{\cdot}5-2{\cdot}4-0{\cdot}6$ *cm.*

$=54{\cdot}5$ *cm.*

Fig. 5.16

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Position of neutral axis. Assuming the neutral axis to be outside the flange,

$$221 \times 18(n-9) = 13 \times 36{\cdot}19(54{\cdot}5-n)$$
$$n = 13{\cdot}82 \text{ cm}$$

But thickness of the slab
$$= 18 \text{ cm}.$$

$\therefore$ Neutral axis lies within the slab.

Now the neutral axis is given by

$$221 \times \frac{n^2}{2} = 13 \times 36{\cdot}19(54{\cdot}5-n)$$

$$n^2 + 4{\cdot}258\, n - 232{\cdot}06 = 0$$

$$\therefore \quad n = 13{\cdot}25 \text{ cm}.$$

$\therefore$ Actual lever arm $= d - \frac{n}{3}$

$$= 54{\cdot}50 - \frac{13{\cdot}25}{3} \text{ cm}.$$

$$= 50{\cdot}08 \text{ cm}.$$

Actual stress in steel $= \frac{22750 \times 100}{36{\cdot}19 \times 50{\cdot}08}$ kg./cm.^{2}

$$= 1255 \text{ kg./cm.}^2$$

Corresponding compressive stress in concrete

$$= \frac{t}{m} \times \frac{n}{d-n}$$

$$= \frac{1255}{13} \times \frac{13{\cdot}25}{54{\cdot}5-13{\cdot}82} \text{ kg./cm.}^2$$

$$= 31 \text{ kg /cm}^2$$

End section. Maximum hogging bending moment

$$= 22750 \text{ kg. m}.$$

Let the section be designed as a doubly reinforced beam on the steel beam theory.

Placing the top and bottom bars at the effective cover of 6 cm., distance between the centres of top and bottom bars

$$= 60 - 12 = 48 \text{ cm}.$$

Allowing a stress of 1000 kg./cm.^{2} in the top and bottom bars

$$A_t = A_c = \frac{22750 \times 100}{1000 \times 48} \text{ cm.}^2$$

$$= 47{\cdot}4 \text{ cm}^2.$$

Provide 10 bars of 26 mm. diameter (53·09 cm.^{2})

These bars may be provided up to the points of contraflexure.

Actual distance between centres of top and bottom bars

$$= 60 - 2[2{\cdot}6 + 2{\cdot}6 + 0{\cdot}6] = 48{\cdot}4 \text{ cm}$$

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Fig. 507

Maximum shear force

$$= 9098 \times \frac{5}{2} \text{ kg.}$$

$$= 22745 \text{ kg.}$$

$$\text{Shear stress} = \frac{v}{ab}$$

$$= \frac{22745}{48{\cdot}4 \times 30} \text{ kg/cm.}^2$$

$$= 15{\cdot}7 \text{ kg./cm.}^2$$

Spacing of four-legged 12 *mm.* diameter stirrups

$$= \frac{A_v f_v a}{S}$$

$$= \frac{4 \times 1{\cdot}13 \times 1250 \times 48{\cdot}4}{22745} \text{ cm.}$$

$$= 12 \text{ cm. centres.}$$

(v) Vertical walls

These walls may be designed either as propped cantilevers or as spanning horizontally. In our case let us design the vertical walls as propped cantilevers.

Maximum pressure intensity at the bottom of the wall

$$= p = 1000 \times 2{\cdot}5 \text{ kg/m.}^2$$

$$= 2500 \text{ kg./m.}^2$$

Fig. 508 (*a*)

Maximum bending moment producing tension near the water face

$$= \frac{ph^2}{15}$$

$$= \frac{2500 \times 2{\cdot}5^2}{15} \text{ kg. m.}$$

$$= 1040 \text{ kg. m.}$$

Maximum bending moment producing tension away from the water face

$$= \frac{ph^2}{33{\cdot}5}$$

$$= \frac{2500 \times 2{\cdot}5^2}{33{\cdot}5} \text{ kg. m.}$$

$$= 466 \text{ kg. m.}$$

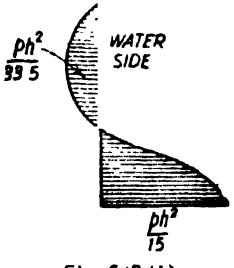

Fig. 508 (*b*)

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Equating the moment of resistance to the maximum bending moment, we have,

$$14{\cdot}11 \times 100\, d^2 = 1040 \times 100$$

$$d = 8{\cdot}6 \text{ cm.}$$

If 12 *mm.* diameter bars be used at clear cover of 25 *mm.*, overall depth required

$$= 8{\cdot}6 + 2{\cdot}5 + 0{\cdot}6 \text{ cm.}$$

$$= 11{\cdot}7 \text{ cm. say } 12 \text{ cm.}$$

Let us provide 12 *cm.* thick wall.

Actual effective depth

$$= 12 - 3{\cdot}1 = 8{\cdot}9 \text{ cm.}$$

∴ Steal requirement for maximum bending moment

$$= \frac{1040 \times 100}{1000 \times 0{\cdot}84 \times 8{\cdot}9} \text{ cm.}^2$$

$$= 13{\cdot}91 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{13{\cdot}91} \text{ cm.}$$

$$= 8{\cdot}12 \text{ say } 8 \text{ cm centres.}$$

Steel for a bending moment of 466 *kg. m.* producing tension away from the water side

$$= \frac{466 \times 100}{1250 \times 0{\cdot}86 \times 8{\cdot}9} \text{ cm.}^2$$

$$= 4{\cdot}87 \text{ cm.}^2$$

Spacing of 12 *mm.* dia bars.

$$= \frac{1{\cdot}13 \times 100}{4{\cdot}87} \text{ cm.}$$

$$= 23{\cdot}2 \text{ say } 23 \text{ cm. centres}$$

Distribution steel $= 0{\cdot}3\%$ of gross area

$$= \frac{0{\cdot}3}{100} \times 12 \times 100 \text{ cm.}^2$$

$$= 3{\cdot}6 \text{ cm.}^2$$

Spacing of 10 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{3{\cdot}6} \text{ cm.}$$

$$= 13{\cdot}9 \text{ cm. say } 13{\cdot}5 \text{ cm.}$$

If the steel be provided near both the faces the spacing will be 27 cm. centres.

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(*vi*) **Columns.** Load transferred to one column from tank and its contents

= half the load on roof beam
+ half the load on floor beam

$$= 1386 \times \frac{5}{2} \times 9098 \times \frac{5}{2}$$

$$= 3465 + 22745 \text{ kg.} = 26210 \text{ kg.}$$

Wind load analysis. Wind load on the vertical walls only will be considered.

Height of wall for considering wind load

= 2·5 *m.* + thickness of roof slab
+ thickness of floor slab

= 2·50 + 0·10 + 0·18 *metres*

= 2·78 *metres.*

Let us allow a wind load of 150 *kg./metre*^{2}

Total wind load transferred to one pair of columns

$$= 150 \times 2{\cdot}78 \times 3 \text{ kg.} = 1250 \text{ kg.}$$

Let us assume that points of contraflexure occur at the middle points of the various panels.

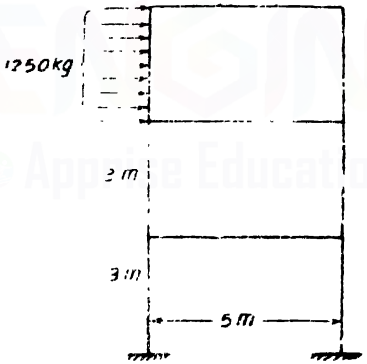

Fig. 509

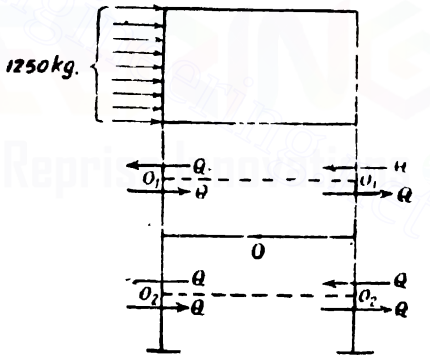

Fig. 510

Shear force Q just above and below a point of contraflexure of a column is given by the condition

$$2Q = 1250 \text{ kg.}$$

$$\therefore \quad Q = 625 \text{ kg.}$$

$\therefore$ B.M at the top of column, just above and below the brace and at the base of the column

$$= 625 \times 1{\cdot}5 \text{ kg. m.} = 937{\cdot}5 \text{ kg. m.}$$

We will design the column section at the base.

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Let the direct load transferred to one column due to wind pressure, at the level of the lower points of contraflexure be R kg.

Taking moments about the lower points of contraflexure, we have

$$R \times 5 = 1250 \left[3 + 1{\cdot}5 \times \frac{2{\cdot}78}{2}\right] \text{ approximately}$$

$\therefore$ $R = 7360$ kg.

Loads at the level of the base of the leeward column

Load from tank and contents = 26210 kg.

Direct load due to wind pressure = 7360 kg.

Weight of columns (30 cm. × 30 cm.)

$= 0{\cdot}3 \times 0{\cdot}3 \times (6 + 2{\cdot}78) 2400$ (approximately) = 1896 kg.

Half the weight of longitudinal and transverse brace (30 cm. × 30 cm.)

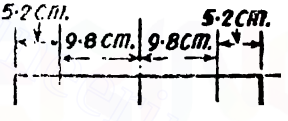
$0{\cdot}3 \times 0{\cdot}3 \left[\frac{5+3}{2}\right] \times 2400$ = 8 4 kg.

Total load = 36330 kg.

Maximum bending moment = 937 5 kg. m.

Now onwards the working stresses to be adopted will be as follows, since the usual mix may be adopted. $c = 40$ kg./cm.^{2} direct and 50 kg./cm.^{2} (bending), $t = 1400$ kg./cm.^{2}

$m = 18$.



Let four bars of 24 m.m. diameter be used.

Clear cover to bars = 4 cm.

$\therefore$ Effective cover $= 4 + 1{\cdot}2$

$= 5{\cdot}2$ cm

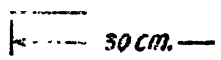


Fig. 511

Equivalent concrete area

$= 30^2 + (18 - 1) \times 4 \times 4{\cdot}52$ cm.^{2}

$= 1207{\cdot}4$ cm.^{2}

Equivalent moment of inertia

$$= \frac{30^4}{12} + (18 - 1) \times 4 \times 4{\cdot}52 \times 9{\cdot}8^2 \text{ cm.}^4$$

$= 97010$ cm.^{4}

.. Extreme stresses in concrete are

$$\frac{36330}{1207{\cdot}4} + \frac{937{\cdot}5 \times 100}{97010} \times 15 \text{ kg./cm.}^2$$

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$$= 30{\cdot}11 \pm 14{\cdot}50 \text{ kg./cm.}^2$$

$$= 44{\cdot}61 \text{ kg./cm.}^2 \text{ and } 15{\cdot}61 \text{ kg./cm.}^2$$

(vi) Braces. Maximum B.M. for the brace

= Sum of column moments just above and below the brace

$$= 937{\cdot}5 + 937{\cdot}5 = 1875 \text{ kg. m.}$$

Let an effective cover of 4 *cm.* be provided. The brace will be designed as a doubly reinforced beam on the steel beam theory

Let the brace section be 30 *cm.* × 30 *cm.*

Distance between centres of top and bottom bars

$$= 30 - 8 = 22 \text{ cm.}$$

$$\therefore \quad A_c = A_t = \frac{1875 \times 100}{1400 \times 22} \text{ cm.}^2$$

$$= 6{\cdot}09 \text{ cm}^2.$$

Provide 4 bars of 16 mm. diameter (8·04 cm.²) at top and 4 bars of 16 mm. diameter at the bottom.

Maximum shear force

$$= \frac{\text{B.M. for the brace}}{\text{Half the span of brace}}$$

$$= \frac{1875}{5} = 375 \text{ kg.}$$

$$\text{Nominal shear stress} = \frac{S}{bd}$$

$$= \frac{375}{30 \times 26} \text{ kg./cm.}^2 = 0{\cdot}48 \text{ kg./cm.}^2$$

Provide nominal two-legged 10 mm. diameter stirrups at a spacing of 20 cm. centres

(vii) Foundation

Let the safe bearing capacity of the soil be 20 *tonnes per metre*²

Column load = 36330 *kg.*

Foundation (10%) = 3630 *kg.*

= 39960 *kg.*

Approximate area of foundation

$$= \frac{39960}{20000} = 1{\cdot}998 \text{ metre}^2$$

Let us try 1·5 *metres* × 1·5 *metres*

$$\text{Eccentricity} = e = \frac{937{\cdot}5}{39960} = 0{\cdot}02 \text{ m.}$$

(This being small, may be neglected.)

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Net upward presure

$$= \frac{36330}{1{\cdot}5 \times 1{\cdot}5} \text{ kg./m}^2 = 16147 \text{ kg./m.}^2$$

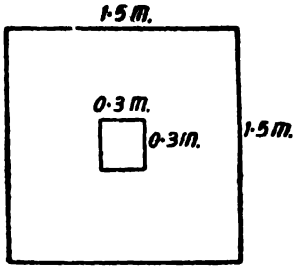

Fig. 512

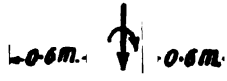

Fig. 513

Cantilever moment per *metre* width

$$= 16147 \times \frac{0{\cdot}6^2}{2} \text{ kg. m.}$$

$$= 2907 \text{ kg. m.}$$

Equating the moment of resistance to the bending moment

$$= 8{\cdot}5 \times 100d^2$$

$$= 2907 \times 100$$

$$d = 18{\cdot}5 \text{ cm.}$$

Providing an effective cover of 8 *cm.* overall depth required

$$= 18{\cdot}5 + 8 = 26{\cdot}5 \text{ cms,}$$

say 27 *cms*

Actual effective depth

$$= 27 - 8 = 19 \text{ cm.}$$

$$A_s = \frac{2907 \times 100}{1400 \times 0{\cdot}87 \times 19} \text{ cm.}^2$$

$$= 12{\cdot}56 \text{ cm.}^2$$

∴ Steel required in the whole width

$$= 12{\cdot}56 \times 1{\cdot}5 \text{ cm.}^2$$

$$= 14{\cdot}84 \text{ cm.}^2$$

Provide 10 bars of 16 mm. diameter in each principal direction.

§185. Small Overhead Circular Tanks

A small overhead circular tank consists of the following :

(*i*) Cylindrical wall,

(*ii*) Circular slab, and

(*iii*) Supporting beams and columns.

The cylindrical wall may be designed for hoop tension. The circular slab may be supported by four beams of similar spans

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connecting the column tops It is usual to design the floor slab to carry the total load W by providing reinforcement in two principal directions. Alternate bars may be bent up near the line of beams. The bending moment for the floor slab may be taken as

$\frac{W}{26}$ *Nm per metre* (*kg. m. per metre*).

The beams and columns may be designed as in the case of overhead square tanks designed earlier.

Design 130 *Fig. 514 shows an arrangement for a circular overhead tank supported on four columns Design the tank.*

Solution Maximum water pressure

$$= 9810 \times 2{\cdot}5 - 24525 \ N/metre^2$$

Maximum hoop tension per *metre* height

$$= \frac{24525 \times 3{\cdot}5}{2} \ N$$

$$= 42919 \ N$$

Hoop steel

$$= \frac{42919}{10000} \ cm.^2$$

$$= 4{\cdot}29 \ cm.^2$$

Spacing of 8 *mm.* ϕ bars

$$= \frac{0{\cdot}5 \times 100}{4{\cdot}29} \ cm.$$

$$= 11 \ cm. \ centres$$

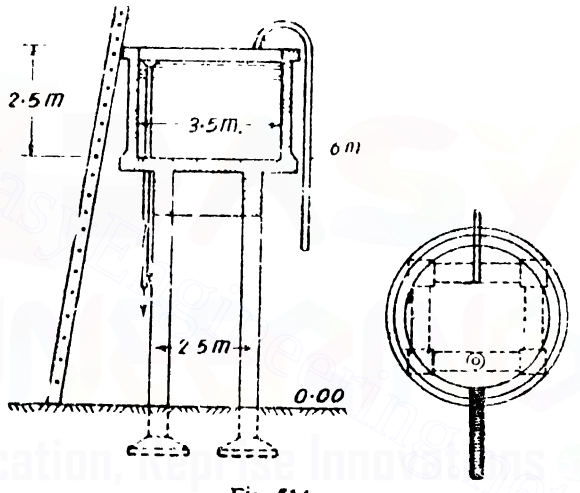

Fig. 514

Let the thickness of the wall be 15 *cm.*

Tensile stress on the equivalent concrete area

$$= \frac{42919}{100 \times 15 + (13-1)\frac{0{\cdot}5 \times 100}{11}} \ N/cm.^2$$

$$= 27{\cdot}6 \ N/cm.^2 \text{ (less than } 120 \ N/cm.^2\text{)}$$

Design of the floor slab

The total load W on the slab consists of the following :

Weight of water $= \frac{\pi \times 3{\cdot}5^2}{4} \times 2{\cdot}5 \times 9810 = 235959 \ N$

Top rib (20 *cm.* × 10 *cm.*) $= \pi \times 3{\cdot}8 \times 0{\cdot}2 \times 0{\cdot}1 \times 25000 = 5969 \ N$

Wall $= \pi \times 3{\cdot}65 \times 0{\cdot}15 \times 2{\cdot}5 \times 25000 = 107500 \ N$

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Bottom slab (18 *cm.* thick) $\pi \times \frac{3{\cdot}8^2}{4} \times 0{\cdot}18 \times 25000 = 51035\ N$

Total $= 400463\ N$

Maximum sagging bending moment for the slab per *metre* width

$$= \frac{W}{26}\ Nm$$

$$= \frac{400463}{26} = 15402\ Nm \text{ per } metre \text{ width}$$

Adopting $c = 700\ N/cm.^2$, $t = 12500\ N/cm.^2$, $m = 13$,

and equating the moment of resistance to the maximum bending moment, we have

$$126{\cdot}4 \times 100\ d^2 = 15402 \times 100$$

$$\therefore \quad d = 11\ cm.$$

Effective depth to top layer of bars $= 11\ cm.$

Dia. of bars (12 *mm.*) $= 1{\cdot}2\ cm.$

$\frac{1}{2}$ dia of bar $= 0{\cdot}6\ cm.$

Cover (25 *mm.*) $= 2{\cdot}5\ cm.$

Maximum overall thickness $= 15{\cdot}3\ cm.$

Let us provide 18 *cm.* overall

Actual effective depth to the centre of top layer of bars

$$= 18 - 4{\cdot}3 = 13{\cdot}7\ cm.$$

$$A_t = \frac{1540200}{12500 \times 0{\cdot}86 \times 13{\cdot}7} = 10{\cdot}4\ cm.^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{10{\cdot}4}\ cm. = 10\ cm.\ centres$$

Beams

It will be assumed that the load transferred to each beam will be a triangular load

$\therefore$ B.M. due to superimposed load

$$= \frac{W}{4} \cdot \frac{l}{6}$$

$$= \frac{400463}{4} \times \frac{2{\cdot}5}{6} \times 100\ N\ cm.$$

$$= 4171490\ N\ cm.$$

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B.M. due to self-weight (30 *cm.* × 40 *cm.*)

$$= 0{\cdot}3 \times 0{\cdot}4 \times 25000 \times \frac{2{\cdot}5^2}{8} \times 100 \ N\ cm.$$

$$= 234375 \ N\ cm.$$

$\therefore$ Max. B.M. $= 4171490 + 234375 = 4405865 \ N\ cm.$

Equating the moment of resistance to the maximum bending moment, we have

$$126{\cdot}4 \times 30\, d^2 = 4405865$$

$\therefore$ $d = 34{\cdot}1$ *cm.*

Effective depth available

$$= 40 - 4 = 36 \ cm.$$

$$= \frac{4405865}{12500 \times 0{\cdot}86 \times 36} = 11{\cdot}38 \ cm.^2$$

Provide 4 bars of 20 mm. diameter (*12·57 cm.*²)

Maximum S.F. due to superimposed load

$$= \frac{W}{8} = \frac{400463}{8} = 50058 \ N$$

S.F. due to self-weight

$$= \frac{0{\cdot}3 \times 0{\cdot}4 \times 25000 \times 2{\cdot}5}{2} \ N = 3750 \ N$$

Total S.F. $= 50058 + 3750 = 53808 \ N$

Nominal shear stress

$$= q = \frac{S}{bd}$$

$$= \frac{53808}{30 \times 36} = 50 \ N/cm.^2 = 0{\cdot}5 \ N/mm.^2$$

Percentage of tensile steel provided

$$= \frac{12{\cdot}57}{30 \times 36} \times 100 = 1{\cdot}2\%$$

For 1·2% steel, $q_c = 0{\cdot}4 \ N/mm.^2 = 40 \ N/cm.^2$

Shear resistance of concrete

$$= q_c bd$$

$$= 40 \times 30 \times 36 = 43200 \ N$$

Shear to be resisted by stirrups

$$= V_s = 50058 - 43200 = 6858 \ N$$

Spacing of 2-legged 10 *mm.* ϕ stirrups

$$= \frac{A_v f_v d}{V_s} = \frac{(2 \times 0{\cdot}5)\ 14000 \times 36}{6858} = 73 \ cm.$$

Provide 2-legged 10 mm. ϕ stirrups at 25 cm. c/c.

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Columns and foundations may be designed as in the square tank supported on columns.

Design 131. *Fig. 515 shows an arrangement for a circular overhead tank supported on four columns. Design the tank.*

Solution. Maximum water pressure

$$= 1000 \times 2{\cdot}5 = 2500 \text{ kg./m.}^2$$

Maximum hoop tension per *metre* height

$$= \frac{2500 \times 3{\cdot}5}{2} \text{ kg.}$$

$$= 4375 \text{ kg.}$$

Hoop steel

$$= \frac{4375}{1000} \text{ cm.}^2$$

$$= 4{\cdot}37 \text{ cm.}^2$$

Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{4{\cdot}37} \text{ cm.}$$

$$= 11 \text{ cm. centres.}$$

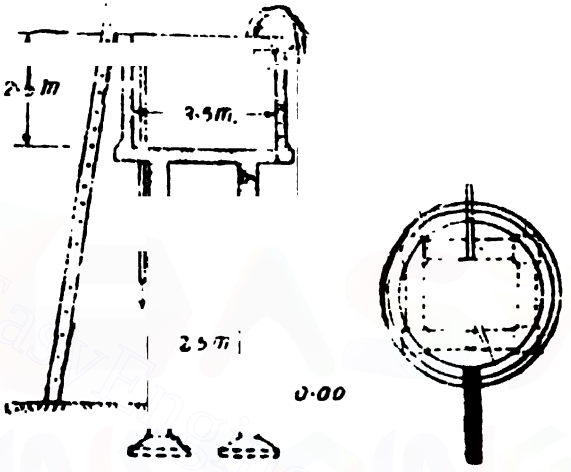

Fig. 515

Let the thickness of the wall be 15 *cm.*

Tensile stress on the equivalent concrete area

$$\frac{4375}{100 \times 15 + (13-1)\frac{0{\cdot}5 \times 100}{11}} \text{ kg./cm.}^2$$

$$= 2{\cdot}82 \text{ kg./cm.}^2 \text{ (less than 12 kg./cm.}^2\text{)}$$

Design of the floor slab

The total load W on the slab consists of the following :

Weight of water	$= \frac{\pi \times 3{\cdot}5^2}{4} \times 2{\cdot}5 \times 1000$	$= 24050$ kg.
Top rib (20 *cm.* × 10 *cm.*)	$= \pi \times 3{\cdot}8 \times 0{\cdot}2 \times 0{\cdot}1 \times 2400$	$= 570$ kg.
Wall	$= \pi \times 3{\cdot}65 \times 0{\cdot}15 \times 2{\cdot}5 \times 2400$	$= 10320$ kg.
Bottom slab (18 *cm.* thick)	$\pi \times \frac{3{\cdot}8^2}{4} \times 0{\cdot}18 \times 2400$	$= 4900$ kg.
Total		$= 39840$ kg.

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Maximum sagging bending moment for the slab per *metre* width

$$= \frac{W}{26} \text{ kgm./m.}$$

$$= \frac{39840}{26} = 1532 \text{ kg. m. per metre width.}$$

Adopting $c = 70$ kg./cm.^{2}, $t = 1250$ kg./cm.^{2}, and $m = 13$ and equating the moment of resistance to the maximum bending moment, we have

$$12{\cdot}64 \times 100\, d^2 = 1532 \times 100$$

$$\therefore \quad d = 11 \text{ cm.}$$

Effective depth to top layer of bars = 11 *cm.*

Dia. of bars (12 *mm.*)	= 1·2 *cm.*
½ dia. of bar	= 0·6 *cm.*
Cover (25 *mm.*)	= 2·5 *cm.*
Maximum overall thickness	= 15·3 *cm.*

Let us provide 18 *cm.* overall

Actual effective depth to the centre of top layer of bars

$$= 18 - 4{\cdot}3 = 13{\cdot}7 \text{ cm.}$$

$$A_t = \frac{1532 \times 100}{1400 \times 0{\cdot}86 \times 13{\cdot}7} \text{ cm.}^2$$

$$= 9{\cdot}29 \text{ cm.}^2$$

$\therefore$ Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{9{\cdot}29} \text{ cm.} = 12 \text{ cm. centres}$$

Beams

It will be assumed that the load transferred to each beam will be a triangular load

$\therefore$ B.M. due to superimposed load

$$= \frac{W}{4} \cdot \frac{l}{6}$$

$$= \frac{39840}{4} \times \frac{2{\cdot}5}{6} \times 100 \text{ kg. cm.}$$

$$= 415{,}000 \text{ kg. cm.}$$

B.M. due to self-weight (30 *cm.* × 40 *cm.*)

$$= (0{\cdot}3 \times 0{\cdot}4 \times 2400) \times \frac{2{\cdot}5^2}{8} \times 100 \text{ kg. cm.}$$

$$= 22500 \text{ kg. cm.}$$

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$\therefore$ Max. B.M. $= 415{,}000 + 22{,}500 = 437{,}500$ *kg. cm.*

Equating the moment of resistance to the maximum bending moment, we have

$$12{\cdot}64 \times 30d^2 = 437500$$

$$d = 33{\cdot}9 \text{ c.}$$

Effective depth available

$$= 40 - 4 = 36 \text{ cm.}$$

$$A_t = \frac{437500}{1250 \times 0{\cdot}86 \times 36} \text{ cm.}^2$$

$$= 11{\cdot}31 \text{ cm.}^2$$

Provide 4 bars of 20 mm. diameter ($12{\cdot}57$ *cm.*^{2})

Maximum S.F. due to superimposed load

$$= \frac{W}{8} = \frac{39840}{8} \text{ kg.} = 4980 \text{ kg}$$

S.F. due to self-weight

$$= \frac{0{\cdot}3 \times 0{\cdot}4 \times 2400 \times 2{\cdot}5}{2} \text{ kg.} = 350 \text{ kg.}$$

Total S.F. $= 4980 + 350 = 5340$ *kg.*

Nominal shear stress $= q_v = \frac{S}{bd} = \frac{5340}{30 \times 36}$ *kg./cm.*^{2}

$$= 4{\cdot}94 \text{ kg./cm.}^2$$

Percentage of tensile steel provided

$$= \frac{12{\cdot}57}{30 \times 36} \times 100 = 1{\cdot}2\ \%$$

For $1{\cdot}2\%$ steel, $q_c = 4$ *kg./cm.*^{2}

Shear resistance of concrete

$$= q_c\, bd$$

$$= 4 \times 30 \times 36 = 4320 \text{ kg.}$$

Shear to be resisted by stirrups

$$V_s = 5340 - 4320 = 1020 \text{ kg.}$$

Spacing of 2-legged 10 *mm.* ϕ stirrups

$$= \frac{A_{sv} f_v d}{V_s} = \frac{(2 \times 0{\cdot}5) 1400 \times 36}{1020} = 49 \text{ cm.}$$

Provide 2-legged 10 mm. ϕ *stirrups @ 25 cm. c/c.*

Columns and foundations may be designed as in the case of the square tank supported on columns.

UNDERGROUND TANKS

Often underground tanks are used in water purification works. The walls of such a tank are subjected to water pressure due to

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water stored in the tank and earth pressure exerted by the surrounding earth. Certain critical cases of loading shall be allowed in the design of such a tank. The extreme cases arise when the above-mentioned pressures act separately. Various practices have been adopted in this regard. One of the reasonable practices would be to design the tank due to the independent effect of earth pressure, and also due to the combined effect of earth and water pressure. In this method it is assumed that the tank is not likely to be filled when the surrounding soil is absent. However, in some contracts one of the conditions may be to test the tank for leakage before the earth is filled round it. In such a case the design should also make provision for the independent effect of water pressure.

The greatest possible level of the water table should also be studied and provision should be made for the tank to be safe when the surrounding earth is likely to be charged with water.

Lateral pressure exerted by wet earth. The exact lateral pressure intensity exerted by wet earth is difficult to determine and various methods have been suggested by designers. One of the methods of estimating the lateral pressure intensity exerted by wet earth is to assume Rankine's formula with an angle of repose of, about 6°. For instance if the soil weighs 18000 $N/metre^3$ lateral pressure of wet earth at a depth of h *metres* can be taken at

$$18000 \times h \times \frac{1-\sin 6^\circ}{1+\sin 6^\circ} \; N/metre^2$$

It is usual to have a monolithic connection between the walls and the base slab. The base slab may be extended beyond the walls so that the weight of the earth on the projecting part of the base slab may assist in preventing the tank from being pushed up due to upthrust when the water table rises.

Design 132. (S.I.). *A dividing wall of an underground water reservoir is to be designed for the worst possible condition of loading. The wall is to be of reinforced concrete fixed at the base and simply propped at the roof. Following data may be used for the design :*

Height of wall above the base = 6 metres

Maximum height of water above the floor of reservoir

= 5·50 metres.

Reaction at the support due to triangular water pressure

$= \frac{1}{5}$ *of the triangular load.*

Fixing moment at the base due to water pressure per unit run of the wall is given by

$$M = \frac{ph^2}{120H^2}\,(8H^2+9Hy+3y^2)$$

where

p = intensity of water pressure at the base

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h=height of water in the reservoir

H=total height of dividing wall

y=free board in the reservoir.

Assuming suitable working stresses, design the dividing wall.

Solution. The worst condition of loading occurs when the reservoir is full only on one side of the dividing wall.

For this case, $H=6$ *metres*

$h=5{\cdot}5$ *metres*

$y=0{\cdot}5$ *metre*

Pressure at the bottom of the wall

$$=p=9810\times 5{\cdot}5=53955\ N/metre^2$$

Fixing moment at the base per *metre* run of the wall, *i.e.* B.M. at the base

$$=\frac{ph^2}{120H^2}(8H^2+9Hy+3y^2)\ Nm.$$

$$=\frac{53955\times 5{\cdot}5^2}{120\times 6^2}[8\times 6^2+9\times 6\times 0{\cdot}5+3(0{\cdot}5^2)]$$

kg. m.

$$=119290\ Nm.$$

Fig. 516 shows the dividing wall. If the right side of wall is subjected to water pressure then the bending moment at the base of the wall $=1192900\ Nm.$ producing tension near the face *A* of the wall.

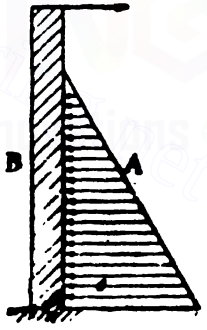

Fig. 516

Triangular load

$$=P=\frac{53955}{2}\times 5{\cdot}5\ N$$

$$=148375\ N$$

Reaction at the top support

$$\frac{P}{5}=\frac{148375}{5}=29675\ N$$

B.M. at any section at depth x *metres* from the top

$$M_x=29675\ x-9810\ \frac{(x-0{\cdot}5)^3}{6}$$

For M_x to be maximum,

$$\frac{dM_x}{dx}=0$$

$$\frac{dM_x}{dx}=29675-\frac{9810}{6}\times 3(x-0{\cdot}5)^2=0$$

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$$(x-0{\cdot}5)^2=6{\cdot}05$$

$$x-0{\cdot}5=2{\cdot}46$$

$$x=2{\cdot}96\ m.$$

At $x=2{\cdot}96\ m.$

$$M_s=29675\times2{\cdot}96-\frac{9810}{c}(2{\cdot}96-0{\cdot}5)^2$$

$$=63500\ Nm.$$

But B.M. at base $=119290\ Nm.$

Adopting $c=7\ kg./mm.^2=700\ N/cm.^2$

$t=100\ kg./mm.^2=10000\ N/cm.^2$

$m=13$

and equating the M.R. to the B.M. at the base

$$141{\cdot}1\times100\ d^2=119290\times100 \qquad \therefore\ d=29{\cdot}1\ cm.$$

Providing an effective cover of 4 *cm.* to the reinforcement, overall depth required $=29{\cdot}1+4=33{\cdot}1\ cm.$

Provide a thickness of 34 *cm.*

Steel required for a B.M. of 119290 *Nm.*

$$=\frac{119290\times100}{10000\times0{\cdot}84\times30}=47{\cdot}3\ cm.^2$$

Spacing of 20 *mm.* ϕ bars

$$=\frac{3{\cdot}14\times100}{47{\cdot}3}=6{\cdot}6\ cm.$$

Provide 20 mm. ϕ bars @ 6 cm. c/c.

Since any one side of the wall is likely to be subjected to water pressure the same reinforcement may be provided near both the faces.

Since at a depth of 2·96 *m.* from the top or 3·04 *m.* from the base, the bending moment is 63500 *Nm.*, the corresponding steel requirement

$$=\frac{63500\times100}{10000\times0{\cdot}84\times30}\ cm.^2$$

$$=25{\cdot}2\ cm.^2$$

Spacing of 20 *mm.* ϕ bars

$$=\frac{3{\cdot}14\times100}{25{\cdot}2}=12{\cdot}46\ cm.$$

say 12 cm. c/c.

This steel, to this extent, shall be provided near each face.

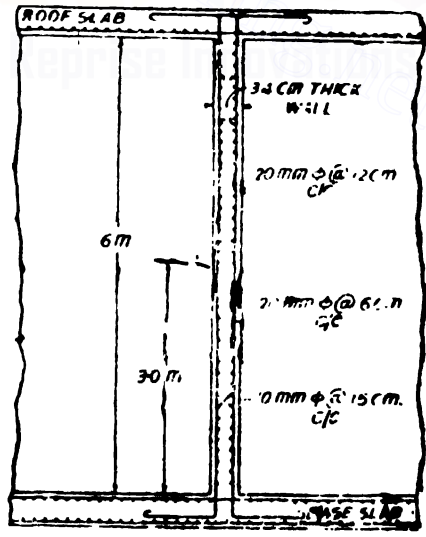

Fig. 517

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Distribution steel $= \frac{0{\cdot}3}{100} \times 34 \times 100 \ cm.^2$

$= 10{\cdot}2 \ cm^2$

Spacing of 10 *mm.* dia. bars

$= \frac{0{\cdot}79 \times 100}{10{\cdot}2} \ cm.$

$= 7{\cdot}7 \ cm.$ say $7{\cdot}5 \ cm.$

$\therefore$ *Provide 10 mm. dia. bars at 15 cm. c/c near each face.*

Design 133. *A dividing wall of an underground water reservoir is to be designed for the worst possible conditions of loading. The wall is to be of reinforced concrete fixed at the base and simply propped at the roof. Following data may be used for the design :*

Height of wall above the base $= 6$ *metres*

Maximum height of water above the floor of reservoir

$= 5{\cdot}50$ *metres*

Reaction at the support due to triangular water pressure

$= \frac{1}{5}$ *of the triangular load.*

Fixing moment at the base due to water pressure per unit run of the wall is given by

$$M = \frac{ph^2}{120H^2}(8H^2 + 9Hy + 3y^2)$$

where

p = *intensity of water pressure at the base*

h = *height of water in the reservoir*

H = *total height of dividing wall*

y = *free board in the reservoir.*

Assuming suitable working stresses, design the dividing wall.

Solution. The worst condition of loading occurs when the reservoir is full only on one side of the dividing wall.

For this case, $H = 6$ *metres*

$h = 5{\cdot}5$ *metres*

$y = 0{\cdot}5$ *metres*

Pressure at the bottom of the wall

$p = 1000 \times 5{\cdot}5 = 55\ 0 \ kg./m.^2$

$\therefore$ Fixing moment at the base per *metre* run of the wall, *i.e.* B.M. at the base

$$= \frac{ph^2}{120H^2}(8H^2 + 9Hy + 3y^2) \ kg. \ m.$$

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$$= \frac{5500 \times 5{\cdot}5^2}{120 \times 6^2}\,[8 \times 6^2 + 9 \times 6 \times 0{\cdot}5 + 3(0{\cdot}5^2)]$$

kg. m.

$$= 12140 \text{ kg. m.}$$

Fig. 518 shows the dividing wall. If the right side of wall is subjected to water pressure then the bending moment at the base of the wall = 12140 *kg. m.* producing tension near the face *A* of the wall.

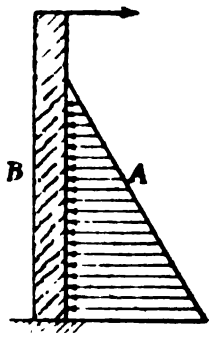

Fig. 518

Triangular load

$$= P = \frac{5500}{2} \times 5{\cdot}5 \text{ kg.}$$

$$= 15125 \text{ kg.}$$

Reaction at the top support

$$= \frac{P}{5} = \frac{15125}{5} = 3025 \text{ kg.}$$

B.M. at any section at depth x *metres* from the top

$$M_x = 3025x - 1000\,\frac{(x-0{\cdot}5)^3}{6}$$

For M_x to be a maximum,

$$\frac{dM_x}{dx} = 0$$

$$\frac{dM_x}{dx} \quad 3025 - \frac{1000}{6} \times 3(x-0{\cdot}5)^2 = 0$$

$$500(x-0{\cdot}5)^2 = 3025$$

$$(x-0{\cdot}5)^2 = 6{\cdot}05$$

$$x - 0{\cdot}5 = 2{\cdot}46$$

$$x = 2{\cdot}96 \text{ m.}$$

At $x = 2{\cdot}96$ *m.*

$$M_x = 3025 \times 2{\cdot}96 - \frac{1000}{6}\,(2{\cdot}96-0{\cdot}5)^3 \text{ kg. m.}$$

$$= 6474 \text{ kg. m.}$$ producing tension away from the face *B*.

But B M. at the base = 12140 *kg. m.*

Adopting $c = 70$ *kg./cm.*^{2}, $t = 1000$ *kg/cm.*^{2} and $m = 13$ and equating the moment of resistance to the maximum bending moment

$$14{\cdot}11 \times 100\,d^2 = 12140 \times 100$$

$$d = 29{\cdot}34 \text{ cm.}$$

Providing an effective cover of 4 *cm.* to the reinforcement overall depth required

$$= 29{\cdot}34 + 4 = 33{\cdot}34 \text{ cm.}$$

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Let us provide 34 *cm.* thick wall.

Steel requirement for a bending moment of 12140 *kg. m.*

$$= \frac{12140 \times 100}{1000 \times 0{\cdot}84 \times 30} \ cm.^2 = 48{\cdot}2 \ cm.^2$$

Spacing of 20 *mm.* diameter bars

$$= \frac{3{\cdot}14 \times 100}{48{\cdot}2} \ cm.$$

$$= 6{\cdot}62 \ cm. \text{ say } 6 \ cm. \text{ centres}$$

Since any one side of the wall is likely to be subjected to water pressure the same reinforcement may be provided near both the faces.

Since at a depth of 2·96 *m.* from the top, *i e*, 3·04 *m* from the base bending moment is 6474 *kg. m.*, the corresponding steel requirement

$$= \frac{6474 \times 100}{1000 \times 0{\cdot}84 \times 30} \ cm.^2$$

$$= 25{\cdot}7 \ cm.^2$$

Spacing of 20 *mm.* diameter bars $= \frac{3{\cdot}14 \times 100}{25{\cdot}7} \ cm.$

$= 12{\cdot}2 \ cm.$

say *12 cm centres*

This steel, to this extent shall be provided near each face.

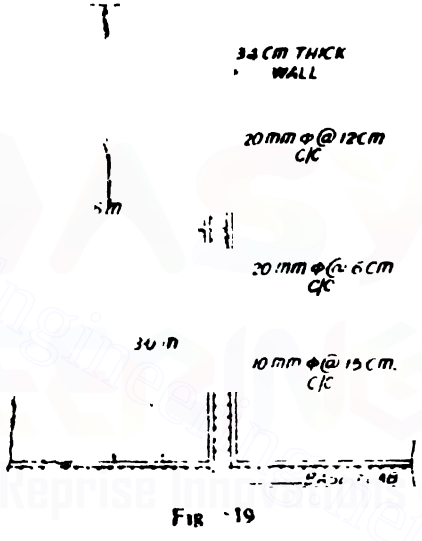

FIG. 19

Distribution steel $= \frac{0{\cdot}3}{100} \times 34 \times 100 \ cm.^2$

$= 10{\cdot}2 \ cm.^2$

Spacing of 10 *mm.* dia bars

$$= \frac{0{\cdot}79 \times 100}{10{\cdot}2} \ cm.$$

$$= 7{\cdot}7 \ cm. \text{ say } 7{\cdot}5 \ cm.$$

∴ *Provide 10 mm. dia. bars at 15 cm. c/c near each face.*

Design 134 (S I.). *Design an unaerground tank of internal dimensions 6 m. × 3 m. × 3 m. The soil surrounding the tank always remains dry. The tank shall be provided with a roof slab. The soil weighs 16000 N per metre³ having an angle of repose of 30°.*

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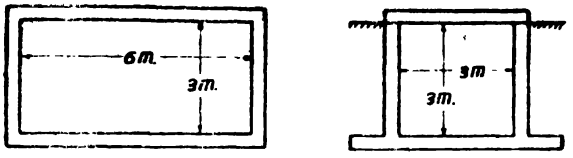


Fig. 520

Solution. Unless otherwise mentioned it will be taken that the soil surrounding the tank is not liable to be removed at any stage.

The tank will therefore be designed for the following critical cases :

Case 1. *When the tank is full.*

Case 2. *When the tank is empty.*

All walls will be designed as propped cantilevers

Case 1. Analysis : *When the tank is full*

Maximum soil pressure

$$16000 \times 3 \times \frac{1-\sin 30^\circ}{1+\sin 30^\circ} \; N/metre^2$$

$$= 16000 \; N/metre^2$$

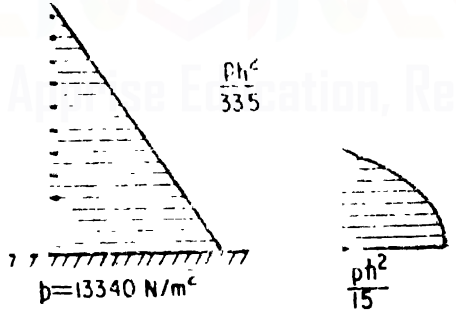
Maximum water pressure

$$= 9810 \times 3 = 29430 \; N/metre^2$$

Net max. pressure

$$= p = 29430 - 16000$$

$$= 13340 \; N/metre^2$$



∴ Max. B.M. producing tension away from the water side

$$= \frac{ph^2}{33{\cdot}5}$$

$$= \frac{13340 \times 3^2}{33{\cdot}5} \; Nm.$$

$$= 3583{\cdot}9 \; Nm.$$

$$= 358390 \; N\,cm.$$

Fig. 521

Maximum bending moment producing tension near the water face

$$= \frac{ph^2}{15} = \frac{13340 \times 3^2}{15} \; Nm. = 8004 \; Nm.$$

$$= 800400 \; N\,cm.$$

Case 2. Analysis : ***When the tank is empty.***

Maximum soil pressure

$$= 16000 \; N/metre^2$$

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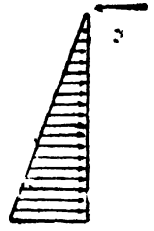
Maximum bending moment producing tension near the water face

$$= \frac{ph^2}{33{\cdot}5}$$

$$= \frac{16000 \times 3^2}{33{\cdot}5}\ Nm$$

$$= 4298{\cdot}5\ Nm.$$

$$= 429850\ N\ cm.$$



$p = 16000\ N/m.$

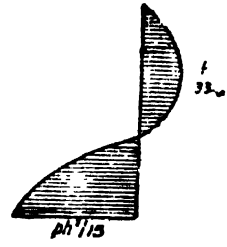


Fig. 522

Maximum bending moment producing tension away from the water side

$$= \frac{ph^2}{15}$$

$$= \frac{16000 \times 3^2}{15}\ Nm.$$

$$= 9600\ Nm.$$

$$= 960000\ N\ cm.$$

The results obtained for the above two cases are tabulated below :

Case	*B M. producing Tension on water face (N cm.)*	*B.M. producing Tension away from water face (N cm)*
Case 1	800400	358390
Case 2	429850	960000

The outer walls of an underground tank must also be designed from cracking stress consideration. Cracking stress means the actual tensile stress in concrete due to bending moment. This stress must be limited to 160 N/cm^2. To satisfy this condition the thickness of the walls must be sufficiently thick. The overall thickness D may be determined by assuming the moment of resistance as $26{\cdot}67\ bD^2\ N/cm^2$

$$[\tfrac{1}{6} f b D^2 = \tfrac{1}{6} \times 160\ bD^2 = 26{\cdot}67\ bD^2]$$

In our case equating the moment of resistance to the maximum bending moment, we have

$$26{\cdot}67 \times 100\ D^2 = 960000$$

$$\therefore \quad D = 18{\cdot}97\ cm.$$

Let us provide a thickness of 20 *cm.*

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Providing an effective cover of 4 *cm.* effective depth

$$= 20 - 4 = 16 \text{ cm.}$$

Steel for a bending moment of 960000 *N cm.*

$$= \frac{960000}{12500 \times 0.86 \times 16} \text{ cm.}^2$$

$$= 5.6 \text{ cm}^2.$$

Provide 12 mm. dia. bars @ 20 cm. centres (5.65 cm.²)

Steel for bending moment of 800400 *N cm.*

$$= \frac{800400}{10000 \times 0.84 \times 16} \text{ cm.}^2 = 5.95 \text{ cm.}^2$$

Provide 12 mm. dia. bars at 18 cm. centres (6.28 cm.²).

Check for Cracking Stresses

Position of actual neutral axis. Taking moments about the neutral axis

$$100 \frac{n^2}{2} + (13-1)6.28(n-4)$$

$$= 100 \frac{(20-n)^2}{2} + (13-1)5.65(20-n-4)$$

Solving, we get

$$n = 8.84 \text{ cm.}$$

Moment of resistance determined from the tension zone may be taken as

Total tension $\times 0.82\, d$

Cracking stress due to a B.M. of 960000 N cm.

This B.M. produces tension away from the water side.

Let the maximum tensile stress be C_t.

Total tension

$$= 100 \times 11.16 \frac{C_t}{2} + (13-1) \times 5.65 \times \frac{7.16}{11.16} C_t \text{ kg.}$$

$$= 601.5\, C_t \text{ N}$$

Equating the M.R. to B.M.

$$601.5\, C_t \times 0.82 \times 16 = 960000$$

$$\therefore \quad C_t = 121.6 \text{ N/cm.}^2 \text{ (less than 160 N/cm.}^2\text{)}$$

Cracking stress due to bending moment of 800400 N cm.

This bending moment produces tension near water face.

Let the tensile stress in concrete be C_t

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Total tension

$$= 1\cdot0 \times 8\cdot84 \times \frac{C_t}{2} + (13-1) \times 6\cdot28 \times \frac{4\cdot84}{8\cdot84} C_t \text{ kg.}$$

$$= 483\cdot3\ C_t$$

Equating the M.R. to the B M.

$$483\cdot3\ C_t \times 0\cdot82 \times 16 = 800400$$

$$C_t = 126\cdot2\ N/cm^2. \text{ (less than } 160\ N/cm.^2)$$

Provide also distribution steel of 8 *mm.* diameter bars at 15 *cm.* centres near each face.

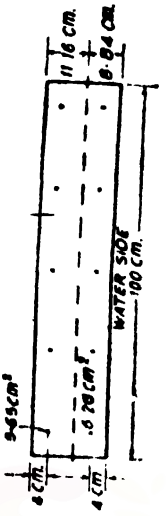

Fig. 523

Design of roof slab

Loads. Dead load (15 *cm.*) $= 3750\ N/m.^2$

Live load $= 1500\ N/m.^2$

Total $= 5250\ N/cm.^2$

Consider a one *metre* wide strip of the slab

Maximum bending moment

$$= \frac{5250 \times 3\cdot2^2}{8}\ Nm.$$

$$= 6720\ Nm. = 672000\ N\ cm.$$

If the usual concrete mix. (*M* 15) be used for the roof slab,

$$85 \times 100\ d^2 = 672000$$

$$d^2 = 8\cdot89\ cm.$$

Providing a cover of 4 *cm.* effective depth available

$$= 15 - 4 = 11\ cm.$$

$$A_t = \frac{672000}{14000 \times 0\cdot87 \times 11}\ cm.^2 = 5\cdot01\ cm.^2$$

Provide 10 mm. ϕ bars @ 16 cm. c/c.

Distribution steel $= \frac{0\cdot3}{100} \times 15 \times 100\ cm.^2 = 4\cdot5\ cm.^2$

Spacing of 8 *mm.* diameter bars

$$= \frac{0\cdot5 \times 100}{4\cdot5}\ cm. = 11\ cm.\ centres$$

Design of the base slab. Case 1. *When the tank is full.*

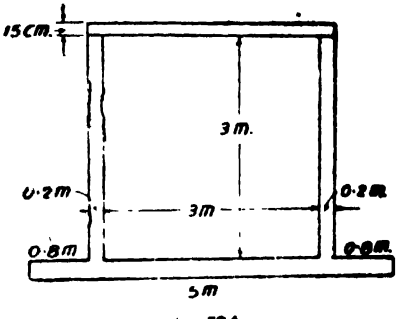

Fig. 524

Consider one *metre run* of the tank.

Weight of Roof slab :

$$3\cdot4 \times 0\cdot15 \times 25000 = 12750\ N/m.$$

Walls :

$$2 \times 0\cdot2 \times 3 \times 25000 = \frac{30000\ N/m.}{42750\ N/m.}$$

(Note Water pressure on the base slab and the weight of the base slab will be directly

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counteracted by ground pressure and will, therefore, not be included in the B.M. calculations).

$$\text{Net upward reaction} = \frac{42750}{5} = 8550 \text{ N/metre.}^2$$

B.M. at the centre due to the above loading

$$= 21375\ (1{\cdot}6 - 1{\cdot}25) \text{ Nm.}$$
$$= 7481{\cdot}25 \text{ Nm.}$$
$$= 748125 \text{ N cm.}$$

producing tension on the water side.

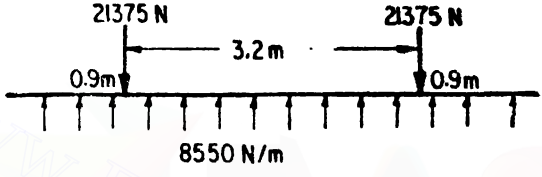

Fig. 525

Water pressure and soil pressure acting on the wall will produce a moment of 800400 *N cm* of the same type.

∴ Total bending moment at the centre

$$= 748125 + 800400 \text{ N cm.}$$
$$= 1548525 \text{ N cm.}$$

producing tension on the water side.

B.M. at the end. Hogging bending moment

$$= 800400 \text{ N cm.}$$

Sagging bending moment

$$= 8550 \times \frac{0{\cdot}9^2}{2} \times 100 = 346275 \text{ N cm}$$

∴ Net. B.M. $= 800400 - 346275 = 454125$ *N cm.*

producing tension on the water side.

Case 2. *When the tank is empty.*

For this condition B.M. at centre due to vertical loads

$$= 748125 \text{ N cm.}$$

producing tension on the water side.

B.M. due to soil pressure on the vertical walls

$$= 960000 \text{ N cm.}$$

producing tension away from water side.

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∴ Net B.M. at the centre

$$= 960000 - 748125 = 211875 \ N\ cm.$$

producing tension away from the water side.

B.M. at the end $= -346275 + 960000 \ N\ cm.$

$$= 1306275 \ N\ cm.$$

producing tension away from the water side.

The results obtained for the above two cases are tabulated below :

Case	*B.M. at End section (N cm.)*	*BM at Mid span (N cm)*	*B.M. produces Tension*
Case 1	454125	1548525	On water side
Case 2	1306275	211875	Away from water side

Let the thickness of the base slab be D *cm.*

From cracking stress consideration, equating the moment of resistance to max. B.M., we have,

$$26{\cdot}67bD^2 = 26{\cdot}67 \times 100 \ D^2 = 1548525$$

$$D = 24 \ cm.$$

Let us provide an overall depth of 25 *cm.*

Let the effective cover be 6 *cm.*

∴ Effective depth $= 25 - 6 = 19$ *cm.*

Steel for a bending moment of 1548525 N *cm.*

$$= \frac{1548525}{10000 \times 0{\cdot}84 \times 19} = 9{\cdot}70 \ cm.^2$$

Spacing of 12 *mm.* diameter bars

$$\frac{1{\cdot}13 \times 100}{9\ 70} \ cm = 11{\cdot}6 \ cm.$$

Provide 12 mm. diameter bars at 11 cm. centres (10·27 cm.²)

Steel for a bending moment of 1306275 N *cm.*

$$= \frac{1306275}{12500 \times 0{\cdot}86 \times 10} \ cm.^2$$

$$= 6{\cdot}39 \ cm.^2$$

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Spacing of 12 *mm* diameter bars

$$\frac{1\ 13\times100}{6\ 39}\ cm=17{\cdot}68\ cm.$$

Provide 12 mm. diameter bars at 17 cm. centres (6·65 *cm.*²)

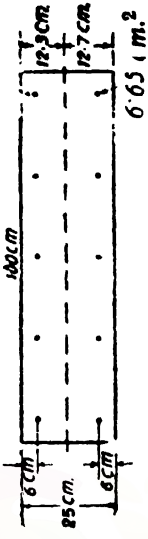

Fig 526

Check for cracking stress

Position of actual neutral axis

Taking moments above the neutral axis, we have,

$$100\frac{n^2}{2}+(13-1)\times10{\cdot}27\ (n-6)$$

$$=100\frac{(25-n)^2}{2}+(13-1)\ 6\ 65{\cdot}(25-n-6)$$

$$n=12{\cdot}3\ cm.$$

Cracking stress due to a bending moment of 1548525 N cm.

This bending moment produces tension on the water face.

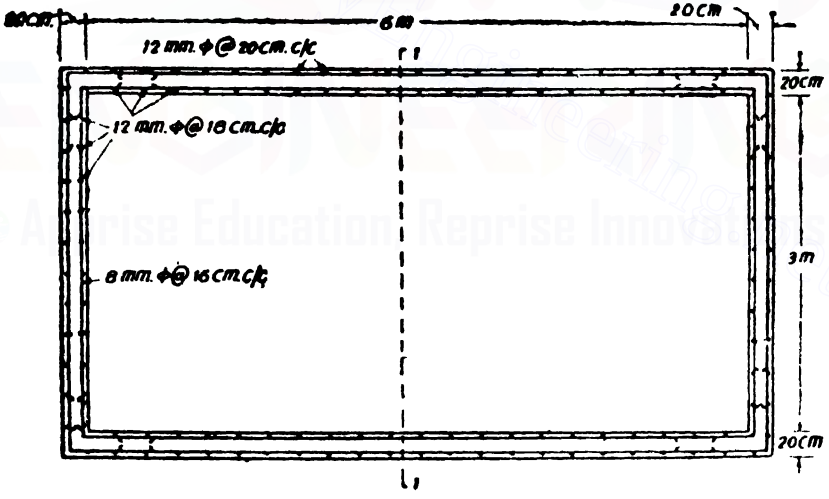

Fig. 527.

Let the tensile stress in concrete be C_t

$$\text{Total tension}=100\times12{\cdot}3\times\frac{C_t}{2}+(13-1)\ 10\ 27\times\frac{8{\cdot}3}{12{\cdot}3}\ C_t\ N$$

$$=708{\cdot}8\ C_t\ N$$

Equating the moment of resistance to the bending moment

$$708{\cdot}8\ C_t\times0{\cdot}82\times19=1548525$$

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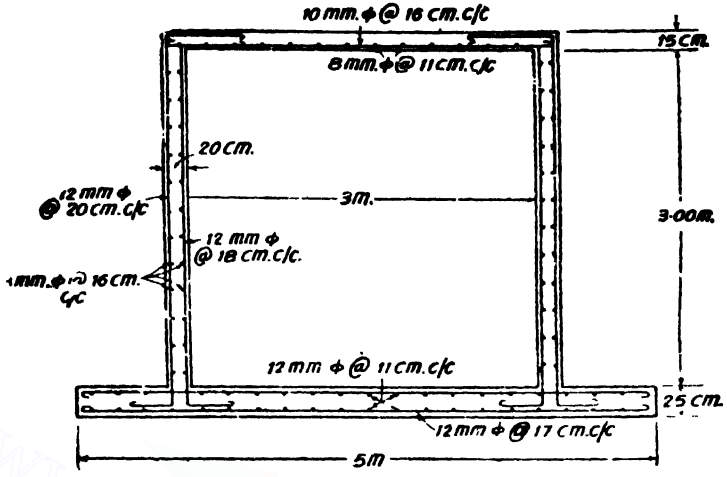

Fig. 528

$C_t = 140{\cdot}2\ N/cm.^2$
(less than 160 $N/cm.^2$)

Cracking stress due to a bending moment of 1306275 N cm.

This bending moment produces tension away from the water side.

Let the tensile stress in concrete be C_t

Total tension $= 100 \times 12{\cdot}7 \times \frac{C_t}{2} + (13-1) \times 6{\cdot}65 \times \frac{8{\cdot}7}{12{\cdot}7} C_t\ N$

$= 687\ C_t\ N.$

Equating the moment of resistance to the bending moment,

$687\ C_t \times 0{\cdot}82 \times 19 = 1306275$

$C_t = 122\ N/cm.^2$

(less than 160 N/cm^2)

Design 135. *Design an underground tank of internal dimensions 6 m. × 3 m. × 3 m. The soil surrounding the tank always remains dry. The tank shall be provided with a roof slab. The soil weighs 1600 kg./metre³ having an angle of repose of 30°. Weight of R.C.C. = 2400 kg./m³.*

Solution. Unless otherwise mentioned it will be taken that the soil surrounding the tank is not liable to be removed at any stage.

The tank will therefore be designed for the following critical cases :

Case 1. *When the tank is full*

Case 2. *When the tank is empty.*

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Fig. 529

All walls will be designed as propped cantilevers.

Case 1. Analysis : *When the tank is full.*

Maximum soil pressure

$$= 1600 \times 3 \times \frac{1-\sin 30^\circ}{1+\sin 30^\circ} \ kg./m.^2$$

$$= 1600 \ kg./m.^2$$

Maximum water pressure

$$= 1000 \times 3 = 3000 \ kg./m.^2$$

Net water pressure

$$= p = 3000 - 1600$$

$$= 1400 \ kg./m.^2$$

$\therefore$ Max. B.M. producing tension away from the water side

$$= \frac{ph^2}{33{\cdot}5}$$

$$= \frac{1400 \times 3^2}{33{\cdot}5} \ kg.\ m.$$

$$= 376{\cdot}1 \ kg.\ m.$$

$$= 37610 \ kg.\ cm.$$

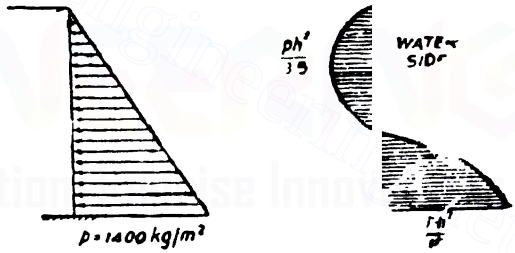

Fig. 530

Maximum bending moment producing tension near the water face

$$= \frac{ph^2}{15} = \frac{1400 \times 3^2}{15} \ kg.\ m.$$

$$= 840 \ kg.\ m. = 84000 \ kg.\ cm.$$

Case 2. Analysis : *When the tank is empty.*

Maximum soil pressure

$$= 1600 \ kg./m.^2$$

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Maximum bending moment producing tension near the water face

$$= \frac{ph^2}{33{\cdot}5}$$

$$= \frac{1600 \times 3^2}{33{\cdot}5} \text{ kg. m.}$$

$$= 429{\cdot}8 \text{ kg. m.}$$

$$= 42980 \text{ kg. cm.}$$

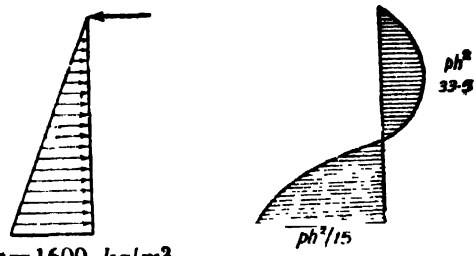

Fig. 531

Maximum bending moment producing tension away from the water side

$$= \frac{ph^2}{15} = \frac{1600 \times 3^2}{15} \text{ kg. m.}$$

$$= 960 \text{ kg. m.}$$

$$= 96000 \text{ kg. cm.}$$

The results obtained for the above two cases are tabulated below :

Case	*B.M. producing Tension on water face (kg. cm.)*	*B.M producing Tension away from water face (kg. cm.)*
Case 1	84000	37510
Case 2	42980	96000

The outer walls of an underground tank must also be designed from cracking stress consideration. Cracking stress means the actual tensile stress in concrete due to bending moment. This stress must be limited to 16 *kg./cm.*^{2} To satisfy this condition the thickness of the walls must be sufficiently thick. The overall thickness D may be determined by assuming the moment of resistance as $2{\cdot}67\ bD^2$.

$$[\tfrac{1}{8} fbD^2 = \tfrac{1}{8} \times 16\ bD^2 = 2{\cdot}67\ bD^2]$$

In our case equating the moment of resistance to the maximum bending moment, we have

$$2{\cdot}67 \times 100\ D^2 = 96000$$

$$D = 19 \text{ cm.}$$

Let us provide a thickness of 20 *cm.*

Providing an effective cover of 4 *cm.* effective depth

$$= 20 - 4 = 16 \text{ cm.}$$

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Steel for a bending moment of 96000 *kg. cm.*

$$= \frac{96000}{1250 \times 0{\cdot}86 \times 16} \ cm.^2$$

$$= 5{\cdot}6 \ cm^2.$$

Provide 12 mm. dia. bars @ 20 cm. centres (5·65 *cm.*^{2})

Steel for bending moment of 84000 *kg. cm.*

$$= \frac{84000}{1000 \times 0{\cdot}84 \times 16} \ cm.^2 = 6{\cdot}25 \ cm.^2$$

Provide 12 mm dia. bars at 18 cm. centres (6·28 *cm.*^{2})

Check for Cracking Stresses

Position of actual neutral axis. Taking moments about the neutral axis

$$100 \ \frac{n^2}{2} + (13-1)6{\cdot}28(n-4)$$

$$= 100 \ \frac{(20-n)^2}{2} + (13-1)5{\cdot}65(20-n-4)$$

Solving, we get

$$n = 8{\cdot}84 \ cm.$$

Moment of resistance determined from the tension zone may be taken as

Total tension $\times 0{\cdot}82 \ d$

Cracking stress due to a B M. of 96000 kg. cm.

This B.M. produces tension away from the water side.

Let the maximum tensile stress be C_t.

Total tension

$$= 100 \times 11{\cdot}16 \ \frac{C_t}{2} + (13-1) \times 5{\cdot}65 \times \frac{7{\cdot}16}{11{\cdot}16} \ C_t \ kg$$

$$= 601{\cdot}5 \ C_t \ kg.$$

Equating the M R. to B.M.

$$601{\cdot}5 \ C_t \times 0{\cdot}82 \times 16 = 96000$$

$$\therefore \qquad C_t = 12{\cdot}16 \ kg/cm^2 \text{ (less than 16 } kg/cm.^2).$$

Cracking stress due to a bending moment of 84000 kg. cm.

This bending moment produces tension near water face.

Let the tensile stress in concrete be C_t

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Total tension

$$= 100 \times 8{\cdot}84 \times \frac{C_t}{2} + (13-1) \times 6{\cdot}28 \times \frac{4{\cdot}84}{8{\cdot}84} C_t \text{ kg.}$$

$$= 483{\cdot}3\ C_t$$

Equating the M.R. to the B.M.

$$483{\cdot}3\ C_t \times 0{\cdot}82 \times 16 = 84000$$

$C_t = 13{\cdot}25$ *kg./cm.*^{2} (less than 16 *kg./cm.*^{2}).

Provide also distribution steel of 8 *mm.* diameter bars at 16 *cm.* centres near each face.

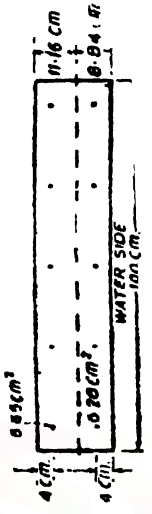

Fig. 532

Design of roof slab

Loads. Dead load (15 *cm.*) = 360 *kg./m.*^{2}

Live load = 150 *kg./m.*^{2}

Total = 510 *kg./m.*^{2}

Consider a one *metre* wide strip of the slab

Maximum bending moment

$$= \frac{510 \times 3{\cdot}2^2}{8} \text{ kg. m.}$$

$$= 652{\cdot}7 \text{ kg. m} = 65270 \text{ kg. cm.}$$

If the usual concrete mix (*M* 150) be used for the roof slab,

$$8{\cdot}5 \times 100\ d^2 = 65270$$

$$d = 8{\cdot}8 \text{ cm.}$$

Providing a cover of 4 *cm.* effective depth available

$$= 15 - 4 = 11 \text{ cm.}$$

$$A_t = \frac{65270}{1400 \times 0{\cdot}87 \times 11} \text{ cm.}^2 = 4{\cdot}87 \text{ cm.}^2$$

Spacing of 10 *mm.* diameter bars

$$= \frac{0{\cdot}79 \times 100}{4{\cdot}87} \text{ cm.} = 16 \text{ cm.}$$

Distribution steel $= \frac{0{\cdot}3}{100} \times 15 \times 100 \text{ cm.}^2$

$$= 4{\cdot}5 \text{ cm}^2$$

Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{4{\cdot}5} \text{ cm} = 11 \text{ cm. centres}$$

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Design of the base slab. Case 1. *When the tank is full.*

Consider one *metre run* of the tank.

Weight of roof slab
$=3{\cdot}4\times0{\cdot}15\times2400=1224$ *kg./m.*
Walls
$=2\times0{\cdot}2\times3\times2400=2880$ *kg./m.*

4104 *kg./m.*

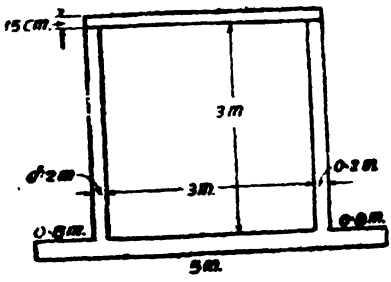

Fig. 533

(Note. Water pressure on the base slab and the weight of the base slab will be directly counteracted by ground pressure and will, therefore, not be included in the B.M. calculations).

Net upward reaction

$$=\frac{4104}{5}=820{\cdot}8\ kg./m^2.$$

B.M. at the centre due to the above loading

$=2052\ (1{\cdot}6-1{\cdot}25)$ *kg. m.*

$=718{\cdot}2$ *kg. m.*

$=71820$ *kg cm.* producing tension on the water side.

Fig. 534

Water pressure and soil pressure acting on the wall will produce a moment of 84000 *kg. cm.* of the same type.

∴ Total bending moment at the centre
$=71820+84000$ *kg. cm.*
$=155820$ *kg. cm.*

producing tension on the water side.

B.M. at the end. Hogging bending moment
$=84000$ *kg. cm.*

Sagging bending moment

$$=820{\cdot}8+\frac{0{\cdot}9^2}{2}\times100=33240\ kg.\ cm.$$

∴ Net B.M. $=84000-33240=50760$ *kg. cm.*

producing tension on the water side.

Case 2. *When the tank is empty.*

For this condition B M. at centre due to vertical loads
$=71820$ *kg. cm.* producing tension on the water side,

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B.M. due to soil pressure on the vertical walls

$= 96000$ *kg. cm. producing tension away from water side.*

$\therefore$ Net B.M. at the centre

$= 96000 - 71820 = 24180$ *kg. cm. producing tension away from the water side.*

B.M. at the ends

$= 33240 + 96000$ *kg. cm.*

$= 129240$ *kg. cm.*

producing tension away from the water side.

The results obtained for the above two cases are tabulated below :

Case	*B.M. at End section (kg. cm.)*	*B.M. at Mid span (kg. cm.)*	*B.M. produces Tension*
Case 1	50760	155820	On water side
Case 2	129240	24180	Away from water side

Let the thickness of the base slab be D *cm.*

From cracking stress consideration, equating the moment of resistance to max. B.M., we have,

$$2{\cdot}67\, bD^2 = 2{\cdot}60 \times 100\ D^2 = 155820$$

$$D = 24 \text{ cm.}$$

Let us provide an overall depth of 25 *cm.*

Let the effective cover be 6 *cm.*

$\therefore$ Effective depth $= 25 - 6 = 19$ *cm.*

Steel for a bending moment of 155820 *kg. cm.*

$$= \frac{155820}{1000 \times 0{\cdot}84 \times 19} = 9{\cdot}76 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$\frac{1{\cdot}13 \times 100}{9{\cdot}76} \text{ cm.} = 11{\cdot}58 \text{ cm.}$$

Provide 12 mm diameter bars at 11 cm. centres (10·27 cm.²)

Steel for a bending moment of 129240 *kg. cm.*

$$= \frac{129240}{1250 \times 0{\cdot}86 \times 19} \text{ cm.}$$

$$= 6{\cdot}33 \text{ cm}^2.$$

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Spacing of 12 *mm.* diameter bars

$$= \frac{1\cdot13 \times 100}{6\cdot33}\ cm. = 17\cdot85\ cm.$$

Provide 12 mm. diameter bars at 17 cm. centres ($6\cdot65\ cm^2$).

Check for cracking stress

Position of actual neutral axis

Taking moments above the neutral axis, we have,

$$100\frac{n^2}{2} + (13-1) \times 10\cdot27\,(n-6)$$

$$= 100\frac{(25-n)^2}{2} + (13-1)\ 6\cdot65\,(25-n-6)$$

$$n = 12\cdot3\ cm.$$

Cracking stress due to a bending moment of 155820 kg. cm.

This bending moment produces tension on the water face.

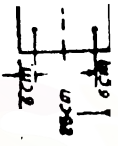

Fig. 335

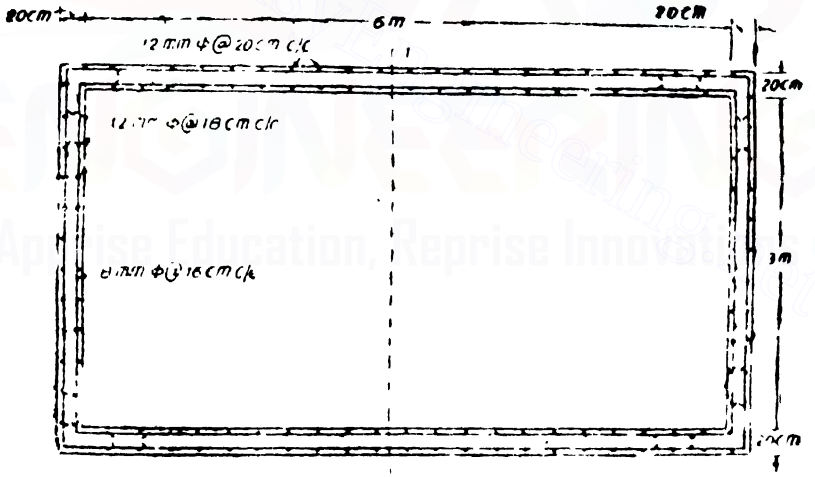

Fig. 536

Let the tensile stress in concrete be C_t

$$\text{Total tension} = 100 \times 12\cdot3 \times \frac{C_t}{2} + (13-1)\ 10\cdot27 \times \frac{8\cdot3}{12\cdot3}\ C_t\ kg$$

$$= 708\cdot8\ C_t\ kg.$$

Equating the moment of resistance to the bending moment

$$708\cdot8\ C_t \times 0\cdot82 \times 19 = 155820$$

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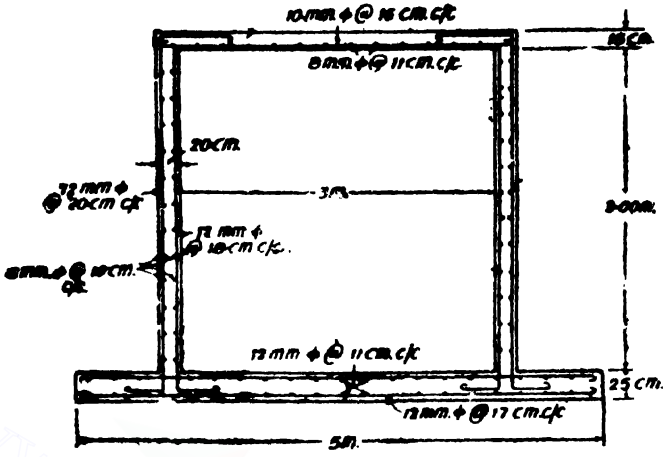

Fig. 53

$C_t = 12.76$ kg./cm.^{2}

(less than 16 kg./cm^2).

Cracking stress due to a bending moment of 129240 kg. cm.

This bending moment produces tension away from the wate. side.

Let the tensile stress in concrete be C_t

Total tension $= 100 \times 12{\cdot}7 \times \frac{C_t}{2} + (13-1) \times 6{\cdot}65 \times \frac{8{\cdot}7}{12{\cdot}7} C_t$ kg.

$= 687\ C_t$ kg.

Equating the moment of resistance to the bending moment,

$687\ C_t \times 0{\cdot}82 \times 19 = 129240$

$C_t = 10{\cdot}92$ kg/cm.^{2}

(less than 16 kg./cm^2).

Design 136. *Design an R.C. tank of internal dimensions 12 m. × 3·5 m. × 3·5 m. The tank is to be provided underground. The soil surrounding the tank is likely to get wet. Adopt suitable working stresses. Soil weighs 18000 N/metre^3. Use M 20 concrete.*

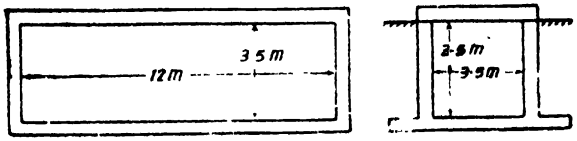

Fig. 538

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Solution. The following cases arise :

Case 1. Tank is full and the surrounding soil is dry.

Case 2. Tank is empty and the surrounding soil is water-logged.

Case 3. Tank is full and the surrounding soil is water-logged.

Case 4. Tank is empty and the surrounding soil is dry. Of these, the first two cases are the extreme cases and the tank should be designed for these cases.

Walls

Case 1. *Tank is full and the surrounding soil is dry.*

Maximum soil pressure

$$= 18000 \times 3{\cdot}5 \times \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} \; N/metre^2$$

$$= 21000 \; N/metre^2$$

Maximum water pressure

$$= 9810 \times 3{\cdot}5 \; N/metre^2$$

$$= 34335 \; N/metre^2$$

Net pressure at the bottom of wall

$$= 34335 - 21000 \; N/metre^2$$

$$= 13335 \; N/m^2.$$

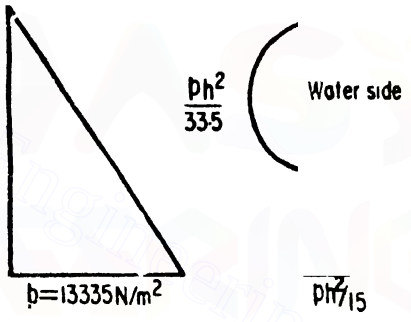

Fig. 539

Consider one *metre* run of the wall.

Maximum bending moment producing tension away from the water face

$$= \frac{ph^2}{33{\cdot}5} = \frac{13335 \times 3{\cdot}5^2}{33\,5}$$

$$= 4876\,2 \; Nm. = 487620 \; N\,cm.$$

Maximum bending moment producing tension near the water face

$$\frac{ph^2}{15}$$

$$\frac{13335 \times 3{\cdot}5^2}{15} \; Nm.$$

$$= 10890{\cdot}2 \; Nm. = 1089020 \; N\,cm.$$

Case 2. *Tank is empty and the surrounding soil is water-logged.*

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Pressure intensity exerted by wet earth at the bottom of the wall

$$= wh\frac{1-\sin 6^\circ}{1+\sin 6^\circ}$$

$$18000 \times 3{\cdot}5 \times \frac{1-\sin 6^\circ}{1+\sin 6^\circ}\ N/m.^2$$

$$= 51090\ N/m.^2$$

Consider one metre run of the wall.

Maximum bending moment producing tension near water face

$$= \frac{ph^2}{33{\cdot}5} = \frac{51090 \times 3{\cdot}5^2}{33{\cdot}5}\ Nm.$$

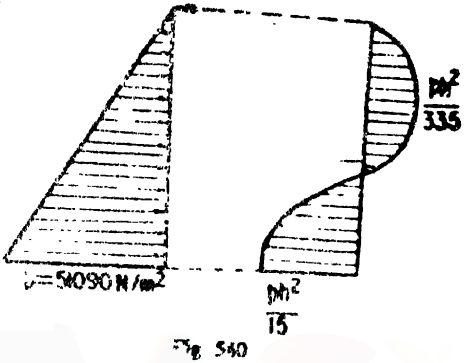

Fig. 540

$$= 18680\ Nm. = 186800\ N\ cm.$$

Maximum bending moment producing tension away from the water face

$$= \frac{ph^2}{15} = \frac{51090 \times 3{\cdot}5^2}{15}\ N\ m.$$

$$= 41730\ Nm. = 4173000\ N\ cm.$$

The bending moments computed above are tabulated below :

Case	B.M. producing Tension near water face (N cm.)	B.M. producing Tension away from water face (N cm.)
Case 1	1089020	437620
Case 2	1868000	4173000

From cracking stress consideration an overall depth to satisfy $M = 26{\cdot}67\ bd^2$ [$\frac{1}{6} f\ bD^2 = \frac{1}{6} \times 160\ bD^2 = 26{\cdot}67\ bD^2$] may be provided.

$$\therefore \quad 26{\cdot}67 \times 100\ D^2 = 4173000$$

$$D = 39{\cdot}53\ cm.$$

Let us provide a thickness of 40 cm.

Let the effective cover to the reinforcement be 4 cm.

$\therefore$ Effective depth $= 40 - 4 = 36$ cm.

Steel for a bending moment of 4173000 N cm.

$$= \frac{4173000}{10000 \times 0{\cdot}84 \times 36}\ cm.^2$$

$$= 13{\cdot}8\ cm^2.$$

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Spacing of 18 *mm.* diameter bars

$$= \frac{2{\cdot}54 \times 100}{13{\cdot}8} \text{ cm} = 18{\cdot}41 \text{ cm.}$$

Hence provide 18 mm. diameter bars at 18 cm. c/c (14·11 cm.²)

Steel for a bending moment of 1868000 *N cm.*

$$= \frac{1868000}{10000 \times 0{\cdot}84 \times 36} \text{ cm.}^2 \quad 6{\cdot}18 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$\frac{1{\cdot}13 \times 100}{6{\cdot}18} \text{ cm.} = 18{\cdot}28 \text{ cm.}$$

Provide 12 mm. diameter bars at 18 cm. centres (6·28 cm.²).

Check for cracking stresses

Position of actual neutral axis

Taking moments about the neutral axi we have

$$\frac{100n^2}{2} + (13-1)\,14{\cdot}11\,(n-4)$$

$$= \frac{100(40-n)^2}{2} + (13-1)\,6{\cdot}28\,(40-n-4)$$

$\therefore \quad n = 19{\cdot}64$ *cm.*

Cracking stress due to a
of 4173000 *N cm.*

Let the maximum tensile stress in concrete be C_t.

Total tension

$$= \frac{100 \times 19{\cdot}64\, C_t}{2} + (13-1)$$

$$14{\cdot}11 \times \frac{15{\cdot}64}{19{\cdot}64}\, C_t\, N$$

$$= 1117\, C_t$$

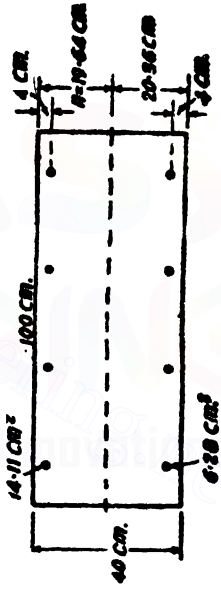

Fig. 5·1

Equating the M.R. to the B.M.

$$1117\, C_t \times 0{\cdot}82 \times 36 = 4173000$$

$$C_t = 126{\cdot}6 \text{ N/cm.}^2$$

(less than 160 *N/cm.²*)

Cracking stress due to a bending moment of 1868000 *N cm.*

Let the maximum tensile stress in concrete be C_t.

Total tension $= \frac{100 \times 20{\cdot}36 \times C_t}{2} + (13-1) \times 6{\cdot}28 \times \frac{16{\cdot}36}{20{\cdot}36}\, C_t$

$$= 1079\, C_t$$

Equating the M.R. to the B.M.

$$1079\, C_t \times 0{\cdot}82 \times 36 = 1868000$$

$$C_t = 58{\cdot}6 \text{ N/cm.}^2$$

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Design of the roof slab

Loads. Dead load (15 *cm.*) : 250×15 $= 3750\ N/m.^2$

Live load $= 1500\ N/m.^2$

Total $= 5250\ N/m.^2$

Consider a one *metre* wide strip of the slab.

Maximum bending moment

$$= \frac{5250 \times 3{\cdot}9^2}{8}\ Nm.$$

$$= 9981\ Nm. = 998100\ N\ cm.$$

If the usual concrete mix. (M 150) be used for the roof slab.

$$85 \times 100\ d^2 = 998100$$

$$d = 10{\cdot}8\ cm.$$

Providing an effective cover of 4 *cm.* the effective depth available $= 15 - 4 - 1\frac{1}{2}\ cm.$

$$A_t = \frac{998100}{14000 \times 0{\cdot}87 \times 11}\ cm.^2 = 7{\cdot}45\ cm.^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{7{\cdot}45} = 15{\cdot}2\ cm.$$

Provide 12 mm. diameter bars at 15 cm. centres.

Distribution steel $= \frac{0{\cdot}3}{100} \times 15 \times 100\ cm.^2$

$$= 4{\cdot}5\ cm.^2$$

Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{4{\cdot}5}\ cm. = 11\ cm.\ centres.$$

Design of the bottom slab

Assume 0·5 *m* thick base. Uplift pressure $= 9810 \times 4 = 39240\ N/m.^2$

Consider one *metre* run of tank.

Weight of roof

$= 4{\cdot}3 \times 0{\cdot}15 \times 25000\ N$

$= 16125\ N$

Weight of two walls

$= 2 \times 0{\cdot}4 \times 3{\cdot}5 \times 25000\ N$

$= 70000\ N$

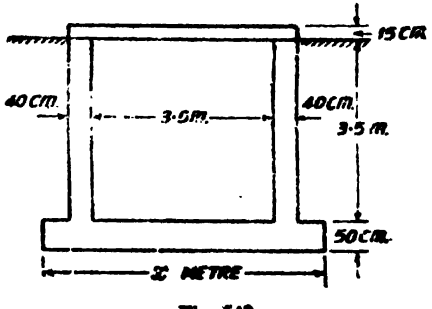

Fig. 542

Base slab $= 0{\cdot}5 \times x \times 25000\ N = 12500\ x\ N$

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Weight of soil on the projecting portions of the base slab

$$=(x-4{\cdot}3)3{\cdot}5\times 18000\ N$$

$$=63000\ (x-4{\cdot}3)\ N$$

Total upthrust $=39240\ x\ N$

Equating the downward and upward forces,

$$16125+70000+12300\ x$$

$$+63000(x-4{\cdot}3)=39240\ x$$

$$\therefore \quad x=5{\cdot}09\ m.$$

Let us provide a base width of 5·5 *metres*

The base slab will be designed for cases 1 and 2 mentioned earlier.

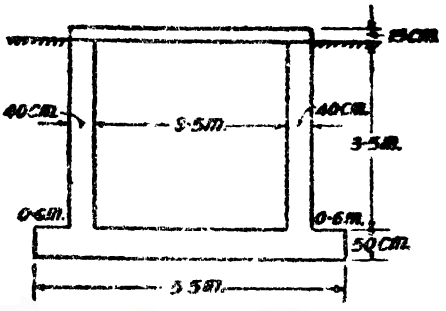

Fig. 543

Case 1. *Tank is full and the surrounding soil is dry. Consider one metre run of the tank.*

Weight of the roof slab $=16125\ N/m.$

Weight of the two walls $=70000\ N/m.$

Total $=86125\ N/m.$

Note Water pressure on the base slab and the weight of the base slab will be directly counteracted by ground pressure and will not therefore be included in the B.M computations.

Upward reaction due to the net load

$$=\frac{86125}{5\cdot5}\ N/m.$$

$$=15659{\cdot}09\ N/m.$$

B M. at the centre due to the above loading

$$=\frac{86125}{2}\ [1{\cdot}95-1{\cdot}375]\ Nm.$$

$$=24761\ Nm.$$

$$=2476100\ N\ cm.\ producing$$

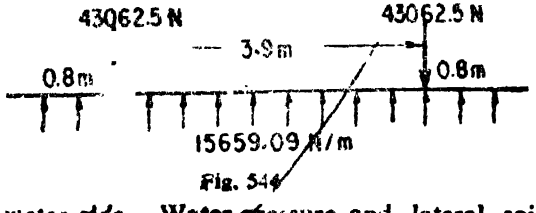

Fig. 544

ension on the water side. Water pressure and lateral soil pressure n the vertical walls will produce a moment of 1089020 *N cm.* of the ame type.

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∴ Total bending moment at the centre

$= -2476100 + 1089020$ *N cm.*

$= -3565120$ *N cm.*

producing tension near water face.

B.M. at the end

Hogging bending moment $= 86840$ *N cm.*

Sagging moment $= \frac{15659.09 \times 0.8^2}{2} \times 100 = 501090$ *N cm.*

Net B.M. 587930 *N cm.*

producing tension on the water face.

Case 2. *Tank is empty and the surrounding soil is water-logged.*

Intensity of upthrust exerted by water in the soil

$= 9810 \times 4 = 39240$ *N/metre*^{2}

Net upward pressure $= 15659.09 + 39240 = 54899.09$ *N/metre*^{2}

Consider one *metre* run of the tank

B.M. at the centre $= \frac{54899.09 \times 5.5}{2} (1.95 - 1.375)$ *Nm.*

$= 86809$ *Nm.*

$= 8680900$ *N cm.*

producing tension on the water side.

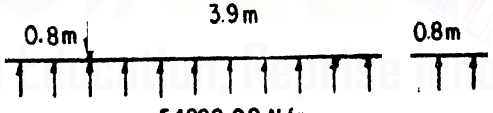

Fig. 545

But B.M. at the centre due to saturated earth pressure on the vertical walls $= 4173000$ *N cm. producing tension away from the water side.*

∴ Net B.M. at the centre

$= 8680900 - 4173000$ *N cm.*

$= 4507900$ *N cm.*

producing tension on the water side.

B.M. at the end

Sagging moment $= 4173000$ *N cm.*

Cantilever moment (sagging)

$= \frac{54899.09 \times 0.8^2}{2} \times 100$ *N cm.*

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$= 1756770\ N\ cm.$

∴ Total B.M. at the end

$= 4173000 + 1756770\ N\ cm.$

$= 5929770\ N\ cm.$ *producing tension away from the water side.*

From cracking stress consideration, equating the moment of resistance to the maximum bending moment, we have

$$26{\cdot}67\ bD^2 = 26{\cdot}67 \times 100\ D^2 = 5929770$$

$$\therefore \quad D = 47{\cdot}1\ cm.$$

Summary of Moments for the base slab

Case	*B.M. at End section (N cm.)*	*B.M. at Mid span (N cm.)*	*M. produces Tension*
Case 1	987930	3565120	On water side
Case 2	5929770		Away from water side
		4507900	On water side

Thickness provided $= 50\ cm.$

Let us provide an effective cover of 6 *cm.*

∴ Effective depth $= 50 - 6 = 44\ cm.$

Steel for a bending moment of

$5929770\ N\ cm.$

$$= \frac{5929770}{10000 \times 0{\cdot}84 \times 44}\ cm.^2 = 16{\cdot}04\ cm.^2$$

Spacing of 18 *mm.* dia bars

$$\frac{2{\cdot}54 \times 100}{16{\cdot}04} = 15{\cdot}83\ cm.$$

Provide 18 mm. diameter bars at 15 cm. c/c (16·93 cm.²)

Steel for a bending moment of 4507900 *N cm.*

$$= \frac{4507900}{10000 \times 0{\cdot}84 \times 44}\ cm.^2$$

$$= 12{\cdot}20\ cm^2.$$

Spacing of 16 *mm* diameter bars

$$\frac{2{\cdot}01 \times 100}{12{\cdot}20} = 16{\cdot}5$$

Provide 16 mm. diameter bars at 16 cm. c/c (12·56 cm².)

Check for cracking stresses

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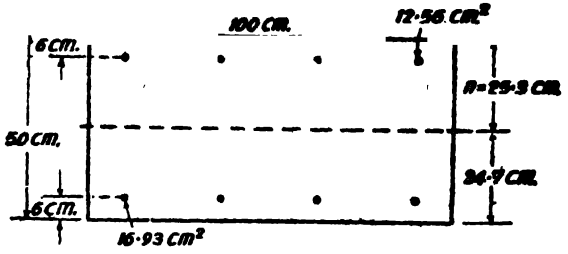

Fig. 546

Position of actual neutral axis

$$100\,\frac{n^2}{2}+(13-1)\times 12{\cdot}56\,(n-6)$$

$$=100\frac{(50-n)^2}{2}+(13-1)\times 16{\cdot}93(50-n-6)$$

$$\therefore \qquad n=25{\cdot}3\ cm.$$

Cracking stress due to a bending moment of 5929770 N cm.

This bending moment produces tension away from the water side.

Let the maximum tensile stress in concrete be C_t.

$$\text{Total tension} = 100\times 24{\cdot}7\times\frac{C_t}{2}+(13-1)\times 16{\cdot}93\times\frac{18{\cdot}7}{25{\cdot}7}C_t$$

$$=1389\ C_t\ N.$$

Equating the M.R. to B.M.

$$1389\ C_t\times 0{\cdot}82\times 44=5929770$$

$$C_t=118{\cdot}3\ N/cm.^2$$

(less than 160 $N/cm.^2$)

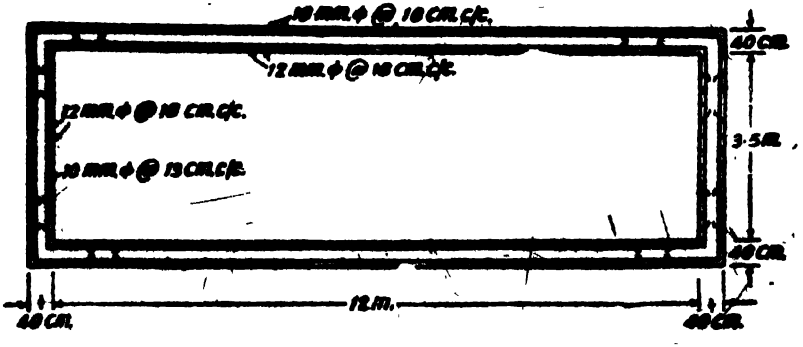

Fig. 547

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$= 1756770\ N\ cm.$

$\therefore$ Total B.M. at the end

$= 4173000 + 1756770\ N\ cm.$

$= 5929770\ N\ cm.$ *producing tension away from the water side.*

From cracking stress consideration, equating the moment of resistance to the maximum bending moment, we have

$$26{\cdot}67\ bD^2 = 26{\cdot}67 \times 100\ D^2 = 5929770$$

$$\therefore \quad D = 47{\cdot}1\ cm.$$

Summary of Moments for the base slab

Case	B.M. at End section (N cm.)	B.M. at Mid span (N cm.)	B.M. produces Tension
Case 1	987930	3565120	On water side
Case 2	5929770		Away from water side
		4507900	On water side

Thickness provided $= 50\ cm.$

Let us provide an effective cover of 6 *cm.*

$\therefore$ Effective depth $= 50 - 6 = 44\ cm.$

Steel for a bending moment of

$5929770\ N\ cm.$

$$= \frac{5929770}{10000 \times 0{\cdot}84 \times 44}\ cm.^2 = 16{\cdot}04\ cm.^2$$

Spacing of 18 *mm.* diameter bars

$$\frac{2{\cdot}54 \times 100}{16{\cdot}04} = 15{\cdot}83\ cm.$$

Provide 18 mm. diameter bars at 15 cm. c/c (16·93 cm.²)

Steel for a bending moment of 4507900 *N cm.*

$$= \frac{4507900}{10000 \times 0{\cdot}84 \times 44}\ cm.^2$$

$$= 12{\cdot}20\ cm^2.$$

Spacing of 16 *mm* diameter bars

$$\frac{2{\cdot}01 \times 100}{12{\cdot}20} = 16{\cdot}5\ cm.$$

Provide 16 mm. diameter bars at 16 cm. c/c (12·56 cm².)

Check for cracking stresses

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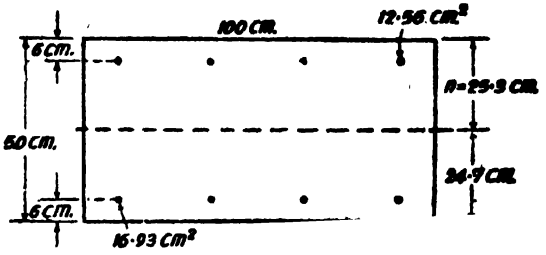

Fig. 546

Position of actual neutral axis

$$100\,\frac{n^2}{2}+(13-1)\times 12{\cdot}56\,(n-6)$$

$$=100\frac{(50-n)^2}{2}+(13-1)\times 16{\cdot}93(50-n-6)$$

$$\therefore \qquad n=25{\cdot}3\ cm.$$

Cracking stress due to a bending moment of 5929770 N cm.

This bending moment produces tension away from the water

Let the maximum tensile stress in concrete be C_t.

$$\text{Total tension} = 100\times 24{\cdot}7\times\frac{C_t}{2}+(13-1)\times 16{\cdot}93\times\frac{18{\cdot}7}{25{\cdot}7}C_t$$

$$=1389\ C_t\ N.$$

Equating the M.R. to B.M.

$$1389\ C_t\times 0{\cdot}82\times 44=5929770$$

$$C_t=118{\cdot}3\ N/cm.^2$$

(less than 160 $N/cm.^2$)

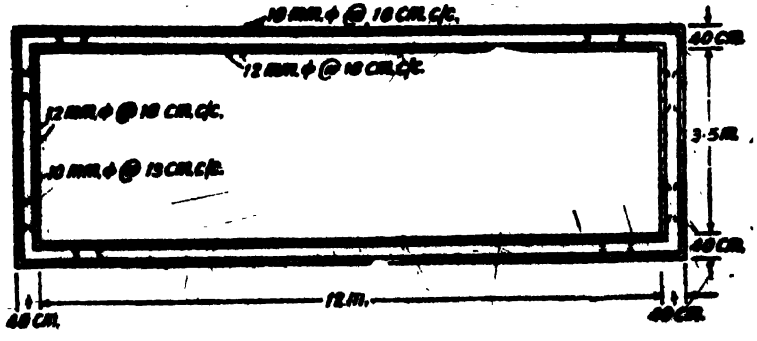

Fig. 547

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$$= \frac{ph^2}{33{\cdot}5} = \frac{5109 \times 3{\cdot}5^2}{33{\cdot}5}$$

$$= 1868 \text{ kg. m.} = 186800 \text{ kg. m.}$$

Maximum bending moment producing tension *away* from the water face

$$= \frac{ph^2}{15} = \frac{5109 \times 3{\cdot}5^2}{15} \text{ kg. m.}$$

$$= 4173 \text{ kg. m} = 417300 \text{ kg. cm.}$$

The bending moments computed above are tabulated below :

Case	*B.M. producing Tension near water face (kg. cm.)*	*B.M. producing Tension away from water face (kg. cm)*
Case 1	114300	51210
Case 2	186800	417300

From cracking stress consideration an overall depth to satisfy $M = 2{\cdot}67\ bD^2 (\frac{1}{8} fbd^2 = \frac{1}{8} \times 16 \times bD^2 = 2{\cdot}67\ bD^2)$ may be provided.

$$\therefore \quad 2{\cdot}67 \times 100 D^2 = 417300$$

$$D = 32{\cdot}54 \text{ cm.}$$

Let us provide a thickness of 40 *cm.*

Let the effective cover to the reinforcement be 4 *cm.*

$\therefore$ Effective depth $= 40 - 4 = 36$ *cm.*

Steel for a bending moment of 417300 *kg. cm.*

$$= \frac{417300}{1000 \times 0{\cdot}84 \times 36} \text{ cm.}^2$$

$$= 13{\cdot}8 \text{ cm}^2$$

Spacing of 18 *mm* diameter bars

$$= \frac{2{\cdot}54 \times 100}{13{\cdot}8} \text{ cm} = 18{\cdot}41 \text{ cm}$$

Hence provide 18 mm. diameter bars at 18 cm. c/c (14·11 cm².)

Steel for a bending moment of 186800 *kg. cm.*

$$= \frac{186800}{1000 \times 0{\cdot}84 \times 36} \text{ cm.}^2 = 6{\cdot}18 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{6{\cdot}18} \text{ cm.} = 18{\cdot}28 \text{ cm.}$$

Provide 12 mm. diameter bars at 18 cm. centres (6·28 cm.²).

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Check for cracking stresses

Position of actual neutral axis

Taking moments about the neutral axis, we have

$$\frac{100n^2}{2}+(13-1)\,14{\cdot}11\,(n-4)$$

$$=\frac{100(40-n)^2}{2}+(13-1)\,6{\cdot}28\,(40-n-4)$$

$\therefore \quad n=19{\cdot}64\ cm.$

Cracking stress due to a bending moment of 417300 kg. cm.

Let the maximum tensile stress in concrete be C_t.

Total tension

$$=\frac{100\times 19{\cdot}64\ C_t}{2}+(13-1)\,14{\cdot}11\times\frac{15\,64}{19{\cdot}64}\,C_t\ kg.$$

$$=1117\ C_t$$

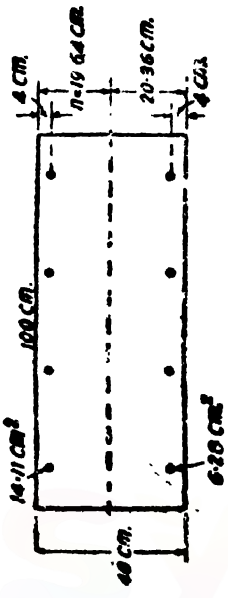

Fig. 552

Equating the M.R. to the B.M.

$$1117\ C_t\times 0{\cdot}82\times 36=417300$$

$$C_t=12{\cdot}66\ kg./cm.^2$$

(less than 16 *kg./cm.*²)

Cracking stress due to a bending moment of 186800 kg. cm.

Let the maximum tensile stress in concrete be C_t.

$$\text{Total tension}=\frac{100\times 20{\cdot}36\times C_t}{2}+(13-1)\times 6{\cdot}28\times\frac{16{\cdot}36}{20{\cdot}36}\,C_t$$

$$=1079\ C_t$$

Equating M.R. to the B.M.

$$1079\ C_t\times 0{\cdot}82\times 36=186800$$

$$C_t=5{\cdot}86\ kg./cm.^2$$

Design of the roof slab

Dead load (15 *cm.*)	**=360 *kg./m.*²**
Live load	**=150 *kg./m.*²**
Total	**=510 *kg./m.*²**

Consider a one *metre* wide strip of the slab.

$$510\times 3{\cdot}9^2\ kg.\ m.$$

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$= 9629 \text{ kg. m.} = 96990 \text{ kg. cm.}$

If the ordinary concrete mix (M 150) be used for the roof slab,

$$8{\cdot}5 \times 100\, d^2 = 96990$$

$$d = 10{\cdot}7 \text{ cm.}$$

Providing an effective cover of 4 cm. the effective depth available $= 15 - 4 = 11$ cm.

$$A_{st} = \frac{96990}{1400 \times 0{\cdot}87 \times 11} \text{ cm.}^2 = 7{\cdot}24 \text{ cm.}^2$$

Spacing of 12 mm. diameter bars

$$= \frac{1{\cdot}13 \times 100}{7{\cdot}24} \text{ cm.} = 15{\cdot}61 \text{ cm.}$$

Provide 12 mm. diameter bars at 15 cm. centres.

Distribution steel $= \frac{0{\cdot}3}{100} \times 15 \times 100 \text{ cm.}^2$

$= 4{\cdot}5 \text{ cm.}^2$

Spacing of 8 mm. diameter bars

$$= \frac{0{\cdot}5 \times 100}{4{\cdot}5} \text{ cm.} = 11 \text{ cm. centres.}$$

Design of the bottom slab

Assume 0·5 *m* thick base. Uplift pressure

$= 1000 \times 4 = 4000 \text{ kg./m.}^2$

Consider one *metre* run of tank.

Weight of roof

$= 4{\cdot}3 \times 0{\cdot}15 \times 2400 \text{ kg.}$

$= 1548 \text{ kg.}$

Weight of two walls

$= 2 \times 0{\cdot}4 \times 3{\cdot}5 \times 2400 \text{ kg.}$

$= 6720 \text{ kg.}$

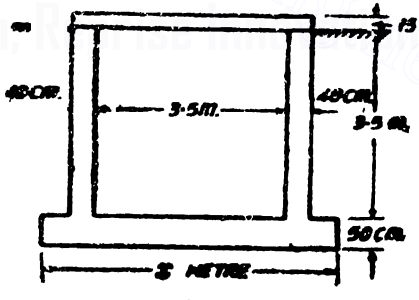

Fig. 553

Base slab $= 0{\cdot}5 \times x \times 2400 \text{ kg} = 1200\, x \text{ kg.}$

Weight of soil on the projecting portions of the base slab

$= (x - 4{\cdot}3) \times 3{\cdot}5 \times 1800 \text{ kg.}$

$= 6300(x - 4{\cdot}3) \text{ kg.}$

Total upthrust $= 4000\, x \text{ kg.}$

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Equating the downward and upward forces,

$$1548 + 6720 + 1200\,x + 6300(x - 0.3) = 4000\,x$$

$$\therefore \quad x = 5.38\ m.$$

Let us provide a base width of 5.5 metres.

The base slab will be designed for cases 1 and 2 mentioned earlier.

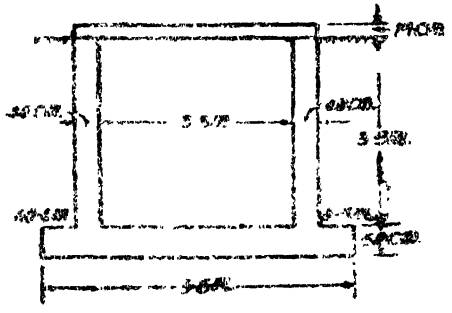

Fig. 554

Case 1. Tank is full. The surrounding soil is dry. Consider one metre run of the tank.

Weight of the roof slab $= 1548\ kg./m.$

Weight of the two walls $= 6720\ kg./m.$

$8268\ kg./m.$

Note. Water pressure on the base slab and weight of the base slab will be directly counteracted by ground pressure and will not therefore be included in the B.M. computations.

Upward reaction due to the net load

$$= \frac{8268}{5.5}\ kg./m.$$

$$= 1503.3\ kg./m.$$

B.M. at the centre due to the above loading

$$= \frac{8268}{2}\,[1.95 - 1.375]\ kg.\ m.$$

$$= 2378\ kg.\ m.$$

$$= 237800\ kg.\ cm.$$ *producing tension on the water side.* Water pressure and lateral soil pressure on the vertical wall will produce a moment of 114300 *kg. cm.* of the same type.

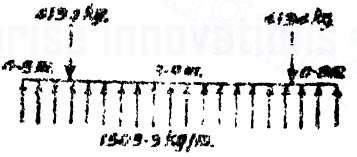

Fig. 555

∴ **Total bending moment at the centre**

$$= 237800 + 114300\ kg.\ cm.$$

$$= 352100\ kg.\ cm.$$

producing tension near water face.

B.M. at the end

Hogging bending moment $= 114300\ kg.\ cm.$

Sagging moment $= \frac{1503.3 \times 0.8^2}{2} \times 100 = 48090\ kg.\ cm.$

Net B.M. $= 66210\ kg.\ cm.$

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producing tension on the water face.

Case 2. *Tank is empty and the surrounding soil is water-logged.*

Intensity of upthrust exerted by water in the soil

$$=1000\times 4=4000 \text{ kg./m.}^2$$

Net upward pressure $=1503{\cdot}3+4000$ kg./m.^{2}

$$=5503{\cdot}3 \text{ kg. m.}^2$$

Consider one *metre* run of the tank

B.M. at the centre

$$=-\frac{5503{\cdot}3\times 5{\cdot}5}{2}(1{\cdot}95-1{\cdot}375) \text{ kg.m.}$$

Fig. 556

$=-8704$ *kg. m.*

$=-870400$ *kg. cm.*

producing tension on the water stae.

But B.M. at the centre due to saturated earth pressure on the vertical walls $=417300$ *kg. cm. producing tension away from the water side.*

∴ Net B.M. at the centre

$$=-870400-417300 \text{ kg. cm.}$$

$$=453100 \text{ kg. cm.}$$

producing tension in the water side.

B.M. at the end

Sagging moment $=417300$ *kg. cm.*

Cantilever moment (sagging)

$$=-\frac{5503{\cdot}3\times 0{\cdot}8^2}{2}\times 100 \text{ kg. cm.}$$

$$=-176100 \text{ kg. cm.}$$

∴ Total B.M. at the end

$$=417300+176100 \text{ kg. cm.}$$

$=593400$ *kg. cm. producing tension away from the water side.*

From cracking stress consideration, equating the moment of resistance to the maximum bending moment, we have

$$2{\cdot}67b\,D^2=2{\cdot}67\times 100\,D^2$$

$$=593400$$

$$D=47{\cdot}1 \text{ cm}$$

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Summary of Moments for the base slab

Case	*B.M. at End section (kg. cm.)*	*B.M. at Mid span (kg. cm.)*	*B.M. produces Tension*
Case 1	66210	352100	On water side
Case 2	593400		Away from water side
		453100	On water side

Thickness provided $= 50$ *cm.*

Let us provide an effective cover of 6 *cm.*

$\therefore$ Effective depth $= 50 - 6 = 44$ *cm.*

Steel for a bending moment of 593400 *kg. cm.*

$$= \frac{593400}{1000 \times 0{\cdot}84 \times 44} \text{ cm}^2 = 16{\cdot}05 \text{ cm.}^2$$

Spacing of 18 *mm.* diameter bars

$$= \frac{2{\cdot}54 \times 100}{16{\cdot}05} \text{ cm.} = 15{\cdot}82 \text{ cm.}^2$$

Provide 18 mm. diameter bars at 15 cm. c/c (16·93 cm.²)

Steel for a bending moment of 453100 *kg. cm.*

$$= \frac{453100}{1000 \times 0{\cdot}84 \times 44} \text{ cm.}^2$$

$$= 12{\cdot}26 \text{ cm.}^2$$

Spacing of 16 *mm.* diameter bars

$$= \frac{2{\cdot}01 \times 100}{12{\cdot}26} \text{ cm.}$$

$$= 16{\cdot}4 \text{ cm.}$$

Provide 16 mm. diameter bars at 16 cm. c/c (12·56 cm.²).

Check for cracking stresses

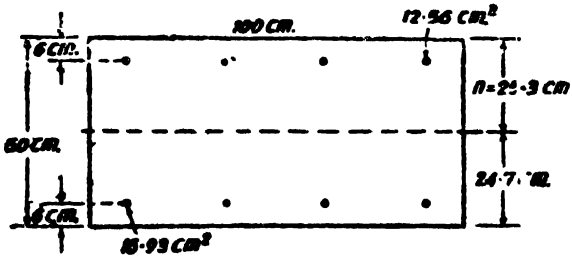

Fig. 557

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Position of actual neutral axis

Taking moments about the neutral axis, we have

$$100\frac{n^2}{2}+(13-1)\times 12{\cdot}56\,(n-6)$$

$$=100\frac{(50-n)^2}{2}+(13-1)\times 16{\cdot}93(50-n-6)$$

$$\therefore \quad n=25{\cdot}3\ cm.$$

Cracking stress due to a bending moment of 593400 kg. cm.

This bending moment produces tension away from the water side.

Let the maximum tensile stress in concrete be C_t.

$$\text{Total tension} = 100\times 24{\cdot}7\times\frac{C_t}{2}+(13-1)\times 16{\cdot}93\times\frac{18{\cdot}7}{25{\cdot}7}C_t$$

$$=1389\ C_t\ kg.$$

Equating the M.R. to B.M

$$1389\ C_t\times 0{\cdot}82\times 44=593400$$

$$C_t=11{\cdot}82\ kg./cm^2$$

(less than 1 $kg./cm^2$)

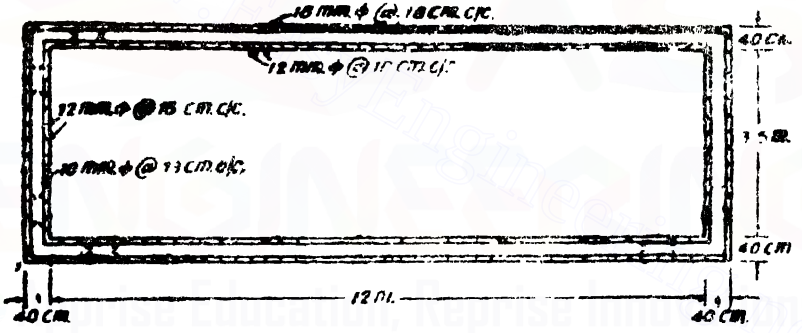
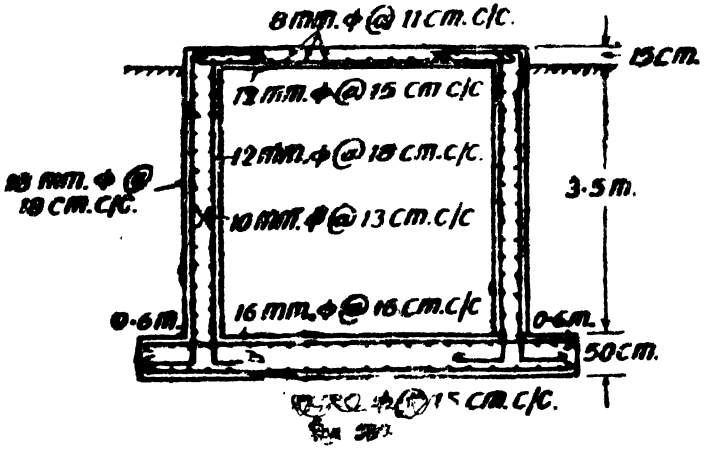


Fig. 558



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Cracking stress due to a bending moment of 453100 kg. cm.

This bending moment produces tension near the water side.

Let the maximum tensile stress in concrete be C_t.

$$\text{Total tension} = \frac{100 \times 25{\cdot}3 \times C_t}{2} + (13-1) \times 12{\cdot}56 \times \frac{19{\cdot}3}{25{\cdot}3} C_t$$

$$= 1389\ C_t$$

Equating the M.R. to the B.M.

$$1389\ C_t \times 0{\cdot}82 \times 44 = 453100$$

$$C_t = 9\ kg./cm.^2$$

§186. Large Underground Reservoir

The design of a large open underground reservoir depends on the nature and the safe bearing capacity of the subsoil and the level of the subsoil water. For cases in which the subsoil is dry or properly well-drained and there is no possibility of uneven settlement the underground reservoir will only consist of four vertical walls forming the side and a watertight floor. The walls are nothing but retaining walls and will be designed as such. The joints between the walls and the floor should also be watertight.

Floor of the reservoir. Even if the depth of water in a reservoir is 6 *metres* the weight of water will produce a pressure of only 6 *tonnes per metre*^{2}, and this pressure can be safely resisted by the soil under the floor. Hence the floor slab will only require nominal reinforcement so as to enable it to span over any possible weak patches of ground. A slab 15 *cm.* to 20 *cm.* thick is often sufficient, reinforced with 12 *mm.* diameter bars at 30 *cm.* centres in both directions in the slab. Sometimes, two such mats are provided and when so provided, it is usual that one mat will be staggered relative to the other.

It is usual to lay the floor in either one or two courses on the alternate bay system. These bays should never exceed 45 *metre*^{2} of area. When two courses are made, the joints of the upper and the lower layer should be staggered and tarred paper (or canvas) is sometimes provided between the two courses.

In cases where the subsoil water is likely to exceed the level corresponding to the bottom of the tank it is usual to provide 8 *cm.* to 10 *cm.* diameter drain pipes at 3 *metres* to 3·5 *metres* centres with open joints in trenches filled with ballast or rubble below the floor and should discharge to a main collecting drain.

Reservoir wall

The walls may be either of mass concrete or reinforced concrete The walls are designed as cantilever retaining walls. These are usually designed to remain stable for the following two conditions :

(*i*) When the reservoir is empty and the walls are loaded by lateral soil pressure.

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(*ii*) When the reservoir is full and the backing is removed for making connections or extensions. Stability computations and the determination of reinforcement are made exactly on lines similar to retaining walls. It is necessary to provide reinforcement in both faces.

Expansion joints should be provided in the wall at 6 *metres* to 9 *metres* intervals.

Horizontal construction joints should be as few as possible. This is done by connecting the wall in as few lifts as possible.

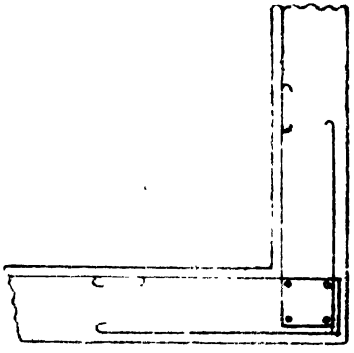

Fig. 560. Horizontal reinforcement in corner of walls.

At corners of walls additional horizontal reinforcement must be provided. A suitable arrangement is shown in Fig. 560.

Design 138 (S.I.). *Design the walls and floor of a reservoir to be provided below ground level in good dry earth, to hold 3 metres depth of water. The safe bearing capacity of the soil is 160 kN per metre*^{2}. *The soil has an angle of repose of 30° having a weight of 16000 N/metre*^{3}.

Solution. Fig 561 shows the proposed arrangement for the retaining walls of the tank.

Let us now investigate the stability of the wall per *metre* run.

Case 1. ***When only water pressure acts.***

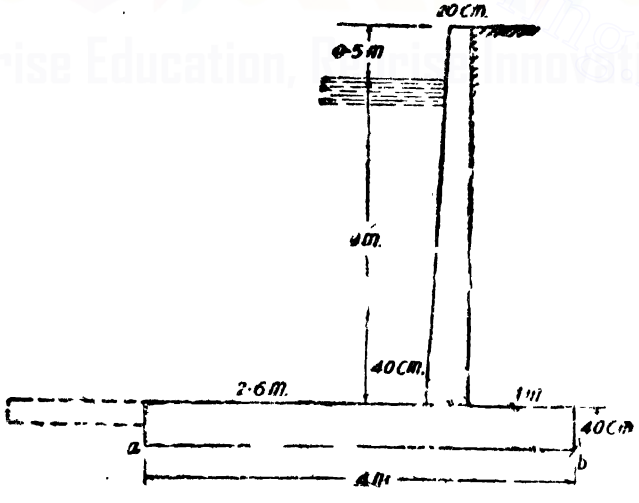

Fig. 561

The relevant calculations are shown in the following table :

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Load due to	*Magnitude of load (N)*	*Distance from a (m.)*	*Moment about a (Nm.)*
Stem			
$0{\cdot}2\times3{\cdot}5\times25000$	17500	2·9	50750
$0{\cdot}2\times\frac{3{\cdot}5}{2}\times25000$	8750	2·73	23887
Wt. of water			
$\frac{3\times0{\cdot}2}{3{\cdot}5}\times\frac{3}{2}\times9810$	2522	$2{\cdot}6+\frac{1}{3}\left(\frac{3\times0{\cdot}2}{3{\cdot}5}\right)$	
		$=2{\cdot}66$	6708
$2{\cdot}60\times3\times9810$	76518	1·30	99473
Base slab			
$4\times0{\cdot}4\times25000$	40000	2	80000
Moment of water pressure			
$9810\times\frac{3^3}{6}$			44145
Total	145290		304563

$$\therefore \quad z=\frac{304963}{145290}=2{\cdot}10\ m.$$

$$\therefore \quad \text{Eccentricity} \quad e=z-\frac{b}{2}$$

$$=2{\cdot}10-2=0{\cdot}10\ m.$$

Maximum and minimum pressures at the base are given by

$$p=\frac{W}{b}\left[1\pm\frac{6e}{b}\right]$$

$$p=\frac{145290}{4}\left[1\pm\frac{6\times0{\cdot}10}{4}\right]\ N/m.^2$$

$$\therefore \quad p_{max}=41770\ N/m.^2$$

$$p_{min}=30870\ N/m.^2$$

Total horizontal pressure exerted by water

$$=P=\frac{wh^2}{2}$$

$$=9810\times\frac{3^2}{2}\ N=44145\ N$$

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Maximum available friction

$$= \mu W$$
$$= 0{\cdot}5 \times 145290 = 72645\ N$$

Factor of safety against sliding

$$= \frac{\mu W}{P}$$
$$= \frac{72645}{44145} = 1{\cdot}6 \text{ (greater than } 1{\cdot}5).$$

Case 2. *When the tank is empty.*

The relevant stability calculations are shown in the table below :

$$\therefore \quad z = \frac{297909}{122250} = 2{\cdot}44\ m.$$

Eccentricity $\quad e = z - \frac{b}{2} = 2{\cdot}44 - 2 = 0{\cdot}44\ m.$

Maximum and minimum pressures at the base are given by

$$p = \frac{122250}{4}\left[1 \pm \frac{6 \times 0{\cdot}44}{4}\right] N/m.^2$$

$$p_{max} = 50730\ N/m.^2$$

and $\quad p_{min} = 10390\ N/m.^2$

Load due to	*Magnitude of load (N)*	*Distance from a (m.)*	*Moment about a (Nm.)*
Stem	17500	2·9	50750
	8750	2·73	23887
Base slab	40000	2	80000
Wt. of soil			
1×3·5×16000	56000	3·5	196000
Total	122250		350637
Deduct for moment of lateral pressure			
$16000 \times \frac{3{\cdot}5^2}{6} \frac{1-\sin 30°}{1+\sin 30°}$			52728
Net moment about *a*			297909

Reinforcement

Case 1. Maximum bending moment at the bottom of the stem due to water pressure alone

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$$= 9810 \times \frac{3^3}{6} \; Nm. = 44145 \; Nm.$$

This bending moment produces tension on the water face

Adopting $c = 700 \; N/cm.^2$, $t = 10000 \; N/cm.^2$, $m = 13$

Equating the M.R. to the B M.

$$141 \times 100 \; d^2 = 44145 \times 100$$

$$d = 17{\cdot}7 \; cm.$$

Providing an effective cover of 4 *cm.*, the effective depth available

$$= 40 - 4 = 36 \; cm.$$

$$A_t = \frac{44145 \times 100}{10000 \times 0{\cdot}84 \times 36} \; cm.^2$$

$$= 14{\cdot}60 \; cm.^2$$

Spacing of 16 *mm.* diameter bars

$$= \frac{2{\cdot}01 \times 100}{14{\cdot}60} \; cm.$$

$$= 13{\cdot}8 \; cm. \; say \; 13 \; cm. \; centres.$$

Case 2. Maximum bending moment at the bottom of the stem due to lateral soil pressure alone

$$= 16000 \times \frac{3{\cdot}5^3}{6} \left(\frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} \right) Nm.$$

$$= 38110 \; Nm.$$

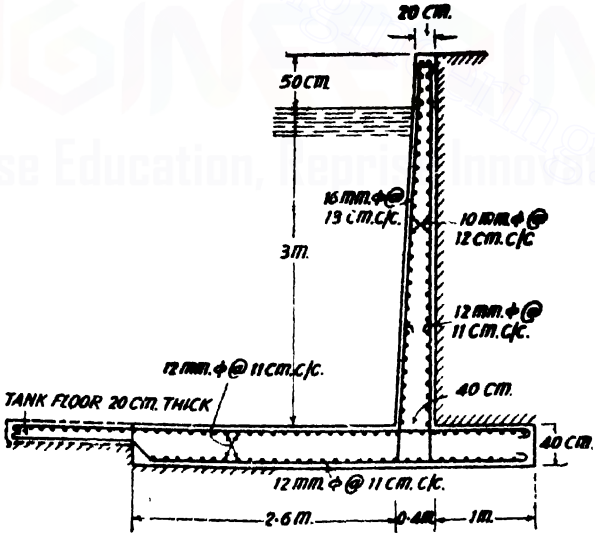

Fig 562

This bending moment produces tension away from the water face.

Hence adopting $c = 700 \; N/cm.^2$, $t = 12500 \; N/cm.^2$, $m = 13$

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$$A_s = \frac{38110 \times 100}{12500 \times 0{\cdot}86 \times 36} \text{ cm.}^2 = 9{\cdot}85 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{9{\cdot}85} \text{ cm.}$$

$$= 11{\cdot}48 \text{ cm. say } 11 \text{ cm. centres.}$$

Distribution steel

$$= 0{\cdot}3\% \text{ of gross area}$$

$$= \frac{0{\cdot}3}{100} \times 40 \times 100 \text{ cm.}^2$$

$$= 12 \text{ cm.}^2$$

Spacing of 10 *mm.* diameter bars

$$= \frac{0{\cdot}79 \times 100}{12} \text{ cm.}$$

$$= 6{\cdot}6 \text{ cm. say } 6 \text{ cm. centres.}$$

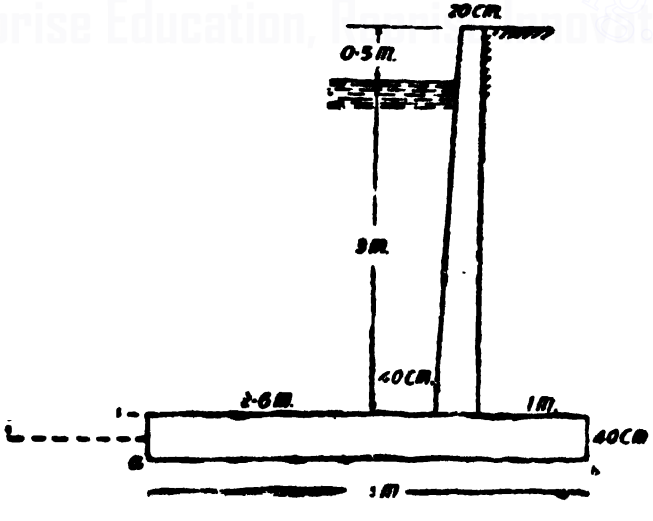
If the distribution steel be provided near both the faces then the spacing will be at 12 cm. centres near each face.

Design 139. *Design the walls and floor of a reservoir to be provided below ground level in good dry earth, to hold 3 metres depth of water. The safe bearing capacity of the soil is 16 tonnes per metre*^{2}*. The soil has an angle of repose of 30° having a weight of 1600 kg/metre*^{3}*.*

Solution. Fig. 563 shows the proposed arrangement for the retaining walls of the tank.

Let us now investigate the stability of the wall per *metre* run.

Case 1. *When only water pressure acts.*



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The relevant calculations are shown in the following table :

Load due to	*Magnitude of load (kg.)*	*Distance from a (m.)*	*Moment (kg. m.)*
Stem			
$0{\cdot}2\times3{\cdot}5\times2400$	1680	2·9	4872
$0{\cdot}2\times\frac{3{\cdot}5}{2}\times2400$	840	2·73	2293
Wt. of water			
$\frac{3\times0{\cdot}2}{3{\cdot}5}\times\frac{3}{2}\times1000$	257	$2{\cdot}6+\frac{1}{3}\left(\frac{3\times0{\cdot}2}{3{\cdot}5}\right)$ $=2{\cdot}66$	684
$2{\cdot}60\times3\times1000$	7800	1·30	10140
Base slab			
$4\times0{\cdot}\times2400$	3840	2	7680
Moment of water pressure			
$1000\times\frac{3^3}{6}$			4500
Total	14417		30169

$$z=\frac{30169}{14417}=2{\cdot}09\ m.$$

Eccentricity $e=z-\frac{b}{2}$

$=2{\cdot}09-2=0{\cdot}09\ m.$

Maximum and minimum pressures at the base are given by

$$p=\frac{W}{b}\left[1\pm\frac{6e}{b}\right]$$

$$p=\frac{14417}{4}\left[1\pm\frac{6\times0{\cdot}09}{4}\right]\ kg./m.^2$$

$\therefore$ $p_{max}=4090\ kg./m.^2$

$p_{min}=3118\ kg./m.^2$

Total horizontal pressure exerted by water

$$=P=\frac{wh^2}{2}$$

$$1000\times\frac{3^2}{2}\ kg.=4500\ kg.$$

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Maximum available friction

$$= \mu W$$
$$= 0{\cdot}5 \times 14417 \text{ kg.} = 7208 \text{ kg.}$$

Factor of safety against sliding

$$= \frac{\mu W}{P}$$
$$= \frac{7208}{4500} = 1{\cdot}6 \text{ (greater than } 1{\cdot}5).$$

Case 2. *When the tank is empty.*

The relevant stability calculations are shown in the table below:

$$z = \frac{291 \quad 2}{11960} = 2\,44 \text{ m.}$$

Eccentricity $e = z - \frac{v}{2} = 2{\cdot}44 - 2 = 0{\cdot}44$ m.

Maximum and minimum pressures at the base are given by

$$p = \frac{11960}{4}\left[1 \pm \frac{6 \times 0{\cdot}44}{4}\right] \text{ kg./m.}^2$$

$p_{max} = 4964$ kg./m.^{2}

and $p_{min} = 1016$ kg./m.^{2}

Load due to	*Magnitude of load (kg.)*	*Distance from a (m.)*	*Moment about a (kg.m.)*
Stem	1680 810	2·9 2·73	4872 2293
Base slab	3840	2	7680
Wt. of soil $1 \times 3{\cdot}5 \times 1600$	5600	3·5	19600
Total	11960		34445
Deduct for moment of lateral pressure $1600 \times \frac{3{\cdot}9^2}{6} \frac{1-\sin 30°}{1+\sin 30°}$			5273
Net moment about *a*			29172

Reinforcement

Case 1. Maximum bending moment at the bottom of the stem due to water pressure alone

$$= 1000 \times \frac{3^3}{6} \text{ kg. m.} = 4500 \text{ kg. m.}$$

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This bending moment produces tension on the water face

Adopting $c = 70\ kg./cm.^2$, $t = 1000\ kg./cm.^2$, $m = 13$

Equating the M.R. to the B.M.

$$14{\cdot}1 \times 100\ d^2 = 4500 \times 100$$

$$d = 17{\cdot}8\ cm.$$

Providing an effective cover of 4 *cm.*, the effective depth available

$$= 40 - 4 = 36\ cm.$$

$$\therefore \quad A_t = \frac{4500 \times 100}{1000 \times 0{\cdot}84 \times 36}\ cm.^2$$

$$= 14{\cdot}88\ cm.^2$$

Spacing of 16 *mm.* diameter bars

$$= \frac{2{\cdot}01 \times 100}{14{\cdot}88}\ cm.$$

$$= 13{\cdot}51\ cm.\ say\ 13\ cm.\ centres.$$

Case 2. Maximum bending moment at the bottom of the stem due to lateral soil pressure alone

$$= 1600 \times \frac{3{\cdot}5^3}{6} = \frac{1 - \sin 30°}{1 + \sin 30°}\ kg.\ m.$$

$$= 3812\ kg.\ m.$$

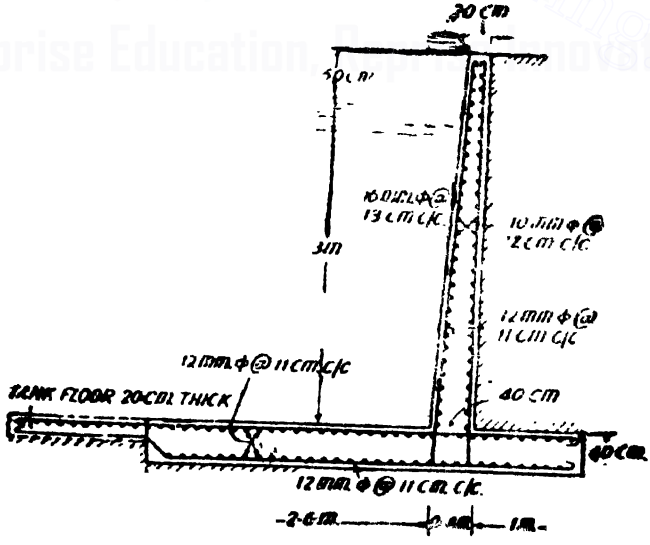

FIG. 564

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This bending moment produces tension away from the water face.

Hence adopting $c=70\ kg/cm.^2$, $t=1250\ kg./cm.^2$, $m=13$

$$A_s = \frac{3812\times 100}{1250\times 0{\cdot}86\times 36}\ cm.^2 = 9{\cdot}85\ cm.^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13\times 100}{9{\cdot}85}\ cm.$$

$$= 11{\cdot}48\ cm.\ say\ 11\ cm.\ centres.$$

Distribution steel

$$= 0{\cdot}3\%\ \text{of gross area}$$

$$= \frac{0{\cdot}3}{100}\times 40\times 100\ cm.^2$$

$$= 12\ cm.^2$$

Spacing of 10 *mm.* diameter bars

$$= \frac{0{\cdot}79\times 100}{12}\ cm.$$

$$= 6{\cdot}6\ cm.\ say\ 6\ cm.\ centres.$$

If the distribution steel be provided near both the faces then the spacing will be at 12 cm. centres near each face.

§187. Box Culverts.

These are provided for conveying water to serve the following requirements :

(*i*) To serve as means for a cross drainage.

(*ii*) To provide a supporting slab for a roadway, under which the cross drainage flows.

Fig. 565 shows the section of a box culvert.

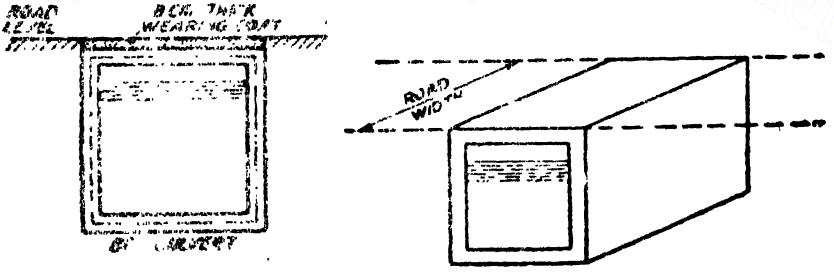

Fig 565

The culvert consists of two horizontal and two vertical slabs built monolithically forming a box with two open ends. Where high embankments are involved an R.C. box culvert will work out to be economical as compared with masonry abutments.

A box culvert should be designed to remain safe for the following cases :

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Case 1. When the top slab carries the dead and live load and the culvert is empty.

Case 2. When the top slab carries the dead and live load and the culvert is full of water.

Case 3. When the sides of the culvert do not carry the live load and the culvert is full.

Design 140 *Design a box culvert having inside dimensions 3·5m. × 3·5 m. The superimposed dead load on the culvert is 1·00 kg / metre². The live load on the culvert is 5000 kg / metre². The soil at the site weighs 1800 kg./ metre³ having an angle of repose of 30°*

Solution. Let us provide a thickness of 30 *cm.* for all the slabs of the culvert. Hence the centre line dimensions of the culvert will be 3·8 *m.* × 3·8 *m.*

As mentioned earlier the culvert will be designed for the following cases :

Case 1. *When the top slab carries the dead and live load and the culvert is empty.*

Loads. Weight of top slab = 24 × 30 = 720 *kg./m.*²
Superimposed dead load = 1200 *kg /m.*²
Live load = 5000 *kg./m.*²

Total load on top slab = 6920 *kg /m.*²

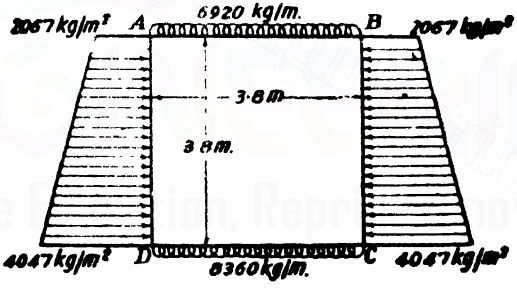

FIG. 566

Weight of two vertical walls
= 2 × 0·3 × 3·8 × 2400 *kg./m.*
= 5472 *kg /m.*

Consider one *metre* run of the culvert. Pressure on the base sl

$$= \frac{6920 \times 3{\cdot}8 + 5472}{3{\cdot}8} \text{ kg./m.}^2$$

= 8360 *kg./m* ²

Note. Weight of the bottom slab will be directly counteracted he corresponding ground reaction.

At any depth *h* from the level of top slab, lateral pressure

$$= p = (5000 + 1200)\frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} + 1800 \times h \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ}$$

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$p = \frac{1}{3}[6200 + 1800\,h]$

At $h = 0$, *i.e.*, at the level of the top slab

$$p = \frac{6200}{3} = 2067 \text{ kg./m.}^2$$

At $h = 3{\cdot}8$, $p = \frac{1}{3}[6200 + 1800 \times 3{\cdot}8]$ kg./m.^{2}

$= 4047$ kg /m.^{2}

Consider one *metre* run of the culvert.

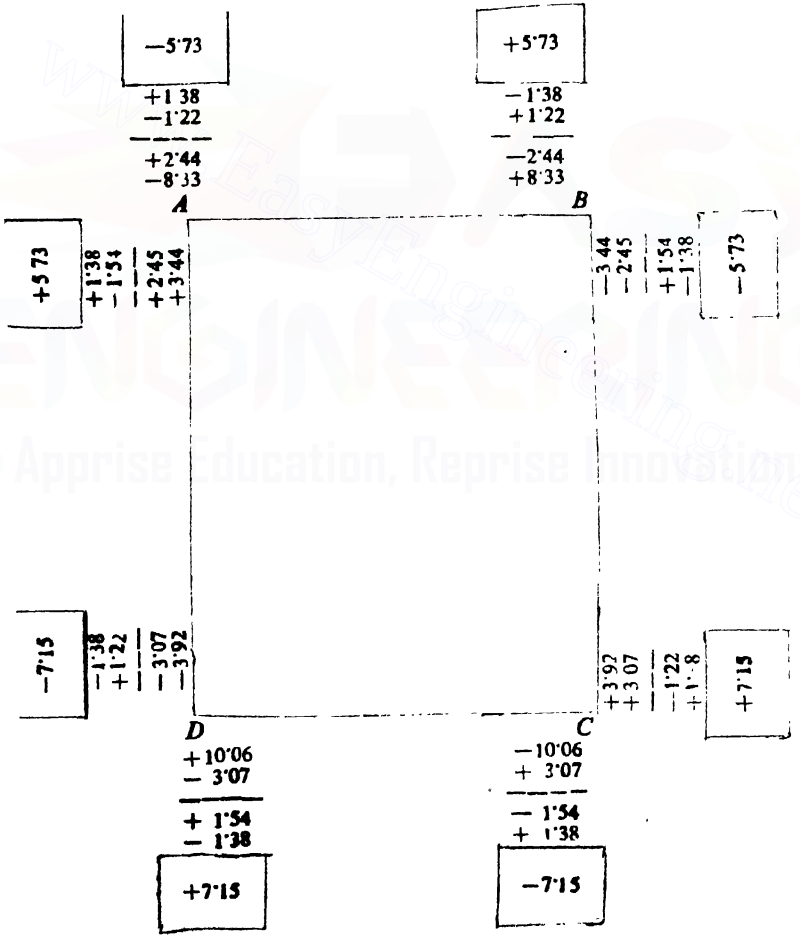
The loading corresponding to this case is shown in Fig. 566.

Considering each panel as fixed at the ends, the fixed end moments are as follows :

$$m_{ab} = -\frac{6920 \times 3{\cdot}8^2}{12} \text{ kg. m.} = -8330 \text{ kg. m.}$$

$= -8{\cdot}33$ *tonne metre*

$m_{ba} = +8{\cdot}33$ *tonne metre*



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$$m_{dc} = +\frac{8360 \times 3{\cdot}8^2}{12} \text{ kg. m.} = +10060 \text{ kg. m.}$$

$$= +10{\cdot}06 \text{ tonne metre}$$

$$m_{cd} = -10{\cdot}06 \text{ tonne metre}$$

$$m_{da} = -\left[\frac{2067 \times 3{\cdot}8^2}{12} + \frac{(4047-2067)\,3{\cdot}8^2}{20}\right] \text{kg. m.}$$

$$= -3918 \text{ kg. m} = -3{\cdot}92 \text{ tonne metre (say)}$$

$$m_{ad} = +\left[\frac{2067 \times 3{\cdot}8^2}{12} + \frac{(4047-2067)\,3{\cdot}8^2}{30}\right] \text{kg. m.}$$

$$= +3441 \text{ kg. m.} = +3{\cdot}44 \text{ tonne metre (say)}$$

$$m_{bc} = -3{\cdot}44 \text{ tonne metre}$$

$$m_{cb} = +3{\cdot}92 \text{ tonne metre.}$$

The moments for the culvert for this case are determined by moment distribution as shown on page 908.

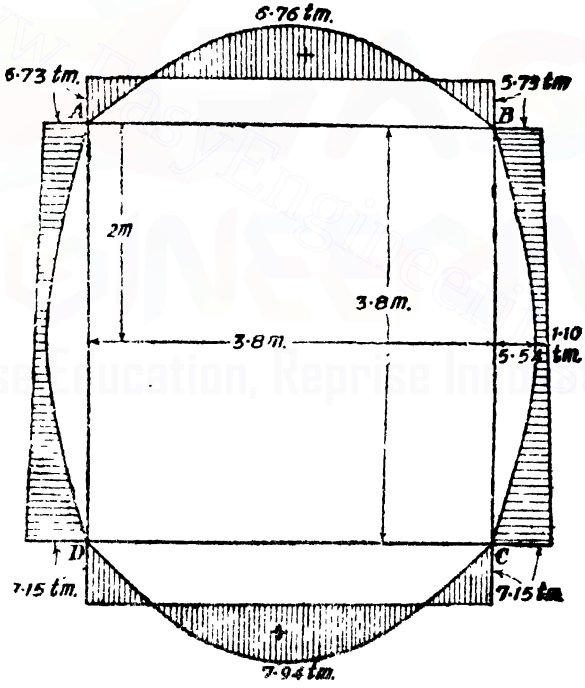

Fig. 567. B.M. diagram for case 1 loading.

Bending moment producing concavity outside has been regarded as positive.

Max. free B.M. for AB

$$= \frac{6{\cdot}920 \times 3{\cdot}8^2}{8} \text{ tm.}$$

$$= 12{\cdot}49 \text{ tm.}$$

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Net B.M. at centre of AB

$$= 12\,49 - 5\cdot73 + 6\cdot76\ tm.$$

Max. free B.M. for CD

$$= -\frac{8\cdot360 \times 3\,8^2}{8}\ tm.$$

$$= -15\cdot09\ tm.$$

$\therefore$ Net B.M. at centre of CD

$$= 15\,09 - 7\cdot15 = 7\,94\ tm.$$

Max. free B.M. for vertical wall

$$= 5\cdot54\ tm.\ \text{at } 2\ m \text{ from } A$$

Net B.M. at 9 m below A

$$= 6\cdot64 - 5\cdot54 = 1\cdot10\ tm.$$

Case 2. *When the top slab carries the dead and live load and the culvert is full of water.*

For this case the net loading on the top and bottom slabs will be the same as in case 1.

For the vertical wall water pressure intensity will vary from zero at the top to $3\cdot8 \times 1000 = 3800\ kg./m.^2$ at bottom

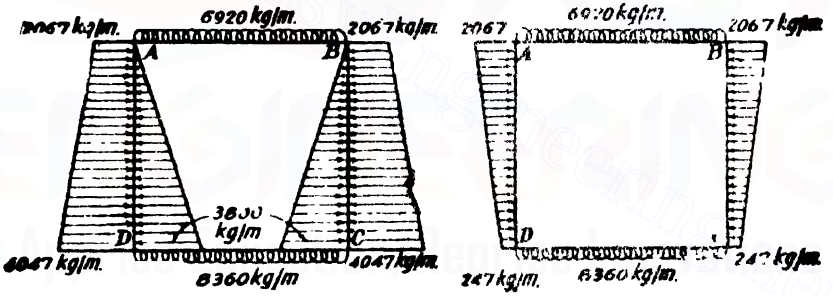

Fig. 568.

Hence the net pressure for the vertical wall will be as follows :

At top : $2067\ kg./m.^2$

At bottom : $4047 - 3800 = 247\ kg./m.^2$

Fixed end moments

$$m_{ab} = -\frac{6920 \times 3\,8^2}{12}\ kg.\ m. = -8330\ kg.\ m = -8\cdot33\ tm.$$

$$m_{ba} = +8\cdot33\ tm.$$

$$m_{dc} = +\frac{8360 \times 3\cdot8^2}{12}\ kg.\ m. = +10060\ kg.\ m. = +10\,06\ tm.$$

$$m_{cd} = -10\cdot06\ tm.$$

$$m_{da} = -\left[\frac{247 \times 3\cdot8^2}{12} + \frac{(2067 - 247) \times 3\cdot8^2}{30}\right] kg.\ m.$$

$$= -1170\ kg.\ m. = -1\cdot17\ tm.$$

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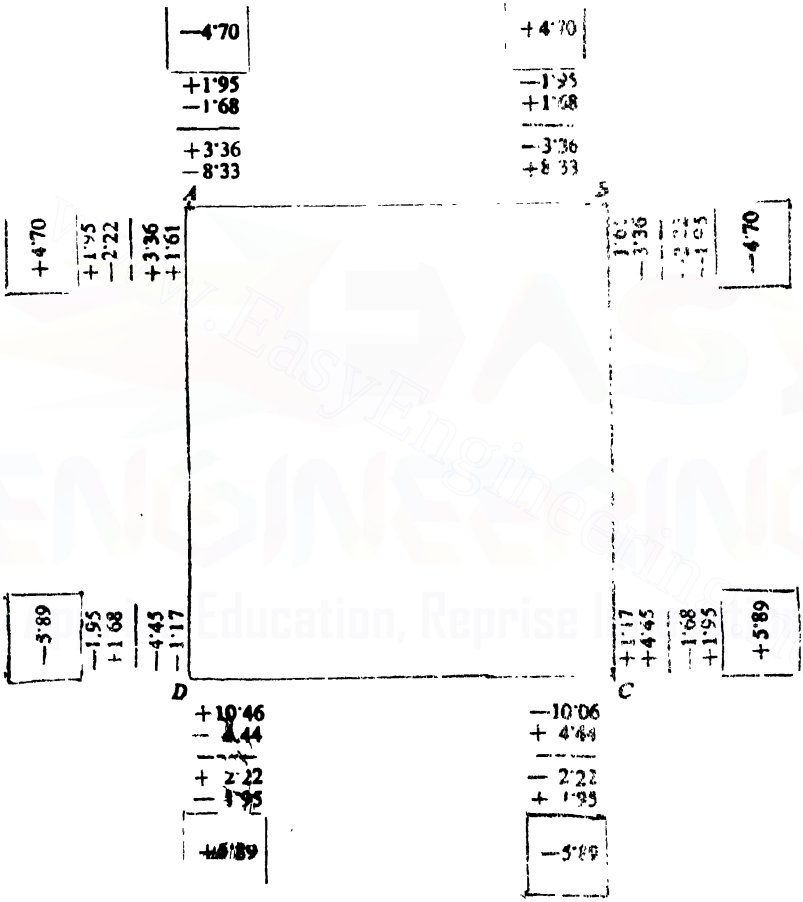
$$m_{ad} = + \left[\frac{247 \times 3{\cdot}8^2}{12} + \frac{(2067-247) \times 3{\cdot}8^2}{12} \right] kg.\ m.$$

$$= +1610\ kg.\ m. = +1{\cdot}61\ tm.$$

$$m_{bc} = -1{\cdot}61\ tm.$$

$$m_{cb} = +1{\cdot}17\ tm.$$

The moments for the culvert for this case are determined by moment distribution as shown below :



Max free B.M. for AB

$$= 12{\cdot}49\ tm.$$

$\therefore$ Net B.M. at centre of AB

$$12{\cdot}49 - 4{\cdot}70 = 7{\cdot}79\ tm.$$

Max. free B.M. for CD

$$= 15{\cdot}09\ tm$$

$\therefore$ Net B.M. at centre of CD

$$= 15{\cdot}09 - 5{\cdot}89 = 9{\cdot}20\ tm.$$

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Max. free B.M. for vertical wall
$=2{\cdot}08$ *tm.* at $1{\cdot}66$ *m.* from *A*

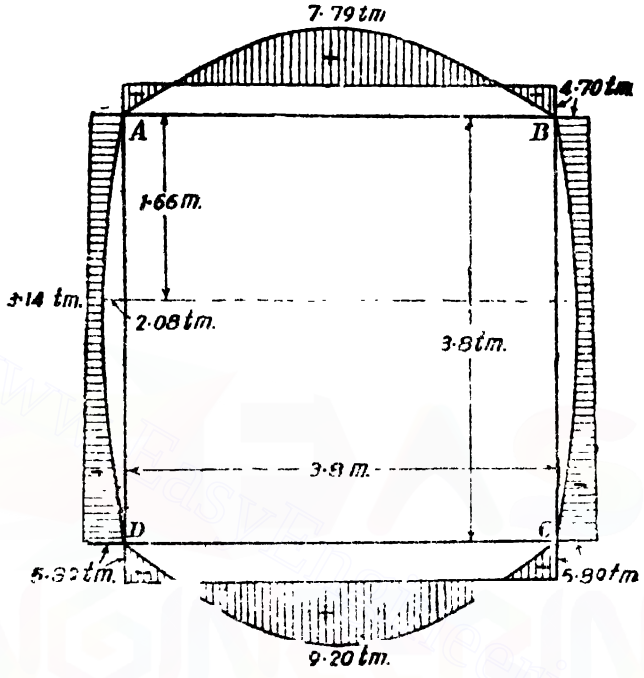

Fig. 569

∴ Net B.M. at $1{\cdot}66$ *m.* below *A*
$=5{\cdot}22-2{\cdot}08$ *tm.*
$=3{\cdot}14$ *tm.*

Case 3. ***When the sides of the culvert do not carry the live load and the culvert is full.***

For this case the net loading on the top and bottom slab be the same as in the previous cases. But the loading on the vertical wall will be as follows :

Water pressure intensity will vary from zero at top to 3800 *kg./m.*^{2} at bottom

Lateral pressure due to superimposed dead lead and soil at depth

$$p = 1200\frac{1-\sin 30^\circ}{1+\sin 30^\circ}+1800\,h\,\frac{1-\sin 30^\circ}{1+\sin 30^\circ}$$
$$=400+600\,h$$

At top $p=400$ *kg./m.*^{2}

At bottom $p=400+600\times 3{\cdot}8=2680$ *kg./m.*^{2}

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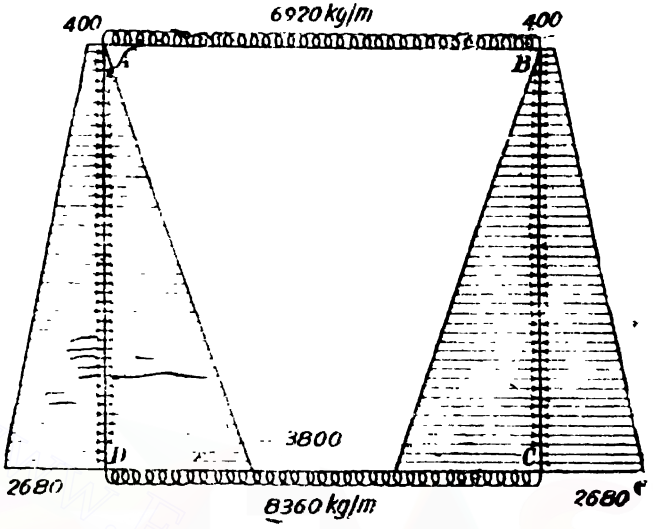

Fig. 570

Fig. 570 shows the loading on the culvert for one *metre* of its length.

Fixed end moments

$$m_{ab} = -\frac{6920 \times 3{\cdot}8^2}{12} \text{ kg. m.} = -8330 \text{ kg. m.} = -8{\cdot}33 \text{ tm.}$$

$$m_{ba} = +8{\cdot}33 \text{ tm.}$$

$$m_{dc} = +\frac{8360 \times 3{\cdot}8^2}{12} \text{ kg. m.} = +10060 \text{ kg. m.} = +10{\cdot}06 \text{ tm.}$$

$$m_{cd} = -10{\cdot}06 \text{ tm.}$$

$$m_{da} = +(3800-2280)\frac{3{\cdot}8^2}{20} - \frac{400 \times 3{\cdot}8^2}{12} \text{ kg. m.}$$

$$= +616 \text{ kg.m.} = 0{\cdot}62 \text{ tm.}$$

$$m_{ad} = -(3800-2280)\frac{3{\cdot}8^2}{30} = \frac{400 \times 3{\cdot}8^2}{12}$$

$$= -251 \text{ kg. m.} = -0{\cdot}251 \text{ tm.}$$

$$m_{bc} = +0{\cdot}25 \text{ tm.}$$

$$m_{cb} = -0{\cdot}62 \text{ tm.}$$

The moments for the culvert for this case are determined by moment distribution as shown on page 914.

Max. free B.M. for AB

$$= -\frac{6{\cdot}92 \times 3{\cdot}8^2}{8} \text{ tm.}$$

$$= +12{\cdot}49 \text{ tm.}$$

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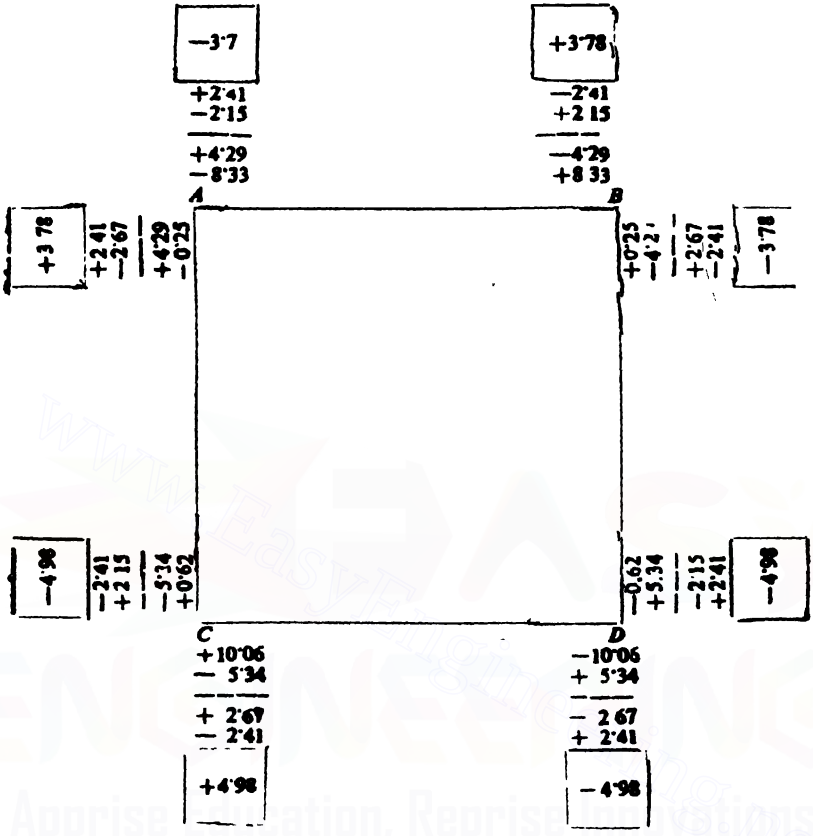

∴ Net B.M. at centre of AB

$$= 12{\cdot}49 - 3{\cdot}78 = 8{\cdot}71 \ tm.$$

Max. free B.M. for CD

$$= \frac{8{\cdot}36 \times 3{\cdot}8^2}{8} \ tm. = 15{\cdot}09 \ tm.$$

∴ Net B.M. at centre of CD

$$= 15{\cdot}09 - 4{\cdot}98 = 10{\cdot}11 \ tm.$$

Max. free B.M. for AD is approximately $\frac{w_1 l^2}{9\sqrt{3}} - \frac{w_2 l^2}{8}$ at nearly $\frac{l}{\sqrt{3}}$ from the top.

$$(3100 - 2280)\frac{3{\cdot}8^2}{9\sqrt{3}} - 400 \times \frac{3{\cdot}8^2}{8}$$

$$= -686 \ kg.\ m.$$

$$= -0{\cdot}69 \ tm.$$

at $\frac{l}{\sqrt{3}} = \frac{3{\cdot}8^2}{\sqrt{3}} = 2{\cdot}2$ m. from top

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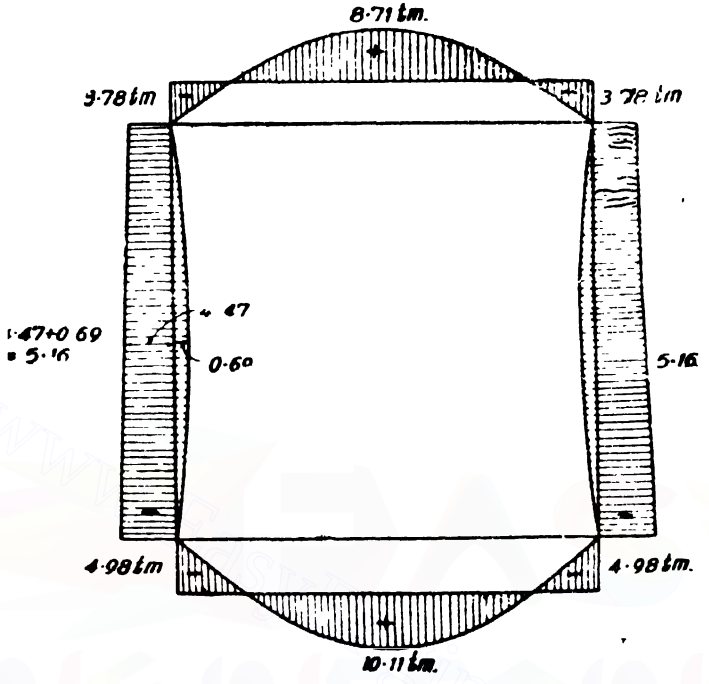
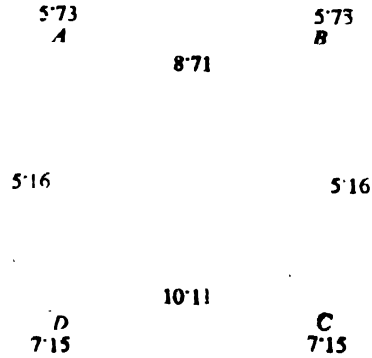


Fig. 571

Resultant B.M. at 2·2 *m*. from top

$$= 4{\cdot}47 + 0{\cdot}69 = 5{\cdot}16 \ tm.$$

The maximum bending moments to which the various sections are subjected during the above three conditions of loading are given below :



It is, therefore, convenient to design the mid span sections of the top and bottom slabs for a bending moment of 10·11 *tm*

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producing tension near the water face. The vertical walls and the end sections of the top and bottom slabs may be designed for a bending moment of 7·15 *tm.* producing tension away from the water side.

Equating the M.R. to the max. B.M.

$$14{\cdot}1 \times 100d^2 = 10{\cdot}11 \times 1000 \times 100$$

$$d = 26 \text{ cm.}$$

Providing an effective cover of 4 *cm.* effective depth available

$$= 30 - 4 = 26 \text{ cm.}$$

Steel for a bending moment of 10·11 *tm.*

$$= \frac{10{\cdot}11 \times 1000 \times 10'}{1000 \times 0{\cdot}84 \times 26} \text{ cm.}^2 = 46{\cdot}28 \text{ cm.}^2$$

Spacing of 22 *mm.* diameter bars

$$\frac{3{\cdot}8 \times 100}{46{\cdot}28} \text{ cm.} = 8 \text{ cm. c/c.}$$

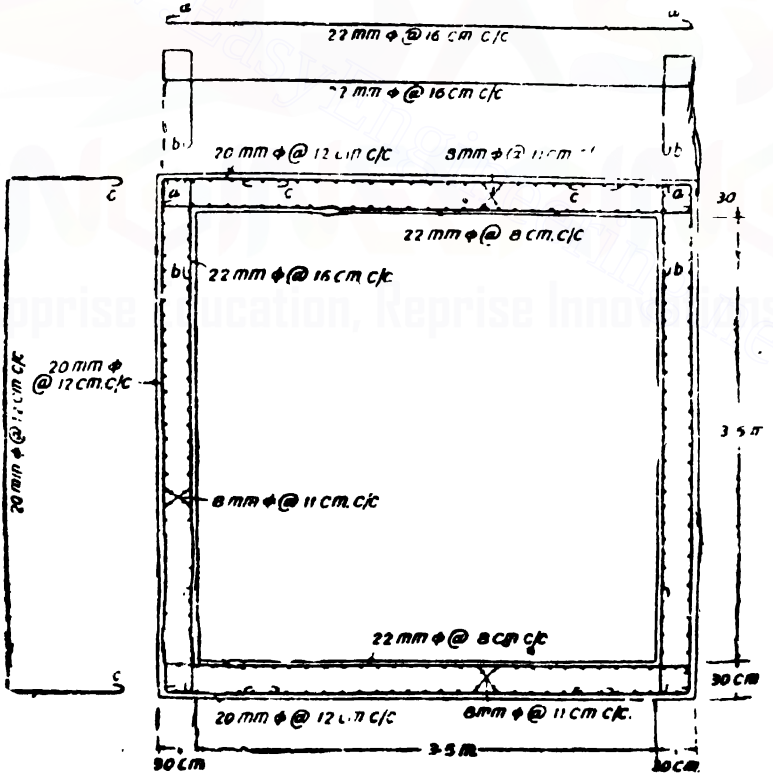

Fig. 572

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Steel for a bending moment of 7·15 *tm.*

$$= \frac{7{\cdot}15 \times 1000 \times 100}{1250 \times 0{\cdot}86 \times 26} \ cm.^2 = 25{\cdot}6 \ cm.^2$$

Spacing of 20 *mm.* diameter bars

$$= \frac{3{\cdot}14 \times 100}{25{\cdot}6} \ cm.$$

$$= 12 \ cm. \ centres.$$

Distribution Steel

$$= \frac{0{\cdot}3}{100} \times 30 \times 100 \ cm.^2$$

$$= 9 \ cm.^2$$

Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{9} \ cm.$$

$$= 5{\cdot}5 \ cm.$$

If the distribution bars be provided near both the faces the spacing of the bars will be at 11 *cm.* centres near each face.

Fig. 572 shows details of reinforcement.

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16

Bunkers and Silos

Bunkers and silos are structures meant for storing materials like cemon coal, wheat, broken stone clinker etc. A bunker is a shallow structure in which the plane of rupture of the material stored meets the surface of the stored material before meeting the opposite side of the structure. See Fig 573. The angle of rupture may be taken at $\frac{90^\circ + \phi}{2}$ with the horizontal. (ϕ is the angle of repose)

For a bunker, the lateral pressure on the walls may be determined by Rankine's theory. The total load of the contents of the bunker is taken to be supported by the floor of the bunker

A silo is a tall structure in which the plane of rupture of the material stored meets the opposite face of the structure before meeting the surface of the stored material

In this case, the entire weight of the material stored will not act on the floor of this structure. A considerable part of this load is resisted by the friction between the material, stored and the wall. See Fig. 574.

If b and h be the breadth and depth of the structure, then the structure can be taken to be a silo if $h > b \tan\left(\frac{90+\phi}{2}\right)$. For instance if $\phi = 30^\circ$ then, for a silo,

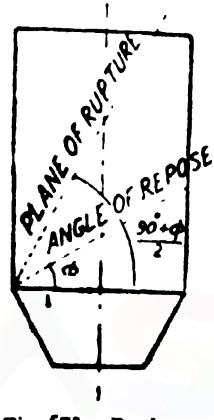

Fig. 573. Bunker or shallow bin.

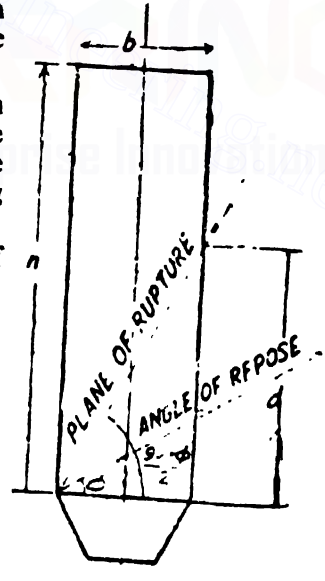

Fig. 574. Silo or deep bin.

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$$h > b \tan 60°$$
$$h > 1\,732\,b$$

Similarly, for $\phi = 25°$, the condition that the structure is a silo is $h > 1{\cdot}5697\ b$.

ters

As mentioned earlier the lateral pressure will be determined by Rankine's theory

Often the bunkers are filled with material retained at a surcharge angle α. The lateral pressure intensity at any depth h is given by

$$p = wh \cos \alpha \quad \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}}$$

This lateral pressure intensity acts in a direction parallel to the free sloping surface of the material stored.

Often a bunker may have to be designed allowing the greatest surcharge angle i.e. if $\alpha = \phi$ then

$$p = wh \cos \phi.$$

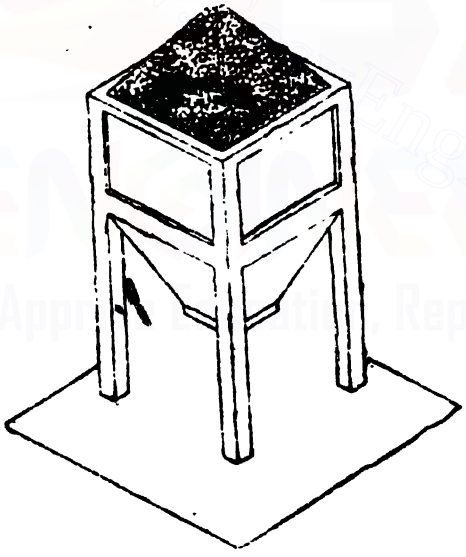

Fig. 575. Square bunker

The horizontal component of this up pressure

$$= p_h = wh \cos^2 \phi$$

Square bunkers

The vertical walls of the bunker may be designed as spanning horizontally as in the case of the square tank resting on beams and columns.

The walls are also subjected to tension whose effect also may be included in computing the reinforcement required.

Design 141 (S.I

Fig. 576 shows an arrangement for a coal bunker Design the bunker.

Solution. Capacity of bunker

	cu m.
$2{\cdot}5 \times 2{\cdot}5 \times 3$	$= 18{\cdot}75$
$2{\cdot}5 \times 2{\cdot}5 \times \frac{1}{2} \times 0{\cdot}72$	$= 1{\cdot}50$
$\frac{1}{3}[2{\cdot}5^2 + 0{\cdot}45^2 + \sqrt{6{\cdot}25 \times 0{\cdot}2025}]$	$= 2{\cdot}53$
	22·78 *cu. m.*

Weight of coal at 9080 *N* per cubic *metre*

$$= 22{\cdot}78 \times 9080\ N = 205020\ N$$

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Design of side walls

Horizontal pressure intensity

$= wh \cos^2 \phi$

$= 9000 \times 3(0{\cdot}866)^2$

$= 20250\ N/m.^2$

Taking the effective span as

$2{\cdot}5 + 0{\cdot}30 = 2{\cdot}80$ *metres*

Maximum bending moment

$$\frac{20250 \times 2{\cdot}8^2}{12} \times 100\ N\ cm.$$

$= 1323000\ N.\ cm.$

Max. tension

$$\frac{20250 \times 2{\cdot}5}{2}\ N$$

$= 25312\ N$

Let the walls be 15 *cm.* thick.

Let the eff. depth be 12 *cm.*

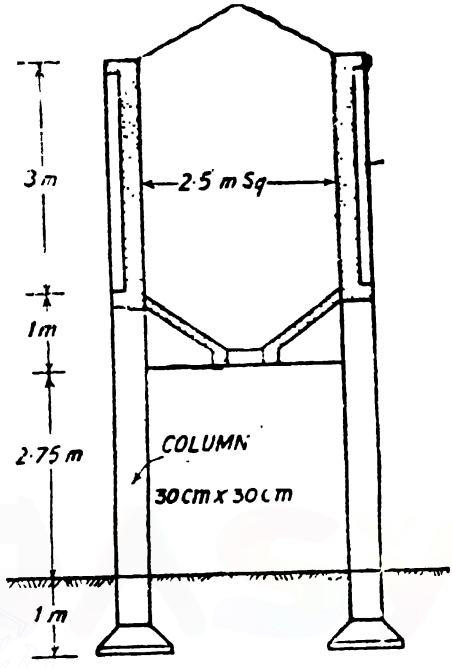

Fig. 576

Resultant B.M. $= 1323000 - 25312\ (12 - 7{\cdot}5)\ N\ cm.$

$= 1209096\ N\ cm.$

Adopting $c = 500\ N/cm.^2$, $t = 14000\ N/cm^2$, $m = 8$, we have $0{\cdot}87\ d$ and $Q = 85$

$\therefore \quad 85 \times 100 \times d^2 = 1209096$

$$d = \sqrt{\frac{1209096}{8500}} = 11{\cdot}92\ cm.$$

Maintain 15 *cm.* overall and 12 *cm.* effective

$$A_s = \frac{1209096}{14000 \times 0{\cdot}87 \times 12}\ cm.^2$$

$= 8{\cdot}27\ cm.^2$

$\therefore$ Spacing of 12 *mm.* dia. bars

$$= \frac{1{\cdot}13 \times 100}{8{\cdot}27}\ cms.$$

$= 13{\cdot}66\ cms.$

Hence provide 12 mm. dia. bars at 13 cms centres

Distribution steel $= 0{\cdot}15\%$ of gross area

$$= \frac{0{\cdot}15}{100} \times 100 \times 15\ cm.^2 = 2{\cdot}25\ cm.^2$$

$\therefore$ **Spacing of 8 *mm.* dia. bars,**

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$$= \frac{0{\cdot}50 \times 100}{2{\cdot}25} = 22 \text{ cm.}$$

Design of the hopper bottom

Weight of coal = 205020 *N*

Sloping bottom (15 *cm.* + 2·5 *cm.* lining) $\left[\frac{2{\cdot}5+0{\cdot}45}{2}\right]$

$$\sqrt{1{\cdot}025^2+1^2} \times 4 \times 0{\cdot}175 \times 25000 = 36960 \text{ N}$$

Total = 241980 *N*

∴ Load on one wall

$$\frac{241980}{4} = 60495 \text{ N}$$

If *T* be the tension in the sloping wall

$$T \sin\theta = 60495 \qquad T = \frac{60495}{\sin\theta}$$

$$\tan\theta = \frac{1}{\cdot 025}$$

$$\theta = 44^\circ\ 18'$$

$$T = \frac{60495}{\sin 44^\circ\ 18'} = 86617 \text{ N}$$

Tension per *metre* run

$$= \frac{86617}{2{\cdot}5} \text{ N/metre} = 34645 \text{ N/metre}$$

∴ Steel required per *metre*

$$\frac{34645}{14000} = 2{\cdot}47 \text{ cm.}^2$$

Provide 8 mm. dia. bars at 20 cms. centres.

Sloping bottom as a slab spanning between corners

The hopper bottom should also be designed to resist bending due to

(*i*) The normal component p_n of the pressure due to coal.

(*ii*) The normal component of the weight of the slab and the lining.

Normal component of the coal pressure

$$= wh(\cos^2\theta + \cos^2\phi \sin^2\theta)$$

$w = 9000 \text{ N/m.}^3$

$$h = 3 + \tfrac{1}{2} + \frac{0{\cdot}72}{2} = 3{\cdot}86 \text{ m.}$$

$\theta = 44^\circ\ 18'$

∴ Normal pressure due to coal

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$= 9000 \times 3{\cdot}86[\cos^2 44° 18' + \cos^2 30° \sin^2 44° 18']$

$= 30500\ N/m.^2$

Normal component of the weight of the sloping slab

$$\frac{(15+2{\cdot}5)}{100} \times 25000 \cos 44° 18\ N/m^2 = 3130\ N/m.^2$$

$\therefore$ Total normal pressure

$= 30500 + 3130$

$= 33630\ N/m.^2$

Effective span

$$\frac{2{\cdot}5 + 0{\cdot}45}{2} + 0{\cdot}15\ \text{metres}$$

$= 1{\cdot}63$ metres

$\therefore$ Max. B M.

$$\frac{33630 \times 1{\cdot}63^2}{12} \times 100\ N\ cm.$$

$= 744596\ N/cm./metre$ width

$85 \times 100d^2 = 744596$

$$\therefore\ d = \sqrt{\frac{744596}{8500}}$$

$= 9{\cdot}36\ cm.$

But the eff. depth available

$15\ cm. - 3\ cm$

$= 12\ cm.$

$$\therefore \frac{744596}{14000 \times 0{\cdot}87 \times 12}\ cm^2$$

$= 5{\cdot}09\ cm^2$

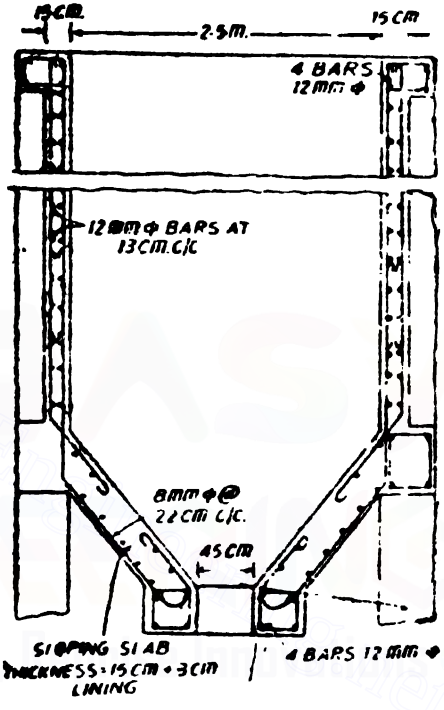

Fig. 577

Spacing of 10 mm bars

$$\frac{0{\cdot}79 \times 100}{5{\cdot}09} = 15{\cdot}5\ cm.$$

Say 10 mm bars at 15 cms centres.

Design 142. *Fig 578 shows an arrangement for a coal bunker. Design the bunker.*

Solution Capacity of the bunker

	cu. m.
$2{\cdot}5 \times 2{\cdot}5 \times 3$	$= 18{\cdot}75$
$2{\cdot}5 \times 2{\cdot}5 \times \frac{1}{3} \times 0{\cdot}72$	$= 1{\cdot}50$
$\frac{1}{3}[2{\cdot}5^2 + 0{\cdot}45^2 + \sqrt{6{\cdot}25 + 0{\cdot}2025}]$	$= 3{\cdot}00$
	23·25 cu. m.

Weight of coal at 900 kg. per cubic metre

$= 23{\cdot}25 \times 900\ kg. = 20925\ kg.$

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Design of side walls

Horizontal pressure intensity

$= wh \cos^2 \phi$

$= 900 \times 3(0{\cdot}866)^2$

$= 2027\ kg./m.^2$

Taking the effective span as

$2{\cdot}5 + 0{\cdot}30 = 2{\cdot}80$ *metres*

Maximum bending moment

$= \frac{2027 \times 2{\cdot}8^2}{12} \times 100\ kg.\ cm.$

$= 132400\ kg.\ cm.$

Max. tension

$= \frac{2027 \times 2{\cdot}5}{2}\ kg.$

$= 2534\ kg.$

Let the walls be 15 *cm* thick.

Let the eff. depth be 12 *cm.*

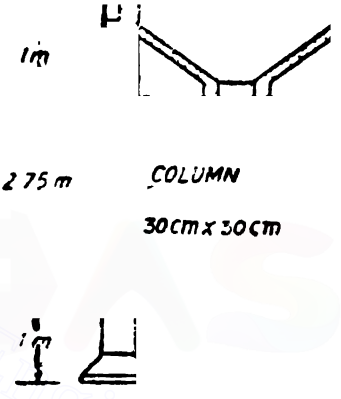

Fig. 578.

Resusltant b.m. $= 132400 - 2534(12 - 7{\cdot}5)\ kg./cm.$

$= 120997\ kg.\ cm.$

Adopting $c = 50\ kg/cm.^2$, $t = 1400\ kg.\ cm.^2$, $m = 18$, we have $a = 0{\cdot}87\ d$ and $Q = 8{\cdot}50$.

$\therefore \quad 8{\cdot}50 \times 100 \times d^2 = 120997$

$\therefore \quad d = \sqrt{\frac{120997}{850}} = 12\ cm.$

Maintain 15 *cm.* overall and 12 *cm.* effective

$$A_t = \frac{120997}{1400 \times 0{\cdot}87 \times 12}\ cm^2$$

$$= 8{\cdot}28\ cm^2$$

$\therefore$ Spacing of 12 *mm.* dia. bars

$$= \frac{1{\cdot}13 \times 100}{8{\cdot}28}\ cms.$$

$$= 13{\cdot}65\ cms.$$

Hence provide 12 mm. dia. bars at 13 cms. centres.

Distribution steel $= 0{\cdot}15\%$ of gross area

$$= \frac{0{\cdot}15}{100} \times 100 \times 15\ cm.^2 = 2{\cdot}25\ cm.^2$$

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$\therefore$ Spacing of 8 *mm.* dia bars

$$= \frac{0{\cdot}50 \times 100}{2{\cdot}25} = 22 \text{ cm.}$$

Design of the hopper bottom

Weight of coal $= 20925$ *kg.*

Sloping bottom (15 *cm.* + 2·5 *cm.* lining) $\left[\frac{2{\cdot}5 \times 0{\cdot}45}{2}\right]$

$\sqrt{1{\cdot}025^2 + 1^2} \times 4 \times 0{\cdot}175 \times 2400 = 2406$ *kg.*

Total 23331 *kg.*

$\therefore$ Load on one wall

$$= \frac{23331}{4} = 5833 \text{ kg.}$$

If T be the tension in the slopping wall

$$T \sin\theta = 5833 \qquad T = \frac{5833}{\sin}$$

$$\tan\theta = \frac{1}{1{\cdot}025}$$

$$\theta = 44^\circ\ 18'$$

$$T = \frac{5833}{\sin 44^\circ 18'} = 8352 \text{ kg.}$$

$\therefore$ Tension per *metre* run

$$= \frac{8352}{2{\cdot}5} \text{ kg./metre} = 3340 \text{ kg./metre}$$

$\therefore$ Steel required per *metre*

$$= \frac{3340}{1400} \text{ cm}^2 = 3{\cdot}34 \text{ cm.}^2$$

Provide 10 mm. dia. bars at 22 cms. centres.

Sloping bottom as a slab spanning between Corners

The hopper bottom should also be designed to resist bending due to

(*i*) The normal component P_n of the pressure due to coal.

(*ii*) The normal component of the weight of the slab and the lining.

Normal component of coal pressure

$$= wh(\cos^2\theta + \cos^2\phi \sin^2\theta)$$

$w = 900 \text{ kg./m.}^2$

$h = 3 + \frac{1}{2} + \frac{0{\cdot}72}{2} = 3{\cdot}86 \text{ m.}$

$\theta = 44^\circ\ 18'$

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$\therefore$ Normal pressure due to coal
$= 900 \times 3\ 86[\cos^2 44° 18' + \cos^2 30° \sin^2 44° 18']$
$= 3050\ kg/m.^2$

Normal component of the weight of the sloping slab

$$\frac{(15+2\ 5)}{100} \times 2400 \cos 44° 18'\ kg/m.^2 = 292\ kg./m.^2$$

$\therefore$ Total normal pressure
$= 3050 + 292 = 3342\ kg/m.^2$

Effective span

$$\frac{2{\cdot}5 + 0\ 45}{2} + 0{\cdot}15\ metres$$

$= 1{\cdot}63\ metres$

$\therefore$ Max. b.m.

$$= \frac{3342 \times 1{\cdot}63^2}{12} \times 100\ kg\ cm.$$

$= 73990\ kg.\ cm.$ *per metre width*

$$8{\cdot}50 \times 100d^2 = 73990$$

$$\therefore d = \sqrt{\frac{73990}{850}}$$

$= 9{\cdot}33\ cm$

But the eff. depth available
$= 15\ cm. - 3\ cm.$
$= 12\ cm.$

$$A_s = \frac{73990}{1400 \times 0\ 87 \times 12}\ cm$$

$= 5{\cdot}04\ cm.^2$

Spacing of 10 *mm.* bars

$$= \frac{0{\cdot}79 \times 100}{5{\cdot}04} = 15{\cdot}7\ cm.$$

Say 10 mm. bars at 15 cm. centres.

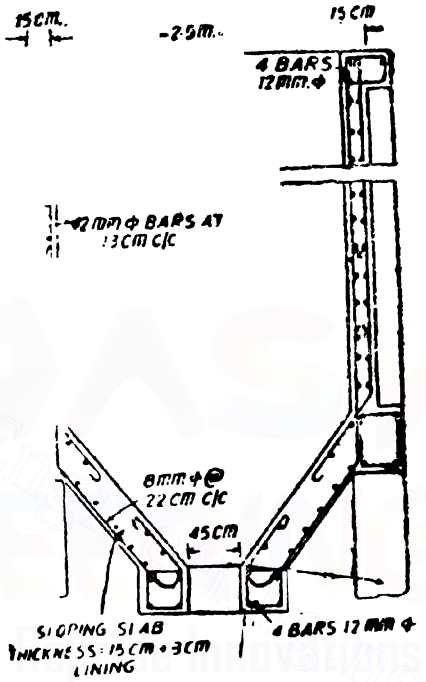

Fig. 579

§188. Circular Bunker

More or less on similar principles the circular bunker may also be designed. The vertical walls are designed for hoop tension. Fig. 580 shows a circular bunker.

Design 143. *Fig. 581 shows an arrangement for a circular coal bunker. Design the bunker*

Solution. Capacity of the bunker.

Conical part $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 1{\cdot}25^2 \times 1{\cdot}25 \tan 30° = 1{\cdot}18\ m^2.$
Cylindrical part $= \pi r^2 h = \pi \times 1{\cdot}25^2 \times 3 = 14{\cdot}72\ m.^3$
Frustum of cone

$$= \frac{\pi h}{12}\left[d_1^2 + d_2^2 + d_1 d_2\right]$$

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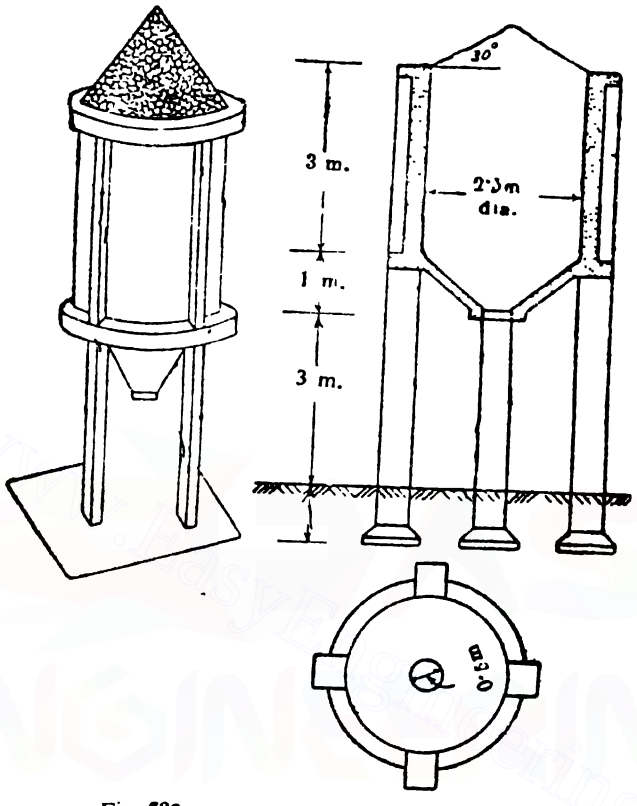

Fig. 580.
Circular Bunker.

Fig. 581

$$= \frac{\pi \times 1}{12}\left[2{\cdot}5^2 + 0{\cdot}5^2 + 2{\cdot}5 \times 0{\cdot}5\right] = 2{\cdot}03\ m.^3$$

Total $= 17{\cdot}93\ m.^3$

Weight of coal at 900 $kg./m.^3$

$= 17{\cdot}93 \times 900\ kg. = 16137\ kg.$ say 16140 $kg.$

Horizontal pressure exerted by coal

$= p = wh \cos^2 \phi\ kg/m^2$

$= 900 \times 3(0{\cdot}866)^2\ kg./m.^2$

$= 2025\ kg./m.^2$

Hoop tension in the cylindrical wall per *metre* height

$$= \frac{pD}{2} = 2025 \times \frac{2{\cdot}5}{2}\ kg. = 2530\ kg$$

$$A_s = \frac{2530}{1400}\ cm.^2 = 1{\cdot}81\ cm.^2$$

Let the walls be 15 *cm.* thick.

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Minimum steel requirement

$$= 0{\cdot}15\% \text{ of gross area}$$

$$\frac{0{\cdot}15}{100} \times 15 \times 100 = 2{\cdot}25 \text{ cm.}^2$$

Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{2{\cdot}25} \text{ cm.} = 22 \text{ cm. centres.}$$

Provide also vertical distribution bars of 8 mm. diameter at 22 cm. centres.

Hopper bottom

Thickness of sloping slab

$$= 15 \text{ cm.} + 3 \text{ cm. lining} = 18 \text{ cm.}$$

Weight of sloping bottom

$$= 1 \times \pi \times 2 \times 0{\cdot}18 \times 2\pi \left[\frac{1{\cdot}25 + 0{\cdot}25}{2}\right] \times 2400$$

$$= 2879 \text{ kg.}$$

Weight of coal $= 16140$ *kg.*

Total load $= 19019$ *kg.*

Let the tension per metre run be T *kg.*

$T \times 2\pi \times 1{\cdot}25 \sin 45^\circ = 19019$ *kg.*

$\therefore \quad T = 3425$ *kg.*

$\therefore$ Steel required $= \frac{3425}{1400}$ *cm.*$^2 = 2{\cdot}45$ *cm.*^{2}

$\therefore$ Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{2{\cdot}45} \text{ cm.} = 20 \text{ cm. centres}$$

Normal pressure on the sloping slab

Normal pressure due to coal

$$= wh(\cos^2\theta + \cos^2\phi \sin\theta)$$

$w = 900$ *kg./m.*^{3}

$$h = 3{\cdot}00 + \frac{3 \tan 30^\circ}{2} + \frac{1}{2}$$

(Corresponding to middle of hopper slab)

$$= 3{\cdot}79 \text{ m.}$$

$\therefore$ Normal pressure due to coal at middle of hopper

$$900 \times 3{\cdot}79 \times (\cos^2 45^\circ + \cos^2 30^\circ \sin^2 45^\circ) \text{ kg./m.}^2$$

$$= 2984 \text{ kg /m.}^2$$

Normal component of weight of 18 *cm.* thick sloping slab.

$$= 24 \times 18 \cos 45 \text{ kg./m.}^2$$

$$= 306 \text{ kg./m.}^2$$

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Total normal pressure

$= 2984 \times 306$ *kg./m*^{2}

$= 3290$ *kg./m*^{2}

Approximate hoop tension per *metre*

$$3290 \times \left(\frac{2{\cdot}5+0{\cdot}5}{2}\right) \text{ kg.}$$

$= 2468$ *kg.*

$$\therefore \quad A_t = \frac{2468}{1400} \text{ cm.}^2$$

$= 1{\cdot}76$ *cm.*^{2}

Provide 8 mm. diameter hoop bars at 22 cm. c/c.

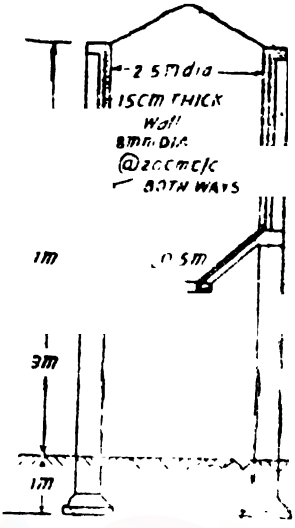

Fig. 582

Design 144. *Design a reinforced concrete continuous trough coal bunker to the following particulars (See Fig. 583):*

(i) Width of bunker = 6 m.
(ii) Height of bunker = 6 m
(iii) Bunker is to be supported by columns arranged in two rows.
Distance between rows of columns = 6 m.
Spacing of columns = 6 m
(iv) Weight of coal = 900 kg /metre^3
(v) Angle of repose for coal = 35°
(vi) Top surface of retained coal : level.
Adopt suitable working stresses.

Solution. Consider 1 *metre* run of the bunker.

Lateral pressure intensity at the bottom of the bunker

$$= p = wh \frac{1-\sin\phi}{1+\sin\phi}$$

$$= 900 \times 6 \times \frac{1-\sin 35^\circ}{1+\sin 35^\circ}$$

$= 1463$ *kg./m.*^{2}

Total lateral pressure

$$= p = \frac{1463}{2} \times 6 \text{ kg.}$$

$= 4389$ *kg.*

Distance *OB*

$= \sqrt{6^2+5^2}$ *metre*

(*See* Fig. 584).

$= 7{\cdot}80$ *metres*

$\therefore$ *AB* = 3 9 *m.*

OC = 5 *metres*

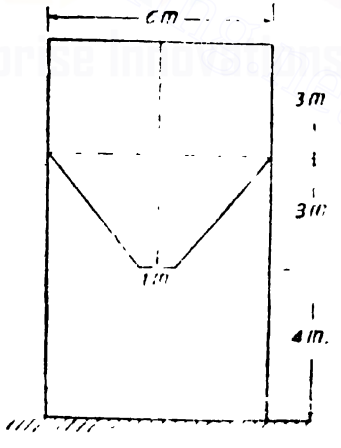

Fig. 583

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Weight of coal in triangle OCB

$$= \frac{5 \times 6}{2} \times 900 \text{ kg.}$$

$$= 13500 \text{ kg}$$

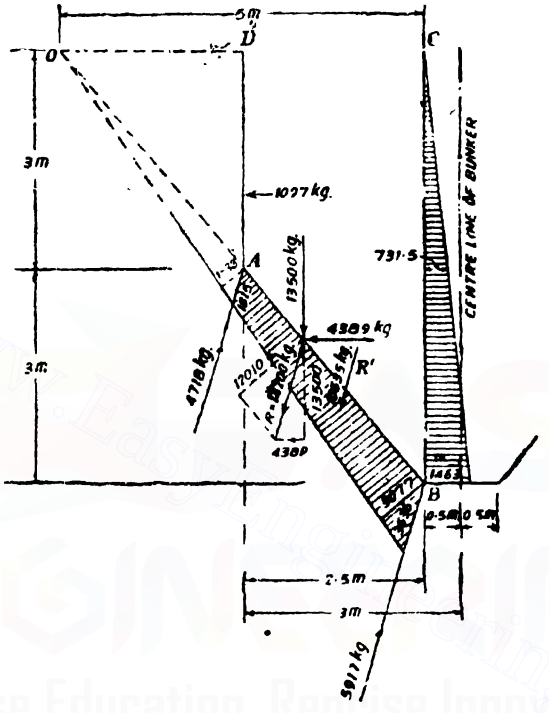

Fig. 584

Resultant force $= R = \sqrt{4389^2 + 13500^2}$ kg.

$= 14200$ kg.

Inclination of sloping slab with the horizontal

$$= \tan^{-1} \frac{6}{\quad} = 50° 12'$$

Inclination of the resultant force with the vertical

$$= \tan^{-1} \frac{4389}{1350\ 0} = 18°\ 1'$$

Component of R normal to sloping slab

$= 14200 \cos (50° 12' - 18°1') \text{ kg.}$

$= 12010$ kg.

Maximum pressure intensity at the mouth of bunker parallel to the resultant R

$$= \frac{14200 \times 2}{7\ 81} \text{ kg./m.}^2 = 3636 \text{ kg /m}^2$$

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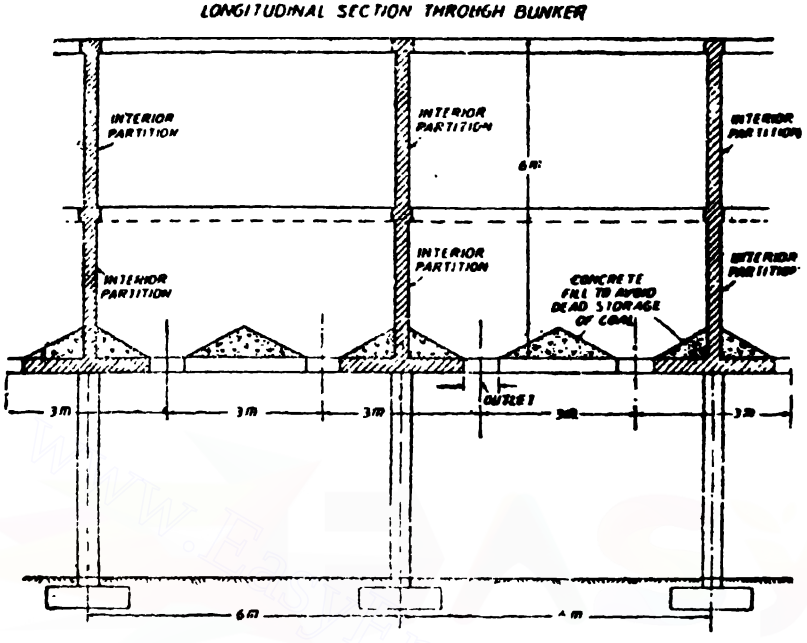

Fig. 585

and maximum pressure intensity normal to the sloping wall at the mouth

$$= 3636 \cos 32° 11'$$

$$= 3077 \ kg./m.^2$$

Pressure intensities at the top of the sloping slab are as follows :

Parallel to R

$$= 3636 \times \frac{(7{\cdot}80 - 3{\cdot}90)}{7{\cdot}80} \ kg/m.^2$$

$$= 1818 \ kg./m.^2$$

Normal to sloping slab

$$3077 \times \tfrac{1}{2} = 1538 \ kg./m.^2$$

It must be noted that R represents the total pressure given by the triangle of base OB.

Hence the total pressure on the sloping slab is given by the trapezium of pressure shown hatched in Fig. 584.

Distance of the centroid of this trapezium from A

$$= \frac{1818 + 2 \times 3636}{1818 + 3636} \times \frac{3{\cdot}9}{3} \ metre$$

$$= 2{\cdot}17 \ m.$$

(or alternatively $\frac{1538 + 2 \times 3077}{1538 + 3077} \times \frac{3{\cdot}9}{3} \ m. = 2{\cdot}17 \ m.$)

Maximum force on the sloping slab

$= R$ = area of the trapezium hatched

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$$= \frac{1818+3636}{2} \times 3{\cdot}9 \text{ kg.}$$

$$= 10635 \text{ kg.}$$

∴ Shears at the top and bottom of the sloping slab are as follows :

$$R_t = \frac{10635 \times (3{\cdot}9 - 2{\cdot}17)}{3{\cdot}9} \text{ kg.} = 4718 \text{ kg.}$$

and $$R_b = \frac{10635 \times 2{\cdot}17}{3{\cdot}9} \text{ kg.} = 5917 \text{ kg.}$$

The vertical and horizontal components of R_t and R_b are :

$H_t = 4718 \sin 18° 1' = 1460$ kg.

$V_t = 4718 \cos 18° 1' = 4486$ kg.

$H_b = 5917 \sin 18° 1' = 1830$ kg.

$V_b = 5917 \cos 18° 1' = 5627$ kg.

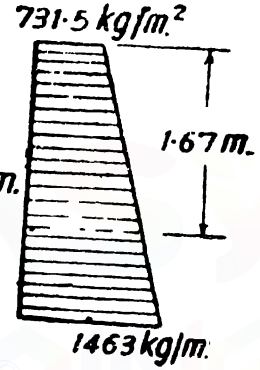

Fig. 586

Let us now check for the side pressures.

Horizontal pressure at 6 *m.* from top $= 1463$ kg./m².

Horizontal pressure at 3 *m.* from top, $= 731{\cdot}5$ kg./m².

Depth of centroid of pressure diagram

$$= \frac{731{\cdot}5 + 2 \times 1463}{731{\cdot}5 + 1463} \times \frac{3}{3} \text{ m.}$$

$= 1{\cdot}67$ *m.* from top of pressure diagram.

Total horizontal load for the bottom 3 *metres* depth

$$= \frac{731{\cdot}5 + 1463}{2} \times 3 \text{ kg.}$$

$$= 3290 \text{ kg.}$$

Horizontal shears :

At top $$H_t = \frac{3290 \times (3 - 1{\cdot}67)}{3} \text{ kg.}$$

$$= 1460 \text{ kg.}$$

∴ $$H_b = 3290 - 1460 = 1830 \text{ kg.}$$

These values tally with the values obtained earlier.

Now let us add the effect of the weight of coal above the one *metre* wide mouth.

Weight of this amount of coal

$$= 6 \times 1 \times 900 \text{ kg.}$$

$$= 5400 \text{ kg.}$$

Horizontal pressure on the vertical sides of the bunker

$$= 731{\cdot}5 \times \frac{3}{2} \text{ kg.} = 1097 \text{ kg.}$$

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Corresponding shears at top and bottom are

$$\frac{1097}{3} = 366 \text{ kg. at top}$$

and $1097 - 366 = 731$ kg. at bottom.

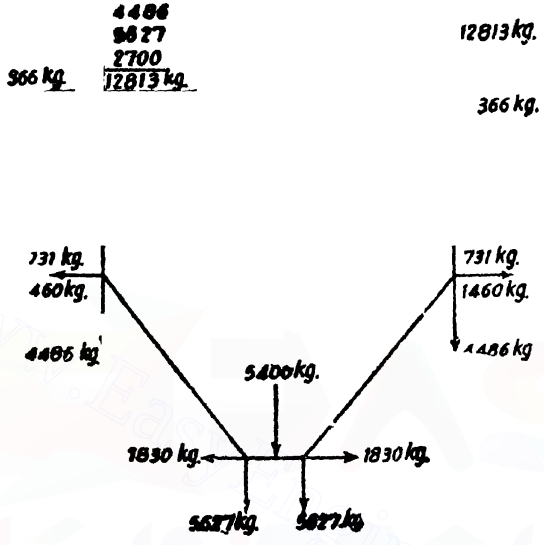

Fig. 587

Fig. 587 shows the various vertical and horizontal forces discussed so far.

The vertical reactions together will be equal to

$$2 \times 12813 = 25626 \text{ kg.}$$

This value should be checked against the weight of coal contained in the bunker.

$$\text{Area of the bunker} = 6 \times 3 + \frac{6+1}{2} \times 3 \text{ metre}^2 = 28{\cdot}5 \text{ metre}^2$$

$\therefore$ Weight of coal for one *metre* run of the bunker

$$= 28{\cdot}5 \times 1 \times 900 \text{ kg.}$$
$$= 25650 \text{ kg.}$$

This fairly agrees with the value 25626 *kg.* obtained above.

In order to determine the maximum tensions and the thrusts acting on the bunker it is necessary to include the self-weight of the bunker.

Weight of the sloping slabs assuming a thickness of 20 *cm.*

$$= 3{\cdot}9 \times 0{\cdot}2 \times 2400 \times 2 \text{ kg.} = 3744 \text{ kg.}$$

This load of 3744 *kg.* will also be added to the load of 16654 *kg.* in order to compute the maximum tension in the sloping slabs.

Fig. 588 shows the maximum forces acting on the bunker.

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Let the tension in each sloping slab be T *kg*.

$\therefore \quad 2T \sin 50° 12' = 20398$ *kg*.

$\therefore \quad T = 13270$ *kg*.

Horizontal component of T

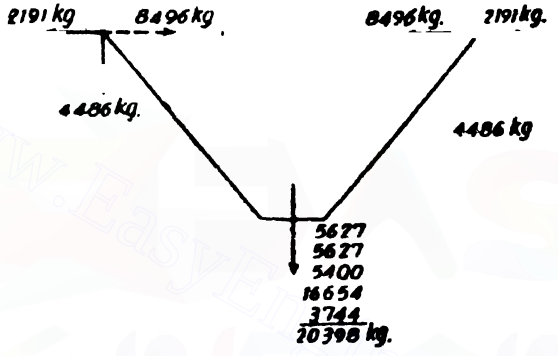

Fig. 588

$= 13270 \cos 50° 12'$

$= 8496$ *kg*.

Design of the sloping continuous slab

This slab can be analysed by moment distribution for the normal loads acting. The slab can be treated as simply supported at the ends A and A_1.

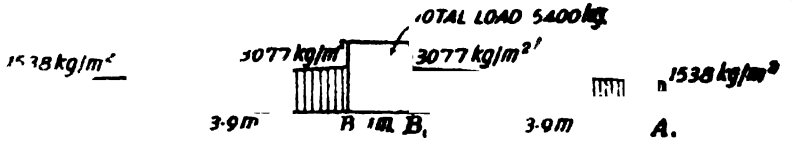

Fig. 589

Distribution factors

Joint	Member	Relative Stiffness	Total Stiffness	Distribution factors
B	BA	$\frac{3}{4} \cdot \frac{I}{3{\cdot}9}$		$\frac{0{\cdot}2}{1{\cdot}2} = \frac{1}{6}$
		$= 0{\cdot}2I$	$1{\cdot}2I$	
	BB_1	$\frac{I}{1} = I$		$\frac{5}{6}$

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Fixed end moments

$$m_{ab} = -\left[\frac{1538 \times 3{\cdot}9^2}{12} + (3077 - 1538)\ \frac{3{\cdot}9^2}{30}\right] kg.m.$$

$$= -2729\ kg.m. = -2{\cdot}73\ tonne\ metre$$

	B		B_1	
	$\frac{1}{6}$ | $\frac{5}{6}$		$\frac{5}{6}$ | $\frac{1}{6}$	
A				A_1
−2·73	+3·12	−0·45	+0·45	−3·12	+2·73
+2·73	+1·37			−1·37	−2·73
0·00	+4·49	−0·45	+0·45	−4·49	0
	−0·67	−3·7	+3·37	+0·67	
		+1·68	−1·68		
	−0·28	−1·40	+1·40	+0·28	
		+0·70	−0·70		
	−0·12	−0·58	+0·58	+0·12	
		+0·29	−0·29		
	−0·05	−0·24	−0·24	+0·05	
		+0·12	−0·12		
	−0·02	−0·10	+0·10	+0·02	
Final moments	0	+3·35	−3·35	+3·35	−3·35	0

$$m_{ba} = +\left[\frac{1538 \times 3{\cdot}9^2}{12} + (3077 - 1538)\ \frac{3{\cdot}9^2}{20}\right] kg.\ m.$$

$$= +3119\ kg.\ m.$$

$$= +3{\cdot}12\ tonne\ metre$$

$$m_{bb_1} = -\frac{5400 \times 1}{12} = -450\ kg.\ m.$$

$$= -0{\cdot}45\ tonne\ metre$$

$$m_{b_1b} = +0{\cdot}45\ tonne\ metre$$

Free B.M. = approximately $\left(\frac{1538 + 3077}{2}\right)\frac{3{\cdot}9^2}{8}\ kg.\ m.$

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$= 4388$ *kg. m.*
$= 4{\cdot}39$ *tonne metre*

Reduce the support moment to 90% of the calculated values to allow for partial fixity at ends.

∴ **Amended support moment**
$= 3{\cdot}35 \times 0{\cdot}9$ *tonne metre*
$= 3{\cdot}015$ *tonne metre*
say $3{\cdot}02$ *tonne metre*

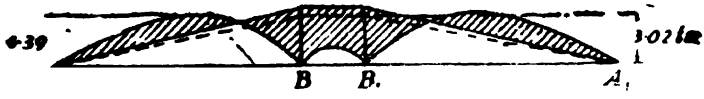

Fig. 590

This value will be used for both positive and negative moments

Self-weight of slab $= 0{\cdot}2 \times 3{\cdot}9 \times 2400$ *kg.*
$= 1872$ *kg.*

B.M. due to coal $= 3{\cdot}02$ *tm.*

B.M. due to wt. of sloping slab

$$= \frac{1872 \times 2{\cdot}5}{12 \times 1000} \text{ tm.} = 0\ 39 \text{ tm.}$$

∴ **Total B.M.** $= 3{\cdot}02 + 0{\cdot}39 = 3\ 41$ *tm.*

Tension in the sloping slab
$= 13270$ *kg.*

Thickness of slab $= 20$ *cm.*

If 16 *mm.* diameter bars be used at a clear cover of 25 *mm.* effective depth available
$= 20 - 2{\cdot}5 - 0{\cdot}8 = 16\ 7$ *cm.*

B.M. $= 3{\cdot}41$ *tm.*
$= 3{\cdot}41 \times 1000 \times 100$ *kg. cm.*
$= 341{,}000$ *kg cm.*

Tension $= 13270$ *kg.*

Result bending moment
$= 341{,}000 - 13270(16{\cdot}7 - 10)$ *kg. cm.*
$= 252080$ *kg. cm.*

Steel for bending moment

$$= \frac{252080}{1400 \times 0{\cdot}87 \times 16{\cdot}7} \text{ cm.}^2$$
$$12{\cdot}4 \text{ cm.}^2$$

Steel for tension $= \frac{13270}{1400}$ *cm.*$^2 = 9{\cdot}5$ *cm.*^{2}

Total steel $12{\cdot}4 + 9{\cdot}5 = 21{\cdot}9$ *cm.*^{2}

Spacing of 16 *mm.* diameter bars

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$$= \frac{2{\cdot}01 \times 100}{21{\cdot}9} \text{ cm.} = 9 \text{ cm. centres.}$$

Check for shear

Shear on slabs $= 5917 \cos (50° 12' - 18° 1')$ $= 5008$ *kg.*

Shear due to self wt. of slab $= \frac{1872}{2}$ $= 936$ *kg.*

Total shear $= 5944$ *kg.*

$\therefore$ Shear stress $= \frac{S}{ab}$

$$= \frac{5944}{0{\cdot}87 \times 16{\cdot}7 \times 100} \text{ kg./cm.}^2$$

$= 4{\cdot}09$ *kg./cm.*^{2}
(less than 5 *kg./cm.*^{2})

Distribution steel $= \frac{0{\cdot}15}{100} \times 20 \times 100$ *cm.*$^2 = 3$ *cm.*^{2}

Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{30} \text{ cm.}$$

$= 16$ *cm. centres.*

Vertical wall of bunker

This may be designed as a propped cantilever.

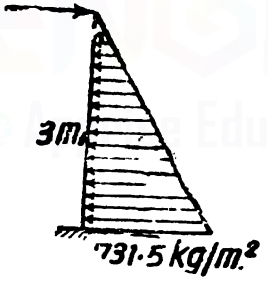

Fig. 591

Fixed end moments

At top $= \frac{731{\cdot}5 \times 3^2}{30}$ *kg. m.*

$= 219{\cdot}5$ *kg. m.*

At bottom $= \frac{731{\cdot}5 \times 3^2}{20}$ *kg. m.*

$= 329{\cdot}2$ *kg. m.*

$\therefore$ Actual moment at top and bottom are

At top, moment $= 0$

At bottom, moment $= 329{\cdot}2 + \frac{219{\cdot}5}{2}$ *kg. m.*

$= 439$ *kg. m.*
$= 43900$ *kg. cm.*

$$A_s = \frac{43900}{1400 \times 0{\cdot}87 \times 16{\cdot}7} \text{ cm.}^2$$

$= 2{\cdot}2$ *cm.*^{2} **(at bottom of vertical wall)**

Approximate positive b.m.

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$$= \frac{731{\cdot}5}{2} \times \frac{3^2}{8} \text{ kg. m.} = 412 \text{ kg. m.}$$

(nearly same as B.M. at bottom)

Reinforcement requirement being small provide nominal steel of *8 mm. ϕ at 16 cm. c/c near each face.*

Vertical wall as girder

Weight of coal in bunker

$= 25650$ *kg./metre* length

The columns in each row are spaced at 6 *metres* centres.

Loads transferred to girder

Load due to coal $= \frac{25650}{2} \times 6$	$= 76950$ *kg.*
Self weight of wall $3 \times 0{\cdot}2 \times 6 \times 2400$	$= 8040$ *kg.*
Sloping slab + bottom slab $(3{\cdot}9 + 0{\cdot}5) \times 0{\cdot}2 \times 6 \times 2400$	$= 12680$ *kg.*
Beam at junction of vertical and sloping slab 100 *cm.* × 50 *cm.* (approximately) $= 1 \times 0\ 5 \times 6 \times 2400$	$= 7200$ *kg.*
	Total $= 104870$ *kg.*

Say 105000 *kg.*

$$\therefore \text{ Max. B.M.} = \frac{Wl}{10}$$

$$= \frac{105000 \times 6}{10} \text{ kg. m.} = 63000 \text{ kg. m.}$$

Equating the M.R. to the B.M., we have

$$8{\cdot}5 \times 20\, d^2 = 63000 \times 100$$

$$b = 192{\cdot}5 \text{ cm.}$$

But overall depth $= 3 + 1 = 4$ *m.*

$$A_t = \frac{63000 \times 100}{1400 \times 0{\cdot}87 \times 390} \text{ cm.}^2$$

$= 16\ 7$ *cm.*^{2} at mid span and over supports.

Provide four bars of 24 mm. diameter (18·1 cm.^{2}*)*

Same amount of steel is to be provided at the top as well as at the bottom.

Shear stress in concrete

$$= \frac{0\ 5 \times 105000}{0{\cdot}87 \times 390 \times 20} \text{ kg./cm.}^2$$

$= 7{\cdot}7$ *kg./cm.*^{2}

Spacing of 2-legged 10 *mm.* diameter stirrups

$$= \frac{2 \times 0{\cdot}79 \times 1400 \times 0{\cdot}87 \times 390}{0{\cdot}5 \times 105000} \text{ cm.}$$

$= 14{\cdot}3$ *cm. say 14 cm. centres.*

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Continuous longitudinal beam at top

Let the section of the beam be 35 *cm.* × 30 *cm.*

This is necessary to stiffen the wall.

Lateral *i.e.* outward pressure transmitted by coal

$= 366 \times 6 = 1596$ *kg.*

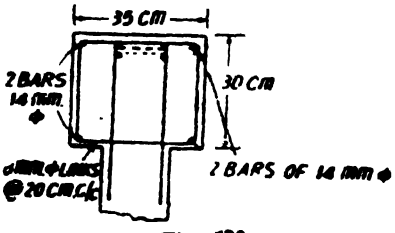

Fig. 592

Horizontal bending moment

$$= -\frac{1596 \times 6}{10} \text{ kg. m.}$$

$$= -958 \text{ kg. m.}$$

Equating the M.R. to B.M.

$$8{\cdot}5 \times 30 \times d^2 = 958 \times 100$$

$$\therefore \quad d = 19{\cdot}4 \text{ cm.}$$

$$\therefore \quad A_t = \frac{958 \times 100}{1400 \times 0{\cdot}87 \times 31} \text{ cm.}^2$$

$$= 2{\cdot}54 \text{ cm.}^2$$

Provide 2 bars of 14 mm. diameter (3·08 cm.²) near each face. Provide also 8 mm. links at 20 cm. c/c.

If it is desired to reduce the horizontal bending moment for this longitudinal beam then cross beams may be provided at intervals of 3 *metres.* Cross beams may also be found useful to carry the conveyor.

Knee beam at junction of vertical and sloping slab

From the arrangement shown in Fig. 593 the section of beam is nearly 100 *cm.* × 40 *cm.*

Inward pull $= 8496 - 2191$ *kg.*

$= 6305$ *kg./m.* of the length of beam.

∴ Horizontal Inward pull for the whole length of the beam

$= 6305 \times 6$ *kg.*

$= 37830$ *kg.*

∴ Horizontal B.M.

$$= -\frac{37830 \times 6}{10} \text{ kg. m.}$$

$$= 22698 \text{ kg. m.}$$

Equating the M.R. to B.M.

$$8{\cdot}5 \times 40\, d^2 = 22698 \times 100$$

$$d = 81{\cdot}7 \text{ cm.}$$

The depth required may also be determined from diagonal tension consideration.

Horizontal shear $= \frac{37830}{2} = 18915$ *kg.*

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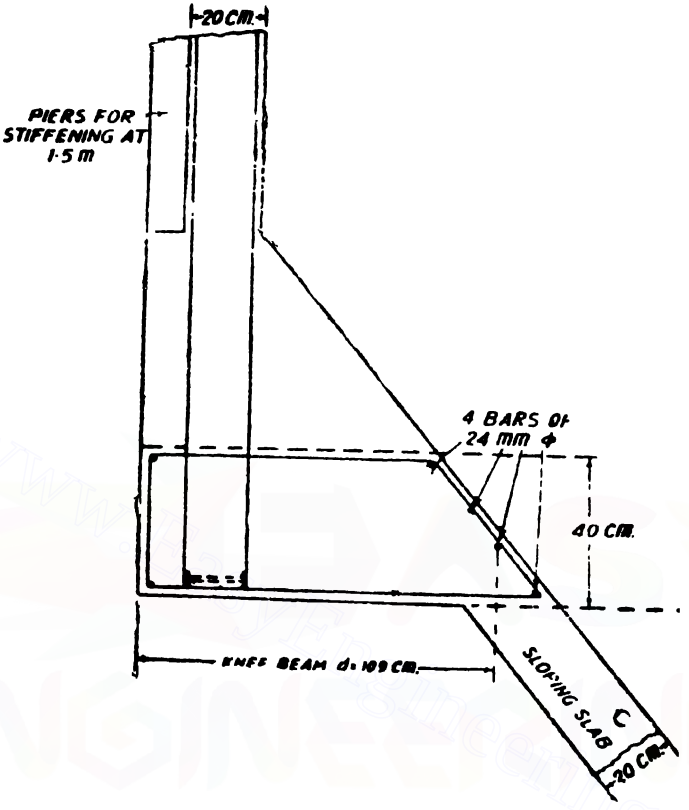

Fig. 593

Limiting the diagonal tensile stress to 5 *kg./cm* ^{2} we have

$$q = 5 = \frac{18915}{0{\cdot}87d \times 40} \text{ kg./cm.}^2$$

$$\therefore \quad d = \frac{18915}{5 \times 0{\cdot}87 \times 40} \text{ cm.}$$

$$= 108{\cdot}8 \text{ cm. say } 109 \text{ cm.}$$

$$\therefore \quad A_t = \frac{22698 \times 100}{1400 \times 0{\cdot}87 \times 109} \text{ cm.}^2$$

$$= 17{\cdot}1 \text{ cm.}^2$$

Provide four bars of 24 mm. diameter.

Interior partition

These may be designed as in the case of the vertical walls.

Transverse strut at bottom of vertical wall

Compressive load transferred by the sloping slab = 37830 *kg.*

When this load occurs no transverse B.M. can occur.

When one bunker is full and the other is empty let us design

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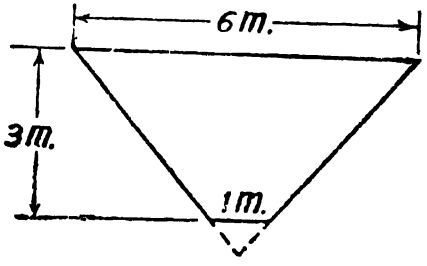

Fig. 594

Average width of partition

$$=6-\frac{(6-1)}{3}\times 1{\cdot}14\ m.$$

$$=4{\cdot}1\ metre$$

Reaction at F is approximately

$=\frac{2}{3}$ of triangular load on $EF+\frac{1}{2}$ uniform load at 731·5 $kg./m.+\frac{1}{3}$ of triangular load varying from 0 to

$1463-731{\cdot}5=731{\cdot}5\ kg./m.$

$=\frac{2}{3}\times 1097\ kg./m.$ of width for 6 *metres*

$$+\left(\frac{2195}{3}+\frac{1}{3}\times 1097\right)\ kg./m.\ \text{for a width of } 4{\cdot}1\ m.$$

Let us increase these figures by 20% due to continuity.

$\therefore$ Total pressure acting on the transverse strut

$$=\left[\frac{2}{3}\times 1097\times 6+\left(\frac{2195}{2}+\frac{1}{3}\times 1097\right)4{\cdot}1\right]\times 1{\cdot}2\ kg.$$

$=12440\ kg.$

Approximate B.M.

$$-\frac{12440\times 6}{10}\ kg.\ m.$$

$=7464\ kg.\ m.$

Direct load $=18915\ kg.$

$\therefore$ Eccentricity $e=\dfrac{7464}{18915}$ 0·39 $m.$

Let the size of the strut be 50 *cm.* wide and 40 *cm.* deep.

$$\frac{e}{B}=\frac{0{\cdot}39\times 100}{50}=0{\cdot}78\ m.$$

the member. For this condition the thrust will be

$$\frac{37830}{2}=18915\ kg.$$

Centroidal distance of the hopper area from the transverse strut

$$=\frac{6+2\times 1}{6+1}\times\frac{3}{3}\ m.$$

$1{\cdot}14\ m.$

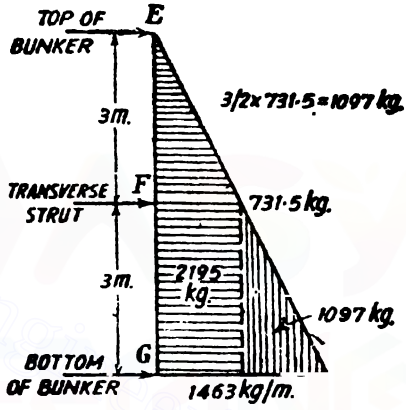

Fig. 595

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Let us try four bars of 24 mm. dia. near each face.

Let the effective cover be 4 *cm.*

Position of neutral axis from B.M. consideration

$$\frac{40\,n^2}{2}+(18-1)\times 18{\cdot}1'n-4)=18\times 18{\cdot}1(46-n)$$

$$n=16{\cdot}74\ cm.$$

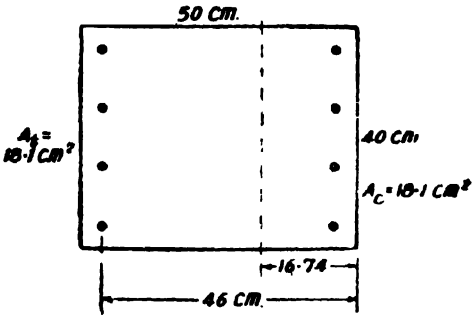

Fig. 596

Let the maximum compressive stress be c *kg./cm.*^{2}

$$\text{M.R.}=40\times 16{\cdot}74\times\frac{c}{2}\left(46-\frac{16{\cdot}74}{3}\right)+17\times 18{\cdot}1\times\frac{12{\cdot}74}{16{\cdot}74}c\,(46-4)\ kg.\ cm.$$

$$=23428\ c\ kg.\ cm.$$

Equating the M.R. to B.M.

$$23428\ c=7464\times 100$$

$$c=31{\cdot}85\ kg./cm.^2$$

Compressive stress due to direct load

$$c'=\frac{\text{Direct load}}{\text{Equivalent uncracked concrete area}}=\frac{W}{bn+(m-1)A_c+mA_s}$$

$$c'=\frac{18915}{40\times 16{\cdot}74+17\times 18{\cdot}1+18\times 18{\cdot}1}\ kg./cm.^2$$

$$=14{\cdot}52\ kg./cm.^2$$

Resultant compressive stress in concrete

$$=c+c'=31{\cdot}85=14{\cdot}52=46{\cdot}37\ kg./cm.^2$$

Tensile stress in steel due to B.M.

$$=\frac{46-16{\cdot}74}{16{\cdot}74}\times 31{\cdot}85\times 18\ kg./cm.^2$$

$$=1002\ kg/cm.^2$$

Compressive stress due to direct load $=mc'$

$$=18\times 14{\cdot}52\ kg./cm.^2$$

$$=261\ kg/cm.^2$$

$\therefore$ Resultant tensile stress in steel

$$=1002-261=741\ kg./cm.^2$$

Shear. Approximate lever arm

= distance between centres of steel

$=42\ cm.$

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Shear stress $= \frac{S}{ab}$

$= \frac{6220}{42 \times 40}$ kg./cm.²

$= 3{\cdot}70$ kg./cm.²

Transverse beams at top

These beams run just above the interior partitions.

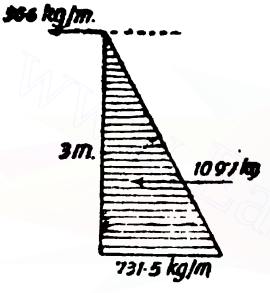

Fig. 597

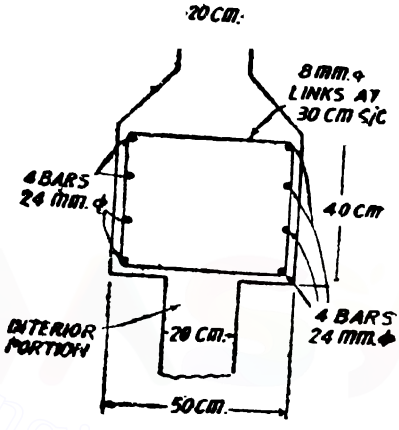

Fig. 598. Section of transverse strut.

Maximum horizontal pressure on the transverse beam at the top $= 366 \times 6$ kg.

$= 2196$ kg.

∴ B.M. for the beam

$= \frac{2196 \times 6}{10}$ kg. m.

$= 1317{\cdot}6$ kg. m.

$= 131760$ kg. cm.

This B.M. is accompanied by a pull of $\frac{366 \times 6}{2}$ kg.

i.e. 1098 kg.

Let the section of the beam be 40 cm. × 25 cm.

Let the effective cover to the reinforcement be 4 cm.

∴ Effective depth

$= 40 - 4 = 36$ cm.

Resultant B.M.

$= 131760 - 1098(36 - 20)$ kg. cm.

$= 114192$ kg. cm.

∴ Steel for B.M.

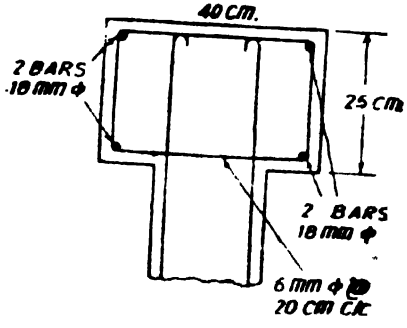

Fig. 599

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$$= \frac{114192}{1400 \times 0.87 \times 36} \text{ cm.}^2$$

$$= 2.61 \text{ cm.}^2$$

Steel for pull $= \frac{1098}{1400} \text{ cm.}^2$

$$= 0.78 \text{ cm.}^2$$

Total steel $= 2.61 + 0.73 = 3.39 \text{ cm.}^2$

Provide two bars of 18 mm. diameter near each face.

Shear stress $= \frac{1098}{0.87 \times 36 \times 25} \text{ kg./cm.}^2$

$$= 1.4 \text{ kg./cm.}^2$$

Provide nominal 6 mm. dia. links at 20 cm. c/c.

Columns

Loads on one column

Due to weight of coal

$= 76950$ *kg.*

Sloping slab $0.2 \times 3.9 \times 6 \times 2400$	11230 *kg.*
Vertical wall $3 \times 0.2 \times 6 \times 2400$	8640 *kg.*
Knee beam $1.09 \times 0.4 \times 6 \times 2400$	6280 *kg.*
Partitions $3 \times 0.2 \times 4.5 \times 2400$ (approx.)	6480 *kg.*
Longitudinal beam at top $0.35 \times 0.3 \times 6 \times 2400$	1510 *kg.*
Transverse beam at top $0.4 \times 0.25 \times 3 \times 2400$	720 *kg.*
	Total $= 111810$ *kg.*

∴ **Total load on one column**

$$= 111810 \text{ kg.}$$
$$= 111.8 \text{ tonnes.}$$

Let the wind load be 100 *kg./m.*^{2}

Wind load on bunker

$$= \frac{6 \times 6 \times 100}{1000} \text{ tonne}$$

$$= 3.6 \text{ tonnes.}$$

Wind load on column has been neglected.

Let the axial load transferred to each column be y due to wind load.

Let us assume the points of contraflexure to occur at the middle points of the columns.

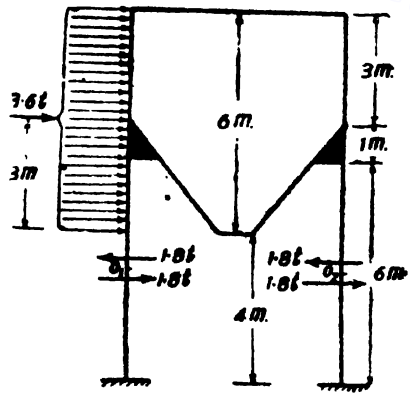

Fig 600

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$\therefore$ Taking moments about a point of contraflexure, we have

$$v \times 6 = 3{\cdot}6 \times (3+1)$$

$\therefore$ $v = 2{\cdot}4$ *tonnes*

Total load on the column

$$= 111{\cdot}8 + 2{\cdot}4 = 114{\cdot}2 \text{ tonnes}$$

Weight of one column 50 *cm.* × 50 *cm.*

$$= 0{\cdot}5 \times 0{\cdot}5 \times 6 \times \frac{2400}{1000} = 3\,6 \text{ tonnes}$$

Total load at the base of column

$$= 114{\cdot}2 + 3\,6 \text{ tonne}$$
$$= 117{\cdot}8 \text{ tonne}$$
say 118 *tonnes*

B.M. at the base $= 1{\cdot}8 \times 3 = 5{\cdot}4$ *tm.*

Eccentricity $e = \frac{5{\cdot}4}{118} \times 100$ *cm.*

$= 4{\cdot}6$ *cm.* (within middle third)

Trial section

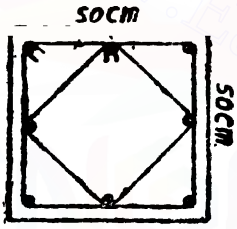

Fig. 601

Let eight bars of 24 mm. diameter be provided.

Clear cover to steel = 4 *cm.*

Effective cover $4 + 1{\cdot}2 = 5{\cdot}2$ *cm.*

Equivalent concrete area

$= A_e = 50 \times 50 + (18-1)8 \times 4{\cdot}52$ *cm.*^{2}

$= 3114{\cdot}6$ *cm.*^{2}

say 3115 *cm.*^{2}

Equivalent moment of inertia

$$= I = \frac{50 \times 50^3}{12} + 17 \times 6 \times 4{\cdot}52 \times 19{\cdot}8^2 \text{ cm.}^4$$

$$= 701560 \text{ cm.}^4$$

Extreme stresses are given by

$$f = \frac{118 \times 1000}{3115} \pm \frac{5{\cdot}4 \times 1000 \times 100}{701560} \times 25 \text{ kg./cm.}^2$$

$$38 \pm 19 \text{ kg./cm.}^2$$

$\therefore$ Max. compressive stress $= 38 + 19 = 57$ *kg./cm.*^{2}

Foundation

This can be designed as in the case of the columns of the square tank (see design 52).

Silos

As mentioned earlier, the plane of rupture of the material stored in a silo or deep bin, meets the opposite side of the bin before meeting the free surface of the material stored.

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Lateral pressures in silos. In the case of a silo the weight of the material is not completely transferred to the bottom of the silo. A part of the load is resisted by friction between the material stored and the side walls of the silo. This results in a reduction of the lateral pressure. The part of the total load transferred to the side walls by friction and the lateral pressure intensity are determined by *Jansen's* and *Airy's* theories.

Consider a small square area a *metre* by a *metre* at a depth of h *metre*. The volume of material standing on this area equals a^2h *metre*^{3}. The weight of this quantity $= wha^2$ *kg.* where w = weight *per cubic metre* of the material. The perimeter of the square is $4a$ *metres* and on each *metre* the total pressure for the height h

$$= \frac{wh^2}{2} \quad \frac{1-\sin\varphi}{1+\sin\varphi}$$

$\therefore$ Total lateral pressure all round the prism

$$= 4a \times \frac{wh^2}{2} \frac{1-\sin\varphi}{1+\sin\varphi}$$

$$= 2awh^2 \frac{1-\sin\varphi}{1+\sin\varphi}$$

where φ = Angle of repose of the material stored.

$\therefore$ The friction of the surrounding material

$$= \mu \times \text{Normal load}$$

$$= \tan\varphi \; 2awh^2 \frac{1-\sin\varphi}{1+\sin\varphi}$$

Hence an upward force to the above extent will resist a part of the downward load which is due to the weight of the prism of the material.

Hence the net load on the base of the prism,

$$= Q = wha^2 - 2awh^2 \frac{1-\sin\varphi}{1+\sin\varphi} \tan\varphi.$$

For Q to be a maximum,

$$\frac{dQ}{dh} = 0$$

$$\therefore \quad wa^2 - 4awh \frac{1-\sin\varphi}{1+\sin\varphi} \tan\varphi = 0$$

$$\therefore \quad a = 4\left[\frac{1-\sin\varphi}{1+\sin\varphi}\right] \tan\varphi \; h$$

For the above value of h the net downward load is maximum and hence the pressure intensity is also a maximum.

Suppose the square area is 1 *metre* × 1 *metre*, $\phi = 30°$ and $w = 850$ *kg./metre*^{3}

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The depth at which the maximum vertical pressure occurs

$$h = \frac{1}{4 \times \frac{1}{3} \times \frac{1}{\sqrt{3}}} \text{ metres.}$$

$$= 1{\cdot}299 \text{ metres.}$$

Max. vertical pressure intensity corresponding to this depth

$$p_v = \frac{Q_{max}}{a^2}$$

$$= wh - \frac{2wh^2}{a} \frac{1-\sin\varphi}{1+\sin\varphi} \tan\varphi$$

$$= 850 \times 1{\cdot}299 - \frac{2 \times 850 \times 1{\cdot}299^2}{1} \times \frac{1}{3} \times \frac{1}{\sqrt{3}} \text{ kg./m.}^2$$

$$= 552 \text{ kg /m.}^2$$

It can be seen that at other depths p_v has a lesser value than this value. At a certain depth p_v may be equal to zero. This is given by

$$wh - \frac{2wh^2}{a} \frac{1-\sin\varphi}{1+\sin\varphi} \tan\varphi = 0$$

$$\therefore \quad h = \frac{a}{2} \frac{1+\sin\varphi}{1-\sin\varphi} \cot\varphi$$

For the example cited above, p_v equals zero at a depth

$$h = \tfrac{1}{2} \, 3 \cdot 3 = 2{\cdot}598 \text{ metre.}$$

For values of h greater than 2·598 *metres* p_v will work out to be negative. This may appear meaningless: but actual observations show that the material contained in the silo sometimes has to be poked well before it can flow.

The mean pressure on the base of the silo will have a definite value for a given set of working conditions. This can be determined by Jansen's theory. This theory assumes that the material is uniform in its texture possessing a definite angle of repose. The theory also assumes a definite value for the coefficient of friction between the material stored and the side walls.

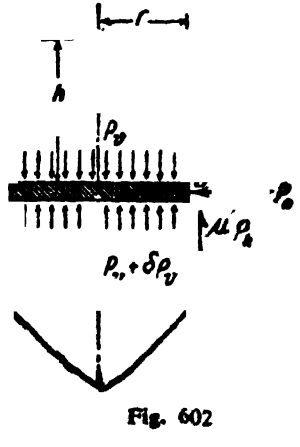

Fig. 602

Fig. 602 shows a silo of radius r.

Let p_v = vertical pressure in $kg./m.^2$ at h *metres* below the free surface.

$p_v + \delta p_v$ = vertical pressure in $kg./m.^2$ at $(h+\delta h)$ *metre* below the free surface

r = radius of the silo in *metre*.

μ' = coefficient of friction between the silo walls and the material stored

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R = Hydraulic mean radius

$$= \frac{\text{Area of cross-section of the silo}}{\text{Perimeter of the section}}$$

$$= \frac{\pi r^2}{2\pi r} = \frac{r}{2}$$

n = Ratio of the horizontal intensity of pressure to the vertical intensity of pressure

$$= \frac{p_h}{p_v}$$

w = weight per *cubic metre* of the material

Consider an elemental disc of the material stored at a depth of h *metres* below the free surface. For the equilibrium of all the vertical forces on this disc, we have

$$\pi r^2\, dh\, w = \pi r^2\, dp_v + \mu'\, p_h\, 2\pi r\, dh$$

Dividing by πr,

$$r\, dh\, w = r\, dp_v + 2\mu'\, p_h\, dh$$

But $p_h = np_v$

$$\therefore \quad r\, dh\, w = r\, dp_v + 2\mu'\, np_v\, dh$$

$$\therefore \quad dh(wr - 2\mu'\, np_v) = r\, dp_v$$

$$\therefore \quad dh = \frac{r}{wr - 2\mu' np_v}\, dp_v$$

Integrating, we get

$$h = -\frac{r}{2\mu' n} \log_e [wr - 2\mu' n\, p_v] + C_1$$

where C_1 is a constant of integration.

But at $h = 0$, $p_v = 0$

$$\therefore \quad 0 = -\frac{r}{2\mu' n} \log_e wr + C_1$$

$$\therefore \quad C_1 = \frac{r}{2\mu' n} \log_e wr$$

$\therefore$ The solution to the differential equation is,

$$h = -\frac{r}{2\mu' n} \log_e [wr - 2\mu' n p_v] + \frac{r}{2\mu' n} \log_e wr$$

$$\therefore \quad h = -\frac{r}{2\mu' n} \log_e \left[\frac{wr - 2\mu' n p_v}{wr}\right]$$

$$\therefore \quad -\frac{2\mu' nh}{r} = \log_e \left[\frac{wr - 2\mu' n p_v}{wr}\right]$$

$$\therefore \quad 1 - \frac{2\mu' n p_v}{wr} = e^{-\frac{2\mu' nh}{r}}$$

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$$\therefore \quad \frac{2\mu' n p_v}{wr} = 1 - e^{\frac{-2\mu' nh}{r}}$$

$$\therefore \quad p_v = \frac{wR}{2\mu' n}\left[1 - e^{\frac{-2\mu' nh}{r}}\right]$$

But Hydraulic mean depth

$$= R = \frac{r}{2}$$

$$\therefore \quad p_v = \frac{wR}{\mu' n}\left[1 - e^{\frac{-\mu' nh}{R}}\right]$$

But $p_h = n p_v$

$$\therefore \quad p_h = \frac{wR}{\mu'}\left[1 - e^{\frac{-\mu' nh}{R}}\right]$$

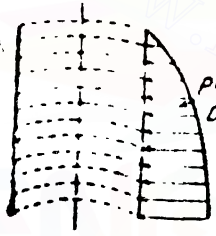

Fig. 603

But $e^{\frac{-\mu' nh}{R}} = e^{\frac{1}{\frac{\mu' nh}{R}}}$

Hence for large values of h the quantity $e^{\frac{1}{\frac{\mu' nh}{R}}}$ becomes a negligibly small quantity. For this case,

$$p_h = \frac{wR}{\mu'}$$

i.e. the lateral pressure intensity for large values of h is a constant.

The expression for p_h being complicated its simplification can be made in the following steps :

(*i*) Find the common logarithm of e and multiply by $\frac{\mu' nh}{R}$ and find the antilog.

(*ii*) Find the reciprocal of the above result and subtract it from 1.

(*iii*) Multiply the above result by $\frac{wR}{\mu'}$ and obtain p_h.

Note. Common logarithm of $e = 0{\cdot}4343$.

Design 145. *A cylindrical silo has an internal diameter of 4·5 metres and 20 metres deep and is filled with wheat weighing 850 kg /m.*[3] *The angle of repose for wheat may be taken as 25°. The coefficient of friction between wheat and the silo walls may be taken as 0·444. Ratio*

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of horizontal to vertical pressure intensity is 0·40. Determine the lateral pressure intensities at intervals of 2 metres and plot the variation of lateral intensity of pressure with depth.

Solution. Lateral pressure intensity at any depth h is given by

$$p_h = \frac{wR}{\mu'}\left[1 - e^{-\frac{\mu' nh}{R}}\right]$$

The lateral pressure intensities at intervals of 2 *metres* are worked out below :

h*=2 *metres

Common logarithm of $e = 0.4343$

$$\mu' = 0.444$$

$$n = 0.40$$

$$R = \frac{\text{Area}}{\text{Perimeter}} = \frac{r}{2}$$

$$= \frac{2.25}{2} = 1.125 \text{ metres}$$

Common logarithm of $e \times \frac{\mu' nh}{R}$

$$= \frac{0.4343 \times 0.444 \times 0.4 \times 2}{1.125}$$

$$= 0.1371$$

Antilog $0.1371 = 1.371$

$$\frac{1}{1.371} = 0.7294$$

$$\therefore \quad 1 - 0.7294 = 0.2706$$

$$\therefore \quad p_h = 0.2706 \times \frac{wR}{\mu'} \text{ kg./m}^2$$

$$= \frac{0.2706 \times 850 \times 1.125}{0.444} \text{ kg./m.}^2$$

$$= 582.8 \text{ kg./m.}^2, \text{ say } 583 \text{ kg./m.}^2$$

h*=4 *metres

Common logarithm of $e \times \frac{\mu' nh}{R}$

$$\frac{0.4343 \times 0.444 \times 0.4 \times 4}{1.125}$$

$$= 0.2742$$

Antilog $0.2742 = 1.880$

$$\frac{1}{1.88} = 0.5319$$

$$1 - 0.5319 = 0.4681$$

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$$p_h = 0{\cdot}4681 \frac{wR}{\mu}$$

$$= \frac{0{\cdot}4681 \times 850 \times 1{\cdot}125}{0{\cdot}444} \text{ kg./m.}^2$$

$$= 1008 \text{ kg./m.}^2$$

h = 6 metres

Common logarithm of $e \times \frac{\mu' nh}{R}$

$$= \frac{0{\cdot}4343 \times 0{\cdot}444 \times 6}{1{\cdot}125}$$

$$= 0{\cdot}4113$$

Antilog $0{\cdot}4113 = 2{\cdot}578$

$$\frac{1}{2{\cdot}578} = 0{\cdot}3879$$

$$1 - 0{\cdot}3879 = 0{\cdot}6121$$

$$\therefore \quad p_h = 0{\cdot}6121 \frac{wR}{\mu'}$$

$$= \frac{0{\cdot}6121 \times 850 \times 1{\cdot}125}{0{\cdot}444} \text{ kg./m.}^2$$

$$= 1318 \text{ kg./m.}^2$$

h = 8 metres

Common logarithm of $e \times \frac{\mu' nh}{R}$

$$= \frac{0{\cdot}4343 \times 0{\cdot}444 \times 8}{1{\cdot}125}$$

$$= 0{\cdot}5484$$

Antilog $0{\cdot}5484 = 3{\cdot}535$

$$\frac{1}{3{\cdot}535} = 0{\cdot}2829$$

$$1 - 0{\cdot}2829 = 0{\cdot}7171$$

$$p_h = 0{\cdot}7171 \frac{wR}{\mu'}$$

$$= \frac{0{\cdot}7171 \times 850 \times 1{\cdot}125}{0{\cdot}444} \text{ kg./m.}^2$$

$$= 1545 \text{ kg./m.}^2$$

h = 10 metres

Common logarithm of $e \times \frac{\mu' nh}{R}$

$$= \frac{0{\cdot}4343 \times 0{\cdot}444 \times 10}{1{\cdot}125}$$

$$= 0{\cdot}6855$$

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Antilog 0·6855 = 4·848

$$\frac{1}{4{\cdot}848} = 0{\cdot}2063$$

$$1 - 0{\cdot}2063 = 0\ 7937$$

$$p_h = 0{\cdot}7937 \frac{wR}{\mu'}$$

$$= \frac{0{\cdot}7937 \times 850 \times 1{\cdot}125}{0{\cdot}444} \ kg./m.^2$$

$$= 1710\ kg./m^2.$$

h = 12 metres

Common logarithm of $e \times \frac{\mu' nh}{R}$

$$= \frac{0{\cdot}4343 \times 0{\cdot}444 \times 0{\cdot}4 \times 12}{1{\cdot}125}$$

$$= 0{\cdot}8226$$

Antilog 0·8226 = 6·646

$$\frac{1}{6{\cdot}646} = 0{\cdot}1505$$

$$1 - 0{\cdot}1505 = 0\ 8495$$

$$p_h = 0{\cdot}8495 \frac{wR}{\mu'}$$

$$= \frac{0{\cdot}8495 \times 850 \times 1{\cdot}125}{0{\cdot}444} \ kg./m.$$

$$= 1820\ kg./m.^2$$

h = 14 metres

Common logarithm of $e \times \frac{\mu' nh}{R}$

$$= 0{\cdot}4343 \times 0{\cdot}444 \times 0{\cdot}4 \times 14$$

$$1{\cdot}125$$

$$= 0{\cdot}9597$$

Antilog 0·9597 = 9·096

$$\frac{1}{9{\cdot}096} = 0{\cdot}1099$$

$$1 - 0{\cdot}1099 = 0{\cdot}8901$$

$$p_h = 0{\cdot}8902 \frac{wR}{\mu'}$$

$$= \frac{0{\cdot}8902 \times 850 \times 1{\cdot}125}{0{\cdot}444} \ kg./m.^2$$

$$= 1917\ kg./m.^2$$

h = 16 metres

Common logarithm of $e \times \frac{\mu' nh}{R}$

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$$= -\frac{0.4343 \times 0.444 \times 0.4 \times 16}{1.125}$$

$$= -1.0968$$

Antilog $1.0968 = 12.49$

$$\frac{1}{12.49} = 0.0801$$

$$1 - 0.0801 = 0.9199$$

$$\therefore \quad p_h = 0.9199 \frac{wR}{\mu'}$$

$$= \frac{0.9199 \times 850 \times 1.125}{0.444} \text{ kg./m.}^2$$

$$= 1981 \text{ kg /m.}^2$$

h = 18 metres

Common logarithm of $e \times \frac{\mu' nh}{R}$

$$= -\frac{0.4343 \times 0.444 \times 0.4 \times 18}{1.125}$$

$$= -1.2339$$

Antilog $1.2339 = 17.14$

$$\frac{1}{17.14} = 0.0584$$

$$1 - 0.0584 = 0.9416$$

$$\therefore \quad p_h = 0.9416 \frac{wR}{\mu'}$$

$$= \frac{0.9416 \times 850 \times 1.125}{0.444} \text{ kg./m.}^2$$

$$= 2028 \text{ kg./m.}^2$$

h = 20 metres

Common logarithm of $e \times \frac{\mu' nh}{R}$

$$= -\frac{0.4343 \times 0.444 \times 0.4 \times 20}{1.125}$$

$$= -1.3710$$

Antilog $1.3710 = 23.50$

$$\frac{1}{23.50} = 3.0426$$

$$1 - 0.0426 = 0.9574$$

$$\therefore \quad p_h = 0.9574 \frac{wR}{\mu'}$$

$$= \frac{0.9574 \times 850 \times 1.125}{0.444} \text{ kg./m.}^2$$

$$= 2062 \text{ kg./m.}^2$$

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Fig. 604 shows the lateral pressure intensity on the silo wall.
When h tends to infinity

$$p_h = \frac{wR}{\mu'}$$

$$= \frac{850 \times 1{\cdot}125}{0{\cdot}444} \ kg./m.^2$$

$$= 2154 \ kg./m.^2$$

and $$p_v = \frac{p_h}{n} = \frac{2154}{0{\cdot}4} = 5385 \ kg./m.^2$$

For $h = 20 \ metres$

$p_h = 2062 \ kg./m.^2$

and $$p_v = \frac{2062}{0{\cdot}4} = 5155 \ kg./m.^2$$

5155 $kg./m.^2$ is equivalent to depth of $\frac{5155}{850}$ *metres* of wheat

$= 6{\cdot}07$ *metres* depth of wheat

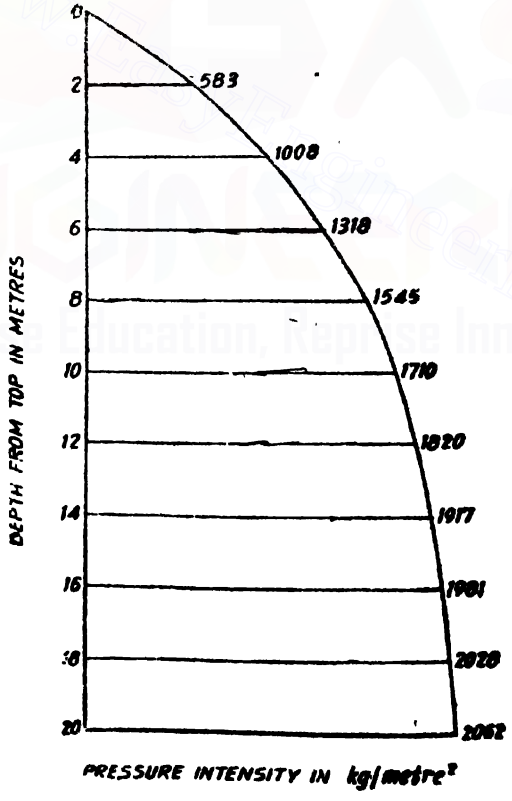

Fig. 604. Lateral pressure for silo wall.

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Hence even though there is 20 *metres* depth of wheat the effective vertical load is only due to 6·07 *metres* of wheat.

20 *metres* depth of wheat corresponds to 850×20

$$= 17000 \text{ kg./metre}^2$$

But the actual load intensity at 20 *metres* depth is only 5155 *kg./m*^{2}. which is only $\frac{5155}{17000} \times 100 = 30{\cdot}3\%$ of the total weight of wheat. This means the balance 69·7% of the total weight is transferred to the silo walls by friction.

It is now easy to determine the steel area required at various levels.

The walls may be designed for hoop tension caused by the lateral pressure.

At any depth *h metres* if the lateral pressure intensity is p_h the hoop tension per *metre* height

$$= \frac{p_h D}{2} \text{ kg.}$$

where D is the diameter of the silo.

The hoop tension per *metre* height and the corresponding steel requirement at various levels are tabulated below :

Depth h (metres)	*Horizontal pressure intensity (p_h kg./m.^{2})*	*Hoop tension per metre height $p_h \frac{D}{2}$ $= p_h \times 2{\cdot}25$ (kg.)*	*Steel requirement per metre height allowing 1400 kg./cm.^{2} (cm.^{2})*	*Reinforcement suggested*
2	583	1312	0·93	
4	1008	2268	1·62	8 *mm.* dia. bars at 20 *cm.* centres
6	1318	2966		
8	1545	3476		
10	1710	3848		
12	1820	4095		
14	1917	4313		
16	1981	4457		
18	2028	4563		8 *mm.* dia. bars at 14 *cm.* centres
20	2062	4640		

Vertical distribution steel

This steel is usually 0·3% of the gross area.

Let the walls be 15 *cm.* thick

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$$\text{Vertical steel} = \frac{0{\cdot}2}{100} \times 15 \times 100 \text{ cm.}^2 \text{ per metre of circumference}$$

$$= 3 \text{ cm.}^2$$

Spacing of 8 *mm* diameter bars

$$= \frac{0{\cdot}5 \times 100}{3} \text{ cm.}$$

$$= 16 \text{ cm. centres.}$$

The horizontal section of the wall should be checked for the stresses caused due to self-weight and weight of material transferred and due to wind pressure.

Consider a horizontal section just above the base of the silo, *i e.*, at a depth of 20 *metres*.

Vertical pressure intensity at this depth

$$= 5155 \text{ kg./m.}^2$$

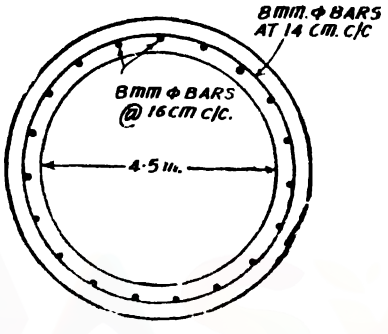

Fig. 605

But weight of wheat per *q. metre* of plan area

$$= 1 \times 850 \times 20 = 17000 \text{ kg /m.}^2$$

∴ Load transferred to vertical walls

$$= (17000 - 5155) \frac{\pi}{4} \times 4{\cdot}5^2 \text{ kg.}$$

Intensity of load on the silo wall

$$11845 \frac{\pi}{4} \times 4{\cdot}5^2 \text{ kg /m}^2$$

$$[4{\cdot}8^2 - 4{\cdot}5^2]$$

$$= 8604{\cdot}0 \text{ kg /m.}^2$$

$$= 8{\cdot}6 \text{ kg /cm}^2$$

Stress due to self-weight of silo wall

$$= 1 \times 20 \times 2400 \text{ kg /m}^2$$

$$= 48000 \text{ kg /m.}^2$$

$$= 4{\cdot}8 \text{ kg /cm.}^2$$

∴ Stress due to self-weight and wheat load

$$= 8{\cdot}6 + 4{\cdot}8 = 13{\cdot}4 \text{ kg /cm}^2$$

Wind load on 20 *metres* height of silo at 100 *kg /metre*^{2}

$$= 20 \times 4{\cdot}8 \times 100 \text{ kg.}$$

$$= 9600 \text{ kg. acting at 10 metres above the base}$$

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$\therefore$ Moment of wind pressure

$$= 9600 \times 10 = 96000 \text{ kg.m.}$$

Moment of inertia of wall section

$$= \frac{\pi}{64}[4{\cdot}8^4 - 4{\cdot}5^4] \text{ metre}^4$$

$$= 6{\cdot}345 \text{ metre}^4$$

$\therefore$ Stress due to wind moment

$$\pm \frac{M}{I} y$$

$$= \frac{96000}{6{\cdot}354} \times \frac{4{\cdot}8}{2} \text{ kg./m.}^2$$

$$= \pm 36250 \text{ kg./m.}^2$$

$$= +3{\cdot}63 \text{ kg./cm}^2$$

$\therefore$ Maximum compressive stress

$$= 13{\cdot}4 + 3{\cdot}63 = 17{\cdot}03 \text{ kg./cm.}^2$$

Minimum compressive stress

$$= 13{\cdot}4 - 3{\cdot}63 = 9{\cdot}77 \text{ kg./cm.}^2$$

Design of the sloping bottom

Vertical pressure exerted by wheat

$$= 5155 \text{ kg./metre}^2$$

Let T be the tension in the sloping bottom per *metre* run. If the sloping slab is at 45° with the horizontal we have

$$T \sin 45° \times \pi \times 4{\cdot}65 = 5155 \times \frac{\pi}{4} \times 1{\cdot}5^2$$

$\therefore$ $T = 7936$ kg.

Steel required for resisting this tension

$$= \frac{7936}{1400} \text{ cm}^2$$

$$= 5{\cdot}67 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{5{\cdot}67} \text{ cm.}$$

$$= 19{\cdot}9 \text{ cm.}$$

say 18 cm. c/c.

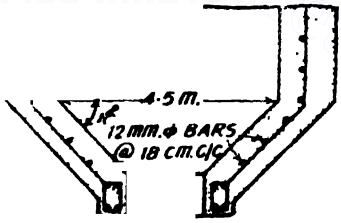

Fig. 606

§189. W. Airy's Theory

Airy deduced formulae to determine the pressure per *metre* run of the horizontal periphery of the silo (and thus the pressure intensity in *kg./metre*^{2}) and the part of the load of the filling caused by friction on the sides.

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Shallow Bins. For this case in which the depth of the grains stored in the bin is small, consider the wedge ACE of the grain.

Let the thickness of the wedge be unity.

The forces acting on the wedge are the following :

(*i*) The weight W of the wedge ACE.

(*ii*) Total reaction on the side AC.

(*iii*) Total reaction on the side AE.

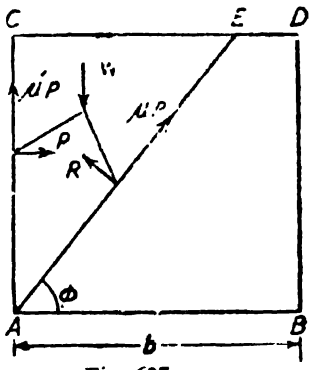

Fig. 607

The reaction on the side AC is the resultant of the normal reaction P and the frictional resistance $\mu' P$ while the resultant reaction on the side AE is the resultant of the normal reaction R and the frictional resistance μR.

Resolving the above forces in equilibrium along the plane AE and perpendicular to the plane we get

$$\mu R + P \cos\theta = (W - \mu' P) \sin\theta$$
$$R - P \sin\theta = (W - \mu' P) \cos\theta$$

From the above relations, it can be shown that

$$P = \left[\frac{\tan\theta - \mu}{(1 - \mu\mu') + (\mu + \mu')\tan\theta}\right] \times W$$

But $$W = \frac{wh^2}{2} \cot\theta$$

$$\therefore \quad P = \frac{\tan\theta - \mu}{(1 - \mu\mu') + (\mu + \mu')\tan\theta} \cdot \frac{wh^2}{2} \cot\theta \quad \ldots (1)$$

The plane of rupture is corresponding to the condition that P must be a maximum, and for this condition,

$$\frac{dP}{d\theta} = 0$$

Differentiating the above expression for P with respect to θ and equating to zero, it can be shown that

$$\tan\theta = \mu + \sqrt{\mu \frac{1 + \mu'}{\mu - \mu'}} \quad \ldots (2)$$

If the value of θ obtained from equation (2) is substituted in equation (1), the lateral pressure exerted by the wedge of unit thickness can be determined If the result is multiplied by the periphery of the bin the total lateral pressure can be determined. Multiplying the total pressure by μ', the weight of the material held by friction at the sides can be calculated The load on the bottom can now be determined by subtracting from the total weight of the grains, the load carried by friction at the sides..

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Deep bins If the depth of the bin is large so that the plane of separation AE strikes the opposite side of the bin before emerging from the free surface of the grain, the angle of inclination θ of the plane of rupture and the total lateral pressure P may be computed on lines similar to the previous case. But for this case the weight of the

$$\text{wedge} = W = \frac{wb}{2}(2h - b \tan \theta)$$

where b = breadth of the bin
and h = depth of grains.

The total pressure can be shown to be

$$P = \frac{wb}{2}(2h - b \tan \theta) \frac{\tan \theta - \mu}{1 - \mu\mu' + (\mu + \mu') \tan \theta}$$

and

$$\tan \theta = \sqrt{\frac{2h}{b} \cdot \frac{1+\mu^2}{\mu+\mu'} + \frac{1+\mu^2}{\mu+\mu'} \cdot \frac{1-\mu\mu'}{\mu+\mu'} - \frac{1-\mu\mu'}{\mu+\mu'}}$$

Design 146. *A concrete bin is 3 metres × 3 metres and contains wheat weighing 850 kg/metre². The coefficient of friction between grain and grain is 0·466. The coefficient of friction between grain and concrete is 0·444. If the depth of wheat be 2·5 metres determine the lateral pressure per metre run of the bin wall.*

Solution. $b = 3$ *metres*

$h = 3{\cdot}5$ *metres*

Assuming the bin to be shallow the plane of rupture is given by

$$\tan \theta = \mu + \sqrt{\mu \frac{1+\mu^2}{\mu+\mu'}}$$

In our case, $\mu = 0{\cdot}466$ and $\mu' = 0{\cdot}444$

$$\therefore \quad \mu + \mu' = 0{\cdot}466 + 0{\cdot}444 = 0{\cdot}910$$

$$\mu^2 = (0{\cdot}466)^2 = 0{\cdot}217$$

$$\therefore \quad \frac{1+\mu^2}{\mu+\mu'} = \frac{1+0{\cdot}217}{0{\cdot}910} = 1{\cdot}337$$

$$\therefore \quad \mu \frac{1+\mu^2}{\mu+\mu'} = 0{\cdot}466 \times 1{\cdot}337$$

$$= 0{\cdot}623$$

$$\therefore \quad \tan \theta = 0{\cdot}466 + \sqrt{0{\cdot}623}$$

$$\therefore \quad \tan \theta = 1{\cdot}25$$

$$\therefore \quad b \tan \theta = 3 \times 1{\cdot}255 \text{ metres}$$

$$= 3{\cdot}765 \text{ metres}$$

But $h = 2{\cdot}5$ *metres*

Since $h < b \tan \theta$, the bin is shallow.

$$\frac{wh^2}{2} \cot \theta \frac{\tan \theta - \mu}{(1 - \mu\mu') + (\mu + \mu') \tan \theta}$$

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$$= \left\{\frac{850\times 2\cdot 5^2}{2\times 1\cdot 225}\right\} \frac{1\cdot 255-0\cdot 466}{(1-0\cdot 446\times 0\cdot 444)+(0\cdot 910\times 1\cdot 225)} \text{ kg./metre run}$$

$$= 863 \text{ kg./metre run}$$

Design 147. *A concrete bin is 3 metres × 3 metres and contains wheat weighing 850 kg./metre³. The coefficient of friction between grain and grain is 0·466. The coefficient of friction between grain and concrete is 0·444. If the depth of wheat be 6 metres determine the lateral pressure per metre run of the bin wall.*

Solution. If the bin had been shallow the plane of rupture would be given by

$$\tan\theta = 1\cdot 255$$

and $$b\tan\theta = 3\times 1\cdot 255$$

$$= 3\cdot 765 \text{ metres}$$

But $h = 6$ *metres*

Since $h > b\tan\theta$, the bin is a deep bin.

The plane of rupture for the material is therefore given by

$$\tan\theta = \sqrt{\frac{2h}{b}\cdot\frac{1+\mu^2}{\mu+\mu'}+\frac{1+\mu^2}{\mu+\mu'}\cdot\frac{1-\mu\mu'}{\mu+\mu'}-\frac{1-\mu\mu'}{\mu+\mu'}}$$

In our case $\mu = 0\cdot 466$ and $\mu' = 0\cdot 444$ and $h = 6$ *metres*

$$\frac{1+\mu^2}{\mu+\mu'} = \frac{1+(0\cdot 466)^2}{0\cdot 466+0\cdot 444}$$

$$= 1\cdot 337$$

$$\frac{1-\mu\mu'}{\mu+\mu'} = \frac{1-0\cdot 466\times 0\cdot 444}{0\cdot 466+0\cdot 444}$$

$$= 0\cdot 871$$

$$\therefore \quad \tan\theta = \sqrt{\frac{2\times 6}{3}\times 1\cdot 337+1\cdot 337\times 0\cdot 871-0\cdot 871}$$

$$\therefore \quad \tan\theta = 1\cdot 681$$

$$\therefore \quad P = \frac{wb}{2}(2h-b\tan\theta)\frac{\tan\theta-\mu}{(1-\mu\mu')+(\mu+\mu')\tan\theta}$$

$$= \frac{850\times 3}{2}(2\times 6-3\times 1\cdot 681)\frac{1\cdot 681-0\cdot 466}{(1-0\cdot 466\times 0\cdot 444)+0\cdot 910\times 1\cdot 681} \text{ kg./metre run}$$

$$= 4637 \text{ kg./metre run.}$$

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17

Bus Stop Shelter Frames

A bus stop frame usually consists of a continuous slab supported on a number of frames of the type shown in Fig. 608 at a spacing of 3 to 4 *metres* centres. The supporting frames are usually pre-cast.

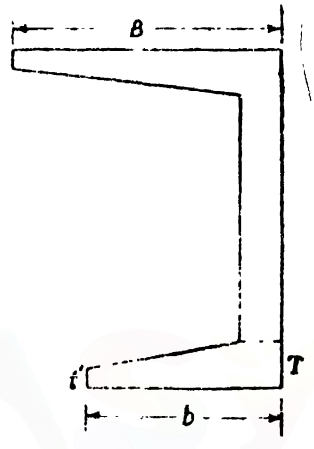

Fig. 608.

The following rules may be adopted in choosing the dimensions of the frame.

(*i*) Maximum thickness of beam at the column end
= 18 *cm.* per *metre* of projection

(*ii*) Minimum thickness of beam at end
= 20 *cm.* to 30 *cm.*

(*iii*) $b = 0.6\ B$ to $0.7\ B$

(*iv*) $t = 30$ *cm.*

(*v*) $T = 60$ *cm.*

Design 148 (S I). *Design a cantilever shed for a bus-stop. The shed is 3·50 metres wide and 3 metres high above the ground level and shall consist of a 11 cm. thick slab supported on frames spaced at 3 metres centres. Use M15 concrete.*

Solution

Design of the slab

Loads

D.L. of 11 *cm.* slab $= 250 \times 11 = 2750\ N/m.^2$

L.L. $= 1500\ N/m.^2$

Total $4250\ N/m.^2$

Consider a 1 *m.* wide strip of the slab.

∴ Maximum bending moment

$$= \frac{4250 \times 3^2}{12}\ Nm.$$

$= 3187{\cdot}5\ Nm. = 318750\ N.\ cm.$

Adopting $c = 5\ N/mm^2 = 500\ N/cm.^2$

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$t = 140\ N/mm.^2 = 14000\ N/cm.^2$

and $m = 18$

We have $a = 0{\cdot}87\,d$ and $Q = 85$

$\therefore \quad 85 \times 100\,d^2 = 318750$

$\therefore \quad d = \sqrt{\dfrac{318750}{8500}} = 6{\cdot}12\ cm.$

Providing a clear cover of 15 *mm.* and using 8 *mm.* ϕ bars,

effective cover $= 15 + 4 = 19\ mm. = 1{\cdot}90\ cm.$

$\therefore$ effective depth $= 11 - 1{\cdot}90 = 9{\cdot}10\ cm.$

$$A_t = \frac{318750}{14000 \times 0{\cdot}87 \times 9{\cdot}10} = 2{\cdot}88\ cm.^2$$

Spacing of 8 *mm.* ϕ bars

$$= \frac{0{\cdot}5 \times 100}{2{\cdot}88} = 17{\cdot}4\ cm.$$

Provide 8 mm. ϕ *@ 17 cm. c/c.*

Provide also distribution steel of 6 mm. ϕ *@ 17 cm. c/c.*

Design of the beam

Consider 1 metre run of the beam

Load from the slab

$= 4250 \times 3\ N/m.$

$= 12750\ N/m.$

D L. of the beam

[30 *cm.* × 30 *cm.* to 30 *cm.* × 66 *cm.*]

$= 0{\cdot}3 \times 0{\cdot}48 \times 25000\ N/m.$

$= 3600\ N/m.$

Total $= 12750 + 3600$

$= 16350\ N/m.$

$\therefore$ Max. B.M. for the cantilevering beam

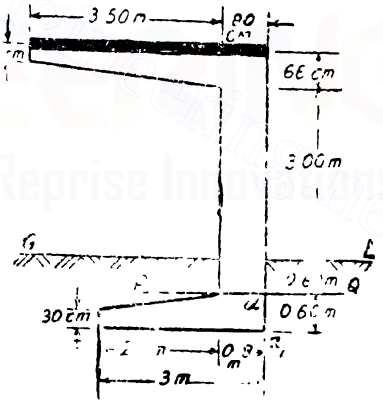

Fig. 609

$$= \frac{16350 \times 3{\cdot}5^2}{2} \times 100\ N\ cm.$$

$= 10014375\ N\ cm.$

$\therefore \quad 85 \times 30\,d^2 = 10014375$

$\therefore \quad d = \sqrt{\dfrac{10014375}{85 \times 30}}$

$= 62\ cms.$

Effective cover $= 4\ cms.$

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Overall depth $= 66\ cm.$

$$A_t = \frac{10014375}{14000 \times 0{\cdot}87 \times 62}\ cm.^2$$

$$= 13{\cdot}26\ cm.^2$$

Let us provide 4 bars of 24 mm. dia.

Max. S.F. $= 16350 \times 3{\cdot}5 = 57225\ N.$

$\therefore\ q_v =$ Nominal shear stress

$$= \frac{S}{bd} = \frac{57225}{30 \times 62} \quad 31\ N/cm.^2$$

$$= 0{\cdot}31\ N/mm.^2$$

Percentage of tensile steel provided

$$= \frac{4 \times 4{\cdot}52}{30 \times 62} \times 100 = 0{\cdot}97\%$$

For 0·97% steel $q_c = 0{\cdot}37\ N/mm.^2$ $\qquad q_v < q_c$

We will provide nominal stirrups of 6 mm. dia. bars at 20 cm. centres.

Design of the column

Let us try 30 *cm.* × 80 *cm.* section.

Load due to	*Magnitude of load (N)*	*Distance from a (m)*	*Moment about a (Nm)*
From the slab : 4250×4·3×3	54825	2·15	117874
Beam : 0·3×0·48×3·5 ×25000	12600	$0{\cdot}8 \frac{66+60}{66+30} \cdot \frac{3{\cdot}5}{3}$ $= 0{\cdot}8 + 1{\cdot}53 = 2{\cdot}33$	29358
Column : 0·3×0·8×4·25 × 25000	25560	0·4	10224
Total	92985		157456

$\therefore$ Distance of centre of gravity of loads from a

$$= \frac{157456}{92985} = 1{\cdot}693\ m.$$

$$= 169{\cdot}3\ cm.$$

Let us try 4 bars of 24 mm. dia. $\quad \therefore\ A_t = 18{\cdot}08\ cm.^2$

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Let the neutral axis be at a distance n from the compression edge. The distance of neutral axis is given by

$$\frac{30n^2}{2} = 18 \times 18{\cdot}08\ (75-n)$$

$$\therefore \qquad n = 30{\cdot}92\ cm.$$

$$\therefore \text{ Lever arm} \qquad = a = 75 - \frac{30{\cdot}92}{3}$$

$$= 64{\cdot}69\ cm.$$

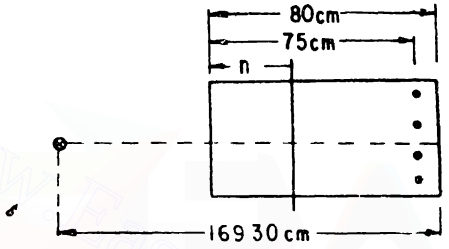

Fig. 610

$$\text{Eccentricity} \qquad = 169\ 3 - (80 - 30{\cdot}92)$$

$$= 120{\cdot}22\ cm.$$

Stress in concrete due to B.M. is given by

$$30 \times 30{\cdot}92 \times \frac{c}{2} \times 64{\cdot}69 = 92985 \times 120{\cdot}22$$

$$\therefore \qquad c = 372{\cdot}6\ N/cm.^2$$

$$\therefore \text{ Stress in steel} \qquad = 18 \times 372\ 6 \times \left[\frac{75 - 30\ 92}{30{\cdot}92}\right] N/cm.^2$$

$$= 9561{\cdot}3\ N/cm.^2$$

Direct stress in concrete

$$= \frac{92985}{30 \times 30\ 92 + 18 \times 18{\cdot}08}\ N/cm.^2$$

$$= 74{\cdot}2\ N/cm.^2$$

$\therefore$ Resultant stress in concrete

$$= 372{\cdot}6 + 74{\cdot}2 = 446{\cdot}8\ N/cm.^2$$

$$= 4{\cdot}468\ N/mm.^2$$

Resultant stress in steel

$$= 9561{\cdot}3 - 18 \times 74{\cdot}2\ N/cm.^2$$

$$= 8225{\cdot}7\ N/cm.^2 \qquad = 82{\cdot}26\ N/mm.^2$$

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$=42\ N/cm.^2$

Percentage of steel $\dfrac{18{\cdot}08}{30\times 59}\times 100 = 1{\cdot}02\%$

For 1·02% steel, $q_c = 0{\cdot}37\ N/mm.^2 = 37\ N/cm.^2$

Shear resistance of concrete

$$= q_c bd = 37\times 30\times 59\ N$$
$$= 65490\ N$$

Shear to be resisted by stirrups

$$= V_s = 74951 - 65490 = 9461\ N$$

Spacing of 2-legged 6 *mm.* ϕ stirrups

$$= \frac{A_w f_v d}{V_s} = \frac{(2\times 0{\cdot}28)14000\times 59}{9461}\ cm.$$
$$= 49\ cm.$$

Provide 2-legged 6 mm. ϕ *stirrups @ 20 cm. c/c.*

Design 149. *Design a cantilever shed for a bus-stop. The shed is 3·50 metres wide and 3 metres high above the ground level and shall consist of a 11 cm. thick slab supported on frames spaced at 3 metres centres. Use M 150 concrete.*

Solution. Design of the slab

D.L. of 11 *cm.* thick slab $= 275\ kg./m^2.$

L.L $= 150\ kg./m^2.$

Total $= 425\ kg./m^2.$

$\therefore$ Max. B.M. $= \dfrac{425\times 3^2}{12}\times 100\ kg.\ cm.$

$= 31875\ kg.\ cm.$

Adopting $c = 50\ kg./cm.^2$, $t = 1400\ kg.\ cm.^2$,

and $m = 18$

We have $a = 0{\cdot}87\ d$ and $Q = 8{\cdot}50$

$\therefore \quad 8{\cdot}50\times 100 d^2 = 31875$

$$= \sqrt{\frac{31875}{850}}$$
$$= 6{\cdot}12\ cm.$$

But effective depth available with an effective cover of 2 *cm.*

$$= 11 - 2 = 9\ cms.$$

$$A_t = \frac{31875}{1400\times 0{\cdot}87\times 9}$$
$$= 2{\cdot}91\ cm.^2$$

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$\therefore$ Spacing of 8 *mm.* dia bars

$$= \frac{0{\cdot}5 \times 100}{2{\cdot}91} = 17{\cdot}1 \text{ cm.}$$

Hence provide 8 mm dia. bars at 16 cm. centres.

Design of the beam

Consider 1 metre run of the beam

Load from the slab

$= 425 \times 3$

$= 1275$ *kg./m*

D.L. of the beam

[30 *cm* × 30 *cm* to 30 *cm.* × 66 *cm*]

$= 0{\cdot}3 \times 0{\cdot}48 \times 2500$

$= 360$ *kg /m.*

Total $= 1275 + 360$

$= 1635$ *kg /m.*

$\therefore$ Max. B.M. for the cantilevering beam

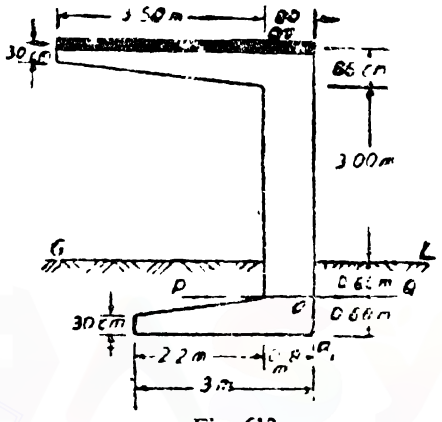

Fig. 613

$$= -\frac{1635 \times 3{\cdot}5^2}{2} \times 100 \text{ kg. cm.}$$

$$= 1001440 \text{ kg. cm.}$$

$\therefore$ $8{\cdot}5 \times 30 \times d^2 = 1001440$

$$d = \sqrt{\frac{1001440}{8{\cdot}5 \times 30}} \text{ cm.}$$

$= 62$ *cms.*

Effective cover $= 4$ *cms.*

Overall depth $= 66$ *cm.*

$$A_t = \frac{1001440}{1400 \times 0{\cdot}87 \times 62} \text{ cm.}^2$$

$= 13{\cdot}26$ *cm.*^{2}

Let us provide 4 bars of 24 mm. dia.

Max. S.F. $= 1635 \times 3{\cdot}5 = 5723$ *kg.*

$\therefore$ Nominal shear stress

$$q_v = \frac{S}{bd} = \frac{5723}{30 \times 62} = 3{\cdot}1 \text{ kg./cm.}^2$$

Percentage of tensile steel

$$= \frac{4 \times 4{\cdot}52}{30 \times 62} \times 100 = 0{\cdot}97\%$$

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For 0·97% steel, $q_c = 3{\cdot}7$ *kg./cm.*^{2}

$\therefore$ $q_v < q_c$

We will provide nominal two-legged stirrups of 6 mm. dia. at 20 cm. centres.

Design of the column

Let us try 30 *cm.* × 80 *cm* section

Load due to	*Magnitude of load (kg.)*	*Distance from a (m)*	*Moment about a (kg. m.)*
From the slab : 425 × 4·3 × 3	5483	2·15	11787
Beam : 0·3 × 0·48 × 3·5 × 2500	1260	$0{\cdot}8+\frac{66+60}{66+30}\cdot\frac{3{\cdot}5}{3}$ $0{\cdot}8+1{\cdot}53=2{\cdot}33$	2936
Column : 0·3 × 0·8 × 4·26 × 2500	2556	0·4	1022
Total	9299		15745

$\therefore$ Distance of centre of gravity of loads from a

$$= \frac{15745}{9299} = 1{\cdot}693 \text{ metres}$$

$$= 169{\cdot}3 \text{ cms.}$$

Let us try 4 bars of 24 mm. dia. $\therefore$ $A_t = 18{\cdot}08$ *cm.*^{2}

Let the neutral axis be at a distance n from the compression edge. The distance of neutral axis is given by

$$\frac{30n^2}{2} = 18 \times 18{\cdot}08\ (75-n)$$

$\therefore$ $n = 30{\cdot}92$ *cm.*

$\therefore$ Lever arm $= a - 75 - \frac{30{\cdot}92}{3}$

$= 64{\cdot}69$ *cm.*

Eccentricity $= 169{\cdot}3 - (80 - 30{\cdot}92)$ *cm.*

$= 120{\cdot}22$ *cm.*

Stress in concrete due to B.M. is given by,

$$30 \times 30{\cdot}92 \times \frac{c}{2} \times 64{\cdot}69 = 9299 \times 120{\cdot}22$$

$\therefore$ $c = 37{\cdot}25$ *kg./cm.*^{2}

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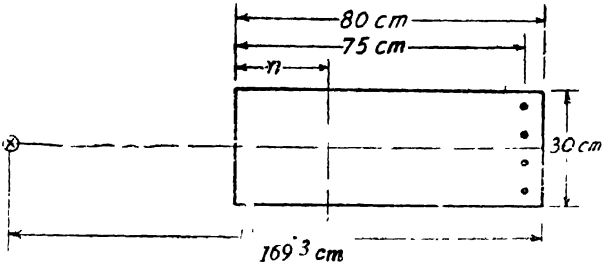

Fig. 614

∴ Stress in steel

$$= t = 18 \times 37{\cdot}25 \frac{(75 - 30{\cdot}92)}{30{\cdot}92} \text{ kg./cm.}^2$$

$$= 956 \text{ kg./cm.}^2$$

Direct stress in concrete

$$= \frac{9299}{30 \times 30{\cdot}92 + 18 \times 18{\cdot}08} \text{kg./cm.}^2$$

$$= 7{\cdot}42 \text{ kg./cm}^2$$

∴ Resultant stress in concrete

$$= 37{\cdot}25 + 7{\cdot}42 = 44{\cdot}67 \text{ kg./cm.}^2$$

Resultant stress in steel

$$= 956 - 18 \times 7{\cdot}42 \text{ kg./cm.}^2$$

$$= 822\ 44 \text{ kg /cm.}^2$$

Design of the foundation

Width of the foundation

$$= b = 0{\cdot}6\ B \text{ to } 0{\cdot}7\ B.$$

$$0{\cdot}6\ B = 0{\cdot}6 \times 4{\cdot}3 = 2{\cdot}58 \text{ metres}$$

$$0{\cdot}7\ B = 0{\cdot}7 \times 4{\cdot}3 = 3{\cdot}01 \text{ metres}$$

Let us provide a base width of 3 *metres*. Let the maximum and minimum depths of the foundation slab be 60 *cm.* and 30 *cm.* respectively

$$\therefore \quad z = \frac{17211}{10401} = 1{\cdot}65$$

$$e = z - \frac{b}{2} = 1{\cdot}65 - 1{\cdot}50 = 0{\cdot}15 \text{ metres}$$

But $$\frac{b}{2} = \frac{3}{6} = 0{\cdot}50 \text{ metres}$$

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Pressure Distribution at the Base

Load due to	*Magnitude of load (kg.)*	*Distance from a_1 (m.)*	*Moment about a_1 (kg. m)*
From the slab	5483	2·15	11787
Beam	1260	2·33	2936
Column	2 56	0·40	1022
Rectangle of concrete : 0·8×0·6×0·3×2500	360	0·4	144
Cantilever : 0 45×0 3×2· ×2500	742	$0\ 84+\frac{0\ 6+0\ 6}{0\ 6+0\ 3}\times\frac{2·2}{3}$ $=0·8+0\ 98=1·78$	1322
Total	10401		17211

The maximum and minimum pressures at the base are given by

$$p = \frac{10401}{3\times 0\ 3}\left[1 \pm \frac{6\times 0\ 15}{3}\right] kg./metre^2$$

$\therefore$ $p_{max} = 15024\ kg./m.^2$

$p_{min} = 8090\ kg./m.^2$

Total pressure on the cantilever

$$= \frac{15024+9939}{2}(0·3\times 2\ 2)\ kg.$$

$= 8237·8\ kg.$

This acts at a distance of

$$\frac{9939+2\times 15024}{9939+15024}\times\frac{2·2}{3}\ metres$$

$= 1·17\ metres$ from the column edge

$\therefore$ Moment due to upward pressure

$= 8237·8\times 1·17\times 100\ kg.\ cm.$

$= 963820\ kg\ cm.$

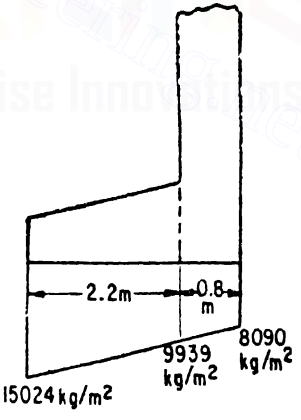

Fig. 615

D.L. of the cantilevering part

$= 742\ kg.$

Moment due to the D.L. of the cantilever

$= 742\times 0·98\ kg.\ m.$

$= 727·2\ kg.\ m.$

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$= 72720$ *kg. cm.*

∴ Net B.M. for the base slab

$= 963820 - 72720 = 891100$ *kg. cm.*

∴ $8{\cdot}5 \times 30\ d^2 = 891100$

∴ $d = 59$ *cm.*

Overall depth required

$= 59 + 4 = 63$ *cm.*

$$A_t = \frac{891100}{1400 \times 0{\cdot}87 \times 59} \text{ cm.}^2$$

$= 12{\cdot}40$ *cm.*^{2}

Provide 4 bars of 24 mm. ϕ *(18·08 cm*^{2}*)*

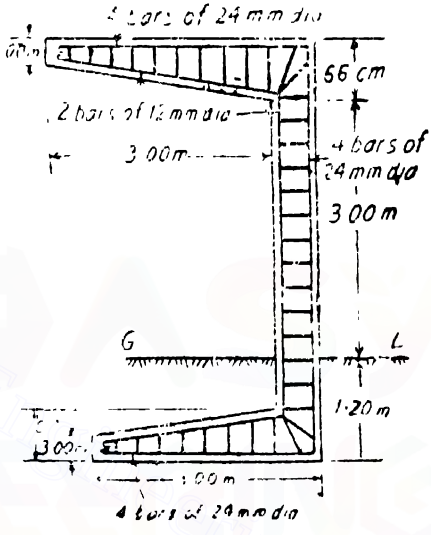

Fig. 61

Check for shear

Maximum S.F.

$= S = 8237{\cdot}8 - 742$ *kg.*

$= 7495{\cdot}8$ *kg.*

say 7496 *kg.*

Nominal shear stress

$$q_v = \frac{7496}{30 \times 59} = 4{\cdot}2 \text{ kg./cm.}^2$$

Percentage of tensile steel provided

$$= \frac{18{\cdot}08}{30 \times 59} \times 100\% = 1{\cdot}02\%$$

For 1·02% steel, $q_c = 3{\cdot}7$ *kg./cm.*^{2}

Shear resistance of concrete

$= q_c bd = 3{\cdot}7 \times 30 \times 59$ *kg.*

$= 6549$ *kg.*

∴ Shear to be resisted by stirrups

$= V_s = 7496 - 6549 = 947$ *kg.*

Spacing of 2-legged 6 *mm.* ϕ stirrups

$$\therefore \quad \frac{A_w f_w d}{V_s} = \frac{(2 \times 0{\cdot}28)\ 1400 \times 59}{947} = 49 \text{ cm}$$

Provide 2-legged 6 mm. ϕ *stirrups @ 20 cm. c/c.*

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18

Frames for Buildings

Introduction

Structurally a building may consist of load bearing walls and floors. The floor slabs may be supported on beams which in turn may be supported on walls or columns. But for a multi-storeyed structure a building frame either of steel or reinforced concrete is made. This frame is designed for all the vertical and horizontal loads transmitted to it. The openings between the columns, where necessary, will be filled with thin brick walls. A frame of this type will consist of columns and beams built monolithically forming a network. This provides rigidity to the connections of members. By this arrangement the bending moments for the members of the structure are reduced. Earthquake loads and other horizontal loads due to wind etc. are evenly distributed to the whole structure. This makes the structure not only safer but economical.

We shall now discuss the frames used in various types of buildings.

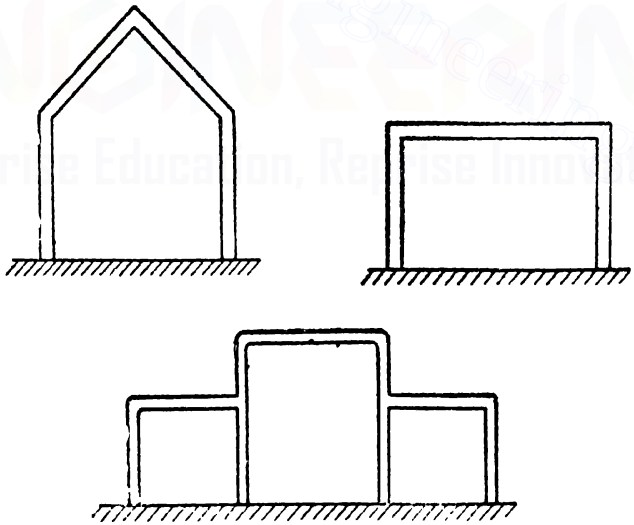

Fig. 617

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§190. Frames for single storey buildings—Portal frames

A portal frame consists of a beam built monolithically with the columns. Such units are used when the span of a building or hall is large. Sometimes instead of providing a horizontal beam, the portal frame may be provided with ridged beams. Fig. 617 shows the types of portal frames used for different occasions.

Other alternative structures used for similar conditions are steel trusses, concrete trusses, sheds and arches.

The portal frame may be analysed by any of the standard methods like the moment distribution method or column analogy method *etc.* The columns of the frame may be designed either for hinged end or fixed end conditions. Footings are provided to the supporting columns.

Design 150 (S.I.). Portal Frame. *The roof of a hall is 36 metres long and 12 metres wide between centres of columns and consists of continuous R.C. slab over rectangular portal frames spaced 3 metres apart. The roof is waterproofed with tar felt. The portals may be assumed to be hinged at their bases. Design the continuous slab and one of the intermediate frames and draw a sectional elevation of the portal frame showing details of the portal frame showing details of reinforcement. The ceiling height is 3·5 metres above the hinge level. Use M 15 concrete.*

Solution. Design of the slab

D.L. of the slab (10 *cm.*)	$=2500\ N/metre^2$
L L. on the slab	$=1500\ N/metre^2$
Tar felt	$=\ 500\ N/metre^2$
Bottom finish	$=\ 250\ N/metre^2$
Total load	$=4750\ N/metre^2$

Consider one *metre* strip of slab

$$\text{Max. b.m.} = \frac{wl^2}{12}$$

$$= \frac{4750 \times 3^2}{12} = 3562{\cdot}5\ Nm.$$

$$= 356250\ N\ cm.$$

Adopting $c=5$ N/mm.^{2} $=500$ N/cm^2; $t=140$ $N/mm.^2$ $=14000\ N/cm.^2$, $m=18$,

we get $a=0{\cdot}87\ d$

and $Q=85$

and equating the moment of resistance to the bending moment,

$$85 \times 100\ d^2 = 356250$$

$$\therefore \quad d = 6{\cdot}47\ cm$$

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But eff. depth available

$$= 10 - 2 = 8 \text{ cms.}$$

$$\therefore \quad A_t = \frac{356250}{14000 \times 0.87 \times 8} \text{ cm.}^2$$

$$= 3.60 \text{ cm.}^2$$

$\therefore$ Spacing of 8 *mm.* dia. bars

$$= \frac{0.50 \times 100}{3.60} = 13.9 \text{ cms.}$$

Say 8 mm. dia. at 14 cms. centres.

Distribution steel $= 0.15\%$ of gross area

$$= \frac{0.15}{100} \times 10 \times 100 = 1.5 \text{ cm.}^2$$

$\therefore$ Spacing of 6 *mm.* dia. bars

$$= \frac{0.28 \times 100}{1.5} = 18.7 \text{ cms.}$$

Hence provide 6 mm. dia. bars at 18 cms. centres.

Portals

Distance between centres of columns

$$= 12 \text{ metres.}$$

Beam section : 45 *cm.* × 75 *cm.* overall

Column section : 45 *cm.* × 60 *cm.*

Load per metre run on the beam

Load from the slab $= 4750 \times 3 = 14250$ *N/m.*

D.L. of the rib $= \frac{(75-10)}{100} \times \frac{45}{100} \times 25000 = 673$ *kg./m*

Total load/*metre* $= 21562$ *N/m.*

say 21·56 *kN per metre.*

Height of the centre line of the beam above the hinge

$$= 3.50 + 0.10 - \frac{0.75}{2} \text{ metres}$$

$= 3.22$ *metres.*

The moments of inertia of the columns and the beams will be approximately in the ratio of

$$I_{ab} : I_{bc} = \frac{45 \times 60^3}{12} : \frac{45 \times 75^3}{12}$$

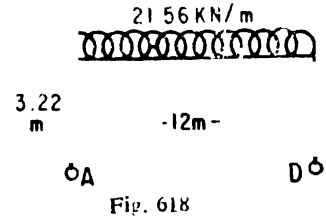

Fig. 618

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$$: \left(\frac{75}{60}\right)^3$$

$$= 1 : 1{\cdot}95$$

Let $I_{ab} = I$ $\therefore$ $I_{bc} = 1{\cdot}95I$

Distribution factors. These have been computed in the following table :

Joint	*Member*	*Relative Stiffness*	*Total Stiffness*	*Distribution factors*
B	*BA*	$\frac{3}{4}\frac{I}{3{\cdot}22} = 0{\cdot}24\ I$	0·40 *I*	$\frac{3}{5}$
	BC	$\frac{1{\cdot}95}{12} I = 0{\cdot}16I$		$\frac{2}{5}$

Fixed end moment for beam *BC*

$$= \frac{wl^2}{12}$$

	B		*C*		
A	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	*D*
0	0·00	−258·72	+258·72	0·00	0
	+155·23	+103·49	−103·49	−155·23	
		−51·75	+51·75		
	+31·05	+20·70	−20·70	−31·05	
		−10·35	+10·35		
	+6·21	+4·14	−4·14	−6·21	
		−2·07	+2·07		
	+1·24	+0·83	−0·83	−1·24	
		−0·42	+0·42		
	+0·25	+0·17	−0·17	−0·25	
0	+193·98	−193·98	+193·98	−193·98	0

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$$= \frac{21.56 \times 12^2}{12} \ kNm.$$

$$= 258.72 \ kNm.$$

The final moments for the frame have been worked out by the moment distribution method shown on page 975.

Sagging b.m. at the centre

$$= \frac{21.56 \times 12^2}{8} - 193.98 \ kNm.$$

$$= +194.1 \ kNm.$$

The points of contraflexure are given by

$$\frac{21.56 \times 12}{2} x - \frac{21.56 \times x^2}{2} - 193.98 = 0$$

$$\therefore \quad x^2 - 12x + 17.99 = 0$$

$$\therefore \quad x = 1.75 \ m \text{ or } 10.25 \ m.$$

$\therefore$ Distance between the points of contraflexure

$$= 10.25 - 1.75 = 8.50 \ m.$$

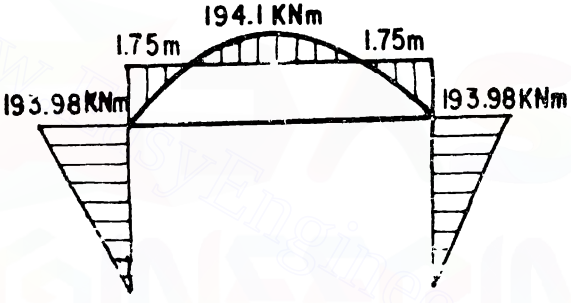

Fig. 61

Design of the beam at mid span Here the beam acts as a T-beam. The width of the flange is given by

$$B = \frac{l_o}{6} + b_w + 6 \ d_s$$

where l_o = distance between points of contraflexure.

$$B = \frac{850}{6} + 45 + (6 \times 10) = 246.7 \ cm.$$

Assuming 4 *cms* effective cover.

Eff. depth $= d = 75 - 4 = 71$ *cms.*

Approximate lever arm

$$a = d - \frac{d_s}{2}$$

$$= 71 - \frac{10}{2} = 66 \ cms.$$

$\therefore$ Approximate steel area required

$$= \frac{194.1 \times 1000 \times 100}{14000 \times 66} \ cm.^2$$

$$= 21 \ cm.^2$$

Let us try 8 bars of 20 mm. diameter

Actual area provided = 25.12 *cm.*^{2}

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Assuming the N.A. to lie below the flange, we have,

$$246{\cdot}7 \times 10(n-5) = 18 \times 25{\cdot}12(71-n)$$

$$n = 15{\cdot}22 \text{ cm.}$$

$$y = \frac{3n-2d_s}{2n-d_s} \cdot \frac{d_s}{3}$$

$$= \frac{(3\times 15{\cdot}22)-(2\times 10)}{(2\times 15{\cdot}22)-10} \times \frac{10}{3} \text{ cm.}$$

$$= 4{\cdot}18 \text{ cm.}$$

$\therefore$ Correct lever arm

$$= d - y$$

$$= 71 - 4{\cdot}18 = 66{\cdot}82 \text{ cm.}$$

$\therefore$ Stress in steel $$= \frac{194{\cdot}1 \times 1000 \times 100}{25{\cdot}12 \times 66{\cdot}82} \text{ N/cm.}^2$$

$$= 11563 \text{ N/cm.}^2$$

Corresponding stress in concrete

$$c = \frac{t}{m} \cdot \frac{n}{d-n}$$

$$= \frac{11563}{18} \times \frac{15{\cdot}22}{71-15{\cdot}22}$$

$$= 175 \text{ N/cm}^2.$$

Corner section B.M. $= 193{\cdot}98 \times 1000 \times 100$ *N cm.*

$= 19398000$ *N cm.*

The section will be designed as a doubly reinforced beam on the steel beam theory.

Let the effective cover to steel centre at the top

$= 4$ *cms.*

Effective cover to bottom steel centre $= 4$ *cms.*

Distance between centres of top steel and bottom steel

$$= 75 - 4 - 4 = 67 \text{ cms.}$$

$\therefore$ $$A_t = A_c = \frac{19398000}{14000 \times 67} \text{ cm.}^2$$

$$= 20{\cdot}7 \text{ cm.}^2$$

Provide 8 bars of 20 mm. dia. (25·12 cm.²) at top and an equal amount of steel at the bottom.

Max. S.F.

$$= \frac{21{\cdot}56 \times 12}{2} \times 1000 \text{ N}$$

$$= 129360 \text{ N}$$

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Nominal shear stress

$$=q_v \quad \frac{S}{bd} \quad \frac{129360}{45\times71} \quad =40\ N/cm.^2$$

$$=0{\cdot}4\ N/mm.^2$$

Percentage of tensile steel

$$=\frac{25{\cdot}12}{45\times71}\times100=0{\cdot}8\%$$

For 0·8% steel $q_c=0{\cdot}34\ N/mm.^2=34\ N/cm.^2$

q_v is nearly equal to q_c

Provide two-legged 8 mm. ϕ stirrups @ 30 cm. c/c.

Design of columns. Column section at the top.

Load from the beam

$$=129360\ N$$

$$\therefore \quad \text{B.M.}=19398000\ N\ cm.$$

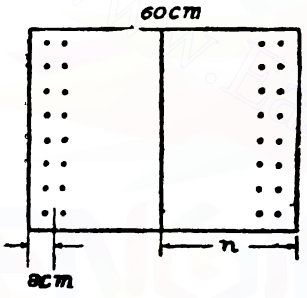

Fig. 620

Let 16 bars of 20 mm. dia. be provided for each of the compression and tension side. Let the effective cover to the centroid of the reinforcement be 8 *cms.*

$$A_c=A_t=50{\cdot}28\ cm.^2$$

Distance of the N.A. from the compression edge is give by,

$$\frac{45\ n^2}{2}+17\times50{\cdot}28(n-8)$$

$$=18\times50{\cdot}28\ (52-n)$$

$$n=23{\cdot}5\ cms.$$

Let the compressive stress in concrete due to the bending moment be $c\ N/cm.^2$

Hence the M.R. of the beam

$$45\times23{\cdot}5\times\frac{c}{2}\left(52-\frac{23{\cdot}5}{3}\right)$$

$$+17\times50\ 28\times\ \frac{15{\cdot}5}{23{\cdot}5}\ c\times44\ N\ cm.$$

$$=41140\ c\ N\ cm.$$

$$\therefore \quad 48140\ c=19398000\ N\ cm.$$

$$\therefore \quad c=403\ N/cm.^2$$

Equivalent uncracked concrete area

$$=bn+(m-1)\ A_c+mA_t$$

$$=45\times23{\cdot}5+17\times50{\cdot}28+18\times50{\cdot}28\ cm.^2$$

$$=2818\ cm.^2$$

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∴ Direct stress in concrete due to direct load

$$= \frac{129360}{2818} = 45{\cdot}9 \ N/cm.^2$$

∴ Resultant stress in concrete

$$= 403 + 45{\cdot}9 \ N/cm.^2$$
$$= 448{\cdot}9 \ N/cm.^2$$

Stress in steel due to b.m.

$$t = \frac{mc}{n}(d-n)$$

$$= \frac{18 \times 403 \times 28{\cdot}5}{23{\cdot}5} \ N/cm.^2$$

$$= 8797 \ N/cm.^2$$

Compressive stress in steel due to direct load

$$= 18 \times 45{\cdot}9 = 826{\cdot}2 \ N/cm.^2$$

∴ Resultant stress in steel

$$= 8797 - 826{\cdot}2 = 7970{\cdot}8 \ N/cm.^2$$

Section of the column just above the hinge

Load from the beam	$= 129360 \ N$
Self wt. of the column $= 0{\cdot}6 \times 0{\cdot}45 \times 3{\cdot}22 \times 25000$	$= 21735 \ N$
Total	$= 151095 \ N$
B.M. at the section	$= 0{\cdot}00$
Load that can be carried on plain concrete at 400 N/cm^2 $= 60 \times 45 \times 400$	$= 1080000 \ N$
But load on the section	$= 151095 \ N$

Hence plain concrete alone can safely resist the load

Hence only nominal reinforcement will be provided.

Design of the hinge

In order to provide a hinge action concrete will be made to reach nearly half the ultimate stress. Let us assume that the ultimate stress for concrete is 1500 $N/cm.^2$ Hence the stress in concrete at the hinge section

$$= \frac{1500}{2} = 750 \ N/cm.^2$$

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∴ Area required at the hinge section

$$= \frac{151095}{750} \text{ cm.}^2$$

$$= 201{\cdot}5 \text{ cm.}^2$$

Let us provide 14 cm. × 14 cm.

Length of the hinge = double the least width

$$= 2 \times 14 = 28 \text{ cms.}$$

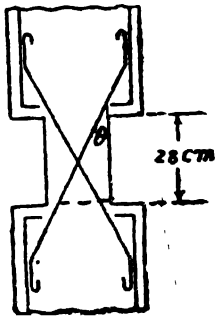

Fig. 621

Shear at the hinge section

= Horizontal thrust

$$= \frac{193{\cdot}98 \times 1000}{3{\cdot}22} N$$

$$= 60242 \ N$$

Let 18 *mm.* dia. bars be used to resist the shear.

Let the minimum eff. cover be 2 *cm.*

If the inclination of this steel with the vertical be θ then

$$\tan \theta = \frac{14-4}{28} = 0{\cdot}357$$

$$\therefore \quad \theta = 19° \ 39'$$

Allowing a stress of 14000 *N/cm.*^{2} the shear resistance of the shear bars

$$14000 A_t \sin \theta = 60242$$

$$A_t = \frac{60242}{14000 \times \sin 19° \ 39'} \text{ cm.}^2$$

$$= 12{\cdot}8 \text{ cm.}^2$$

Provide 8 bars of 16 mm. dia. (16·08 *cm.*^{2})

Of these 8 bars, four bars will be made to resist the shear by diagonal tension and the other four bars will be made to resist the shear by diagonal compression.

Design of the foundation

Load on columns	= 151095 *N*
Wt. of foundation @ 10% column load	= 15110 *N*
Total	= 166205 *N*

Moment about the base 60242×1

$$= 60242 \text{ Nm.}$$

∴ Eccentricity of the load

$$e = \frac{60242}{166205} \text{ m.}$$

$$= 0{\cdot}36 \text{ m.}$$

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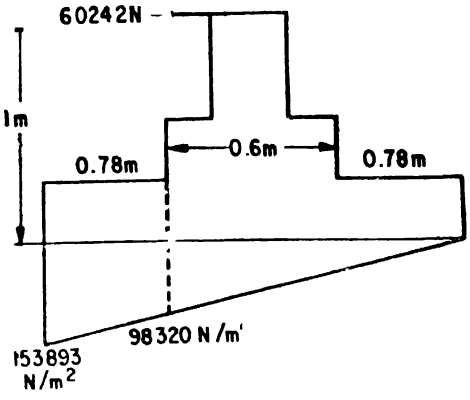

Fig. 622

Hence in order tension may be just avoided

Breadth $= b = 6e = 6 \times 0{\cdot}36 = 2{\cdot}16$ *metres*

Let the foundation area be 1 *metre* $\times$ 2·16 *metres*.

Total upward load = Total downward load

$$\frac{p}{2} \times 1 \times 2{\cdot}16 = 166205$$

$$\therefore \quad p = 153893 \ N/m.^2$$
$$= 153{\cdot}9 \ kN/m.^2$$

Safe bearing capacity of the soil may be assumed as 200 $kN/m.^2$

$$p' = \frac{1{\cdot}38}{2{\cdot}16} \times 153893 = 98320 \ N/m.^2$$

Total pressure on the cantilevering part

$$= \frac{p+p'}{2} \times (0{\cdot}78 \times 1)$$

$$= \frac{(98320+153893)}{2}(0{\cdot}78 \times 1) \ N$$

$$= 98363 \ N$$

This acts at a distance of $\frac{p'+2p}{p'+p} \times \frac{0{\cdot}78}{3}$ *metre*

$$= \frac{98320+2\times153893}{98320+153893} \times \frac{0{\cdot}78}{3}$$

$= 0{\cdot}42$ *metre* from the column edge

$\therefore$ B.M. due to upward pressure

$$= 98363 \times 0{\cdot}42 = 41312{\cdot}5 \ Nm.$$

$\therefore$ Depth of foundation slab is given by $Qbd^2 = M$

$$85 \times 100d^2 = 41312{\cdot}5 \times 100$$

$$\therefore \quad d = 22 \ cms.$$

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Providing 8 *cm.* eff. cover, the total depth required = 30 *cms.* Let us provide 32 *cms.* overall and 24 *cms* effective.

$$A_t = \frac{41312{\cdot}5 \times 100}{14000 \times 0{\cdot}87 \times 22} \text{ cm.}^2$$

$$= 15{\cdot}42 \text{ cm.}^2$$

Provide 8 bars of 16 mm. dia. (16·08 $cm.^2$).

Design 151 *Fig. 623 shows an intermediate frame provided for a building. These frames are spaced at 3·5 metres centres. Design the roof slab and an intermediate portal frame. The live load on the roof slab may be taken as 150 kg./metre². Adopt suitable working stresses.*

Solution. Roof slab

Let the thickness of the slab be 10 *cm.*

Fig. 623

Loads

Dead load of slab 10×24

$= 240$ *kg./m.*²

Live load = 150 *kg./m.*²

Total = 390 *kg./m.*²

Consider a one *metre* wide strip of the slab.

Maximum B.M. $= \frac{390 \times 3{\cdot}5^2}{12}$ *kg. m.*

$= 398{\cdot}1$ *kg. m.* = 39810 *kg. cm.*

Adopting $c = 50$ *kg./cm.*², $t = 1400$ *kg /cm.*², $m = 18$ and equating the moment of resistance to the maximum bending moment. we have,

$$8{\cdot}5 \times 100d^2 = 39810$$

$$\therefore \quad d = 6{\cdot}84 \text{ cm.}$$

Providing 2 *cm.* effective cover, the effective depth available

$$= 10 - 2 = 8 \text{ cm.}$$

$$\therefore \quad A_t = \frac{39810}{1400 \times 0{\cdot}87 \times 8} \text{ cm.}^2 = 4{\cdot}09 \text{ cm.}^2$$

Spacing of 10 *mm.* diameter bars

$$= \frac{0{\cdot}79 \times 100}{4{\cdot}09} \text{ cm} = 19{\cdot}3 \text{ cm.}$$

Provide 10 mm. bars @ 18 cm. centres.

Distribution steel $= \frac{0{\cdot}15}{100} \times 10 \times 100$ *cm.*²

$= 1{\cdot}5$ *cm.*²

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Spacing of 6 *mm.* diameter bars

$$\frac{0{\cdot}28 \times 100}{1{\cdot}5} \ cm.$$

$$= 18{\cdot}7 \ cm. \ say \ 18 \ cm. \ centres.$$

Design of Portal frame

Depth of beam about $\frac{1}{15}$ of span

$$= \frac{1}{15} \times 10 \times 100 \ cm. = 67 \ cm.$$

Let the beam section be 30 *cm.* × 70 *cm.*

Let the column section be 30 *cm.* × 50 *cm.*

Load per metre run of beam

From slab : $390 \times 3{\cdot}5 = 1365 \ kg./m.$

Rib : $\frac{(70-10)}{100} \times 0{\cdot}3 \times 2400 = 432 \ kg./m.$

$1797 \ kg./m.$

Let I_{ab} and I_{bc} be the moments of inertia of the sections of column and beam respectively

$$\therefore \quad I_{ab} : I_{bc} = 30 \times \frac{50^3}{12} : 30 \times \frac{70^3}{12}$$

$$= 1 : \left(\frac{7}{5}\right)^3 = 1 : 2{\cdot}744$$

Let $I_{ab} = I$

$\therefore \quad I_{bc} = 2 \ 744 \ I$

Distribution factors. These are computed in the following table :

Joint	*Member*	*Relative Stiffness*	*Total Stiffness*	*Distribution factors*
B	BA	$\frac{I}{5} = 0{\cdot}200 \ I$	$0{\cdot}474 \ I$	$\frac{0{\cdot}2}{0{\cdot}474} = 0{\cdot}42$
	BC	$\frac{2 \ 744 \ I}{10} = 0{\cdot}274 \ I$		$\frac{0{\cdot}274}{0{\cdot}474} = 0{\cdot}58$

Fixed end moments

$$m_{bc} = -\frac{1797 \times 10^2}{12} \ kg. \ m.$$

$$= -14970 \ kg. \ m. = -1497 \ tm.$$

$$m_{cb} = +14{\cdot}97 \ tm.$$

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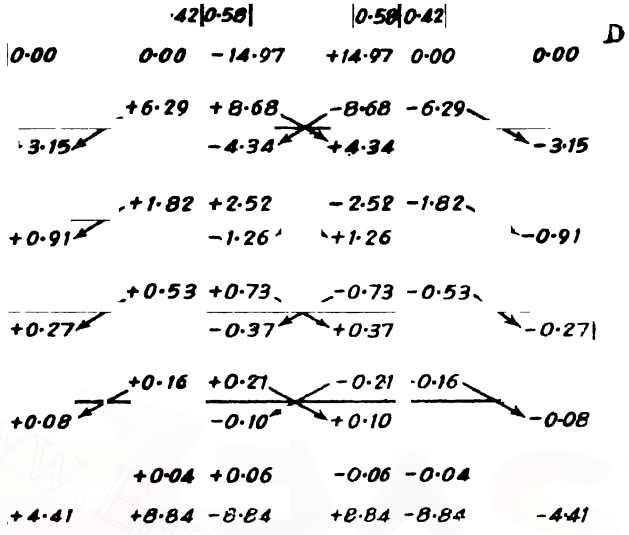

Fig. 624

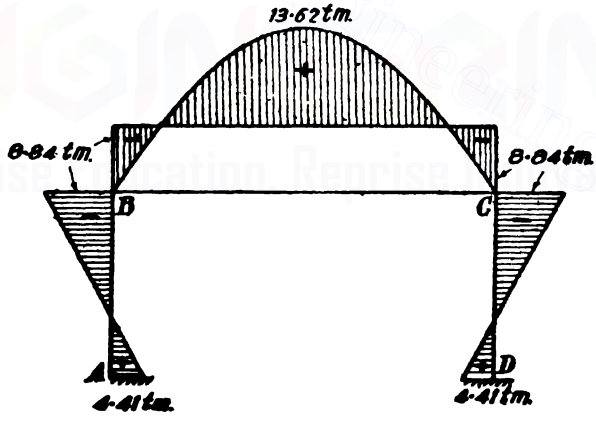

Fig. 625

The frame is now solved by moment distribution (see Fig. 624).

Free B.M. for span *BC*

$$\frac{1797 \times 10^2}{8 \times 1000}\ tm. = 22{\cdot}46\ tm.$$

∴ Net B.M. at the centre of span *BC*

= 22·46 − 8 84 = 13·62 *tm.*

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Design of Beam at mid span section. Maximum B.M. 13·62 *tm.* (sagging moment).

The section of beam will be designed as a *T* beam.

Overall depth of beam = 70 *cm*

Providing an effective cover of 4 *cm.* the effective depth

$$= 70 - 4 = 66 \text{ cm.}$$

Approximate lever arm $= a \quad d - \frac{d_s}{2} = 66 - \frac{10}{2} = 61 \text{ cm.}$

Approximate steel requirement

$$= \frac{13{\cdot}62 \times 1000 \times 100}{1400 \times 61} \text{ cm.}^2 = 15{\cdot}95 \text{ cm.}^2$$

Provide 4 bars of 22 mm. diameter (15·21 cm·²)

Width of flange. This will be taken at the least of the following :

(*i*) $\frac{1}{3}$ span $= \frac{10}{3} \times 100 = 333$ *cm.*

(*ii*) Spacing of beams $= 3{\cdot}5$ *m.* $= 350$ *cm.*

(*iii*) $12d_s + b_r = 12 \times 10 + 30 \quad = 150$ *cm.*

$\therefore$ Width of flange $= 150$ *cm.*

Position of neutral axis. Let us assume that the neutral axis lies below the flange.

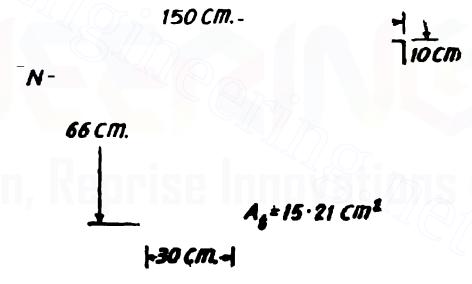

Fig. 626

Taking moments about the neutral axis, we have,

$150 \times 10(n - 5) = 18 \times 15{\cdot}21(66 - n)$

Solving, we get

$n = 14{\cdot}4$ *cm.*

Actual lever arm $a = d - y$

$$y = \frac{3n - 2d_s}{2n - d_s} \quad \frac{d_s}{3}$$

$$\frac{3 \times 14{\cdot}4 - 2 \times 10}{2 \times 14{\cdot}4 - 10} \times \frac{10}{3} \text{ cm.}$$

$$= 4{\cdot}11 \text{ cm.}$$

$\therefore \quad a = 66 - 4{\cdot}11 = 61{\cdot}89$ *cm.*

$\therefore$ Actual stress in steel

$$= \frac{13{\cdot}62 \times 1000 \times 100}{15{\cdot}21 \times 61{\cdot}89} \text{ kg./cm.}^2$$

$$= 1446 \text{ kg./cm.}^2$$

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Percentage of overstress

$$= \frac{46}{1400} \times 100\% = 3{\cdot}3\%$$

Since the overstress is less than 5% the section can be still considered safe.

Corresponding stress in concrete $c = \frac{t}{m} \cdot \frac{n}{d-n}$

$$= \frac{1446}{18} \times \frac{144}{66-144} \text{ kg./cm.}^2$$

$$= 22{\cdot}4 \text{ kg/cm.}^2$$

Design of beam at end section. Maximum bending moment

$$= 8{\cdot}84 \text{ tm.}$$

Here the section must be designed as a rectangular section.

Equating the moment of resistance to the bending moment, we have,

$$8{\cdot}5 \times 30d^2 = 8{\cdot}84 \times 1000 \times 100$$

$$d = 58{\cdot}9 \text{ cm.}$$

Eff. depth available $= 66$ cm.

$$A_t = \frac{8{\cdot}84 \times 1000 \times 100}{1400 \times 0{\cdot}87 \times 66} \text{ cm.}^2 = 11 \text{ cm.}^2$$

Provide four bars of 20 mm. diameter (12·57 cm.²)

Shear. Max. S.F. $= 1797 \times \frac{10}{2}$ kg.

$$= 8985 \text{ kg.}$$

Shear stress $= \frac{S}{ab}$

$$= \frac{8985}{0{\cdot}87 \times 66 \times 30} \text{ kg./cm.}^2 = 5{\cdot}22 \text{ kg./cm.}^2$$

Spacing of two-legged 10 mm. diameter stirrups

$$= \frac{A_w \, t_w \, a}{S}$$

$$= \frac{2 \times 0{\cdot}79 \times 1400 \times 0{\cdot}87 \times 66}{8985} \text{ cm.}$$

$$= 14{\cdot}14 \text{ cm. say } 14 \text{ cm. c/c.}$$

Column section at top. B.M. $= 8{\cdot}84$ tm. $= 8840$ kg. m.

Direct load $= 8985$ kg.

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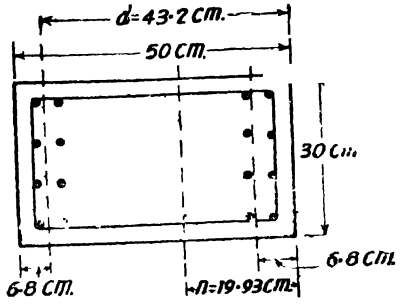

Fig. 627

Trial section. *Provide 8 bars of 22 mm. diameter near each face.*

Clear cover to outer bars = 4 *cm*

Distance between the two rows of bars = 12 *mm.*

Effective cover to the centroid of steel = 4 + 2·2 + 0·6 *cm.*
= 6·8 *cm.*

Effective depth
= 50 − 6·8 = 43·2 *cm.*

$A_c = A_t = 30{\cdot}41$ *cm.*^{2}

Analysis for bending moment

Position of neutral axis Taking moments about the neutral axis, we have

$$\frac{30n^2}{2} + 17 \times 30{\cdot}41\,(n - 6{\cdot}8) = 18 \times 30{\cdot}41\,(43{\cdot}2 - n)$$

$$\therefore \quad n^2 + 70{\cdot}95n = 1811{\cdot}3$$

$$\therefore \quad n = 19{\cdot}93 \text{ cm.}$$

Let the maximum compressive stress in concrete due to bending be c *kg /cm.*^{2}

$\therefore$ M.R. of the section

$$= \frac{30 \times 19{\cdot}93 \times c}{2}(43{\cdot}2 - 6{\cdot}64) + 17 \times 30{\cdot}41 \times \frac{13{\cdot}13}{19{\cdot}93} \times c \times 36{\cdot}4 \text{ kg. cm.}$$

$$= 23330c = 8{\cdot}84 \times 1000 \times 100$$

$$\therefore \quad c = 37{\cdot}9 \text{ kg./cm.}^2$$

Corresponding tensile stress in steel

$$t = 18 \times \frac{43{\cdot}2 - 19{\cdot}93}{19{\cdot}93} \times 37{\cdot}9 \text{ kg./cm.}^2$$

$$= 796{\cdot}4 \text{ kg /cm.}^2$$

Compressive stress in concrete due to direct load

$$c' = \frac{\text{Direct load}}{\text{Equivalent uncracked concrete area}}$$

$$= \frac{W}{bn + (m-1)A_c + m\,A_t}$$

$$= \frac{8985}{30 \times 19{\cdot}93 + 17 \times 30{\cdot}41 + 18 \times 30{\cdot}41} \text{ kg./cm.}^2$$

$$= 5{\cdot}41 \text{ kg./cm.}^2$$

$\therefore$ **Resultant compressive stress in concrete**

$$= c + c' = 37{\cdot}9 + 5{\cdot}41 \text{ kg./cm.}^2$$

$$= 43{\cdot}31 \text{ kg./cm.}^2$$

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Resultant tensile stress in steel

$$= t - mc'$$
$$= 796{\cdot}4 - 18 \times 5{\cdot}41 \ kg./cm.^2$$
$$= 699 \ kg./cm.^2$$

Column section at base. Load from beam 8985 *kg.*

Weight of column $= 0{\cdot}3 \times 0{\cdot}5 \times 5 \times 2400$ 1800 *kg.*

Total load $= 10785$ *kg.*

B.M. at base $= 4{\cdot}41$ *tm.* $= 4410$ *kg. m.*

Trial section. *Provide four bars of 22 mm. diameter near each face.*

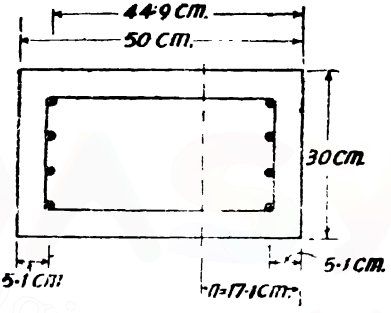

Fig. 628

Clear cover to bars $= 4$ *cm.*

$\therefore$ Effective depth

$$= 4 + 1{\cdot}1 = 5{\cdot}1 \ cm.$$

$\therefore$ Effective depth

$$= 50 - 5{\cdot}1 = 44{\cdot}9 \ cm.$$

Analysis for bending moment

Position of neutral axis. Taking moments about the neutral axis, we have,

$$\frac{30n^2}{2} + 17 \times 15{\cdot}21 \ (n - 5{\cdot}1)$$
$$= 18 \times 15{\cdot}21 \ (44{\cdot}9 - n)$$

Solving, we get, $n = 17{\cdot}1$ *cm.*

Let the maximum compressive stress in concrete due to bending moment be c *kg./cm.*^{2}

M.R of the section,

$$= 30 \times 17{\cdot}1 \frac{c}{2}\left(44{\cdot}9 - \frac{17{\cdot}1}{3}\right) + 17 \times 15{\cdot}21 \times \frac{(17{\cdot}1 - 5{\cdot}1)}{17{\cdot}1} \times c \times 39{\cdot}8$$
$$= 17281c = 4410 \times 100$$

$\therefore$ $c = 25{\cdot}5 \ kg./cm.^2$

Corresponding tensile stress in steel

$$t = 18 \times \frac{(44{\cdot}9 - 17{\cdot}1)}{17{\cdot}1} \times 25{\cdot}5 \ kg./cm.^2$$
$$= 746 \ kg./cm.^2$$

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Compressive stress in concrete due to direct load

$$c' = \frac{10785}{30 \times 17{\cdot}1 + 17 \times 15{\cdot}21 + 18 \times 15{\cdot}21} \ kg./cm.^2$$

$$= 10{\cdot}3 \ kg/cm.^2$$

$\therefore$ Resultant compressive stress in concrete

$$= c + c' = 25{\cdot}5 + 10{\cdot}3 = 35{\cdot}8 \ kg./cm.^2$$

$\therefore$ Resultant tensile stress in steel $= t - mc'$

$$= 746 - 18 \times 10{\cdot}3 = 561 \ kg./cm.^2$$

Foundation. Load at column base $= 10785 \ kg.$

Approximate weight of foundation (10%) $= 1079 \ kg.$

$11864 \ kg.$

Moment $= 4410 \ kg.\ m.$

Eccentricity $= e = \frac{4410 \times 100}{11854} \ cm.$

$= 37 \ cm.$

Foundation plan will be chosen to satisfy the condition that the centre of gravity of loads coincides with the centroid of footing area. By this arrangement the pressure intensity on the footing will be uniform.

Let the bearing capacity of soil be 10000 $kg./metre^2$

$\therefore$ Area of foundation

$$= \frac{11864}{10000} = 1{\cdot}186 \ metres^2$$

Let the width of foundation be 60 *cm.*

Provide also a pedestal 60 *cm.* $\times$ 80 *cm.* and 50 *cm.* deep.

Length of footing $= \frac{1{\cdot}186}{0{\cdot}6} \ m. = 2 \ metres$

Fig. 629 shows the arrangement for the footing.

Net upward pressure

$$= \frac{10785}{0{\cdot}6 \times 2} \ kg./m.^2$$

$$= 8988 \ kg./m.^2$$

Maximum B.M. $= 8988 \times 0{\cdot}6 \times \frac{0{\cdot}97^2}{2} \ kg.\ m.$

$$= 2537 \ kg.\ m.$$

Equating the M.R. to B.M., we have

$$8{\cdot}5 \times 60d^2 = 2537 \times 100 \ kg.\ cm.$$

$\therefore$ $d = 22{\cdot}3 \ cm.$

Providing an effective cover of 6 *cm.*

Overall depth required $= 22{\cdot}3 + 6 + 28{\cdot}3 \ cm.$

Let us provide 30 *cm.* depth.

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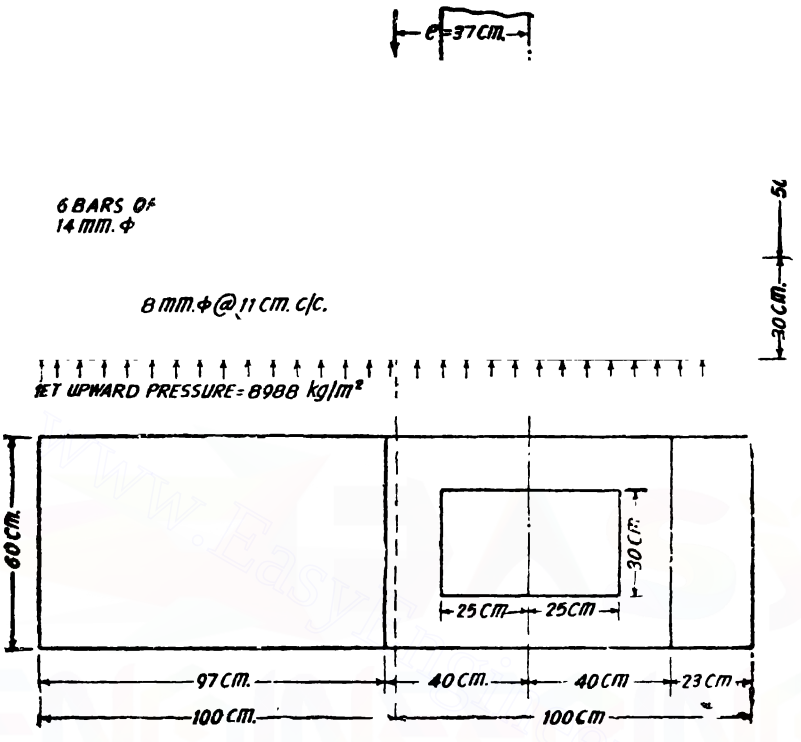

Fig. 629

Depth from punching shear stress consideration.

Allowing a punching stress of 10 *kg./cm.*²

$$2 \times 60 \times D \times 10 = 10785$$

$$\therefore \quad D = 9 \text{ cm.}$$

Hence we will provide 30 *cm.* overall.

Effective depth $= 30 - 6 = 24$ *cm.*

$\therefore$ Area of steel required

$$= A_t = \frac{2537 \times 100}{1400 \times 0{\cdot}87 \times 24} \text{ cm.}^2$$

$$= 8{\cdot}68 \text{ cm.}^2$$

Provide 6 bars of 14 mm. diameter (9·24 *cm.*²)

Distribution steel $= \frac{0{\cdot}15}{100} \times 30 \times 100 \text{ cm.}^2 = 4{\cdot}50 \text{ cm.}^2$

Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{4{\cdot}5} \text{ cm.}$$

$$= 11 \text{ cm. centres.}$$

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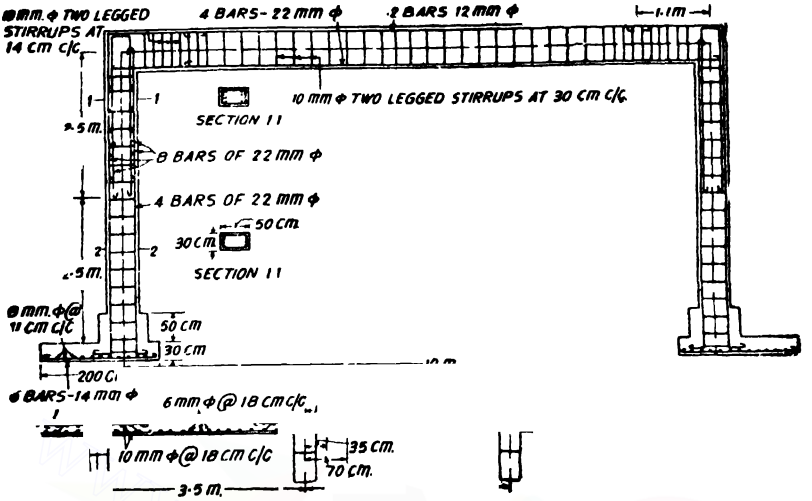

Fig. 630

Design 152. *Fig. 631 shows an intermediate frame proposed for a shed.*

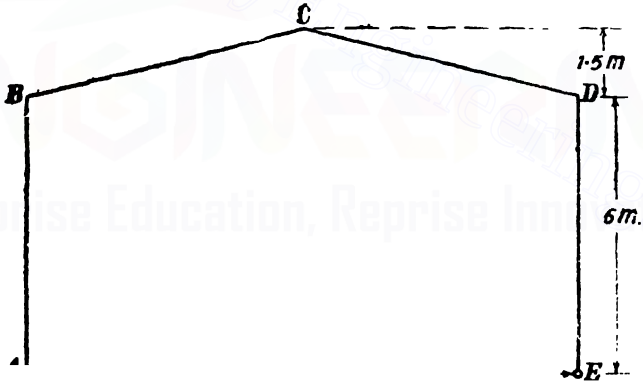

Fig. 631

Design the frame to the following requirements :

(*i*) *All members have the same overall section.*
(*ii*) *Spacing of frames = 3 metres.*
(*iii*) *Live load on roof including wind pressure = 200 kg./metre*^{2}
(*iv*) *Columns are hinged at their bases.*
Adopt suitable working stresses.
(*v*) *Roof slab is not monolithic with frames.*

Solution. Let us assume that the frames will support a 10 *cm.* thick slab. The loading on the member *BC* per *metre* of its length will consist of the following :

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Weight of slab	: $24 \times 10 \times 3$	= 720 *kg./m.*
Beam (40 *cm.* × 80 *cm.*)	: $20{\cdot}4 \times 0{\cdot}8 \times 2400$	= 768 *kg./m.*
Live load	: 200×3	= 600 *kg./m.*
		Total = 2088 *kg./m.*

Let θ be the inclination of BC with the horizontal

$$\tan \theta = \frac{1{\cdot}5}{6} = 0{\cdot}25$$

$$\therefore \quad \theta = 14^\circ\ 2'$$

$$\sin \theta = 0{\cdot}2425, \cos \theta = 0{\cdot}9702$$

Length $BC = \sqrt{6^2 + 1{\cdot}5^2} = 6{\cdot}19$ *metres*

Normal component of the load on the roof

$$= 2088 \times 0{\cdot}9702 \text{ kg /m.}$$
$$= 2026 \text{ kg./m.}$$

Fixed end moments for BC

$$m_{bc} = -\frac{wl^2}{12}$$

$$= -\frac{2026 \times 6{\cdot}19^2}{12 \times 1000} \text{ tonne metre}$$

$$= -6{\cdot}47 \text{ tonne metre}$$

$$m_{cb} = +6{\cdot}47 \text{ tm.}$$

Distribution factors. *These have been computed in the following table* :

Joint	*Member*	*Relative stiffness*	*Total stiffness*	*Distribution factors*
B	*BA*	$\frac{3}{4} \cdot \frac{I}{6} = 0{\cdot}125\ I$	$0{\cdot}287\ I$	$\frac{0{\cdot}125}{0{\cdot}287} = 0{\cdot}43$
	BC	$\frac{I}{6{\cdot}19} = 0{\cdot}162\ I$		$\frac{0\ 162}{0\ 287} = 0{\cdot}57$
C	*CB*	$0{\cdot}162\ I$	$0{\cdot}324\ I$	0·5
	CD	$0{\cdot}162\ I$		0 5

The moments will first be worked out assuming that *B*, *C* and *D* are restrained against lateral displacements. These are shown in Fig. 632.

Moments due to sway

Let due to the deformation the joint *B* be horizontally displaced to B_1 so that $BB_1 = \delta$.

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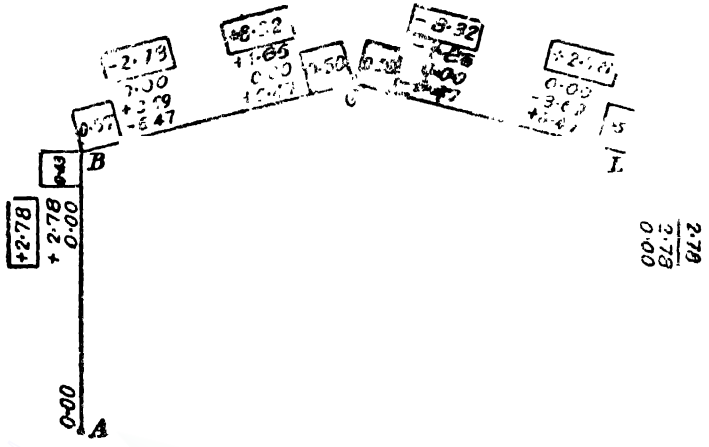

Fig. 632

Let C be vertically displaced to C_1.

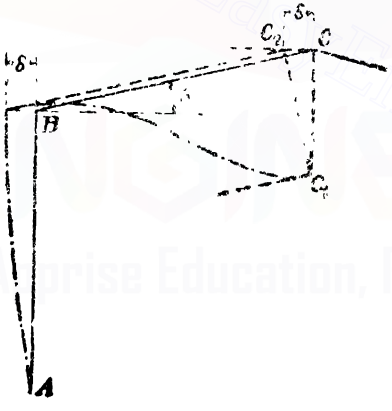

Fig. 633

$\therefore$ Relative distortion of C normal to the member BC

$= \delta_1 = C_1C_2 = \delta \text{ cosec } \theta$

$= \delta \text{ cosec } 14° 2'$

$= 4{\cdot}12\,\delta$

Hence corresponding to this distortion fixing moment for BC

$$= \frac{6EI\delta_1}{BC^2}$$

But, fixing moment at B for BA

$$\frac{3EI\delta}{AB^2}$$ since A is hinged.

The above moments are in the ratio

$$\frac{2\delta_1}{BC^2} : \frac{\delta}{AB^2}$$

$$\frac{8{\cdot}24\delta}{6{\cdot}19^2} : \frac{\delta}{6^2} \text{ or } 7{\cdot}74 : 1$$

say $77{\cdot}4 : 10$

Due to the above selected moments of 77·4 *kg. m.* and 10 *kg. m.* in the members BC and BA the moment distribution is again worked out.

The moments shown in Fig. 634 are not the correct sway moments. These moments are to be corrected by a factor K. The

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corrected sway moments should be added on to the non-sway moments to get the resultant moments.

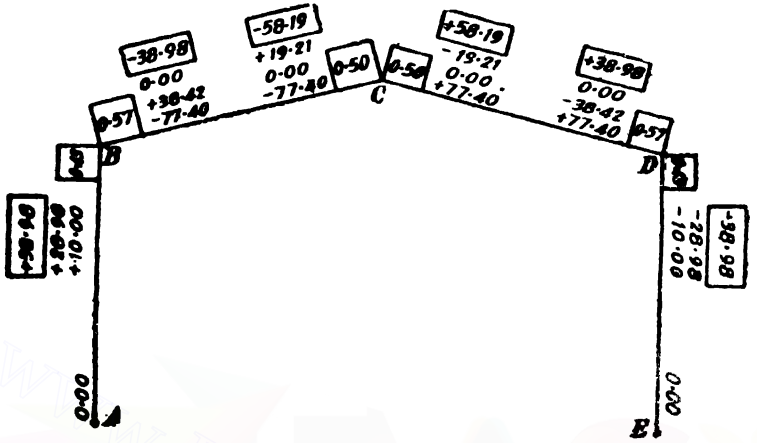

Fig. 634

Hence the final moments are as follows :

$$M_{ab} = 0$$
$$M_{ba} = +2{\cdot}78 + 38{\cdot}98\ K$$
$$M_{bc} = -2{\cdot}78 - 38{\cdot}98\ K$$
$$M_{cb} = +8{\cdot}32 - 58{\cdot}19\ K$$

If H_{ba} and H_{bc} are the horizontal reactions at BC then for the equilibrium, $H_{ba} + H_{bc} = 0$

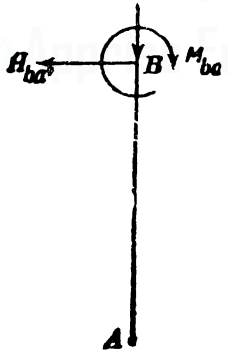

Fig. 635

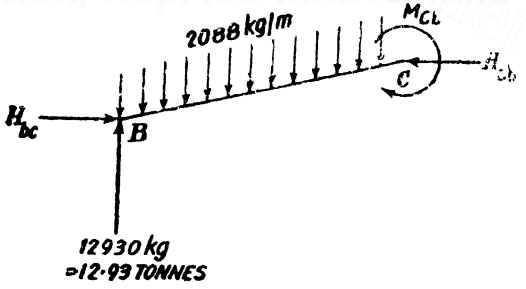

Fig. 636

$$H_{ba} = -\frac{M_{ba} + M_{ab}}{AB} = \frac{-(2{\cdot}78 + 38{\cdot}98\ K) + 0}{6}$$
$$= -0{\cdot}463 - 6{\cdot}497\ K \text{ tonnes}$$
$$H_{bc} = \frac{M_{cb} + M_{bc} + \text{Moment of forces on } BC \text{ about } C}{\text{Vertical height of } BC}$$

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$$= \frac{(+8{\cdot}32 - 58{\cdot}19\,K) + (-2{\cdot}78 - 38{\cdot}98\,K) + 12{\cdot}93(6-3)}{1{\cdot}5}$$

tonnes

$$= 29{\cdot}55 - 64{\cdot}78\,K$$

Since $H_{ba} + H_{bc} = 0$, we have

$$-0{\cdot}463 - 6{\cdot}497\,K + 29{\cdot}55 - 64{\cdot}78\,K = 0$$

$$\therefore \quad K = 0{\cdot}408$$

∴ The resultant moments are,

$$M_{ab} = 0$$
$$M_{ba} = +2{\cdot}78 + 38{\cdot}98 \times 0{\cdot}408 = +18{\cdot}69 \text{ } tm.$$
$$M_{bc} = -18{\cdot}69 \text{ } tm.$$
$$M_{cb} = +8{\cdot}32 - 58{\cdot}19 \times 0{\cdot}408 = -15{\cdot}42 \text{ } tm.$$

Design of Members

Member BC

B.M. at B $= -18{\cdot}69$ *tm.* (hogging)

B.M. at C $= 15{\cdot}42$ *tm.* (sagging)

Free B.M. at the middle point of CD

$$= \frac{(2088 \times 6{\cdot}19)^2}{8 \times 1000} \text{ } tm.$$

$$= 9{\cdot}70 \text{ } tm. \text{ (sagging)}$$

Fig 637

Net B M. at the middle point of BC

$$= \frac{-18{\cdot}69 + 15{\cdot}42}{2} + 9{\cdot}70 = +8{\cdot}06 \text{ } tm.$$

Adopting $c = 50$ *kg/cm*^{2}, $t = 1400$ *kg./cm.*^{2} and $m = 18$ and equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}5 \times 40\,d^2 = 18{\cdot}69 \times 1000 \times 100$$

$$\therefore \quad d = 74{\cdot}13 \text{ } cm.$$

Overall depth provided

$$= 80 \text{ } cm.$$

Providing an effective cover of 4 *cm.* the effective depth available:

$$= d = 80 - 4 = 76 \text{ } cm.$$

$$A_s = \frac{18{\cdot}69 \times 1000 \times 100}{1400 \times 0{\cdot}87 \times 76} \text{ } cm.^2 = 20{\cdot}19 \text{ } cm.^2$$

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Provide 6 bars of 22 mm. diameter (22·81 cm.²)

Shear force at B for BC

$$S = V\cos\theta - H\sin\theta$$

where V = Vertical reaction at B = 12·93 *t*.

and H = Horizontal reaction at B

= Horizontal reaction at A

$$= \frac{M_{ba} + M_{ab}}{AB} = \frac{18{\cdot}69 + 0}{} \text{ tonnes}$$

$$= 3{\cdot}115 \text{ tonnes}.$$

$\therefore$ $S = 12{\cdot}93 \cos 14^\circ 2' - 3{\cdot}115 \sin 14^\circ 2'$

$= 11{\cdot}78$ *tonnes*.

$\therefore$ Shear stress $= \frac{S}{ab}$

$$= \frac{11{\cdot}78 \times 1000}{0{\cdot}87 \times 76 \times 40} \text{ kg./cm.}^2 = 4{\cdot}45 \text{ kg./cm.}^2$$

Provide two-legged 10 mm. diameter stirrups at 30 cm centres.

Section at C. B.M. = 15·42 *tm.*

$$A_s = \frac{15{\cdot}42 \times 1000 \times 100}{1400 \times 0{\cdot}87 \times 76} = 16{\cdot}66 \text{ cm.}^2$$

Provide 6 bars of 20 mm. diameter.

Column section at top

Direct load = 12·93 *tonnes*

B.M. = 18·69 *tm.*

Let 12 bars of 22 mm. diameter be provided (with 6 bars near each face).

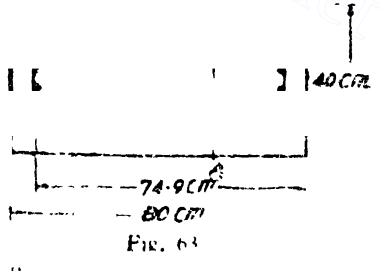

Fig. 63

Clear cover to steel

= 4 *cm.*

$\therefore$ Effective cover

= 4 + 1·1 = 5·1 *cm*

$\therefore$ Effective depth = 80 − 5·1 = 74·9 *cm*

Taking moments about the neutral axis

$$\frac{40n^2}{2} + (18-1)22{\cdot}81(n - 5{\cdot}1) = 18 \times 22{\cdot}81(74{\cdot}9 - n)$$

$\therefore$ $n^2 + 39{\cdot}91\, n - 1674 = 0$

Solving, we get $n = 25{\cdot}2$ *cm.*

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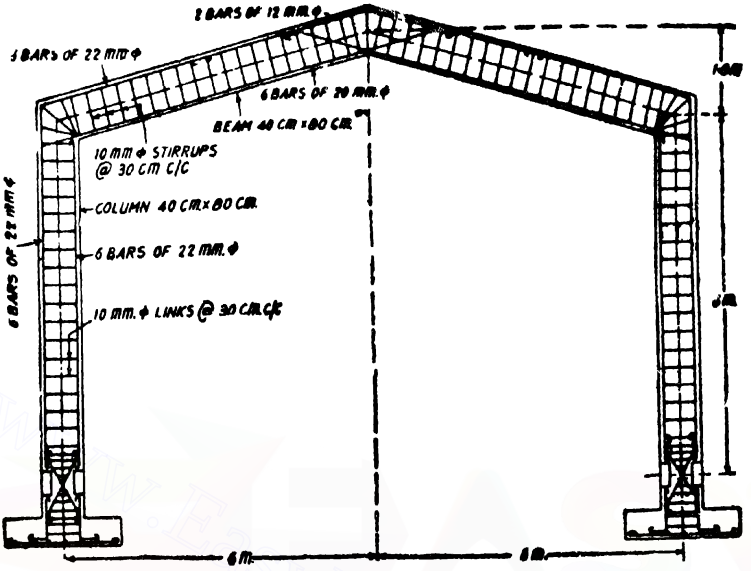

Fig. 639

Stress in concrete

Stress due to B.M.

Let the maximum compressive stress in concrete, due to B.M. be c *kg./cm.*^{2} M.R. = B.M.

$$\therefore \quad 40 \times 25{\cdot}2 \times \frac{c}{2}\left(74{\cdot}9 - \frac{25{\cdot}2}{3}\right) + 17 \times 22{\cdot}81 \times \left(\frac{25{\cdot}2 - 5{\cdot}1}{25{\cdot}2}\right) c \times (74{\cdot}9 - 5{\cdot}1)$$

$$55110\, c = 18{\cdot}69 \times 1000 \times 100$$

$$\therefore \quad c = 33{\cdot}9 \text{ kg /cm.}^2$$

Stress due to direct load

$$= c' = \frac{\text{Direct load}}{\text{Equivalent uncracked concrete area}}$$

$$= \frac{W}{bn + (m-1)A_c + mA_s}$$

$$= \frac{12{\cdot}93 \times 1000}{40 \times 25{\cdot}2 + 17 \times 22{\cdot}81 + 18 \times 22{\cdot}81} \text{ kg /cm.}^2$$

$$= 7{\cdot}2 \text{ kg /cm.}^2$$

$\therefore$ Resultant compressive stress in concrete

$$= 33{\cdot}9 + 7{\cdot}2 = 41{\cdot}1 \text{ kg./cm.}^2$$

Stress in steel

Tensile stress due to B.M.

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$$d - n_{mc} = \frac{(74{\cdot}9 - 25{\cdot}2)}{25{\cdot}2} 18 \times 33{\cdot}9 \ kg./cm.^2$$

$$= 1203 \ kg/cm^2$$

Compressive stress due to direct load $= t' = mc'$

$$= 18 \times 7{\cdot}2 \ kg./cm.^2$$

$$= 129{\cdot}6 \ kg./cm.^2 \text{ say } 130 \ kg./cm.^2$$

$\therefore$ Resultant stress in steel $= 1203 - 130 = 1073 \ kg./cm.^2$

Hinge section and foundation may be designed as in design 150.

§191. Analysis of simple portal frames and continuous beams

Ramamrutham's method

In the usual method of moment distribution the moments are worked out by the principle of successive approximations. This method has some disadvantages. The process may often be lengthy and tedious and the moments arrived at do not represent their absolute values. The following method is devised by the author to get over the above difficulties. The method consists in arriving at the formulae for the final moments. These formulae will be found to be simple for their application.

Fig. 640

Consider a continuous girder $ABCD$.

Let the fixed end moments for the spans be as follows :

$m_{ab} = p$ $\quad m_{bc} = r$ $\quad m_{cd} = t$

$m_{ba} = q$ $\quad m_{cb} = s$ $\quad m_{dc} = u$

Let $m_{ba} + m_{bc} = q + r = m$

and $m_{cb} + m_{cd} = s + t = m'$

m is called the resistant moment at B.

Similarly m' is called the restraint moment at C.

Let $2a$ and $2b$ be the distribution factors for BA and BC at B.
Let $2b'$ and $2a'$ be the distribution factors for CB and CD at C.

Now in terms of m and m' the moment distribution can be carried. This is shown in the table on page 999.

Let M_{ab} be the final moment at A.

Let $M_{ab} = p + C_{ab}$ (See moment table).

where,

$$C_{ab} = -am + ab'm' - ab'bm + abb'^2m' - ab'^2b^2m + ab^2b'^3m' - ab'^3b^2m + ab^3b'^4m' - \ldots$$

$$= -am\{1 + (bb') + (bb')^2 + (bb')^3 + \ldots\} + ab'm\{1 + (bb') + (bb')^2 + (bb')^3 + \ldots\}$$

$$= [-am + ab'm'][1 + (bb') + (bb')^2 + (bb)^3 + \ldots]$$

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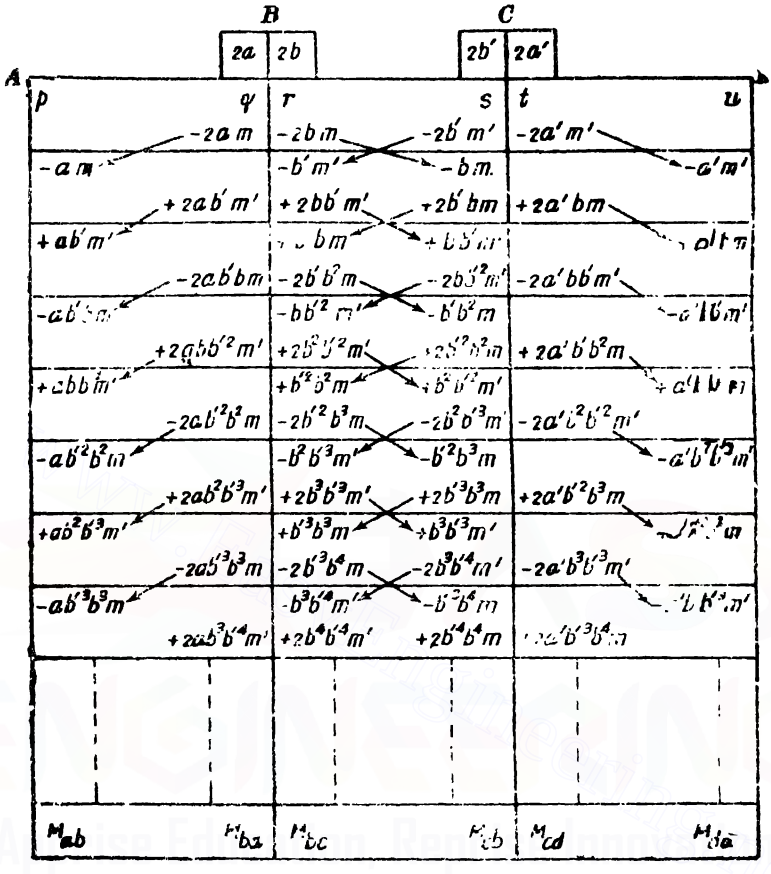

Fig. 641

b and b' being less than unity the product bb' is also less than unity. Hence summing the geometric progression in the above expression for all the infinite terms, we have,

$$C_{ab} = [-am + ab'm'] \left[\frac{1}{1 - bb'}\right]$$

$$\therefore \quad C_{ab} = \frac{am - ab'm'}{bb' - 1}$$

Obviously the final moment at B for the panel BA is given by

$$M_{ba} = q + 2C_{ab}$$

Since $M_{ba} + M_{bc} = 0$ we also know the moment M_{bc}, once M_{ba} is is determined.

Similarly final moment at D for DA is given by

$$M_{da} = u + C_{dc}$$

where

$$C_{dc} = \frac{a'm' - a'bm}{bb' - 1}$$

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and $M_{cd} = t + 2C_{dc}$

The correcting moments C_{ab} and C_{dc} given by the above formulae may be remembered as follows :

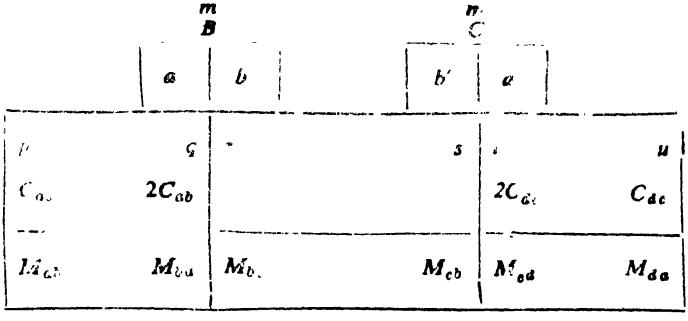

Fig. 642

The fixed end moments p, q, r, s, t, u are entered first. In the compartments at B and C enter one-half the distribution factors, *i.e.*, in our case a and b are entered in the compartments at B and b' and a' are entered in the compartments at C. The quantities $q + r - m$ and $s + t - m'$ are entered at B and C *above* the table. The factors a, b, b' and a' will be called the *correcting factors*. These are just one-half of the usual distribution factors.

The following nomenclature will be followed :

Correcting factors a is called the *left factor*.
Correcting factor a' is called the *right factor*.
The product ab' is called the *left product*.
The product $a'b$ is called the *right product*.
The product bb' is called the *middle product*.

Now

Correcting moment C_{ab}

$$= \frac{am - ab'm'}{bb' - 1}$$

$$\therefore \quad C_{ab} = \frac{\text{left factor} \times m - \text{left product} \times m'}{\text{middle product} - 1}$$

Similarly correcting moment C_{dc}

$$= \frac{a'm' - a\,bm'}{bb' - 1}$$

$$\therefore \quad C_{dc} = \frac{\text{right factor} \times m' - \text{right product} \times m}{\text{middle product} - 1}$$

The formulae given above in the worded form may be easily remembered. By applying the correcting moments as corrections to the initial fixed end moments the final moments are easily determined.

The following examples illustrate the application of this method.

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Design 153. *Determine the support moments at A, B, C, D for the continuous girder shown in Fig. 643.*

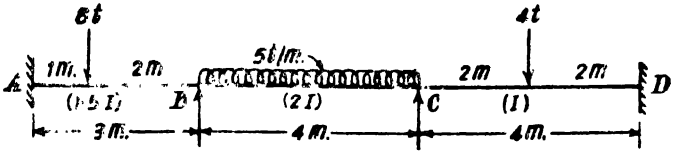

Fig. 643

Solution. Fixed end moments. (Follow Hardy Cross sign conventions, *i.e.*, clockwise moments are positive)

$$m_{ab} = -\frac{8 \times 1 \times 2^2}{3^2} = -3{\cdot}56 \text{ tm.}$$

$$m_{ba} = +\frac{8 \times 1^2 \times 2}{3^2} = +1{\cdot}78 \text{ tm.}$$

$$m_{bc} = -\frac{5 \times 4^2}{12} = -6{\cdot}67 \text{ tm.}$$

$$m_{cb} = +\frac{5 \times 4^2}{12} = +6{\cdot}67 \text{ tm.}$$

$$m_{cd} = -\frac{4 \times 4}{8} = -2{\cdot}00 \text{ tm.}$$

$$m_{dc} = +\frac{4 \times 4}{8} = +2{\cdot}00 \text{ tm.}$$

Correcting factors. These are determined in the following table :

Joint	Member	Relative stiffness	Total stiffness	Distribution factors	Correcting factors
B	BA	$\frac{1{\cdot}5I}{3} = \frac{I}{2}$	I	$\frac{1}{2}$	$\frac{1}{4}$
	BC	$\frac{2I}{4} = \frac{I}{2}$		$\frac{1}{2}$	$\frac{1}{4}$
C	CB	$\frac{2I}{4}$	$\frac{3I}{4}$	$\frac{2}{3}$	$\frac{1}{3}$
	CD	$\frac{I}{4}$		$\frac{1}{3}$	$\frac{1}{6}$

Restraint moment at B

$$= m = m_{ba} + m_{bc}$$
$$= +1{\cdot}78 - 6{\cdot}67 = -4{\cdot}89 \text{ tm.}$$

Restraint moment at C

$$= m' = m_{cb} + m_{cd}$$
$$= +6{\cdot}67 - 2{\cdot}00 = +4{\cdot}67 \text{ tm.}$$

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	A		$m=-4\cdot89$ B			$m'=+4\cdot67$ C		D
			$\frac{1}{4}$	$\frac{1}{4}$		$\frac{1}{3}$	$\frac{1}{6}$	
	$-3\cdot56$	$+1\cdot78$		$-6\cdot67$	$+6\cdot67$		$-2\cdot10$	$+2\cdot00$
	$+1\cdot76$	$+3\cdot52$					$-2\cdot14$	$-1\cdot07$
Final moments	$-1\cdot80$	$+5\cdot30$		$-5\cdot30$	$+4\cdot14$		$-4\cdot14$	$+0\cdot93$

Correcting moment C_{ab}

$$= \frac{\text{left factor} \times m - \text{left product} \times m'}{\text{middle product} - 1}$$

$$= \frac{\frac{1}{4}(-4\cdot89) - \frac{1}{4} \times \frac{1}{3}(4\cdot67)}{\frac{1}{4} \times \frac{1}{3} - 1}$$

$$= +1\cdot758 \text{ say} +1\cdot76 \ tm.$$

Correcting moment C_{ba}

$$= 2 \times C_{ab} = 2 \times 1\cdot758$$

$$= +3\cdot516 \text{ say} +3\cdot52 \ tm.$$

Correcting moment C_{dc}

$$= \frac{\text{right factor} \times m' - \text{right product} \times m}{\text{middle product} - 1}$$

$$= \frac{\frac{1}{6} \times 4\cdot67 - \frac{1}{4} \times \frac{1}{4}(-4\cdot89)}{\frac{1}{4} \times \frac{1}{3} - 1} \ tm.$$

$$= -1\cdot072 \ tm \text{ say} -1\cdot07 \ tm.$$

Correcting moment $C_{cd} = 2C_{dc}$

$$= 2 \times (-1\cdot072) = -2\cdot144 \ tm.$$

$$\text{say} -2\cdot14 \ tm.$$

Applying the correcting moments the final moments are obtained.

Design 154 (S.I.). Determine the support moments at *A, B, C, D* for the continuous girder shown in Fig. 644.

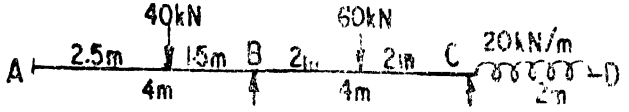

Fig. 644

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Solution. *Fixed end moments*

$$m_{ab} = -\frac{40 \times 2{\cdot}5 \times 1{\cdot}5^2}{4^2} = -14{\cdot}06 \ kNm.$$

$$m_{ba} = +\frac{40 \times 2{\cdot}5^2 \times 1{\cdot}5}{4^2} = +23{\cdot}43 \ kNm.$$

$$m_{bc} = -\frac{60 \times 4}{8} = -30 \ kNm$$

$$m_{cb} = +\frac{60 \times 4}{8} = +30 \ kNm.$$

$$m_{cd} = -\frac{20 \times 2^2}{12} = -6{\cdot}67 \ kNm.$$

$$m_{dc} = +\frac{20 \times 2^2}{12} = +6{\cdot}67 \ kNm.$$

Correcting factors

Let $I_{ab} = 2I$ $\therefore$ $I_{bc} = 2I$ and $I_{cd} = I$

The correcting factors are determined in the table below :

Joint	*Member*	*Relative stiffnes*	*Total relative stiffness*	*Distribution factors*	*Correcting factors*
B	*BA*	$\frac{2I}{4} = \frac{I}{2}$	I	$\frac{1}{2}$	$\frac{1}{4}$
	BC	$\frac{2I}{4} = \frac{I}{2}$		$\frac{1}{2}$	$\frac{1}{4}$
C	*CB*	$\frac{2I}{4} = \frac{I}{2}$	I	$\frac{1}{2}$	$\frac{1}{4}$
	CD	$\frac{I}{2}$		$\frac{1}{2}$	$\frac{1}{4}$

Restraint moment at $B = m = +23{\cdot}43 - 30{\cdot}00 = 6{\cdot}57 \ kNm.$

Restraint moment at $C = m' = +30{\cdot}00 - 6{\cdot}67 = +23{\cdot}33 \ kNm.$

		$m = -6{\cdot}57$		$m' = +23{\cdot}33$		
		B		*C*		
		$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	
A						*D*
	$-14{\cdot}06$	$+23{\cdot}43$	$-30{\cdot}00$	$+30{\cdot}00$	$-6{\cdot}67$	$+6{\cdot}67$
	$+3{\cdot}31$	$+6{\cdot}61$			$-13{\cdot}32$	$-6{\cdot}66$
Final moments	$-10{\cdot}75$	$+30{\cdot}04$	$-30{\cdot}04$	$+19{\cdot}99$	$-19{\cdot}99$	$+0{\cdot}01$

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$$\text{Correcting moment } C_{ab} = \frac{\text{left factor} \times m. - \text{left product} \times m'}{\text{middle product} - 1}$$

$$= \frac{\frac{1}{2}(-6{\cdot}57) - \frac{1}{2} \cdot \frac{1}{2}(+23{\cdot}33)}{\frac{1}{2} \cdot \frac{1}{2} - 1}$$

$$= +3{\cdot}307 \text{ } kNm \text{ say} + 3{\cdot}31 \text{ } kNm.$$

$$\text{Correcting moment } C_{ba} = 2\ C_{ab} = 2 \times 3{\cdot}307 = +6{\cdot}614 \text{ } kNm.$$

$$\text{Correcting moment } C_{cd} = \frac{\text{right factor} \times m' - \text{right product} \times m}{\text{middle product} - 1}$$

$$= \frac{\frac{1}{2}(+23{\cdot}33) - \frac{1}{2} \cdot \frac{1}{2}(-6{\cdot}57)}{\frac{1}{2} \cdot \frac{1}{2} - 1}$$

$$= -6{\cdot}659 \text{ } kNm. \text{ say} -6{\cdot}66 \text{ } kNm.$$

$$\text{Correcting moment } C_{ed} = 2\ C_{dc} = 2(-6{\cdot}659) = -13{\cdot}318 \text{ } kNm.$$

$$= -13{\cdot}32 \text{ } kNm.$$

Applying the correcting moments the final moments are obtained.

Continuous girder with a hinged end. In the example solved above both ends of the continuous girder were fixed. Even when one end or both ends of the girder should be hinged, the analysis can be made easily.

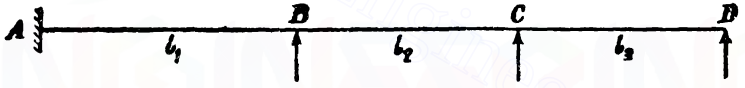

Fig. 645

Suppose the end D of the girder $ABCD$ is hinged.

The following points should be noted :

(*i*) As in the moment distribution method the hinged end should be first corrected.

(*ii*) Relative stiffness of CD should be taken as $\frac{3}{4}\frac{I_{cd}}{l_3}$

(*iii*) Correcting moments C_{ab} and C_{dc} are determined by the formulae derived earlier.

(*iv*) Correction C_{ab} is applied at A ; correction 2 C_{ab} is applied at B for the panel BA ; correction C_{dc} is not applied at D; correction 2 C_{dc} should be applied at C for the panel CD.

The following example illustrates the method for such a continuous girder.

Design 155. *Determine the support moments for the continuous girder shown in Fig. 646.*

Solution. Fixed end moments

$$m_{ab} = -\frac{3 \times 4^2}{12} = -4{\cdot}00 \text{ } tm.$$

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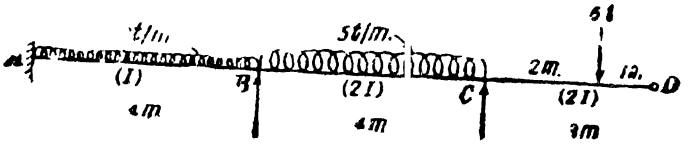

Fig. 646

$$m_{ba} = +4{\cdot}00 \ tm.$$

$$m_{bc} = -\frac{6 \times 4^2}{12} = -8{\cdot}00 \ tm.$$

$$m_{cb} = +8{\cdot}00 \ tm.$$

$$m_{cd} = -\frac{6 \times 2 \times 1^2}{3^2} = -1{\cdot}33 \ tm.$$

$$m_{dc} = +\frac{6 \times 2^2 \times 1}{3^2} = +2{\cdot}67 \ tm.$$

Correcting factors. These have been determined in the following table :

Joint	Member	Relative stiffness	Total stiffness	Distribution factors	Correcting factors
B	BA	$\frac{I}{4}$	$\frac{3I}{4}$	$\frac{1}{3}$	$\frac{1}{6}$
	BC	$\frac{2I}{4}$		$\frac{2}{3}$	$\frac{1}{3}$
C	CB	$\frac{2I}{4} = \frac{I}{2}$	I	$\frac{1}{2}$	$\frac{1}{4}$
	CD	$\frac{3}{4} \cdot \frac{2I}{3} = \frac{I}{2}$		$\frac{1}{2}$	$\frac{1}{4}$

The fixed end moments are entered in the moment table as shown. The end D is corrected so as to make the B M. at D zero by applying a correction $-2{\cdot}67 \ tm.$ Obviously the carry over moment $-\frac{2{\cdot}67}{2} = -1{\cdot}34 \ tm.$ is transferred to the end C. The moments at this stage are added. The restraint moments m and m' at B and C are computed with these moments.

Restraint moment at B : $m = +4{\cdot}00 - 8{\cdot}00 = -4{\cdot}00 \ tm.$

Restraint moment at C : $m' = +8{\cdot}00 - 2{\cdot}67 = +5{\cdot}33 \ tm.$

Now the correcting moments are computed

$$C_{ab} = \frac{\text{left factor} \times m - \text{left product} \times m'}{\text{middle product} - 1}$$

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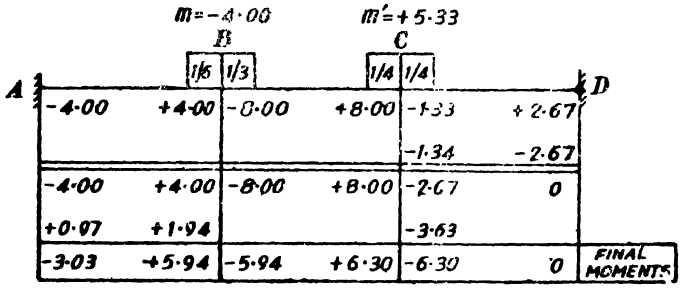

Fig. 647

$$= \frac{\frac{1}{6}(-4{\cdot}00) - \frac{1}{6} \times \frac{1}{4}(5{\cdot}33)}{\frac{1}{3} \times \frac{1}{4} - 1} \; tm.$$

$$= +0{\cdot}9695 \; tm. \text{ say } +0{\cdot}97 \; tm.$$

$$C_{ba} = 2\, C_{ab} = 2 \times 0{\cdot}9695$$

$$= +1{\cdot}9390 \; tm \text{ say } +1{\cdot}94 \; tm.$$

$$C_{dc} = \frac{\text{right factor} \times m' - \text{right product} \times m}{\text{middle product} - 1}$$

$$= \frac{\frac{1}{4}(5{\cdot}33) - \frac{1}{4} \times \frac{1}{3}(-4{\cdot}00)}{\frac{1}{3} \times \frac{1}{4} - 1} \; tm.$$

$$= -1{\cdot}817 \; tm. \text{ say } -1{\cdot}82 \; tm.$$

$$C_{cd} = 2C_{dc} = 1(-1{\cdot}817) = -3{\cdot}634 \; tm$$

$$\text{say } -3{\cdot}63 \; tm.$$

Of these correcting moments, the correcting moment C_{dc} should not be applied. The correcting moments C_{ab} C_{ba} and C_{cd} are applied and the final moments are determined as shown in the moment table.

Continuous beam with sinking supports. For this case also the method explained above can be applied. The additional fixed end moments due to sinking of supports are added to the corresponding fixed end moments due to external loads. The correcting moments are applied as usual.

Portal frames with two columns and a horizontal beam. These frames are similar to continuous girders with three spans. Hence the analysis will be similar to the analysis of continuous beams. The effect of the sway, if any, should also be considered and the sway moments may also be determined by the same principle. The following two problems illustrate the application of the method.

Design 156. *Determine the moments at A, B, C and D for the portal frame ABCD shown in Fig. 648 and hence find the reactions at the supports. All members have the same flexural rigidity.*

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Solution. Fixed end moments

$m_{ab} = m_{ba} = m_{cd} = m_{dc} = 0$

$m_{bc} = -\frac{6 \times 8^2}{12} = -32.00$ *tm.*

$m_{cb} = +32$ *tm.*

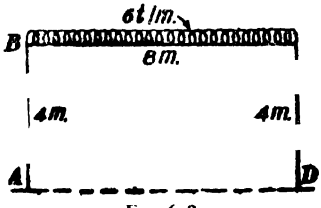

Fig. 648

Correcting factors. These have been computed in the following table :

Joint	Member	Relative stiffness	Total stiffness	Distribution factors	Correcting factors
B	BA	$\frac{I}{4} = \frac{2I}{8}$	$\frac{3I}{8}$	$\frac{2}{3}$	$\frac{1}{3}$
	BC	$\frac{I}{8}$		$\frac{1}{3}$	$\frac{1}{6}$
C	CB	$\frac{I}{8}$	$\frac{3I}{8}$	$\frac{1}{3}$	$\frac{1}{6}$
	CD	$\frac{I}{4} = \frac{2I}{8}$		$\frac{2}{3}$	$\frac{1}{3}$

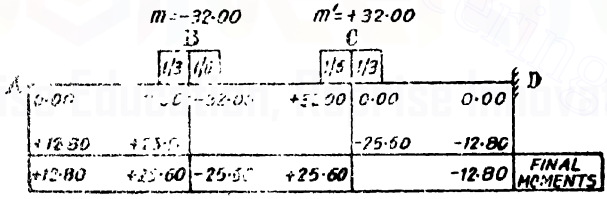

Fig. 649

Restraint moment at $B = 0 - 32{\cdot}00 = -32{\cdot}00$ *tm.*

Restraint moment at $C = +32{\cdot}00 + 0 = +32{\cdot}00$ *tm.*

$$C_{ab} = \frac{\frac{1}{3}(-32) - \frac{1}{18}(+32)}{\frac{1}{36} - 1}$$

$$= +12{\cdot}80 \text{ tm.}$$

$$C_{ba} = 2\,C_{ab} = 2 \times 12{\cdot}80 = 25{\cdot}6 \text{ tm.}$$

$$C_{dc} = \frac{\frac{1}{3}(32) - \frac{1}{18}(-32)}{\frac{1}{36} - 1} \text{ tm.} = -12{\cdot}80 \text{ tm.}$$

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$C_{cd} = 2\ C_{de} = 2(-12{\cdot}8) = -25{\cdot}6\ tm.$

Applying these correcting moments to the fixed end moments the final moments are obtained.

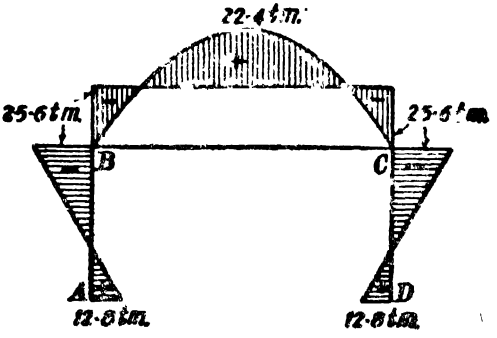

Fig. 650. B.M. diagram for the frame.

Each vertical reaction

$$= \frac{6 \times 8}{2} = 24\ t \uparrow$$

Horizontal reaction at $A = H_a$

$$= \frac{M_{ab} + M_{ba} + \text{moment of forces on } AB \text{ about } B}{\text{Vertical height } AB}$$

$$= \frac{+12{\cdot}80 + 25{\cdot}60}{4}\ tonnes = +9{\cdot}6\ t \rightarrow$$

Similarly $H_d = -9{\cdot}6t \leftarrow$.

Design 157. *Determine the support moments for the portal frame shown in Fig. 651.*

Solution. Fixed end moments

$$m_{ab} = -\frac{6 \times 4}{8}$$

$$= -3{\cdot}00\ tm.$$

$$m_{ba} = +3{\cdot}00\ tm.$$

$$m_{bc} = -\frac{4 \times 4^2}{12}$$

$$= -5{\cdot}33\ tm.$$

$$m_{cb} = +5{\cdot}33\ tm.$$

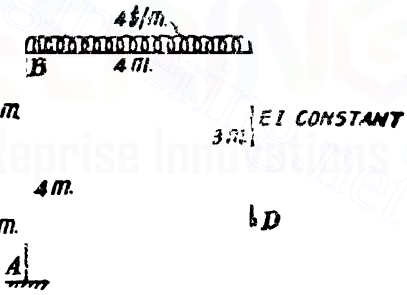

Fig. 651

Correcting factors. These have been computed in the table shown on page 1009.

Ignoring the sway of the frame the moments are first determined.

$$C_{ab} = -\frac{\frac{1}{4}(-2{\cdot}33) - \frac{1}{16}(5{\cdot}33)}{\frac{1}{16} - 1}\ tm.$$

$$= +0{\cdot}977\ tm.\ \text{say} + 0{\cdot}98\ tm.$$

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Joint	Member	Relative stiffness	Tota stiffness	Distribution factors	Correcting factors
B	BA	$\frac{I}{4}$		$\frac{1}{2}$	$\frac{1}{4}$
	BC	$\frac{I}{4}$	$\frac{2I}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
C	CB	$\frac{I}{4}$		$\frac{1}{2}$	$\frac{1}{4}$
	CD	$\frac{3}{4} \cdot \frac{I}{3} = \frac{I}{4}$	$\frac{2I}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

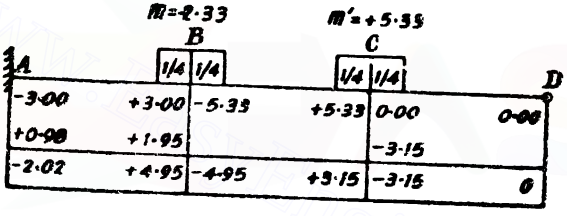


Fig. 652.

$$C_{ba} = +0{\cdot}977 \times 2 = +1{\cdot}954 \text{ tm.}$$

say $+1{\cdot}95$ tm.

$$C_{da} = \frac{\frac{1}{4}(5{\cdot}33) - \frac{1}{16}(-2{\cdot}33)}{1 \quad -1} \text{ tm.}$$

$$= -1{\cdot}577 \text{ tm. say } -1{\cdot}58 \text{ tm.}$$

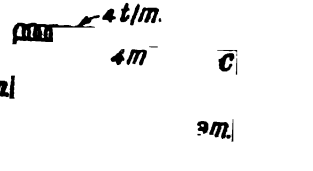
$$C_{cd} = -1{\cdot}577 \times 2 = -3{\cdot}154 \text{ tm.}$$

say $-3{\cdot}15$ tm.

Horizontal reaction at A

$$= -\frac{-2{\cdot}02 + 4{\cdot}95 - 6 \times 2}{4}$$

$$= -2{\cdot}2675 \text{ t} \leftarrow\!-\!-\!-$$



Horizontal reaction at D

$$= -\frac{-3{\cdot}15 + 0}{3}$$

$$= -1{\cdot}05 \text{ t} \leftarrow\!-\!-$$

Fig .653

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Horizontal Force acting left to right
$= 6{\cdot}0000\ t$

Horizontal Forces acting right to left
$= 2{\cdot}2675 + 1{\cdot}05 = 3{\cdot}3175$

Sway force $= 2{\cdot}6825\ t \longrightarrow$

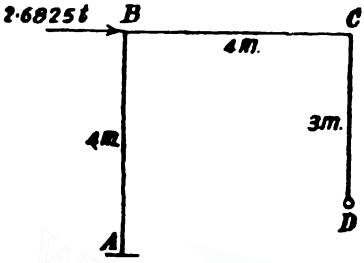

Fig. 654

Sway analysis. Ratio of initial equivalent moments at B and C for the panels BA and CD

$$= \frac{2I_1}{l_1^2} : \frac{I_2}{l_2^2}$$

$$= \frac{2I}{16} : \frac{I}{9}$$

$$= \frac{I}{8} : \frac{I}{9}$$

say 90 : 80.

With these arbitrary moments, the moments at the supports are determined.

$m = -90{\cdot}00$ $\quad$ $m' = -80{\cdot}00$

	B			C		
	1/4	1/4		1/4	1/4	
A						D
-90·00	-90·00	0·00		0·00	-80·00	0
	+18·67	+37·33			+30·67	
-71·33	-52·67	+52·67		+49·33	-49·33	COL x

Fig. 665

$$C_{ab} = \frac{\frac{1}{4}(-90) - \frac{1}{16}(-80)}{\frac{1}{16} - 1}\ tm$$

$$= +18{\cdot}667\ tm.\ \text{say}\ +18{\cdot}67\ tm.$$

$$C_{ba} = 2 \times 18{\cdot}667 = +37{\cdot}334\ tm.$$
say $+37{\cdot}33\ tm.$

$$C_{da} = \frac{\frac{1}{4}(-80) - \frac{1}{16}(-90)}{\frac{1}{16} - 1}\ tm.$$

$$= +15{\cdot}333\ tm.$$
say $+15{\cdot}33\ tm.$

$$C_{cd} = 2 \times 15{\cdot}333 = 30{\cdot}666\ tm.$$
say $+30{\cdot}67\ tm.$

Let the moments arrived at with arbitrary initial moments be due to a sway force of S.

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Horizontal thrust at A

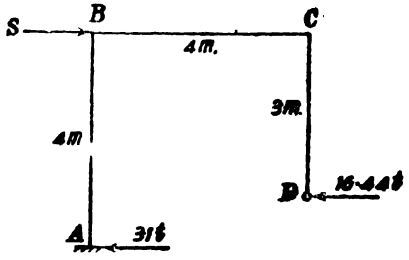

Fig. 656

$$= -71{\cdot}33 \times 52{\cdot}57$$

$$= -31\ t \leftarrow$$

Horizontal thrust at D

$$= -\frac{49{\cdot}30+0}{3}$$

$$= -16{\cdot}44\ t \leftarrow$$

Resolving the forces on frame horizontally, we get,

$$S - 16{\cdot}44 + 31$$

$$= 47{\cdot}44\ t$$

For a sway force of 47·44 t the sway moments are as shown in the table above. The actual sway force being 2·6825 t the corresponding sway moments are $\frac{2{\cdot}6825}{47{\cdot}44}$ times the sway moments shown above. Thus the actual sway moments may be determined. These moments should be added on to the moments obtained neglecting the sway of the frame in order to obtain the final moments.

	A	B	C	D
Column x	71·33—52·47	+52·67 +49·33	−49·33	0
$\frac{2{\cdot}6825}{47{\cdot}44}$ × col. x	−4·03 − 2·98	+2·38 +2·79	−2·79	0
Non-sway moments	−2·02 +4·95	−4·95 +3·15	−3·15	0
Final moments	−6·05 + 1·97	1·97 +5·94	−5·94	0

Reactions. Horizontal thrust at A

$$= \frac{-6{\cdot}05+1{\cdot}97-6\times 2}{4}\ t$$

$$= -4{\cdot}02\ t \leftarrow$$

Horizontal thrust at D

$$= \frac{-5{\cdot}94+0}{3}$$

$$= -1{\cdot}98\ t \leftarrow$$

Forces acting left to right

$$= 6{\cdot}00\ t$$

Forces acting right to left

$$= 1{\cdot}98 + 4{\cdot}02 = 6{\cdot}00\ t$$

Vertical reaction at A

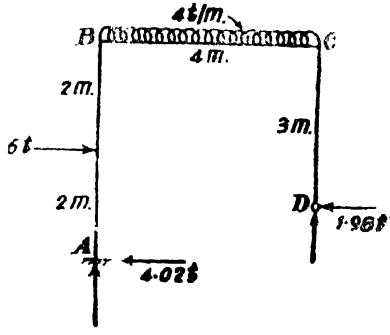

Fig. 657

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$$\therefore V_{BC} = -\frac{M_{bc} + M_{cb} + \text{moment of forces on } BC \text{ about } C}{\text{span } BC}$$

$$= -\frac{-1{\cdot}97 + 5{\cdot}94 - 4 \times 4 \times 2}{4} \text{ tonnes}$$

$$= +7{\cdot}0075\ t \quad \text{say } 7{\cdot}01\ t \uparrow$$

$$V_d = 4 \times 4 - 7{\cdot}01 = +8{\cdot}99\ t \uparrow$$

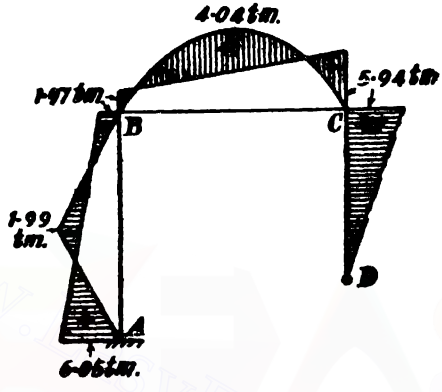

Fig. 658. B.M. Diagram for the frame

Four span continuous beam. A four span continuous beam can also be analysed in a similar manner.

Fig. 659

Let $ABCDE$ be a continuous girder consisting of four spans AB, BC, CD and DE. Considering each span as a separate fixed beam, let the fixed end moments for the various spans be as follows :

$m_{ab} = p$ $\quad m_{bc} = x$ $\quad m_{cd} = t$ $\quad m_{de} = v$

$m_{ba} = q$ $\quad m_{cb} = s$ $\quad m_{dc} = u$ $\quad m_{ed} = w$

Let the restraint moments at B, C and D be M, N and Q respectively

Let the distribution factors at B be $2a$, $2b$

Distribution factors at C be $2c$, $2d$

Distribution factors at D be $2e$, $2f$

Let also $bM + eQ = R$ and $bc + de = r$

In terms of quantities introduced above the moment dis-

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tribution has been carried out in the following table, in the usual manner.

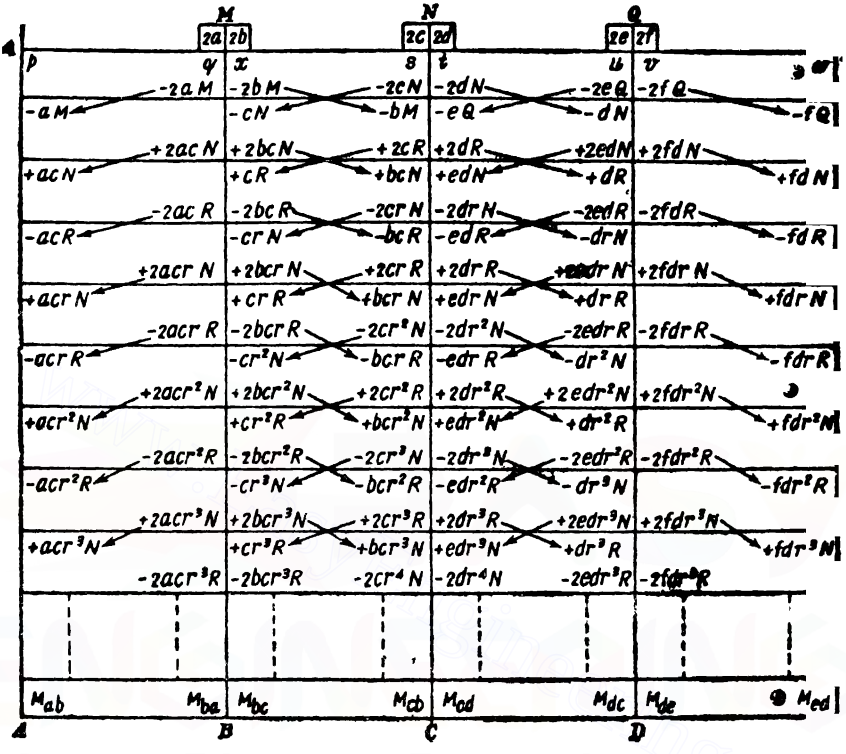

Fig. 660

Now the final moments can be determined by the summation of the series of terms appearing at each end of each panel.

For instance the final moment at A for the panel AB is given by

$$M_{ab} = p + C_{ab}$$

where

$$\begin{aligned} C_{ab} &= -aM + acN - acR + acrn - acrR \\ &\quad + acr^2N - acr^2R + acr^3N \ldots\ldots \\ &= -aM + acN(1 + r + r^2 + r^3 + \ldots\ldots) \\ &\quad - acR(1 + r + r^2 + r^3 + \ldots\ldots) \end{aligned}$$

Since r is less than unity the geometric series

$$1 + r + r^2 + r^3 + \ldots\ldots \text{upto infinite terms}$$

$$= \frac{1}{1-r}$$

$$\therefore \quad C_{ab} = -aM + \frac{(acN - acR)}{1-r}$$

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$$= \frac{-aM + aMr + acN - acR}{1-r}$$

$$= \frac{aM - aMr - acN + acR}{r-1}$$

Putting $R = bM + eQ$

and $r = bc + de$

$$C_{ab} = \frac{aM - aM(bc+de) - acN + ac(bM+eQ)}{r-1}$$

$$= \frac{aM - acN + aceQ - adeM}{r-1}$$

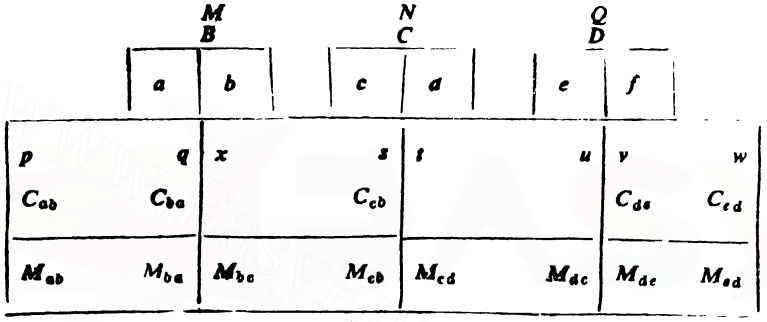

Fig. 661

The formula obtained above can be remembered in a worded form.

Let (a, b), (c, d), (e, f) be the correcting factors at the joints B, C and D.

The following nomenclature will be adopted :

(first) left factor

ac product of first two left factors

ace product of first three left factors

f : (first) right factor

fd : product of first two right factors

fdb : product of first three right factors

bc : left middle product

de : right middle product

$bc + de$: Σ middle product.

$$\therefore \quad C_{ab} = \frac{1}{\Sigma \text{ Mid-product} - 1}\Big[\text{left factor} \times M - \text{product of first two left factors} \times N + \text{product of first three left factors} \times Q - \text{left factor} \times \text{right middle product} \times M\Big]$$

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End moment $M_{ab} = w + C_{ab}$

Similarly the correcting moment C_{ab} is given by,

$$C_{ab} = \frac{fQ - fdN + fdbM - fbcQ}{r-1}$$

C_{ab}

$$= \frac{1}{\Sigma \text{ Mid-product} - 1}\Big[\text{right factor} \times Q - \text{product of first two right factors} \times N + \text{product of first three right factors} \times M - \text{right factor} \times \text{left middle product} \times Q\Big]$$

End moment $M_{ba} = q + C_{ba}$

$$C_{ba} = 2\,C_{ab}$$

End moment $M_{ba} = -M_{ba}$

End moment M_{cb}. $M_{cb} = s + C_{cb}$

In the moment distribution table formed studying the series for C_{ab} and C_{ba}, we have,

$$C_{cb} = [-bM + bcN - bcR + bcrN - bcrR + bcr^2N - bcr^2R + bcr^3N] + [-2cN + 2cR - 2crN + 2crR - 2cr^2N + 2cr^2R - 2cr^3N + 2cr^3R - 2cr^4N \ldots]$$

$$C_{cb} = \frac{b}{a} C_{ab} - \frac{C_{ba}}{a} - 2M$$

$$= \frac{b}{a} C_{ab} - \frac{2C_{ab}}{a} - 2M$$

$$C_{ab} = \left[\frac{b-2}{a}\right] C_{ab} - 2M$$

The formulae obtained are written below :

$$C_{ab} = \frac{aM - acN + aceQ - adeM}{r-1}$$

$$C_{ba} = 2C_{ab}$$

$$C_{cb} = \left[\frac{b-2}{a}\right] C_{ab} - 2M$$

$$C_{de} = 2C_{ed}$$

$$C_{ed} = \frac{fQ - fdN + fdbM - fbcQ}{r-1}$$

Studying the formulae in the worded form, the above formulae be easily written. The following example illustrates the use of the above formulae.

Design 158. *Find the end moments of the various spans of the continuous beam shown in Fig. 662.*

Solution. Fixed end moments

$$m_{ab} = -\frac{2 \times 4^2}{12} = -2{\cdot}67 \text{ tm.}$$

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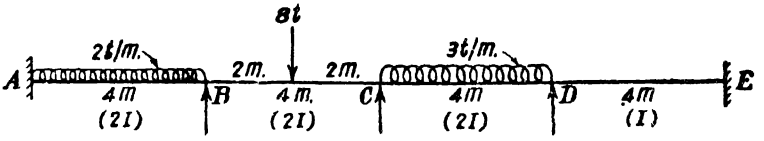

Fig. 662

$$m_{ba} = +2{\cdot}67 \ tm.$$

$$m_{bc} = -\frac{8 \times 4}{8} = -4{\cdot}00 \ tm.$$

$$m_{cb} = +4{\cdot}00 \ tm.$$

$$m_{cd} = -\frac{3 \times 4^2}{12} = -4{\cdot}00 \ tm.$$

$$m_{de} = +4{\cdot}00 \ tm.$$

$$m_{da} = 0 = m_{ed}.$$

Correcting factors. These are computed in the following table :

Joint	Member	Relative stiffness	Total stiffness	Distribution factors	Correcting factors
B	BA	$\frac{2I}{4} = \frac{I}{2}$	I	$\frac{1}{2}$	$\frac{1}{4}$
	BC	$\frac{2I}{4} = \frac{I}{2}$		$\frac{1}{2}$	$\frac{1}{4}$
C	CB	$\frac{I}{2}$	I	$\frac{1}{2}$	$\frac{1}{4}$
	CD	$\frac{I}{2}$		$\frac{1}{2}$	$\frac{1}{4}$
D	DC	$\frac{2I}{4}$	$\frac{3I}{4}$	$\frac{2}{3}$	$\frac{1}{3}$
	DE	$\frac{I}{4}$		$\frac{1}{3}$	$\frac{1}{6}$

In our case $a = \frac{1}{2}$, $b = \frac{1}{2}$, $c = \frac{1}{2}$, $d = \frac{1}{2}$, $e = \frac{1}{3}$, $f = \frac{1}{3}$

$$r = bc + de = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{3} = \frac{7}{12}$$

$$C_{ab} = \frac{\frac{1}{2}(-1{\cdot}33) - \frac{1}{2} \times \frac{1}{2}(0{\cdot}00) + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3}(4{\cdot}00) - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{3}(-1{\cdot}33)}{\frac{7}{48} - 1}$$

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		M=-1·33		N= 0·00		Q=+4·00		
	A	B		C		D		E
		1/4	1/4	1/4	1/4	1/3	1/6	
	-2·67	+2·67	-4·00	+4·00	-4·00	+4·00	0·00	0·00
	+0·26	+0·52		+0·85			-1·43	-0·72
FINAL MOMENTS	-2·41	+3·19	-3·19	+4·85	-4·85	+1·43	-1·43	-0·72

Fig. 663

$= +0{\cdot}2592$ *tm.* say $0{\cdot}26$ *tm.*

$C_{ba} = 2C_{ab} = 2 \times 0{\cdot}2592 = +0{\cdot}5184$ *tm.* say $+0{\cdot}52$ *tm.*

$$C_{ab} = \left[\frac{b-2}{a}\right] C_{ab} - 2M$$

$$= \frac{\frac{1}{4}-2}{\frac{1}{4}} (0{\cdot}2592) - 2 \times (-1{\cdot}33) \text{ tm.}$$

$= +0{\cdot}8456$ *tm.* say $+0{\cdot}85$ *tm.*

$$C_{cd} = \frac{\frac{1}{8}(4{\cdot}00) - \frac{1}{8} \times \frac{1}{4}(0{\cdot}00) + \frac{1}{8} \times \frac{1}{4} \times \frac{1}{4}(1{\cdot}33) - \frac{1}{8} \times \frac{1}{4} \times \frac{1}{4}(+4{\cdot}00)}{\frac{7}{48} - 1}$$

$= -0{\cdot}715$ *tm.*

$C_{dc} = 2(-0{\cdot}715) = -1{\cdot}430$ *tm.*

Applying these correcting moments at the appropriate ends of panels the final moments are obtained.

§192. Analysis of Multi-storeyed frames

A multi-storeyed frame is a complicated statically indeterminate structure. The exact analysis by the moment distribution method is very lengthy and difficult It is proposed to discuss some of the methods adopted in the analysis of multi-storeyed frames.

§193. Method of substitute frames (*Two cycle method*)

In this method only a part of the frame is considered for the analysis. The part considered is called a *substitute frame.* The moments for each floor are separately computed. It will be assumed that the moments transferred from one floor to another are small. Each floor will be taken as connected to columns above and below with their far ends fixed. The frame taken this way is analysed for the moments and shears in the beams and columns.

Consider the multi-storeyed frame shown in Fig. 664.

Generally it is sufficient to consider two adjacent spans on each side of the joint considered

Suppose we consider the second floor of the frame.

For the second floor the manner of loading for maximum positive and negative bending moments at *G*, and for the maximum moment at *A* are shown in Figs. 665, 666 and 667.

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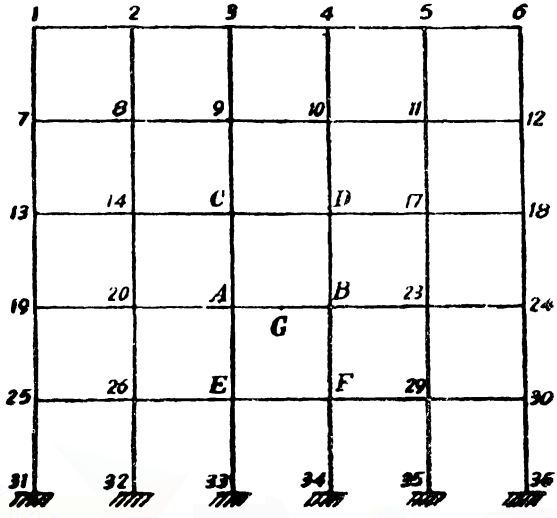


Fig 664

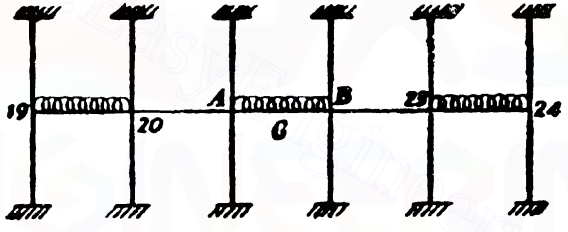
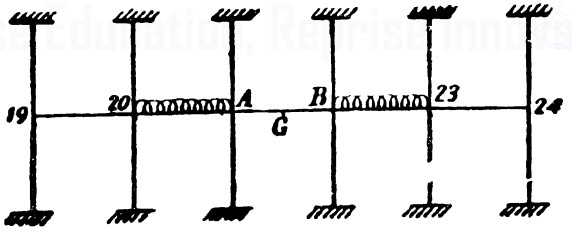


Fig. 665. Loading for maximum positive B M. at G



ig. 666. Loading for maximum negative B.M. at G.

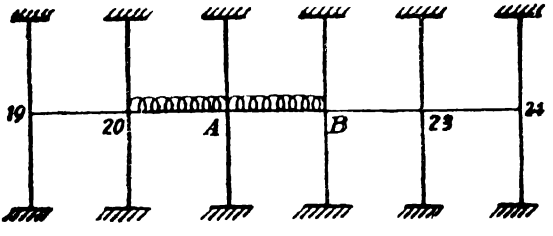


Fig. 667. Loading for maximum negative B.M. at A.

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The columns will carry the maximum bending moment when any one series of alternate spans should be loaded.

The moment distribution for the substitute frame is performed only for two cycles and hence, the method is sometimes referred to, as the *two cycle method.*

Design 159. *Fig. 668 shows an intermediate frame of a multi-storeyed structure. If the spacing of the frames be 3·5 metres, analyse the frame if the dead load and the live load of the slab are 350 kg./metre² and 450 kg /metre². Self-weight of beams may be taken at the following values :*

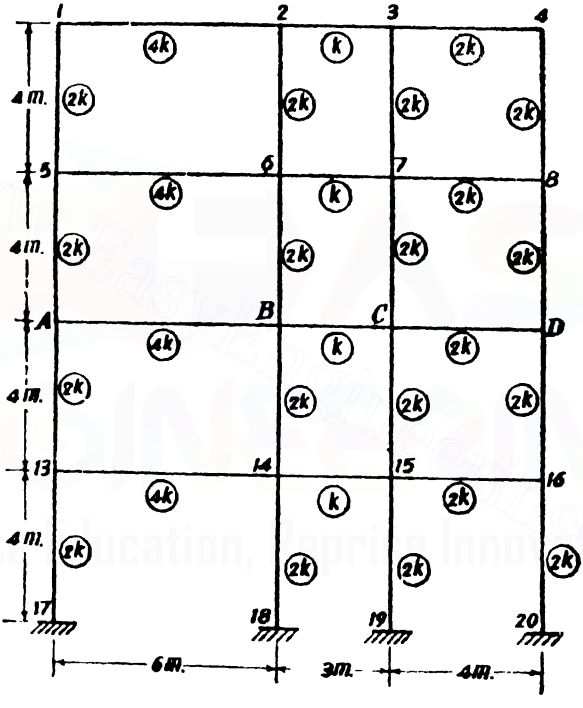

Fig. 668

Beams of span 6 metres = 450 kg /m.
Beams of span 3 metres = 300 kg /m.
Beams of span 4 metres = 400 kg /m.
The relative stiffness of the various members are marked in the figure.

Solution It is proposed to analyse the second floor beams. The other floor beams may also be analysed similarly.

Loads Dead load of floor per *metre* run of the girder
$= 350 \times 3.5 = 1225$ *kg /m*
Live load $= 450 \times 3.5 = 1575$ *kg /m.*

Fixed end comments. The fixed end moments due to dead and live load are tabulated below :

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Member	*D.L. per metre (kg /m.)*	*Live load per metre (kg./m.)*	*F.E M. due to Dead load (kg. m.)*	*F.E.M. due to Dead and Live load (kg. m.)*
AB	1675	1575	5025	9750
BC	1525	1575	1144	2325
CD	1625	1575	2167	4267

Fig. 669 shows the substitute frame to be considered.
This frame will be analysed by *two cycle* moment distribution.

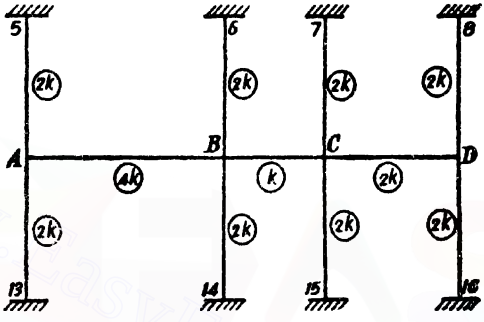

Fig. 669

Condition for Maximum Negative Moments at the Various Joints

Joint A. Loading : D.L. on *ABCD*
L.L. on *AB*

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
Distribution factors	1/2	4/9	1/9	1/7	2/7	1/3
(*i*) FEM due to D.L.			−1144	+1144		
(*ii*) FEM due to total load	−9750	+9750				
(*iii*) Distribution +carry over	−1912					
	−11662					
(*iv*) Distribution	+5831					
(*v*) Net B.M. at *A*	−5831					

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Joint B. D.L. on *ABCD*
L.L. on *ABC*

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
Distribution factors	1/2	4/9	1/9	1/7	2/7	1/3
(*i*) FEM due to D.L.					−2167	
(*ii*) FEM due to total load	−9750	+9750	−2325	+2325		
(*iii*) Distribution +carry over		+2438	−11			
		+1218	−2336			
(*iv*) Distribution		−4379	−1095			
(*v*) Net moment at B		+7809	−3431			

Joint C. D.L. on *ABCD*
L.L. on *BCD*

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
Distribution factors	1/2	4/9	1/9	1/7	2/7	1/3
(*i*) FEM due to D.L.	−5025	+5025				
(*ii*) FEM due to total load			−2325	+2325	−4267	+4267
(*iii*) Distribution +carry over				−150	−711	
				+2175	−4978	
(*iv*) Distribution				+401	+802	
(*v*) Net moment at C				+2576	+4176	

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Joint D. D.L. on *ABCD*
LL on *CD*

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
Distribution factors	1/2	4/9	1/9	1/9	2/7	1/3
(*i*) FEM due to D.L.				−1144	+1144	
(*ii*) FEM due to total load					−4277	+ 267
(*iii*) Distribution +carry over						+ 446
						+4713
(*iv*) Distribution						−1571
(*v*) Net moment at D						+3142

Explanation for the two cycle moment distribution. When it ir required to find the maximum negatjve moment at a joint, then the spans meeting at the joint are loaded with dead and live load. The other spans are loaded with dead load alone.

For instance suppose we require the maximum negative moment at *A*.

The span *AB* is loaded with dead and live load while the span *BC* is loaded with dead load alone. Effect of loading on span *CD* is ignored Distribution is done at joint *B* and the carryover effect is transferred to the joint *A*. Now after adding the moments at *A* the distribution process is done at *A*, and the final moment at *A* is obtained. (Distribution at the joint *B* will not be recorded in the table).

Similarly for the maximum negative B.M. at *B*, the spans *BA* and *BC* meeting at *B* are loaded with dead and live load. The span *CD* is loaded with dead load alone. Distribution is done at the joints *A* and *C* and the corresponding carryover moments are transferred to the joint *B*. Now again the distribution process is done at *B* and the final moments are obtained. (Distribution at the joints *A* and *C* will not be recorded in the table).

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Condition for Maximum Positive B.M. at Mid spans

Mid span of AB: D.L. on *ABCD*
L.L. on *AB* and *CD*

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
Distribution factors	1/2	4/9	1/9	1/7	2/7	1/3
(*i*) FEM due to D.L.			1144	+1144		
(*ii*) FEM due to total load	− 9750	+9750			−4267	+4267
iii) Distribution moment	+ 4875	−3825		+446		
(*iv*) Carry over moment	− 1913	+2438	+223			
(*v*) Distribution	+956	−1183				
Total	· 5832	+7180				

Free B.M. at the centre of span *AB*

$$\frac{wl^2}{8}$$

$$=(1675+1575)\times \frac{6^2}{8}\ kg.\ m.=14625\ kg.\ m.$$

Net B.M. at the centre of span *AB*

$$=4625-\frac{5832+7180}{2}\ kg\ m.=+8119\ kg\ m.$$

Explanation of the two cycle distribution carried out. For maximum positive B.M. at the mid span of *AB* the spans *AB* and *CD* will be loaded with dead and live load while the span *BC* will be loaded with dead load alone. Now the distribution process is performed at joints *A*, *B* and *C*. These moments are carried over from *A* to *B* and *B* to *A* and from *C* to *B*. Now agai the distribution process is performed. The moments entered are all added and the final moments at *A* and *B* for the span *AB* are determined.

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Mid span of BC : D.L. on *ABCD* L.L. on *CD*

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
Distribution factors	1/2	4/9	1/9	1/7	2/7	1/3
(*i*) FEM due to D.L.	−5025	+5025			−2167	+2167
(*ii*) FEM due to total load			−2325	+2325		
(*iii*) Distribution moments	+2513		−300	−23		−722
(*iv*) Carry over moments		+1257	−12	−150	−361	
(*v*) Distribution			−178	+73		
Total			−2815	+222		

Free B.M. at the centre of span *BC*

$$= \frac{wl^2}{8} = (1525+1575) \times \frac{3^2}{8} \text{ kg. m.}$$

$$= 3488 \text{ kg m.}$$

∴ Net B.M. at the centre of span *BC*

$$= 3488 - \frac{2815+2225}{2} \text{ kg. m.} = 968 \text{ kg. m.}$$

Explanation of the two cycle distribution performed. For maximum positive bending moment at the mid span of *BC*, the spans *AB* and *CD* should be loaded with dead load alone, while the span *BC* is loaded with dead and live load. Distribution process is performed at *A*, *B*, *C* and *D*. The effect of these moments should be carried over from *A* to *B*, from *B* to *C*, from *C* to *B* and from *D* to *C*. Now again the distribution process is performed at *B* and *C*.

Mid span of CD : D.L. on *ABCD* L.L. on *CD*

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
Distribution factors	1/2	4/9	1/9	1/7	2/7	1/3
(*i*) FEM due to D.L.	−5025	+5025	−1144	+1114		
(*ii*) FEM due to total load					−4267	+4267
(*iii*) Distribution moments			−431		+892	−1422
(*iv*) Carry over moments				−216	−711	+446
(*v*) Distribution					+265	−149
Total					−3821	+3142

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Free B M. at the centre of span CD

$$= \frac{wl^2}{8} = \frac{(1625+1575)\times 4^2}{8} \text{ kg. m.} = 6400 \text{ kg. m.}$$

$\therefore$ Net B.M. at the centre of span CD

$$= 6400 - \frac{3821+3142}{2} \text{ kg. m.} = 2118 \text{ kg. m.}$$

Condition for maximum negative moment at the centre of span BC.

Loads : D L. on $ABCD$ L L on AB and CD

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
Distribution factors	1/2	4/9	1/9	1/7	2/7	1/3
(*i*) FEM due to D.L.			−1144	+1144		
(*ii*) FEM due to total load	−9750	+9750			− 4267	+4267
(*iii* Distribution moments	+4875	−3825	−956	+446	+892	−1422
(*iv*) Carry over moments		+2438	+223	−478	−711	
(*v*) Distribution			−296	+170		
Total			−2173	+1282		

Free B.M. at the centre of span BC

$$= \frac{\text{Dead load intensity} \times l^2}{8}$$

$$= 1525 \times \frac{1^2}{8} \text{ kg. m.} = 1716 \text{ kg. m.}$$

$\therefore$ Net B.M. at the centre of span BC

$$= 171 - \frac{2173+1282}{2} \text{ kg. m.}$$

$$= 716 - 1728 \text{ kg} \quad - 12 \text{ kg. m.}$$

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(*i*) **Bending moments in columns**

Loading : D.L. on *ABCD*

L L. on *AB* and *CD*

Joint	A	B		C		D
Column distribution factors						
Just above floor	1/4	2/9		2/7		1/3
Just below floor	1/4	2/9		2/7		1/3
Horizontal members	AB	BA	BC	CB	CD	DC
Distribution factors	1/2	4/9	1/9	1/7	2/7	1/3
(*i*) FEM due to D.L			−1144	+1144		
(*ii*) FEM due to total load	−9750	+9750			−4267	+4267
(*iii*) Distribution +carry over	−1912	+2438	+223	−478	−711	+446
(*iv*) Summation	−11662	+12188	−921	+666	−4968	+4713
(*v*) Distribution to columns						
Just above floor	+2916	−2504		+1229		−1571
Just below floor	+2916	−2504		+1229		−1571

Explanation of the two cycle distribution carried in the above table : For maximum bending moment in columns, alternate spans must be loaded with live and dead load. Let the spans *AB* and *CD* be loaded with dead and live load. The span *BC* is loaded with only dead load. Due to the distribution at the joints *A*, *B*, *C* and *D* the corresponding carry over moments are transferred to the appropriate farther joints. The *FEM* and the carry over moments are added. Now the moments obtained are distributed to the columns.

Now a second analysis is necessary with the spans *AB* and *CD* loaded with only dead load and the span *BC* loaded with dead and live load.

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(*ii*) **Bending moments in columns**
Loading : Dead load on *ABCD*
Live load on *BC*

Joint	A	B		C		D
Column Distribution factors						
Just above floor	1/4	2/9		2/7		1/3
Just below floor	1/4	2/9		2/7		1/3
Horizontal members	AB	BA	BC	CB	CD	DC
Distribution factors	1/2	4/9	1/9	1/7	2/7	1/3
(*i*) FEM due to D.L.	−5025	+5025			−2167	+2167
(*ii*) FEM due to total load			−2 25	+2325		
(*iii*) Distribution + carry over	−600	+1256	−11	−150	−361	−22
(*iv*) Summation	−5625	+6881	−2336	+2175	−2528	+2145
(*v*) Distribution to columns						
Just above floor	+1406	−1010		+101		−715
Just below floor	+1406	−1010		+101		−715

Now that the analysis is complete the beams and columns at the floor level may be designed.

§194. Analysis of Frames Subjected to Horizontal Forces

A frame is liable to be subjected to horizontal forces due to wind pressure and seismic effects. Such forces introduce direct loads on columns and bending moments in all the members of the frame. Since these effects are small, an elaborate and rigorous method of analysis is not required. Moments and shears due to the above forces may be determined by approximate methods.

The following approximate methods are in practice.
(*i*) the portal method and (*ii*) the cantilever method.

(1) **The portal method.** It will be assumed that (*a*) points of contraflexure occur at the middle points of the members of the frame and (*b*) horizontal shear taken by each interior column is double the horizontal shear taken by each of the external columns. By making the above two assumptions the structure can be easily analysed.

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Fig. 670 shows a multistoreyed structure. It is assumed that the horizontal force on each storey height is equally divided and transmitted to the top and bottom of the storey. Let P_1, P_2, P_3, P_4 be the horizontal forces transmitted at A_1, A_2, A_3, A_4

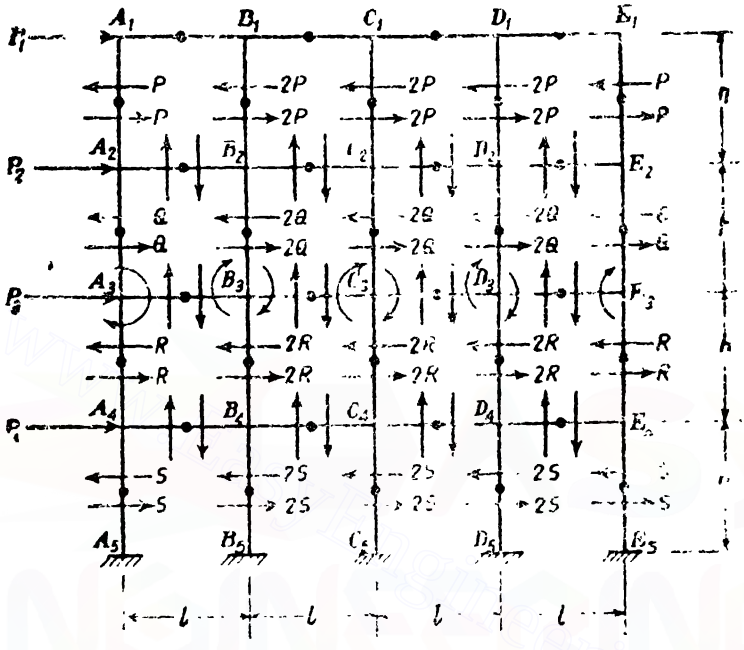

Fig. 670

Now consider the vertical members A_1A_2, B_1B_2, C_1C_2, D_1D_2 and E_1E_2. Let the horizontal shear for the membe A_1A_2 or E_1E_2 be P. Horizontal shear for each of the intermediate members B_1B_2, C_1C_2 and D_1D_2 will be equal to $2P$. P can be determined by the condition, $P_1 = P + 2P + 2P + 2P + P = 8P$

Similarly consider the vertical members, A_2A_3, B_2B_3, C_2C_3, D_2D_3 and E_2E_3

Let Q be the horizontal shear in each of the exterior members. The horizontal shear in each of the interior members will be $2Q$. Q can be determined by the condition,

$$P_1 + P_2 = Q + 2Q + 2Q + 2Q + Q = 8Q$$

$$Q = \frac{P_1 + P_2}{8}$$

Similarly, for the vertical members A_3A_4, B_3B_4, C_3C_4, D_3D_4 and E_3E_4, $P_1 + P_2 + P_3 = 8R$

$$R = \frac{P_1 + P_2 + P_3}{8}$$

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Similarly, for the vertical members A_4A_5, B_4B_5, C_4C_5, D_4D_5 and E_4E_5, $P_1+P_2+P_3+P_4=8S$

$$\therefore \quad S=\frac{P_1+P_2+P_3+P_4}{8}$$

Now the moments for the columns at top and bottom of each storey can be easily determined.

For instance,

$M_{a1a2}=M_{a2a1}=\frac{Ph}{2}$ (numerically)

$M_{a2a3}=M_{a3a2}=Qh$ (numerically)

$M_{b1b2}=M_{b2b1}=2P\frac{h}{2}=Ph$ (numerically)

$M_{b2b3}=M_{b3b2}=2Q\frac{h}{2}=Qh$ (numerically)

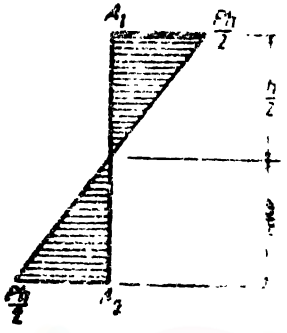

Fig. 671

Moments in beams. Consider the beam end at A_3

B.M. in column just above $A_3=\frac{Qh}{2}$

B.M. in column just below $A_3=\frac{Qh}{2}$

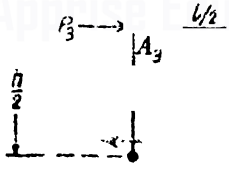

Fig. 672

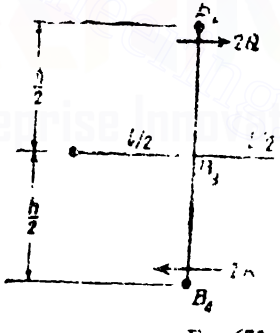

Fig. 673

Hence the end A_3 of the beam A_3B_3 is subjected to a clockwise moment of $\frac{(Q+R)\,h}{2}$

Similarly consider the joint B_3.

Here the clockwise moment at the joint B_3

$$=2Q\frac{h}{2}+2R\frac{h}{2}$$

$$=(Q+R)h$$

This moment will be transmitted equally to the two beams B_3A_3 and B_3C_3 meeting at B_3.

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$\therefore$ B.M. at the end of each beam meeting at B_3

$$= (Q + R)\frac{h}{2}$$

$$\text{Shear for any beam} = \frac{\text{B.M. at the beam end}}{\text{half span of the beam}}$$

Vertical reaction in columns

Let the end moment of a beam be M. The end moments of a beam will induce a pull $= \frac{2M}{l}$ in the windward column and a push $\frac{2M}{l}$ in the leeward column connected to the beam.

These reactions neutralize for any intermediate column.

Hence only the exterior columns will be subjected to these vertical reactions.

Hence, at each floor level pull in the exterior windward column

$$= \frac{2M}{l}$$

Push in the exterior leeward column $= \frac{2M}{l}$

(*ii*) The cantilever method

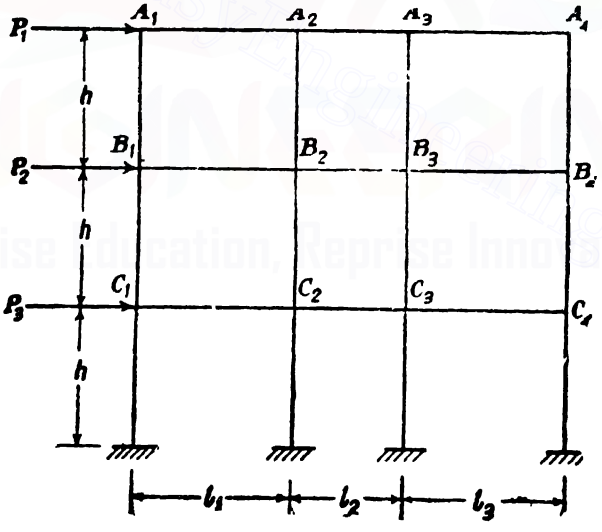

Fig. 674

This method introduces the following two assumptions :

(*a*) Points of contraflexure occur at the middle points of the various members.

(*b*) Direct stresses in the columns are proportional to their distances from the centroidal vertical axis of the frame.

Fig. 674 shows a multi-storeyed frame.

Let us now consider the top storey of this frame.

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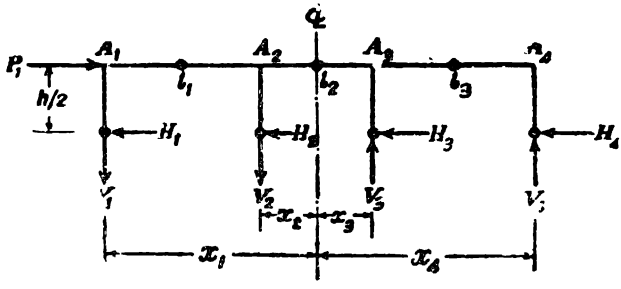

Fig. 675

The part of the storey just above the points of contraflexure of the columns is shown in Fig. 675.

Let a_1, a_2, a_3, a_4 be the sectional areas of the members A_1B_1, A_2B_2, A_3B_3 and A_4B_4.

Let H_1, H_2, H_3, and H_4 be the horizontal shears in the above members

We have the following conditions for this part of the structure

$$H_1+H_2+H_3+H_4=P_1 \qquad ...(i)$$

$$\frac{\left(\frac{V_1}{a_1}\right)}{x_1}-\frac{\left(\frac{V_2}{a_2}\right)}{x_2}-\frac{\left(\frac{V_3}{a_3}\right)}{x_3}=\frac{\left(\frac{V_4}{a_4}\right)}{x_4} \qquad ...(ii)$$

where x_1, x_2, x_3 and x_4 are the centroidal distances of the columns A_1B_1, A_2B_2, A_3B_3 and A_4B_4 from the vertical centroidal axis of the frame.

Taking moments about the point of intersection of the verticle centroidal axis and the top beam,

$$(H_1+H_2+H_3+H_4)\frac{h}{2} = -V_1x_1+V_2x_2+V_3x_3+V_4x_4$$

$$\therefore \quad V_1x_1+V_2x_2+V_3x_3+V_4x_4 = \frac{P_1h}{2} \qquad ...(iii)$$

From equations (*ii*) and (*iii*) V_1, V_2, V_3 and V_4 can be determined.

Now, taking moments about the point of contraflexure in A_1A_2 (middle point of A_1A_2)

$$H_1\frac{h}{2}=V_1\ \frac{l_1}{2}$$

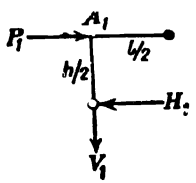

Fig. 676

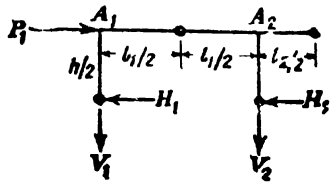

Fig. 677

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$$\therefore \qquad H_1 = \frac{l_1}{H_2} V_1$$

Similarly taking moments about the point of contraflexure in A_2A_3 (Mindle point of A_2A_3)

$$H_1 \frac{h}{2} + H_2 \frac{h}{2} = V_1 \left(l_1 + \frac{l_2}{2} \right) + V_2 \frac{l_2}{2}$$

$$= V_1 l_1 + (V_1 + V_2) \frac{l_2}{2}$$

$$\therefore \qquad (H_1 + H_2) = \frac{2 \left[V_1 l_1 + (V_1 + V_2) \frac{l_2}{2} \right]}{h}$$

Since H_1 is already known H_2 can be determined.
Similarly H_3 and H_4 can be determined.

Since H_1, H_2, H_3 and H_4 are now known, the bending moments for the beams and columns can be computed.

This is the analysis of the uppermost storey. Now the lower storey and the other storeys can be analysed similarly.

§195. Kani's Rotation Contribution method

This is a method developed by Gasper Kani of Germany. This method is an excellent extension of the slope deflection method. Since this new method has been recognized as a very useful method it is proposed to discuss this method in detail.

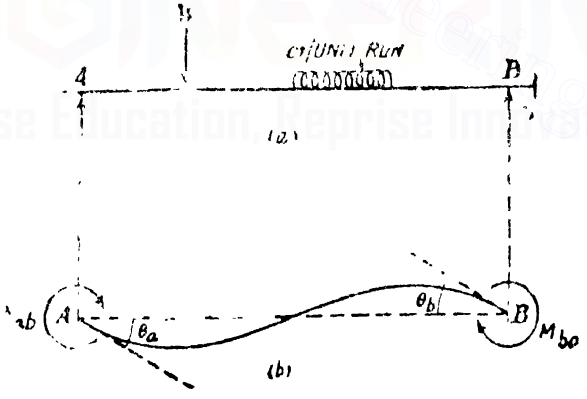

Fig. 678

Let AB represent one of the spans of a framed or continuous structure. Let the span be carrying a loading.

Fig. 678 (*b*) shows the deformed shape of the member AB. Let the ends A and B undergo rotations θ_a and θ_b respectively. Let us assume that lateral displacements of the ends do not occur. Let M_{ab} and M_{ba} represent the end moments for the span AB. Let us follow the following sign conventions regarding end moments and rotations :

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(*i*) Clockwise end moments are positive.
(*ii*) Clockwise rotations at ends are positive.

In this method the final end moments are split up into certain components. By an orderly successive approximation process these components are computed.

The components to which the final end moments M_{ab} and M_{ba} can be split up are shown in Fig. 679.

The moments are determined by going through the following stages :

1. The ends A and B of the member are first regarded as fixed. Corresponding to this condition the fixed end moments M_{ab} at A and M_{ba} at B are determined [Fig. 679 (*b*)].

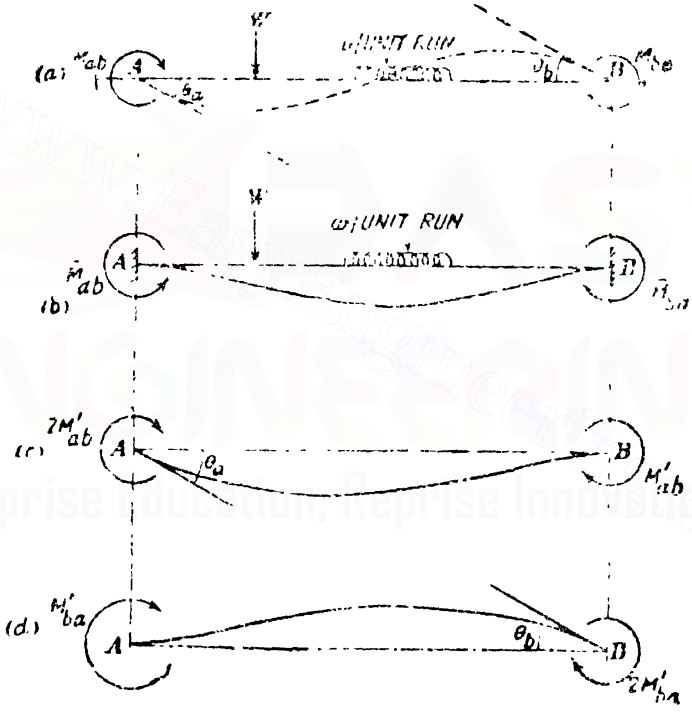

Fig. 679

2. Now maintaining the fixity of the end B, the end A is rotated through an angle θ_a by the application of a moment $2M'_{ab}$ at A. For this condition a moment of M'_{ab} is induced at the end B. The moment M'_{ab} is called the **rotation contribution of the end A.** [See Fig. 679 (*c*)].

3. In this stage, the end A is considered as fixed, and the end B is rotated through an angle θ_b by the application of a moment $2M'_{ba}$ at B. For this condition a moment of M'_{ba} is induced at the end A. The moment M'_{ba} is called **the rotation contribution of the end B.** [See Fig 679 (*d*)].

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Thus, the final moments M_{ab} and M_{ba} can be expressed as follows :

$$\left.\begin{aligned} M_{ab} &= \bar{M}_{ab} + 2M'_{ab} + M'_{ba} \\ \text{and} \quad M_{ba} &= \bar{M}_{ba} + 2M'_{ba} + M'_{ab} \end{aligned}\right\} \quad \ldots(i)$$

For the member AB, when we refer to the final moment M_{ab} at A, the end A may be referred to as the **near end** and the end B as the **far end.**

Similarly, when we refer to the final moment M_{ba} at B, the end B may be referred to as the near end and the end A as the far end.

Relations shown in eq. (i) above may be worded as follows :

Moment at the near end of a member

= Sum of (a) the fixed end moment at the near end due to the loading on the member,

(b) twice the rotation contribution of the near end,

(c) the rotation contribution of the far end.

Fig. 680 shows a multi-storeyed frame. If no lateral joint displacements occur for the members, the equations of the type obtained above are applicable to all the members.

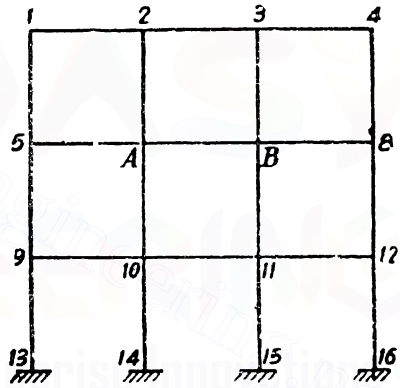

Fig 680

Consider the various members meeting at the joint A.

The end moments at A for the members meeting at A are therefore given by

$$\left.\begin{aligned} M_{a\text{-}b} &= \bar{M}_{a\text{-}b} + 2M'_{a\text{-}b} + M'_{b\text{-}a} \\ M_{a\text{-}10} &= \bar{M}_{a\text{-}10} + 2M'_{a\text{-}10} + M'_{10\text{-}a} \\ M_{a\text{-}5} &= \bar{M}_{a\text{-}5} + 2M'_{a\text{-}5} + M'_{5\text{-}a} \\ M_{a\text{-}2} &= \bar{M}_{a\text{-}2} + 2M'_{a\text{-}2} + M'_{2\text{-}a} \end{aligned}\right\} \quad \ldots(ii)$$

For the equilibrium of the joint A, the sum of all the end moments at A must be equal to zero.

i.e. $\Sigma M_{ab} = 0$

$\therefore$ $\Sigma M_{ab} = \Sigma \bar{M}_{ab} + 2\Sigma M'_{ab} + \Sigma M'_{ba} = 0 \quad \ldots(iii)$

where, $\Sigma \bar{M}'_{ab}$ = algebraic sum of the fixed end moments at A for all the members meeting at A.

$\Sigma M'_{ab}$ = algebraic sum of the rotation contributions at A for all the members meeting at A.

$\Sigma M'_{ba}$ = algebraic sum of the rotation contributions at the far end joints with respect to the joint A.

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For the frame shown in Fig. 680.

$$\Sigma\overline{M}_{ab} = \overline{M}_{ab} + \overline{M}_{a\_10} + \overline{M}_{a\_5} + \overline{M}_{a\_2}$$
$$\Sigma M'_{ab} = M'_{ab} + M'_{a\_10} + M'_{a\_5} + M'_{a\_2}$$
$$\Sigma M'_{ba} = M'_{ba} + M'_{10\_a} + M'_{5\_a} + M'_{2\_a}$$

From eq. (*iii*), we have,

$$\Sigma M'_{ab} = (-\tfrac{1}{2})\{\Sigma\overline{M}_{ab} + \Sigma M'_{ba}\} \qquad ...(iv)$$

But we know for the member AB, with the end B fixed, the moment required at A so as to rotate the joint A by θ_a is given by

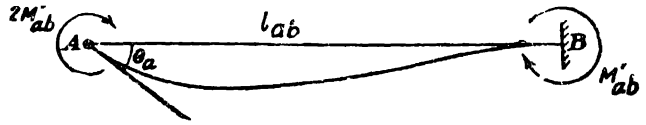

Fig. 681

$$2M'_{ab} = \frac{4EI_{ab}}{l_{ab}}\theta_a = 4EK_{ab}\theta_a$$

where $K_{ab} = \dfrac{I_{ab}}{l_{ab}}$

and E = Young's modulus

$\therefore$ $M'_{ab} = 2EK_{ab}\theta_a$

Now consider the members meeting at the joint A. The end rotation at A of each member meeting at A equals θ_a. Assuming that the Young's moduli to be the same for all the members, we have,

$$\Sigma M'_{ab} = 2E\theta_a \Sigma K_{ab}$$

$\therefore$ $$\frac{M'_{ab}}{\Sigma M'_{ab}} = \frac{K_{ab}}{\Sigma K_{ab}}$$

$\therefore$ $$M'_{ab} = \left\{\frac{K_{ab}}{\Sigma K_{ab}}\right\} \Sigma M'_{ab} \qquad ...(v)$$

From equations (*iv*) and (*v*), we get,

$$M'_{ab} = \left\{-\frac{1}{2}\,\frac{K_{ab}}{\Sigma K_{ab}}\right\}\{\Sigma\overline{M}'_{ab} + \Sigma M'_{b_1}\}$$

The ratio $\left\{-\dfrac{1}{2}\,\dfrac{K_{ab}}{\Sigma K_{ab}}\right\}$ is called the **rotation factor** for the member AB at the joint A.

Let U_{ab} be the rotation factor at the joint A for the member AB.

$\therefore$ $$M'_{ab} = U_{ab}\{\Sigma\overline{M}_{ab} + \Sigma M'_{ba}\} \qquad ...(vi)$$

Now consider equation (*vi*). The summation $\Sigma\overline{M}_{ab}$ can be computed. This is therefore a known quantity. If trial values be assumed for the far end rotation contributions the approximate value of the near end rotation contribution can be computed from equation (*vi*).

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By successive application of equation (*vi*) to the various joints the rotation contributions can be determined.

For instance as a first approximation assuming that the rotation contributions of the far ends of the members meeting at *A* to be zero, the rotation contribution at *A* for the member *AB* is given by

$$M_{AB} = U_{AB} \Sigma \bar{M}_{AB}$$

By a similar approximation the rotation contributions at other joints are also determined. With the approximate values of the rotation contributions computed, it is possible to again determine a more correct value of the rotation contribution at *A* for the member *AB* using the equation

$$M'_{AB} = U_{AB}(\Sigma \bar{M}_{AB} + \Sigma M'_{BA})$$

The process mentioned above is repeated so that more and more accurate values of the rotation contributions are obtained. After attaining the desired degree of accuracy in the values of the rotation contributions the final moments can be easily computed from the relations like,

$$M_{AB} = \bar{M}_{AB} + 2M'_{AB} + M'_{BA}$$

Some important points to be remembered

(a) If an end of a member is fixed the rotation at the end being zero, the rotation contribution at the end is also zero.

(b) If an end of a member is hinged or pinned, it will be convenient to consider the end as fixed and to take the relative stiffness $\frac{3}{4}\frac{I}{l}$.

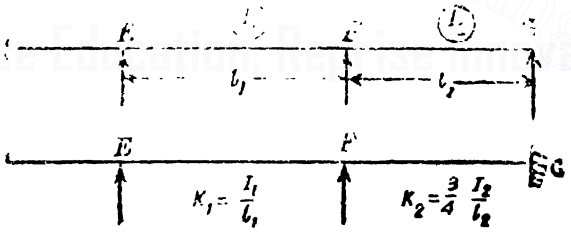

Fig. 682

Design 160. *Determine the end moments at A, B, C, D for the continuous beam shown in Fig. 683.*

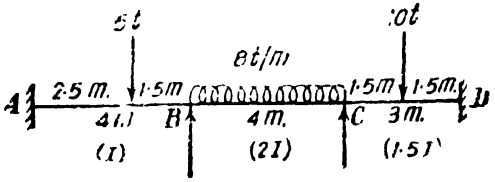

Fig. 683

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Solution.

Let us first determine the fixed end moments

$$\bar{M}_{ab} = -\frac{5 \times 2{\cdot}5 \times 1{\cdot}5^2}{4^2} \text{ tm.}$$

$$= -1{\cdot}76 \text{ tm.}$$

$$\bar{M}_{ba} = +\frac{5 \times 2{\cdot}5^2 \times 1{\cdot}5}{4^2} \text{ tm.}$$

$$= +2{\cdot}93 \text{ tm.}$$

$$M_{bc} = -\frac{8 \times 4^2}{12} \text{ tm.}$$

$$= -10{\cdot}67 \text{ tm.}$$

$$\bar{M}_{cb} = +10{\cdot}67 \text{ tm.}$$

$$M_{cd} = -\frac{10 \times 3}{8} \text{ tm.}$$

$$= -3{\cdot}75 \text{ tm.}$$

$$M_{dc} = +3{\cdot}75 \text{ tm.}$$

Rotation factors at B and C

These have been computed in the following table.

Joint	Member	Relative stiffness K	Rotation factor $C = \frac{K}{2K}$
B	BA	$\frac{I}{4} = 3I$	$\frac{1}{2}\left(\frac{3}{4}\right) = \frac{3}{8}$
	BC	$\frac{2I}{4} = 2I$	$\frac{1}{2}\left(\frac{2}{4}\right) = \frac{2}{8}$
C	CB	$\frac{2I}{4} = 2I$	$\frac{1}{2}\left(\frac{2}{4}\right) = \frac{2}{8}$
	CD	$\frac{1{\cdot}5I}{3} = 2I$	$\frac{1}{2}\left(\frac{2}{4}\right) = \frac{2}{8}$

Sum of fixed end moments at B

$$\Sigma M_{ba} = M_{ba} + \bar{M}_{bc}$$

$$= +2{\cdot}93 - 10{\cdot}67 = -7{\cdot}74 \text{ tm.}$$

Sum of fixed end moments at C

$$= \Sigma M_{cb} = \bar{M}_{cb} + M_{cd}$$

$$= +10{\cdot}67 - 3{\cdot}75 = +6{\cdot}92 \text{ tm.}$$

The scheme for proceeding with the method of moment distribution is shown in Fig. 6.34. The joints B and C are enclosed in square boxes. Sum of the fixed end moments at B is written within the smaller square for the joint B. Similarly sum of the fixed end moments at C, is written within the smaller square for the joint C

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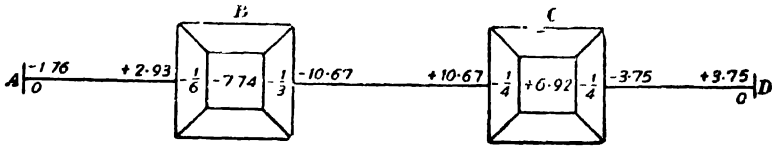

Fig. 684

The rotation factors $-\frac{1}{6}$ and $-\frac{1}{3}$ for members BA and BC at B and the rotation factors $-\frac{1}{4}$ and $-\frac{1}{4}$ for the members CB and CD at C are entered as shown in Fig. 684. The fixed end moments are written above the horizontal line outside the square boxes Rotation contribution at A as well as D equals zero since these ends are fixed.

The rotation contributions can now be determined by a successive iteration process as explained below.

Now consider the joint B

Applying equation (*vi*) to this joint, we have,

$$M'_{ba} = U_{ba}\{\Sigma \overline{M}_b + \Sigma M'_{ab}\}$$

and $$M'_{bc} = U_{bc}\{\Sigma \overline{M}_b + \Sigma M'_{ab}\}$$

Sum of fixed end moments at B $= -7{\cdot}74$ *tm.*

Sum of rotation contributions of the far ends

At A : 0 (since A is fixed)
At C : 0 (assumed) $= 0$

Total $-7{\cdot}74$ *tm.*

$$M'_{ba} = U_{ba}(-7{\cdot}74)$$
$$= -\tfrac{1}{6}(-7{\cdot}74) = +1{\cdot}29 \text{ tm}$$
$$M'_{bc} = U_{bc}(-7{\cdot}74)$$
$$= -\tfrac{1}{3}(-7{\cdot}74) = +2{\cdot}58 \text{ tm}.$$

(These calculations could also be done approximately for the first trial).

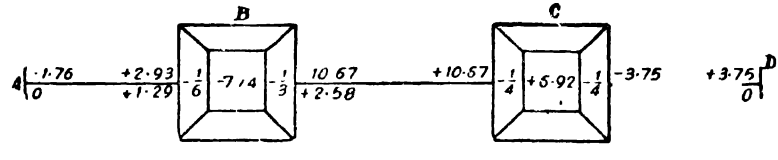

Fig. 685

These rotation contributions at B are written below the horizontal at their appropriate places as shown in Fig. 685.

Now consider joint C

Rotation contributions M'_{cb} and M'_{cd} are determined by equation (*vi*).

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Sum of fixed end moments at C $= +6{\cdot}92$ *tm.*

Sum of rotation contributions of far ends :

At B : 2·58 (value calculated at joint B)
At D : 0 (fixed end) $= +2{\cdot}58$ *tm.*

Total $= +9\ 50$ *tm.*

$$\therefore \quad M'_{cb} = U_{cb}(9{\cdot}50)$$
$$= -\tfrac{1}{4}(9{\cdot}50) = -2{\cdot}38 \text{ tm.}$$
$$M'_{cd} = U_{cd}(9{\cdot}50)$$
$$= -\tfrac{1}{4}(9{\cdot}50) = -2{\cdot}38 \text{ tm.}$$

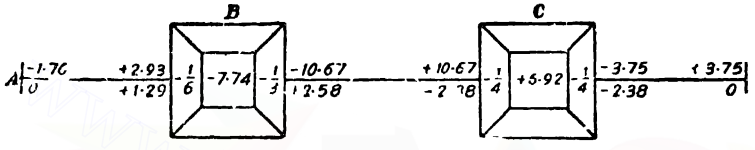

Fig. 686

These rotation contributions at C are written below the horizontal line at their appropriate places as shown in Fig. 686.

Now consider joint B

Since the rotation contributions of the far ends of members meeting at B are known (or approximately known), more accurate values of rotation contributions at the joint B can be determined by again using equation (*vi*).

Sum of fixed end moments at B $= -7{\cdot}74$ *tm.*

Sum of rotation contributions of the far ends :

At A : 0
At C : −2·38 $= -2{\cdot}38$ *tm.*

Total $= -10{\cdot}12$ *tm.*

$$M'_{ba} = U_{ba}(-10{\cdot}12)$$
$$= -\tfrac{1}{6}(-10{\cdot}12) = 1{\cdot}69 \text{ tm.}$$
$$M'_{bc} = U_{bc}(-10{\cdot}12)$$
$$= -\tfrac{1}{3}(-10{\cdot}12) = +3{\cdot}37 \text{ tm.}$$

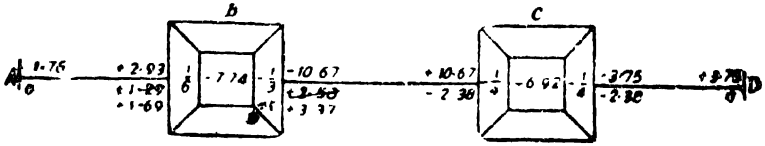

Fig. 687

These values of the rotation contributions supersede the values +1·29 and +2·58 obtained earlier. Hence the values obtained

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earlier are struck off. After striking these figures the values +1·69 and +3·37 are entered as shown in Fig. 687.

Now consider joint C

Now values of rotation contributions are determined as explained earlier.

Sum of fixed end moments at C = + 6·92 *tm.*

Sum of rotation contributions of far ends :

At B : +3·37
At D : 0

= + 3·37 *tm.*

Total = +10·29 *tm.*

$$M'_{CB} = -\frac{1}{4}(+10.29) = -2.57 \text{ tm}$$

$$M'_{CD} = -\frac{1}{4}(+10.29) = -2.57 \text{ tm}$$

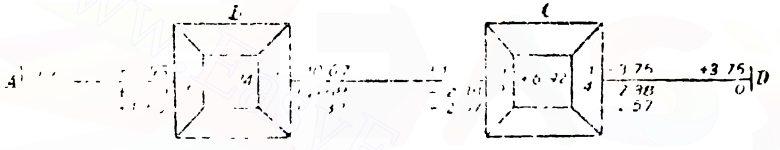

Fig. 688

The previously determined values −2·38 and −2·38 are struck off and the more correct values −2·57 and −2·57 are written as shown in Fig. 688.

Now consider joint B

Sum of fixed end moments at B = − 7·74 *tm.*

Sum of rotation contributions of far ends :

At A : 0
At C : −2·57

= − 2·57 *tm*

Total = −10·31 *tm*

$$M'_{BA} = -\frac{1}{8}(-10.31) = +1.72 \text{ tm}$$

$$M'_{BC} = -\frac{1}{8}(-10.31) = +1.14 \text{ tm.}$$

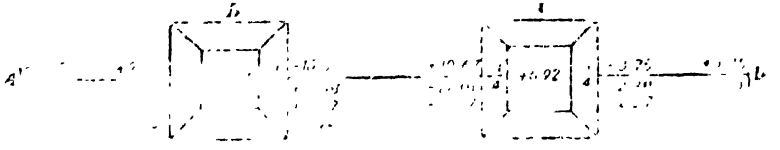

Fig. 689

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These values are entered at the joint B, after striking off the previously obtained values $+1{\cdot}69$ *tm.* and $+3{\cdot}37$ *tm.* as shown in Fig. 689.

Now consider joint C

Sum of fixed end moments at C $= +\ 6{\cdot}92$ *tm.*

Sum of rotation contributions of far ends :

At B : $+3{\cdot}44$
At D : 0 $= +\ 3{\cdot}44$ *tm.*

Total $= +10{\cdot}36$ *tm.*

$$\therefore \quad M'_{cb} = U_{cb}(10{\cdot}36)\ tm.$$
$$= -\tfrac{1}{2}(10{\cdot}36) = -2{\cdot}59\ tm.$$
$$M'_{cd} = U_{cd}(10{\cdot}36)$$
$$= -\tfrac{1}{2}(10{\cdot}36) = -2{\cdot}59\ tm.$$

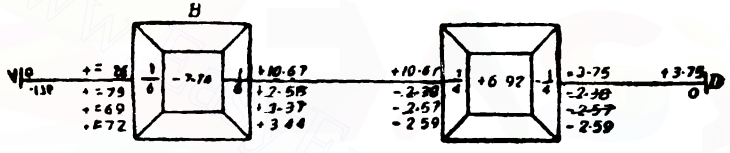

Fig. 690

The previously determined values $-2{\cdot}57$ and $-2{\cdot}57$ at C are struck off and the new values $-2{\cdot}59$ and $-2{\cdot}59$ are entered as shown in Fig. 690.

Now, since we know the actual values of the rotation contributions the final end moments can be easily determined.

In general for any member,
Final end moment at the near end
= Fixed end moment at the near end
+ twice the rotation contribution of the near end
+ rotation contribution of the farther end

The end moments for the continuous girder may be determined as shown in the following tabular form :

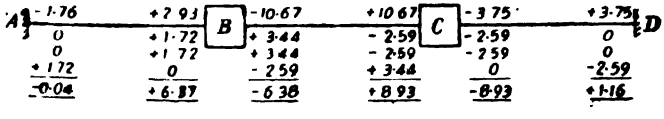

Fig. 691

We know for the equilibrium of the joints

$$M_{ba} + M_{bc} = 0$$
$$M_{cb} + M_{cd} = 0$$

The results obtained above satisfy this requirement.

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Continuous girder with a hinged or pinned end. If the end of an end span of a continuous girder is pinned or hinged, it will be convenient to consider that this end is fixed and that the relative stiffness of the span is $\frac{3}{4}\frac{I}{l}$ where I is the moment of inertia of the section of the beam and l the span length. The following example illustrates this condition.

Design 161. *Determine the support moments for the continuous girder shown in Fig. 692.*

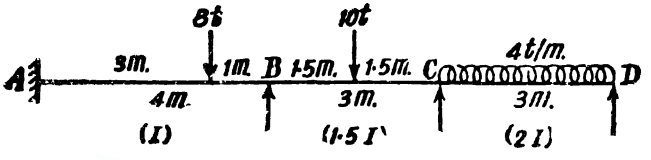

Fig. 692

Solution. Fixed moments

$$\bar{M}_{ab} = -\frac{8 \times 3 \times 1^2}{4^2} = -1{\cdot}50 \; tm.$$

$$\bar{M}_{ba} = +\frac{8 \times 3^2 \times 1}{4^2} = +4{\cdot}50 \; tm.$$

$$\bar{M}_{bc} = -\frac{10 \times 3}{8} = +3{\cdot}75 \; tm.$$

$$\bar{M}_{cb} = +\frac{10 \times 3}{8} = +3{\cdot}75 \; tm.$$

$$\bar{M}_{cd} = -\frac{4 \times 3^2}{8} = -4{\cdot}50 \; tm. \text{ (End } D \text{ is hinged)}$$

Rotation factors. These are determined as shown in the following table :

Joint	*Member*	*Relative stiffness* K	ΣK	*Rotation factor* $U = \frac{K}{\Sigma_c K}(-\frac{1}{2})$
B	*BA*	$\frac{I}{4}$	$\frac{3I}{4}$	$\frac{1}{3}(-\frac{1}{2}) = -\frac{1}{6}$
	BC	$\frac{1{\cdot}5I}{3} = \frac{2I}{4}$		$\frac{2}{3}(-\frac{1}{2}) = -\frac{1}{3}$
C	*CB*	$\frac{1{\cdot}5I}{2} = \frac{I}{2}$	I	$\frac{1}{2}(-\frac{1}{2}) = -\frac{1}{4}$
	CD	$\frac{3}{4} \times \frac{2I}{3} = \frac{I}{2}$		$\frac{1}{2}(-\frac{1}{2}) = -\frac{1}{4}$

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Sum of fixed end moments at B

$$= \bar{M}_{ba} + \bar{M}_{bc} = +4\ 50 - 3\cdot75 = +0\cdot75\ tm.$$

Sum of fixed end moments at C

$$= \bar{M}_{cb} + \bar{M}_{cd} = +3\ 75 - 4\ 50 = -0\cdot75\ tm.$$

Now the rotation contributions at the various joints are determined as shown below. It should be noted that the rotation contribution at A for AB is zero since A is a fixed end. Since the span CD is also substituted with a span, whose end D is considered as fixed, the rotation contribution at the end D is also written as zero.

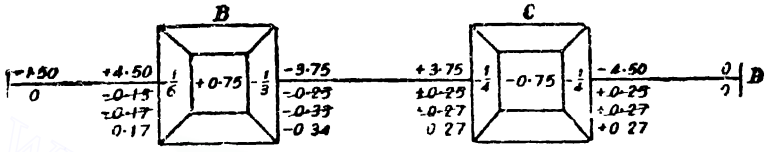

Fig. 693

Now that the rotation contributions are known the final moments are easily determined.

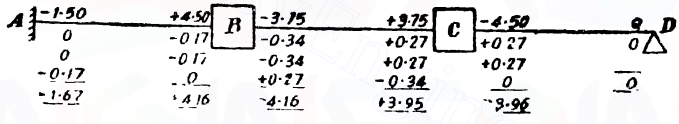

Fig. 694

Since the end D is actually hinged the bending moment at D is zero and hence no computations are done at this end in the above table.

Overhang in a continuous girder

The overhanging part of the girder should be considered as having a stiffness equal to zero. The rotation contributions are computed in the usual manner. The following problem illustrates this condition.

Design 161. *Find the support moments for the continuous beam shown in Fig 695.*

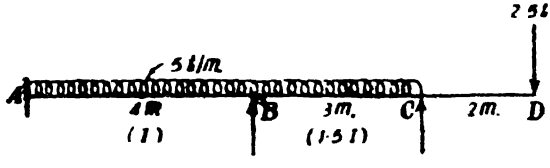

Fig. 695

Solution. The rotation contributions are determined in the usual manner. These are shown in Fig. 696.

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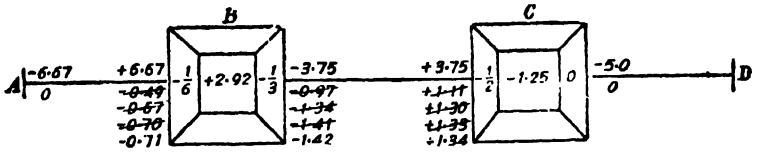

Fig. 696

The final end moments are determined as shown below :

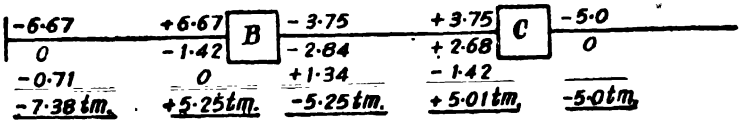

Fig. 697

Members with relative lateral displacement

Fig. 698 shows a beam AB whose ends have undergone lateral displacement so that the relative lateral displacement, between the ends equals δ.

Fig. 698

The fixed end moments due to this condition of the beam

$$= M'_{ab} = M''_{ba} = \frac{6EI\delta}{l^2}$$

where I = moment of inertia of the beam section
l = span length
and E = Young's modulus

When such displacements occur for a member of a frame the final moments at A and B are given by

$$M_{ab} = \overline{M}_{ab} + 2M'_{ab} + M'_{ba} + M''_{ab}$$

$$M_{ba} = \overline{M}_{ba} + 2M'_{ba} + M'_{ab} + M''_{ba}$$

The quantity $M''_{ab} = M''_{ba}$ is called the **displacement contribution** of the member AB.

The expression for the final moment at the near end of a member can be expressed as the algebraic sum of

(*a*) the fixed end moment at the near end
(*b*) twice the rotation contribution of the near end
(*c*) rotation contribution of the far end
(*d*) the displacement contribution.

If A happens to be a joint where several members meet (Fig. 693) then for the condition of equilibrium of the joint.

$$\Sigma M_{ab} = 0$$

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$$\therefore \quad \Sigma\bar{M}_{ab}+2\Sigma M'_{ab}+\Sigma M'_{ba}+\Sigma M''_{ab}=0$$

$$\therefore \quad \Sigma M'_{ab}=(-\tfrac{1}{2})\{\Sigma\bar{M}_{ab}+\Sigma M_{ba}+\Sigma M''_{ab}\}$$

But $\quad M'_{ab}=2EK_{ab}\theta_a$

$$\therefore \quad \Sigma M'_{ab}=2E\theta_a\Sigma M_{ab}$$

$$\therefore \quad \frac{M'_{ab}}{\Sigma M'_{ab}}=\frac{K_{ab}}{\Sigma K_{ab}}$$

$$\therefore \quad M'_{ab}=\frac{K}{\Sigma M_{ab}}\Sigma M'_{ab}$$

$$\therefore \quad M'_{ab}=\frac{K_{ab}}{\Sigma K_{ab}}(-\tfrac{1}{2})\{\Sigma\bar{M}_{ab}+\Sigma M'_{ba}+\Sigma M''_{ab}\}$$

$$=U_b\{\Sigma\bar{M}_{ab}+\Sigma M'_{ba}+\Sigma M''_{ab}\} \qquad ...(vi)$$

From such equations for each rotation contribution, the rotation contributions can be determined by the iterative process followed earlier. If the lateral displacement δ is a known quantity the displacement contribution is also a known quantity. Hence at each end of a member the resultant fixed end moment can be computed.

For instance for a span AB resultant fixed end moment at A

$$=\bar{M}_{ab}+M''_{ab}$$

Similarly, resultant fixed end moment at B

$$=\bar{M}_{ba}+M''_{ab}$$

If the lateral displacement δ is unknown, then additional equations are to be used to evaluate the results. This case will be dealt with at a later stage.

Design 163. *Determine the support moments for the continuous girder shown in Fig. 699, if the support B sinks by 0·25 cm. For all members take I=3500 cm.⁴ and E=2000 t/cm.².*

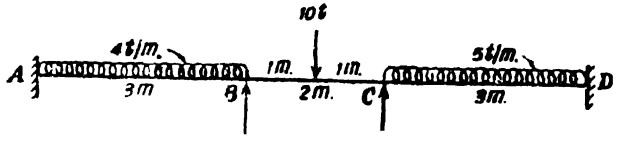

Fig. 699

Solution. Fixed end moments

Span AB

Resultant fixed end moment at A

$$=\bar{M}_{ab}-\frac{6EI\delta}{l_1^2}$$

$$=-\frac{4\times3^2}{12}-\frac{6\times2000\times3500\times0{\cdot}25}{3^2(100)^3}\ tm.$$

$$=-3{\cdot}00-1{\cdot}17=-4{\cdot}17\ tm.$$

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Resultant fixed end moment at B

$$= +3\cdot00 - 1\cdot17 = +2\cdot83 \ tm.$$

Span BC. Resultant fixed end moment at B

$$= \bar{M}_{bc} + \frac{6EI\delta}{l_2^2}$$

$$= -\frac{10 \times 2}{8} + \frac{6 \times 2000 \times 3500 \times 0\cdot25}{2^2 \ 100^3} \ tm.$$

$$= -2\cdot50 + 2\cdot63 = +0\cdot13 \ tm.$$

Resultant fixed end moment at C

$$= +2\cdot50 + 2\cdot63 \ tm. = 5\cdot13 \ tm.$$

Span CD. Fixed end moment at C

$$= \bar{M}_{cd}$$

$$= -\frac{5 \times 3^2}{12} = -3\cdot75 \ tm.$$

Fixed end moment at D

$$= \bar{M}_{dc} = +3\cdot75 \ tm.$$

Rotation factors. These have been computed in the following table :

Joint	Member	Relative stiffness K	Total stiffness ΣK	Rotation factor $\frac{K}{\Sigma K}\left(-\frac{1}{2}\right)$
B	BA	$\frac{I}{3} = \frac{2I}{6}$	$\frac{5I}{6}$	$\frac{2}{5}\left(-\frac{1}{2}\right) = -\frac{1}{5}$
	BC	$\frac{I}{2} = \frac{3I}{6}$		$\frac{3}{5}\left(-\frac{1}{2}\right) = -\frac{3}{10}$
C	CB	$\frac{I}{2} = \frac{3I}{6}$	$\frac{5I}{6}$	$\frac{3}{5}\left(-\frac{1}{2}\right) = -\frac{3}{10}$
	CD	$\frac{I}{3} = \frac{2I}{6}$		$\frac{2}{5}\left(-\frac{1}{2}\right) = -\frac{1}{5}$

Now the rotation contributions are computed. This computation is shown below

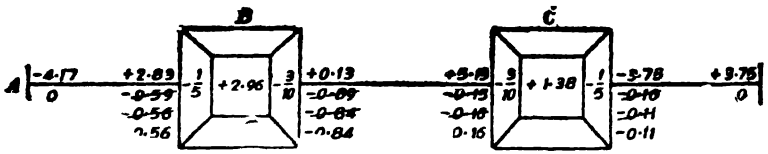

Fig. 700

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The final moments are now computed in the usual manner. This computation is shown below.

A	-4·17	+7·65	+0·13	+5·13	C	-3·75	+3·75
	0	-0·56	-0·84	-0·16		-0·11	0
	-0·56	-0·56	-0·84	-0·84		-0·11	0
	-4·73 tm.	+1·71 tm.				0	-0·11
						-3·97 tm.	+3·64

Fig. 701

Portal frames and Multistoreyed frames without lateral sway

Such frames are analysed on the same basis as that of the continuous girder. When a frame consists of symmetrically provided vertical and horizontal members and carries a symmetrical vertical loading the frame will not undergo any lateral sway. Analysis of frames in which wind load effect is ignored is an example of this category.

Design 164. *Determine the moments at ABCD for the portal frame loaded as shown in Fig. 702.*

Solution. Fixed end moments

$$\bar{M}_{ab} = 0 = \bar{M}_{ba} = \bar{M}_{cd} = \bar{M}_{dc}$$

$$\bar{M}_{bc} = -\frac{10 \times 6^2}{12}$$

$$= -30 \text{ tm.}$$

$$\bar{M}_{cb} = +30 \text{ tm.}$$

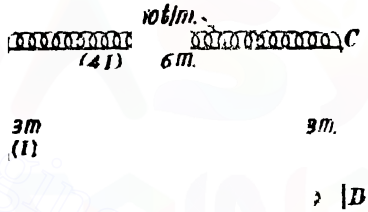

Fig. 702

Rotation factors

Joint	Member	Relative stiffness K	Total stiffness ΣK	Rotation factor $\frac{K}{\Sigma K}\left(-\frac{1}{2}\right)$
B	BA	$\frac{I}{3}$	$\frac{3I}{3}$	$\frac{1}{3}(-\frac{1}{2}) = -\frac{1}{6}$
	BC	$\frac{4I}{6} = \frac{2I}{3}$		$\frac{2}{3}(-\frac{1}{2}) = -\frac{1}{3}$
C	CB	$\frac{4I}{6} = \frac{2I}{3}$	$\frac{3I}{3}$	$\frac{2}{3}(-\frac{1}{2}) = -\frac{1}{3}$
	CD	$\frac{I}{3}$		$\frac{1}{3}(-\frac{1}{2}) = -\frac{1}{6}$

Now the rotation contributions are computed as usual. These are shown in Fig. 703.

The final moments can now be computed. These computations are shown below in Fig. 704.

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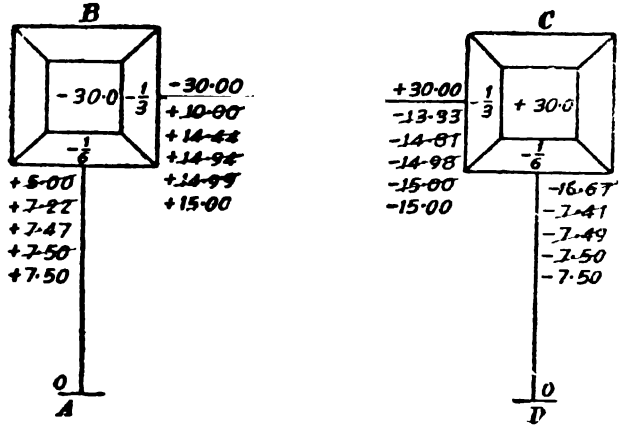

Fig. 703

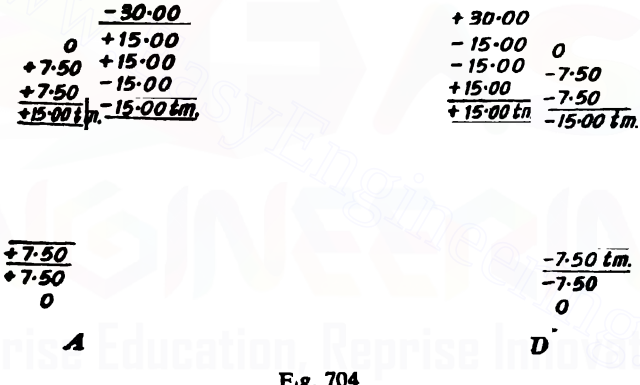

Fig. 704

Simplified analysis of symmetrical frames without lateral sway

Case 1. *When the axis of symmetry passes through the centres of beams*

Let AB be a horizontal member of the frame, through whose centre the axis of symmetry passes.

Let M_{ab} and M_{ba} be the end moments. Due to symmetry of deformation, M_{ab} and M_{ba} are numerically equal but are opposite in their sense.

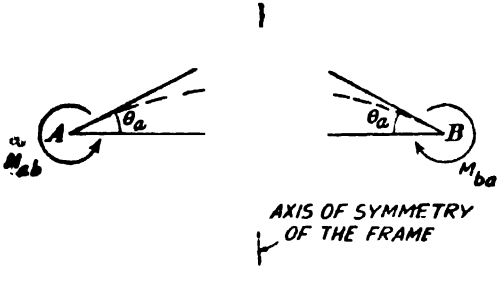

Fig. 705

Slope at A θ_a = Slope due to M_{ab} + Slope due to M_{ba}

$$= \frac{M_{ab}l}{3EI} + \frac{M_{ba}l}{6E}$$

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since each rotation at A is anticlockwise for the case shown in Fig. 705.

$$\therefore \quad \theta_a = \frac{M_{ab} l}{2EI}$$

Since $M_{ab} = M_{ba}$ numerically.

Let this member AB be replaced by a member AB' whose end A will undergo the rotation θ_a due to a moment M_{ab} applied at A, the end B' being restrained

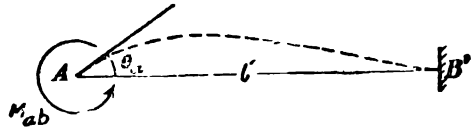

Fig. 706

$$\therefore \quad \theta_a = \frac{M_{ab} l'}{4EI'}$$

where l' is the span of the substitute member and I' the moment of inertia of the section of the substitute member. (Fig. 706).

Hence for the equality of conditions between the original member AB and the substitute AB',

$$\theta_a = \frac{M_{ab} l}{2EI} = \frac{M_{ab} l'}{4EI'}$$

$$\frac{I}{l} = \frac{2I'}{l'}$$

$$K = 2K'$$

$$\therefore \quad K' = \frac{K}{2}$$

Hence if K is the relative stiffness of the member AB this member can be replaced by the substitute member AB' having a relative stiffness $\frac{K}{2}$. Since the member AB and other similar horizontal members through the centres of which the symmetrical axis passes bend symmetrically, the frame may be replaced with fixed ends at the middle points of these beams. The following problems illustrate this case.

Design 165. *Analyse the portal frame ABCD shown in Fig. 702.*

Solution. Since the symmetrical vertical axis passes through the middle of the beam BC, only one-half of this frame will be considered. See the substitute frame which we will now discuss (Fig. 702).

$$K_{ba} = \frac{I}{3}$$

$$K'_{bc} = \frac{I}{2} \times \frac{4I}{6} = \frac{I}{3}$$

Rotation factors at B are obviously $-\frac{1}{4}$ and $-\frac{1}{4}$.

The rotation contributions are now easily determined as shown in Fig. 708.

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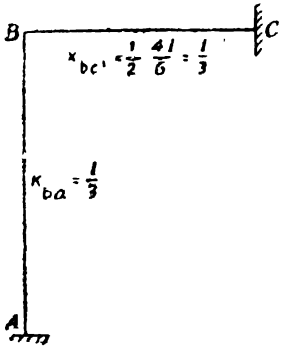

Fig. 707

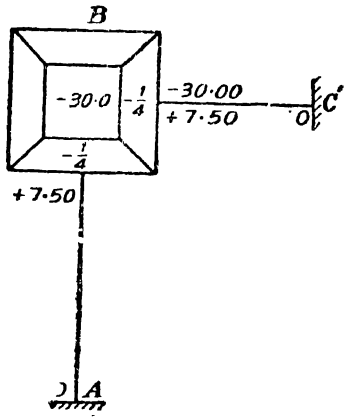

Fig. 708

The final moments can now be easily determined. These are shown in Figs. 709 and 710.

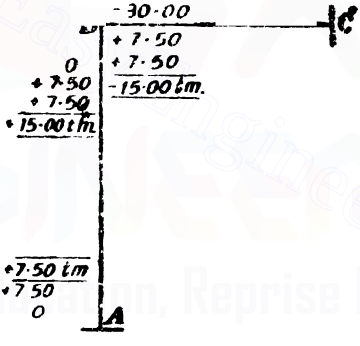

Fig. 709

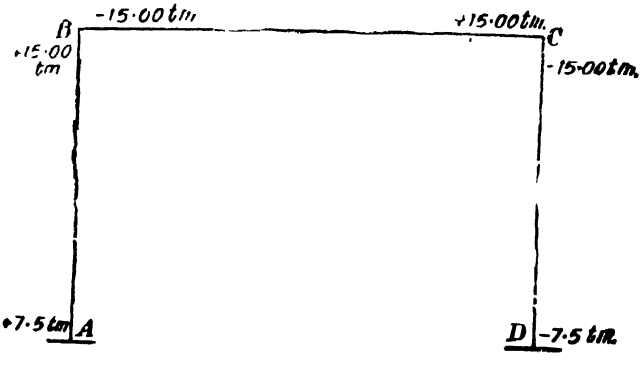

Fig. 710

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Design 166. *Analyse the frame shown in Fig. 711, taking advantage of the symmetry of the frame and loading.*

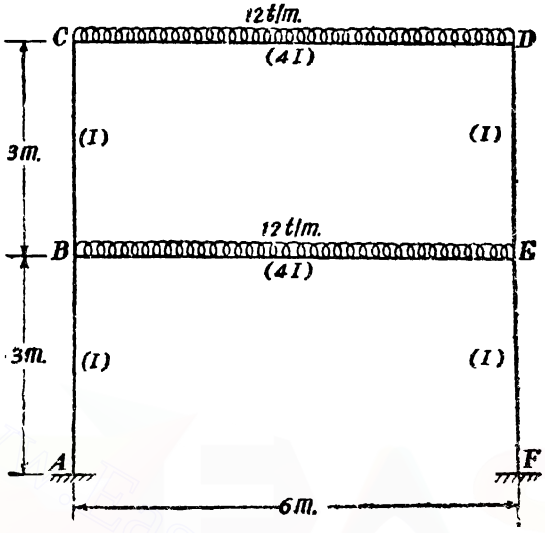

Fig. 711

Solution. Fixed end moments

$$M_{cd} = -\frac{12 \times 6^2}{12} = -36 \text{ tm.}$$

$$M_{bc} = -36 \text{ tm.}$$

The substitute frame to be considered is shown in Fig. 712.

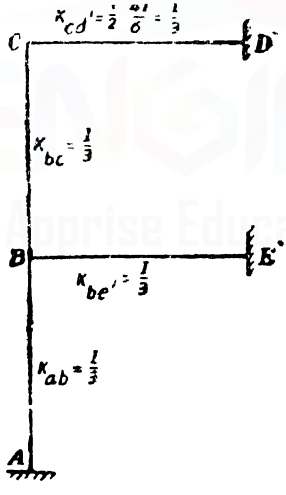

Fig. 712

For the substitute frame relative stiffness of CD'

$$= K'_{cd} = \frac{1}{2} K_{cd}$$

$$= \frac{1}{2} \cdot \frac{4I}{6} = \frac{I}{3}$$

Similarly the relative stiffness of the substitute member BE'

$$= K'_{be} = \frac{1}{2} K_{bc} = \frac{I}{3}$$

Now the rotation contributions at C and B can be computed. This computation is shown in Fig 713.

Now the end moments can be calculated easily. This calculation is shown in Fig. 714.

Fig. 715 shows the final moments for the frame.

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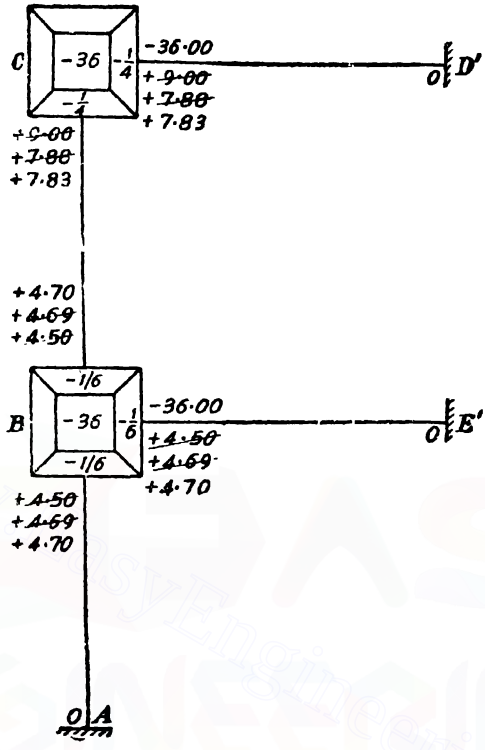

Fig. 713

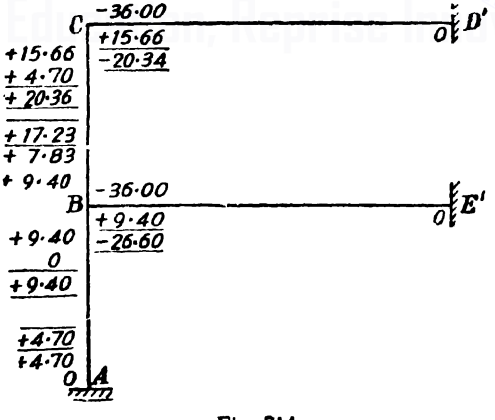

Fig. 714

Case 2. *When the axis of symmetry passes through the column.*

This case occurs when the number of bays is an even number.

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Since the frame is symmetrical and the loading is also symmetrical the joints in the axis of symmetry will not rotate.

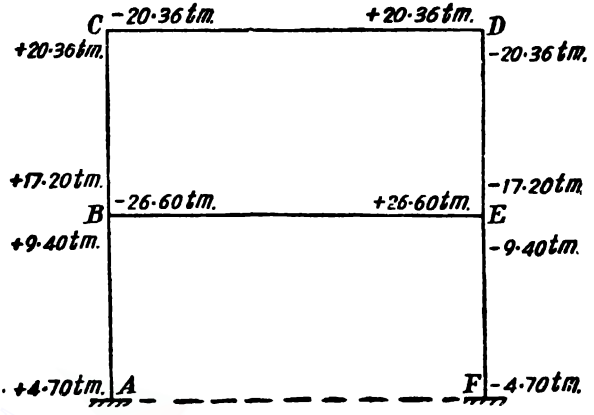

Fig. 715

For instance, let *DEF* represent the column lying in the axis of symmetry

If the joint *E* rotates the deformed shape of members meeting at *E* is shown in Fig. 716.

If the deformation of the members must also be symmetrical the column cannot deform as shown at *E*. The column *DEF* should therefore lie on the axis of symmetry Hence all joints lying on the axis of symmetry can be considered as encastered. Hence it is sufficient if half the frame is analysed. The following problems illustrate this case.

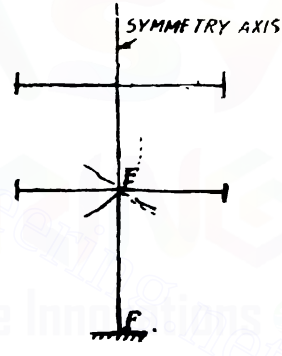

Fig. 716

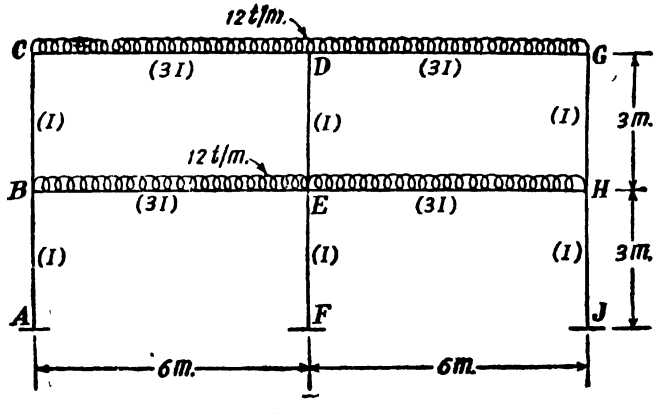

Fig. 717

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Design 167 *Analyse the frame shown in Fig. 71 taking advantage of the symmetry of frame and loading.*

Solution We will only consider one-half of the frame

Since the structure contains two bays *i.e.* an even number of bays the joints at *D* and *E* will be considered as encastered.

Hence the substitute frame which we will consider is as shown in Fig. 718.

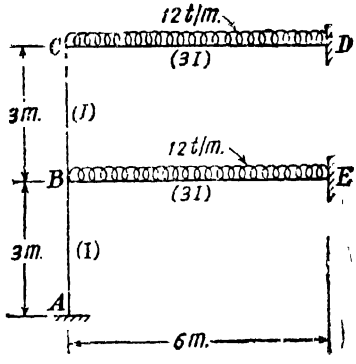

Fig. 718

Fixed end moments

$$\overline{M}_{cd} = -\frac{12 \times 6^2}{12} = -36 \text{ tm.}$$

and $\overline{M}_{bc} = +36$ tm.

The rotation contributions at *C* and *B* can now be easily computed. These are shown in Fig. 719.

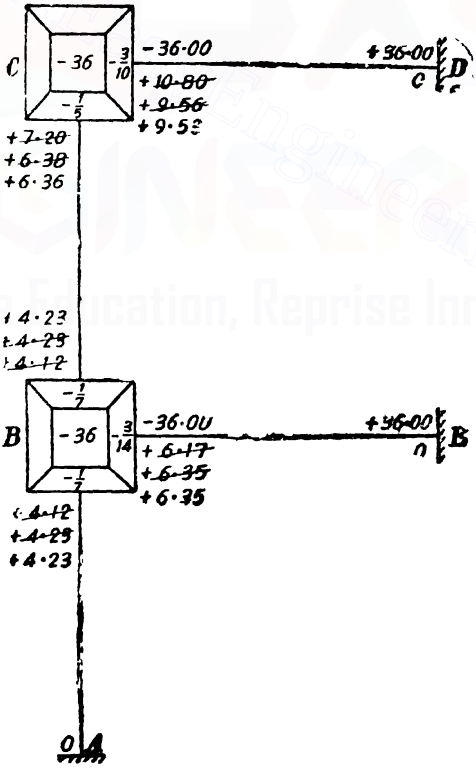

Fig. 719

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The rotation contributions being known the final moments can be easily evaluated. These are shown in Fig. 720.

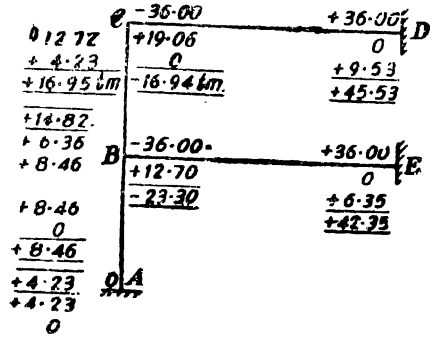

Fig 720

The moments at all the joints for the whole frame are shown in Fig. 721.

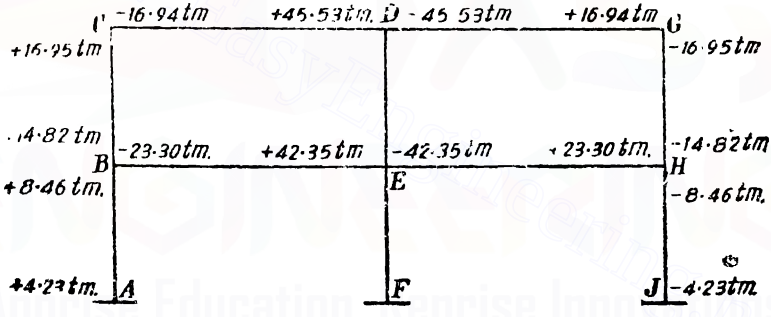

Fig 721

Design 168. *Analyse the frame shown in Fig. 722.*

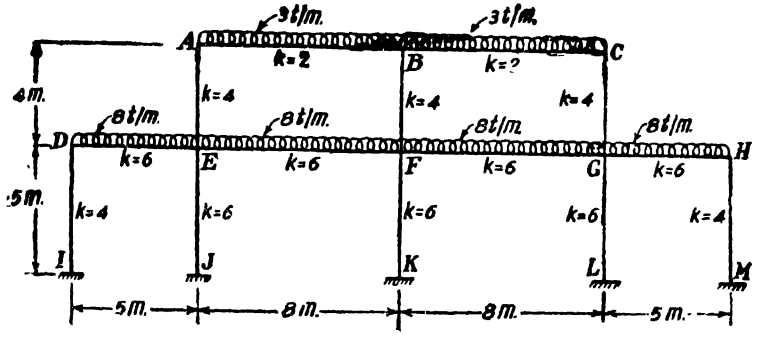

Fig. 722

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Solution. We will only consider one-half of the frame, as the frame is symmetrical and symmetrically loaded. Since in each storey there are even number of bays the joints B and F will be considered as encastered. Hence the substitute frame which we will consider is as shown in Fig. 723.

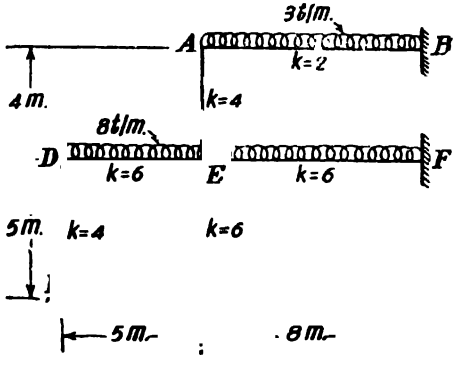

Fig. 723

Fixed end moments

$$\overline{M}_{ab} = -\frac{3 \times 8^2}{12} \text{ tm.}$$

$$= -16 \text{ tm.}$$

$$\overline{M}_{ba} = +16 \text{ tm.}$$

$$\overline{M}_{ef} = -16 \text{ tm.}$$

$$\overline{M}_{fe} = +16 \text{ tm.}$$

$$\overline{M}_{de} = -\frac{8 \times 5^2}{12}$$

$$= -16{\cdot}67 \text{ tm.}$$

$$\overline{M}_{ed} = +16{\cdot}67 \text{ tm.}$$

Rotation factors. These are computed as shown in the following table :

Joint	Member	Relative stiffness K	Total stiffness ΣK	Rotation factor $\frac{K}{\Sigma K}(-\frac{1}{2})$
A	AE	4	6	$\frac{4}{6}(-\frac{1}{2}) = -\frac{1}{3}$
	AB	2		$\frac{2}{6}(-\frac{1}{2}) = -\frac{1}{6}$
E	EA	4	22	$\frac{4}{22}(-\frac{1}{2}) = -\frac{1}{11}$
	EF	6		$\frac{6}{22}(-\frac{1}{2}) = -\frac{3}{22}$
	EJ	6		$\frac{6}{22}(-\frac{1}{2}) = -\frac{3}{22}$
	ED	6		$\frac{6}{22}(-\frac{1}{2}) = -\frac{3}{22}$
D	DE	6	10	$\frac{6}{10}(-\frac{1}{2}) = -\frac{3}{10}$
	DI	4		$\frac{4}{10}(-\frac{1}{2}) = -\frac{1}{5}$

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Now the rotation contributions can be easily computed. These computations are shown in Fig. 724.

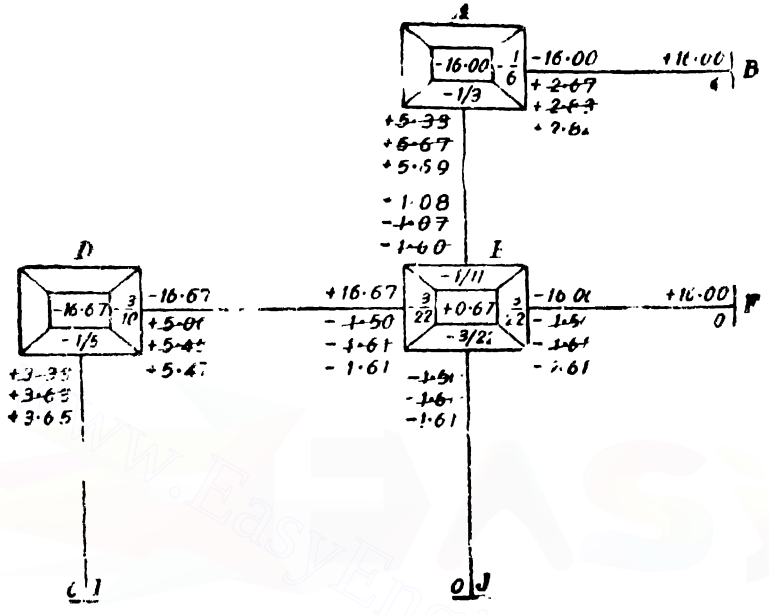

Fig. 724

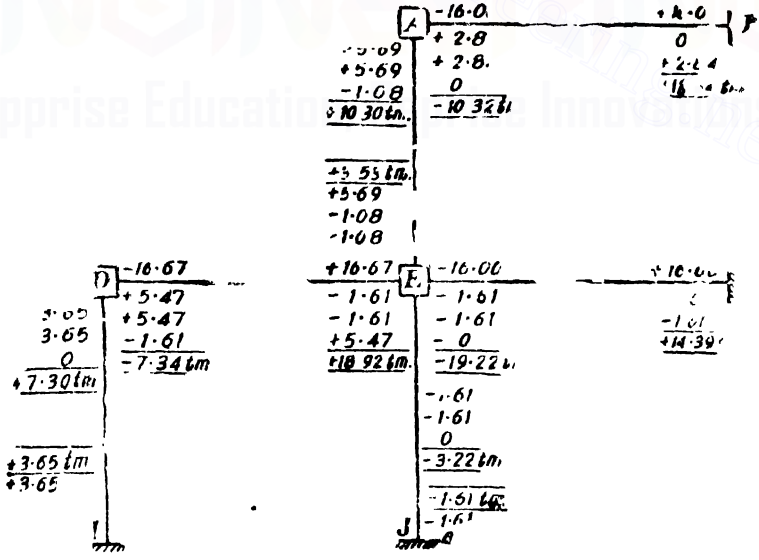

Fig. 725

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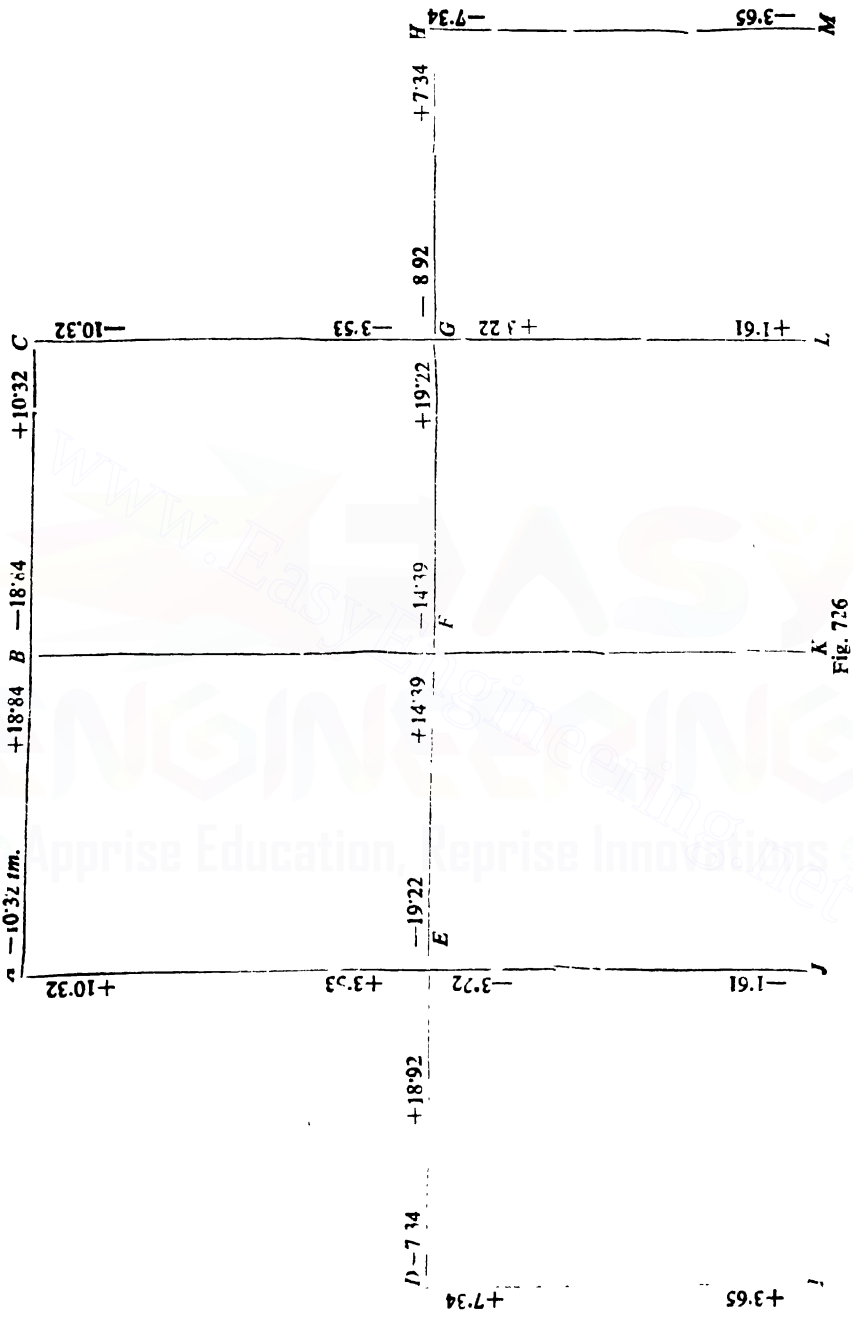

Fig. 726

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The order of computations of the rotation contributions is as follows :

Joint A, Joint D, Joint E.

Three cycles of computations have been made.

After determining rotation contributions the final moments can be easily determined. These are shown in Figs. 725 and 726.

Analysis of frames with lateral sway—Vertical loading

Let PQ represent a vertical member of any storey of a multi-storeyed frame

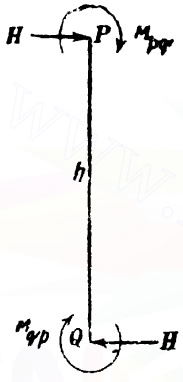

Fig. 727

Let M_{pq} and M_{qp} be the end moments at P and Q.

Let the horizontal forces exerted by the frame on the member PQ at P and Q be H.

Let h be the height of the member PQ.

For the condition of equilibrium of the member PQ, we have,

$$M_{pq}+M_{qp}+Hh=0$$

$$\therefore \quad H=-\frac{M_{pq}+M_{qp}}{h}$$

Now H represents the shear force at any section of the member PQ

Let PQ, EF, GH, JK ..be the vertical members of a storey.

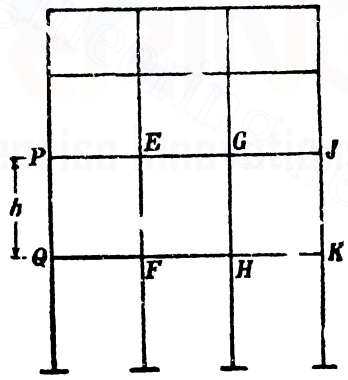

Fig. 728

Applying the above relation to all the columns of the storey,

$$\Sigma H=-\frac{\Sigma M_{pq}+\Sigma M_{qp}}{h}$$

If, in general, PQ, EF, GH, JK are the columns of rth storey then,

$\Sigma H=S_r=$ sum of shear forces in all the columns of the rth storey.

$$\therefore \quad S_r=-\frac{(\Sigma M_{pq}+\Sigma M_{qp})}{h_r} \text{ for the } r^{th} \text{ storey} \qquad ...(vii)$$

Where h_r = height of the columns of the rth storey.

In the above summation,

ΣM_{pq} represents the sum of the end moments at the upper ends of all the columns of the rth storey.

ΣM_{qp} represents the sum of the end moments at the lower ends

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of all the columns of the rth storey.

The quantity S_r may be called the *storey shear* for the rth storey.

Obviously the storey shear for the rth storey should be equal to the sum of all horizontal external loads above the rth storey. But in our case where the external loading is vertical, for each storey the storey shear equals zero. Further all columns of the rth storey have the same height h_r.

$\therefore \quad \Sigma M_{pq} + \Sigma M_{qp} = 0$, for the rth storey ...(viii)

We also know the general expressions for the end moments of the member PQ, namely

$$M_{pq} = \bar{M}_{pq} + 2M'_{pq} + M'_{qp} + M''_{pq}$$

and $\quad M_{qp} = \bar{M}_{qp} + 2M'_{qp} + M'_{pq} + M''_{pq}$

where $\quad M'_{pq}$ = rotation contribution of the end P

M'_{qp} = rotation contribution of the end Q

M''_{pq} = displacement contribution of PQ

$\bar{M}_{pq}$ = fixed end moment at P

and $\quad M_{pq}$ = fixed end moment at Q

Since the loading on the frame is vertical,

$$\bar{M}_{pq} = 0 \text{ and } M_{qp} = 0$$

$\therefore \quad M_{pq} + M_{qp} = 3M'_{pq} + 3M'_{qp} + 2M''_{pq}$

But $\quad \Sigma M_{pq} + \Sigma M_{qp} = 0$ ...(vii)

$\therefore \quad 3\Sigma M'_{pq} + 3\Sigma M'_{qp} + 2\Sigma M''_{pq} = 0$

$\therefore \quad \Sigma M''_{pq} = \frac{3}{2}(\Sigma M'_{pq} + \Sigma M'_{qp})$ (ix)

The above equation is the relation between the rotation contributions and the displacement contributions.

Now, we know, for any member the displacement contribution

$$= \frac{6EI\delta}{l^2}$$

Consider all the columns of a storey.

For all these columns,

The relative lateral displacement δ is the same. The length l of each column is the same. The Young's modulus E is assumed to be the same.

$\therefore$ Displacement contribution for a column of a storey is proportional to the moment of inertia of the section of the column.

$\therefore \quad M''_{pq} \propto I$

But relative stiffness $K = \frac{I}{l}$

$\therefore \quad M''_{pq} \propto K$ since I has the same value for all the columns of the storey

$$\frac{M''_{pq}}{\Sigma M''_{pq}} = \frac{K_{pq}}{\Sigma K_{pq}}$$

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$$M''_{pq} = \frac{K_{pq}}{\Sigma K_{pq}} \cdot M''_{p} \qquad \ldots(x)$$

From equations (*ix*) and (x), we have,

$$M''_{pq} = \frac{K_{pq}}{\Sigma K_{pq}} \left(-\frac{3}{2}\right)\left[\Sigma M'_{pq} + \Sigma M'_{qp}\right] \qquad \ldots(xi)$$

The quantity $\frac{K_{pq}}{\Sigma K_{pq}} \left(-\frac{3}{2}\right)$ is called the *displacement factor* for the member PQ.

It may be noted that in equation (*xi*) $\Sigma M'_{pq} + \Sigma M'_{qp}$ represents the sum of rotation contributions of the top and bottom ends of all the columns of the storey considered.

ΣK_{pq} represents the sum of the relative stiffnesses of all the columns of the storey considered.

Obviously the sum of the displacement factors of all the columns of a storey will be equal to $-\frac{3}{2}$.

The various equations obtained earlier are give below :

$$M_{pq} = \overline{M}_{pq} + 2M'_{pq} + M'_{qp} + M''_{pq} \qquad \ldots(i)a$$

$$M'_{pq} = U_{pq}(\Sigma \overline{M}_{pq} + \Sigma \overline{M'}_{pq} + \Sigma M''_{pq}) \qquad \ldots(vi)a$$

$$M''_{pq} = V_{pq}(\Sigma M'_{pq} + \Sigma M'_{qp}) \text{ for the storey considered} \qquad \ldots(xi)$$

where V_{pq} = displacement factor for PQ.

By the application of the above relations – Equations (*vi*)*a* and (*xi*) and similar relations for all the storeys, the rotation and distribution contributions may be determined by iteration. Once the rotation and displacement contributions are known the final moments may be determined by equations of the type (*i*)*a*.

Design 169. *Determine the moments at ABCD for the portal frame shown in Fig. 729.*

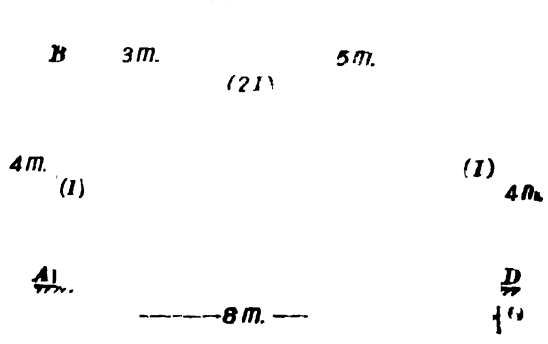

Fig. 729

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Solution. Fixed end moments

$$\overline{M}_{ab} = \overline{M}_{ba} = \overline{M}_{cd} = \overline{M}_{dc} = 0$$

$$\overline{M}_{bc} = -\frac{1 \times 3 \times 5^2}{8^2} \text{ tm.}$$

$$= -1875 \text{ tm.}$$

$$\overline{M}_{cb} = +\frac{16 \times 3^2 \times 5}{8^2} \text{ tm.} = +11{\cdot}25 \text{ tm.}$$

Rotation factors. These are computed in the following table :

Joint	Member	Relative stiffness K	Total relative stiffness ΣK	Rotation factor $\frac{K}{\Sigma K}\left(-\frac{1}{2}\right)$
B	BA	$\frac{I}{4}$	$\frac{2I}{4}$	$\frac{1}{2}\left[-\frac{1}{2}\right] = -\frac{1}{4}$
	BC	$\frac{2I}{8} = \frac{I}{4}$		$\frac{1}{2}\left[-\frac{1}{2}\right] = -\frac{1}{4}$
C	CB	$\frac{2I}{8} = \frac{I}{4}$	$\frac{2I}{4}$	$\frac{1}{2}\left[-\frac{1}{2}\right] = -\frac{1}{4}$
	CD	$\frac{I}{4}$		$\frac{1}{2}\left[-\frac{1}{2}\right] = -\frac{1}{4}$

Displacement factors. These are computed in the following table :

Vertical member	Relative stiffness K	Total stiffness ΣK	Displacement factors $\frac{K}{\Sigma K}\left[-\frac{3}{2}\right]$
AB	$\frac{I}{4}$	$\frac{2I}{4}$	$\frac{1}{2}\left[-\frac{3}{2}\right] = -\frac{3}{4}$
CD	$\frac{I}{4}$		$\frac{1}{2}\left[-\frac{3}{2}\right] = -\frac{3}{4}$

The fixed end moments are entered as shown in the commencement scheme (Fig. 730)

Total fixed end moments at B and C are entered within the square boxes The rotation factors are entered at their usual places. The displacement factors are written by the side of each column.

Now the iteration process starts. The various computations may follow the following order :

Joint B, Joint C, member AB and member CD.

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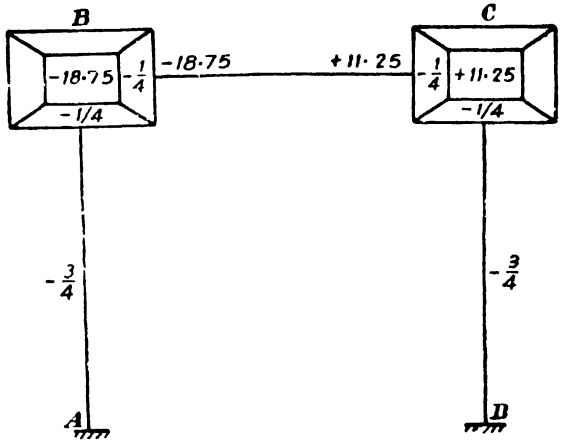

Fig. 730

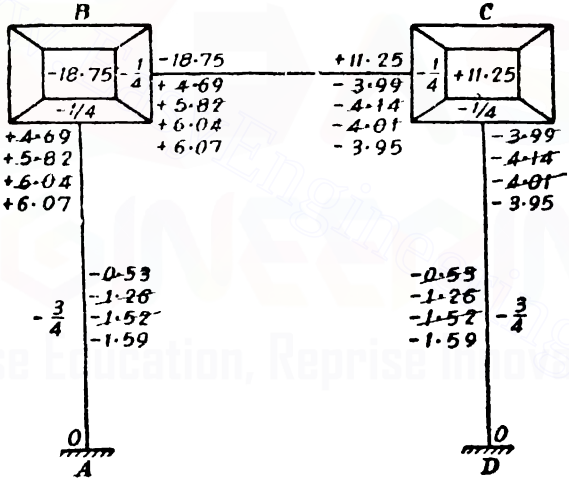

Fig. 731

Explanation of the iteration process

Cycle 1

Joint B Sum of fixed end moments		= −18·75
Rotation contributions of far ends :		
At *A*		= 0
At *C* (assumed)		= 0
Sum of displacement contributions :		
BC		= 0
AB (assumed)		= 0
	Total	−18·75

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$M'_{bc} = -\frac{1}{2}(-18{\cdot}75) = +4{\cdot}69$ *tm.*
$M'_{ba} = -\frac{1}{2}(-18{\cdot}75) = +4{\cdot}69$ *tm.*

Joint C. Sum of fixed end moments		$= +11{\cdot}25$
Rotation contributions of far ends :		
At *B*		$= +\ 4{\cdot}69$
At *D*		$= +\ 0$
Sum of displacement contributions :		
CB		$=\ 0$
CD (assumed)		$=\ 0$
	Total	$= +15{\cdot}94$

$\therefore \quad M'_{ab} = -\frac{1}{2}(15{\cdot}94) = -3{\cdot}99$ *tm.*
$M'_{cd} = -\frac{1}{2}(15{\cdot}94) = -3{\cdot}99$ *tm.*

Storey one (there is only one storey)		
Rotation contribution at top of column *AB*		$= +\ 4{\cdot}69$
Rotation contribution at top of column *CD*		$= -\ 3{\cdot}99$
Rotation contribution at bottom of columns		$=\ 0$
	Total	$= +\ 0{\cdot}70$ *tm.*

$M''_{ab} = -\frac{1}{2}(0{\cdot}70) = \ \ 0{\cdot}53$ *tm.*
$M''_{cd} \ \ -\frac{1}{2}(0{\cdot}70) = -0{\cdot}53$ *tm.*

Cycle 2

Joint B. Sum of fixed end moments		$= -18{\cdot}75$
Rotation contributions of far ends :		
At *A*		$=\ 0$
At *C*		$= -3{\cdot}99$
Sum of displacement contributions :		
BC		$=\ 0$
BA		$= -\ 0{\cdot}53$
	Total	$= -23{\cdot}27$

$\therefore \quad M'_{bc} = -\frac{1}{2}(-23{\cdot}27) = +5{\cdot}82$ *tm.*
$M'_{ba} = -\frac{1}{2}(-23{\cdot}27) = +5{\cdot}82$ *tm.*

Joint C. Sum of fixed end moments		$= +11{\cdot}25$
Rotation contributions of far ends :		
At *B*		$= +\ 5{\cdot}82$
At *D*		$=\ 0$
Sum of displacement contributions :		
CB		$=\ 0$
CD		$= -\ 0.53$
	Total	$= +16{\cdot}54$

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$\therefore$ $M'_{ab} = -\frac{1}{4}(16{\cdot}54) = -4{\cdot}14$ *tm.*

$M'_{cd} = -\frac{1}{4}(16{\cdot}54) = -4{\cdot}14$ *tm.*

Storey one

Rotation contributions at top of column *AB*	= + 5·82
Rotation contributions at top of column *CD*	= − 4·14
Rotation contributions at bottom of columns	= 0
Total	= + 1·68

$\therefore$ $M''_{ab} = -\frac{3}{4}(1{\cdot}68) = -1{\cdot}26$ *tm.*

$M''_{cd} = -\frac{3}{4}(1{\cdot}68) = -1{\cdot}26$ *tm.*

Cycle 3

Joint B. Sum of fixed ends moments	= −18·75
Rotation contributions of far ends :	
At *A*	= 0
At *C*	= − 4·14
Sum of displacement contributions :	
BC	= 0
BA	= − 1·26
Total	= −24·15

$\therefore$ $M'_{bc} = -\frac{1}{4}(-24{\cdot}15) = +6{\cdot}04$ *tm.*

$M'_{ba} = -\frac{1}{4}(-24{\cdot}15) = +6{\cdot}04$ *tm.*

Joint C. Sum of fixed end moments	= +11·25
Rotation contribution of far ends :	
At *B*	= + 6·04
At *D*	= 0
Sum of displacement contributions	
CB	= 0
CD	= − 1·26
Total	= +16·03

$\therefore$ $M'_{cb} = -\frac{1}{4}(16{\cdot}03) = -4{\cdot}01$ *tm.*

$M'_{cd} = -\frac{1}{4}(16{\cdot}03) = -4{\cdot}01$ *tm.*

Storey one. Rotation contribution at top of column *AB*	= + 6·04
Rotation contribution at top of column *CD*	= − 4·01
Rotation contributions at bottom of columns	= 0
Total	= + 2·03

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$\therefore$ $M''_{ab} = -\frac{3}{4}(2{\cdot}03) = -1{\cdot}52$ *tm.*

$M''_{cd} = -\frac{3}{4}(2{\cdot}03) = -1{\cdot}52$ *tm.*

Cycle 4

Joint B. Sum of fixed end moments		$= -18{\cdot}75$
Rotation contributions of far ends :		
At *A*		$= 0$
At *C*		$= -4{\cdot}01$
Sum of displacement contributions :		
BC		$= 0$
BA		$= -1{\cdot}52$
	Total	$= -24{\cdot}28$

$\therefore$ $M'_{bc} = -\frac{1}{2}(-24{\cdot}28) = +6{\cdot}07$ *tm.*

$M'_{ba} = -\frac{1}{2}(-24{\cdot}28) = +6{\cdot}07$ *tm.*

Joint C. Sum of fixed end moments		$= +11{\cdot}25$
Rotation contributions of far ends :		
At *B*		$= +6{\cdot}07$
At *D*		$= 0$
Sum of displacement contributions		
CB		$= 0$
CD		$= -1{\cdot}52$
	Total	$= +15{\cdot}80$

$\therefore$ $M'_{cb} = -\frac{1}{2}(15{\cdot}80) = -3{\cdot}95$ *tm.*

$M'_{cd} = -\frac{1}{2}(15{\cdot}80) = -3{\cdot}95$ *tm.*

Storey one

Rotation contribution at top of column *AB*		$= +6{\cdot}07$
Rotation contribution at top of column *CD*		$= -3{\cdot}95$
Rotation contributions at bottom of columns		$= 0$
	Total	$= +2{\cdot}12$

$M''_{ab} = -\frac{3}{4}(2{\cdot}12) = -1{\cdot}59$ *tm.*

$M''_{cd} = -\frac{3}{4}(2{\cdot}12) = -1{\cdot}59$ *tm.*

Now that the rotation contributions and displacement contributions are known, the final moments can be easily computed. These calculations are shown in Fig. 752.

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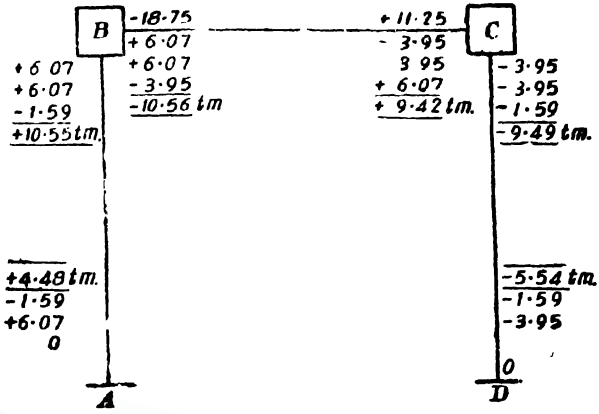

Fig. 732

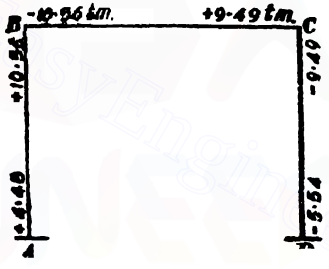

Fig. 733. Final moments

Design 170. *Determine the end moments at the various joints of the frame shown in Fig. 734.*

Solution. Fixed end moments

$$\bar{M}_{ab} = -\frac{6 \times 4^2}{12} = -8{\cdot}00 \text{ tm.}$$

$$\bar{M}_{ba} = +8{\cdot}00 \text{ tm.}$$

$$\bar{M}_{bc} = -8{\cdot}00 \text{ tm.}$$

$$\bar{M}_{cb} = +8{\cdot}00 \text{ tm.}$$

$$\bar{M}_{cd} = -\frac{6 \times 3^2}{12} = -4{\cdot}50 \text{ tm.}$$

$$\bar{M}_{dc} = +4{\cdot}50 \text{ tm.}$$

$$\bar{M}_{ef} = -\frac{8 \times 4^2}{12} = -10{\cdot}67 \text{ tm.}$$

$$\bar{M}_{fe} = +10{\cdot}67 \text{ tm.}$$

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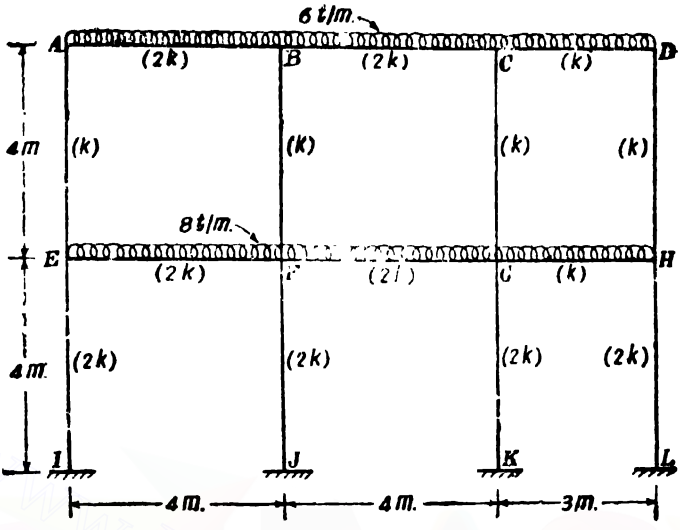

Fig. 734

$\overline{M}_{fo} = -10{\cdot}67$ *tm.*

$\overline{M}_{of} = +10{\cdot}67$ *tm.*

$$\overline{M}_{gh} = -\frac{8 \times 3^2}{12} = -6{\cdot}00 \text{ tm.}$$

$\overline{M}_{hg} = +6{\cdot}00$ *tm.*

Displacement factors

These have been computed in the following table :

Storey	*Vertical member*	*Relative stiffness*	*Total Relative stiffness*	*Displacement factor*
No. 1 (Ground storey)	IE	2K	8K	$\frac{2}{8}\left[-\frac{3}{2}\right] = -\frac{3}{8}$
	JF	2K		$\frac{2}{8}\left[-\frac{3}{2}\right] = -\frac{3}{8}$
	KG	2K		$\frac{2}{8}\left[-\frac{3}{2}\right] = -\frac{3}{8}$
	LH	2K		$\frac{2}{8}\left[-\frac{3}{2}\right] = -\frac{3}{8}$
No. 2	EA	K	4K	$\frac{1}{4}\left[-\frac{3}{2}\right] = -\frac{3}{8}$
	FB	K		$\frac{1}{4}\left[-\frac{3}{2}\right] = -\frac{3}{8}$
	GC	K		$\frac{1}{4}\left[-\frac{3}{2}\right] = -\frac{3}{8}$
	HD	K		$\frac{1}{4}\left[-\frac{3}{8}\right] = -\frac{3}{8}$

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Rotation factors

These have been computed in the following table.

The rotation contributions and displacement contributions can now be determined. See Fig. 7.35 and explanatory tables.

Joint	*Member*	*Relative stiffness*	*Total stiffness*	*Rotation factor*
A	AB	2 K	5 K	$\frac{2}{5}\left(-\frac{1}{2}\right) = -\frac{1}{5}$
	AF	K		$\frac{1}{5}\left(-\frac{1}{2}\right) = -\frac{1}{10}$
B	BA	2 K	5 K	$\frac{2}{5}\left(-\frac{1}{2}\right) = -\frac{1}{5}$
	BC	2 K		$\frac{2}{5}\left(-\frac{1}{2}\right) = -\frac{1}{5}$
	BF	K		$\frac{1}{5}\left(-\frac{1}{2}\right) = -\frac{1}{10}$
C	CB	2 K	4 K	$\frac{2}{4}\left(-\frac{1}{2}\right) = -\frac{1}{4}$
	CD	K		$\frac{1}{4}\left(-\frac{1}{2}\right) = -\frac{1}{8}$
	CG	K		$\frac{1}{4}\left(-\frac{1}{2}\right) = -\frac{1}{8}$
D	DC	K	2 K	$\frac{1}{2}\left(-\frac{1}{2}\right) = -\frac{1}{4}$
	DH	K		$\frac{1}{2}\left(-\frac{1}{2}\right) = -\frac{1}{4}$
E	EA	K	5 K	$\frac{1}{5}\left(-\frac{1}{2}\right) = -\frac{1}{10}$
	EF	2 K		$\frac{2}{5}\left(-\frac{1}{2}\right) = -\frac{1}{5}$
	EI	2 K		$\frac{2}{5}\left(-\frac{1}{2}\right) = -\frac{1}{5}$
F	FE	2 K	7 K	$\frac{2}{7}\left(-\frac{1}{2}\right) = -\frac{1}{7}$
	FB	K		$\frac{1}{7}\left(-\frac{1}{2}\right) = -\frac{1}{14}$
	FG	2 K		$\frac{2}{7}\left(-\frac{1}{2}\right) = -\frac{1}{7}$
	FJ	2 K		$\frac{2}{7}\left(-\frac{1}{2}\right) = -\frac{1}{7}$
G	GF	2 K	6 K	$\frac{2}{6}\left(-\frac{1}{2}\right) = -\frac{1}{6}$
	GC	K		$\frac{1}{6}\left(-\frac{1}{2}\right) = -\frac{1}{12}$
	GH	K		$\frac{1}{6}\left(-\frac{1}{2}\right) = -\frac{1}{12}$
	GK	2 K		$\frac{2}{6}\left(-\frac{1}{2}\right) = -\frac{1}{6}$
H	HG	K	4 K	$\frac{1}{4}\left(-\frac{1}{2}\right) = -\frac{1}{8}$
	HD	K		$\frac{1}{4}\left(-\frac{1}{2}\right) = -\frac{1}{8}$
	HK	2 K		$\frac{2}{4}\left(-\frac{1}{2}\right) = -\frac{1}{4}$

Now the iteration process is completed to determine the rotation contributions and the displacement contributions (See Fig. 7.35). The relevant calculations for the various cycles in the iteration process are shown in the following tables A and B.

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Table A. Order of Calculations

First cycle	*Second cycle*	*Third cycle*
Joint A	**Joint A**	**Joint A**
−8·00	−8·00	−8·00
0	−0·53	−0·36
0	+0·93	+0·94
0	+0·17	+0·09
−8·00	−7·43	−7·33
$m'_{ab} = -\frac{1}{3}(-8{\cdot}00) = +2{\cdot}67$	$m'_{ab} = -\frac{1}{3}(-7{\cdot}43) = +2{\cdot}48$	$m'_{ab} = -\frac{1}{3}(-7{\cdot}33) = +2{\cdot}44$
$m'_{ae} = -\frac{1}{6}(-8{\cdot}00) = +1{\cdot}33$	$m'_{ae} = -\frac{1}{6}(-7{\cdot}43) = +1{\cdot}24$	$m'_{ae} = -\frac{1}{6}(-7{\cdot}33) = +1{\cdot}22$
Joint B	**Joint B**	**Joint B**
0	0	0
+2·67	+2·48	+2·44
+2·67	−0·74	−0·49
	−0·11	−0·09
	+0·17	+0·09
	+1·80	+1·95
$m'_{ba} = -\frac{1}{5}(2{\cdot}67) = -0{\cdot}53$	$m'_{ba} = -\frac{1}{5}(1{\cdot}80) = -0{\cdot}36$	$m'_{ba} = -\frac{1}{5}(1{\cdot}95) = -0{\cdot}39$
$m'_{bc} = -\frac{1}{5}(2{\cdot}67) = -0{\cdot}53$	$m'_{bc} = -\frac{1}{5}(1{\cdot}80) = -0{\cdot}36$	$m'_{bc} = -\frac{1}{5}(1{\cdot}95) = -0{\cdot}39$
$m'_{bf} = -\frac{1}{10}(2{\cdot}67) = -0{\cdot}27$	$m'_{bf} = -\frac{1}{10}(1{\cdot}80) = -0{\cdot}18$	$m'_{bf} = -\frac{1}{10}(1{\cdot}95) = -0{\cdot}20$

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First cycle	Second cycle	Third cycle
Joint C	**Joint C**	**Joint C**
	+3·50	+3·50
	−0·36	−0·39
	−1·03	−0·96
	−0·34	−0·33
+3·50	+0·17	+0·09
−0·53		
+2·97	+1·94	+1·91
$m'_{cb} = -\frac{1}{4}(2{\cdot}97) = -0{\cdot}74$	$m'_{cb} = -\frac{1}{4}(1{\cdot}94) = -0{\cdot}49$	$m'_{cb} = -\frac{1}{4}(1{\cdot}91) = -0{\cdot}48$
$m'_{cd} = -\frac{1}{8}(2{\cdot}97) = -0{\cdot}37$	$m'_{cd} = -\frac{1}{8}(1{\cdot}94) = -0{\cdot}24$	$m'_{cd} = -\frac{1}{8}(1{\cdot}91) = -0{\cdot}24$
$m'_{cg} = -\frac{1}{8}(2{\cdot}97) = -0{\cdot}37$	$m'_{cg} = -\frac{1}{8}(1{\cdot}94) = -0{\cdot}24$	$m'_{cg} = -\frac{1}{8}(1{\cdot}91) = -0{\cdot}24$
Joint D	**Joint D**	**Joint D**
	+4·50	
	−0·24	+4·50
	−0·58	−0·24
	+0·17	−0·62
+4·50		−0·09
−0·37	+3·85	
+4·13		+0·73
$m'_{de} = -\frac{1}{4}(4{\cdot}13) = -1{\cdot}03$	$m'_{dc} = -\frac{1}{4}(3{\cdot}85) = -0{\cdot}96$	$m'_{dc} = -\frac{1}{4}(3{\cdot}73) = -0{\cdot}93$
$m'_{dh} = -\frac{1}{4}(4{\cdot}13) = -1{\cdot}03$	$m'_{dh} = -\frac{1}{4}(3{\cdot}85) = -0{\cdot}96$	$m'_{dh} = -\frac{1}{4}(3{\cdot}73) = -0{\cdot}93$

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First cycle	*Second cycle*	*Third cycle*
Joint E	**Joint E**	**Joint E**
	−10·67	−10·67
	+ 1·24	+ 1·22
	− 0·23	− 0·18
	+ 0·17	+ 0·09
−10·67	+ 0·08	+ 0·07
+ 1·33	− 9·41	− 9·47
− 9·34		
$m'_{ea} = -\frac{1}{16}(-9\cdot34) = +0\cdot93$	$m'_{ea} = -\frac{1}{16}(-9\cdot41) = +0\cdot94$	$m'_{ea} = -\frac{1}{16}(-9\cdot47) = +0\cdot95$
$m'_{ef} = -\frac{1}{8}(-9\cdot34) = +1\cdot87$	$m'_{ef} = -\frac{1}{8}(-9\cdot41) = +1\cdot88$	$m'_{ef} = -\frac{1}{8}(-9\cdot47) = +1\cdot89$
$m'_{eg} = -\frac{1}{8}(-9\cdot34) = +1\cdot87$	$m'_{eg} = -\frac{1}{8}(-9\cdot41) = +1\cdot88$	$m'_{eg} = -\frac{1}{8}(-9\cdot47) = +1\cdot89$
Joint F	**Joint F**	**Joint F**
	0	0
	+1·88	+1·89
	−0·18	−1·20
	−0·68	−0·65
0	+0·17	+0·09
− 0·27	+0·08	+0·07
+ 1·87	+1·27	+1·20
+1·60		
$m'_{fe} = -\frac{1}{7}(1\cdot60) = -0\cdot23$	$m'_{fe} = -\frac{1}{7}(1\cdot27) = -0\cdot18$	$m'_{fe} = -\frac{1}{7}(1\cdot20) = -0\cdot17$
$m'_{fb} = -\frac{1}{16}(1\cdot60) = -0\cdot11$	$m'_{fb} = -\frac{1}{16}(1\cdot27) = -0\cdot09$	$m'_{fb} = -\frac{1}{16}(1\cdot20) = -0\cdot09$
$m'_{fg} = -\frac{1}{7}(1\cdot60) = -0\cdot23$	$m'_{fg} = -\frac{1}{7}(1\cdot27) = -0\cdot18$	$m'_{fg} = -\frac{1}{7}(1\cdot20) = -0\cdot17$
$m'_{fj} = -\frac{1}{7}(1\cdot60) = -0\cdot23$	$m'_{fj} = -\frac{1}{7}(1\cdot27) = -0\cdot18$	$m'_{fj} = -\frac{1}{7}(1\cdot20) = -0\cdot17$

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First cycle	Second cycle	Third cycle
Joint G	**Joint G**	**Joint G**
	+4·66	+4·67
	−0·18	−0·17
	−0·24	−0·24
	−0·58	−0·62
	+0·17	+0·09
	+0·08	+0·07
+4·67	+3·92	+3·80
−0·23		
−0·37		
+4·07		
$m'_{gf} = -\frac{1}{6}(4·07) = -0·68$	$m'_{gf} = -\frac{1}{6}(3·92) = -0·65$	$m'_{gf} = -\frac{1}{6}(3·80) = -0·63$
$m'_{ge} = -\frac{1}{1\,2}(4·07) = -0·34$	$m'_{ge} = -\frac{1}{1\,2}(3·92) = -0·35$	$m'_{ge} = -\frac{1}{1\,2}(3·80) = -0·32$
$m'_{gh} = -\frac{1}{1\,2}(4·07) = -0·34$	$m'_{gh} = -\frac{1}{1\,2}(3·92) = -0·33$	$m'_{gh} = -\frac{1}{1\,2}(3·80) = -0·32$
$m'_{gk} = -\frac{1}{6}(4·07) = -0·68$	$m'_{gk} = -\frac{1}{6}(6·92) = -0·65)$	$m'_{gk} = -\frac{1}{6}(3·80) = -0·63$
Joint H	**Joint H**	**Joint H**
	+6·00	+6·10
	−0·35	−0·32
	−0·95	−0·93
	−0·17	+0·09
	−0·08	+0·07
+6·00	+4·96	+4·91
−0·34		
−1·03		
+4·63		
$m'_{hg} = -\frac{1}{6}(4·63) = -0·58$	$m'_{hg} = -\frac{1}{6}(4·96) = -0·62$	$m'_{hg} = -\frac{1}{6}(4·91) = -0·61$
$m'_{hd} = -\frac{1}{6}(4·63) = -0·58$	$m'_{hd} = -\frac{1}{6}(4·96) = -0·62$	$m'_{hd} = -\frac{1}{6}(4·91) = -0·61$
$m'_{hl} = -\frac{1}{4}(4·63) = -1·16$	$m'_{hl} = -\frac{1}{4}(4·96) = -1·24$	$m'_{hl} = -\frac{1}{4}(4·91) = -0·91$

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Table B. Sway Correction (Displacement Contributions)

Storey	*After first cycle*	*After second cycle*	*After third cycle*
2 Upper storey	+1·33 −0·27 −0·37 −1·03 +0·93 −0.11 −0·34 −0·58 ——— −0 44 ———	+1·24 −0·18 −0·24 −0·96 +0·94 −0·09 −0·33 −0 62 ——— −0·24 ———	+1·22 −0·20 −0·24 −0·93 +0·95 −0·09 − 0·32 −0·61 ——— −0·22 ———
	$m''_{ae} = -\frac{3}{8}(-0{\cdot}44) = +0{\cdot}17$	$m''_{ae} = -\frac{3}{8}(-0{\cdot}24) = +0{\cdot}09$	$m''_{ae} = -\frac{3}{8}(-0{\cdot}22) = +0{\cdot}08$
	$m''_{bf} = -\frac{3}{8}(-0{\cdot}44) = +0{\cdot}17$	$m''_{bf} = -\frac{3}{8}(-0{\cdot}24) = +0\ 09$	$m''_{bf} = -\frac{3}{8}(-0{\cdot}22) = +0{\cdot}08$
	$m''_{cg} = -\frac{3}{8}(-0{\cdot}44) = +0{\cdot}17$	$m''_{cg} = -\frac{3}{8}(-0{\cdot}24) = +0{\cdot}09$	$m''_{cg} = -\frac{3}{8}(-0{\cdot}22) = +0{\cdot}08$
	$m''_{dh} = -\frac{3}{8}(-0{\cdot}44) = +0{\cdot}17$	$m''_{dh} = -\frac{3}{8}(-0{\cdot}24) = +0{\cdot}09$	$m''_{dh} = -\frac{3}{8}(-0\ 22) = +0{\cdot}08$

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Storey	After first cycle	After second cycle	After third cycle
1 Lower storey	+1·87	+1·88	+1·89
	−0·23	−0·18	−0·17
	−0·68	−0·65	−0·63
	−0·16	−1·24	−1·23
	———	———	———
	−0·20	−0·19	−0·14
	———	———	———
	$m''_{cd} = -\frac{3}{8}(-0·2) = +0·08$	$m''_{cd} = -\frac{3}{8}(-0·19) = +0·07$	$m''_{cd} = -\frac{3}{8}(-0·14) = +0·05$
	$m''_{fg} = -\frac{3}{8}(-0·2) = +0·08$	$m''_{fg} = -\frac{3}{8}(-0·19) = +0·07$	$m''_{fg} = -\frac{3}{8}(-0·14) = +0·05$
	$m''_{gk} = -\frac{3}{8}(-0·2) = +0·08$	$m''_{gk} = -\frac{3}{8}(-0·19) = +0·07$	$m''_{gk} = -\frac{3}{8}(-0·14) = +0·05$
	$m''_{hl} = -\frac{3}{8}(-0·2) = +0·08$	$m''_{hl} = -\frac{3}{8}(-0·19) = +0·07$	$m''_{hl} = -\frac{3}{8}(-0·14) = +0·05$

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Now the final moments can be computed. These have been computed below :

Horizontal members

$M_{ab} = -\ 8{\cdot}00 + 2{\cdot}44 + 2{\cdot}44 - 0{\cdot}39 = -\ 3{\cdot}51$ *tm.*
$M_{ba} = +\ 8{\cdot}00 - 0{\cdot}39 - 0{\cdot}36 + 2{\cdot}44 = +\ 9{\cdot}66$ *tm.*
$M_{bc} = -\ 8{\cdot}00 - 0{\cdot}39 - 0{\cdot}39 - 0{\cdot}48 = -\ 9{\cdot}26$ *tm.*
$M_{cb} = +\ 8{\cdot}00 - 0{\cdot}48 - 0{\cdot}48 - 0{\cdot}39 = +\ 6{\cdot}65$ *tm.*
$M_{cd} = -\ 4{\cdot}50 - 0{\cdot}24 - 0{\cdot}24 - 0{\cdot}93 = -\ 5{\cdot}91$ *tm.*
$M_{dc} = +\ 4{\cdot}50 - 0\ 93 - 0{\cdot}93 - 0{\cdot}24 = +\ 2{\cdot}40$ *tm.*
$M_{ef} = -\ 10{\cdot}67 + 1{\cdot}89 + 1{\cdot}89 - 0{\cdot}17 = -\ 7{\cdot}06$ *tm.*
$M_{fe} = -\ 10{\cdot}67 - 0{\cdot}17 - 0{\cdot}17 + 1{\cdot}89 = +\ 12{\cdot}22$ *tm.*
$M_{fg} = -\ 10{\cdot}67 - 0{\cdot}17 - 0{\cdot}17 - 0{\cdot}63 = -\ 11{\cdot}64$ *tm.*
$M_{gf} = +\ 4{\cdot}67 - 0{\cdot}63 - 0{\cdot}63 - 0{\cdot}17 = +\ 3{\cdot}24$ *tm.*

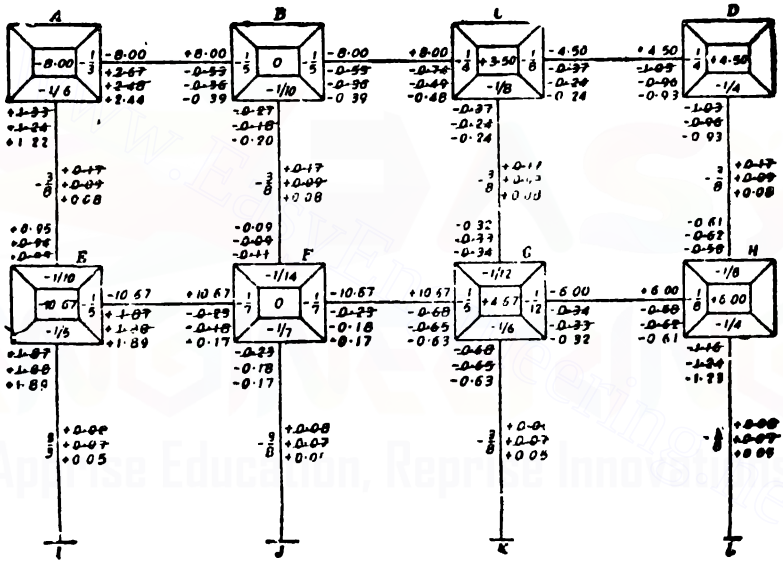

Fig. 735

Vertical members

$M_{ae} = +1{\cdot}22 + 1{\cdot}22 + 0{\cdot}95 + 0{\cdot}08 = +3{\cdot}47$ *tm.*
$M_{ea} = +0{\cdot}95 + 0{\cdot}95 + 1{\cdot}22 + 0{\cdot}08 = +3{\cdot}20$ *tm.*
$M_{bf} = -0{\cdot}20 - 0{\cdot}20 - 0{\cdot}09 + 0{\cdot}08 = -0{\cdot}41$ *tm.*
$M_{fb} = -0{\cdot}09 - 0{\cdot}09 - 0{\cdot}20 + 0{\cdot}08 = -0{\cdot}30$ *tm.*
$M_{cg} = -0{\cdot}24 - 0{\cdot}24 - 0{\cdot}32 + 0{\cdot}08 = -0{\cdot}72$ *tm.*
$M_{gc} = -0{\cdot}32 - 0{\cdot}32 - 0{\cdot}24 + 0{\cdot}08 = -0{\cdot}80$ *tm.*
$M_{dh} = -0{\cdot}93 - 0{\cdot}93 - 0{\cdot}61 + 0{\cdot}08 = -2{\cdot}39$ *tm.*
$M_{hd} = -0{\cdot}61 - 0{\cdot}61 - 0{\cdot}93 + 0{\cdot}08 = -2{\cdot}07$ *tm.*
$M_{ei} = +1{\cdot}89 + 1{\cdot}89 + 0{\cdot}05 = +3{\cdot}83$ *tm.*
$M_{fj} = -0{\cdot}17 - 0{\cdot}17 + 0{\cdot}05 = -0{\cdot}29$ *tm.*
$M_{gk} = -0{\cdot}63 - 0{\cdot}63 + 0{\cdot}05 = -1{\cdot}21$ *tm.*
$M_{hl} = -1{\cdot}23 - 1{\cdot}23 + 0{\cdot}05 = -2{\cdot}41$ *tm.*

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19

Reinforced Concrete Chimneys

One of the structures for which concrete is most suited is a chimney. A chimney may be regarded as a vertical cantilever supported at its base. It is subjected to the following loads :

(*i*) The self-weight of the chimney, and

(*ii*) Wind load on the chimney.

Design formulae

The following notations have been used in the formulae derived.

W = Self-weight of the chimney above any section under consideration (N) or ($kg.$)

M = B.M. at the section considered ($N\,cm.$) or ($kg.\,cm.$)

m = Modular ratio between steel and concrete

c = Maximum compressive stress intensity in concrete taken at the mean circumference ($N/cm.^2$) or ($kg./cm.^2$)

f_s = Tensile stress intensity in steel ($N/cm.^2$) or ($kg./cm.^2$)

D = mean diameter of the chimney ($cm.$)

r = Mean radius of the chimney ($cm.$)

t_s = Thickness of an equivalent steel shell of mean radius r, whose cross-section equals to the total area of reinforcement ($cm.$)

t = Total thickness of the chimney wall ($cm.$)

t_c = Thickness of the concrete only ($cm.$)

A_s = Total area of steel at the section considered ($cm.^2$)

n_1 = Ratio of the distance of the neutral axis from the mean circumference on the compression side to the mean diameter D

l = Distance between the centre of compression and centre of tension.

See Fig 736. Since the intensity of compressive stress in the concrete at the mean circumference is c, the corresponding compressive stress in the steel at this point will be mc. The stress diagram across a section on the equivalent steel section is shown in the figure.

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From the geometry of the stress diagram, we have

$$\frac{mc}{f_c} = \frac{n_1 D}{D - n_1 D} = \frac{n_1}{1 - n_1}$$

$$n_1 = \frac{1}{1 + \frac{mc}{\quad}} \qquad \ldots(i)$$

From the above relation the position of the neutral axis can be determined.

Let the angle subtended by the portion in compression at the centre be 2α.

We have $\cos\alpha = \dfrac{\frac{D}{2} - n_1 D}{\frac{D}{2}} = (1 - 2n_1)$

$$\therefore \quad \cos\alpha = 1 - 2n_1 \qquad \ldots(ii)$$

From the above relation for any given value of n_1, the quantity $\cos\alpha$ can be determined. Hence α and $\sin\alpha$ also can be determined. The neutral axis may be located for any given combination of c, f_c and m

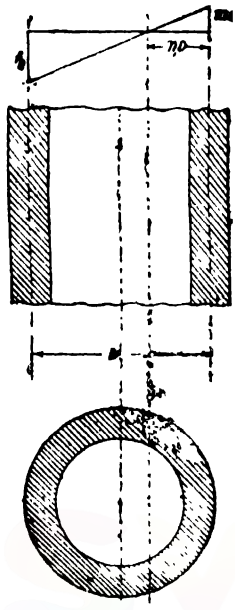

Fig. 735

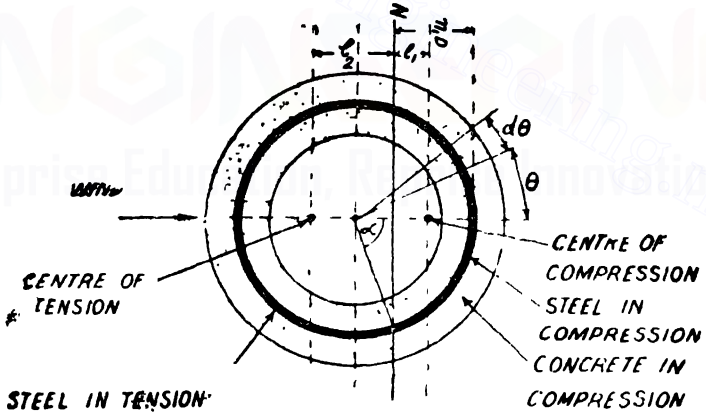

Fig. 737

Consider a small element subtending an angle $d\theta$ at the centre as shown in the figure.

Length of this element $= r d\theta$

We have for this element,

Actual area of concrete $= t_c\, r d\theta$

Actual area of steel $= t_s\, r d\theta$

This element is at a distance of $r(\cos\theta - \cos\alpha)$ from the neutral axis. But the distance between the neutral axis and the most

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distant fibre in the compression zone equals $r(1-\cos\theta)$ and at this distance the stress in concrete is c.

$\therefore$ Stress in concrete in the element

$$=\frac{r(\cos\theta-\cos\alpha)}{r(1-\cos\alpha)}\,c.$$

and the stress in steel in the element

$$=\frac{r(\cos\theta-\cos\alpha)}{r(1-\cos\alpha)}\,mc.$$

$\therefore$ Total force on the elemental area

$=dP=$ Force on concrete + Force on steel

$\therefore$ $$dP=t_c\,rd\theta\,\frac{\cos\theta-\cos\alpha}{1-\cos\alpha}\,c+t_s\,rd\theta\,\frac{\cos\theta-\cos\alpha}{1-\cos\alpha}\,mc.$$

$\therefore$ $$dP=(t_c+mt_s)\,crd\theta\,\frac{\cos\theta-\cos\alpha}{1-\cos\alpha}$$

$\therefore$ Total compression $$P=(t_c+mt_s)\,cr\,2\int_0^{\alpha}\frac{\cos\theta-\cos\alpha}{1-\cos\alpha}\,d\theta$$

on integrating, we get

$$P=(t_c+mt_s)\,cr\,\frac{}{1-\cos\alpha}(\sin\alpha-\alpha\cos\alpha)\ldots(iii)$$

Again, $$dP=(t_c+mt_s)\,cr\,d\theta\,\frac{\cos\theta-\cos\alpha}{1-\cos\alpha}$$

The force dP acts at a distance of $r(\cos\theta-\cos\alpha)$ from the neutral axis. Hence the moment of the force dP about the neutral axis

$$dM_c=(t_c+mt_s)\,cr\,d\theta\,\frac{\cos\theta-\cos\alpha}{1-\cos\alpha}\,.\,r(\cos\theta-\cos\alpha)$$

$\therefore$ $$dM_c=(t_c+mt_s)\,cr^2\,\frac{(\cos\theta-\cos\alpha)^2}{1-\cos\alpha}\,d\theta$$

$\therefore$ Moment of P about the neutral axis is

$$M_c=\frac{(t_c+mt_s)\,cr^2}{1-\cos\alpha}\int(\cos\theta-\cos x)^2\,d\theta$$

Completing the integration, we get,

$$M_c=(t_c+mt_s)\,cr^2\,\frac{}{1}$$

$$\left[\alpha\cos^2\alpha-\frac{}{}\sin\alpha\cos\alpha+\frac{}{}\right.$$

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$$\therefore \quad l_1 = \frac{M_c}{P} = \frac{\alpha \cos^2 \alpha - \frac{3}{2} \sin \alpha \cos \alpha + \frac{\alpha}{2}}{\sin \alpha - \alpha \cos \alpha} r \qquad \ldots(iv)$$

On similar lines corresponding equations may also be obtained for the tension side. As usual, the concrete in the tension will not be considered.

Let us now consider a small element subtending an angle $d\theta$.

Area of steel in the element $= t_s \, r \, d\theta$

The stress intensity on this element

$$= \frac{r(\cos\theta + \cos\alpha)}{r(1+\cos\alpha)} f_s$$

$$= \left[\frac{\cos\theta + \cos\alpha}{1+\cos\alpha}\right] f_s$$

$\therefore$ The total tension on the element

$$dT = t_s \, r d\theta \left[\frac{\cos\theta + \cos\alpha}{1+\cos\alpha}\right] f_s$$

$\therefore$ Total tension on the tension zone

$$T = \frac{t_s f_s r}{1+\cos\alpha} 2 \int_0^{(\pi-\alpha)} (\cos\theta + \cos\alpha) \, d\theta$$

On completing the integration, we have,

$$T = t_s f_s r \frac{2}{1+\cos\alpha} [\sin\alpha + (\pi-\alpha)\cos\alpha] \qquad \ldots(v)$$

Again $$dT = \frac{t_s f_s r}{(1+\cos\alpha)} (\cos\theta + \cos\alpha) \, d\theta.$$

This force acts at a distance of $r(\cos\theta + \cos\alpha)$ from the neutral axis.

Hence the moment of dT about the neutral axis,

$$= dM_t = \frac{t_s f_s r}{(1+\cos\alpha)} (\cos\theta + \cos\alpha) \, d\theta$$

$$r(\cos\theta + \cos\alpha$$

$$\therefore \quad dM_t = \frac{t_s f_s r^2}{(1+\cos\alpha)} (\cos\theta + \cos\alpha)^2 \, d\theta$$

$\therefore$ Moment of T about neutral axis,

$$= M_t = \frac{t_s f_s r}{1+\cos\alpha} \cdot 2 \int_0^{\pi-\alpha} (\cos\theta + \cos\alpha)^2 \, d\theta$$

Completing the integration, we get,

$$M_t = \frac{t_s f_s r^2}{1+\cos\alpha} 2[(\pi-\alpha)\cos^2\alpha + \tfrac{3}{2}\sin\alpha\cos\alpha + \tfrac{1}{2}(\pi-\alpha]$$

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$$l_2 \quad \frac{M}{T} = \frac{(\pi-\alpha)\cos^2\alpha + \frac{3}{2}\sin\alpha\cos\alpha + \frac{1}{2}(\pi-\alpha)}{\sin\alpha+(\pi-\alpha)\cos\alpha} r \qquad ...(vi)$$

From equations (*iv*) and (*vi*), l_1 and l_2 can be determined.

$$\text{Lever arm} = l_1 + l_2.$$

Wind Load Effect. In a chimney the wind load on the chimney produces tensile and compressive stresses. The weight of the chimney will only produce the compressive stresses. Hence the compressive and tensile stresses which we have been considering in the above discussions, are due to the combined effect of the self-weight of the chimney and the stresses due to wind load.

Consider any horizontal section XY. The forces acting above the section are the following :

(*i*) Horizontal wind load producing a bending moment M.

(*ii*) Central vertical load W due to the weight of the chimney above the section XY. These forces are resisted and hence kept in equilibrium by the forces P and T and the shear resistance.

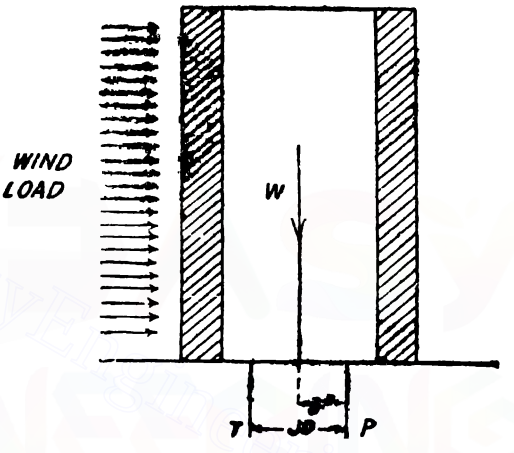

Fig. 738

Let $\frac{\text{lever arm}}{D} = j.$

Let the distance between W and $P = zD$.

Taking moments of these forces about P, we have,

$$T\,j\,D = M - WzD$$

But $$T = t_s f_s r \frac{2}{1+\cos\alpha} [\sin\alpha+(\pi-\alpha)\cos\alpha]$$

or $$T = C_T t_s f_s r \qquad ...(vii)$$

where $$C_T = \frac{2}{1+\cos\alpha} [\sin\alpha+(\pi-\alpha)\cos\alpha]$$

$$C_T t_s f_s r j D = M - WDz$$

$$t_s s = \frac{M - WDz}{C_T f_s jD}$$

But total area of steel

$$= A_s = 2\pi r t_s$$

$$A_s = \frac{2\pi (M - WzD)}{C_T f_s jD} \qquad ...(viii)$$

Resolving the forces vertically, we get,

$$P - T = W$$

But $$P = (t_s + m t_s) cr \frac{1}{1-\cos\pi} [\sin\alpha - \alpha\cos\alpha]$$

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$$P = C_p cr\,(t_c + mt_s)$$

where $$C_p = \frac{2}{1-\cos\alpha}(\sin\alpha - \alpha\cos\alpha)$$

Substituting for P and T,
we have $C_p\,cr\,(t_c+mt_s) - C_T\,t_s\,f_s\,r = W$

$$\therefore \quad t_s = \frac{W+(C_T f_s - C_p\,mc)\,rt_s}{C_p\,cr}$$

Total thickness of the chimney wall
$$= t = t_c + t_s$$

$$\therefore \quad t = \frac{W+(C_T f_s + C_p\,mc)\,rt_s}{C_p\,cr} + t_s$$

Putting $$r = \frac{D}{2} \text{ and } t_s = \frac{A_s}{\pi D} \text{ we get}$$

$$t = \frac{2W+(C_T f_s - C_p\,mc)\frac{A_s}{\pi}}{C_p\,cD} + \frac{A_s}{\pi D}.$$

Design (S.I.) 171. *An R.C. chimney having a mean diameter of 2·5 metres is reinforced with sixty bars of 16 mm. diameter. Assuming an effective wind pressure of 1400 N/metre² on projected area, determine the maximum stresses in concrete and steel at a section 25 metres from the top. Assume wt. of RCC = 24000 N/m³.*

Solution. Let the thickness of the chimney wall be 15 *cm.*

Bending moment at a depth of 25 *metres*
$$= (25 \times 2{\cdot}65 \times 1400)\frac{25}{2} \times 100 \text{ } N\,cm.$$
$$= 11,59,00000 \text{ } N\,cm.$$

Self-weight of the chimney
$$= \pi \times 2{\cdot}5 \times 0{\cdot}15 \times 25 \times 24000 \text{ } N$$
$$= 70680 \text{ } N$$
A_s = area of steel $= 60 \times 2{\cdot}01$ *cm.*²
$$= 120{\cdot}6 \text{ } cm.^2$$

Let us assume $n_1 = 0{\cdot}4$

we have, $$C_p = \frac{2}{1-\cos\alpha}(\sin\alpha - \alpha\cos\alpha)$$

$$\cos\alpha = 1 - 2n_1 = 1 - 0{\cdot}8 = 0{\cdot}2$$
$$\therefore \quad \alpha = 78^\circ\,28' = 1{\cdot}3695 \text{ } radian$$
$$\therefore \quad C_p = \frac{2}{1-0{\cdot}2}(\sin 78^\circ\,28' - 1{\cdot}3695 \times 0{\cdot}2)$$
$$= 1{\cdot}765$$
$$C_T = \frac{2}{1+\cos\alpha}[\sin\alpha + (\pi - \alpha)\cos\alpha]$$
$$= \frac{2}{1+0{\cdot}2}[\sin 78^\circ\,28' + (3{\cdot}1416 - 1{\cdot}3695) \times 0{\cdot}2]$$
$$= 2{\cdot}224$$

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It is observed that the factor j is practically constant and in almost all cases $j=0{\cdot}7854$.

$$l_1=\frac{\alpha\cos^2\alpha-\frac{3}{2}\sin\alpha\cos\alpha+\frac{\alpha}{2}}{\sin\alpha-\alpha\cos\alpha}\cdot\frac{D}{2}$$

For the value of α known, we get

$l_1=0{\cdot}316\ D$

$\therefore$ Distance between centre of chimney and centre of compression

$=zD$

$=(0{\cdot}5D-0{\cdot}4D)+0{\cdot}316D$

$=0{\cdot}416D$

$\therefore \quad z=0{\cdot}416$

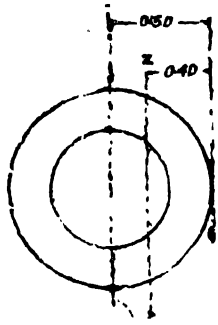

Fig. 739

$$A_s=\frac{2\pi(M-WzD)}{C_t f_s j D}$$

$$\therefore\ 120{\cdot}6=\frac{2\pi\,(11,59,00000-706800\times0{\cdot}416\times250}{2{\cdot}224\times f_s\times0{\cdot}7854\times250}$$

$$\therefore \quad f_s=5060\ N/cm.^2$$

But we know, $t=\dfrac{2W+(C_t f_s-C_p\ cm)\dfrac{A_s}{\pi}}{C_p\ cD}-\dfrac{A_s}{\pi D}$

Substituting the values of t, W, C_T, Cr, f_s, m, A_s we get $c=188\ N/cm.^2$

$$n_1=\frac{1}{1+\dfrac{f_s}{mc}}=\frac{1}{1+\dfrac{5060}{18\times188}}=0{\cdot}401$$

This value agrees well with the assumed value $n=0{\cdot}4$.

Hence the stresses computed may be taken as correct.

Design 172. *An R.C. chimney having a mean diameter of 2·5 metres is reinforced with sixty bars of 16 mm. diameter. Assuming an effective wind pressure of 140 kg./metre² on projected area, determine the maximum stresses in concrete and steel at a section 25 metres from the top.*

Solution. Let the thickness of the chimney wall be 15 *cm.*

Bending moment at a depth of 25 *metres*

$$=(25\times2{\cdot}65\times140)\frac{25}{2}\times100\ kg.\ cm.$$

$$=11,590,000\ kg.\ cm.$$

Self-weight of the chimney

$=\pi\times2{\cdot}5\times0{\cdot}15\times25\times2400\ kg.$

$=70680\ kg.$

$A_s=$ area of steel $=60\times2{\cdot}01\ cm.^2$

$=120{\cdot}6\ cm.^2$

Let us assume $n_1=0\ 4$

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we have $C_p = \frac{2}{1-\cos\alpha}(\sin\alpha - \alpha\cos\alpha)$

$\cos\alpha = 1 - 2n_1 = 1 - 0{\cdot}8 = 0{\cdot}2$

$\therefore \quad \alpha = 78^\circ\ 28' = 1{\cdot}3695$ *radian*

$\therefore \quad C_p = \frac{2}{1-0{\cdot}2}(\sin 78^\circ\ 28' - 1{\cdot}369\,5 \times 0{\cdot}2)$

$= 1\ 765$

$C_T = \frac{2}{1+\cos\alpha}[\sin\alpha + (\pi - \alpha)\cos\alpha]$

$= \frac{2}{1+0{\cdot}2}[\sin 78^\circ\ 28' + (3{\cdot}1416 - 1{\cdot}3695) \times 0{\cdot}2]$

$= 2{\cdot}224$

It is observed that the factor j is practically constant and in almost all cases $j = 0{\cdot}7854$.

$$l_1 = \frac{\alpha\cos^2\alpha - \frac{3}{2}\sin\alpha\cos\alpha + \frac{\alpha}{2}}{\sin\alpha - \alpha\cos\alpha} \cdot \frac{D}{2}$$

For the value of α known, we get

$l_1 = 0{\cdot}316\ D$

$\therefore$ Distance between centre of chimney and centre of compression

$= zD$

$= (0{\cdot}5D - 0{\cdot}4D) + 0{\cdot}316D$

$= 0{\cdot}416D$

$\therefore \quad z = 0{\cdot}416$

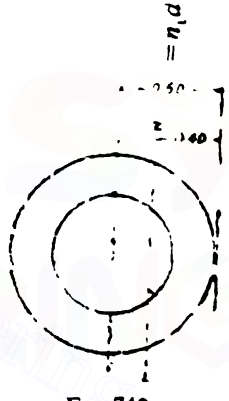

Fig 740

$$A_s = \frac{2\pi(M - WzD)}{C_t f_s j D}$$

$$\therefore \quad 120{\cdot}6 = \frac{2\pi(11{,}590{,}000 - 70680 \times 0{\cdot}416 \times 250}{2{\cdot}224 \times f_s \times 0{\cdot}7854 \times 250}$$

$$\therefore \quad f_s = 506\ kg./cm^2$$

But we know, $t = \frac{2W + (C_t f_s - C_p\ cm)\frac{A_s}{\pi}}{C_p\, cD} + \frac{A_s}{\pi D}$

Substituting the values of t, W, C_T, C_r, f_s, m, A_s

We get $\quad c = 18{\cdot}8\ kg/cm.^2$

$$n_1 = \frac{1}{1 + \frac{f_s}{mc}} = \frac{1}{1 + \frac{506}{18 \times 18{\cdot}8}} = 0{\cdot}401$$

This value agrees well with the assumed value $n = 0{\cdot}4$.

Hence the stresses computed may be taken as correct.

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20
Bridges

Bridges may be of reinforced concrete or steel construction. We will discuss in this chapter about the design of reinforced concrete bridges. An R.C. bridge usually consists of *T* beam supporting continuous slabs. The beams are supported on intermediate piers and end abutments. In India bridges for roads are designed as per the recommendations of the Indian Roads Congress. Rail bridges are designed on the recommendations of specifications of the Indian Railway standard code of practice.

It is proposed to first deal with the design of bridges subjected to simplified loadings.

Design 173 (S.I.). *A reinforced concrete (single span) T beam bridge is to consist of 5 beams 1·75 metres apart. The span of the bridge is 12 metres clear with end bearings of 60 cm. The live load on the bridge is 17000 N per sq. m. including impact. The carriageway over the bridge is to be 7 metres wide with a footpath of 1·5 metres width on either side. The loading on the footpath may be taken as 3900 N per square metre. Design the bridge.*

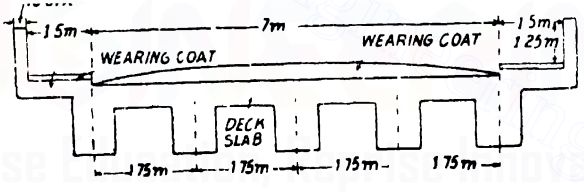

Fig. 741

Solution. Fig. 741 shows an arrangement for the bridge. A parapet 10 *cm.* thick and 1·25 *metres* high is provided at each side.

Design of footpath slab. The bending moment for the footpath slab will consist of the following :

Consider one *metre* run of the bridge.

(*i*) B.M. due to self-weight of parapet

$$0{\cdot}1 \times 1{\cdot}25 \times 25000 \times 1{\cdot}55 = 4844 \ Nm.$$

(*ii*) B.M. due to weight of wearing coat (7·5 *cm.*)

$$1{\cdot}5 \times 0{\cdot}075 \times 24000 \times \frac{1{\cdot}5}{2} = 2025 \ Nm$$

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(*iii*) B.M. due to wt. of footpath (15 *cm.*)
$1{\cdot}6 \times 0{\cdot}15 \times 2{\cdot}000 \times 0{\cdot}8$ = 4800 *Nm.*

(*iv*) B.M. due to live load

$$3900 \times 15 \times \frac{1{\cdot}5}{2} = 4388 \text{ Nm.}$$

∴ Max. B.M. for the footpath slab = 16057 *Nm.*

Taking $c = 5$ *N/mm*^{2} $= 500$ *N/cm.*^{2} ; $t = 140$ *N/mm.*^{2} $= 14000$ *N/cm.*^{2} and $m = 18$ and equating the moment of resistance to the maximum bending moment, we have,

$$85 \times 100\, d^2 = 16057 \times 100$$

$$d = 13{\cdot}7 \text{ cm.}$$

Providing 12 *mm.* ϕ bars at a clear cover of 15 *mm.*
effective cover $= 15 + 6 = 21$ *mm.* $= 2{\cdot}1$ *cm.*
∴ Effective depth available

$$= 15 - 2{\cdot}1 = 12{\cdot}9 \text{ cm.}$$

∴
$$A_t = \frac{16057 \times 100}{14000 \times 0{\cdot}87 \times 12{\cdot}9} \text{ cm.}^2$$

$$= 10{\cdot}2 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{10{\cdot}2} \text{ cm.}$$

$$= 11 \text{ cm. c/c.}$$

Distribution steel $= 0{\cdot}15\%$ of gross area

$$= \frac{0{\cdot}15}{100} \times 15 \times 100 \text{ cm.}^2$$

$$= 2{\cdot}25 \text{ cm.}^2$$

Spacing of 8 *mm.* dia. bars

$$= \frac{0{\cdot}5 \times 100}{2{\cdot}25} \text{ cm.}$$

$$= 22 \text{ cm. c/c.}$$

Design of the deck slab

Loads

Dead load of deck slab (15 *cm.*)	$= 250 \times 15$	$= 3750$ *N/m.*^{2}
Wearing coat (7·5 *cm.* average)	$= 240 \times 7{\cdot}5$	$= 1800$ *N/m.*^{2}
Live Load		$= 17000$ *N/m.*^{2}
	Total	$= 22550$ *N/m.*^{2}

Consider a one *metre* wide strip of the deck slab.
Max. bending moment

$$= \frac{wl^2}{10}$$

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$$= \frac{22550 \times 1{\cdot}75^2}{10} \ Nm.$$

$$= 6905{\cdot}9 \ Nm.$$

$$= 690590 \ N \ cm.$$

Providing 2·5 *cm.* clear cover, the effective depth to the centre of 12 *mm.* diameter bars

$$= 15{\cdot}00 - 2{\cdot}5 - 0{\cdot}60 = 11{\cdot}9 \ cm.$$

$$\therefore \quad A_t = \frac{690590}{14000 \times 0{\cdot}87 \times 11{\cdot}9} \ cm.^2$$

$$= 4{\cdot}76 \ cm.^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{4{\cdot}76} \ cm.$$

$$= 23 \ cm. \ centres$$

Design of beams

Load per metre run on an intermediate T beam

Load from deck slab	$= 22550 \times 1{\cdot}75$	$= 39462 \ N/m$
Dead load of the rib (112 *cm.* overall, 36 *cm.* wide rib)	$= 0{\cdot}97 \times 0{\cdot}36 \times 25000$	$= 8730 \ N/m.$
		$= 48192 \ N/m.$

Load per metre run on the end beam

From cantilevering slab

Weight of parapet	$= 0{\cdot}10 \times 1{\cdot}25 \times 25000$	$= 3125 \ N/m.$
Wearing coat (7·5 *cm.*)	$= 0{\cdot}075 \times 1{\cdot}5 \times 24000$	$= 2700 \ N/m.$
Footpath slab	$= 0{\cdot}15 \times 1{\cdot}6 \times 25000$	$= 6000 \ N/m.$
Live load on the footpath slab	$= 3900 \times 1{\cdot}5$	$= 5850 \ N/m.$

Load from the deck slab

$$= \frac{39462}{2} \qquad = 19731 \ N/m.$$

Dead load of the rib $= 8730 \ N/m.$

Total $= 46136 \ N/m.$

But load on an intermediate beam $= 48192 \ N/m.$

Hence the intermediate beams carry heavier loads than the end beams. We shall therefore design an intermediate beam.

$$\text{Maximum B.M.} = \frac{48192 \times 12{\cdot}6^2}{8} \ Nm.$$

$$= 956370 \ Nm.$$

$$= 95637000 \ N \ cm.$$

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Assuming 26 *mm.* diameter bars are likely to be provided in two rows and providing a clear cover of 4 *cm.* to the outer layer of bars, the effective depth to the centroid of the tensile reinforcement

$$= d = \text{overall depth} - \text{cover} - \text{dia. of bar} - \text{half the spacing between layers}$$

$$= 112 - 4 - 2{\cdot}6 - 0{\cdot}6 \text{ cm.}$$

$$= 104{\cdot}8 \text{ cm}$$

Approximate lever arm

$$= d - \frac{d_s}{2}$$

$$= 104{\cdot}8 - \frac{15}{2} \text{ cm.}$$

$$= 97{\cdot}3 \text{ cm.}$$

Approximate steel requirement

$$= \frac{95637000}{14000 \times 97{\cdot}3} \text{ cm.}^2$$

$$= 70{\cdot}20 \text{ cm.}^2$$

Let us provide 14 bars of 26 mm. diameter arranged in two layers with 7 bars in each layer.

Area of steel provided $= 74{\cdot}34 \text{ cm.}^2$

Width of flange of the T beam

$$B = \frac{l}{6} + b_r + 6\,ds$$

$$= \frac{12{\cdot}6 \times 100}{6} + 36 + (6 \times 15)$$

$$= 210 + 36 + 90 = 336 \text{ cm.}$$

B shall not be taken greater than the breadth of web + half the sum of the clear distance to the adjacent beams on either side.

On this basis = *c* to *c* distances between beams

$$\therefore \quad B = 175 \text{ cm.}$$

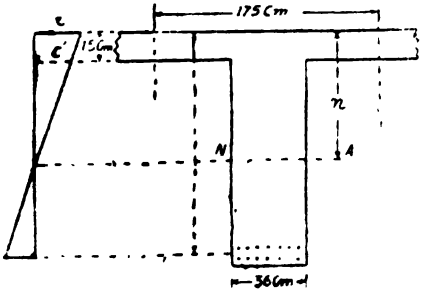

Fig. 742

Assuming the neutral axis, to be situated below the slab, we have,

$$175 \times 15\,(n - 7{\cdot}5) = 18 \times 74{\cdot}34\,(104{\cdot}8 - n)$$

$$\therefore \quad n = 35{\cdot}4 \text{ cm.}$$

See Fig. 742

$$c' = \frac{35{\cdot}4 - 15}{35{\cdot}4}\,c$$

$$= 0{\cdot}58\,c$$

Lever arm $= d -$

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where $= \frac{c+2c'}{c+c'} \cdot \frac{d_0}{3}$

$= \frac{c+1{\cdot}16\,c}{1{\cdot}58c} \cdot 15$ *cm.*

$= 6{\cdot}8$ *cm.*

$\therefore$ Actual lever arm $= 104{\cdot}8 - 6{\cdot}8 \quad 98$ *cm.*

Actual stress in steel $= \frac{98637000}{74{\cdot}34 \times 98}$ *N/cm.*^{2}

$= 13539$ *N/cm.*^{2}

Corresponding compressive stress in concrete

$$f_c = \frac{13539}{18} \times \frac{35{\cdot}4}{104{\cdot}8 - 35{\cdot}4} \text{ N/cm.}^2$$

$= 384$ *N/cm.*^{2}

Shear

Maximum S.F. $= S = \frac{48192 \times 12}{\quad}$ *N*

$= 289152$ *N*

Nominal shear stress

$$= q_v = \frac{S}{bjd}$$

$$\frac{289152}{36 \times 104{\cdot}8} = 77 \text{ N/cm.}^2$$

$= 0{\cdot}77$ *N/mm.*^{2}

Percentage of tensile steel after 50% curtailment of steel

$$\frac{37{\cdot}17 \times 100}{36 \times 104{\cdot}8} = 0{\cdot}98\%$$

For 0·98% steel $q_c = 0{\cdot}37$ *N/mm.*$^2 = 37$ *N/cm.*^{2}

Shear resistance of concrete

$= q_c\, bjd = 37 \times 36 \times 104{\cdot}8 = 139594$ *N*

$\therefore$ Shear to be resisted by reinforcement

$= V_s = 289152 - 139594 = 149558$ *N*

Let four bars be bent up at a distance of 1·414 $a = 1{\cdot}414 \times 98 = 138$ *cm.* from the support.

Shear resistance of four bars

$= 0{\cdot}707\ A_w f_w = 0{\cdot}707 \times 4 \times 5{\cdot}31 \times 14000$ *N*

$= 210234$ *N*

Let 3 more bars be bent up at a further distance of 138 *cm.*

Maximum S.F. at centre when the live load covers half the span

$$\frac{(17000 \times 1{\cdot}75) \times 6^2}{2 \times 12} \text{ N}$$

$= 44625$ *N*

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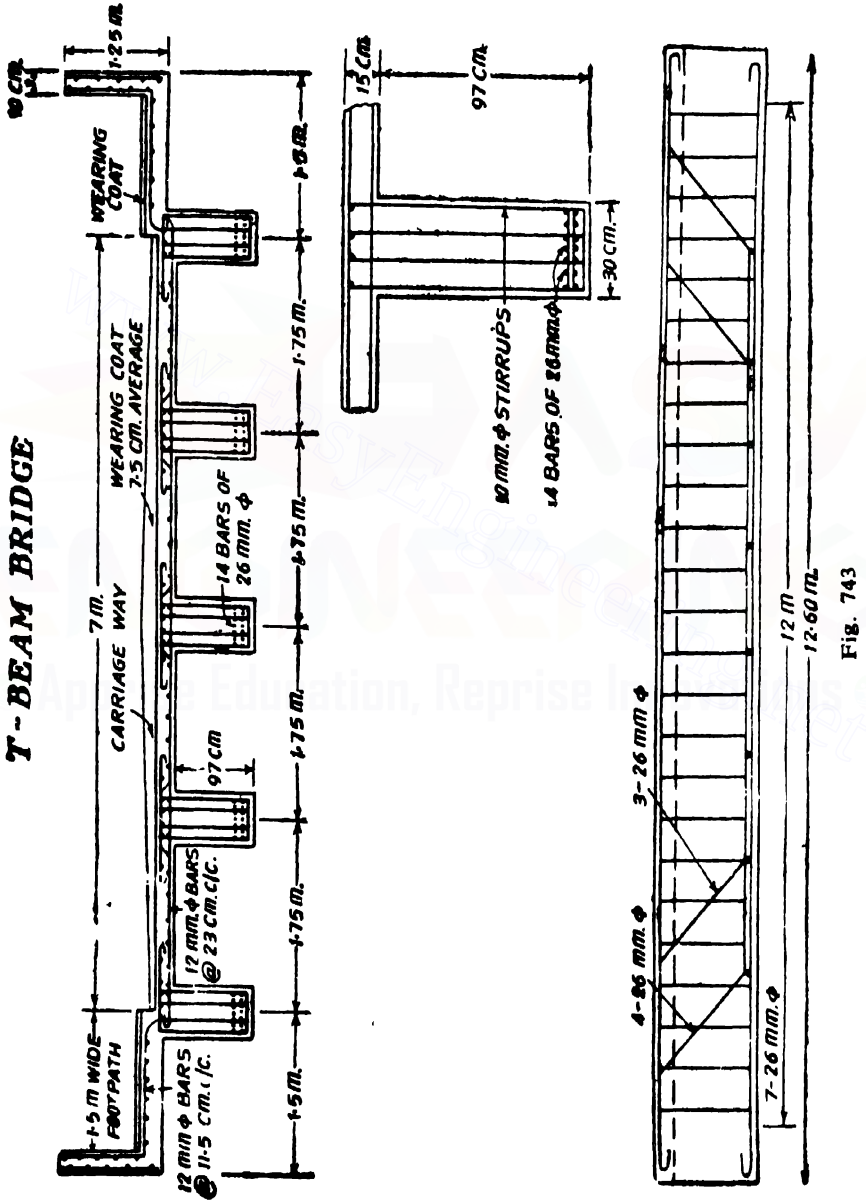

Fig. 743

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Nominal shear stress due to this shear force

$$= \frac{446{\cdot}5}{35 \times 104{\cdot}8} = 12 \text{ N/cm.}^2 = 0{\cdot}12 \text{ N/mm}^2$$

Design 174 *A reinforced concrete (single span) T beam bridge is to consist of 5 beams 1·75 metres apart. The span of the bridge is 12 metres clear with end bearings of 60 cm. The live load on the bridge is 1700 kg. per square metre including impact. The carriage-way over the bridge is to be 7 metres wide with a footpath of 1·5 metres width on either side. The loading on the footpath may be taken as 390 kg. per square metre. Design the bridge.*

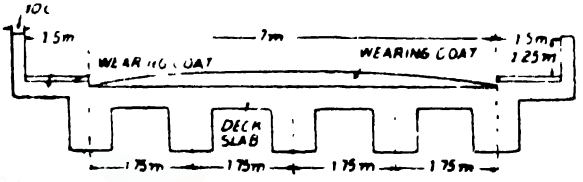

Fig. 744

Solution. Fig. 744 shows an arrangement for the bridge. A parapet 10 *cm.* thick and 1·25 *metres* high is provided at each side.

Design of footpath slab. The bending moment for the footpath slab will consist of the following :

Consider one *metre* run of the bridge.

(*i*) B.M. due to self-weight of parapet $0{\cdot}1 \times 1{\cdot}25 \times 2400 \times 1{\cdot}55 =$ 465 *kg. m.*

(*ii*) B.M. due to weight of wearing coat (7·5 *cm.*)

$$1{\cdot}50 \times 0{\cdot}075 \times 2300 \times \frac{1{\cdot}5}{2} = 194 \text{ kg. m.}$$

(*iii*) B M. due to wt. of footpath (15 *cm.*) $1{\cdot}60 \times 0{\cdot}15 \times 2400 \times 0{\cdot}8 =$ 461 *kg. m.*

(*iv*) B M. due to live load

$$390 \times 1{\cdot}5 \times \frac{1{\cdot}5}{2} = 439 \text{ kg. m.}$$

$\therefore$ Max. B.M. for the footpath slab = 1559 *kg. m.*

Adopting $c = 50$ *kg/cm.*^{2}, $t = 1400$ *kg/cm.*^{2}, $m = 18$ and equating the moment of resistance to the maximum bending moment, we have,

$$8{\cdot}50 \times 100 \times d^2 = 1559 \times 100$$

$$d = 13{\cdot}5 \text{ cm.}$$

With an effective cover of 2 *cm.*, effective depth available = 15 − 2 = 13 *cm.* (almost equal to the requirement)

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$$\therefore \quad A_s = \frac{1559 \times 100}{1400 \times 0{\cdot}87 \times 13} \ cm.^2$$

$$= 9{\cdot}85 \ cm.^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{9{\cdot}85} \ cm.$$

$$= 11{\cdot}5 \ cm. \ c/c.$$

Distribution steel $= 0{\cdot}15\%$ of gross area

$$= \frac{0{\cdot}15}{100} \times 15 \times 100 \ cm.^2$$

$$= 2{\cdot}25 \ cm.^2$$

Spacing of 8 *mm.* dia. bars

$$= \frac{0{\cdot}5 \times 100}{2{\cdot}25} \ cm.$$

$$= 22 \ cm. \ c/c.$$

Design of the deck slab

Loads

Dead load of deck slab (15 *cm.*) $= 24 \times 15$	$=$	360 *kg./m.*^{2}
Wearing coat (7·5 *cm.* average) $= 24 \times 7{\cdot}5$	$=$	180 *kg./m.*^{2}
Live load	$=$	1700 *kg./m.*^{2}
Total	$=$	2240 *kg./m.*^{2}

Consider a one *metre* wide strip of the deck slab.

Max. bending moment

$$= \frac{wl^2}{10}$$

$$= \frac{2240 \times 1{\cdot}75^2}{10} \ kg. \ m.$$

$$= 685{\cdot}8 \ kg. \ m.$$

$$= 68580 \ kg. \ cm.$$

Providing 2·5 *cm.* clear cover, the effective depth to the centre of 12 *mm.* diameter bars

$$= 15{\cdot}00 - 2{\cdot}5 - 0{\cdot}60 = 11{\cdot}9 \ cm.$$

$$\therefore \quad A_s = \frac{68580}{1400 \times 0{\cdot}87 \times 11{\cdot}9} \ cm.^2$$

$$= 4{\cdot}73 \ cm.^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{4{\cdot}73} \ cm.$$

22 cm centres

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Design of beams

Load per metre run on an intermediate T beam

Load from deck slab	$=2240\times1{\cdot}75$	$=3920$ *kg./m.*
Dead load of the rib (112 *cm.* overall, 36 *cm.* wide rib)	$=0{\cdot}97\times0{\cdot}36\times2400=$	838 *kg./m.*
		4758 *kg./m.*

Load per metre run on the end beam

From cantilevering slab

Weight of parapet	$=0{\cdot}10\times1{\cdot}25\times2400=$	300 *kg./m.*
Wearing coat (7·5 *cm.*)	$=0{\cdot}075\times1{\cdot}5\times2300=$	259 *kg./m.*
Footpath slab	$=0{\cdot}15\times1{\cdot}6\times2400=$	576 *kg./m.*
Live load on the footpath slab	$=390\times1{\cdot}5=$	585 *kg./m.*
Load from the deck slab	$=\frac{3920}{2}=$	1960 *kg./m.*
Dead load of the rib		838 *kg./m.*
	Total	4518 *kg./m.*

But load on an intermediate beam $=4758$ *kg./cm.*

Hence the intermediate beams carry heavier loads than the end beams. We shall therefore design an intermediate beam.

Maximum B.M. $=\frac{4758\times12{\cdot}6^2}{8}$ *kg. m.*

$=94430$ *kg. m.*

$=9{,}443{,}000$ *kg. cm.*

Assuming 26 *mm.* diameter bars are likely to be provided in two rows and providing a clear cover of 4 *cm.* to the outer layer of bars, the effective depth to the centroid of the tensile reinforcement

$=d=$ overall depth $-$ cover $-$ dia. of bar $-$ half the spacing between layers

$=112-4-2{\cdot}6-0{\cdot}6$ *cm.*

$=104{\cdot}8$ *cm.*

Approximate lever arm

$=d-$

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$$= 104{\cdot}8 - \frac{1{\cdot}5}{2}\ cm.$$

$$= 97{\cdot}3\ cm.$$

Approximate steel requirement

$$= \frac{9{,}443{,}000}{1400 \times 97{\cdot}3}\ cm.^2$$

$$= 69{\cdot}33\ cm^2$$

Let us provide 14 bars of 26 mm. diameter arranged in two layers with 7 bars in each layer.

Area of steel provided = 74·34 *cm.*^{2}

Width of flange of the T beam

This will be taken as the lesser of the following :

(*i*) spacing of beams = 1·75 *m.* = 175 *cm.*

(*ii*) $\frac{l}{b} + b_r + 6d_s$

$$= \frac{1260}{d} + 36 + 6 \times 15 = 336\ cm.$$

∴ Effective width of flange of the T beam

$$= 175\ cm.$$

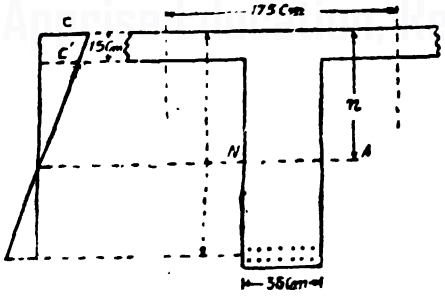

Fig. 745

Assuming the neutral axis to be situated below the slab, we have,

$$175 \times 15(n - 7{\cdot}5) = 18 \times 74{\cdot}34(104{\cdot}8 - n)$$

∴ $n = 35{\cdot}4\ cm.$

See Fig. 745.

$$c' = \frac{35{\cdot}4 - 15}{35{\cdot}4}\ c$$

$$= 0{\cdot}58\ c$$

Lever arm $= d - y$

where

$$y = \frac{c + 2c'}{c + c'} \cdot \frac{d_s}{3}$$

$$= \frac{c + 1{\cdot}16c}{1{\cdot}58c} \cdot \frac{15}{3}\ cm.$$

$$= 6{\cdot}8\ cm.$$

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$\therefore$ Actual lever arm $= 104{\cdot}8 - 6{\cdot}8 = 93$ *cm.*

Actual stress in steel $= \frac{9,443,000}{74{\cdot}34 \times 93}$ *kg./cm.*^{2}

$= 1296$ *kg./cm.*^{2}

Corresponding compressive stress in concrete

$$-c = \frac{1296}{18} \times \frac{35{\cdot}4}{104{\cdot}8 - 35{\cdot}4} \text{ kg./cm.}^2$$

$= 36{\cdot}72$ *kg./cm.*^{2}

Shear

Maximum S.F. $-S = \frac{4758 \times 12{\cdot}6}{2}$ *kg.*

$= 29975$ *kg.*

Maximum nominal shear stress

$$= q = \frac{S}{bd}$$

$$= \frac{29975}{36 \times 104{\cdot}8} \text{ kg./cm.}^2$$

$= 7{\cdot}94$ *kg./cm.*^{2}

Let the seven bars of the upper layer be utilized to resist the shear.

Let four bars be bent up at a distance of $1{\cdot}414a$

$= 1{\cdot}414 \times 98 = 138{\cdot}57$ *cm.*

say 138 *cm.* from each support

Shear resistance of four bars

$= 0{\cdot}707\ A_w t_w$

$= 0{\cdot}707 \times 4 \times 5{\cdot}31 \times 1400$ *kg.*

$= 21020$ *kg.*

Balance shear force to be resisted

$= 29975 - 21020 = 8955$ *kg.*

Spacing of two-legged 10 *mm.* diameter stirrups

$$= \frac{A_w t_w a}{S'}$$

$$= \frac{2 \times 0{\cdot}79 \times 1440 \times 98}{8955} \text{ cm.}$$

$= 24$ *cm. centres.*

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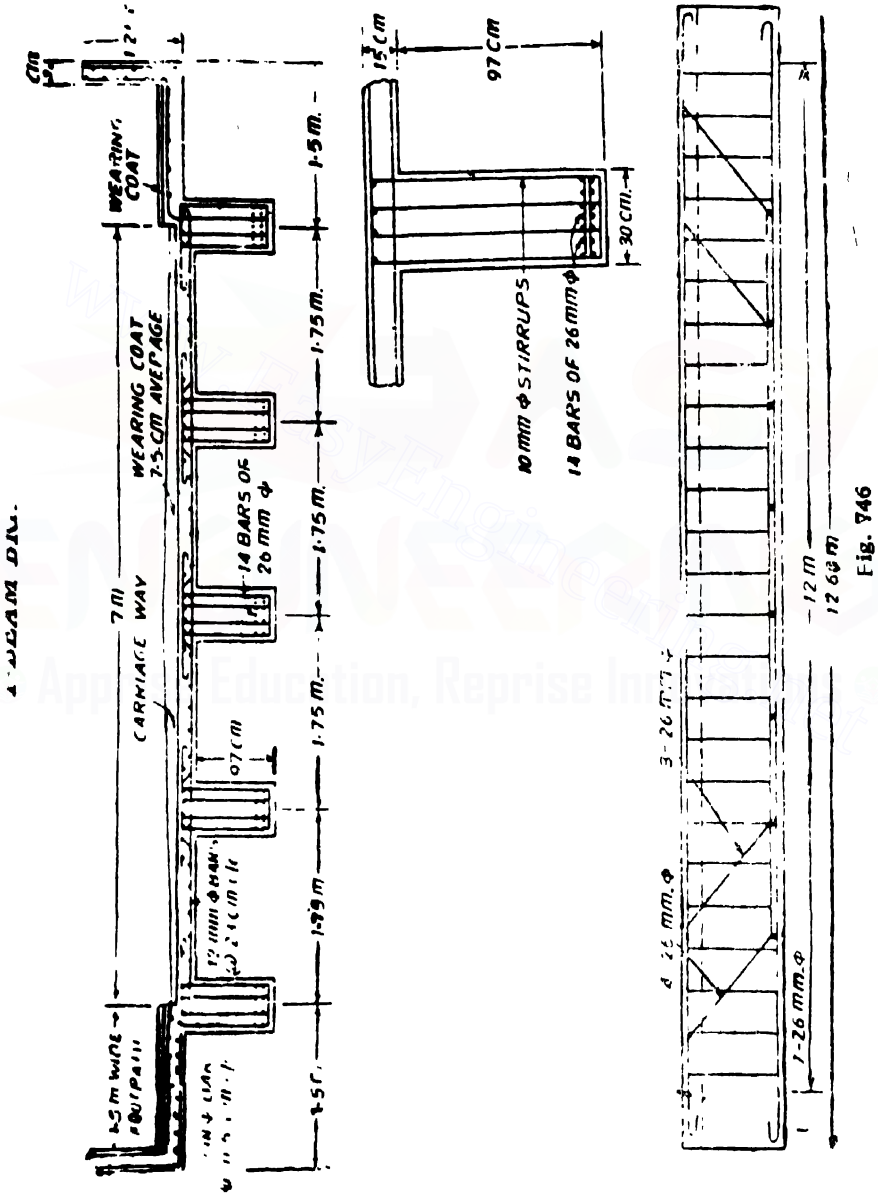

Fig. 746

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Maximum S.F. at the centre

Due to dead load there will be no shear force at the centre. But when the live load covers one-half of the span of the beam the shear force at the centre will be maximum.

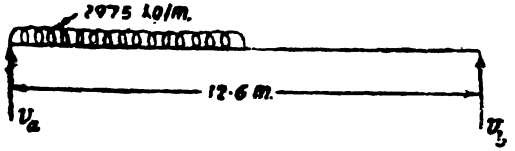

Fig. 747

Live load per *metre* run of the beam

$$= 1700 \times 1 \cdot 75 = 2975 \text{ kg./m.}$$

Max. S.F. at the centre

$$= v_b$$

$$= \frac{2975 \times 6 \cdot 3^2}{2 \times 12 \cdot 6} \text{ kg.}$$

$$= 4685 \text{ kg.}$$

Nominal stress due to this shear force

$$= \frac{4685}{36 \times 104 \cdot 8} \text{ kg./cm}^2.$$

$$= 1 \cdot 24 \text{ kg./cm}^2.$$

Provide nominal stirrups.

Design 175. *Design an R.C. road bridge of clear span 12 metres and clear roadway 7 metres as shown in the accompanying figure to the following requirements :*

(*i*) *Total uniformly distributed live load = 48 tonnes.*

(*ii*) *Concentrated knife edge live load per traffic lane of 3·5 metres width each = 12 tonnes for bending moment and 15 tonnes for shear force respectively.*

(*iii*) **Impact.** *Impact allowances shall be as follows :*

For slab = 50%

For beams = 25%

(*iv*) **Working stresses.** *The following working stresses shall be adopted :*

Tensile stress in steel reinforcement	= 1400 kg./cm.²
Compressive stress in concrete in flexure	= 70 kg./cm.²
Diagonal tensile stress for concrete	= 7 kg./cm²
Local bonds stress	= 13 kg./cm.²
Modular ratio	= 13

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Distribution steel. *This shall be provided as follows :*

40% of Main steel for spans upto 1 25 metres

50% of Main steel for spans upto 1 80 metres

55% of Main steel for spans upto 2 40. metres

60% of Main steel for spans upto 3 metres.

In no case need the area of distribution steel exceed 10·5 cm.² per metre width.

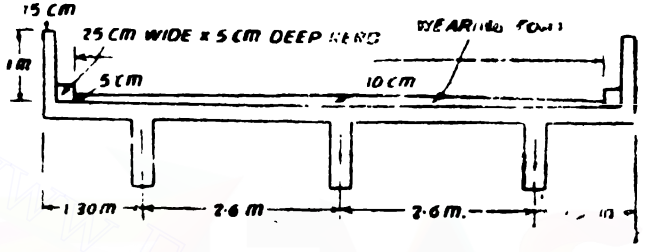

Fig. 748

Solution. For the given set of working stresses it is first necessary to compute the corresponding design coefficients.

Depth of critical neutral axis

$$\frac{mc}{t} = \frac{n}{d-n} = \frac{n_1}{1-n_1} \qquad \text{where } n = n_1 d$$

$$\frac{13 \times 70}{1400} = \frac{n_1}{1-n_1}$$

$$\therefore \quad n_1 = \frac{13}{33}$$

Lever arm

$$a = \left(1 - \frac{n_1}{3}\right) d$$

$$= \left(1 - \frac{13}{99}\right) d$$

$$= 0{\cdot}87\, d$$

$$\text{M.R.} = Q\, bd^2$$

where

$$Q = \frac{1}{2} \times 0{\cdot}87 \times \frac{13}{33} \times 70 = 11{\cdot}99$$

$$\therefore \quad \text{M.R.} = 11{\cdot}99\, bd^2$$

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Design of the slab

Assume the thickness of the slab to be 22 *cm.*

$\left(\text{about } \frac{1}{12} \text{ span of slab}\right)$

Average thickness of the wearing coat $= \frac{7.5}{2} = 3.75$ *cm.*

Dead load

Weight of wearing coat $\frac{7.5}{100} \times 2300$ $= 173$ *kg./m.*^{2}

Weight of slab $\frac{22}{100} \times 2400$ $= 528$ *kg./m.*^{2}

$= 701$ *kg./m.*^{2}

Live load

Uniformly distributed live load

$= \frac{48 \times 1000}{7 \times 12}$ *kg. metre*$^2 = 572$ *kg./m.*^{2}

Knife edge load $= \frac{12 \times 1000}{3.5}$ *kg./m.* for B.M.

$= 3429$ *kg /m.* for B.M.

and $\frac{15 \times 1000}{3.5}$ *kg /m.* for shear

$= 4286$ *kg./m.* for shear.

B.M. Calculations per metre strip of slab

Deck slab

A. Max. bending moment due to dead load at mid support $= \frac{wl^2}{8} = \frac{701 \times 2.6^2}{10} \times 100$

$= 59240$ **kg. cm.**

Max. bending moment due to dead load at the centre of span $= \frac{wl^2}{10} = \frac{701 \times 2.6^2}{10} \times 100$

$= 47400$ **kg. cm.**

B. **B.M. due to uniformly distributed live load**

Over the support $= \frac{572 \times 2.6^2}{8} \times 100(1.5) = 72510$ **kg. cm.**

At the centre of each span

$= \frac{572 \times 2.6^2}{10} \times 100(1.5) = 58010$ **kg. cm.**

C. **B.M. due to knife edge load**

Over the support $= \frac{wl}{5}$

$\frac{3429 \times 26}{5} \times 100(1.5) = 267400$ **kg. cm.**

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At the centre of each span

$$= \frac{Wl}{5} = 267400 \text{ kg .cm.}$$

Total B.M. at the middle support

$$= 59240 + 72510 + 26740 \text{ kg. cm.}$$

$$= 399{,}150 \text{ kg. cm.}$$

Total B.M. at the centre of each span

$$= 47400 + 58010 + 267400 = 372810 \text{ kg. cm.}$$

Cantilever bending moment

B.M. due to weight of parapet

$$= 0{\cdot}15 \times 1 \times 2400 \times 122{\cdot}5 \quad = 44110 \text{ kg. cm.}$$

B.M. due to weight of kerb

$$= 0{\cdot}25 \times 0{\cdot}15 \times 2300 \times 102{\cdot}5 = 8839 \text{ kg. cm.}$$

B.M. due to wearing coat

$$= \frac{7{\cdot}5}{100} \times \frac{90}{100} \times 2300 \times 45 \quad = 6985 \text{ kg. cm.}$$

B.M. due to self-weight of the slab

$$= 0{\cdot}22 \times 1{\cdot}30 \times 2400 \times 65 \quad = 44610 \text{ kg. cm.}$$

B.M. due to uniformly distributed live load

$$= 572 \times \frac{0{\cdot}9^2}{2} \times 100 \times 1{\cdot}5 \quad = 34750 \text{ kg. cm.}$$

B.M. due to knife edge load

$$= 3429 \times 90 \times 1{\cdot}5 \quad = 462900 \text{ kg. cm.}$$

Total cantilever moment $= 602194$ kg. cm.

We find that the cantilever bending moment is the greatest bending moment

Equating the moment of resistance to the maximum bending moment, we have,

$$11{\cdot}99 \times 100\, d^2 = 602194$$

$$d = 22{\cdot}4 \text{ cm.}$$

Clear cover to bridge slab reinforcement

$$= 25 \text{ mm.}$$

Using 18 mm. diameter bars, minimum overall depth required

$$= 22{\cdot}4 + 2{\cdot}5 + 0{\cdot}9 \text{ cm.}$$

$$= 25{\cdot}8 \text{ cm.}$$

Let us make the slab 30 cm. thick.

Increase in bending moment due to 8 cm. increase in thickness

Deck slab

Increase of bending moment over the middle support

$$(8 \times 24) \times \frac{(2{\cdot}6)^2}{8} \times 100 = 16230 \text{ kg. cm.}$$

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Increase of bending moment at the centre of each span

$$=(8\times 24)\times \frac{(2\cdot 6)^2}{10}\times 100=12980 \text{ kg. cm.}$$

Cantilever

Increase of cantilever bending moment

$$=(8\times 24)\times \frac{1\cdot 3^2}{2}\times 100=16230 \text{ kg. cm.}$$

Total bending moment

Over the middle support

$$=399150+16230=415380 \text{ kg. cm.}$$

At the centre of each span

$$=372810+12980=385790 \text{ kg. cm.}$$

Cantilever moment $=602194+16230=618424$ *kg. cm.*

Equating the moment of resistance to the maximum bending moment, we have,

$$11\cdot 99\times 100\, d^2=618424$$

$$\therefore \quad d=22\cdot 7 \text{ cm.}$$

Actual effective depth available (with 18 *mm.* bars at a clear cover of 25 *mm.*)

$$=30-2\cdot 50-0\cdot 9=26\cdot 6 \text{ cm.}$$

Steel requirement

A_s for cantilever moment

$$=\frac{618424}{1400\times 0\cdot 87\times 26\cdot 6} \text{ cm.}^2$$

$$=19\cdot 1 \text{ cm.}^2$$

Spacing of 11 *mm.* diameter bars

$$=\frac{2\cdot 54\times 100}{19\cdot 1} \text{ cm.}$$

$$=13 \text{ cm. centres}$$

A_s at mid span $=\frac{385790}{1400\times 0\cdot 87\times 26\cdot 6}$ *cm.*^{2}

$$=11\cdot 9 \text{ cm.}^2$$

Spacing of 18 *mm.* diameter bars

$$=\frac{2\cdot 54\times 100}{11\cdot 9} \text{ cm.}$$

$$=21 \text{ cm. centres}$$

A_s over middle support

$$=\frac{415380}{400\times 0\cdot 87\times 26\cdot 6} \text{ cm.}^2$$

$$=12\cdot 82 \text{ cm.}^2$$

Spacing of 18 *mm.* diameter bars

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$$= \frac{2{\cdot}54 \times 100}{12{\cdot}82} \text{ cm.}$$

$$= 20 \text{ cm. centres}$$

Studying the spacings of reinforcement computed above, we may adopt the following :

Over mid support and at mid span : *18 mm. diameter bars at 20 cm. centres.*

Distribution steel

Span of slab $= 2{\cdot}6$ *m.*

For the specifications mentioned in the problem the distribution steel requirement $= 60\%$ of main steel

$$= 0{\cdot}60 \times \left(\frac{2{\cdot}45 \times 1200}{20} \right) \text{ cm.}^2$$

$$= 7{\cdot}62 \text{ cm.}^2$$

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{7{\cdot}62} \text{ cm.}$$

$$= 14{\cdot}8 \text{ cm}$$

say 14·5 cm. centres.

Check for shear stress

Shear force at the inner edge of the end beam

$$= \left(30 \times 24 + 7{\cdot}5 \times 23 \right) \times \frac{2{\cdot}6}{2}$$

$$+ 572 \times \frac{2{\cdot}6}{2} (1{\cdot}5) + 4886(1{\cdot}5) \text{ kg.}$$

$$= 8705 \text{ kg.}$$

Shear stress $= \frac{S}{ab}$

$$= \frac{8705}{0{\cdot}87 \times 26{\cdot}6 \times 100} \text{ kg./cm.}^2$$

$$= 3{\cdot}76 \text{ kg./cm.}^2$$

(less than 7 *kg./cm.*^{2})

Shear force at the outer edge of the end beam

Parapet	$0{\cdot}15 \times 1 \times 2400$	...	...	= 360 *kg.*
Kerb	$0{\cdot}25 \times 0{\cdot}15 \times 2300$	...	...	= 86 *kg.*
Slab	$1{\cdot}30 \times 0{\cdot}30 \times 2400$	...	...	= 936 *kg*
Wearing coat	$0{\cdot}90 \times 0{\cdot}075 \times 2300$	...	..	= 155 *kg.*
Uniformly distributed live load	$572 \times 0{\cdot}9 \times 1{\cdot}5$	...	.	= 772 *kg.*
Knife edge load	$4286 \times 1{\cdot}5$	...	...	= 6429 *kg.*
			Total	8738 *kg.*

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Shear stress $= \frac{S}{ab}$

$= \frac{8738}{0{\cdot}87 \times 26{\cdot}6 \times 100}$ *kg /cm.*^{2}

$= 3{\cdot}78$ *kg /cm.*^{2}

(less than 7 *kg./cm* ^{2})

Check for bond stress

$$S_b = \frac{}{a\Sigma o}$$

$$\frac{8738}{0{\cdot}87 \times 26{\cdot}6 \times \frac{100}{10} \times \pi \times 1{\cdot}8} \text{ kg./cm.}^2$$

$= 6{\cdot}68$ *kg./cm*^{2}.

Beams

Loads on the middle beam

Effective span (60 *cm.* bearing)

$= 12 + 0{\cdot}6 = 12{\cdot}6$ *metres*

Overall depth of beam

$= \frac{1}{12}$ of span

$= \frac{1}{12} \times 12{\cdot}6 \times 100$ *cm.* $= 105$ *cm*

$\therefore$ Depth of rib $= 105 - 30 = 75$ *cm.*

Width of rib $= 40$ *cm.* (assumed)

Dead load

Weight of rib	$= 0{\cdot}40 \times 0{\cdot}75 \times 2400$	$= 720$ *kg./m.*
D.L. from slab	$= (30 \times 24 + 7{\cdot}5 \times 2{\cdot}3) \times 2{\cdot}6$	$= 2321$ *kg./m.*
	Total dead load	$= 3041$ *kg /m.*

Live load

Uniformly distributed live load

$= 573 \times 2{\cdot}6 \times 1{\cdot}25 = 1859$ *kg./m.*

Knife edge load $= 12 \times 1000 \times \left(\frac{2{\cdot}6}{3{\cdot}5}\right) \times 1{\cdot}25 = 11142$ *kg.*

Maximum B M. $= \left(3041 + 1859\right) \frac{12{\cdot}6^2}{8} \times 100$

$+ \frac{11142 \times 12{\cdot}6}{8} \times 100$ *kg. cm.*

$= 13{,}234{,}000$ *kg. cm.*

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Loads on the end beam

Parapet = 360 *kg./m.*
Kerb = 96 *kg./m.*
Slab = 936 *kg./m.*
Wearing coat = 155 *kg./m.*
D.L. of deck slab $= \frac{2321}{2}$ *kg./m.* = 1160 *kg./m.*
Rib = 720 *kg./m.*
Uniformly distributed live load
$= 572(0{\cdot}9+1{\cdot}3)\times 1{\cdot}25 = 1573$ *kg./m.*

Total distributed load = 4990 *kg./m.*

Knife edge load $= \left(\frac{0{\cdot}9+1{\cdot}3}{3{\cdot}5}\right)\times 12000\times 1{\cdot}25$

$= 9429$ *kg.*

Maximum bending moment

$$= 4990\times\frac{12{\cdot}6^2}{8}\times 100+\frac{9429\times 12{\cdot}6}{4}\times 100 \text{ kg. cm.}$$

$= 12{,}874{,}000$ *kg. cm.*

We find that the middle beam is subjected to greater bending moment than the end beam.

Let us design the middle beam.

Assuming that 30 *mm.* diameter bars are likely to be used in two layers and allowing a clear cover of 4 *cm.* to the outer layer of bars, the effective depth available

$= d$ = over depth − clear cover − diameter of bar
− half the spacing between layers
$= 105-4-3{\cdot}0-0{\cdot}6$ *cm.*
$= 97{\cdot}4$ *cm.*

Approximate lever arm

$$= d-\frac{d_s}{2}$$

$= 97{\cdot}4-15 = 82{\cdot}4$ *cm.*

Approximate steel requirement

$$= \frac{13{,}234{,}000}{1400\times 82{\cdot}4} \text{ cm.}^2$$

$= 114{\cdot}8$ *cm.*^{2}

Provide 16 bars of 30 mm. diameter (113·1 cm.^{2})

Width of flange

This is taken at the least of the following :

(*i*) Spacing of beams $= 2{\cdot}6$ *m.* $= 260$ *cm.*
(*ii*) $\frac{1}{3}$ span $= \frac{1}{3}\times 12{\cdot}6\times 100 = 420$ *cm.*

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(*iii*) $12\,d_s + b_r = 12 \times 30 + 40 = 400$ *cm.*

$\therefore$ Width of flange $= 260$ *cm.*

Assuming that the neutral axis is situated outside the flange, taking moments about the neutral axis, we have,

$$260 \times 30(n - 15) = 13 \times 113{\cdot}1(97{\cdot}4 - n)$$

$$\therefore \qquad n = 28{\cdot}08 \text{ cm.}$$

But thickness of flange

$$= 30 \text{ cm.}$$

$\therefore$ Neutral axis lies within the flange.

Hence the depth of neutral axis is given by

$$\frac{260\,n^2}{2} = 13 \times 113{\cdot}1(97{\cdot}4 - n)$$

$$\therefore \qquad n^2 + 11{\cdot}31\,n = 1102$$

$$n = 28 \text{ cm.}$$

$$\text{Actual lever arm} = a = d - \frac{n}{3}$$

$$= 97{\cdot}4 - \frac{28}{} \text{ cm.}$$

$$= 88{\cdot}1 \text{ cm.}$$

Actual stress in steel

$$= -\frac{13{,}234{,}000}{113{\cdot}1 \times 88{\cdot}1} \text{ kg./cm.}^2$$

$$= 1327 \text{ kg./cm.}^2$$

Corresponding maximum compressive stress in concrete

$$= \frac{1327}{13} \times \frac{28}{97{\cdot}4 - 28} \text{ kg./cm.}^2$$

$$= 41{\cdot}2 \text{ kg./cm.}^2$$

Maximum shear force

$$= (3041 + 1850) \times \frac{12{\cdot}6}{2} + 15000 \times \frac{2{\cdot}6}{3{\cdot}5} 1{\cdot}25 \text{ kg.}$$

$$= 44800 \text{ kg.}$$

Shear stress $= \frac{S}{ab} = \frac{44800}{88{\cdot}1 \times 40} \text{ kg./cm.}^2 = 12{\cdot}7 \text{ kg./cm.}^2$

Let us provide shear reinforcement in the form of bent up bars.

Perimeter of bars required at the bottom near the supports from local bond stress consideration

$$= \Sigma o \quad \frac{S}{as}$$

$$= \frac{44800}{88{\cdot}1 \times 13} \text{ cm.}$$

$$= 39{\cdot}12 \text{ cm.}$$

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$\therefore$ Number of bars required

$$= \frac{39{\cdot}12}{9{\cdot}42} = 5 \text{ bars}$$

Let eight bars of the top layer be utilized for shear resistance.

Let four bars be bent at a distance of

$$1{\cdot}414a = 1{\cdot}414 \times 88{\cdot}1 = 124{\cdot}6 \text{ cm. say } 125 \text{ cm.}$$

from the support. Four more bars may be bent at $125+125=250$ *cm.* from the support.

Shear resistance of four bars of 30 *mm.* diameter

$$= 0{\cdot}707\ A_w\ f_w$$
$$= 0{\cdot}707 \times (4 \times 7{\cdot}07) \times 1400 \text{ kg.}$$
$$= 27990 \text{ kg.}$$

Balance shear to be resisted

$$= 44800 - 27990 \text{ kg.}$$
$$= 16180 \text{ kg.}$$

Spacing of two-legged 10 *mm.* diameter bars

$$= \frac{A_w\ f_w\ a}{S'}$$

$$= \frac{2 \times 0{\cdot}79 \times 1400 \times 88{\cdot}1}{16810} \text{ cm.}$$

$$= 11{\cdot}6 \text{ cm. say } 11 \text{ cm. centres}$$

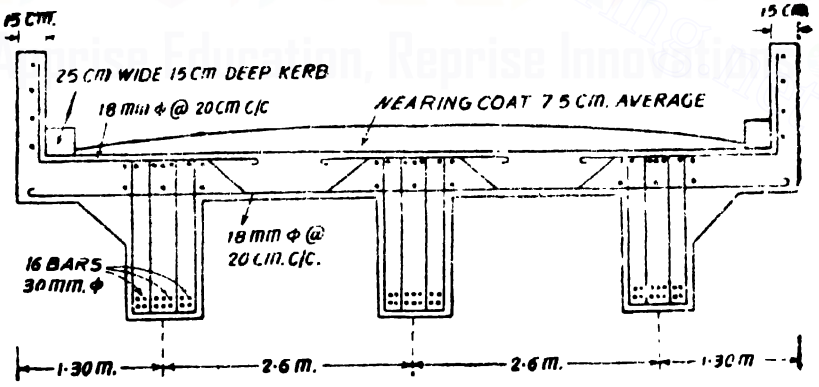

Fig 7·9

§196. Indian Roads Congress Specifications

In India all road bridges are designed according to I.R.C. loading. This code classifies bridges and culverts into the following categories :

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(*i*) I.R.C. Class AA loading

Bridges subjected to very heavy loading on certain specified areas and highways come under this category. This loading is to be adopted within certain municipal limits, in certain existing or contemplated industrial areas, in other specified areas and along certain specified highways. Bridges designed for class *AA* loading should be checked for class *A* loading also, as under certain conditions, heavier stresses may be obtained under class *A* loading.

(*ii*) I.R.C. class A loading

Permanent bridges and culverts are designed for this loading.

(*iii*) I R.C. class B loading

This loading is to be normally adopted for temporary structures and for bridges in specified areas. Structures with timber spans are to be regarded as temporary structures for the purposes of this clause.

Details of I.R.C. loading

The design live load shall consist of standard wheeled or tracked vehicles or trains of vehicles as illustrated in Figs. 750 to 758. The trailers attached to the driving unit are not to be considered as detachable.

Within the kerb to kerb width of the roadways the standard vehicle or train shall be assumed to travel parallel to the length of the bridge and to occupy any position which will produce maximum stresses provided that the minimum clearances between a vehicle and the roadways face of a kerb and between two passing or crossing vehicles as shown in Figs. 751 to 758 are not encroached upon.

For each standard vehicle or train, all the axles of a unit of vehicles shall be considered as acting simultaneously in a position causing maximum stresses.

Vehicles in adjacent lanes shall be taken as headed in the direction providing maximum stresses.

The spaces on the carriageway left uncoverd by the standard train or vehicle shall not be assumed as subjected to any additional live load.

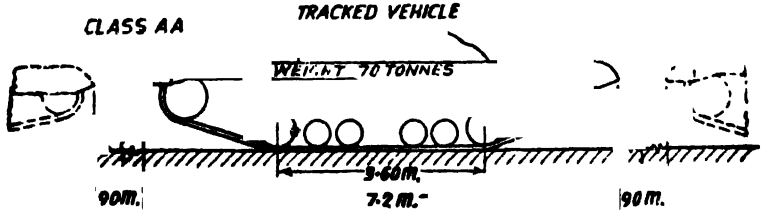

Fig. 750

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Notes : 1. The nose to tail spacing between two successive vehicles shall not be less than 90 *m*.

2. For multilane bridges and culverts one train of class *AA* tracked or wheeled vehicles whichever creates severer conditions shall be considered for every two traffic lane widths.

No other live load shall be considered on any part of the said 2-lane width carriageway of the bridge when the above mentioned train of vehicles is crossing the bridge.

3. The maximum loads for the wheeled vehicles shall be 20 *tonnes* for a single axle, or 40 *tonnes* for a bogie of two axles spaced not more than 1·2 *metres* centres.

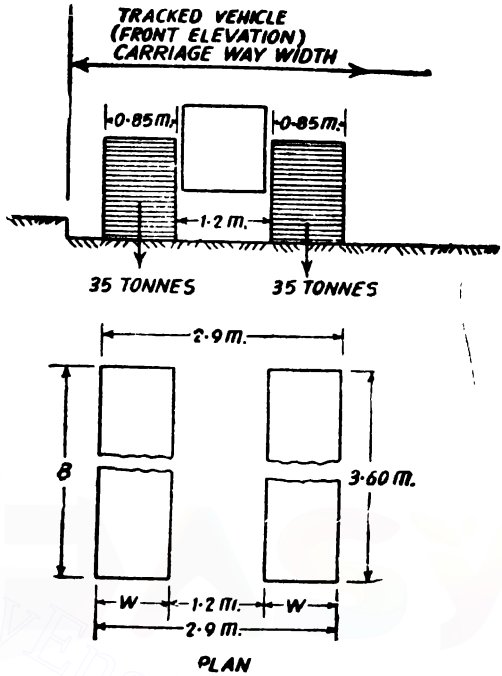

Fig. 751

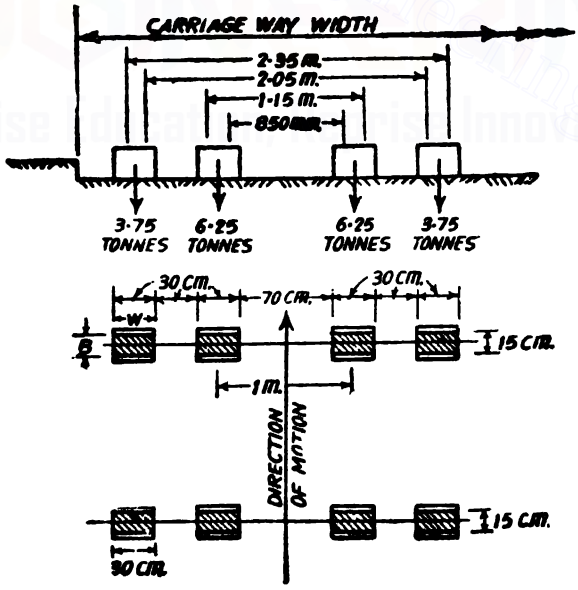

Fig. 752

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4. The minimum clearance between the road face of the kerb and the outer edge of the wheel or track, *C*, shall be taken as follows :

Number of lanes	*Carriageway width*	*Maximum values of C*
Single lane bridges	3·8 *metres* and above	0·3 *m.*
Multilane bridges	less than 5·5 *metres* 5·5 *metres* or above	0·6 *m.* 1·2 *m.*

Class A Train of Vehicles

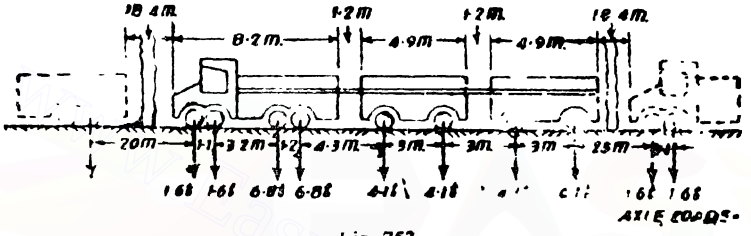

Fig. 753

Notes

1. The nose to tail distance between successive trains shall not be less than 18·4 *metres.*

2. No other live load shall cover any part of the carriageway when a train of vehicles (or trains of vehicles in multilane bridge) is crossing the bridge.

3. The ground contact area of the wheel shall be taken as follows :

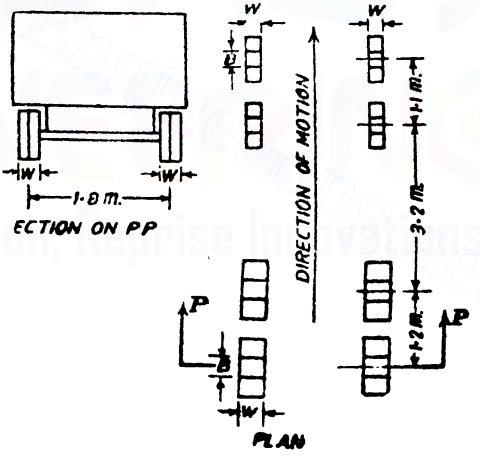

Fig. 754

Axle load (tonnes)	*Ground contact area*	
	B (*mm.*)	*W* (*mm.*)
11·4	250	500
6·8	200	380
2·7	150	200

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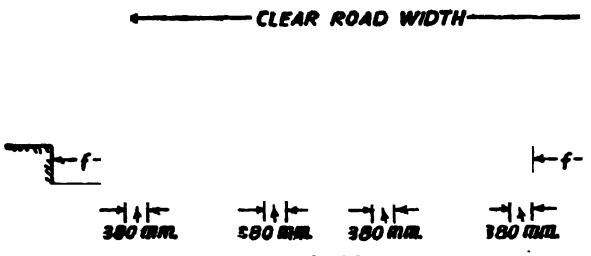


Fig. 755

4. The minimum clearance f between the outer edge of the wheel and the road way face of the kerb, and the minimum clearance g, between the outer edge or passing or crossing vehicles on multilane bridges shall be as given below :

Clear carriage way width		
5·5 *metres* to 7·5 *metres*	Uniformly increasing from 0·4 *m.* to 1·2 *m.*	150 *mm.* for all carriage-way widths.
Above 7·5 *metres*	1·2 *metre*	

Class B Train of Vehicles

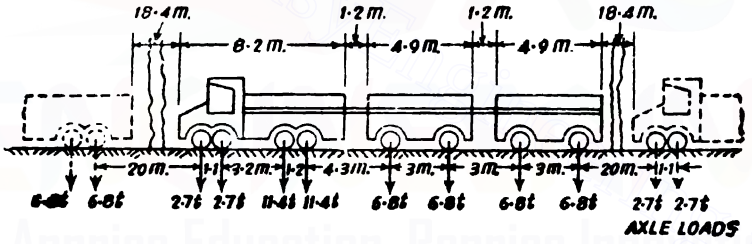


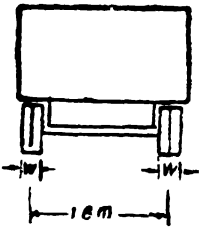
Fig. 756

Notes

1. The nose to tail distance between successive trains shall not be less than 18·4 *metres*.

2. No other live load shall cover any part of the carriage way when a train of vehicles (or trains of vehicles in multilane bridge) is crossing the bridge.

3. The ground contact area of the wheel shall be taken as follows :



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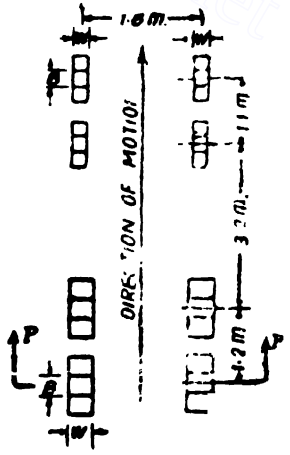

Fig. 757

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Axle load (tonnes)	Ground contact area	
	B (mm.)	M (mm.)
6·8	200	380
4·1	150	300
1·6	125	175

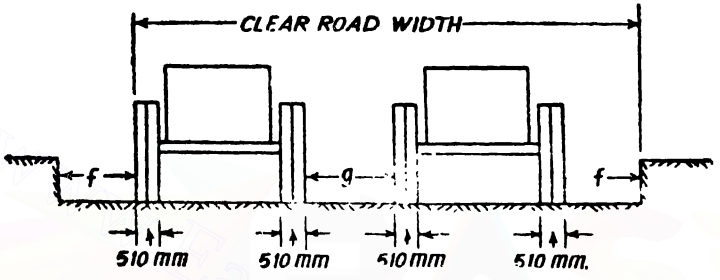

Fig. 758

4. The minimum clearance *f* between outer edge of the wheel and the road way face of the kerb, and the minimum clearance *g* between the outer edges of passing or crossing vehicles on multilane bridges shall be as given below :

Clear carriage width		
5·6 *metres* to 7·5 *metres*	Uniformly increasing from 0·4 *m.* to 1·2 *m.*	150 *mm.* for all carriage-way widths
Above 7·5 *metres*	1·2 *metres*	

Method of application of live loads for the design of deck slabs (excluding pre-cast slabs of width less than width of dispersion).

Any rational method may be adopted for calculating the effect of the live loads on deck slabs but the following methods are recommended.

Distribution of concentrated loads on slabs

(*a*) If a solid slab is supported on two opposite edges and carries concentrated loads, it should be designed to resist the maximum bending moment caused by the loading system. Such bending moment shall be assumed to be resisted by an effective width of slab (measured parallel to the supported edges), the effective width being calculated as follows :

(*i*) For a single concentrated load, the effective width shall be calculated in accordance with the following equation :

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$$e = kx\left(1 - \frac{x}{l}\right) + W$$

where e = the effective width of the slab on which the load rests.

l = the effective span in the case of simply supported slabs and equal to the clear span in the case of continuous slabs.

and x = the distance of the centre of gravity of the concentrated load from the nearer support.

W = the breadth of concentration area of the load, *i.e.*, the dimension of the tyre or track contact area over the road surface of the slab in a direction at right angles to the span plus twice the thickness of the wearing coat or surface finish above the structural slab.

K = a constant having the following values depending upon the ratio $\frac{l'}{l}$ where l' is the width of the slab :

$\frac{l'}{l}$	Values of K		$\frac{l'}{l}$	Values of K	
	Simply supported slab	Continuous slab		Simply supported slab	Continuous slab
0·1	0·40	0·40	1·1	2·60	2·28
0·2	0·80	0·80	1·2	2·64	2·36
0·3	1·16	1·16	1·3	2·72	2·40
0·4	1·48	1·44	1·4	2·80	2·48
0·5	1·72	1·68	1·5	2·84	2·48
0·6	1·90	1·84	1·6	2·88	2·52
0·7	2·12	1·96	1·7	2·92	2·56
0·8	2·24	2·08	1·8	2·96	2·60
0·9	2·36	2·16	1·9	3·00	2·60
1·0	2·48	2·24	2 and above	3·00	2·60

Provided that the effective width shall not exceed the actual width of the slab. And provided further that in case of a load near the supported edge of a slab, the effective width shall not exceed the above value nor half the above value plus the distance of the load from the unsupported edge.

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(*ii*) For two or more concentrated loads in a line in the direction of the span, the bending moment per unit width of the slab shall be calculated separately for each load according to its appropriate effective width of slab calculated in (*i*) above.

(*iii*) For two or more loads not in a line in the direction of the span :

If the effective width of a slab for one load overlaps the effective width of slab for an adjacent load the resultant effective width for the two loads shall be taken as equal to the sum of the respective effective widths for each load minus the width of overlap, provided that the slab so designed is tested for the two loads acting separately.

(*b*) If a cantilever solid slab carries concentrated loads, it should be designed to resist the maximum bending moment caused by the loading system. Such bending moment shall be assumed to be resisted by an effective width of slab (measured parallel to the supported edge) as follows :

(*i*) For a single concentrated load, the effective width shall be calculated in accordance with the following equation :

$$e = 1{\cdot}2x + W$$

where e = the effective width

x = the distance of the centre of gravity of the concentrated load from the face of the cantilevering support.

and W = the breadth of concentration area of the load, *i.e.*, the dimensions of the tyre or track contact area over the road surface of the slab in a direction parallel to the supporting edge of the cantilever plus twice the thickness of the wearing coat or surface finish above the structural slab.

Provided that the effective width of the cantilever slab shall not exceed one-third the length of the cantilever slab measured parallel to the support. And provided further that when the concentrated load is placed near one of the two extreme ends of the length of cantilever slab in the direction parallel to the support, the effective width shall not exceed the above value, nor shall it exceed half the above value plus the distance of the concentrated load from the nearer extreme end measured in the direction parallel to the fixed edge.

(*ii*) **For two or more concentrated loads :**

If the effective width of slab for one load overlaps the effective width of slab for an adjacent load, the resultant effective width for the two loads shall be taken as equal to the sum of the respective effective widths for each load minus the width of overlap, provided that the slab so designed is tested for the two loads acting separately.

(*c*) For slabs (ribbed or through slab) other than solid slabs, the effective width shall depend on the ratio of the transverse and longitudinal flexural rigidities of the slab. When this ratio is one,

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that is when the transverse and longitudinal flexural rigidities are approximately equal, the value of the effective width as found for solid slabs may be used. But as the ratio decreases, a proportionately smaller value should be taken.

(*d*) More accurate method of analysis for cases of slabs covered by (*a*) (*b*) and (*c*) above and for all other cases of slabs may be used with the approval of the engineer-in-charge.

Dispersion of loads along the span The effective length of slab on which a wheel load or track load acts shall be taken as equal to the dimension of the tyre contact area over the wearing surface of the slab in the direction of the span plus twice the overall depth of the slab inclusive of the thickness of the wearing surface.

The effect of concentrated loads on slabs spanning in one or two directions may be calculated from the influence fields of such loads or by any other rational method such as that of M. Pigeaud or Dr. Marcus, the value of Poisson's ratio being taken as 0·15.

Dispersion of loads through fills on arch bridges. The dispersion through the fills above the arch shall be assumed at 45 degrees both along and perpendicular to the span in the case of arch bridges.

Reduction in the intensity of live load stresses on bridges accommodating more than two traffic lanes.

The position and number of loaded lanes shall be such as to produce the maximum stresses in all cases.

Where maximum stresses are produced in any member by simultaneously loading more than two traffic lanes the intensities of the resultant live load stresses shall be reduced by 10 per cent for each additional loaded traffic lane in excess of the two lanes subject to a maximum reduction of 20 per cent and subject also to the condition that the stresses as thus reduced are not lower than the stresses resulting from simultaneous loading on two traffic lanes.

Foot way and kerb loading (the provisions under the clause do not apply to foot bridges).

For all parts of bridge floors accessible only to pedestrians and animals and for all foot ways the loading shall be 400 $kg/metre^2$. Where crowd loads are likely to occur such as on bridges located near towns, which are either centres of pilgrimage or where large congregational fairs are held seasonally, the intensity of footway loading shall be increased from 400 $kg/metre^2$ to 500 $kg./metre^2$.

Kerbs 0·6 *metres* or more in width, shall be designed for the above loads in addition to the lateral loading specified. If the kerb width is less than 0·6 *metre* no live load shall be applied in addition to the load specified.

In calculation of stresses in members of structures with cantilevered footways, the footways shall be considered as loaded on one side or both sides or unloaded, whichever condition gives the maximum stresses.

In bridges designed for any of the I.R.C. loadings described above the main girders, trusses, arches, or other members support-

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ing the footways shall be designed for the following live loads per square *metre* of footways area, the loaded length of footway taken in each case being such as to produce the worst effects on the member under consideration.

(*a*) For effective spans of 7·5 *metres* or less, 400 *kg./metres*^{2} or 500 *kg./metre*^{2} as the case may be.

(*b*) For effective spans of over 7·5 *metres* but not exceeding 30 *metres* the intensity of load shall be determined according to the equation :

$$P = P' - \left\{ \frac{40L - 300}{9} \right\}$$

(*c*) For effective spans of over 30 *metres* the intensity of load shall be determined according to the equation :

$$P = \left\{ P' - 260 + \frac{4800}{L} \right\} \left\{ \frac{1{\cdot}5 - W}{15} \right\}$$

where, $P' = 400$ *kg /metre*^{2} or 500 *kg./metre*^{2} as the case may be.

P = the live load in *kg.* per *metres*^{2}.

L = the effective span of main girder, truss or arch in *metres*.

and W = width of the footway in *metres*.

Impact. Provision for impact or dynamic action shall be made by an increment of the live load by an impact allowance expressed as a fraction or a percentage of the applied live load. The impact fraction shall be determined from the following equations which are applicable for spans between 3 *metres* and 45 *metres*.

For class AA loading. The impact percentage shall be taken as follows :

(*a*) For spans less than 9 metres

(*i*) For tracked vehicles : 25 percent for spans up to 5 *metres* linearly reducing to 10 per cent for spans of 9 *metres*.

(*ii*) For wheeled vehicles : 25 per cent.

(*b*) For spans of 9 metres or more

For R.C. bridges

(*i*) For tracked vehicles : 10 percent up to spans of 40 *metres* and in accordance with the curve indicated in Fig. 759 for spans in excess of 40 *metres*.

(*ii*) For wheeled vehicles : 25 percent for spans up to 12 *metres* and in accordance with the curve indicated in Fig. 759 for spans in excess of 12 *metres*.

For steel bridges

(*i*) Tracked vehicles 10 per cent for all spans.

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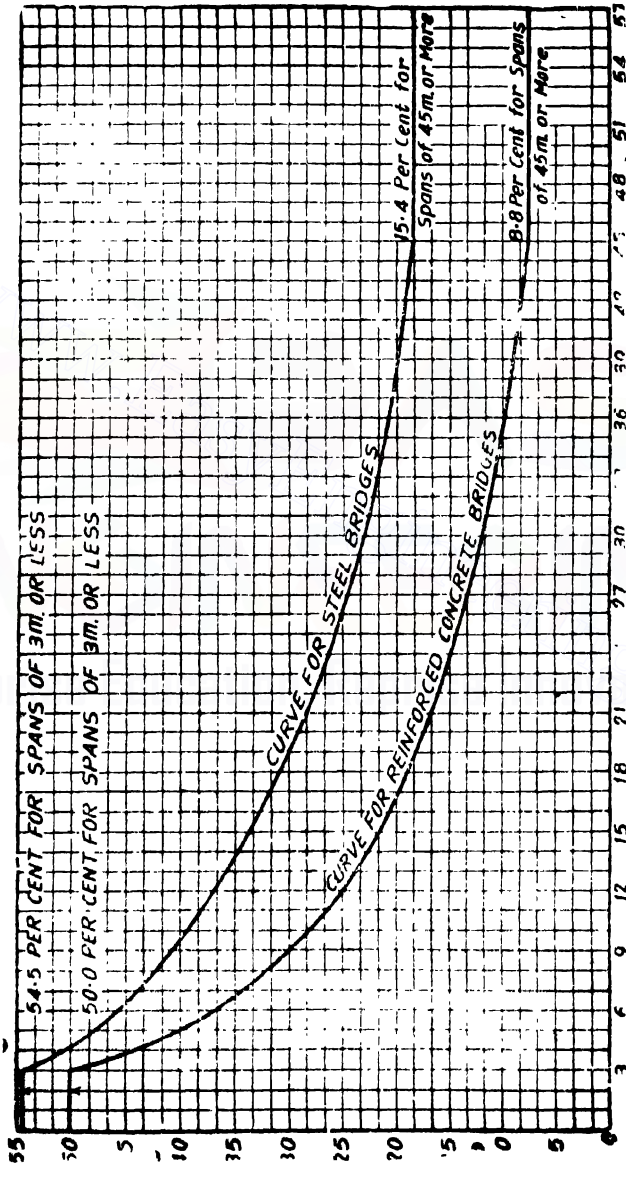

Impact percentage curves for highway bridges for Class A and Class B Loadings
Fig. 759

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(*ii*) **Wheeled vehicles :** 25 percent for spans up to 23 *metres* and in accordance with the curve indicated in Fig. 759 for spans in excess of 23 *metres*.

For class A or class B loading In the members of any bridge designed either for class A or class B loading the impact percentage shall be determined from the curves indicated in Fig. 759. The impact fraction shall be determined from the following equations which are applicable for spans between 3 *metres* and 45 *metres*.

(*i*) Impact factor fraction for R.C. bridges

$$= \frac{4{\cdot}5}{6+L}$$

(*ii*) Impact factor fraction for steel bridges

$$= \frac{9}{13{\cdot}5+L}$$

where L is the length in *metres* of the span as specified below :

(*a*) For spans simply supported or continuous or archers	Effective span on which the load is placed.
(*b*) For bridges having cantilever arms without suspended spans.	Effective overhang of the cantilever arm reduced by 25 percent for loads on the cantilever arm and the effective span between supports for loads on main span.
(*c*) For bridges having cantilever arms with suspended spans.	Effective overhang of the cantilever arm+half the length of the suspended span for loads on the cantilever arm, the effective length of the suspended span for loads on the suspended span and the effective span between supports for loads on the main span.

In any bridge structure where there is a filling of not less than 0·6 *metre* including the road crust the impact percentage to be allowed in the design shall be assumed to be one-half of what is stipulated above.

For calculating the pressure on bearings and on the top surface of the bed blocks, full value of the appropriate impact percentage shall be allowed but, for the design of piers, abutments and structures, generally below the top of the bed block, the appropriate impact percentage shall be multiplied by the factor given below :

(*a*) For calculating the pressure at the bottom surface of the bed block. ... 0·5

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(*b*) For calculating the pressure on the top 3 *metres* of the structure below the bed block. ... 0·5 decreasing uniformly to zero

(*c*) For calculating the pressure on the portion of the structure more than 3 *metres* below the bed block. zero

Kerbs

(*a*) The section given in Fig. 760 shall be adopted for the road kerb.

(*b*) Roadway kerbs shall be adequately anchored into the deck structure to resist a local lateral force of 750 *kg.* per *metre* of the kerb, applied horizontally at the top of the kerb.

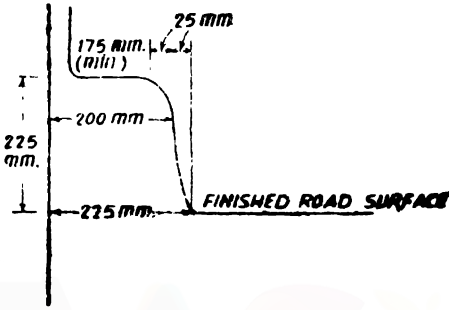

Fig. 760

Note. The horizontal force need not be considered in the design of the main structural members of the bridge.

(*c*) Safety kerbs shall not be less than 600 *mm.* wide.

Width of roadway and footway

(*a*) For high level bridges constructed for the use of road traffic only, the width of the roadway shall not be less than 3·8 *m.* for a single lane bridge and shall be increased by a minimum of 3 *metres* for every additional lane of traffic on multiple lane bridges. Road bridges shall be either one lane, two lanes or four lanes. Three lane bridges shall not be constructed. In the use of four lanes or multiples of two lanes a central verge of at least 1·2 *metres* width shall be provided.

(*b*) Causeways or submersible bridges shall provide for at least two lanes of traffic as specified in (*a*) above unless one lane traffic is specially permitted for the design.

(*c*) For bridges constructed for the use of combined road and tramway traffic the roadway width given in (*a*) above shall be increased by 4 *metres* for a single track tramway and by 7·6 *metres* for a double track tramway.

(*d*) For a bridge on a horizontal curve the roadway width shall be increased by an amount not less than that required by the relevant Road Standards.

(*e*) When a footway is provided its width shall not be less than 1·5 *metre*.

Railings, Parapets or Guide Posts

(1) High level bridges

(*a*) Substantial railings or parapets along each side of the bridge shall be provided for the protection of traffic. Consideration shall be given to the architectural features of the railing or

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parapet to obtain proper proportioning of its various members and its harmony with structure as a whole. Consideration shall be given also to avoiding, as far as is consistent with safety and appearance, obstruction of the view from passing motor cars.

(*b*) Railings or parapets shall have a minimum height above the adjacent roadway or footway surface, of one *metre* less one-half the horizontal width of top rail or top of the parapet. They shall be designed to resist a lateral horizontal force and a vertical force each of 150 *kg.* per *metre* run applied simultaneously at the top of the railing or parapet. These forces shall also be considered in the design of the main structural members if the bridge is provided with footpaths. Where, however, footpaths are not provided, these forces need not be considered in the design of main structural members.

(*c*) The clear distance from the lower rail to the top of the kerb shall not exceed 150 *mm.* unless the space is filled by vertical or inclined members, the clear distance between which is not more than 150 *mm.* The strength of the lower rail shall be at least as great as that of the top rail. The space between the lower rail and the top rail shall be filled by means of vertical, horizontal or inclined members, the clear distance between which shall be fixed, with due regard to the safety of persons and animals using the structure.

(2) Submersible Bridges

Railing shall be either collapsible or removable :

(*a*) Collapsible railings shall be used where it is necessary to put up the railings immediately the bridge is open to traffic after a submerging flood. Care shall be taken in the structural design of these railings to ensure that they sit well in their grooves and are not liable to be dislodged by floods.

(*b*) Removable railings may be adopted when there is no danger to the traffic using the bridges for short periods without railing. Care shall be taken in the structural design of these railings to ensure that the various members are interchangeable and can be easily removed and refitted.

(*c*) Collapsible or removable railings shall be designed to resist as far as possible the same forces as specified in (1) *b* for railings or parapets on high level bridges.

Wind Load

All structures shall be designed for the following lateral wind forces. These forces shall be considered to act horizontally and in such a direction that the resultant stresses in the member under consideration are the maximum.

The wind force on a structure shall be assumed as a horizontal force of the intensity specified below and acting on an area calculated as follows :

(*a*) For deck structures

The area of the structure as seen in elevation including the floor system and railing, less area of performations in the hand railing

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or parapet walls (while calculating the wind pressure on live load, or clear distance between the trailers of a train of vehicles shall not be omitted).

(*b*) For a through or half-through structure

The area of the elevation of the windward truss as specified in (*a*) above, plus half the area of the elevation above the deck level of all other trusses or girders.

The following table gives the wind pressure intensities corrresponding to various velocities of wind The pressure given in this table shall however be doubled for bridges situated in the areas such as the *Kathiawar Peninsula*, and the Bengal and Orissa coasts.

Table of Wind Pressure and Wind Velocities

H	*V*	*P*	*H*	*V*	*P*
0	80	40	30	147	141
2	91	52	40	155	157
4	100	63	50	162	171
6	107	73	60	168	183
8	113	82	70	173	193
10	118	101	80	177	202
15	128	107	90	180	210
20	135	119	100	183	217
25	142	130	110	186	224

In the above table,

H = the average height in *metres* of the exposed surface above the mean retarding surface (ground or bed level or water level).

V = Horizontal velocity of wind in *kilometres* per hour at height *H*.

P = Horizontal wind pressure in *kg./metre*^{2} at height *H*.

The lateral wind force against any exposed moving live load shall be considered as acting at 1·5 *metre* above the roadway and shall be assumed to have the following values :

Highway bridges, ordinary ... 300 *kg./metre* run

Highway bridges carrying tramway ... 450 *kg./metre* run

while calculating the wind pressure of live load, the clear distance between the trailers of a train of vehicles shall not be omitted.

The bridges shall not be considered to be carrying any live load when the wind velocity at deck level exceeds 130 *kilometres* per hour.

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The total assumed wind force as calculated according to the above paragraphs shall however not be less than 450 *kg.* per *metre* run in the plane of the loaded chord and 225 *kg.* per *metre* run in the plane of the unloaded chord on through or semi-through truss, latticed or similar span and not less than 450 *kg.* per *metre* run on deck spans.

A wind pressure of 240 *kg.* per *metre*^{2} on the unloaded structure applied as specified in the above paragraphs shall be used if it produces greater stresses than those produced by the combined wind forces mentioned above.

LONGITUDINAL FORCES

In all road bridges, provision shall be made for longitudinal forces arising from any one or more of the following causes :

(*a*) Tractive effort caused through acceleration of the driving wheels ;

(*b*) Banking effect resulting from the application of brakes to braked wheels ; and

(*c*) Frictional resistance offered to the movement or free bearings due to change of temperature or any other cause.

Note : *Braking effect is invariably greater than the tractive effort.*

The braking effect on a simply supported span of a continuous unit of spans or any other type of bridge unit shall be assumed to have the following value :

(*a*) *In the case of a single lane or a two lane bridge.* 20 percent of the first train load plus 10 percent of loads of the succeeding trains or part thereof, the train loads in one lane only being considered for the purposes of this clause. Where the entire first train is not on the full span, the braking force shall be taken as equal to 20 percent of the loads actually on the span.

(*b*) *In the case of bridges having more than two lanes.* As in (*a*) above for the first two lanes plus 5 percent of the loads on the lanes in excess of two.

Note : *The loads in this clause shall not be increased on account of impact.*

The force due to braking effect shall be assumed to act along a line parallel to the roadway and 1·2 *metres* above it. While transferring the force to the bearings, the change in the vertical reaction at the bearings should be taken into account.

The longitudinal force at any free bearing shall be limited to the sum of dead and live load reactions at the bearing multiplied by the appropriate coefficient of friction. The coefficient of friction at the bearing shall be assumed to have the following values :

For roller bearing	...0·03
For sliding bearings of hard copper alloy	...0·15

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For sliding bearings of steel on cast iron or steel on steel	...0·25
For sliding bearings of steel on ferro asbestos	.. 0·20
For other types of bearings of proved utility if permitted at the discretion of the Engineer-in-charge	As may be permitted by the Engineer in-charge on examination of available data

For simply supported reinforced concrete and prestressed concrete superstructure, the span upto which plate bearing can be used shall be limited to 15 *metres*.

The longitudinal force at the fixed bearings shall be taken as the algebraic sum of the longitudinal forces at the free bearings in the bridge unit under consideration and the force due to the braking effect on the wheels, mentioned in (*a*) and (*b*) above.

The effects of braking force on bridge structures without bearings such as arches, rigid frames *etc.*, shall be calculated in accordance with approved methods of analysis of indeterminate structures

The effects of longitudinal forces and all other horizontal forces should be calculated up to a level where the resultant passive earth resistance of the soil below the deepest scour level (floor level in case of a bridge having pucca floor) balances these forces.

Centrifugal forces. In case a road bridge is situated on a curve, all portions of the structure affected by the centrifugal action of moving vehicles are to be proportioned to carry safely the stress induced by this action in addition to all other stresses to which they may be subjected.

The centrifugal force shall be determined from the following equations :

$$C = \frac{WV^2}{127\,R}$$

where C = Centrifugal forces in *tonnes* acting normally to the traffic (1) at the point of action of the wheel loads or (2) uniformly distributed over every *metre* length on which a uniformly distributed load acts

W = Live load (1) in *tonnes* in case of wheel loads, each wheel load being considered as acting over the respective ground contact length and (2) in *tonnes* per *metre* in case of a uniformly distributed live load

V = The design speed of vehicles using the bridge in *kilometres* per hour and

B = The radius of curvature in *metres*.

The centrifugal force shall be considered to act at a height of 1·2 *metres* above the level of the carriageway.

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No increase for impact effect shall be made on the stresses due to centrifugal action.

The overturning effect of the centrifugal force on the structure as a whole shall also be duly considered.

Temperature Effects. Provision shall be made for stresses or movements from variations in temperature.

The rise and fall in temperature shall be fixed for the locality in which the structure is to be constructed and shall be figured from an assumed temperature at the time of erection.

Due consideration shall be given to the lag between air temperature and the interior temperature of massive concrete members or structures.

Except where stated otherwise, the following range of temperature shall generally be assumed in the design :

(*a*) **Metal structures**

Moderate climate : from −18°C to + 50°C
Extreme climate : from −35°C to + 50°C

(*b*) **Concrete structures**

	Temperature rise	Temperature fall
Moderate climate :	17°C	17°C
Extreme climate :	25°C	25°C

But, in both cases (*a*) and (*b*) above, intermediate values can be allowed at the discretion of the Engineer responsible for the design.

The coefficient of expansion per degree centigrade shall be taken as 0·0000117 for steel and reinforced concrete structures and 0·0000108 for plain concrete structures

Secondary Stresses. These are additional stresses brought into play due either to the movement of supports or to the deformations in the geometrical shape of the structure or its members resulting from causes such as rigidity of end connection or loads applied at intermediate points of trusses or restrictive shrinkage of concrete floor beams.

All bridges shall be designed and constructed in a manner such that the secondary stresses are reduced to a minimum, and they shall be allowed for in the design

For reinforced concrete members the shrinkage coefficient for purposes of design may be taken as 2×10^{-4}.

Erection Stress. Allowance shall be made in the design for stresses set up in any member during erection ; such stresses may be different from those which the member will be subjected to during actual working.

Seismic Force. If a bridge is situated in a region subject to earthquake, allowance shall be made in the design for the seismic force and earthquake-resistant features shall be embodied in the structural details of the design.

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The seismic force shall be taken as a horizontal force equal to the approximate fraction (specified below) of the weight of dead and live loads acting above the section under consideration (parts of the structure embedded in soil shall not be considered to produce any horizontal force.) The allowance for horizontal acceleration may be taken as follows :

Regions liable to minor damage : $\frac{1}{20}$ of Gravity

Regions liable to severe damage : $\frac{1}{10}$ of Gravity

For bridges situated in epicentral tracts where large devastations have occurred in the past due to earthquakes, the percentages shall be fixed by the engineer responsible for the design, due regard to the local conditions regarding the intensity of earthquakes generally experienced in these regions.

The horizontal forces due to seismic force shall be taken to act through the centre of gravity of all the loads under consideration. The direction of this force should be such that the resultant stresses in the member under consideration are maximum.

Seismic and wind forces shall not be considered to act simultaneously.

The magnitude of the seismic force shall not be reduced on account of reduction in weight due to buoyancy obtained in a submerged mass.

SOME USEFUL DESIGN REQUIREMENTS

Genera.

(*i*) Size of bars. The maximum size of reinforcement shall be 45 *mm.* diameter.

Excepting for wire mesh and similar reinforcement the diameter of any reinforcing bar including transverse ties, spirals, stirrups and all secondary reinforcements, shall generally be not less than 6 *mm.*

The diameter of longitudinal reinforcing bars in columns shall not be less than 12 *mm.*

The diameter of wires under tensile stress in connected mesh and similar reinforcement in slabs shall be not less than 3 *mm.*

(*ii*) Distance between bars. The horizontal distance between two parallel reinforcing bars shall not be less than the greatest of the following dimensions :

(*a*) Diameter of the bar—if the diameters are equal.

(*b*) The diameter of the largest bar—if the diameters are unequal.

(*c*) 6 *mm.* more than the normal maximum size of coarse aggregate used in concrete.

Note : In order to comply with the provisions of this clause above, the size of the coarse aggregate for the concrete around congested reinforcement may be reduced. This does not preclude the use of large size aggregate where reinforcement is not congested.

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When immersion vibrators are intended to be used sufficient space shall be left between groups of bars to enable the vibrator to be inserted.

The minimum vertical distance between two horizontal main reinforcing bars shall be 12 *mm.* or the maximum diameter of the coarse aggregate or the maximum size of bar, whichever is greater.

When contact of bars along the lap length cannot be avoided such bars shall preferably be grouped in the vertical plane. In no case, however, shall there be more than three bars in contact.

The pitch of bars or wires of main tensile reinforcement in slabs shall not exceed 300 *mm.* or twice the effective depth of the slab whichever is smaller.

All mesh reinforcement shall be of such dimensions as will enable the coarsest material in the concrete to pass easily through the meshes of such reinforcement.

(*iii*) Cover

The thickness of concrete cover (exclusive of plaster or other decorative finish) shall be as follows :

(*i*) At each end of a reinforcing bar, neither less than 25 *mm.* nor less than twice the diameter of such bar.

(*ii*) For longitudinal reinforcing bar in a column or beam, neither less than 40 *mm.* nor less than the diameter of such bar.

(*iii*) In no case, the cover to any reinforcement including stirrups of binders be less than 25 *mm.* nor less than the diameter of such bar.

In the case of bars which are not round, the diameter shall be taken as the diameter of the circle giving equivalent area.

The cover of concrete in direct contact with earth faces shall be 25 *mm* more than what has been specified in the above clauses.

For reinforced concrete members, totally immersed in sea water, the cover shall be 40 *mm.* and for members subjected to sea spray, the cover of concrete shall be 50 *mm.* more than what has been specified in the above clauses.

Increased cover thickness, as may be decided by the designer shall be provided when surfaces on concrete members are exposed to the action of harmful chemicals, acid vapours, saline atmosphere, sulphurous smoke (as in case of bridges over steam operated railways) *etc.* It may, however, be noted that dense and strong concrete is a better guarantee for the protection of reinforcement against the harmful effects of chemicals than an increase in the thickness of cover.

For controlled concrete with specified cube strength of 250 *kg./cm.*^{2} or more, the additional thickness of cover may be reduced to half.

(*iv*) Moment of inertia for calculating relative stiffness

To determine the relative stiffness of members of statically in

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determinate structures, the moment of inertia may be calculated by considering

(*a*) the entire concrete section, ignoring the reinforcement,

or (*b*) the entire concrete section, including the reinforcement on the basis of modular ratio,

or (*c*) the compression area of the concrete section combined with the reinforcement, on the basis of the modular ratio.

(*v*) Temperature and shrinkage effects

The values of the coefficients of thermal expansion and shrinkage of concrete shall be taken as follows :

(*a*) Thermal expansion :

For reinforced concrete 117×10^{-7} per degree centigrade

For plain concrete 108×10^{-7} per degree centigrade

(*b*) Shrinkage : 2×10^{-4}

Every simply supported beam shall be provided with means to permit both rotation and longitudinal expansion caused by the design loads and forces.

BEAMS AND SLABS

(*i*) Effective span

In the case of free supports on line bearings, the effective span shall be the distance between the centres of bearings.

In case of free support not on line bearings, the effective span shall be $l+d$ or L whichever is smaller.

Where L = distance from centre to centre of supports.

l = clear span

d = Effective depth of beam or slab.

In case of restraint at the supports, the effective span may be taken as equal to the clear distance between the faces of the supports.

(*ii*) Effective depth

The effective depth of a beam or slab shall be the depth from edge of the compression section to the centroid of the tension reinforcement.

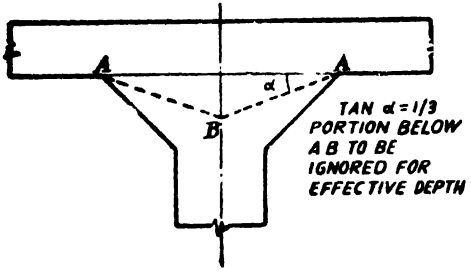

Fig. 761

Where haunches are provided, no portion of the haunch lying below a plane which makes a slope of 1 : 3 as shown in Fig. 761 shall be considered as adding to the effective depth.

However, for calculating the moments in a haunched slab or beam, the actual variation of moments of inertia of the span shall be considered.

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(*iii*) Compression Reinforcement in Beams

The compression reinforcement should be effectively anchored in two directions at right angles over the distance where it is required to act in compression, at points not further apart, centre to centre than twelve times the diameter of the anchored bar. The subsidiary reinforcement used for this purpose should pass round, or be hooked over both the compressive and tensile reinforcement The amount of steel in compression shall preferably not exceed 4 percent. If it does, only 4 percent shall be allowed for in the calculation of the moment of resistance of the beam. The percentage shall be calculated as follows :

(*a*) In rectangular beams, on the total cross-sectional area ;

(*b*) In T-beams or L-beams, on an area equal to the total depth multiplied by the width of the beam web.

(*iv*) Curtailment of Bars

To prevent large changes in the moments of resistance, the points at which the bars are curtailed shall be suitably spread.

In simply supported spans, at least 20 percent of the steel required to resist the maximum bending moment shall be carried over the supports along the tension side of the beam or slab. For a span continuous beyond a support, at least 10 percent of the steel required to resist the maximum positive bending moment shall be similarly carried over the support.

(*v*) Curved or sloped reinforcement

Where the alignment of the reinforcement deviates from the normal to the plane of bending, as in the case of a beam with curved or slopped soffit, only the area of reinforcement effective in the direction normal to the plane of bending shall be considered

(*vi*) Lateral support in Beams

In flanged beams with length between adequate lateral restraints greater than thirty times the width of the compression flange, the permissible compressive stress in concrete shall be reduced by a factor equal to $1{\cdot}5-\frac{L}{60\,B}$ where L is the unsupported length and B is the width of the compression flange of the beam.

(*vii*) Skin Reinforcement in Deep Beams

If the depth of web of a beam exceeds 1·5 *metres* skin longitudinal reinforcement of an area at least 0·5 percent of the web area shall be provided near each face. Spacing of such bars shall not exceed 200 *mm*

(*viii*) Dispersion of live loads on Longitudinal Beams

The dispersion of the live load along the span length through the wearing coat, deck slab and filling shall be considered.

When longitudinal beams are connected together by transverse members like deck slab, cross girders, diaphragms, soffit slab, *etc.* the distribution of bending moments between longitudinals shall be calculated by one of the following methods :

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(*a*) Finding the reactions on the longitudinals assuming the supports of the deck slab as unyielding. The method is applicable where there are only two longitudinals with no soffit slabs.

(*b*) Distributing the loads between longitudinals by Courbon's method strictly within its limitation, *i.e.* when the effective width of the deck is less than half the span and when the stiffness of cross girders is very much greater than that of the longitudinals ; and

(*c*) Distributing the loads between the longitudinals by any rational method of grid analysis, *e.g.*, the method of harmonic analysis as given by *Hendry* and *Jaeger* or *Morice* and *Little's version of the isotrophic plate theory of Guyon and Massonet etc.*

In calculating the shear force on sections of longitudinal beams, wheel loads or track loads shall be allocated to respective longitudinal beams, by any rational method. Alternatively, the following method shall be followed :

For loads at or within 5·5 metres of either supports

The reaction on the longitudinal beams shall be the greater of the results obtained by

(*a*) Assuming the deck slab as simply supported or continuous as the case may be, the supports being assumed unyielding, and

(*b*) Following one of the three methods given in (*a*), (*b*) and (*c*) of paragraph (*viii*).

For loads more than 5·5 metres away from either supports

Distribution of the loads between the longitudinals for the purpose of finding shear forces shall be assumed to be the same as for bending moments given in paragraph (*viii*) (*a*), (*b*) and (*c*).

(*ix*) Distribution of live loads on Transverse Beams

Dispersion of live loads along the span length through the wearing coat, deck slab and filling shall not be considered.

(*x*) Distribution of live loads on intermediate Transverse floor beams

Distribution of loads between longitudinal beams for the purpose of finding bending moments and shears in intermediate transverse floor beams shall be made by one of the methods given in paragraphs (*viii*) (*a*), (*b*) and (*c*).

(*xi*) T-Beams and L-Beams

For *T*-beams or *L*-beams, the slab shall be considered as an integral part of the beam if adequate bond and shear resistance is provided at the junction of the slab and the web of the beam.

For the purpose of calculation of stress at any section of a *T*-beam or *L*-beam or in the calculation of its moment of inertia, the effective width of the slab, to function as the compression flange of the beam, shall be the least of the following :

(*a*) In case of *T*-beams

1. One-fourth of the effective span.

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2. Spacing of centres of ribs of beams.
3. The breadth of the rib + twelve times of thickness of the slab.

(*b*) In case of *L*-beams

1. One-tenth the effective span of the beam.
2. The breadth of the rib + half the clear distance between the ribs
3. The breadth of the rib + four times the thickness of the slab.

If *T*-form or *L*-form is used only for the purposes of providing additional compression area, such as in continuous beams over supports, the flange thickness shall not be less than one-half the width of the web and a total flange width not more than four times the width of the web. For effective stress transfer, it is desirable to splay the junction of web and the flange so as to form an angle of not less than 110° as shown in Fig. 762.

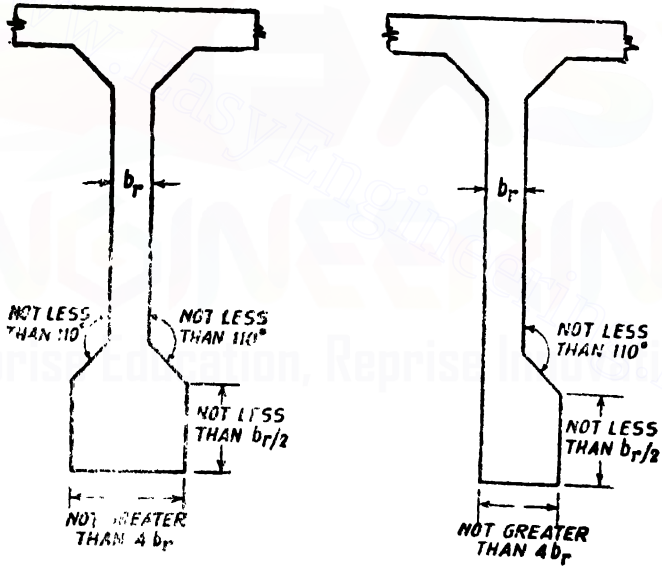

Fig. 762

Where the principal reinforcement in a slab which is considered as the flange of a *T*-beam or *L*-beam is parallel to the beam, transverse reinforcement shall be provided at the top of the flange. This reinforcement shall be equal to 90 percent of the main reinforcement of the slab at its mid span unless it is specially calculated. The length of such reinforcement bars shall be as indicated in Fig. 763.

(*xii*) Distribution reinforcement in slabs

For solid slabs spanning in one direction, distributing bars shall be provided at right angles to the main tensile bars to provide

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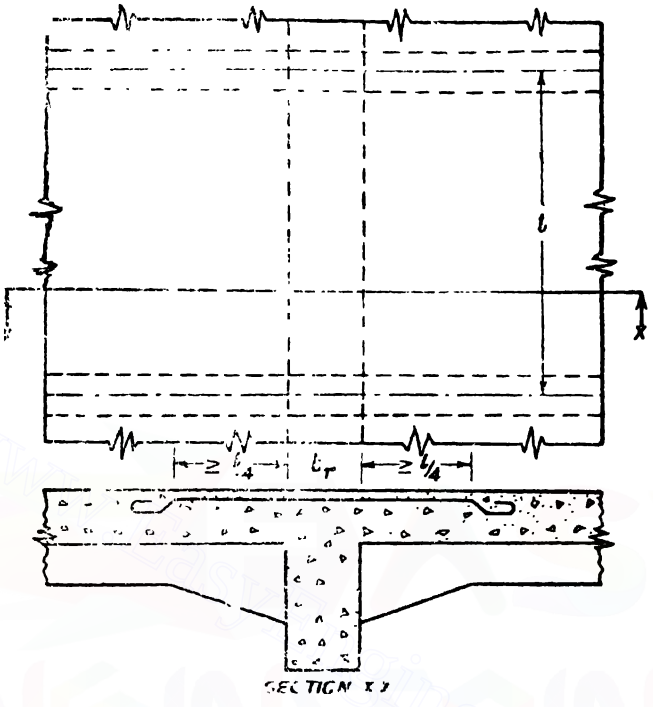

Fig. 763

for the lateral distribution of loads. The distribution reinforcement shall be such as to provide a resistance moment in the direction perpendicular to the span equal to 0·3 times the moment due to concentrated live loads plus 0·2 times the moment due to other loads such as dead load, shrinkage, temperature, *etc.*

In cantilever slabs, the distribution steel to resist a moment equal to that of 0·3 times the live load moment+0·2 times the dead load moment shall be provided half at top and half at the bottom of the slab.

The pitch of distributing bars shall not be greater than three times the effective depth of the slab or 450 *mm.* whichever is less.

Balanced cantilever and continuous construction

Moments. At sections where the dead load moment is of a sign opposite to that of live load moment, the design bending moment for the condition of loading, shall be taken as equal to the live load moment plus 0·7 dead load moment, with proper signs. This condition of loads shall be allowed for in design in addition to the other normal load conditions. However, at least 20 percent of the maximum negative reinforcement shall be carried out right across the span even if not required by design calculations.

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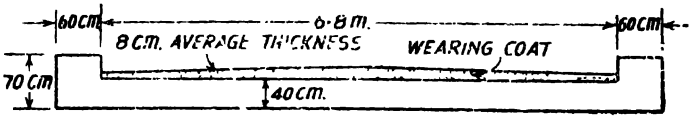
Design 176. *Design a simple slab bridge to the following requirements :*

(*i*) *Clear span = 5·50 metres*
(*ii*) *Clear width of carriage way = 6·80 metres*
(*iii*) *Live load : Class A Loading*
(*iv*) *Concrete mix : M 200*

Solution. Let the thickness of the slab be 40 *cm.* Allowing an effective cover of 3 *cm.*, the effective depth of the slab

$$= 40 - 3 = 37 \text{ cm.}$$



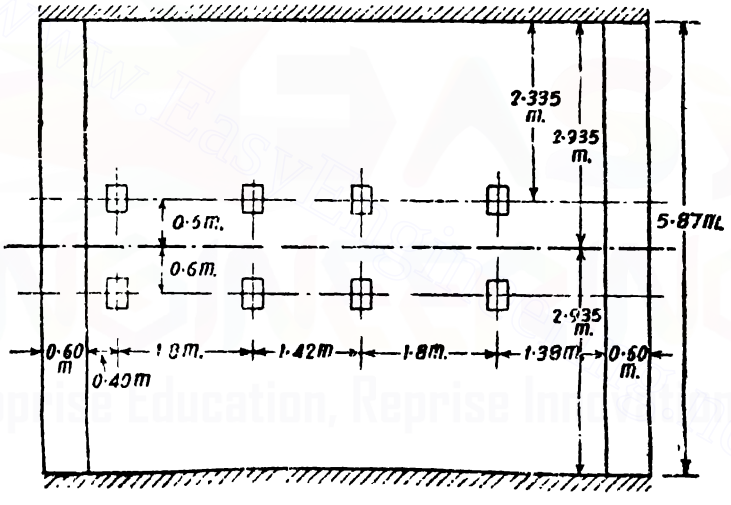


Fig. 761

Effective span = clear span + effective depth
= 5·50 + 0·37 = 5·88 *metres*

Let the average thickness of the wearing coat be 8 *cm.*
Let the width of the kerb be 60 *cm.*

Overall width of bridge
$$= 6{\cdot}80 + 0{\cdot}60 + 0{\cdot}60 = 8{\cdot}0 \text{ m.}$$

B.M. Calculations

(*a*) **Dead Loads**

Weight of wearing coat	$= 8 \times 23$	$= 184$ *kg./m.*
Weight of slab	$= 40 \times 24$	$= 960$ *kg./m.*
		144 *kg./m.*[2]

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B.M. due to dead load, for a one *metre* wide strip

$$= \frac{1144 \times 5{\cdot}87^2}{8} \times 100 \ kg \ cm.$$

$$= 492700 \ kg. \ cm.$$

(*b*) Live Load

Since the clear width of the roadway is 6·8 *metres*, provision is made for two lanes of traffic.

Clear distance g between two trains. This distance is to be determined as follows :

For a clear width of roadway of 5·5 *m*, $g = 0{\cdot}4$ *m*.

For a clear width of roadway of 7·5 *m*, $g = 1{\cdot}2$ *m*.

∴ For a clear width of roadway of 6·8 *m*,

$$g = 0{\cdot}4 + \frac{(1{\cdot}2 - 0{\cdot}4)}{(7{\cdot}5 - 5{\cdot}5)} (6{\cdot}8 - 5{\cdot}5) \ metre$$

$$= 0{\cdot}92 \ metre.$$

Minimum clearance *f* between the outer edge of the wheel and the roadway face of the kerb = 150 *mm*.

$$= 15 \ cm.$$

Ground contact area

Width of contact area = $W = 500$ *mm.* = 50 *cm.*

Length of contact area = $B = 250$ *mm.* = 25 *cm.*

Distance between centres of wheels of two trains

$$= 0{\cdot}92 + 0{\cdot}50 = 1{\cdot}42 \ metre$$

Distance between roadway face of kerb and centre of wheels

$$= 0{\cdot}15 + \frac{0{\cdot}50}{2} \ m.$$

$$= 0{\cdot}40 \ m.$$

For the maximum bending moment condition the heavy loads namely axle loads of 11·4 *tonnes* will be placed symmetrically with respect to the span For this position of the heavy axle loads the other wheel loads are off the span. Fig. 764 shows the position of the heavy loads.

Impact factor

$$= \frac{4{\cdot}5}{6{\cdot}0 + L}$$

$$= \frac{4{\cdot}50}{6{\cdot}0 + 5{\cdot}87}$$

$$= 0{\cdot}38.$$

Distribution of concentrated load on slab

For a single concentrated load, the effective width shall be taken equal to $e = bx \left(1 - \frac{x}{l}\right) + W$

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In our case l = effective span = 5·87 *m*.

x = Distance of load from the nearer support.

= 2·335 *m*.

k = constant depending on the ratio $\frac{l'}{l}$

l' = width of slab = 8 *metres*

$\therefore \quad \frac{l'}{l} = \frac{8}{5{\cdot}87} = 1{\cdot}36$

Referring to table on page 1112.

For $\frac{l'}{l}$ 1·3, $k = 2{\cdot}72$

for $\frac{l'}{l}$ = 1·4, $k = 2{\cdot}80$

for $\frac{l'}{l}$ = 3·36, $k = 2{\cdot}72 + \frac{(2{\cdot}80 - 2{\cdot}72)}{1{\cdot}4 - 1{\cdot}3}(1{\cdot}36 - 1{\cdot}30)$

$= 2{\cdot}768$

W = breadth of concentration area

= width of tyre + 2 × thickness of wearing coat

$= 0{\cdot}50 + 2 \times \frac{8}{100}$

$= 0{\cdot}66$ *m*.

$\therefore$ Effective width $= e = 2{\cdot}768 \times 2{\cdot}335 \left(1 - \frac{2{\cdot}335}{5{\cdot}87}\right) + 0{\cdot}66$ *m*.

$= 4{\cdot}55$ *metre*

This means the effective widths for heavy loads will overlap.

$\therefore$ Effective width for all the heavy loads

$= (0{\cdot}6 + 0{\cdot}4) + 1{\cdot}8 + 1{\cdot}42 + 1{\cdot}80 + 1{\cdot}38 + 0{\cdot}60$ *m*.

$= 8$ *metres*

$\therefore$ Load to be considered per *metre* width

$$\frac{2 \times 11{\cdot}4}{8} = 2{\cdot}85 \text{ tonnes}$$

When the bridge is subjected to only one train of loads the corresponding effective width

$= 0{\cdot}60 + 0{\cdot}40 + 1{\cdot}80 + 4{\cdot}55$ *metre*

$= 5{\cdot}075$ *metre*

Since the effective width is more, the critical condition is therefore the previous case *viz*, when both the trains of loads are present.

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Dispersion of load along the span

Effective length of slab on which the wheel load acts shall be takea as equal to the dimension of the tyre contact area over the wearing surface of the slab in the direction of the span plus twice the overall depth of the slab inclusive of the thickness of the wearing surface.

∴ In our case the effective length on which the wheel load is applied $= 25\ cm. + 2(40+8)\ cm.$

$= 121\ cm. = 1{\cdot}21\ m.$

But distance between wheels

$= 1\ 20\ m.$

The effective length of dispersion is almost equal to wheel space.

The loading for a one *metre* wide strip is therefore taken as shown in Fig. 765.

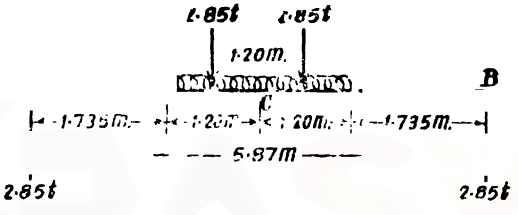

Fig. 765

∴ Max. B.M.

$$= 2{\cdot}85 \times \frac{5{\cdot}87}{} - 2{\cdot}85 \times \frac{1{\cdot}2}{} \ m.$$

$= 6{\cdot}65\ tm.$

$= 665{,}000\ kg.\ cm.$

B M. due to impact $= 0{\cdot}38 \times 665{,}000\ kg.\ cm.$

$= 252{,}700\ kg.\ cm.$

Total bending moment per *metre* width

$= 492{,}700 + 665{,}000 + 252{,}700\ kg.\ cm.$

$= 1{,}410{,}400\ kg.\ cm.$

Adopting $c = 70\ kg/cm.^2,\ t = 1400\ kg./cm.^2$

and $$m = \frac{2800}{3c} = \frac{2800}{3 \times 70} = 13$$

For this set of stresses,

$n_1 = 0{\cdot}394\ d$

$a = 0{\cdot}869\ d$ say $0{\cdot}87\ d.$

$Q = 12$

Equating the moment of resistance to the maximum bending moment, we have,

$$12 \times 100\ d^2 = 1{,}410{,}400$$

$$d = 34{\cdot}28\ cm.$$

Effective depth available providing 25 *mm.* clear cover and using 18 *mm.* diameter bars

$= 40 - 2{\cdot}5 - 0{\cdot}9 = 36{\cdot}6\ cm.$

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$$A_t = \frac{1410400}{1400 \times 0.87 \times 36.6} \ cm.^2$$

$$= 31.63 \ cm.^2$$

∴ Spacing of 18 *mm.* diameter bars

$$= \frac{2.54 \times 100}{31.63} \ cm.$$

$$= 8 \ cm. \ centres$$

Distribution steel

Distribution steel should be computed for resisting 0·3 times the live load bending moment and 0·2 times the dead load moment.

∴ B.M. for which the distribution steel should be computed.

$$= 0.3 \times (665{,}000 + 252{,}700) + 0.2 \times 192700 \ kg. \ cm.$$

$$= 373{,}850 \ kg. \ cm.$$

If 12 *mm.* diameter bars be used, the effective depth to the centre of these bars

$$= 40 - 2.5 - 1.8 - 0.6 \ cm.$$

$$= 35.1 \ cm.$$

∴

$$A_t = \frac{373{,}850}{1400 \times 0.87 \times 35.1} \ cm.^2$$

$$= 8.75 \ cm.^2$$

∴ Spacing of 12 *mm.* diameter bars

$$= \frac{1.13 \times 100}{8.75} \ cm.$$

$$= 12.9 \ cm \text{ say } 12.5 \ cm \ centres.$$

Shear

The greatest shear force will occur when the loads are closest to the support.

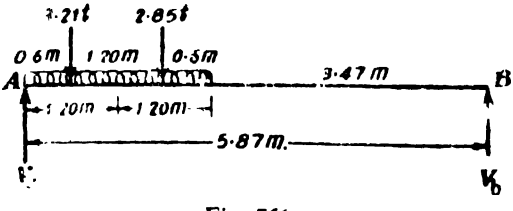

Fig. 766

Effective width of slab for the load near the support

$$= e = kx \left(1 - \frac{x}{l}\right) + W$$

$$= 2.768 \times 0.6 \left(1 - \frac{0.6}{5.87}\right) + 0.66 \ metre$$

$$= 2.15 \ metres.$$

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Now also the effective widths will overlap. When both trains are present, the effective width

$$=0{\cdot}6+0{\cdot}4+1{\cdot}8+1{\cdot}42+1{\cdot}8+\frac{2{\cdot}15}{2} \text{ metre}$$

$$=7{\cdot}095 \text{ metres}$$

$\therefore$ Load to be considered per *metre* width

$$=\frac{2\times 11{\cdot}4}{7{\cdot}095} \text{ tonnes}$$

$$=3{\cdot}21 \text{ tonnes}$$

When the bridge is subjected to only one train of loads the corresponding effective width

$$=0{\cdot}60+0{\cdot}40+1{\cdot}80+\frac{2{\cdot}15}{2}$$

$$=3{\cdot}875 \text{ metres}$$

(this is more than for the case when both the trains are present)

Effective width of slab for the second load

$$=e=kx\left(1-\frac{x}{l}\right)+W$$

$$=2{\cdot}768+1{\cdot}80\left(1-\frac{1{\cdot}80}{5\,87}\right)+0{\cdot}66 \text{ metre.}$$

$$=4{\cdot}12 \text{ metres.}$$

This shows that the effective widths of the two trains will overlap.

$\therefore$ Effective width when both trains are present

$$=0{\cdot}6+0{\cdot}40+1{\cdot}80+1{\cdot}42+1{\cdot}80+1{\cdot}38+0{\cdot}60=8 \text{ metres}$$

$\therefore$ Load to be considered per *metre* width

$$=\frac{2\times 11{\cdot}4}{8}=2{\cdot}85 \text{ t.}$$

When the bridge carries only one train of load, the effective width will be still greater and this condition therefore will not be considered.

Fig. 766 shows the loading for which the S.F. is to be computed.

Vertical Reaction at A

$$=V_a=\frac{2{\cdot}85\times(4{\cdot}07)+3{\cdot}21\times 5{\cdot}27}{5{\cdot}87} \text{ tonnes}$$

$$=4{\cdot}86 \text{ tonnes}$$

S.F. due to L.L.	$= 4{\cdot}86\ t$
S.F. due to impact $=0{\cdot}38\times 4{\cdot}86$	$= 1{\cdot}85\ t$
S.F. due to dead load $=1{\cdot}144\times\frac{5{\cdot}87}{2}$	$= 3\ 36\ t$
Max. S.F.	$=10{\cdot}27\ t$

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Shear stress $\frac{S}{ab}$

$$\frac{10{\cdot}07 \times 1000}{0{\cdot}87 \times 36{\cdot}6 \times 100} \text{ kg./cm.}^2$$

$$= 3{\cdot}16 \text{ kg./cm.}^2$$

(less than the permissible stress of 7 *kg./cm.*^{2})

Let alternate bars be bent at one-seventh of the span

Bond stress $= \frac{S}{a\Sigma o}$

$$= \frac{10{\cdot}07 \times 1000}{0{\cdot}87 \times 36{\cdot}6 \times \frac{100}{16} \times 5{\cdot}05} \text{ kg./cm.}^2$$

$$= 10 \text{ kg /cm.}^2$$

(less than the permissible stress of 13 *kg./cm.*^{2})

Kerb

Kerbs 0·6 *metre* or more in width shall be designed for a live load of 400 *kg.* per *metre*^{2} and also a horizontal load of 750 *kg./m.* run.

Design for vertical loads

Loads on kerb will therefore consist of the following :

D.L. of kerb .. $0{\cdot}6 \times 0{\cdot}70 \times 2400 = 1008$ *kg /metre*

Railing ... = 60 *kg./metre (assumed)*

Live load $400 \times 0{\cdot}6 \times 1$ = 240 *kg./metre*

Total load per *metre* run = 1308 *kg./metre*

B.M. $= \frac{1308 \times 5{\cdot}87^2}{8} \times 100$ *kg. cm.*

$= 563100$ *kg. cm.*

Effective depth available

$= 70 - 2{\cdot}5 - 0{\cdot}9$ *cm.* (18 *mm.* dia. bars)

$= 66{\cdot}6$ *cm.*

$\therefore$ $A_t = \frac{563100}{1400 \times 0{\cdot}86 \times 66{\cdot}6}$ *cm.*^{2}

$= 6{\cdot}94$ *cm.*^{2}

Provide 3 bars of 18 mm. diameter.

Design for horizontal loads

Horizontal load per *metre* run

$= 750$ *kg.*

B M. due to this force

$= 750 \times (70 - 40)$ *kg. cm.*

$= 22500$ *kg. cm.*

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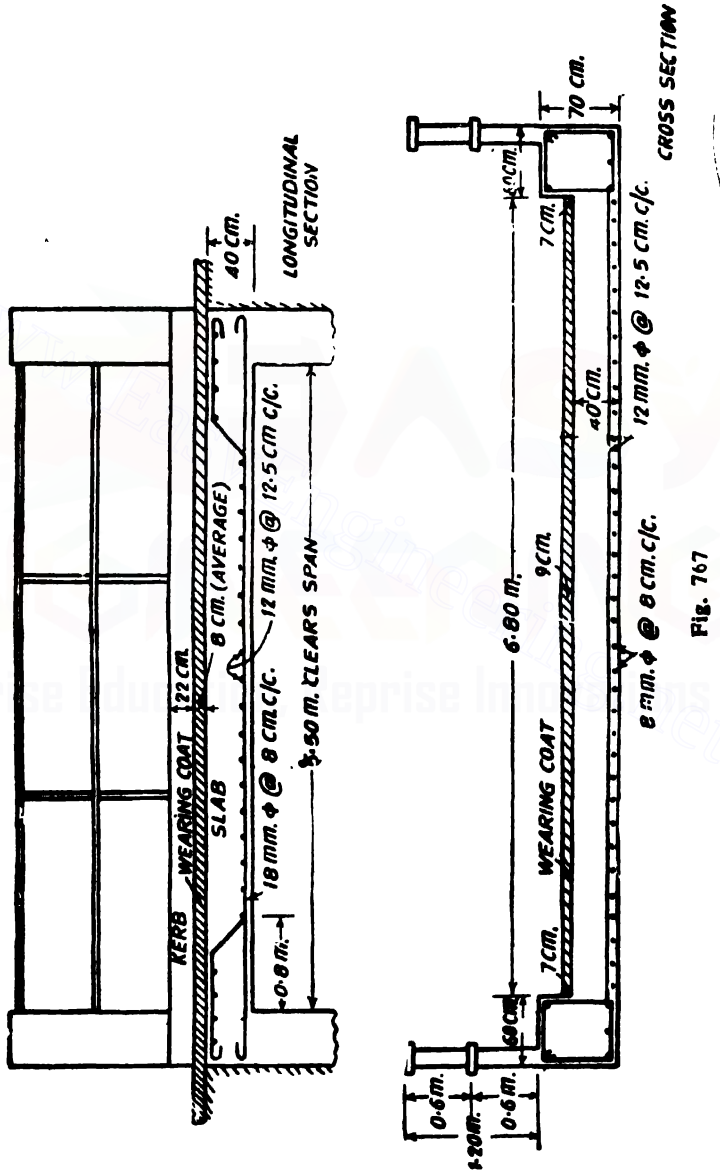

Fig. 767

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Effective depth $60-3=57$ *cm*

$$A_s = \frac{22500}{1400 \times 0.87 \times 57} \text{ cm}^2$$

$= 0.3241$ *cm.*^{2} (very small quantity)

Provide nominal 8 mm. diameter vertical links at 25 cm. centres.

Design 177. *Design a reinforced concrete T-beam bridge for carrying a two-lane traffic, the detailed requirements of which are given below :*

(*i*) *Effective span* $= 12$ *metres*

(*ii*) *Width of the carriage way* $= 6.8$ *metres*

(*iii*) *Thickness of the wearing coat* $= 8$ *cm.*

(*iv*) *Live Load : IRC—Class A loading.*

Fig. 768 shows the axle loads for this class of loading.

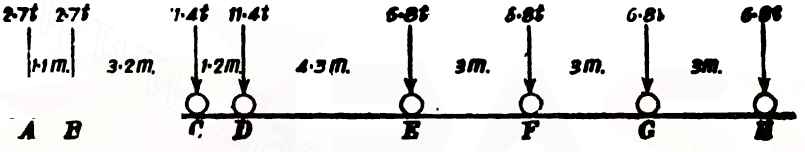

Fig. 768

The nose to tail distance between successive trains shall not be less than 18·4 metres.

No other live load shall cover any part of the carriageway when a train of vehicles (or trains of vehicles in multilane bridge) is crossing the bridge.

The ground contact area of the wheels will be as follows (see Fig. 769).

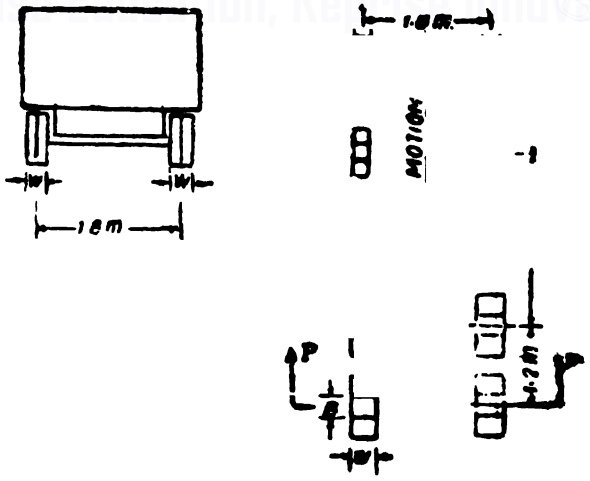

Fig. 769

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Axle load (tonne)	Ground contact area	
	B cm.	W cm.
11·4	25	50
6·8	20	38
2·7	15	20

The minimum clearance f, between the outer edge of the wheel and the roadway face of the kerb and the minimum clearance g between the outer edges of passing or crossing vehicles on multilane bridges shall be as given below :

Clear carriage way	g	f
5·5 m to 7·5 m.	Uniformly increasing from 0·4 m. to 1·2 m.	150 mm.
Above 7·5 m.	1·2 m	150 mm.

(v) *Impact factor* $= \frac{4.5}{6+L}$ *subject to a maximum of* 0·5 (*L is the span in metres*)

(vi) *The bending moments shall be considered on the following basis :*

In the beams the maximum bending moment occurs under the wheel C or D.

In the slab, the maximum bending moment occurs when the wheel loads C and D are in one panel.

In the slab for a centrally placed load,

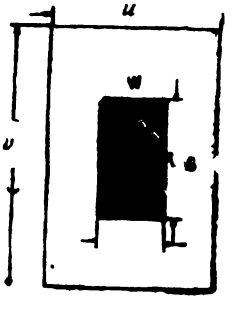

Fig 770

$\frac{v}{L}$	$\frac{u}{l}$	m_1	m_2
0·0	0·36	0·173	0·1
0·3	0·36	0·128	0·026
0·4	0·36	0·106	0·0 4

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B.M. along short span at the centre $=(m_1+0{\cdot}15\ m_2)\ W$

B M. along long span at the centre $=(m_2+0\ 15\ m_1)\ W$

where W is the wheel load and m_1 *and* m_2 *have the following values. (See Fig. 770) where* $u = W+2t$ *and* $v = B+2t$ *where t = thickness of wearing coat.*

Continuity of the slab may be taken into account by reducing the moments determined above to 80 percent of the determined values and assuming these values at the continuous supports also.

(vii) Effective width of the slab in cm. from shear considerations may be taken as

$$27{\cdot}6\sqrt{d+(B+2t)\left(1-\frac{2\ 21}{\sqrt{d}}\right)}$$

where d = *thickness of the slab in cm.*

t = *thickness of wearing coat in cm.*

B = *wheel contact dimension in cm.*

(viii) The width of the rib of the T-beams may be taken as 35 cm. and the spacing of the beams as 2·15 metres.

(ix) Three diaphragms—one at the centre and one each at the ends—may be provided to stiffen the floor system and the width of the diaphragm may be assumed as 30 cm.

(x) Concrete Mix : M-200 grade.

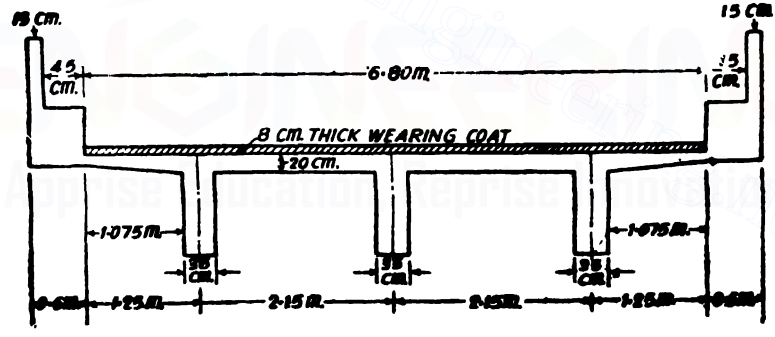

Fig. 771

Solution. Fig. 771 shows an arrangement proposed for the bridge

Since the slab is continuous over the supporting beams, the positive and negative bending moments will be taken at 80 percent of the maximum positive free bending moment The maximum free bending moment due to live load will be determined by keeping the heaviest load (C or D) at the centre of one intermediate panel. (See Fig. 772)

Dispersion of the wheel load

$$L = \frac{12}{2} = 6\ m.$$

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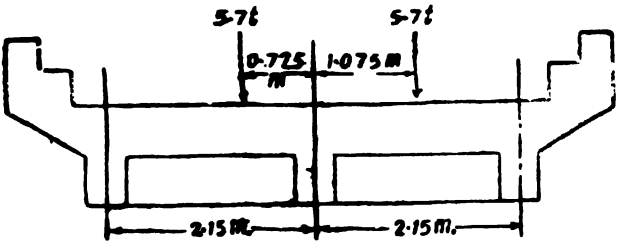

Fig. 772

since a diaphragm is present at the centre

$$l = 2{\cdot}15\ m.$$

Let the thickness of the wearing coat be 8 *cm.*

$$t = 8\ cm.$$

Ground contact dimensions

$$B = 25\ cm.$$

$$W = 50\ cm.$$

$$v = B + 2t = 25 + 2 \times 8 = 41\ cm.$$

$$u = W + 2t = 50 + 2 \times 8 = 66\ cm.$$

$$\frac{v}{L} = \frac{41}{6 \times 100}$$

$$= 0\ 068 \text{ say } 0{\cdot}07$$

$$\frac{u}{l} = \frac{66}{2{\cdot}15 \times 100}$$

$$= 0{\cdot}307 \text{ say } 0{\cdot}31$$

The smallest value of $\frac{u}{l}$ given is 0·36. Hence m_1 and m_2 will be taken corresponding to $\frac{v}{L} = 0{\cdot}07$ and $\frac{u}{l} = 0{\cdot}36$.

Hence, we get, $m_1 = 0\ 173$ and
$m_2 = 0{\cdot}11$

∴ B.M. along the shorter span at the centre

$$= (m_1 + 0{\cdot}15\ m_2)\ W$$

$$= (0{\cdot}173 + 0{\cdot}15 \times 0{\cdot}11) \times 5{\cdot}7 \times 1000\ kg.\ m.$$

$$= 1080\ kg.\ m.$$

$$= 108{,}000\ kg.\ cm.$$

Impact factor $= \frac{4{\cdot}5}{6 + L}$

(L in this case is the span of the slab)

$$\frac{4{\cdot}5}{6 + 2\ 15} = 0{\cdot}552$$

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But the impact factor to be adopted is not to exceed 0·5. Hence we will adopt an impact factor of 0·5.

$\therefore$ Impact moment $= 0{\cdot}5 \times 108{,}000$ *kg. cm.*
$= 54{,}000$ *kg. cm.*

D.L. Moment

Assume the slab to be 20 *cm.* thick
Dead load of slab per *metre*^{2} $= 20 \times 24 = 480$ *kg./metre*^{2}
Dead load of wearing coat $= 8 \times 23 = 184$ *kg./metre*^{2}

Total dead load $= 664$ *kg./metre*^{2}

$$\text{Dead load moment} = \frac{664 \times 2{\cdot}15^2}{8} \text{ kg. m.}$$

$= 383{\cdot}6$ *kg. m.*
$= 38360$ *kg. cm.*

Total B.M. $= 108000 + 54000 + 38360$ *kg. cm.*
$= 200360$ *kg. cm.*

Allowing for continuity, the actual bending moment to be taken for design $= 80\%$ of the max free moment
$= 0{\cdot}8 \times 200360$ *kg. cm.*
$= 160{,}288$ *kg. cm.*

Using *M* 200 concrete mix, the following working stresses will be adopted :

$c = 70$ *kg /cm.*^{2}
$t = 1400$ *kg./cm.*^{2}

$$m = \frac{2800}{3c} = \frac{2800}{3 \times 70} = 13$$

$$n_c = 0{\cdot}394\, d, \quad a = 0{\cdot}87\, d, \quad Q = 12$$

Equating the moment of resistance to the maximum bending moment, we have,

$12 \times 100\, d^2 = 160{,}288$
$\therefore \quad d = 11{\cdot}6$ *cm.*

Providing 25 *mm.* clear cover and using 12 *mm.* diameter bars the effective depth available

$= 20 - 2{\cdot}5 - 0{\cdot}6$ *cm.*
$= 16{\cdot}9$ *cm.*

$$A_t = \frac{160288}{1400 \times 0{\cdot}87 \times 16{\cdot}9} \text{ cm.}^2$$

$= 7{\cdot}79$ *cm.*^{2}

Spacing of 12 *mm.* diameter bars

$$= \frac{1{\cdot}13 \times 100}{7{\cdot}79} \text{ cm.}^2$$

$= 14{\cdot}51$ *cm.*

say 14 cm. centres.

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Distribution steel

This reinforcement is provided so as to provide a moment of resistance = 0·2 times the live load moment + 0·3 times the dead load moment

$= 0{\cdot}2 \times 38360 + 0{\cdot}2(108{,}000 + 54000)$ *kg. cm.*

$= 43910$ *kg cm.*

If 8 *mm.* diameter bars be used as distribution steel, the effective depth to the centre of this steel

$= 20 - 2{\cdot}5 - 1{\cdot}2 - 0{\cdot}4 = 15{\cdot}9$ *cm.*

$$\therefore \quad A_t = \frac{43910}{1400 \times 0{\cdot}87 \times 15{\cdot}9} \text{ cm.}^2$$

$= 2{\cdot}27$ *cm.*^{2}

Spacing of 8 *mm.* diameter bars

$$= \frac{0{\cdot}5 \times 100}{2{\cdot}27} = 22 \text{ cm.}$$

***Provide** 8 mm. dia. bars at 20 cm c/c.*

Check for shear

Dead load shear $= \frac{664 \times 2{\cdot}15}{2} = 714$ *kg*

Live load shear with impact effect $= \frac{5{\cdot}7 \times 1000}{2} \times 1{\cdot}5 = 4275$ *kg.*

Total $= 4989$ *kg.*

Effective width from shear consideration

$$b = 27{\cdot}6\sqrt{d + (B + 2t)\left(1 - \frac{2{\cdot}21}{\sqrt{d}}\right)}$$

$$= 27{\cdot}6\sqrt{20 + (25 + 2 \times 8)\left(1 - \frac{2{\cdot}21}{\sqrt{20}}\right)}$$

$= 176$ *cm.*

Shear stress $= \frac{S}{ab}$

$$= \frac{4989}{0{\cdot}87 \times 15{\cdot}9 \times 176} \text{ kg./cm.}^2$$

$= 1{\cdot}74$ *kg/cm.*^{2}

(less than the permissible stress of 7 *kg./cm.*^{2})

Design of girders. The load transferred to a girder will be a maximum when one row of the wheels of one train is exactly on the girder and one row of wheels of a passing train is closest to the girder. The position of the train loads for producing the severe effects on the girder marked *A* is shown in Fig. 773.

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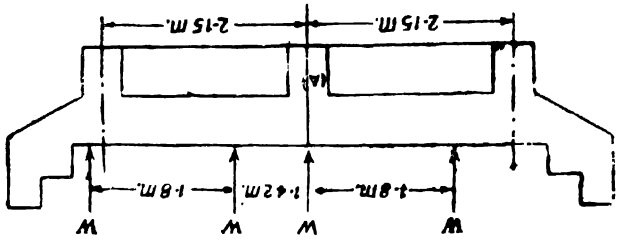

Fig. 773

The minimum clearance g between the outer edges of the crossing or passing vehicles shall be as follows :

Clear carriage width	*Clearance*
5·5 *m* to 7·5 *m*	0·4 *m* to 1·2 *m*
above 7·5 *m*.	1·2 *m*

In our case the carriageway being 6·8 *metres*, the clearance

$$=0{\cdot}40+\left(\frac{1{\cdot}2-0{\cdot}4}{7{\cdot}5-5{\cdot}5}\right)(6{\cdot}8-5{\cdot}5)\ \textit{metres}$$

$$=0{\cdot}40+0{\cdot}52=0{\cdot}92\ \textit{metres}$$

∴ Distance between the centre lines of these wheels

$$=0{\cdot}92+0{\cdot}50=1{\cdot}42\ \textit{metres}$$

If W is one of the wheel loads of one axle then the corresponding load transferred to the girder marked A

$$=\left[\frac{2{\cdot}15+1{\cdot}42}{2{\cdot}15}\right]W+\left[\frac{2{\cdot}15-1{\cdot}8}{2{\cdot}15}\right]W$$

$$=1{\cdot}502\ W$$

But the axle load $=2\ W$

∴ The girder is subjected to $\frac{1{\cdot}502}{2}=0\ 751$ times the axle loads. Multiplying the given axle loads by 0·751 we get the following load system to be considered for the girder A.

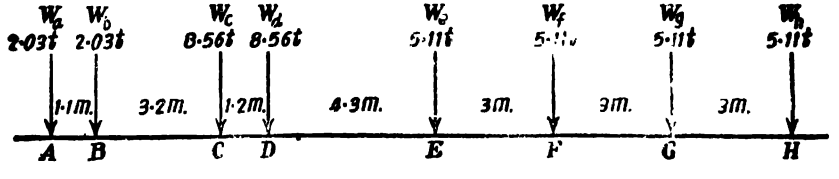

Fig. 774

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$W_a = 0{\cdot}751 \times 2{\cdot}7 = 2{\cdot}03\ t$
$W_b = 0{\cdot}751 \times 2{\cdot}7 = 2{\cdot}03\ t$
$W_c = 0{\cdot}751 \times 11{\cdot}4 = 8{\cdot}56\ t$
$W_d = 0{\cdot}751 \times 6{\cdot}8 = 8{\cdot}56\ t$
$W_e = 0{\cdot}751 \times 6{\cdot}8 = 5{\cdot}11\ t$
$W_f = 0{\cdot}751 \times 6{\cdot}8 = 5{\cdot}11\ t$
$W_g = 0{\cdot}751 \times 6{\cdot}8 = 5{\cdot}11\ t$
$W_h = 0{\cdot}751 \times 6{\cdot}8 = 5{\cdot}11\ t$

It is further given that the maximum bending moment occurs under the wheels C or D.

Since the span of the beam is 12 *metres* effective and since the absolutely maximum bending moment occurs near the centre of the girder, we find that the loads W_a, W_b, W_c, W_d and W_e alone will be on the span corresponding to the maximum bending moment condition.

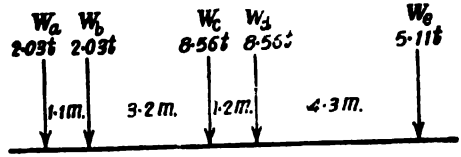

Fig. 775

We find from the above that the absolutely maximum bending moment will occur under the load $W_d = 8{\cdot}56\ t$.

The load system will now be placed on the span such that the load W_d and the resultant of the above load system are equidistant from the middle point of the girder

Distance between the load W_e and the resultant of the above load system is given by

$$\frac{8{\cdot}56 \times 4{\cdot}3 + 8{\cdot}56 \times 5{\cdot}5 + 2{\cdot}03 \times 8{\cdot}7 + 2{\cdot}03 \times 9{\cdot}8}{5{\cdot}11 + 8{\cdot}56 + 8{\cdot}56 + 2{\cdot}03 + 2{\cdot}03}\ metres$$

$$= 4{\cdot}62\ metres$$

$\therefore$ Distance between the resultant load and the load W_d
$= 4{\cdot}62 - 4{\cdot}30 = 0{\cdot}32\ m.$

Hence, the position of the load W_d for absolute maximum B. M. condition is as shown in Fig. 776.

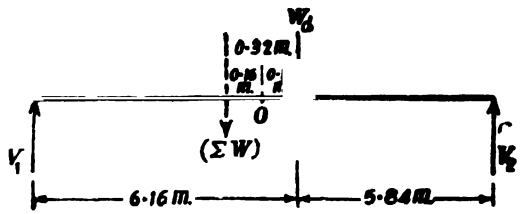

Fig. 776

Hence, the load W_d should be at a distance of 0·16 m. from the left end.

$\Sigma W = 5{\cdot}11 + 8{\cdot}56 + 8{\cdot}56 + 2{\cdot}03 + 2{\cdot}03$
$= 26{\cdot}29\ t$

Reaction $V_1 = \dfrac{26{\cdot}29 \times 6{\cdot}16}{12}\ tonnes$

$= 13{\cdot}50\ t$

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$$V_2 = 26{\cdot}29 - 13{\cdot}50 = 12{\cdot}79\ t$$

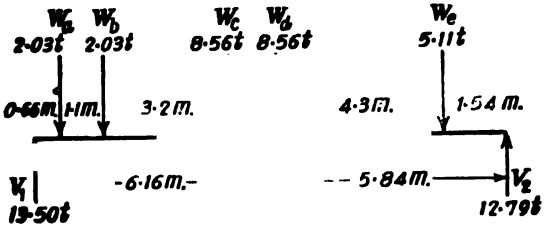

Fig. 777

Absolutely maximum bending moment

$$= 12{\cdot}79 \times 5{\cdot}84 - 5{\cdot}11 \times 4{\cdot}3\ tm.$$

$$= 74\ 69 - 21{\cdot}97 = 52{\cdot}72\ tm$$

Impact factor $= \dfrac{4{\cdot}5}{6+L}$ (L is the span of the beam)

$$= \frac{4{\cdot}5}{6+12} = 0{\cdot}25$$

Impact moment $= 0{\cdot}25 \times 52{\cdot}72\ tm.$

$$= 13\ 18\ tm.$$

Dead load per metre run

Slab	$20+24\times2{\cdot}15$	$=1032\ kg/m.$
Wearing coat	$8\times23\times2{\cdot}15$	$=\ 396\ kg/m.$
Weight of rib (35 *cm.* × 100 *cm.*)	$=0{\cdot}3\times1\times2400$	$=\ 840\ kg./m.$
Total dead load		$=2268\ kg./m.$

D. L. moment $= \dfrac{2268 \times 12^2}{8}$

$$= 40820\ k.\ gm.$$

$$= 40{\cdot}82\ tm.$$

Total B.M. $= 52{\cdot}2 + 13{\cdot}18 + 40{\cdot}82\ tm.$

$$= 106{\cdot}72\ tm.$$

Assuming an effective cover of 10 *cm.* making allowances for two or three layers of bars, approximate effective depth

$$d = 120 - 10 = 110\ cm.$$

Approximate lever arm

$$= d - \frac{d_s}{2}$$

$$= 110 - 10 = 100\ cm.$$

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Approximate steel area required

$$= \frac{106\,72 \times 1000 \times 100}{1400 \times 105} \text{ cm.}^2$$

$$= 76 \text{ cm}^2.$$

Let us try 12 bars of 28 mm diameter ($73\,92$ cm^2) These bars will be arranged in two layers with 6 bars per layer.

Let the clear distance between the two layers be 12 *mm*.

Effective depth provided

$= 120 - 2{\cdot}8 - 2{\cdot}8 - 0\,6$ *cm.*

$= 113{\cdot}8$ *cm.*

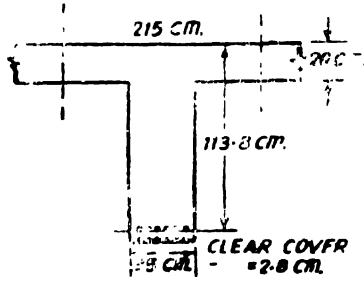

Fig. 778

Width of flange

This may be taken at the least of the following :

(*i*) Centre to centre of beam $= 2{\cdot}15$ *m.* $= 215$ *cm.*

(*ii*) $\frac{1}{3}$ span $= \frac{12}{3} = 4$ *m.* $= 400$ *cm.*

(*iii*) $12d_s + b_r = 12 \times 20 + 35 = 275$ *cm.*

$\therefore$ Width of flange $= 215$ *cm.*

Position of Neutral axis

Assuming the neutral axis to be situated below the flange, we have,

$$215 \times 20(n - 10) = 13 \times 73\,92(113{\cdot}8 - n)$$

$$n = 29 \text{ cm.}$$

$$c' = \frac{29 - 30}{29} \quad c = 0{\cdot}31c$$

$$y = \frac{c + 2c'}{c + c'} \cdot \frac{d_s}{3}$$

$$= \frac{c + 0{\cdot}62c}{c + 0{\cdot}31c} \times \frac{20}{3} \text{ cm.}$$

$$= \frac{1{\cdot}62}{1{\cdot}31} \times \frac{20}{3} \text{ cm.}$$

$$= 8\,24 \text{ cm.}$$

$\therefore$ Actual lever arm $= a = d - y$

$$= 113{\cdot}8 - 8{\cdot}24 = 105{\cdot}56 \text{ cm.}$$

Actual stress in steel

$$= \frac{106{\cdot}72 \times 1000 \times 100}{73\,92 \times 105{\cdot}56} \text{ kg /cm.}^2$$

$$= 1368 \text{ kg./cm.}^2$$

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Corresponding maximum compressive stress in concrete

$$=\left(\frac{29}{113{\cdot}8-29}\right)\left(\frac{1368}{13}\right) kg./cm.^2$$

$$=36\ kg./cm.^2$$

Shear

Fig. 779 shows the position of the loads for absolute maximum positive shear force.

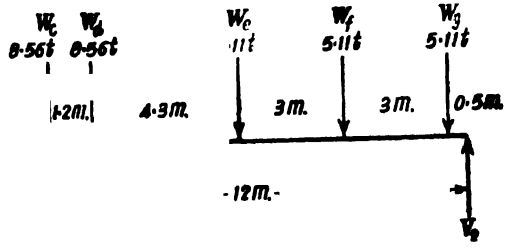

Fig. 779

$$S_{max}=V_1$$

$$=8\ 56+\frac{8{\cdot}56\times10{\cdot}8+5{\cdot}11\times6{\cdot}5+5{\cdot}11\times3{\cdot}5+5\ 11\times0{\cdot}5}{12}\ tonnes$$

$$=20\ 74\ tonnes$$

Impact shear $=0{\cdot}25\times20{\cdot}74=5{\cdot}19\ tonnes$

Dead load shear $=\frac{2268\times6}{1000}\ tonnes$

$=13{\cdot}61\ tonnes.$

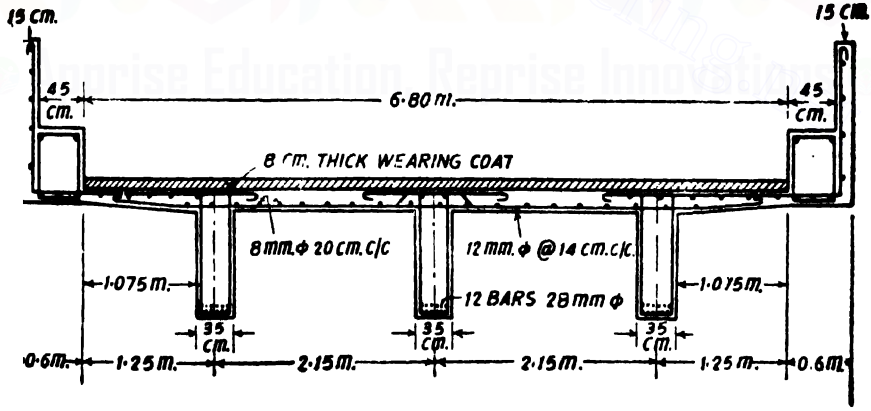

Fig. 780

∴ Total shear force

$$=20{\cdot}74+5{\cdot}19+13{\cdot}61=39{\cdot}54\ tonnes$$

Shear stress $=\frac{S}{ab}$

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$$= \frac{39\,54 \times 1000}{105{\cdot}56 \times 35}\ kg./cm.^2$$

$$= 10\,7\ kg./cm.^2$$

Spacing of four-legged 10 *mm.* diameter stirrups

$$= p = \frac{A_w f_w a}{S}$$

$$= \frac{4 \times 0{\cdot}79 \times 1400 \times 105{\cdot}56}{39{\cdot}54 \times 1000}\ cm$$

$$= 11{\cdot}81\ cm.\ say\ 11\ cm.\ c/c.$$

Or alternatively bars may be bent up to resist the shear, as explained in the previous designs.

Design 178. *Design a reinforced concrete foot bridge to the following requirements :*

(*i*) *Clear road width*	$= 2{\cdot}75$ *metres*
(*ii*) *Effective span*	$= 12$ *metres*
(*iii*) *Live load on the bridge*	$= 400\ kg./m.^2$

Solution. Fig. 781 shows the proposed arrangement. The bridge will consist of a central girder with cantilevered roadway. Kerbs 30 *cm.* × 30 *cm.* will be provided.

Width of rib $= \text{about } \frac{1}{30}$ of the span

$$= \frac{12 \times 100}{30} = 40\ cm.$$

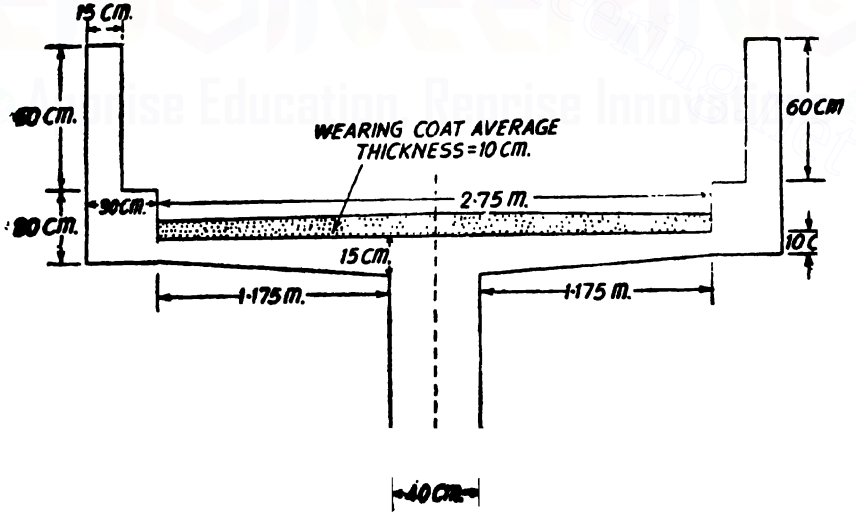

Fig. 781

Slab. Assume the slab thickness to vary from 10 *cm.* at each end to 15 *cm.* at the junction with the rib.

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Load. Per *met e* width of slab :

Kerb : $0{\cdot}3 \times 0{\cdot}3 \times 2400 = 216$ *kg./m.*

Wearing coat : $0{\cdot}1 \times 1{\cdot}175 \times 1 \times 2300 = 270$ *kg./m.*

Slab : $0{\cdot}1 \times 1 \times 1{\cdot}175 \times 2400 = 282$

$\frac{1}{2} \times 4{\cdot}05 \times 1{\cdot}175 \times 2400 = 70$ } $= 352$ *kg./m.*

Parapet $0{\cdot}15 \times 0{\cdot}6 \times 2400 = 216$ *kg./m.*

Dead load moment $= 216(1{\cdot}175 + 0{\cdot}15)$

$$+270\left(\frac{1{\cdot}175}{2}\right)$$

$$= 282 \times 1{\cdot}175$$

$$+70 \times \frac{1}{3} \times 1{\cdot}175$$

$$+216(1{\cdot}175 \times 0{\cdot}225)$$

$$= 940{\cdot}3 \text{ kg. m.}$$

Live load moment $= 400 \times \frac{1{\cdot}1/5^2}{2}$

$$= 276{\cdot}10 \text{ kg. m.}$$

$\therefore$ Total bending moment

$$= 940{\cdot}3 + 276{\cdot}1$$

$$= 1216{\cdot}2 \text{ kg. m.}$$

Use M 150 grade mix.

Equating the moment of resistance to the maximum bending moment, we have

$$8{\cdot}50 \times 100\, d^2 = 1216{\cdot}2 \times 100$$

$$d = 11{\cdot}96 \text{ cm.}$$

Providing an effective cover of 3 *cm.*, effective depth available

$$= 15 - 3$$

$$= 12 \text{ cm.}$$

$\therefore$ Area of steel required

$$= A_s = \frac{1216{\cdot}2 \times 100}{1400 \times 0{\cdot}87 \times 12} \text{ cm.}^2$$

$$= 8{\cdot}32 \text{ cm.}^2$$

Provide 10 mm. diameter bars at 8 cm. centres

Distribution steel $= \frac{0{\cdot}15}{100}\left[\frac{10+15}{2} \times 100\right] \text{cm.}^2$

$$= 1{\cdot}875 \text{ cm.}^2$$

Provide 6 mm. diameter bars at 15 cm. centres.

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Shear. Maximum shear force

$$= 216 + 270 + 352 + 216 + 400 \times 1{\cdot}175 \text{ kg.}$$
$$= 1524 \text{ kg.}$$

$\therefore$ Shear stress $= \dfrac{S}{ab}$

$$= \frac{1524}{0{\cdot}17 \times 12 \times 40} \text{ kg./cm.}^2$$
$$= 3{\cdot}65 \text{ kg/cm.}^2$$

This is less than the safe shear stress of 5 *kg./cm.*^{2}

Main girder. Consider one *metre* run of the girder

Dead load from the two cantilevering portions $= 2 \times [216 + 270 + 352 + 215] = 2108$ *kg*

Dead load of slab and wearing coat above the beam $= 0{\cdot}1 \times 0{\cdot}4 \times 2300 + 0{\cdot}15 \times 0{\cdot}4 \times 2400 = 236$ *kg.*

$\therefore$ Total dead load from the slab

$$= 2108 + 236$$
$$= 2344 \text{ kg.}$$

Let the overall depth of the beam be

$$= 100 \text{ cm.}$$

$\therefore$ Depth of rib $= 100 - 15$
$= 85$ *cm.*

$\therefore$ Dead load of rib $= 0{\cdot}4 \times 0{\cdot}85 \times 2400$
$= 816$ *kg.*

$\therefore$ Total dead load $= 2344 + 816$
$= 3160$ *kg.*

Live load. The live load moment for the girder should be calculated due to a certain amount of live load P per square *metre* of the slab. This shall be taken according to the following condition

$$P = P' - \frac{40L - 300}{9}$$

where P' = the given live load
$= 400 \text{ kg/metres}^2$
L = effective span of the beam
$= 12$ *metres*

$\therefore$ $$P = 400 - \left(\frac{40 \times 12 - 300}{9}\right) \text{ kg./metre}^2$$
$$= 380 \text{ kg./metre}^2$$

$\therefore$ **Live load per *metre* run of the girder**

$$= 380 \times 2{\cdot}75$$
$$= 1045 \text{ kg./m.}$$

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∴ Total load per *metre* run of the girder
$$= 3160 + 1045$$
$$= 4205 \text{ kg./m}$$

∴ Maximum bending moment
$$= \frac{wl^2}{8}$$
$$= \frac{4205 \times 12^2}{8} \times 100 \text{ kg. cm.}$$
$$= 7569000 \text{ kg. cm.}$$

The economic overall depth of beam may be taken as
$$= \sqrt{\frac{M}{20b_r}}$$
$$= \sqrt{\frac{7569000}{20 \times 40}} \text{ cm.}$$
$$= 97 \text{ cm.}$$

Overall depth provided
$$= 100 \text{ cm.}$$

Providing 6 *cm.* effective cover, effective depth
$$= 94 \text{ cm.}$$

d_s = Average flange thickness
$$= \frac{15+10}{2}$$
$$= 12.5 \text{ cm.}$$

Approximate lever arm
$$= d - \frac{d_s}{2}$$
$$= 94 - \frac{12.5}{2}$$
$$= 87.75 \text{ cm.}$$

Approximate area of steel required
$$= \frac{7569000}{1400 \times 87.75} \text{ cm}^2$$
$$= 61.65 \text{ cm.}^2$$

Let us provide 12 bars of 25 mm. diameter
(Area of 12 bars $= 58.92 \text{ cm.}^2$)

Width of flange

This will be taken as the lesser of the following

(*i*) $\frac{1}{3}$ span $= \frac{1200}{3}$
$$= 400 \text{ cm.}$$

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(*ii*) $12\,d_s + b_r$

$= 12 \times 12{\cdot}5 + 40$

$= 190$ *cm.*

∴ Flange width of T beam

$= 190$ *cm.*

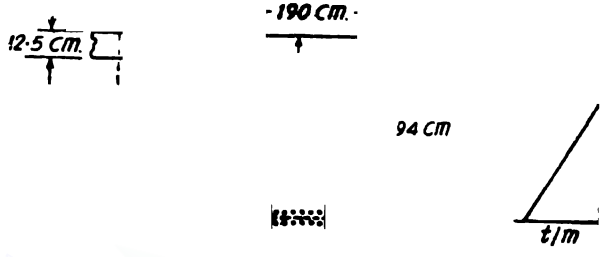

Fig. 782

Assume the neutral axis to be below the flange.

Taking moments about the neutral axis, we have

$190 \times 12{\cdot}5\ (n - 6{\cdot}25) = 18 \times 58{\cdot}92\ (94 - n)$

Solving $n = 33{\cdot}3$ *cm.*

$$y = \frac{3n - 2d_s}{2n - d_s} \cdot \frac{d_s}{3}$$

$$= \frac{3 \times 33{\cdot}3 - 2 \times 12{\cdot}5}{2 \times 33{\cdot}3 - 12{\cdot}5} \cdot \frac{12{\cdot}5}{3}\ \text{cm.}$$

$= 5{\cdot}77$ *cm.*

∴ Actual lever arm $= a = d - y$

$= 94 - 5{\cdot}77$

$= 88{\cdot}23$ *cm.*

∴ Actual stress in steel

$$= \frac{7569000}{58{\cdot}92 \times 88{\cdot}23}\ \text{kg./cm.}^2$$

$= 1465$ *kg./cm.*^{2}

Overstress $= 56$ *kg./cm.*^{2}

Percentage overstress

$$= \frac{56}{1456} \times 100$$

$= 3{\cdot}85\%$

(where the overstress is less than 5 per cent the design may still be considered as safe)

Stress in concrete

$$= \frac{n}{d - n} \cdot \frac{t}{m}$$

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$$= \frac{33{\cdot}3}{94-33{\cdot}3} \times \frac{1456}{18} \text{ kg./cm.}^2$$

$$= 44{\cdot}4 \text{ kg./cm.}^2$$

(less than 50 kg./cm.^{2}

Maximum shear force

$$= 4205 \times \frac{12}{2} \text{ kg.}$$

$$= 25230 \text{ kg.}$$

$\therefore$ Shear stress $= \frac{S}{ab}$

$$= \frac{25230}{88{\cdot}23 \times 40} \text{ kg./cm.}^2$$

$$= 7{\cdot}15 \text{ kg./cm.}^2$$

Shear force corresponding to a shear stress of 5 kg./cm.^{2}

$$S_o = 5 \times a \times b$$

$$= 5 \times 88{\cdot}23 \times 40 \text{ kg.}$$

$$= 17646 \text{ kg.}$$

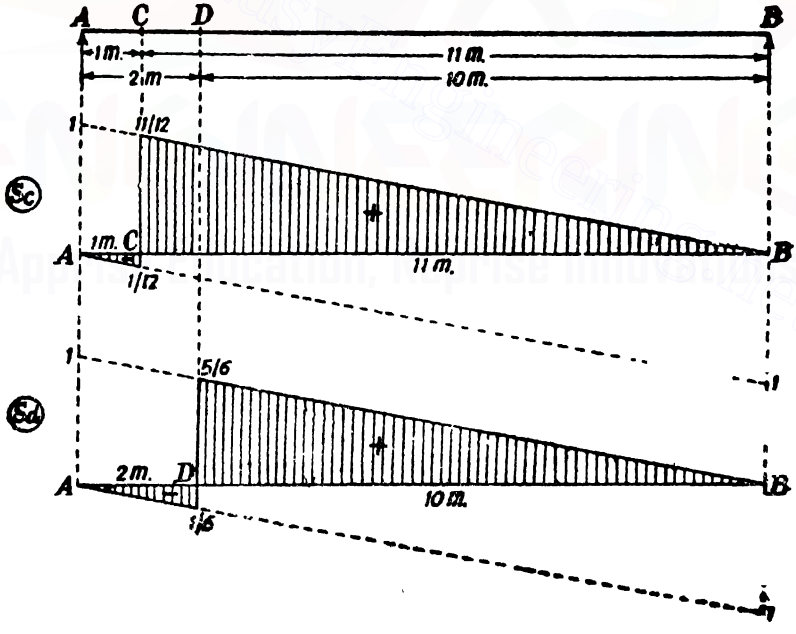

Fig. 783

Consider two sections C and D at 1 *metre* and 2 *metres* from the left support.

Fig. 783 shows the influence line diagram for the shear force at these sections.

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D.L. on the girder $= 3160$ *kg./m.*

L.L. on the girder $= 1045$ *kg./m.*

Shear force at 1 metre from the support

Dead load shear $= 3160 \times \frac{12}{2} - 3160 \times 1$ *kg.*

$= 18960 - 3160$ *kg.*

$= 15800$ *kg.*

Live load shear $= 1045 \times \frac{1}{2} \times 11 \times \frac{11}{12}$ *kg.*

$= 5270$ *kg.*

$\therefore$ Total shear force

$= 15800 + 5270$

$= 21070$ *kg.*

Shear force at 2 metres from the support

Dead load shear $= 3160 \times \frac{12}{2} - 3160 \times 2$ *kg.*

$= 18960 - 6320$ *kg.*

$= 12640$ *kg.*

Live load shear $= 1045 \times \frac{1}{2} \times 10 \times \frac{5}{6}$ *kg.*

$= 4360$ *kg.*

$\therefore$ Total shear force

$= 12640 + 4360$ *kg.*

$= 17000$ *kg.*

Shear value of 3 bent up bars

$= 0{\cdot}707\ A_w\ t_w$

$= 0{\cdot}707 \times 3 \times 4{\cdot}91 \times 1400$ *kg.*

$= 14570$ *kg.*

Let 3 bars be bent up at a distance of $1{\cdot}414\ a = 1{\cdot}414 \times 88{\cdot}23 = 125$ *cm.* from the support and let 3 more bars be bent up at $2{\cdot}828a = 250$ *cm.* from the support.

Net shear at the support for which stirrups are needed

$= 23230 - 14570$ *kg.*

$= 10660$ *kg.*

Spacing of two-legged 8 *mm.* diameter stirrups

$= \frac{A_w\ t_w\ a}{S}$

$= \frac{2 \times 0{\cdot}5 \times 1400 \times 88{\cdot}23}{10660}$ *cm.*

$= 11{\cdot}6$ *cm. say 11 cm. centres*

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Check for bond. Perimeter of bars required at the bottom near the supports

$$= \Sigma_0 = \frac{S}{a\, s_b} = \frac{25230}{82{\cdot}23 \times 10} \text{ cm.}$$

$$= 28{\cdot}6 \text{ cm.}$$

Perimeter provided by 6 bars of 25 mm.

$$= 6 \times 7{\cdot}86 = 47{\cdot}16 \text{ cm.}$$

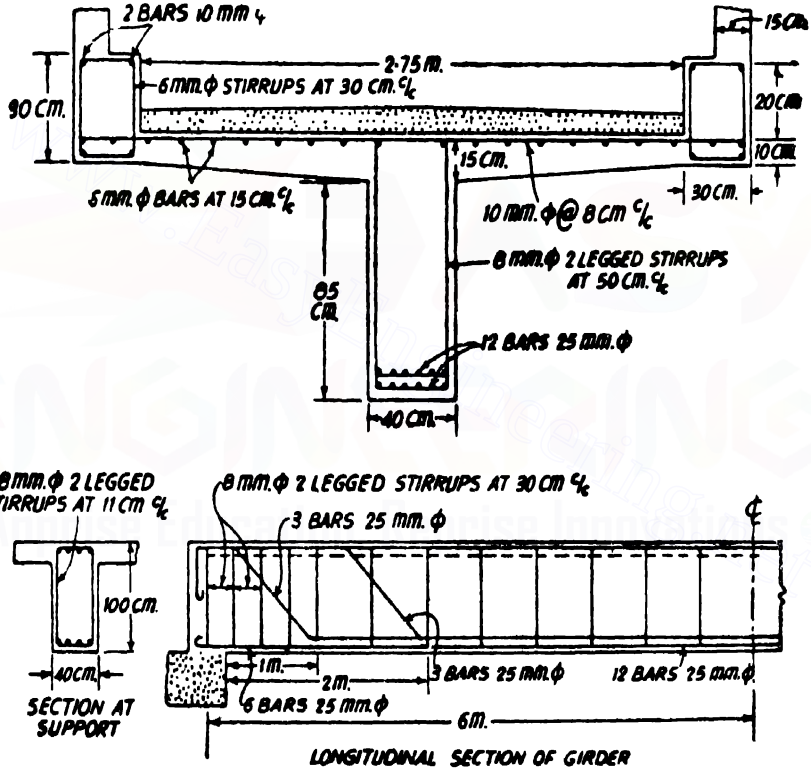

Fig. 784

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21

Roads and Pavings

A road has to satisfy three main requirements. A road must offer the user with a permanent tough, even surface laid to suitable gradient so as to remain self-draining and having a texture to provide adequate friction. The road has an important role to play in spreading the wheel loads and preventing overstressing of the subgrade soil. A road has to safeguard the subgrade from the effects of weather like rain and other seasonal influences which may lead to swelling or settlement or frost-heave.

Even though riding quality is of utmost importance to the road user, the satisfactory and successful performance of a road depends mainly on the efficient protection of the subgrade soil from overstressing due to traffic, and deterioration due to weather. Compared with other road making materials, concrete has the advantage of bending more rigid than the subgrade, having a little tensile strength too. Consequently a concrete slab of a small thickness possesses sufficient structural rigidity to distribute or spread the wheel loads over considerable area of the subgrade ; on the contrary, other road making materials need a greater depth of construction in order to achieve the same effect. Concrete serves as a waterproof umbrella to protect the subgrade from the destructive effect of weather. But it may be realized that concrete while having the advantages mentioned above is liable to displacements resulting from temperature changes, changes in moisture and initial shrinkage. As an effect of friction on the lower side of the concrete slab such movements may produce tensile stresses even when the slab is not subjected to traffic load ; and in case the slabs are laid continuously for great lengths, cracking is most likely to occur.

It may therefore be realized in the design of concrete roads, that attention must be given to the provision of suitable joints for expansion, contraction and the effects of warping of the slab. Since, when slab is subjected to contraction, tensile stresses are induced, the reinforcement (if provided) and the spacing between the joints must be interrelated. This is a very important aspect to be considered in concrete road design. The joints whether they are meant for expansion or contraction or warping, need careful treatment.

These joints are the most necessary features in the road design but are also the most undesirable features from other points of view. Joints have the tendency to spoil the riding qualities of the road.

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They interfere with the process of concrete laying. They are undoubtedly the surest source of weakness For a successful performance, the joints need sealing to prevent the ingress of water and at the same time they must remain flexible

A very important feature in road design is the structural strength of the slab itself. The strength of the slab depends on a number of factors like the quality of concrete, the thickness of the slab and the quantity and arrangement of the reinforcement. The amounts of linear and warping movements and the maximum advisable spacing between joints are dependent on the quality and thickness of the concrete The strength requirement of a slab is estimated by studying the traffic intensity and the strength of the subgrade soil. The traffic intensity is somehow not a very precise factor. It can be gauged in terms of the number of vehicles passing per unit time It can also be gauged in terms of the heaviest vehicle likely to move on the slab and whether this is likely when the slab is in its most unfavourable condition from the point of view of temperature, moisture and shrinkage. A subgrade soil may not possess a uniform strength over a considerable range. This is partly as a result of the condition of the soil presented in nature and partly as a consequence of the disturbances caused by excavations carried out. Thus it may be realised that the design of a road is quite a complex process involving a number of indeterminate factors. It is known by actual practice that for economy it is not possible to design concrete roads to a standard aimed at perfection. Cracking, to some extent, is generally considered acceptable provided the extent of cracking is restricted suitably. A high factor of safety need not be introduced as it may be done in other structures. The failure of a concrete road (though should be avoided) is not likely to have such serious and dire physical consequences that are associated with the failure of an important superstructure. The consequences of a road failure are mainly restricted to the cost and inconvenience of repair. Road design is practically concerned with striking a judicious compromise between first construction cost and subsequent maintenance and repair costs.

A number of attempts have been made to evolve computation methods of concrete road design the most notable among them being the method evolved by Westergaard. The theory presented by Westergaard is applicable to slabs of finite extent and is based on the assumption that the concrete slab acts as a homogeneous unit having elastic properties and that the reaction offered by the subgrade is proportional to its deflection This latter assumption appears to be a very weak point in the theory as it is well known that shrinkage temperature and moisture gradients can produce warping strains which may interfere appreciably with the uniformity of contact between the slab and the subgrade. The theory offers expressions for the stresses produced in the slab due to loading at three important, typical positions namely at the interior, on one edge and at one corner. The findings of Westergaard are surely of qualitative interest. For instance, the findings show that the tensile stress in a slab due to corner loading is approximately equal to the stress pro-

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duced due to an edge load ; on the contrary the stresses caused by interior loads are not more than about half such values. Stresses due to corner loading become very severe in the case of acute-angled slabs which of course should be avoided in all cases.

The Subgrade Soil

The subgrade soil has the role of providing uniformity of support. The stresses produced in a concrete slab increase approximately in proportion as the uniformity of support from the subgrade decreases. Non-uniformity of support is due to variation in conditions of the soil and other factors such as warping of the slab due to moisture shrinkage and temperature gradients. Clayey soils vary in volume according to the moisture content and hence variation in the moisture content (notably in the period immediately after the construction of the road) may cause uneven support. The real strength of the subgrade soil is less important since the resultant increased deformations generally produce more gradual blending in the road slab over larger areas and accordingly produce little increase in the stress in the slab.

The foremost consideration is the attainment of thorough and uniform compaction to prevent uneven settlements. In the case of clayey soil it is better to skim off the last 10 *cm.* immediately prior to putting down the base. This will help in reducing the variation of the natural moisture content of the soil.

The base has the function to provide a clear working surface, easily trimmed to levels and gradients, over which the slab of concrete can be laid without causing any damage to the subgrade. The structural advantage or benefit from a base of any reasonable thickness is ignorable compared with the structural merits of the concrete slab. It may therefere be realised that it is generally uneconomical to

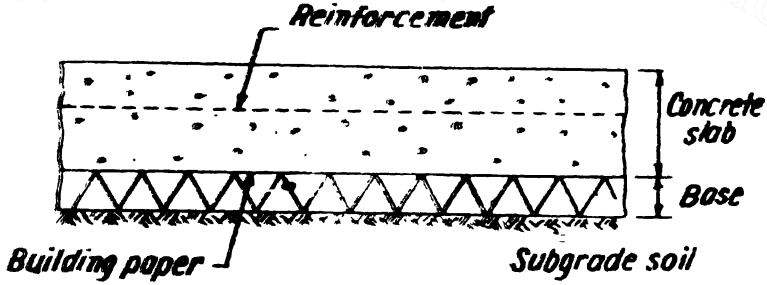

Fig. 785. Concrete road section.

increase the thickness of the base with a view to decrease the thickness of the slab. Generally 7·5 *cm.* to 8 *cm* is an adequate thickness for the base. (In silts, chalks and clayey soils where additional thickness may be provided so as to keep the underside of the base 35 *cm.* or 40 *cm.* below the ground level as a precaution against frost heave. Frost heave is very likely when there is severe and prolonged

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frost in location where the ground water level is close to the surface. The base will be of broken stone or other durable and hard material, properly graded and mechanically compacted so as to form a solid compacted mass. The upper surface of the base is blended well so as to provide an even smooth surface which in turn is covered with waterproof paper satisfactorily lapped at all joints. The combined effect of the smooth surface provided to the base and the waterproof paper appreciably decreases the friction between the base and the lowerside of the concrete slab, thereby allowing the slab to shrink relatively freely on initial hardening of the concrete, earlier to its attaining its full tensile strength. The waterproof paper also prevents the loss of mortar and grout into the base which would reduce the strength of the concrete in the lower part of the concrete slab. If the paper is provided as a rough and unblinded base, it would be of little use only as the weight of the wet concrete causes the stones in the base and the aggregates in the concrete to puncture the paper resulting in the mortar running through into the base and thus weakening the lower side of the slab.

§197. Reinforcement

Usually the purpose of the reinforcement (when provided) is not necessarily to increase the structural strength of the slab in bending, and thus, no special advantage is gained by using high tensile steel as against the ordinary mild steel, since it is the elastic modulus and not the ultimate strength of steel which determines the quantity of steel required. On this basis it would appear that it can make little difference whether the reinforcement is placed near the top or near the bottom. Where the weight of the reinforcement is appreciable, it is found convenient to provide the reinforcement at top and bottom. Examples of this case are embankments or sub-grades of very plastic clays or organic matter.

However, in the case where a single layer of reinforcement is provided, it is provided near the top of the slab, for the reasons given below :

Westerguard's analysis shows that the maximum tensile stresses occur in the slab due to corner and edge loading where cantilever action results. Severe warping which is produced as a consequence of initial hardening of the slab will also produce cantilever action

Figs. 786 and 787 indicate the above two effects. Moreover, the reinforcement near the top of the slab is less likely in the disturbance of placing and vibrating the concrete to be worked down on to the base. For road work the reinforcement is generally provided in made up fabric which should always be obtained in flat mats and never in rolls. Reinforcements in rolls are liable to spring in the concrete in the plastic state before setting and therefore move considerably out of the required position. Normally the fabric is made-up into sizes to accommodate the widths of construction and the positions of joints so as to avoid waste and unnecessary lapping.

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§198. The Concrete Slab

In order to achieve good concrete roads we must use properly designed concrete mixes, and the concrete well compacted to give a 28 day strength of at least 280 *kg /cm²*. Whatever care is taken in respect of other aspects of design and construction work, the road will not be satisfactory if the concrete slab itself is not of uniform thickness and quality throughout. Substantial form work has to be provided accurately set to line and level The concrete has to be compacted thoroughly. Proper finishing and smoothing should be done from the point of view of drainage and ridability Normally a skid-resistant finish is given by brushing. Generally curing is effected by covering the concrete with an impervious membrane of resins and gums applied in solution by spray equipment. The membrane assists in retaining the moisture in the concrete for about two weeks without disturbing the finish or weakening the mortar surface. Later on the member gets removed entirely by evaporation.

Now-a-days air-entrained concrete has been also used in the construction of roads. Air-entrained concrete has a number of properties like resistance to frost attack, easier workability, lesser segregation and bleeding in transport.

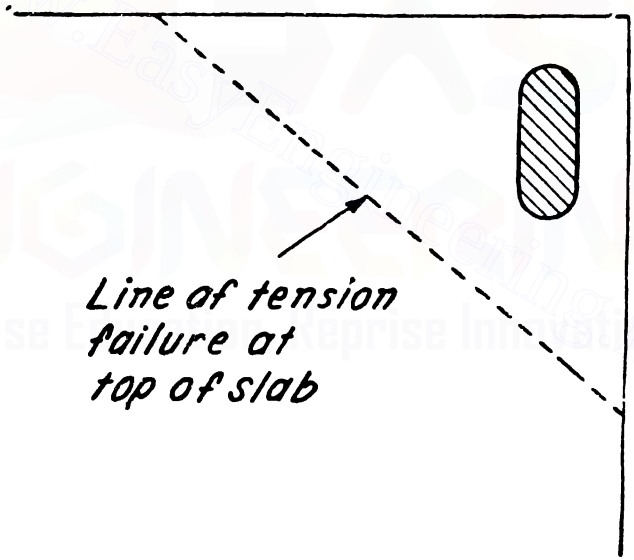

Fig. 786

§199. *Joints in Slab*

From the point of view of road user all joints in concrete roads are undesirable. Unless the joints are formed perfectly, they spoil the riding quality of the road. Joints no doubt are the weakest part in the whole of the construction If the slabs on both sides of the joint are not exactly at the same level or if there is any irregularity of the concrete surface at this point severe impact forces will occur.

It is necessary to seal all joints against ingress of water and grit. All rebates to accommodate sealers should normally be about

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2 *cm.* wide and 2·5 *cm.* deep. The top arises of the rebate should be rounded off to a pencil radius. While rounding the arises it is necessary to make a check by straight edge across all joints to see that the road surface is really flush and has in no way been interfered with by the joint formation. It is very necessary that the sealer has got to be watertight to prevent the water from getting

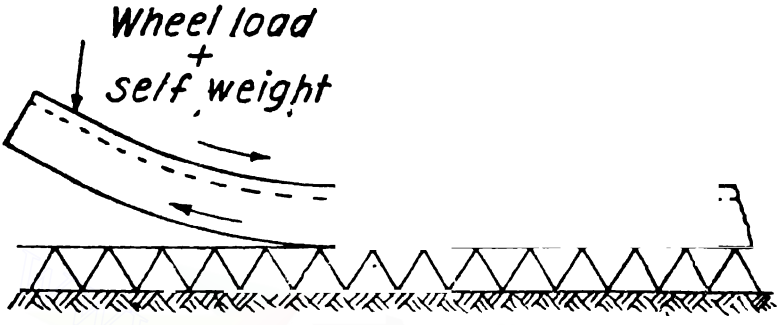

Fig. 787

down to the subgrade soil. The sealer has to stick to the edge of the concrete slab under condition of tension and should be able to stretch without cracking It has to be hard enough also to keep out grit and stones which are liable to be crushed in the joint, when under compression, resulting in the spalling of concrete or in the expansion joints, decreasing their efficiency. The scales should be such that it is possible to melt it on heating so that it can be applied readily. On the contrary, it should not soften sufficiently in hot weather and squeeze up and out of the joint and liable to be spread by the moving traffic. The road surface may get disfigured and the joint may open up in cold weather.

A rubber bitumen compound is the most satisfactory and durable material used for the purpose of sealing the joints. The surface of the concrete to which it is applied should be dry and coated with a special primer rod so as to give it the property of adhesion. Generally we find that seals 25 *mm.* deep give adequate protection with good adhesion and suitable stretching properties without producing unreasonable upward squeeze. Various grades and hardnesses of rubber bitumen compounds are available to suit various situations. For instance, in tropical climates or on roads where stones or grit are liable to be brought in by industrial traffic or otherwise a harder bitumen is desirable But on susceptible clay soils the need for successful sealing against water ingress may outweigh other considerations and therefore a softer and a more pliable sealer may be provided. Many forms of joints have been tried but the joints shown in Fig. 788 known as *expansion joints*, *complete contraction joints* and *partial contraction joints* satisfy all the requirements.

Expansion joints. The spacing of expansion joints depends on (*i*) the net expansion of concrete due to extreme temperature

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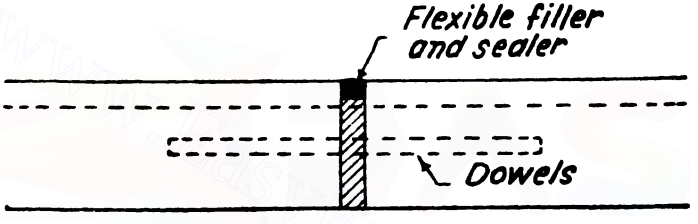
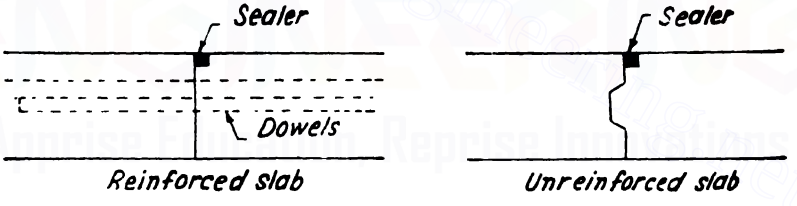
conditions, and (*ii*) the thickness and compressibility of the flexible filler. An expansion joint is provided to prevent compression failures which may appear in the form of upward buckling of adjacent slabs. Besides the effect of temperature expansion can also be caused by moisture content of concrete. While considering the net expansion of the concrete, some allowance is made to take into account of the initial shrinkage of concrete on hardening which may even exceed the amount of expansion. But the cracks which are produced by shrinkage may later on be filled with grit so that after a period of some years they are no more free to accommodate the displacements arising from temperature changes

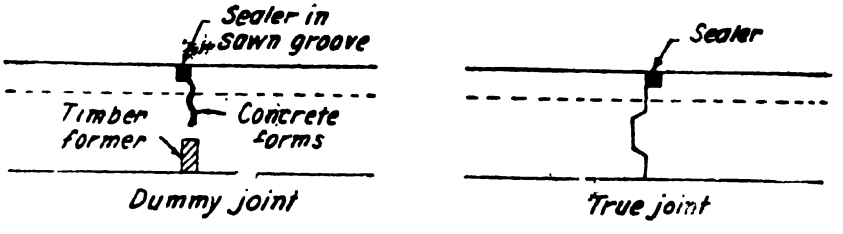
It is worthy to note that the temperature gradient in a slab in the hottest weather may be such that it can cause the slab to warp up making it hog. This may produce eccentric transmission of end thrusts between adjacent slabs.



(*a*) Expansion Joint



(*b*) Complete contraction joint



(*c*) Partial contraction joints

Fig. 7.8.

When expansion joints are provided at great distance apart, considerable tensile stresses may be produced in the slab on cooling. But in the case of unreinforced roads this is not so, as it is the usual practice to provide complete contraction joints at about 4·5 to 5

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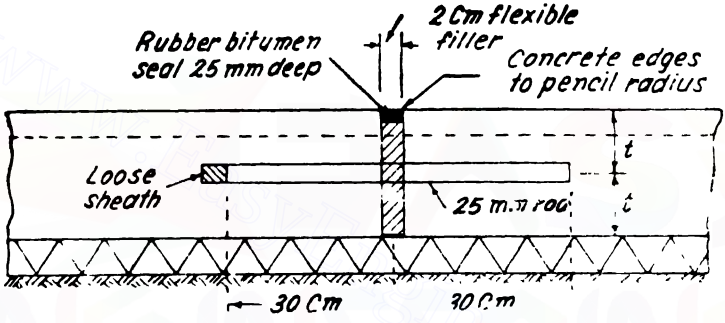
metres centres, in order to prevent the development of stresses due to shrinkage or warping stresses which lead to cracks in the unreinforced slab. Therefore it is normally permissible to provide the expansion joints at a greater spacing in unreinforced slabs than in reinforced slabs. In fact the shrinkage, at 5 *m.* centres, between plain concrete slabs may be sufficient to avoid the need of any expansion joints altogether, if the joints are properly sealed against ingress of grit.

The expansion joints may be 2 *cm* thick and spaced at about 30 *metres* or more for normal road work. In thin slabs having less reinforcement available for drawing the ends in, on cooling, the spacing should be decreased to 15 *metres* intervals. The edges of the slab are their weak parts and hence it is generally necessary to provide load-transfer devices. (See Fig. 789).



Expansion joint showing dowels as load transfer device.

Fig. 789

The dowel bars are connected into the slab on one side of the joint by direct bond between the dowel and the concrete, while on the other side of the joint there is freedom between the dowel and the concrete in order to allow for taking up the movement due to expansion. This freedom of movement is attained by dipping the dowels first into a special bitumen solution and then leaving them to dry an upright position before used in the work. The bitumen solution is a mixture of the following

200 penetrations bitumen	..	66% by weight
Light creosote oil	.	14% by weight
Solvent naphtha	.	20% by weight

In order to provide freedom at the free end of the dowels, a metal or card board sheath 7·5 *cm* long is provided with the end space filled 2 *cm.* of flexible material to accommodate the necessary movement. The dowel bars are usually 25 *mm.* diameter, 60 *cm.* long provided at 30 *cm.* centres. (For conditions of very heavy loading the spacing may be reduced to 25 *cm.* centres.) Where thin slabs are used in conditions of light traffic it is not a practice to provide the dowels and such dowels may do more harm than good. It is very important that the dowels must be set accurately in position and at right angles to the line of the expansion joint. Otherwise,

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they will bind on the concrete as the joint moves decreasing the efficiency of the joint. Good care is to be taken in compacting the concrete between the dowels because the obstruction caused by the dowels is likely to interfere with successful compaction.

The flexible filler provided in the expansion joints must be compressible at small pressure intensities so that the joint may readily close. It must also be resilient to regain a reasonable amount when the joint reopens. If this is not satisfied the sealer may not give adequate support and may be worked down into the joint allowing grit and stones to be forced into the upper part of the joint. Moreover considerable displacements might have taken place before the seal is first applied and in its molten condition it must be prevented from running down the side of the flexible filler. Care is taken to ensure that the flexible filler is arranged continuously and for the entire depth of the joint (except at the top where the sealer is poured). Otherwise the concrete is liable to run into the intervening spaces preventing the joint from closing up on expansion and producing very high stresses resulting in spalling and damage to road. It is equally important to ensure that the flexible filler is provided truly vertical, otherwise, on expansion the slab on one side of the joint may ride over the other side.

Contraction joint. It is necessary that at all construction joints a potential discontinuity is required. There are many ways of achieving this. When done on a small scale, in which concreting is split up into small bays a bituminous solution is painted sometimes on the hardened surface of the concreted panel first ; alternatively a strip of building paper is interposed. In situations where contraction joints are formed by a definite discontinuity in the concreting operation, it is usual to form the rebate for the sealer entirely on one side of the joint. See Figs. 788 (*b*) and (*c*).

Complete contraction joint. See Fig. 788 (*b*). It may be noted that in the reinforced concrete slabs the reinforcement at complete contraction joints is interrupted ; but at partial-contraction joints reinforcement is continued right through. This has the effect that while on expansion in hot weather the entire movement is accommodated at the expansion joints, on contraction in cool weather the movement is shared equally by the opening-up of the expansion and complete contraction joints. Very little opening-up takes place at the intermediate partial-contraction joints since the reinforcement holds these together. This arrangement is not satisfactory since the complete contraction joints do not contain a flexible filler to support the sealer and in course of time the joints become inefficient and a source of trouble. It is often considered better to avoid the use of complete contraction joints in reinforced slabs and rely wholly upon expansion joints.

On the other hand, in unreinforced slabs, complete contraction joints are necessary at regular intervals to avoid development of cracks due to shrinkage of concrete. Shrinkage cracks are liable to

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occur at intervals of about 10 *metres* if an unreinforced slab is concreted for an indefinite length. Due to incidence of traffic the intervals of shrinkage cracks may be reduced to 5 *metres*. Hence it is proper to provide in unreinforced slabs, complete contraction joints at 5 *m.* intervals. By providing these joints the slab will be relieved of stresses which would otherwise be induced as a result of restrained warping effects.

Partial contraction joints. It is necessary to provide partial contraction joints in reinforced slabs at every 5 *metres*. In small scale work it will be convenient to provide each joint with a tongue and groove rebated face while with continuous laying equipment, dummy joints are formed.

Longitudinal joints. Wherever the road width exceeds about 5 *metres*, these are provided. Where the road width exceeds 10 *metres* the longitudinal joints are spaced about 3 to 4 *metres* apart to coincide with the lanes of traffic. These joints do not need any provision for expansion unless the road is hemmed in on both sides between retaining walls or buildings.

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22*

Prestressed Concrete

§200. Introduction

A prestressed concrete member is a member of concrete in which internal stresses are introduced in a planned manner, so that the stresses resulting from the superimposed loads are counteracted to a desired degree.

Concrete has a high compressive strength compared to its very low tensile strength. While it is not difficult to make concrete which can have a crushing strength of over 400 $kg./cm.^2$ (40 N/mm^2) on 15 $cm.$ cubes (after 28 days curing) it is very difficult to obtain a tensile strength greater than 25 $kg./cm^2$ (2·5 $N/mm.^2$). This is why concrete is reinforced with steel bars in such directions where tensile resistance has to be offered In reinforced concrete members due to the low tensile resistance of concrete, a large quantity of steel reinforcement is provided so that the entire tensile resistance is taken as offered by the reinforcement. But, no special means are provided to prevent the concrete offering tensile stresses ; hence concrete will also offer a tensile resistance along with the steel reinforcement and in this process will develop minute tensile cracks. Hence, under the usual superimposed working loads the concrete present in the tension zone of a reinforced concrete member will develop cracks not being able to conform to the strains taking place in the steel. A reinforced concrete member has other weaknesses too The dimensions of the member may also have to be determined from diagonal tensile stress consideration. In the case of long beams, where large shear forces exist the beam sizes have got to be large to limit the diagonal tensile stresses under certain limits A reinforced concrete member is also found to develop cracks even in the absence of loads, due to the shrinkage of concrete in the hardening stage.

In a prestressed concrete member, all the above defects and weaknesses are made up.

A very important aspect to be borne in mind is that a prestressed concrete member is essentially a concrete member only and the tendons (reinforcements) are considered as the necessary device

*This chapter deals with the elementary principles of prestressed concrete. For more details and designs the readers may refer to the text-book Prestressed Concrete by the author.

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to supply the prestress to the concrete. If prestress can be introduced by external jacking, then the tendons will not be necessary for a prestressed concrete member. The tendons therefore serve as the *means to supply the prestressing force.*

In order to get the maximum advantage of a prestressed concrete member, it is necessary to use not only *high strength concrete* but also *high tensile steel* wires.

Concrete used for prestressed work should have a cube strength of 350 *kg./cm.*^{2} (35 *N/mm.*^{2}) for post-tensioned system and 450 *kg./cm.*^{2} (45 *N/mm.*^{2}) for pretensioned system.

High strength concrete is necessary for the following reasons :

(*i*) Since large prestressing forces are applied to the member by the tendons, high bearing stresses are developed at the ends by the anchoring devices. The anchorages are generally designed to be meant for use only for high strength concrete work.

(*ii*) Bursting stresses liable to at the ends of the beam cannot be satisfactorily resisted by low strength concrete.

(*iii*) When the stress transfer to concrete has to take place by bond action, the concrete should have a high bond stress which can be offered only by high strength concrete.

(*iv*) Shrinkage cracks will be very little when high strength concrete is used.

(*v*) Due to the high modulus of elasticity of high strength concrete the elastic and creep strains are very small resulting in smaller loss of prestress in the steel reinforcement.

High tensile steel wires used in prestressed concrete work are called *tendons.*

High tensile steel should be used for the following reasons :

The mild steel used in ordinary reinforced concrete has a yield point of 2000 *kg./cm.*^{2} (200 *N/mm.*^{2}) to 3000 *kg./cm.*^{2} (300 *N/mm.*^{2}) If such a steel is used and if even it is subjected to a stress say 2000 *kg./cm.*^{2} (200 *N/mm.*^{2}) at the stage of tensioning, we find that due to creep and shrinkage of concrete the net tensile stress left over will be extremely low.* In the design of prestressed concrete member, the estimated loss of prestress due to shrinkage of concrete and creep of concrete and steel is at the order of nearly 2000 *kg./cm.*^{2} (200 *N/mm.*^{2}) But high tension steel has an ultimate strength of 21000 *kg./cm.*^{2} (2100 *N/mm.*^{2}) and if initially stressed to say

*Even as early as in 1888, the idea of prestressing concrete existed. In 1888 *Doehring Koenen, Mandl, Lund* and many others tried to produce prestressed concrete members using ordinary mild steel bars. These beams were found satisfactory initially but after a short time developed cracks and failed even when not loaded This remained a mystery for nearly 30 years. Later on *Freyssinet* and *Dischinger* independently found the cause of failure of the *Doehring* beams. They found that the failure was due to the fact that shrinkage and creep was so large that no more strain was left in the steel. Hence the prestress disappeared resulting in the failure of the beam.

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10,000 *kg /cm.*^{2} (1000 *N/mm.*^{2}) there will still be large stress in the reinforcement after making deduction for the loss of prestress.

§201. Assumptions in the Design of Prestressed Concrete Members

Prestressed concrete members are analysed and designed on the basis of the assumptions given below :

(*i*) A transverse plane section of the member will remain a plane after bending also.

(*ii*) Within the limits of the deformations taking place Hooke's law is applicable to concrete and steel components.

(*iii*) The stress in the reinforcement does not change along the length of the reinforcement. Stress changes take place for the concrete component only. Variation of stress in the reinforcement due to changes in the external loading is ignorable.

§202. General Principles

Fig. 790 shows a simply supported prestressed concrete beam of rectangular section prestressed by a tendon provided through its centroidal longitudinal axis. Let the beam be subjected to an external load system. Let P be the prestressing force supplied by the tendon. Due to this prestressing force, the compressive stress induced in concrete $= f_a = -\frac{P}{A}$ where A is the sectional area of the member. If due to the dead load and external loads, the bending moment at a section is M, then the extreme stresses at the section due to bending moment alone

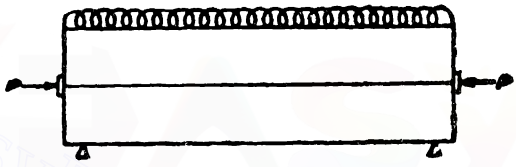

Fig. 790

$$-f_b = \pm\frac{M}{Z}$$

where Z is the section modulus of the beam section.

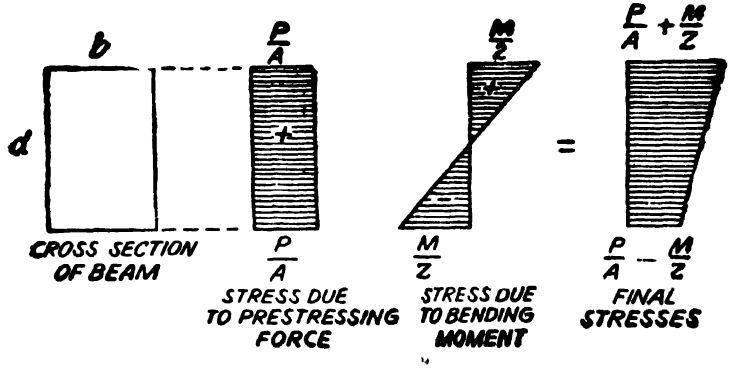

Fig. 791

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Hence, the final extreme stresses on the beam section are given by

$$f = f_a \pm f_b = \frac{P}{A} \pm \frac{M}{Z}$$

Stress at the extreme top edge $= \frac{P}{A} + \frac{M}{Z}$

and stress at the extreme bottom edge $= \frac{P}{A} - \frac{M}{Z}$

The stresses due to direct load, bending and the final stresses are shown in Fig 791.

Design 179 *A prestressed concrete beam 40 cm. × 60 cm. in section has a span of 6 metres and is subjected to a uniformly distributed load of 1600 kg./metre including the self-weight of the beam. The prestressing tendons which are located along the longitudinal centroidal axis provide an effective prestressing force of 96 tonnes. Determine the extreme fibre stresses in concrete at the mid span section.*

Solution. Area of beam section $= A = 40 \times 60 = 2400$ *cm.*^{2}

Section modulus $= Z = \frac{bd^2}{6} = \frac{40 \times 60^2}{6} = 24000$ *cm.*^{3}

Maximum bending moment $= M = \frac{wl^2}{8} = \frac{1000 \times 6^2}{8} \times 100$ *kg. cm.*

$= 720{,}000$ *kg. cm.*

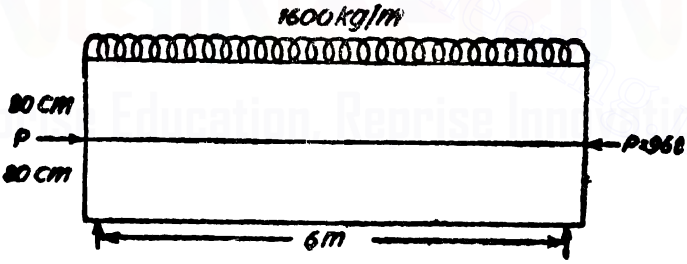

Fig. 792

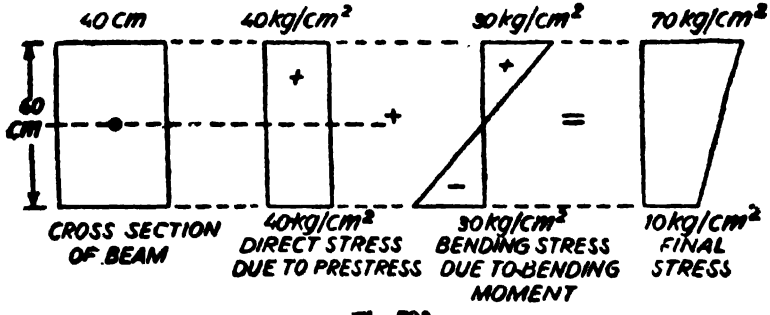

Fig. 793

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Direct stress due to prestressing force $= f_a = \frac{P}{A} = \frac{96000}{2400}$

$= +40 \ kg./cm^2$

Bending stress at the extreme fibres $= \pm \frac{M}{Z} = \frac{720000}{24000}$

$= \pm 30 \ kg./cm^2$

∴ Final stresses for the extreme fibres will be as follows :

Final stress for the extreme top fibre

$= +40 + 30 = +70 \ kg/cm^2.$

Final stress for the extreme bottom fibre

$= +40 - 30 = +10 \ kg/cm^2$

Design 180. (S.I.). *A prestressed concrete beam 40 cm. × 60 cm. in section has a span of 6 metres and is subjected to a uniformly distributed load of 16000 N/metre including the self-weight of the beam. The pre-stressing tendons which are located along the longitudinal centroidal axis provide an effective prestress of 960 kN. Determine the extreme fibre stresses in concrete at the mid span section.*

Solution. Area of beam section $= A = 40 \times 60 = 2400 \ cm.^2$

Section modulus $= Z = \frac{bd^2}{6} = \frac{40 \times 60^2}{6}$

$= 24000 \ cm.^3$

Maximum bending moment due to external loading $= M = \frac{wl^2}{8}$

$= \frac{16000 \times 6^2}{8} \times 100 \ N \ cm.$

$= 7200000 \ N \ cm.$

Direct stress due to prestressing force

$= f_a = \frac{P}{A} = \frac{960000}{2400} = +400 \ N/cm.^2$

Bending stress at the extreme fibres

$= f_b = \pm \frac{M}{Z} = \pm \frac{7200000}{24000} = \pm 300 \ N/cm.^2$

Final stresses for the extreme fibres will be as follows :

Final stress for the extreme top fibre

$= +400 + 300 = +700 \ N/cm.^2$

$= +7 \ N/mm.^2$

Final stress for the extreme bottom fibre

$= +400 - 300 = +100 \ N/cm.^2$

$= +1 \ N/mm.^2$

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203. Tendon Placed at an Eccentricity

It will be advantageous to place the tendon at an eccentricity.

Fig. 794 shows a beam of rectangular section prestressed by a tendon placed longitudinally, at an eccentricity e from the centroidal longitudinal axis. Let the beam be subjected to an external load system. Let P be the prestressing force supplied by the tendon. Let due to the dead load and external loads the bending moment at a section be M.

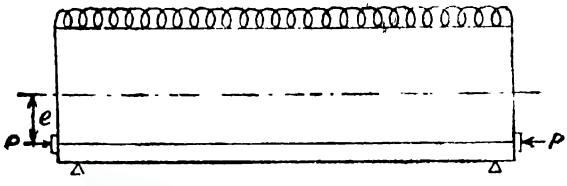

Fig. 794

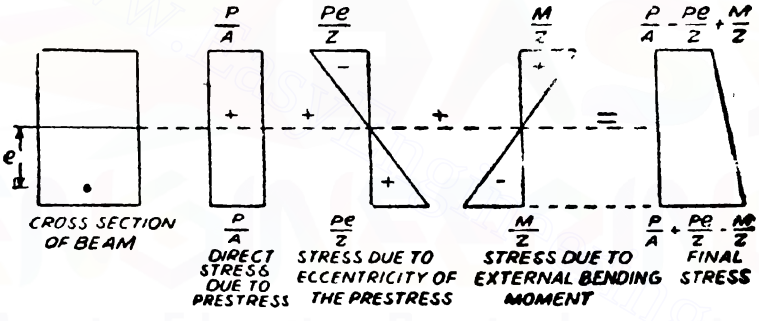

Fig. 795

The stresses on the section consist of the following :

(*i*) Direct stress due to prestressing force $= +\frac{P}{A}$

(*ii*) Extreme stresses due to eccentricity of the prestressing force $= \mp \frac{Pe}{Z}$

(*iii*) Extreme stresses due to bending moment $= \pm \frac{M}{Z}$

Final Stresses :

Stress at the extreme top edge $= \frac{P}{A} - \frac{Pe}{Z} \quad \frac{M}{Z}$

Stress at the extreme bottom edge $= \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$

These are shown in Fig. 795.

By providing an eccentricity to the tendon, a hogging moment

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Pe is developed which will produce stresses which will counteract the stresses due to external bending moment.

Design 181. *A prestressed concrete beam 40 cm. × 60 cm. in section has a span of 6 metres and is subjected to a uniformly distributed load of 1600 kg/metre including the self-weight of the beam. The prestressing tendons are located at the lower third point and provide an effective prestressing force of 96 tonnes. Determine the extreme fibre stresses in concrete at the mid span section.*

Solution. Area of beam section $= A = 40 \times 60 = 2400 \ cm.^2$

$$\text{Section modulus} = Z = \frac{bd^2}{6} = \frac{40 \times 60^2}{6}$$

$$= 24000 \ cm.^3$$

$$\left.\begin{matrix}\text{Maximum bending moment} \\ \text{due to external loading}\end{matrix}\right\} = M = \frac{wl^2}{8}$$

$$= \frac{1600 \times 6^2}{8} \times 100 \ kg. \ cm.$$

$$= 720{,}000 \ kg. \ cm.$$

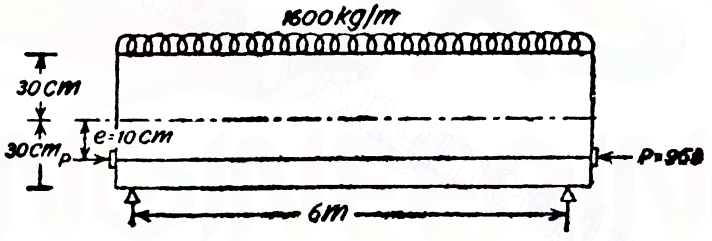

Fig. 796

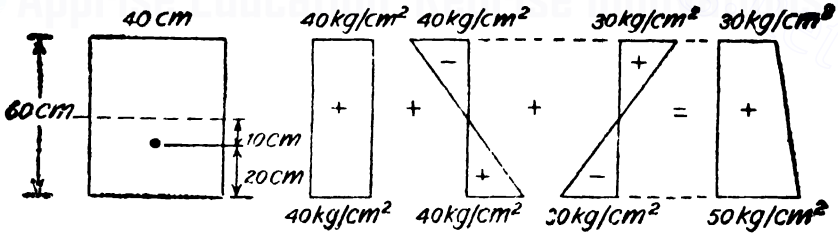

Fig. 797

$$\text{Direct stress due to prestressing force} = \frac{P}{A} = \frac{96000}{2400}$$

$$= +40 \ kg. \ cm.^2$$

$$\left.\begin{matrix}\text{Extreme fibre stress due to eccentri-} \\ \text{city of the prestressing force}\end{matrix}\right\} = \mp \frac{Pe}{Z}$$

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$$= \mp \frac{96000 \times 10}{24000}$$

$$= \mp 40 \text{ kg./cm.}^2$$

Extreme fibre stress due to external bending moment

$$= \pm \frac{M}{Z} = \pm \frac{720000}{24000} = \pm 30 \text{ kg. cm.}^2$$

Final stresses for the extreme fibres are as follows :

Final stress for the extreme top fibre

$$= +40 - 40 + 30 = +30 \text{ kg./cm.}^2$$

Final stress for the extreme bottom fibre

$$= +40 + 40 - 30 = +50 \text{ kg./cm.}^2$$

Design 182 (S.I.). *A prestressed concrete beam 40 cm. × 60 cm in section has a span of 6 metres and is subjected to a uniformly distributed load of 16000 N/metre including the self-weight of the beam. The prestressing tendons are located at the lower third point and provide an effective prestressing force of 960 kN. Determine the extreme fibre stresses in conrete at the mid span section.*

Solution Area of the beam section

$$= A = 40 \times 60 = 2400 \text{ cm.}^2$$

Section modulus $= Z = \frac{40 \times 60^2}{6} = 24000 \text{ cm.}^2$

Maximum B.M. due to external loading

$$= M = \frac{wl}{8} = \frac{16000 \times 6^2}{8} \times 100 \text{ N cm.}$$

$$= 7200000 \text{ N cm.}$$

Direct stress due to prestressing force

$$= \frac{P}{A} = + \frac{960000}{2400} \text{ N/c} : ^2$$

$$= +400 \text{ N/cm.}^2$$

Extreme fibre stress due to eccentricity of the prestressing force

$$= \mp \frac{Pe}{Z} = \mp \frac{960000 \times 10}{24000} = \mp 400 \text{ N/cm.}$$

Extreme fibre stress due to external bending moment

$$= \pm \frac{M}{Z} = \pm \frac{7200000}{24000} = \pm 300 \text{ N/cm.}^2$$

Final stresses

Stress at top

$$= +400 - 400 + 300 = +300 \text{ N/cm.}^2$$

$$= +3 \text{ N/mm.}^2$$

Stress at bottom

$$= +400 + 400 - 300 = +500 \text{ N/cm.}^2$$

$$= +5 \text{ N/mm.}^2$$

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§104. Prestressed Beam with Bent Tendon

Sometimes it is convenient to provide bent tendons. By providing bent tendons, the tendons will exert an upward pressure on the concrete beam and will therefore counteract a part of the external downward loading. Fig. 798 shows a prestressed beam with a bent tendon. The effect of providing a bent tendon can be best considered by studying the concrete as a free body subjected to forces as shown in Fig. 799. For the sake of the discussion let us assume that the tendon forms a sharp bend and that there is no frictional loss along the tendon. Considering the concrete as a free body we find an upward force $2P \sin \theta$ is also available at mid span. This upward force can to some extent counteract a part of the external load. For this beam,

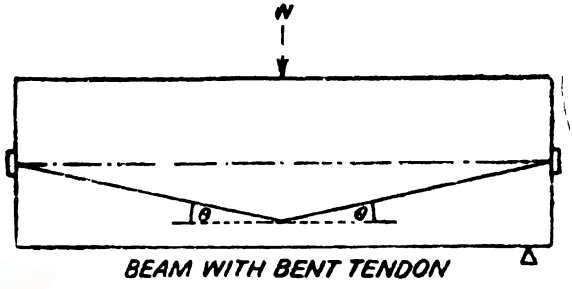

Fig. 798

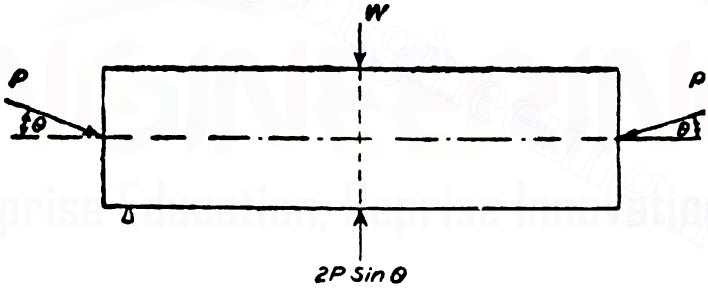

Fig. 799

the net downward load at the centre will be $(W - 2P \sin \theta)$

The axial longitudinal force provided by the tendon

$= -P \cos \theta = -P$ approximately since θ is a small angle

Direct stress on the section $= \dfrac{P \cos \theta}{A} = \dfrac{P}{A}$ approximately.

$$\text{Net B.M.} = M = \frac{(W - 2P \sin \theta)l}{4} + \frac{wl^2}{8}$$

where w = Dead load per unit length of the beam.

$$\text{Extreme fibre stress} = \frac{P}{A} \pm \frac{M}{Z}$$

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With a little reasoning one may realise that the profile of the tendon should follow the shape of the bending moment diagram for the given external loads in order it may offer considerable and effective upward forces. For instance, if the loading on the beam is a uniformly distributed load the tendon may be provided along a parabolic profile.

Design 183 *A prestressed concrete beam 40 cm. wide and 60 cm. deep has a span of 6 metres. The beam is prestressed with a tendon bent as shown in Fig. 800. The external load on the beam consists of a concentrated load of 18 tonnes at midspan. If the effective prestressing force is 120 tonnes, calculate the extreme stresses for the mid span section.*

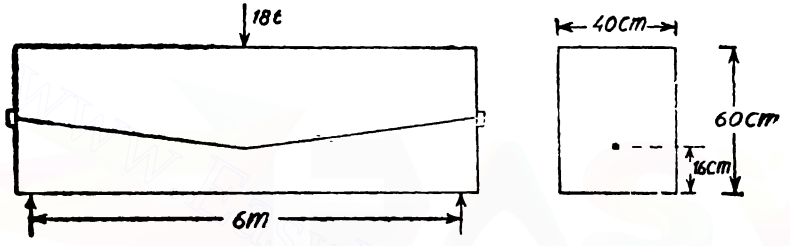
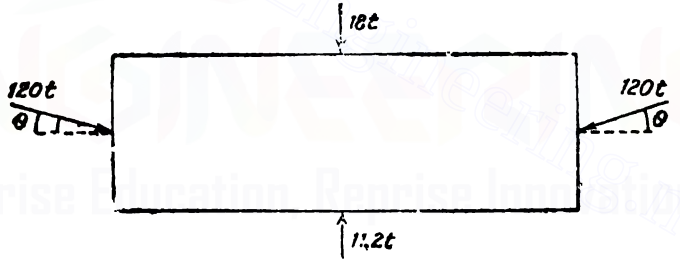


Fig. 800



ig. 801

Solution. Let the tendon be at θ with the horizontal.

$$\tan\theta = \frac{14}{300}$$

The inclined tendon will provide an upward force at the mid span.

This upward force $= 2P \sin\theta$

$= 2P \tan\theta$ (since θ is small)

$= 2 \times 120 \times \frac{14}{300} t = 11{\cdot}2\ t.$

Net vertical load at mid span $= 18 - 11{\cdot}2 = 6{\cdot}8\ t.$

B.M. due to net vertical load $= \frac{6{\cdot}8 \times 1000 \times 6}{4} \times 100\ kg.\ cm.$

$= 1{,}020{,}000\ kg.\ cm.$

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Self-weight of beam $= 0{\cdot}4 \times 0{\cdot}6 \times 2400 = 576$ *kg. per m.*

B.M. due to self-weight $= -\dfrac{576 \times 6^2}{8} \times 100$ *kg. cm*

$= 259{,}200$ *kg. cm.*

Total B.M. $= 1{,}020{,}000 + 2{,}59{,}200 = 1{,}279{,}200$ *kg. cm.*

Section modulus of the section $= Z = \dfrac{bd^2}{6} = \dfrac{40 \times 60^2}{6}$

$= 24000$ *cm.*^{3}

Direct stress due to prestressing force $= + \dfrac{P}{A} = \dfrac{120000}{40 \times 60}$

$= +50$ *kg./cm.*^{2}

Extreme fibre stress due to B.M. $= \pm \dfrac{M}{Z} = \pm \dfrac{1{,}279{,}200}{24000}$ *kg./cm.*^{2}

$= \pm 53{\cdot}3$ *kg./cm.*^{2}

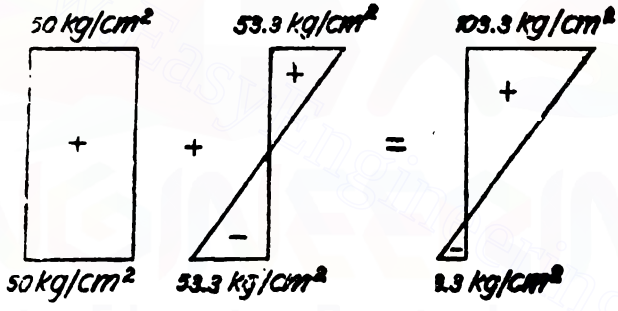

Fig. 802

Resultant extreme stresses are,

$+50 + 53{\cdot}3 = +103{\cdot}3$ *kg./cm.*^{2} (*compressive*) in the top fibre.

and $+50 - 53{\cdot}3 = -3{\cdot}3$ *kg /cm.*^{2} (*tensile*) in the bottom fibre.

Design 184. *Fig. 893 shows a prestressed concrete beam provided with bent tendon. The beam carries a point load of 16 t at the centre. Determine the stress distribution for the end section and the mid-section of the beam. The dead load of the beam is 576 kg./m.*

Solution. Area of beam section $= A = 40 \times 60 = 2400$ *cm.*^{2}

Section modulus $= Z = \dfrac{40 \times 60^2}{6} = 24000$ *cm.*^{3}

Let θ be the inclination of the tendon with the horizontal.

$$\sin \theta = \tan \theta = \frac{15}{300} = \frac{1}{20}$$

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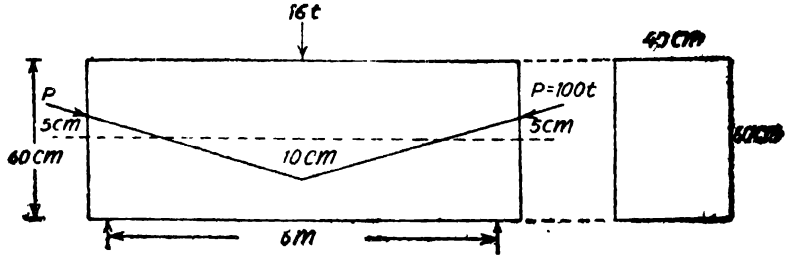

Fig. 803

Analysis of end section

Extreme stress $= \frac{P\cos\theta}{A} \pm \frac{P\cos\theta \times e}{Z} = \frac{P}{A} \pm \frac{Pe}{Z}$

[Since cos θ is nearly equal to unity]

$$= \frac{100{,}000}{2400} \pm \frac{100{,}000 \times 5}{24000} \text{ kg./cm.}^2$$

$$= 41{\cdot}7 \pm 20{\cdot}8 \text{ kg./cm.}^2$$

Stress at top $= 41{\cdot}7 + 20{\cdot}8 = +62{\cdot}5$ kg./cm.^{2}

Stress at bottom $= 41{\cdot}7 - 20{\cdot}8 = +20{\cdot}9$ kg./cm.^{2}

Analysis of mid section

External downward point load at centre $= 16t \downarrow$

Upward point load provided by tendon $= 2P \sin\theta$

$$= 2 \times 100 \times \frac{1}{20} = 10t \uparrow$$

∴ Net downward point load $= 16 - 10 = 6t \downarrow$

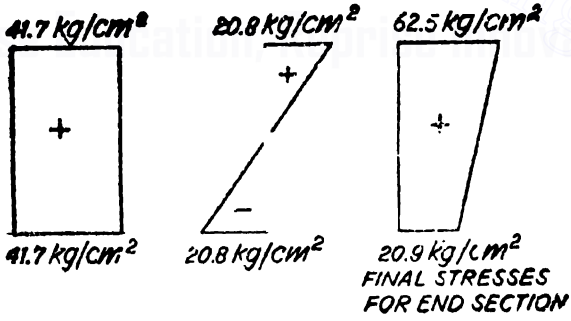

Fig. 804

B M. due to net downward point load

$$= \frac{6000 \times 6}{4} \times 100 \text{ kg-cm.} = +900{,}000 \text{ kg. cm.}$$

B.M. due to dead load

$$= \frac{576 \times 5^2}{8} \times 100 \text{ kg. cm.} = +259{,}200 \text{ kg. cm.}$$

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B.M. due to eccentricity of the prestressing force at the end
$= +100{,}000 \times 5$ *kg. cm.* $= +500{,}000$ *kg. cm.*
Total B.M. $= +1659200$ *kg. cm.*

Extreme stress $= \frac{P}{A} \pm \frac{M}{Z}$

$= \frac{100{,}000}{2400} \pm \frac{1659200}{24000}$ *kg./cm*^{2}

$= 41{\cdot}7 \pm 69{\cdot}1$ *kg./cm.*^{2}

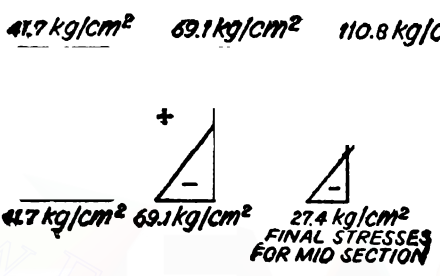

Fig. 805

$\therefore$ Stress at top $= +41{\cdot}7 + 69{\cdot}1$
$= +110{\cdot}8$ *kg./cm.*^{2} (*compressive*)

Stress at bottom $= +41{\cdot}7 - 69{\cdot}1$
$= -27{\cdot}4$ *kg./cm.*^{2} (*tensile*)

Design 185. *Fig. 806 shows a prestressed concrete beam provided with bent tendon. The beam carries a point load of 16 t at the centre. Determine the stress distribution for the end section and the midsection. The beam weighs 576 kg. per m*

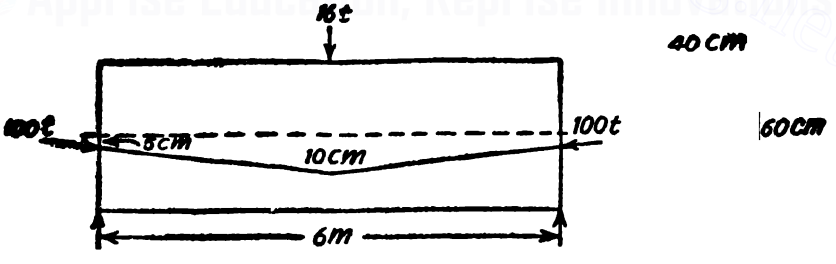

Fig. 806

Solution. Area of beam section
$= A = 40 \times 60 = 2400$ *cm.*^{2}

Section modulus $= Z = \frac{40 \times 60^2}{6} = 24000$ *cm.*^{3}

Let θ be the inclination of the tendon with the horizontal

$\tan \theta = \sin \theta = \frac{5}{300} = \frac{1}{60}$

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Analysis of the end section

Extreme stress $= \frac{P}{A} \mp \frac{Pe}{Z}$

$= \frac{100{,}000}{2400} \mp \frac{100{,}000 \times 5}{24000}$ kg./cm.²

$= 41{\cdot}7 \mp 20{\cdot}8$ kg./cm.²

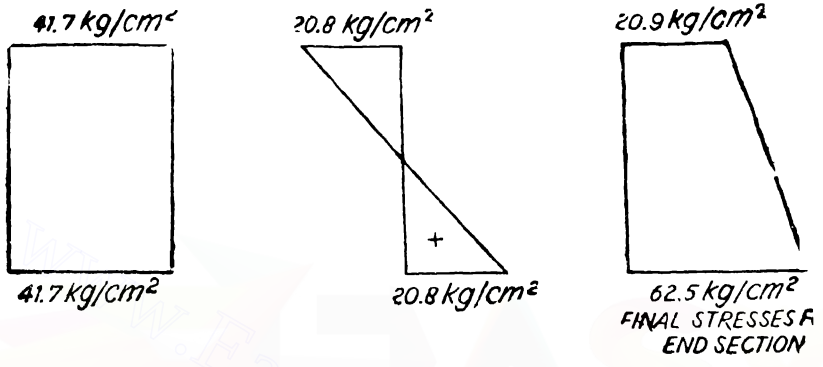

Fig. 807

∴ Stress at bottom $= +41{\cdot}7 + 20{\cdot}8 = +62{\cdot}5$ kg./cm.²

Stress at top $= +41{\cdot}7 - 20{\cdot}8 = +20{\cdot}9$ kg./cm.²

Analysis of the mid section

External downward point load at centre $= 16t \downarrow$

Upward point load by tendon $= 2P \sin\theta$

$= 2 \times 100 \times \frac{1}{60} = \frac{10}{3}\ t \uparrow$

Net downward point load $= 16 - \frac{10}{3} = \frac{38}{3}\ t \downarrow$

B.M. due to net downward point load

$= + \frac{38}{3} \times 1000 \times \frac{6}{4} \times 100$ kg. cm.

$= +1{,}900{,}000$ kg. cm.

B.M. due to dead load $= + \frac{576 \times 6^2}{8} \times 100$ kg. cm.

$= +259{,}200$ kg. cm.

B.M. due to eccentricity of the prestressing force at the end

$= -100 \times 5 \times 1000 = -500{,}000$ kg. cm.

∴ Total B.M. $= +1659200$ kg. cm.

∴ Extreme Stress $= \frac{P}{A} \pm \frac{M}{Z}$

$= \frac{100{,}00}{2400} \pm \frac{1659200}{24000}$ kg./cm.²

$= +41{\cdot}7 \pm 69{\cdot}1$ kg./cm.²

$= 110{\cdot}8$ kg./cm.² at top and

$-27{\cdot}4$ kg./cm.² at bottom

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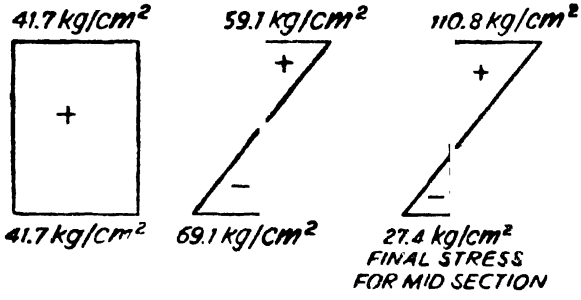

Fig. 808

§205. Tendon with Parabolic Profile

Fig. 809 (*i*) shows a restressed beam carrying an external uni-

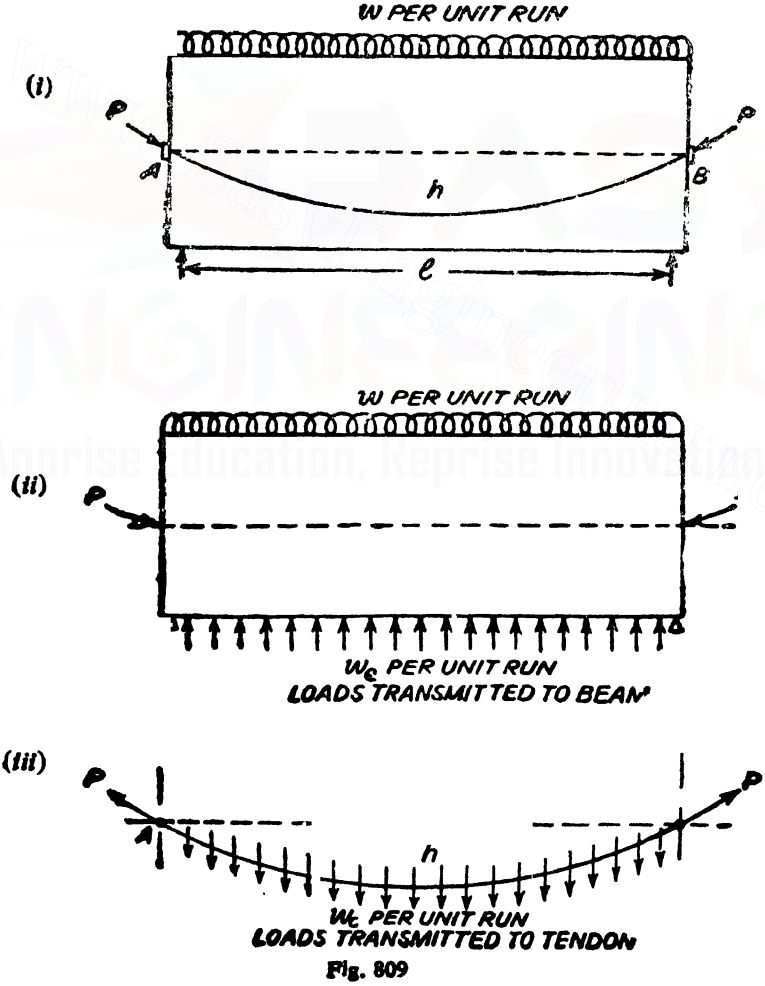

Fig. 809

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formly distributed load w per unit run. The tendon is provided with a parabolic profile. When the cable is provided with a parabolic profile, it will exert uniform upward pressure w_c per unit run on the beam and will therefore receive a downward uniformly distributed load w_c per unit run.

We have studied in the theory of cables carrying uniformly distributed load, on the whole span.

Horizontal reaction at each end of the cable

$$= P_o = \frac{w_c l^2}{8h}$$

where h is the dip of the cable centre below the ends of the cable.

Making the approximation $P_o = P$

$$\text{we get } P = \frac{w_c l^2}{8h}$$

$$\frac{8h P}{l^2}$$

Hence a parabolic tendon carrying a tension P will provide an upward uniform load or pressure $w_c = \frac{8h P}{l^2}$ per unit length of the beam.

Hence the upward uniform pressure supplied by the parabolic tendon will counteract a part of the external downward uniformly applied load.

The net downward loading on the beam is therefore equal to $(w - w_c)$ per unit run.

Design 186. *Fig. 810 shows a prestressed concrete beam provided with a tendon having a parabolic profile. If the total external loading on the beam is 3500 kg./metre on the whole span, calculate the extreme stresses for the mid span section. The tendon carries a prestressing force of 100t.*

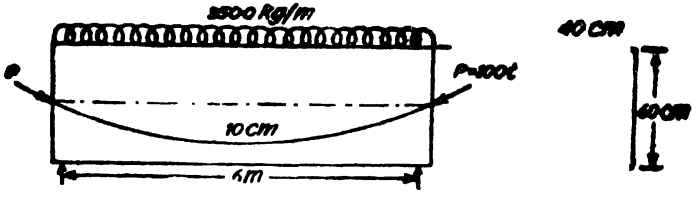

Fig 810

Solution. Area of beam section A

$= 40 \times 60 - 2400\ cm.^2$; section modulus

$= Z = \frac{40 \times 60^2}{6}$

$= 24000\ cm.^3$

Span of tendon $= l = 6\ m.$

Dip of tendon $= h = 0{\cdot}10\ m.$

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Upward uniformly distributed pressure provided by the cable

$$= w_c = \frac{8hP}{l^2} = \frac{8 \times 0{\cdot}10 \times 100 \times 1000}{6 \times 6} \text{ kg./m.}$$

$$= 2222{\cdot}2 \text{ kg.m.}$$

$\therefore$ Net downward load on the beam $= 3500 - 2222{\cdot}2$

$$= 1277{\cdot}8 \text{ kg./m.}$$

$\therefore$ Maximum B.M. $= M = \frac{1277{\cdot}8 \times 6^2}{8} \times 100$ kg. cm.

$$= 575{,}010 \text{ kg. cm.}$$

Extreme stresses for mid section

$$= \frac{P}{A} \pm \frac{M}{Z}$$

$$= \frac{100{,}000}{2400} \pm \frac{575010}{24000} \text{ kg./cm.}^2$$

$$= 41{\cdot}67 \pm 23{\cdot}96 \text{ kg./cm.}^2$$

$\therefore$ Stress at top $= 41{\cdot}67 + 23{\cdot}96 = 65{\cdot}63$ kg./cm.^{2}

and stress at bottom $= 41{\cdot}67 - 23{\cdot}96 = 17{\cdot}71$ kg./cm.^{2}

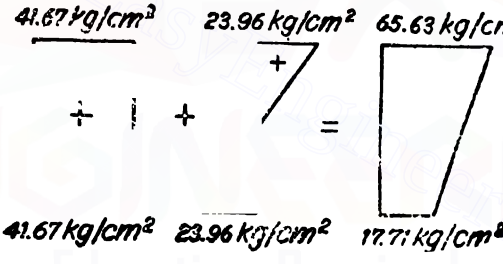

Fig. 811

§206. Load Balancing Method

The examples given above explain an important point that the cable profile can be adjusted so that the cable may exert upward forces counteracting to some extent the downward external loading. If a beam is so designed that the upward forces transmitted by the cable exactly neutralize the external applied loading the method of design is called load balancing method

Design 187. *Determine the profile of a load balancing cable for a beam of span 6 metres carrying on all inclusive load of 4000 kg/m. The prestressing force in the tendon is 120 tonnes. The beam section is 40 cm. × 60 cm.*

Solution. Let h be the dip of the cable.

Upward pressure provided by the parabolic cable

$$= \frac{8Ph}{l^2}$$

$$= \frac{8 \times 120{,}000\, h}{6 \times 6} \text{ kg./m.} = \frac{80{,}000\, h}{3} \text{ kg./m.}$$

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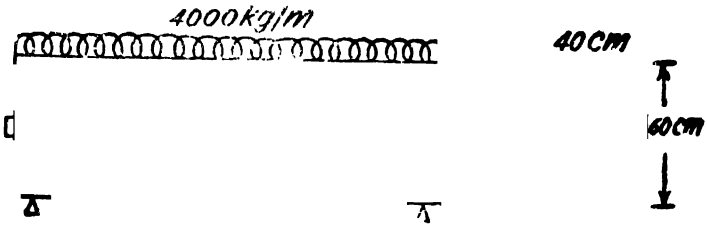

Fig. 812

In order this upward pressure may fully balance the external loading,

$$\frac{80,000}{3} h = 4000$$

$$\therefore \quad h = \frac{4000 \times 3}{80,000} = \frac{3}{20} = 0{\cdot}15 \ m.$$

Hence the cables should be provided with a central dip of 15 *cm*. The uniform stress on the section

$$= \frac{P}{A} = \frac{120,000}{40 \times 60} = 50 \ kg./cm.^2$$

Design 188. *If the prestressing force in the tendon is P for the beam shown in Fig. 813, find the central dip h required to fully balance a concentrated load W applied at mid span.*

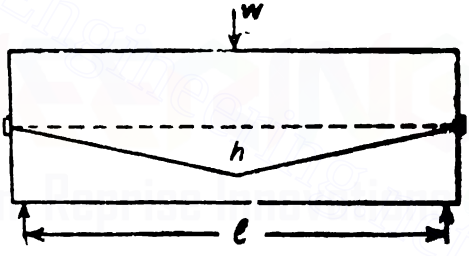

Fig. 813

Solution. Let θ be the inclination of the tendon with the horizontal.

$\therefore$ Upward force provided by the tendon at the centre

$$= 2P \sin \theta = 2P \tan \theta$$

$$= 2P \cdot \frac{h}{\left(\frac{l}{2}\right)} = \frac{4Ph}{l}$$

If this force should balance the applied downward load,

$$\therefore \quad \frac{4Ph}{l} = W$$

$$\therefore \quad h = \frac{Wl}{4p}$$

Design 189. *Find the cable dip h for the beam shown in Fig. 814 so that the applied external loads may be balanced. The tension in the cable is P.*

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Solution. Let θ be the inclination of the end parts of the cable with the horizontal.

$$\therefore \quad \tan\theta \quad \frac{h}{l} = \frac{3h}{l} = \sin\theta \text{ approximately}$$

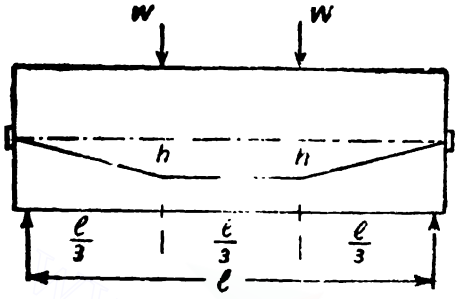

Fig. 814

Upward force provided at each one-third span point by the cable

$$= P \sin\theta$$

In order the external load may be balanced,

$$P \sin\theta = W$$

$$\therefore \quad P\frac{3h}{l} = W$$

$$\therefore \quad h = \frac{Wl}{3P}$$

Design 190. *A rectangular concrete beam 10 cm. wide by 25 cm. deep spanning over 8 metres is prestressed by a straight cable carrying an effective prestressing force of 25 tonnes, located at an eccentricity of 4 cm. If the beam supports a live load of 120 kg./m. (i) Calculate the resultant stresses at the central cross-section of the beam. Concrete weighs 2400 kg/metre³. (ii) Find the magnitude of the prestressing force with an eccentricity of 4 cm. which can balance the stress due to dead and live loads at the bottom fibre of the section at the centre of the span.*

Solution. $A = 10 \times 25 = 250\ cm^2$, $Z = \frac{10 \times 25^2}{6} = 1041{\cdot}67\ cm.^3$

Dead load of the beam $= w_d = 0{\cdot}10 \times 0{\cdot}25 \times 2400 = 60\ kg./m.$

Live load on the beam $= w_l = 120\ kg/m.$

(i) *When the prestressing force is 25 tonnes*

$$\text{Direct stress} = \frac{P}{A} = +\frac{25000}{250} = +100\ kg./cm.^2$$

Extreme stress due to eccentricity of the prestressing force

$$= \mp\frac{Pe}{Z} = \mp\frac{25000 \times 4}{1041{\cdot}67} = \mp 96\ kg.\ cm^2$$

B.M. due to dead and live loads

$$= M = (60+120)\frac{8^2}{8} \times 100\ kg/cm.$$

$$= 144000\ kg./cm.$$

Extreme stress due to bending moment

$$= \pm\frac{M}{Z} = \pm\frac{144000}{1041{\cdot}67} = \pm 138{\cdot}24\ kg./cm.^2$$

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Final stress at top $= +100 - 96 + 138{\cdot}24 = +124{\cdot}24$ *kg./cm.*^{2}

Final stress at bottom $= +100 + 96 - 138{\cdot}24 = +57{\cdot}76$ *kg /cm.*^{2}

(ii) Prestressing force required when the final stress at bottom is zero

Let P be the prestressing force required.

For the condition of zero stress at the bottom,

$$\frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} = 0$$

$$250 + \frac{P \times 4}{1041{\cdot}67} - 138{\cdot}24 = 0$$

Solving we get $P = 17632{\cdot}68$ *kg.*

Design 191. *A concrete beam of rectangular section 20 cm. wide by 60 cm. deep is prestressed by a parabolic cable located at an eccentricity of 10 cm. at mid span and zero at the supports. If the beam has a span of 10 metres and carries a uniformly distributed live load of 400 kg./m., find the effective force necessary in the cable for zero shea stress in the beam. For this condition, calculate the stress at mid section. Take weight of concrete equal to 2400 kg./metre.*^{3}

Solution. Sectional area $= A = 0{\cdot}20 \times 0{\cdot}60 = 0{\cdot}12$ *metre*^{2}

Dead load of the beam $= w_d = 0{\cdot}12 \times 2400 = 288$ *kg./m.*

Live load on the beam $= w_l = 400$ *kg./m.*

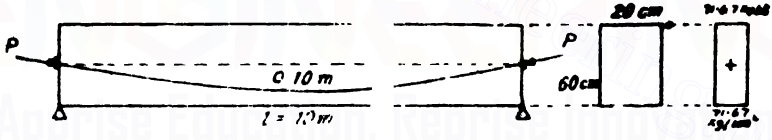

Fig. 815

Total loading on the beam $= w = w_d + w_l = 288 + 400 = 688$ *kg /m.*

Let w_c be the upward loading transmitted by the cable.

For the condition of zero shear $w = w_c$

Let P be the cable force.

$$\therefore \quad P = \frac{w_c l^2}{8h} = \frac{668 \times 10 \times 10}{8 \times 0{\cdot}1} = 86000 \text{ kg.}$$

Stress across the section is uniform.

$$\text{Uniform stress} = \frac{P}{A} = \frac{86000}{20 \times 60} = 71{\cdot}67 \text{ kg./cm.}^2$$

Design 192. *A prestressed concrete rectangular beam 20 cm. × 60 cm. is prestressed with a force of 156500 kg. applied at 18 cm. from the bottom, the force finally reducing to 136100 kg. The span of the beam = 12·20 metres and carries two equal live loads 4500 kg.*

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each at a distance of 4·60 metre from each support. Find the extreme fibre stresses at mid span under (i) initial prestress and no live load, and (ii) final condition. Assume specific weight of concrete = 2400 kg./metre³.

Solution.

$A = 30 \times 60 = 1800\ cm.^2, Z = \frac{30 \times 60^2}{6} = 18000\ cm.^3$

D.L. of the beam $= 0{\cdot}30 \times 0{\cdot}60 \times 2400 = 432\ kg./m.$

D.L. moment $= M_d = \frac{432 \times 12{\cdot}2^2}{8} \times 100 = 803736\ kg.\ cm.$

Eccentricity $= e = 30 - 18 = 12\ cm.$

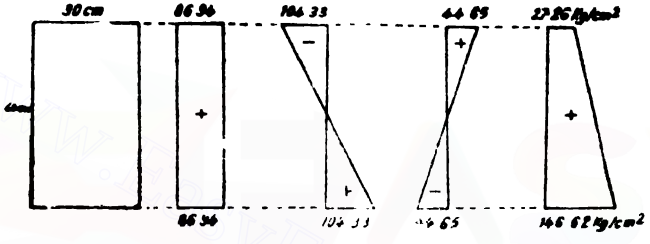

Fig. 816

Case (i). When the beam is subjected to initial prestressing force and dead load.

Direct stress $= + \frac{P_o}{A} = + \frac{156500}{1800} = +86{\cdot}94\ kg./cm.^2$

Extreme stress due to eccentricity of the prestressing force

$= \mp \frac{P_o e}{Z} = \mp \frac{156500 \times 12}{18000}$

$= \mp 104{\cdot}33\ kg./cm.^2$

Extreme stress due to dead load moment

$= \mp \frac{803736}{18000} = \pm 44{\cdot}65\ kg./cm.^2$

Final stres at top $= +86{\cdot}94 - 104{\cdot}33 + 44{\cdot}65$

$= +27{\cdot}26\ kg./cm.^2$

Final stress at bottom $= 86{\cdot}94 + 104{\cdot}33 - 44{\cdot}65$

$= +146{\cdot}62\ kg./cm.^2$

Case (ii). When the beam is subjected to final prestressing force dead load and live load.

L.L. moment $= 4500 \times 4{\cdot}6 \times 100 = 2070000\ kg.\ cm.$

Direct stress $= + \frac{P}{A} = + \frac{136100}{1800} = +75{\cdot}61\ kg./cm.^2$

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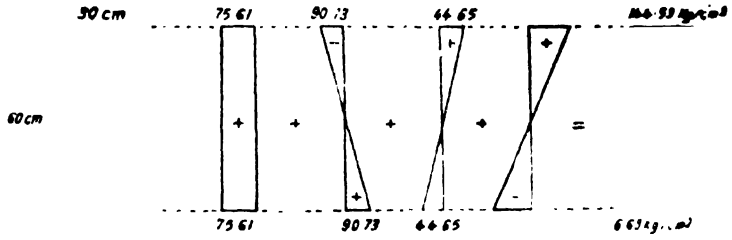

Fig. 817

Extreme stress due to eccentricity of the prestressing force

$$= \mp \frac{Pe}{Z} = \mp \frac{136100 \times 12}{18000} = \mp 90{\cdot}73 \text{ kg./cm.}^2$$

Extreme stress due to D.L. Moment $= \pm 41{\cdot}65$ kg /cm.^{2}

Extreme stress due to L L. Moment

$$= \pm \frac{2070000}{18000} = \pm 115 \text{ kg./cm.}^2$$

Final stress at top $= +75{\cdot}61 - 90{\cdot}73 + 44{\cdot}65 + 115$

$= +144{\cdot}53$ k ./cm.^{2}

Final stress at bottom $= +75{\cdot}61 + 90{\cdot}73 - 44{\cdot}65 - 115$

$= +6{\cdot}69$ kg./cm.^{2}

Design 193 *A pretensioned beam of rectangular section, 8 cm. wide by 12 cm deep is to be designed to support concentrated loads of 400 kg. each at one-third span points over an effective span of 3 metres The permissible stresses in concrete are limited to zero and 14 kg./cm^2 tension at transfer and working load respectively If 3 mm. diameter wires initially stressed to 140 kg /mm^2 are used, find the number of wires required and the eccentricity of the prestressing force assuming 20 percent loss in prestress Weight of concrete = 2400 kg./ metre^3*

Solution.

$A = 8 \times 12 = 96$ cm.^{2}, $Z = \frac{8 \times 12^2}{6} = 192$ cm.^{3}

Dead load of the beam $= 0{\cdot}08 \times 0{\cdot}12 \times 240 = 23{\cdot}04$ kg./m.

Case 1. Analysis at transfer

Let P_o be the prestressing force at transfer. Let the eccentricity be e cm.

Dead load bending moment

$$= M_d = \frac{wl^2}{8} = \frac{23{\cdot}04 \times 3^2}{8} \times 100 = 2592 \text{ kg. cm.}$$

Direct stress $+\frac{P_o}{A} = +\frac{P_o}{96}$ kg./cm.^{2}

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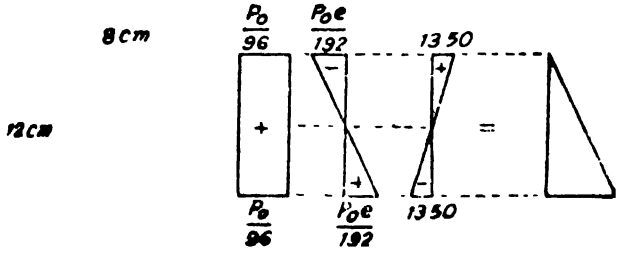

Fig. 818

Extreme stress due to eccentricity of the prestressing force

$$= \mp \frac{P_o e}{Z} = \mp \frac{P_o e}{192} \text{ kg./cm.}^2$$

Extreme stress due to dead load moment

$$= \pm \frac{M_d}{Z} = \pm \frac{2592}{192} \text{ kg./cm.}^2 = \pm 13{\cdot}50 \text{ kg./cm.}^2$$

For the condition of zero stress at the top edge

$$\frac{P_o}{96} - \frac{P_o e}{192} + 13{\cdot}15 = 0$$

$$\therefore \quad \frac{P_o}{96} - \frac{P_o e}{192} = -13{\cdot}50 \qquad \ldots(i)$$

Case 2. Analysis at working conditions

Live load bending moment $= M_l = 400 \times 1 \times 100 = 40{,}000$ *kg. cm.*

Final prestressing force $= P = 0{\cdot}8\ P_o$

Direct stress $= + \frac{P}{A} = + \frac{0{\cdot}8 P_o}{A}$

Extreme stress due to eccentricity of the prestressing force

$$= \mp \frac{Pe}{Z} = \mp \frac{0{\cdot}8 P_o e}{192} \text{ kg./cm.}^2$$

Extreme stress due to dead load bending moment

$$= \pm 13{\cdot}50 \text{ kg./cm.}^2$$

Extreme stress due to live load bending moment

$$= \pm \frac{M_l}{Z} = \pm \frac{40{,}000}{192} = \pm 208{\cdot}33 \text{ kg./cm.}^2$$

For the condition that the final stress at the bottom edge is 14 *kg /cm.*^{2} (*tensile*),

$$\frac{0{\cdot}8 P_o}{96} + \frac{0{\cdot}8 P_o e}{192} - 13{\cdot}50 - 208{\cdot}33 = -14$$

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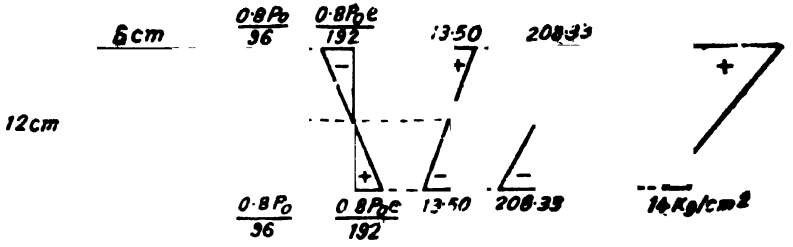

Fig. 819

$$\therefore \quad \frac{0{\cdot}8P_0}{96} + \frac{0{\cdot}8P_0e}{192} = 20{\cdot}83$$

$$\therefore \quad \frac{P_0}{96} + \frac{P_0e}{192} = 259{\cdot}79 \qquad \ldots(ii)$$

But, $$\frac{P_0}{96} - \frac{P_0e}{192} = -13{\cdot}50 \qquad \ldots(i)$$

Adding equations (*i*) and (*ii*)

$$\frac{P_0}{48} = 246{\cdot}29 \qquad P_0 = 11821{\cdot}9 \text{ kg.}$$

Substituting in equation (*i*)

$$\frac{11821{\cdot}9}{96} - \frac{11821{\cdot}9e}{192} = -13{\cdot}50$$

$$\frac{11821{\cdot}9}{192}e = 136{\cdot}64$$

$$\therefore \quad e = \frac{136{\cdot}64 \times 192}{11821{\cdot}9} = 2{\cdot}22 \text{ cm.}$$

Area of tendons $$= A_s = \frac{11821{\cdot}9}{14000} = 0{\cdot}844 \text{ cm.}^2$$

Area of one wire of 3 *mm.* diameter $= 0{\cdot}071$ *cm.*^{2}

∴ Number of 3 *mm* diameter wires required

$$= \frac{0{\cdot}844}{0{\cdot}071} = 12 \text{ wires.}$$

Design 194. *A post-tensioned beam 20 cm. × 30 cm. is pre stressed with a tendon 520 mm.*^{2} *area stretched to a stress of 10500 kg /cm.*^{2} *The tendon passes through a hole 5 cm wide and 7·5 cm. deep, left in the beam, the centre of the hole being 7·5 cm. from the the bottom. Loss of prestress at the time of anchoring is 5%. Find the stresses in concrete immediately after prestressing.*

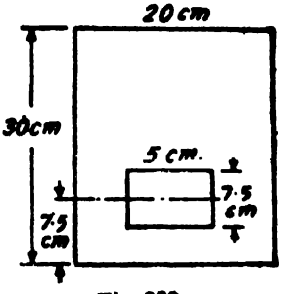

Fig. 820

Solution. Area of the beam section.

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$$= A = (20 \times 30) - (5 \times 7{\cdot}5)$$
$$= 562{\cdot}5 \ cm.^2$$

Depth of centroidal axis from the upper edge

$$= y = \frac{(20 \times 30 \times 15) - 5 \times 7{\cdot}5 \times 22{\cdot}5}{562{\cdot}5}$$
$$= 14{\cdot}5 \ cm.$$

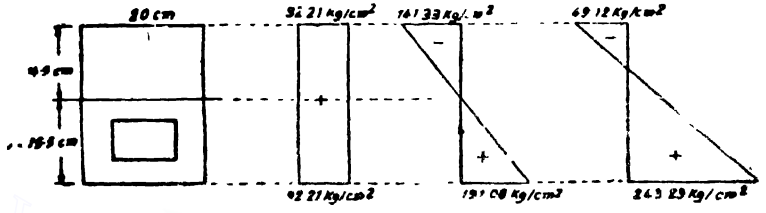

Fig. 821

Moment of inertia about the centroidal axis

$$= \frac{20 \times 14{\cdot}5^3}{3} + \frac{20 \times 15{\cdot}5^2}{3} - \left[\frac{5 \times 7{\cdot}5^3}{12} + 5 \times 7{\cdot}5 \times 8^2\right]$$
$$= 42574 \ cm.^4$$

Prestress in tendons after 5% anchorage loss

$$= 0{\cdot}95 \times 10500 = 9975 \ kg/cm.^2$$

Net prestressing force

$$P = 5{\cdot}20 \times 9975 = 51870 \ kg.$$

Direct stress

$$= \frac{P}{A} = \frac{51870}{562{\cdot}5} = +92{\cdot}21 \ kg./cm.^2$$

Extreme stress at top due to eccentricity of P

$$= -\frac{Pe}{I} y_t = -\frac{51870 \times 8}{42574} \times 14{\cdot}5$$
$$= -141{\cdot}33 \ kg./cm.^2$$

Extreme stress at bottom due to eccentricity of P

$$= +\frac{Pe}{I} y_b = +\frac{51870 \times 8}{42574} \times 15{\cdot}5$$
$$= +151{\cdot}08 \ kg./cm.^2$$

Resultant stress in the extreme top fibres

$$= +92{\cdot}21 - 141{\cdot}33$$
$$= -49{\cdot}12 \ kg./cm.^2 \ (tensile)$$

Resultant stress in the extreme bottom fibres

$$= +92{\cdot}21 + 151{\cdot}08$$
$$= +243{\cdot}29 \ kg./cm.^2 \ (compressive)$$

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Design 195. *A beam of symmetrical I-section spanning 8 metres has a flange width of 20 cm. and a flange thickness of 6 cm. respectively. The overall depth of the beam = 40 cm. Thickness of the web = 8 cm. The beam is prestressed by a parabolic cable with an eccentricity of 15 cm. at the centre and zero at the supports with an effective force of 10 t. The live load on the beam is 200 kg./m. Draw the stress distribution diagram at the mid span section for the following condition :*

(i) Prestress + self-weight

(ii) Prestress + self-weight + live load

Take weight of concrete = 2400 kg / metre.^{3}

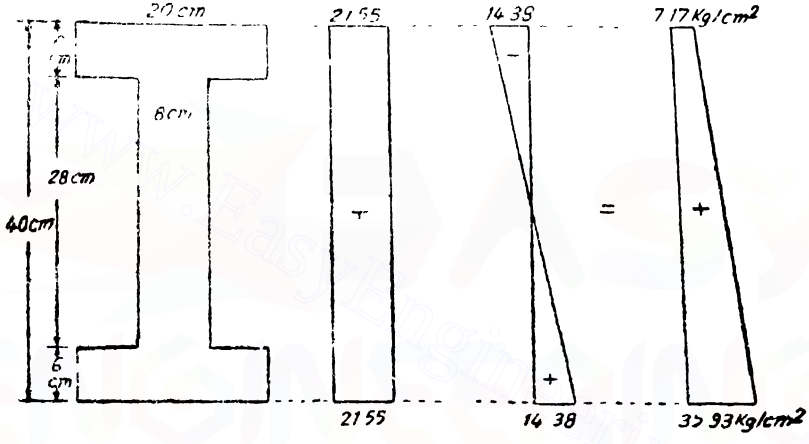

Fig. 822

Solution. $A = (2 \times 20 \times 6) + (8 \times 28) = 464\ cm^2$;

$$I = \frac{20 \times 40^3}{12} - \frac{12 \times 28^3}{12} = 84714{\cdot}66\ cm^4$$

Section modulus $= Z = \dfrac{84714{\cdot}66}{20} = 4235{\cdot}73\ cm.^3$

Dead load of the beam

$$= \frac{464}{100 \times 100} \times 2400 = 111{\cdot}36\ kg./m.$$

Upward uniformly distributed load transmitted by the cable

$$= w_o = \frac{8Pe}{l^2} = \frac{8 \times 10{,}000 \times 0{\cdot}15}{8 \times 8}$$

$$= 187{\cdot}5\ kg./m.$$

Case (i). Beam subjected to prestress and self-weight

Direct stress $= + \dfrac{P}{A} = + \dfrac{10{,}000}{464} = +21{\cdot}55\ kg./cm.^2$

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Net upward uniformly distributed loading
$= 187{\cdot}5 - 111{\cdot}36 = 76{\cdot}14$ *kg /m.*

Net bending moment at centre

$$= -\frac{76{\cdot}14 \times 8^2}{8} \times 100$$

$= 60912$ *kg. cm.* (*hogging*)

Extreme stress due to net bending moment.

$$= \mp \frac{60912}{4235{\cdot}73} = \mp 14{\cdot}38 \text{ kg./cm.}^2$$

Final stress at top $= 21{\cdot}55 - 14{\cdot}38 = +7{\cdot}17$ *kg./cm.*^{2}

Final stress at bottom $= 21{\cdot}55 + 14{\cdot}38 = 35{\cdot}93$ *kg /cm.*^{2}

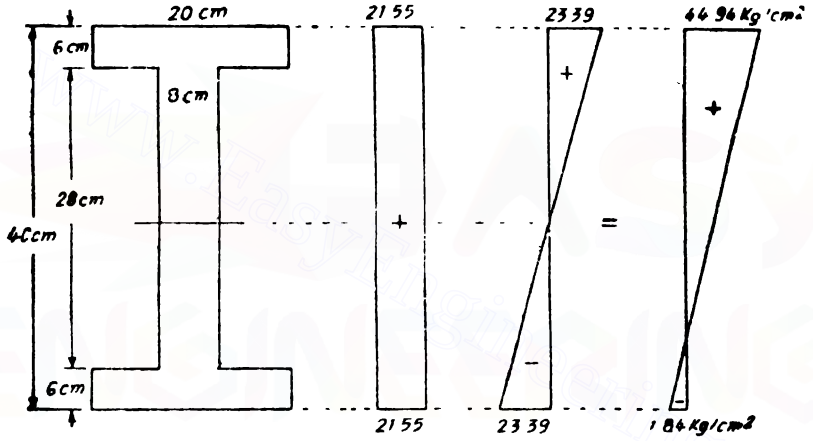

Fig. 823

Case (*ii*). *Beam subjected to prestress, self-weight and live load*

At this stage the net downward uniformly distributed loading
= D.L. + L.L. − upward *udl* transmitted by cable
$= 111{\cdot}36 + 200 - 187{\cdot}5 = 123{\cdot}86$ *kg /m.*

B.M. at centre $= \frac{123{\cdot}86 \times 8^2}{8} \times 100 = 99088$ *kg. cm.*

(*sagging moment*)

Direct stress $= +21{\cdot}55$ *kg./cm.*^{2}

Extreme stress due to B.M.

$$= \pm \frac{99088}{4235{\cdot}73} = \pm 23{\cdot}39 \text{ kg./cm.}^2$$

Final stress at top $= 21{\cdot}55 + 23{\cdot}39 = +44{\cdot}94$ *kg./cm.*^{2}

Final stress at bottom
$= +21{\cdot}55 - 23{\cdot}39 = -1{\cdot}84$ *kg./cm.*^{2}

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Design 196. *A prestressed concrete I beam supports a live load of 400 kg./m over a simply supported span of 8 metres. The beam has an overall depth of 40 cm. The thicknesses of each flange and web are 6 cm and 8 cm. respectively. The width of each flange = 20 cm. The beam is to be prestressed by an effective prestressing force of 23·5 tonnes applied at a suitable eccentricity such that the resultant stress at the bottom of beam at centre of span is zero.*

(i) Find the eccentricity required for the force

(ii) If the tendon is concentric, what should be the magnitude of the prestressing force for the resultant stress to be zero at the bottom fibre of the central section ?

Solution. $A = (2 \times 20 \times 6) + (8 \times 28) = 464\ cm.^2$

Moment of inertia of the section about the neutral axis

$$= I = \frac{20 \times 40^3}{12} - \frac{12 \times 28^3}{12} = 84714{\cdot}66\ cm^4$$

$\therefore$ Section modulus,

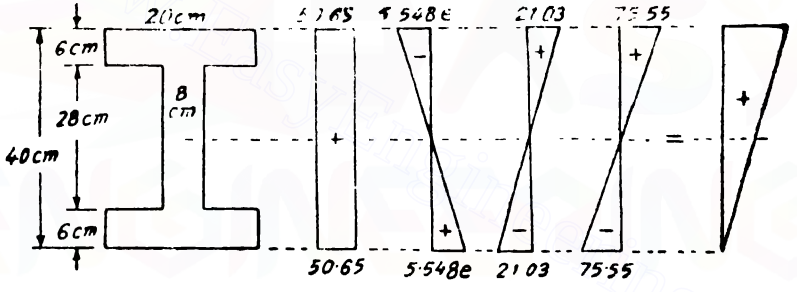

Fig. 824

$$Z = \frac{84714{\cdot}66}{20} = 4235{\cdot}73\ cm.^3$$

Dead load of the beam

$$= \frac{464}{100 \times 100} \times 2400 = 111{\cdot}36\ kg./m.$$

Dead load bending moment

$$= \frac{111{\cdot}36 \times 8^2}{8} \times 100\ kg.\ cm. = 89088\ kg.\ cm.$$

Live load bending moment

$$= \frac{400 \times 8^2}{8} \times 100\ kg\ \ cm. = 320000\ kg.\ cm.$$

Case 1. When the tendon is placed eccentrically.

Let the eccentricity of the prestressing force be e *cm.*

Direct stress $= + \frac{P}{A} = + \frac{23500}{464} = +50{\cdot}65\ kg./cm.^2$

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Extreme stress due to eccentricity of the prestressing force

$$= \mp \frac{Pe}{Z} = \mp \frac{23500\ e}{4235\cdot73} = \mp 5\cdot548\ e\ kg./cm.^2$$

Extreme stress due to dead load bending moment

$$= \pm \frac{M_d}{Z} = \pm \frac{89\cdot188}{4235\cdot73} = \pm 21\cdot03\ kg./cm.^2$$

Extreme stress due to live load bending moment

$$= \pm \frac{M_l}{Z} = \pm \frac{320000}{4235\cdot73} = \pm 75\cdot55\ kg./cm.^2$$

For the condition of zero stress at the bottom edge,

$$50\cdot65 + 5\cdot548\ e - 21\cdot03 - 75\cdot55 = 0$$

$$\therefore \quad e = 6\cdot58\ cm.$$

Case 2. When the tendon is placed concentrically

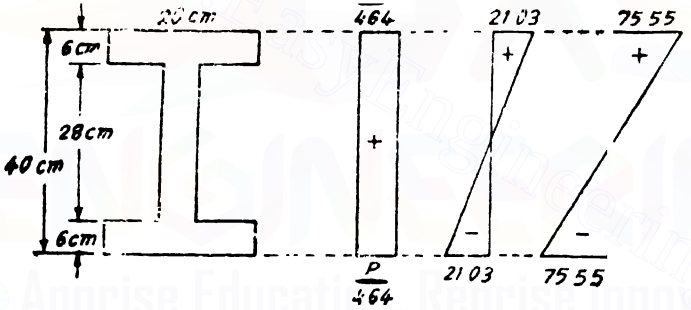

Fig. 825

Let the prestressing force applied concentrically be P kg.

Direct stress $= + \frac{P}{464}\ kg./cm.^2$

Extreme stress due to dead load bending moment

$$= \pm 21\cdot03\ kg./cm.^2$$

Extreme stress due to live load bending moment

$$= \pm 75\cdot55\ kg./cm.^2$$

For the condition of zero stress at the bottom edge,

$$\frac{P}{464} - 21\cdot03 - 75\cdot5 = 0$$

$$\therefore \quad \frac{P}{464} = 96\cdot53$$

$$\therefore \quad P = 44790\ kg.$$

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Design. 197. *A prestressed concrete Tee beam is to be designed to support a superimposed load of 440 kg./metre over a span of 5 metres. The Tee beam is made up of a flange 40 cm. wide and 4 cm. thick. The rib is 10 cm. wide by 20 cm. deep. The stress in the concrete must not exceed 150 kg./cm.^{2} in compression and zero in tension at any stage. Check for adequacy of the section provided and calculate the minimum prestressing force necessary and the corresponding eccentricity. Assume 20 percent loss of prestress after transfer.*

Solution.

Area of the section $= A = (40 \times 4) + (20 \times 10) = 360$ *cm.*^{2}

Depth of centroidal axis from the top edge

$$= \frac{(160 \times 2) + (200 \times 14)}{360} = 8{\cdot}67 \text{ cm.}$$

Moment of inertia about the centroidal axis

$$= I = \frac{40 \times 4^3}{12} + 40 \times 4 \times 6{\cdot}67^2 + \frac{10 \times 4{\cdot}67^3}{3} + \frac{10 \times 15{\cdot}33^3}{3}$$

$$= 19680 \text{ cm.}^4$$

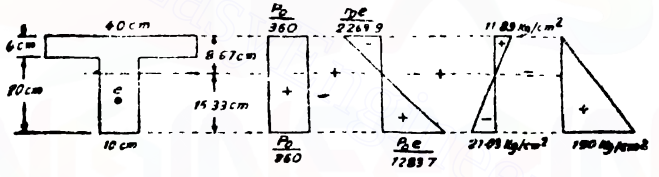

Fig. 826

$$Z_t = \frac{19680}{8{\cdot}67} = 2269{\cdot}9 \text{ cm.}^3; \quad Z_b = \frac{19680}{15{\cdot}33} = 1283{\cdot}7 \text{ cm.}^3$$

Dead load of the beam

$$= w_d = \frac{360}{100 \times 100} \times 2400 = 86{\cdot}4 \text{ kg./m.}$$

Live load on the beam

$$= w_l = 440 \text{ kg./m.}$$

Let P_o = Initial prestressing force
e = eccentricity.

Case 1. When the beam is subjected to the initial prestressing force and dead load.

We will determine P_o and e for the condition that the maximum compressive stress at the bottom edge is 150 *kg./cm.*^{2} and the stress at the top edge is zero.

Dead load bending moment

$$= M_d = \frac{86{\cdot}4 \times 5^2}{8} \times 100 \text{ kg. cm.} = 27000 \text{ kg. cm.}$$

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Direct stress $= + \frac{P_o}{A} = + \frac{P_o}{360}$ *kg /cm*^{2}

Extreme stress at top due to eccentricity of the prestressing force

$$= - \frac{P_o e}{Z_t} = - \frac{P_o e}{2269{\cdot}9} \text{ kg./cm.}^2$$

Fxterme stress at bottom due to eccentricity of the prestressing force $= + \frac{P_o e}{Z_b} = + \frac{P_o e}{1283{\cdot}7}$ *kg /cm.*^{2}

Extreme stress at top due to D.L. moment

$$= + \frac{M_d}{Z_t} = + \frac{27000}{2269{\cdot}9} = 11{\cdot}89 \text{ kg./cm.}^2$$

Extreme stress at bottom due to D.L. moment

$$= - \frac{M_d}{Z_b} = - \frac{27000}{1283{\cdot}7} = -21{\cdot}03 \text{ kg./cm.}^2$$

For the condition of zero stress at top,

$$\frac{P_o}{360} - \frac{P_o e}{2269{\cdot}9} + 11{\cdot}89 = 0 \qquad \ldots(i)$$

For the condition that the stress in the bottem to reach 150 *kg./cm.*^{2},

$$\frac{P_o}{360} + \frac{P_o e}{1283{\cdot}7} - 21{\cdot}03 = 150$$

$$\therefore \quad \frac{P}{360} + \frac{P_o e}{1283{\cdot}7} = 171{\cdot}03 \qquad \ldots(ii)$$

But $$\frac{P}{360} - \frac{P_o e}{2269{\cdot}9} = -11{\cdot}89 \qquad \ldots(i)$$

From equations (*i*) and (*ii*), we get

$P_o = 19508{\cdot}4$ *kg.* and $e = 7{\cdot}69$ *cm.*

Case 2. ***When the beam is subjected to the final prestressing force, dead load and live load.***

Taking $P_o = 19508{\cdot}4$ *kg.* and $e = 7{\cdot}69$ *cm.* we will investigate for the safety of the design for this case.

Final prestressing force

$$= P = 0{\cdot}8\, P_o = 0{\cdot}8 \times 19508{\cdot}4 = 15606{\cdot}72 \text{ kg.}$$

Live load bending moment

$$= M_l = \frac{440 \times 5^2}{8} \times 100 = 137500 \text{ kg. cm.}$$

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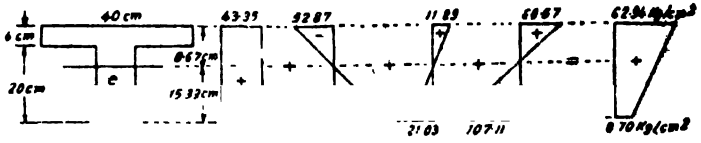

Fig. 827

Direct stress

$$= + \frac{P}{A} = + \frac{15606{\cdot}72}{360} = + 43{\cdot}35 \ kg./cm.^2$$

Extreme stress at top due to eccentricity of the prestressing force

$$\frac{Pe}{Z_t} = \frac{15606{\cdot}72 \times 7{\cdot}69}{2269{\cdot}9} = - 52{\cdot}87 \ kg./cm.^2$$

Extreme stress at bottom due to eccentricity of the prestressing force

$$= + \frac{Pe}{Z_t} = + \frac{15606{\cdot}72 \times 7{\cdot}69}{1253{\cdot}7} = + 93{\cdot}49 \ kg./cm.^2$$

Extreme stress at top due to dead load moment

$$= 11{\cdot}89 \ kg./cm.^2$$

Extreme stress at bottom due to dead load moment

$$= - 21{\cdot}03 \ kg./cm.^2$$

Extreme stress at top due to live load moment

$$= + \frac{137500}{2269{\cdot}9} = + 60{\cdot}57 \ kg./cm.^2$$

Extreme stress at bottom due to live load moment

$$= - \frac{137500}{1283{\cdot}7} = - 107{\cdot}11 \ kg./cm.^2$$

Final stress at top

$$= + 43{\cdot}35 - 52{\cdot}87 + 11{\cdot}89 + 60{\cdot}57 + = + 62{\cdot}94 \ kg./cm.^2$$

Final stress at bottom

$$= + 43{\cdot}35 + 93{\cdot}49 - 21{\cdot}03 - 107{\cdot}11 = + 8{\cdot}70 \ kg./cm.^2$$

Design 198. *A prestressed concrete I-beam has its upper flange 75 cm. wide and 20 cm. deep, lower flange 40 cm. wide and 30 cm. deep and a web of depth 50 cm. and width 15 cm. It is supported over a span of 30 metres and carries a uniformly load of 400 kg./m , exclusive of self-weight. It is prestressed with 120 wires of 5 mm. diameter, with their centroid 10 cm. above the bottom edge and initially tensioned to 10,000 kg./cm². Assuming 15 percent losses in prestress, determine the extreme fibre stresses at mid span. at various stages.*

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Solution.

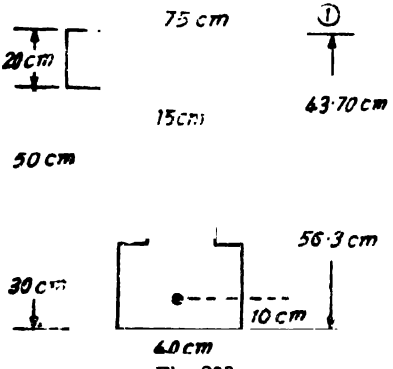

Fig. 828

Properties of section. See table below.

Component	*Area a* $cm.^2$	*Centroidal distance y from 1—1* *cm*	*ay* $cm.^3$	ay^2 $cm.^4$	I_{self} $cm.^4$
Top Flange	1500	10	15000	150000	$\frac{75\times 20^3}{12}=50,000$
Web	750	65	33750	1518750	$\frac{15\times 50^3}{12}=155,250$
Bottom Flange	1200	85	102000	8670000	$\frac{40\times 30^3}{12}=90,000$
Total	3450		150750	10338750	296250

$$\bar{y}=\frac{\Sigma ay}{A}=\frac{150750}{3450}=43\cdot70\ cm.$$

Moment of inertia about the axis 1—1

$$=I_{1-1}=\Sigma I_{self}+\Sigma ay^2$$
$$=296250+10338750=10635000\ cm.^4$$

$$I_{1-1}=I_{aa}+A\bar{y}^2$$

$$\therefore\quad I_{aa}=I_{1-1}-A\bar{y}^2$$
$$=10635000-3450\times 43\cdot70^2=4046570\ cm.^4$$

$$Z_t=\frac{I_{aa}}{y_t}=\frac{4046570}{43\cdot70}=92599\ cm.^3$$

$$Z_b=\frac{I_{aa}}{y_b}=\frac{4046570}{56\cdot30}=71875\ cm.^3$$

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Dead load of the beam $= w_d = \frac{3450}{100 \times 100} \times 2400 = 828$ *kg./m.*

Eccentricity $e = 56{\cdot}30 - 10 = 46{\cdot}30$ *cm.*

The extreme fibre stresses are calculated for the following cases :

Case 1. Beam subjected to initial prestressing force and dead load.

D.L. moment $= \frac{828 \times 30^2}{8} \times 100$ *kg. cm.* $= 9315000$ *kg. cm.*

Initial prestressing force

$$P_o = 120 \times \frac{\pi}{4} (0{\cdot}5)^2\ 10{,}000 = 235620 \text{ kg.}$$

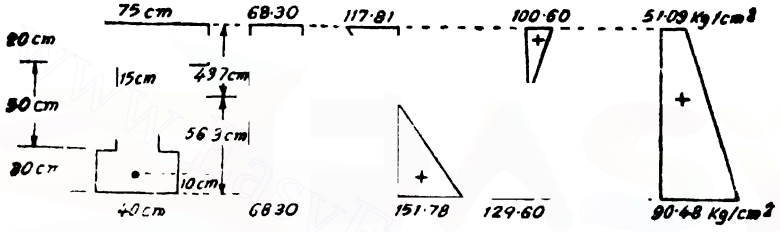

Fig. 829

Direct stress $= + \frac{P_o}{A} = + \frac{235620}{3450} = +68{\cdot}30$ *kg./cm.*²

Extreme stress at top due to eccentricity of the prestressing force

$$- \frac{P_o e}{Z_t} = - \frac{235620 \times 46{\cdot}3}{92599} = -117{\cdot}81 \text{ kg./cm.}^2$$

Extreme stress at bottom due to eccentricity of the prestressing force

$$+ \frac{P_o e}{Z_b} = + \frac{235620 \times 46{\cdot}3}{71875} = +151{\cdot}78 \text{ kg./cm.}^2$$

Extreme stress at top due to D.L. moment

$$= + \frac{M_d}{Z_t} = + \frac{9315000}{92599} = +100{\cdot}60 \text{ kg./cm.}^2$$

Extreme stress at bottom due to D.L. moment

$$= - \frac{M_d}{Z_b} = - \frac{9315000}{71875} = -129{\cdot}60 \text{ kg./cm.}^2$$

Final stress at top $= +68{\cdot}30 - 117{\cdot}81 + 100{\cdot}60 = +51{\cdot}09$ *kg./cm.*²

Final stress at bottom

$$= +68{\cdot}30 + 151{\cdot}78 - 129{\cdot}60 = +90{\cdot}48 \text{ kg./cm.}^2$$

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Case 2. Beam subjected to final prestressing force, dead load and live load.

L.L. moment $= M_l = \frac{400 \times 30^2}{8} \times 100 = 4500000$ *kg. cm.*

Final prestressing force $P = 0{\cdot}85 \times 235620 = 200277$ *kg.*

Direct stress $= + \frac{P}{A} = + \frac{200277}{3450} = + 58{\cdot}05$ *kg./cm.*^{2}

Extreme stress at top due to eccentricity of the prestressing force,

$$= - \frac{Pe}{Z_t} = - \frac{200277 \times 46{\cdot}3}{92599} = - 100{\cdot}14 \text{ kg./cm.}^2$$

Extreme stress at bottom due to eccentricity of the prestressing force,

$$= + \frac{Pe}{Z_b} = + \frac{200270 \times 46{\cdot}3}{71875} = + 129{\cdot}01 \text{ kg.cm.}^2$$

Extreme stress at top due to D.L. moment $= + 100{\cdot}60$ *kg./cm.*^{2}

Extreme stress at bottom due to D L. moment

$= - 129{\cdot}60$ *kg./cm.*^{2}

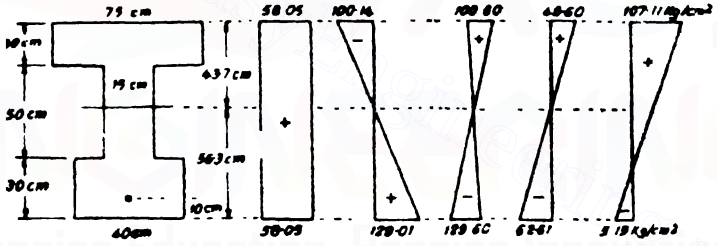

Fig. 830

Extreme stress at top due to L.L. moment

$$= + \frac{M_l}{Z_t} = + \frac{4500000}{92599} = + 48{\cdot}60 \text{ kg./cm.}^2$$

Extreme stress at bottom due to L.L. moment

$$= - \frac{M_l}{Z_b} = - \frac{4500000}{71875} = - 62{\cdot}61 \text{ kg./cm.}^2$$

Final stress at top $= + 58{\cdot}05 - 100{\cdot}14 + 100{\cdot}60 + 48{\cdot}60$

$= + 107{\cdot}11$ *kg./cm.*^{2}

Final stress at bottom $= + 58{\cdot}05 + 129{\cdot}01 - 129{\cdot}60 - 62{\cdot}61$

$= - 5{\cdot}15$ *kg./cm.*^{2}

§207. Systems of Prestressing

Classification of prestressed concrete members. There are many ways of classifying prestressed concrete members based on the method of design, construction and application of prestress. These are explained below.

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(*a*) **Externally and internally prestressed members.** A member can be prestressed either by external reaction offered by rigid abutments or by tensioned tendon. The former method is called external prestressing and the latter method is called internal prestressing.

Fig. 831

In the external prestressing instead of providing a tendon from which the prestress can be applied, the necessary prestressing force can be applied by compressing the members by jacking against abutments. Fig. 831 shows an arrangement in which the prestress is externally applied. This method is difficult to be adopted, though this principle is adopted in the case of arches. It is difficult to rely on this method since the prestressed transferred to the member is likely to be lost due to any possible outward displacements of the abutments. It is also found that shrinkage and creep of concrete are likely to affect the initially applied prestress. It is, however, possible to produce the most desirable reactions by jacking in the case of continuous beams. But still the method is difficult to be practised. Even slight vertical deformations of the supports will disturb the stresses seriously.

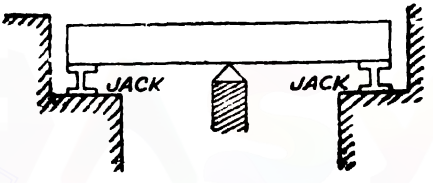

Fig. 832

(*b*) **Linear or circular prestressing.** The term circular prestressing is applied to prestressing circular structures like cylindrical tanks, silos and pipes. In this case the tendons are provided in the form of rings. Linear prestressing is a term applied to prestressing straight members like beams and slabs.

(*c*) **Pre-tensioning and post-tensioning**

In pre-tensioned members, the tendons are tensioned even *before*

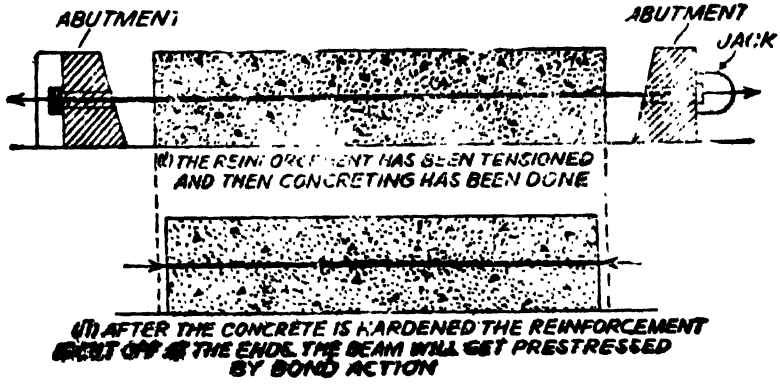

Fig. 833

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casting the concrete. One end of the reinforcement (*i.e.*, tendon) is secured to an abutment while the other end of the reinforcement is pulled by using a jack and this end is then fixed to another abutment. The concrete is now poured. After the concrete has cured and hardened the ends of the reinforcement are released from the abutments. The reinforcement which tends to resume its original length will compress the concrete surrounding it by bond action. The prestress is thus transmitted to concrete entirely by the action of bond between the reinforcement and the surrounding concrete.

A post-tensioned member is one in which the reinforcement is tensioned *after* the concrete has fully hardened. The beam is first cast leaving ducts for placing the tendons. The ducts are made in a number of ways—by leaving corrugated steel tube in the concrete, by providing steel spirals, sheet metal tubing, rubber hose or any other *duct forming unit* in the form work. When the concrete has hardened and developed its strength the tendon is passed through the duct. One end is provided with an anchor and is fixed to one end of the member.

Now the other end of the tendon is pulled by a jack which is butting against the end of the member. The jack simultaneously pulls the tendon and compresses the concrete. After the tendon is subjected to the desired stress, this end of the tendon is also properly anchored to the concrete. To avoid crushing of concrete due to excessive bearing stress, a distribution plate is provided at each end.

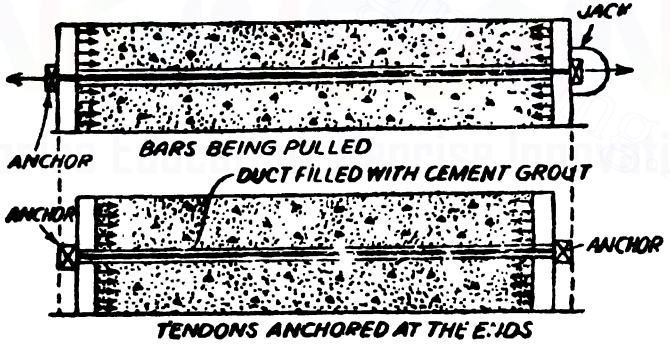

Fig. 834

Systems of prestressing. A system of prestressing means the actual process adopted in making a prestressed beam. A system of prestressing involves the process of tensioning the tendons and securing them firmly to the concrete. Some of the systems commonly followed are described below :

Pre-tensioning system. The simplest method of making a pre-tensioned member is to pull the tendons between abutments or bulkheads which are secured or anchored firmly against the ends of stressing bed. The tendons are cut off at each end after the concrete Now the prestress is transferred to the concrete. The

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bulkhead at the ends and the bed should be designed to resist the prestress and the eccentricity.

§208. **The Hoyer system** is usually adopted for the production of pre-tensioned members in a large scale. This method consists of first stretching the wires between two bulkheads at a large distance apart. The concrete is now poured so that a number of beams can be produced in one line, by providing suitable shuttering between them. After the concrete has hardened the wires are released from the bulkheads. The wires between the different units in one line of beams will also be cut off (Fig. 835). The prestress is transferred to the concrete through bond between the tendons and the concrete.

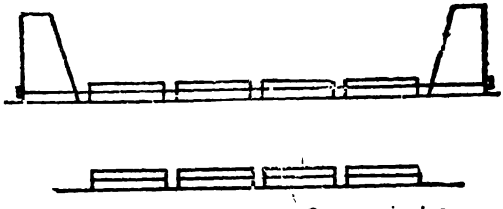

Fig. 835. Hoyer system of pretensioning.

There are similar systems based on the above principle. These systems are put to use for usually pre-cast beams. Pre-tensioning system was widely used in earlier days but has since been replaced by post-tensioning system. Pre-tensioning systems are found uneconomical for large spans.

In order to grip the pre-tensioned wires properly to the bulkheads the devices shown in Fig. 836 are followed.

Post-tensioning systems. The basic principle in all post-tensioning systems is to introduce prestresses in the concrete member cast previously by tightening the tendons accommodated in the ducts which are formed while casting the beam. As the tendons are pulled using a jack against the end of the concrete member, the desired prestressing force is obtained. After the wires are pulled, they are anchored in their stretched position against the end of the concrete member by a suitable wedging device. The various systems followed differ in the arrangement of wires, in the process of jacking and in the anchorage devices.

In order to ensure the correct prestress in the reinforcement it is preferable to measure the load applied by the jack as well as the consequent extension of the reinforcement. Extension measurements give an idea as to how much of the steel is being properly stretched. For instance the sides of the ducts may obstruct the stretching of the reinforcement particularly at the end remote from the jack and this part of the reinforcement may not receive the full tension. This defect

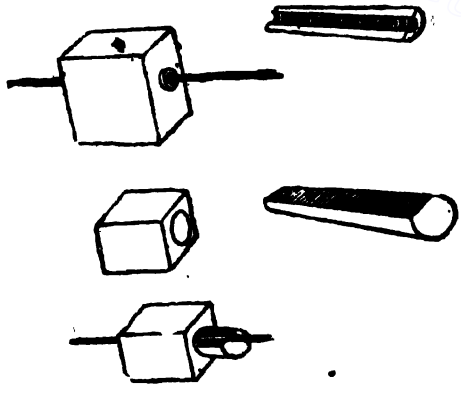

Fig. 836

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is liable to occur when curved cables are provided. On the other hand, it may be possible that the extension might have taken place due to a certain part of the tendon being overstressed For this reason, it is necessary to measure the prestressing force applied.

Excessive bearing stresses will be produced at the ends of the members due to the anchoring devices which bear against the concrete. In this part of the member it will be necessary to provide additional reinforcement to prevent local splitting and failure by shear.

A number of systems are available each protected by patent rights. The systems are adopted for linear prestressing and with some modifications for circular prestressing also. Some of the commonly followed systems of post-tensioning are described below :

§209. The Freyssinet System. The Freyssinet system was the first to be introduced among the post-tensioning systems. High tension steel wires 5 *mm.* to 8 *mm.* diameter about 12 in number are arranged to form a group into cables with a spiral spring inside. The spiral spring provides the means for a proper clearance between the wires and thus provides a channel which can be cement grouted. It further assists to transfer the reaction to concrete. The whole thing is enclosed in a thin metal steel.

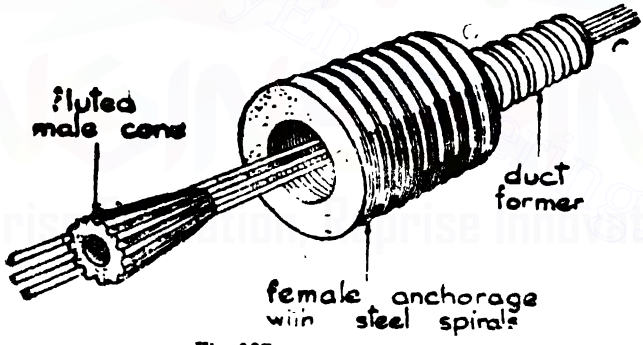

Fig. 837. Freyssinet system.

The anchorage consists of a cylinder of ordinary good quality concrete and is provided with corrugations on the outside. It has a central conical hole and is provided with heavy hoop reinforcement. These cylinders are kept in the proper position and the conical plugs are pushed into the conical holes after the cables are tightened. The central hole passing axially through the plug permits cement grout to be injected through it. In this way the space between the wires will be filled with the grout. This provides additional restraint against the slipping of the tendons.

Advantages of the system

(*i*) Securing the wires is not expensive.

(*ii*) The desired stretching force is obtained quickly.

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(*iii*) The plugs may be left in the concrete and they do not project beyond the ends of the member.

Disadvantages of the system

(*i*) All the wires of a cable are stretched together. Hence the stresses in the wires may not be exactly the same.

(*ii*) The greatest stretching force applied to a cable is from 25 *t* to 50 *t*. This may not be sufficient.

(*iii*) The jacks used are heavy and expensive.

§210. The Magnel Blaton System

In this system a cable of rectangular section is provided, which contains layers of wires 5 *mm*. to 8 *mm* diameter. The wires are arranged with four wires per layer (up to sixteen layers deep for a cable of sixty four wires). The wires in the same layer and the wires in adjacent layers are separated with a clearance of 4 *mm*. The geometric pattern of the wires is maintained in the same form throughout the length of the cable by providing grills or spacers at regular intervals. The grills do not offer any appreciable fricitional resistance to the wires which can be moved relative to each other during the tensioning process The wires are anchored by wedging, two at a time into sandwich plates. The sandwich plates are about 25 *mm*. thick and are provided with two wedge-shaped grooves on its two faces. The wires are taken two in each groove and tightened. Then a steel wedge is driven between the tightened wires to anchor them against the plate. A complete anchorage unit may consist of one to eight sandwich plates, number of wires depending on the number of wires in the cable. Each plate can anchor eight wires. The various sandwich plates forming a unit are arranged one above the other against a distribution plate. The wires are tightened by jacking two wires at a time.

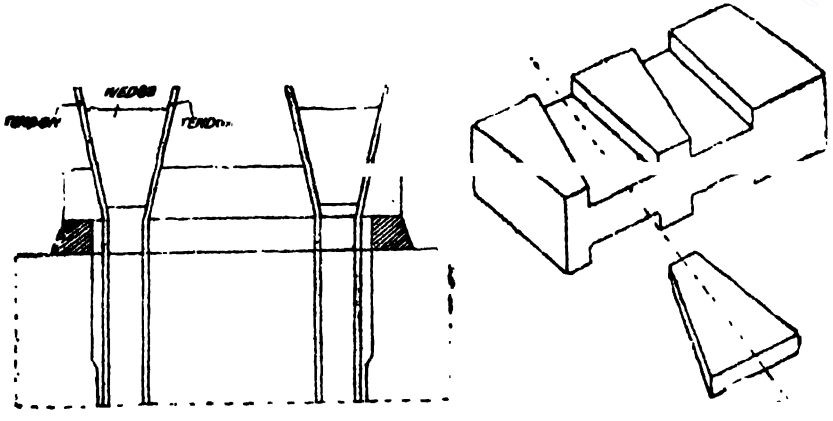

Fig. 838

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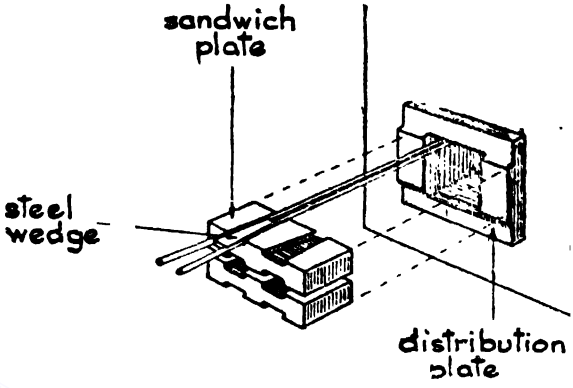

Fig 839. Magnel Blaton System

§211. Gifford Udall System

This method offers the following three methods of pre-stressing :

First method. This is earliest of the three methods of this system.

In this method the wires are stressed and anchored one by one in a separated cylinder using small wedging grips called *udall grips*. Each grip consists of two-half cones. The

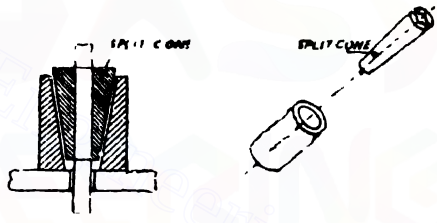

Fig. 840

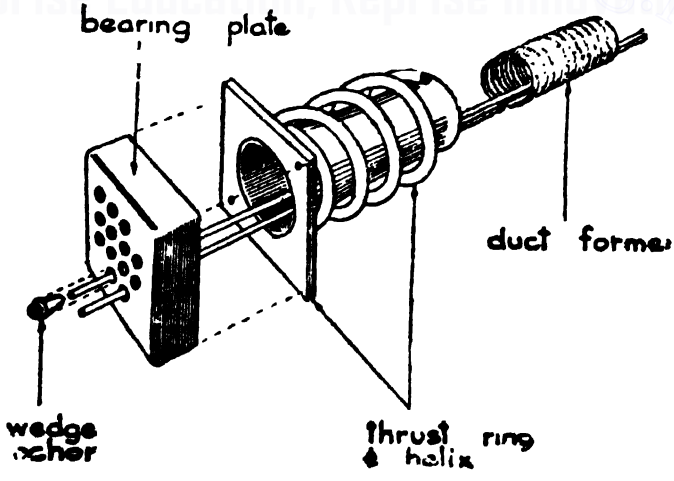

Fig. 841. Gifford-Udal System.

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bearing plate bears against a thrust ring which is cast into the concrete. The duct end is encircled by a helix. Anchorages are supplied to suit cables of 2, 4, 6 and 12 wires.

Second method. In this method, the wires are anchored by wedges which fit directly into tapered recesses made in the bearing plate. The bearing plate bears against a tube unit containing the tube unit and the helix. This tube unit is cast into the concrete. Anchorages are supplied for cables of 8 to 12 wires. This arrangement is compact and minimizes the congestion of the steel wires in the anchor block.

Third method In this method a tendon of 28 *mm.* diameter H.T. type is used This needs a duct of only 40 *mm.* diameter. This tendon is stronger than sixteen 6 *mm.* wires. The anchoring is done by 9 *cm.* diameter grip with a three segment interlocking wedge. This method uses a barrel which bears directly against a tube unit which is cast into concrete

§212 P.S.C. Monowire System

In this system also the wires are tensioned individually. The anchorage consists of a single piece *collet sleeve* wedging in a conical hole A steel truncated guide leads each wire from the cable to the anchorage point along a gentle curvature. In addition to the guide a central block is also provided to anachor the central wires.

§213. C.C.L. Standard System

This is another system where the wires are tensioned individually. As many wires as may be required (not exceeding 12) are used in a circular cable. The size of the duct depends on the number of wires —4 *cm.* diameter for 8 wire-cables and 5 *cm.*

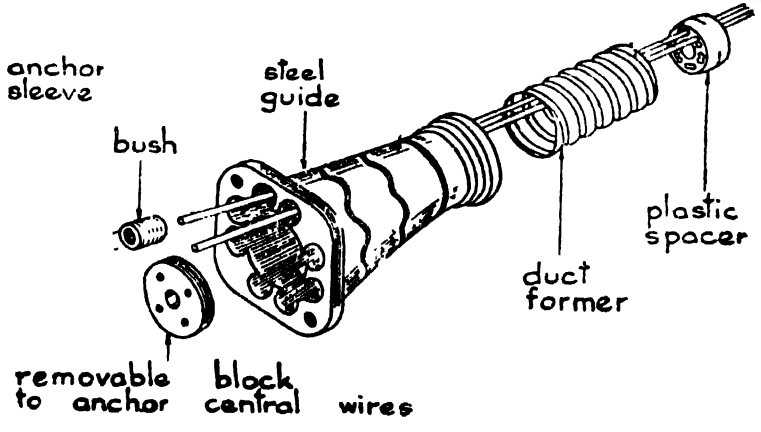

Fig. 842. P.S.C. Monowire System.

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diameter for 12-wire cable. At intervals of 60 *cm.* spacers are provided to keep the wires separated and to prevent the outer wires from touching the sheathing. This not only reduces the friction but also allows satisfactory grouting. A special C.C.L. anchor grip is used to anchor each wire separately. The anchor grip con-

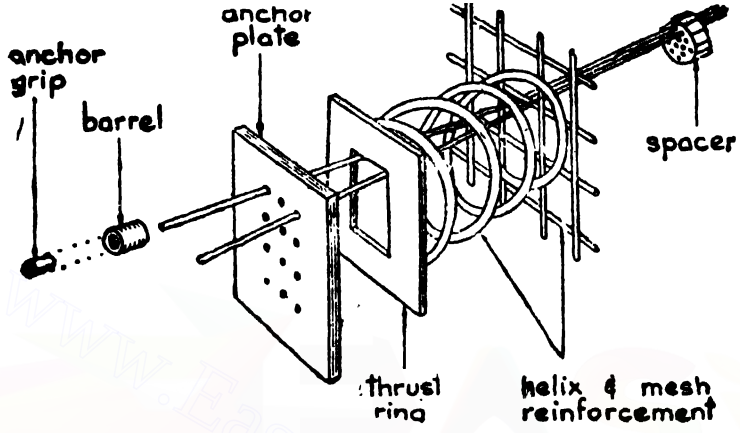
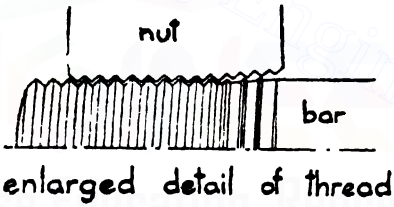


Fig. 843. C.C.L. Standard System.



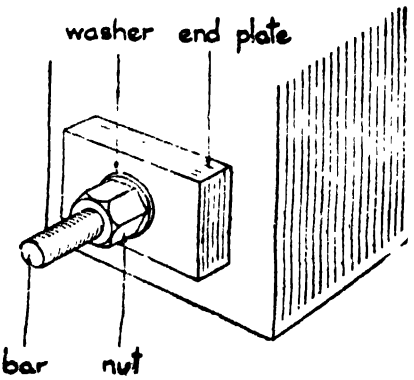


Fig. 844. *Lee-Mccall System.*

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sists of steel wedges which fit over the wire and are accommodated in a steel barrel having a tapered hole.

The barrels are 20 *mm.* diameter and 25 *mm.* long. The wedges are made either in two pieces or 4 segments held together by a steel circlip to form one unit. These units bear against a drilled anchor plate through which the wires are passed. The anchor plate bears against a thrust ring which is cast into the concrete. The thrust ring is of mild steel 6 *mm.* thick and is made rectangular with a central rectangular hole. To prevent local bursting effect a helix is provided behind the thrust ring. A vertical mesh is also provided behind the helix.

The bearing plate is 18 *mm* thick and has 9 *mm.* diameter holes provided at a spacing of 25 *mm.* centres in parallel rows to accommodate the wires.

§214. Lee-Mccall System

This is a system in which high tensile alloy steel bars (silico-manganese steel) are used as the prestressing tendons These bars are provided in 22 *mm*, 25 *mm*, 28 *mm*, and 30 *mm*. diameter and in lengths upto 20 *metres*. The anchoring of the bars is done by screwing special threaded nuts. These nuts bear against a distribution plate provided at the end of the beam.

§215. Loss of Prestress

The steel wires of a prestressed concrete member do not retain all the preliminary prestress. A certain amount of loss of prestress always takes place. A loss of prestress will affect the stress distribution on the section of the member. It is therefore necessary to estimate the probable loss of prestress that may be incurred in a prestressed member.

Losses of prestress take place in a prestressed concrete member due to many causes. In general these losses may be classified into the following :

(*i*) Loss of prestress during the tensioning process

(*ii*) Loss of prestress at the anchoring stage

(*iii*) Losses occurring subsequently.

1. Loss of prestress during the tensioning process due to friction There always exists a certain amount of friction in the jacking and anchoring system and on the walls of the duct where the wires fan out at the anchorages, with the result, the actual stress in the tension is less than what is indicated by the pressure gauge. The losses due to friction in the jack and at the anchorages are different for different systems of prestressing. The proprietors of these systems have recommended suitable loss coefficient which may be adopted.

Considerable frictional loss takes place due to friction between the tendon and the material surrounding it, namely the concrete or

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the sheathing. This loss due to friction may be classified into (*i*) loss due to length effect and (*ii*) loss due to curvature effect.

(*i*) *Loss due to length effect*

The length effect means the extent of friction met with in a straight tendon due to slight imperfections of the duct. In practice the duct meant for the straight tendon is not absolutely straight. Hence the cable will touch the duct or concrete. This loss is also called the loss due to *wobbling effect, or wave effect.*

The prestressing force P_x in the tendon at any distance x from the jack is given by

$$P_x = P_o\, e^{-kx} \qquad ...(i)$$

where P_o = prestressing force at the jacking end and k = a coefficient called *wobble correction factor.*

The following table gives the values of k for the different systems :

Type of duct	*Freyssinet system*		*Magnel system*		*Lee-McCall system*	
	μ	*k/metre*	μ	*k/metre*	μ	*k/metre*
(*i*) Ducts made by withdrawing steel rods.	0·55	0	0·30	0·0033	0·55	0·0016
(*ii*) Unstiffened rubber core.	0 55	0 00 66	0·30	0·0016	0·55	0·0033
(*iii*) Stiffened rubber core.	0 55	0 00 16	0·30	0 0016	0 55	0·0016
(*iv*) Metal sheathing	0 35	0 00 33	0·30	0·0016	0·30	0·0016

(*ii*) *Loss due to curvature effect*

In the case of curved ducts, the loss of prestress depends upon the radius of curvature R of the duct and the coefficient of friction between the duct surface and the tendon.

The tension at any point of the cable is give by

$$P_x = P_o\, e^{-\frac{\mu x}{R}} \qquad ...(ii)$$

It is usual to combine the length effect and the curvature effect and the tension in the tendon at any point is given by

$$P_x = P_o\, e^{-\left(kx + \frac{\mu x}{R}\right)} \qquad ...(iii)$$

But $e^{-\left(kx+\frac{\mu x}{R}\right)} = 1 - \left(kx + \frac{\mu x}{R}\right) + \left(kx + \frac{\mu x}{R}\right)^2$

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Ignoring higher powers of $\left(kx+\frac{\mu x}{R}\right)$, we get

$$P_x=P_0\left[1-kx-\frac{\mu x}{R}\right] \qquad ...(iv)$$

The quantity $\left[1-kx-\frac{\mu x}{R}\right]$ is called the *tension ratio*.

I.S. recommendations. As per the I.S. code, for straight or moderately curved structure with curved or straight cables, the value of the prestressing force P_x at a distance x from the tensioning end shall be calculated by the formula,

$$P_x=P_0\, e^{-(\mu\alpha+k_f x)} \qquad ...(v)$$

where, P_0 = prestressing force in the prestressed steel at the tensioning end,

μ = coefficient of friction in curve,

α = the cumulative angle in radians through which the tangent to the cable profile has turned between any two points under consideration,

and k_f = a friction coefficient for wave effect.

For small values of $(\mu\alpha+k_f x)$ the formula given above can be expressed as,

$$P_x=P[1-\mu\alpha-k_f x] \qquad ...(vi)$$

The friction coefficient k_f for wave effect is normally taken as 0·0015 per *metre*. However, where heavy vibrations, thin walled ducts and other adverse conditions are involved, this figure may be increased up to 0·015 *per metre*, and where the clearance between the duct and the cable is sufficiently large to eliminate the wave effect and the construction of the sheath is of heavy gauge steel tube with water tight joints, and such other arrangements which prevent deformation of the duct profile during vibration of concrete, the coefficient may be reduced to zero.

The values of P_x and P_0 are those acting in the direction of the tangent to the curve of the cable.

Unless otherwise proved by tests, the values of the coefficient of friction μ shall be assumed as :

0·55 for steel moving on smooth concrete,

0·35 for steel moving on steel fixed to duct,

0·25 for steel moving on steel fixed to concrete,

0·25 for steel moving on lead,

0·18 to 0·30 for multilayer wire rope cable in rigid rectangular steel sheath, and

0·15 to 0·25 for multilayer wire rope cables with spacer plates providing lateral separation.

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An expression for the loss of prestress can be obtained from equation (*vi*)

$$P_x = P_o(1-\mu\alpha-k_f x)$$

$$\therefore \quad \frac{P_o-P_x}{P_o} = (\mu\alpha+k_f x) \qquad \therefore \quad \frac{\left[\frac{P_o-P_x}{A}\right]}{\left[\frac{P_o}{A}\right]} = (\mu\alpha+k_f x)$$

$$\therefore \quad \textit{Loss of stress} = (\mu\alpha+k_f x) \times \textit{Initial stress} \qquad \ldots(vii)$$

In circular or similar constructions, where circumferential steel is tensioned by jacks, losses due to friction may be calculated by raducing the above values of μ by 20 percent

If the initial tension applied to a prestressing tendon is partially released, the action of friction in the reverse direction causes an alteration in the distribution of stress along the length of the tendon. In such cases, the effect of the reverse friction shall be taken into consideration.

2. **Loss of prestress at the anchoring stage.** This loss is due to the fact that the anchorage fixtures themselves are subjected to a stretch. It is also possible that the friction wedges holding the wires may slip a little. The amount of this slip may be taken on the average as 2·50 *mm*. But in the case of direct bearing anchorages the slip may be on the average only 0·75 *mm*.

The loss of prestress due to deformation, slip, *etc.* is given by

$$\Delta f_s = \frac{\Delta_s E_s}{L}$$

where Δ_s = effective slip

E_s = Young's modulus for the tendon

L = length of the tendon.

Loss due to slip in anchorage is of special importance with short members and the necessary additional elongation may be provided for at the time of tensioning to compensate for this loss.

3. **Loss of prestress occurring subsequently.** The losses which occur subsequent to prestressing are the following :

(*i*) Loss of stress due to shrinkage of concrete,
(*ii*) Loss of stress due to creep of concrete,
(*iii*) Loss of stress due to elastic shortening of concrete,
(*iv*) Loss of stress due to creep of steel (stress relaxation).

The first three types of losses mentioned above take place due to the reduction in the length of concrete resulting in the reduction of the initial extension of the steel.

The loss of stress due to elastic shortening of concrete is maximum in pretensioned members. In the case of post tensioned

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members these losses occur only when a number of cables are progressively stressed one after another.

(*i*) *Loss of stress due to shrinkage of concrete.*

Shrinkage means a contraction of concrete due to chemical changes and drying. This depends only on the interval of time and the moisture conditions, but is independent of the stresses in the member due to loads. The value of the shrinkage strain lies from 0 to 0·001. Mainly, the shrinkage of concrete is proportional to the quantity of water used in mixing the concrete. By minimizing the water cement ratio and the proportion of cement the shrinkage can be reduced. This may be achieved by using well graded, larger sized aggregates.

For all purposes of design the shrinkage strain may be taken as 0·0002 to 0·0004, for the normal mixes adopted in prestressed concrete construction. Loss of stress due to shrinkage of concrete = Shrinkage strain $\times E_s$

where, E_s = Young's modulus for steel

Suppose the shrinkage strain is 0·0003, corresponding to this strain the loss of prestress in the tendons would be $0{\cdot}0003 \times 2{\cdot}1 \times 10^6$ *kg./cm.*^{2}, = 630 *kg./cm.*^{2}. If the initial prestress in the tendons be 10500 *kg./cm.*^{2}, the percentage loss of prestress due to shrinkage

$$= \frac{630}{10500} \times 100\% = 6\%.$$

I.S. recommendations. As per the I.S. code the loss of prestress due to shrinkage of concrete shall be the product of the modulus of elasticity of steel E_s and the shrinkage strain of concrete calculated as follows :

For pre-tensioning, total residual shrinkage strain $= 3 \times 10^{-4}$

For post-tensioning,

$$\text{Residual shrinkage strain} = \frac{2 \times 10^{-4}}{\log_{10}(T+2)}$$

where, T is the age of concrete at transfer in days.

These values may be increased by 50 percent in dry atmospheric conditions, subject to a maximum value for total shrinkage strain of 3×10^{-4}.

(*ii*) *Loss of stress due to creep of concrete.*

Creep of concrete means the deformation of concrete which depends upon the interval of time to which the member is loaded. For instance, suppose a concrete member is subjected to an external load which causes a deformation. If the member remains loaded for a long time it is seen that the deformation increases though by a small amount. This additional deformation of a stressed member which depends on the interval for which the member is remaining in a stressed state, is called creep. Let e_i be the instantaneous strain of the member immediately after the member is subjected to a load

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and let e_t be the eventual strain after a considerable period of constant stress.

The ratio $C_c = \frac{e_t}{e_i}$ is called the *coefficient of creep*. The value of this coefficient varies considerably as studied from different tests. The creep strain goes on increasing for a period of nearly two years after which there is no more appreciable additional strain. The coefficient C_c is taken from 2·5 to 3 The lower value may be taken in the case of post-tensioned members due to late application of prestress. But in the case of pretensioned members, which receive the prestress early, the coefficient of creep is found to be equal to 3 and in a few cases, slightly greater than 3.

The loss of prestress due to creep

$$= (C_c - 1)\, m f_c$$

where C_c = creep coefficient

f_c = original prestress in concrete

and m = modular ratio.

For instance, if the initial prestress in concrete is 80 *kg./cm.*^{2} and $C_c = 2{\cdot}5$, the loss of prestress in the steel due to creep equals $(2{\cdot}5-1)\, 6 \times 80 = 720$ *kg./cm.*^{2}, the percentage loss of prestress would be $\frac{720}{10{\cdot}00} \times 100\%$ or 6·9%.

I.S. recommendations. As per the I S code, the loss of prestress due to creep of concrete under a load shall be determined for all the permanently applied loads including the prestress.

The creep loss due to live load stresses, erection stresses and other stresses of short duration may be ignored.

The loss of prestress due to creep of concrete is obtained as the product of the modulus of elasticity of the prestressing steel (E_s) and the integrated ultimate creep strain of the concrete along the line of centre of gravity of the prestressing steel over its entire length.

The total creep strain during any specific period shall be assumed, for all practical purposes, to be the creep strain due to a sustained stress equal to the average of the stresses at the beginning and end of the period.

The ultimate creep strain for a sustained unit stress shall be calculated as

$$1{\cdot}75 \left(\frac{1{\cdot}25 - h}{0{\cdot}25}\right)\left(\sqrt{\frac{400}{F_c}}\right)\left(\frac{F_c}{F_r}\right) \times 10^{-6} \text{ cm./cm. per kg./cm.}^2$$

where, h = the average ambient humidity of the environment, being taken generally as 1·0 for water, 0·9 to 1·0 for misty atmosphere, 0·75 to 0·90 for humid atmosphere, 0·75 to

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0·50 for average humidity and 0·50 to 0·20 for dry conditions

Note : The average ambient humidity of the environment assumed in the design shall be maintained for a minimum period of 60 days after transfer either by natural conditions or by artificial means.

(*iii*) *Loss of prestress due to elastic shortening of concrete.*

(*a*) *Pretensioned member.* Due to the prestress transferred to the concrete, the concrete will shorten. This results in a corresponding shortening of the steel. The prestress in the steel is automatically reduced as it is allowed to shorten slightly.

Let the intensity of the compressive stress be f_c

Let the Young's modulus for concrete be E_c

Strain in concrete $= e_c = \dfrac{f_c}{E_c}$

If A_c is the area of concrete and P_o be the prestressing force

We have, $f_c = \dfrac{P_o}{A_c}$ and $e_c = \dfrac{P_o}{A_c E_c}$

Since the steel will also shorten as the concrete shortens, the loss of strain in steel = strain in concrete $= \dfrac{P_o}{A_c E_c}$

$\therefore$ Loss of prestress

$$= \Delta f_s = \frac{P_o}{A_c E_c} E_s$$

where E_s = Young's modulus for steel

$\therefore$ $\Delta f_s = m \dfrac{P_o}{A_c}$ where, $m = \dfrac{E_s}{E_c}$ = modular ratio.

In the above expression P_o the prestress prevailing finally may not be known But for a sufficiently correct estimation of the loss of prestress, it is enough if the initially applied prestress P_o be adopted.

(*b*) *Post-tensioned member.* Suppose only a single tendon had been provided in a member, the concrete gets shortened as the tendon is jacked against it. Hence after tightening, no more shortening of concrete can take place In fact the actual tension in the tendon is recorded after the elastic shortening of concrete has already taken place. The tendon therefore undergoes no loss of prestress.

Suppose more than one tendon be provided in a member. The various tendons provided are not simultaneously tensioned. They will only be tensioned in succession. For instance let the first tendon be tightened A certain amount of elastic shortening takes place and the tension measured is only after the elastic shortening of concrete has taken place. If now the second tendon be tightened, a further elastic shortening of concrete takes place and the tension recorded for the second tendon, no doubt, is after this

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shortening of concrete has taken place. But the elastic shortening of concrete due to the tightening of the second tendon will result in a loss of prestress in the first tendon. Similarly as the third tendon is tightened a further elastic shortening of concrete takes place and the first two tendons will undergo loss of prestress. Hence as the tendons are tightened in succession, only the tendon which is tightened last will not undergo any loss of prestress, but all other tendons will undergo loss of prestress. The tendon which is tightened first will undergo the maximum loss of prestress. For a given number of tendons, the actual loss of prestress in each of these tendons caused by the elastic shortening of concrete can be computed. The percentage loss of prestress may therefore be computed. Alternatively the average loss of prestress may be calculated. This is one-half the loss of prestress experienced by the first tendon. The percentage loss of prestress may also be based on the average loss of prestress. If it is desired to make good these losses, then the individual tendons may be subjected to greater initial tension, so that, as the last tendon is tightened, the tensions left in the tendons may have a desired value.

(iv) *Loss of prestress due to creep in steel (relaxation of stress)*

Steel also possesses the property of creep. The percentage of creep varies from 1% to 5%. It may be normally taken that this percentage is 3%.

I.S. recommendations. The I.S. code states that the magnitude of the loss of prestress due to relaxation of steel is taken to vary 2 to 8 percent of the average initial stress depending on the quality of the steel and the order of initial prestress.

Temporary overstressing for the purpose of reducing the loss of

Percentage of loss of prestress

Cause of loss of prestress	*Percentage loss of prestress*	
	Post-tensioned member	*Pretensioned member*
Shrinkage of concrete	6	7
Creep of concrete	5	6
Creep in steel	3	2
Elastic shortening and bending of concrete	1	3
Total	15	18

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prestress due to relaxation of steel may be permitted provided such overstressing does not exceed 85 percent of the ultimate tensile strength, F_s, of the prestress steel. For values of reduction in the loss of prestress, manufacturer's recommendations shall be followed.

Total loss of prestress. The total loss of prestress depends on many factors such as properties of concrete and steel, moisture and curing condition the magnitude and system of prestress. The table given above shows the percentage loss of prestress for post-tensioned and pretensioned members.

Design 199. *A post-tensioned beam 40 cm.×60 cm., and 10 metres long is provided with straight tendons which are tensioned to 10500 kg./cm.² at the jacking end. Find the loss of prestress due to wobbling effect (i) at mid span and (ii) at the end remote from the jack. Take k=0·30 per 100 m.*

Solution

$$P_x = P_o(1-kx)$$

$$\therefore \quad f_x = f_o(1-kx) = f_o - f_o kx$$

$$\therefore \quad \text{Loss of stress} = (f_o - f_x) = f_o kx$$

$$k = 0{\cdot}3 \text{ per } 100 \text{ } m = \frac{0{\cdot}3}{100} \text{ per } m = 0{\cdot}003 \text{ per metre}$$

(i) *Loss of stress at mid span*

$$= f_o kx$$
$$= 10500 \times 0{\cdot}003 \times 5 \text{ kg./cm.}^2$$
$$= 157{\cdot}5 \text{ kg./cm.}^2$$

$$\text{percentage loss} = \frac{157{\cdot}5}{10500} \times 100\% = 1{\cdot}5\%.$$

(ii) *Loss of stress at the end remote from the jack*

$$= f_o kx$$
$$= 10500 \times 0{\cdot}003 \times 10$$
$$= 315 \text{ kg./cm.}^2$$

$$\text{percentage loss} = \frac{315}{10500} \times 100\% = 3\%.$$

Design 200 *A post-tensioned cable of a beam 10 metres long is initially tensioned to a stress of 10,000 kg./cm.² at one end. If the tendons are curved so that the slope is 1 in 24 at each end, with a cross-sectional area of 6 cm.², calculate the loss of prestress due to friction given the following data :*

Coefficient of friction between duct and cable = 0·3.

Friction coefficient for wave effect = 0·0015/metre.

Solution.

Let θ be the inclination of the cable with the horizontal at each end.

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$\tan\theta = \frac{1}{24}$ $\quad\therefore\quad \theta = \frac{1}{24}$ radian (since θ is small.)

$\therefore$ Total angle of turn of cable

$$= \alpha = 2\ \theta = \frac{2}{24} = \frac{1}{12} \text{ radian.}$$

$$P_x = P_o[1 - \mu x - k_f x]$$

$$\therefore \quad [P_o - P_x] = (\mu\alpha + k_f x)\, P_o$$

$$\therefore \quad \frac{(P_o - P_x)}{A_t} = (\mu\alpha + k_f x)\frac{P_o}{A_t}$$

$\therefore$ *Loss of stress* = $(\mu\alpha + k_f x) \times$ stress in steel at the tensioning end.

$$= [0{\cdot}3 \times \frac{1}{12} + 0{\cdot}0015 \times 10] \times 10{,}000 \text{ kg./cm.}^2$$

$$= 400 \text{ kg./cm.}^2$$

Design 201. *A simply supported post-tensioned concrete beam of span 15 m. has a rectangular cross-section 30 cm. × 80 cm. The prestress at the ends is 1,30,000 kg. with zero eccentricity at supports and an eccentricity of 25 cm. at the centre, the cable profile being parabolic. Assuming $k_f = 0{\cdot}15$ per 100 metres, and $\mu = 0{\cdot}35$, determine the loss due to friction at the centre of the beam.*

Solution. Equation to the cable with one end as origin is,

$$y = \frac{4h}{l^2} x(l - x)$$

Slope at any point $= \frac{dy}{dx} = \frac{4h}{l^2}(l - 2x)$

Slope at end $= \frac{4h}{l^2}(l - 0) = \frac{4h}{l} = \frac{4 \times 0{\cdot}25}{15} = \frac{1}{15}$

Slope at centre $= 0$

Change of slope $= \frac{1}{15}$

Angle of deviation of cable from the end to the centre

$$= \alpha = \frac{1}{15} \text{ radian (since } \alpha \text{ is small)}$$

$$k_f = \frac{0{\cdot}15}{100} = 0{\cdot}0015 \text{ per metre}$$

$$\mu = 0{\cdot}35$$

$$P_x = P_o(1 - \mu\alpha - k_f x)$$

At $\quad x = \frac{l}{2} = 7{\cdot}5 \text{ m.}$

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$$P_o = 130000\ (1 - 0{\cdot}35 \times \frac{1}{15} - 0{\cdot}0015 \times 7{\cdot}5)$$

$$= 125504\ kg.$$

$\therefore$ Loss of prestressing force

$$= 130000 - 125504 = 4496\ kg.$$

$\therefore$ *Percentage loss of prestress*

$$= \frac{4496}{130000} \times 100 = 3{\cdot}46\%.$$

Design 202. *A straight pretensioned, concrete member 15 metres long with a cross-section of 40 cm. × 40 cm. is concentrically prestressed with 9 cm.² of steel wires which are anchored to the bulkheads with a stress of 10500 kg./cm.². Taking the modular ratio as 6, determine the loss of prestress due to elastic shortening of concrete at the transfer of prestress.*

Solution.

Prestressing force $P_o = 10500 \times 9\ kg.$

$$= 94500\ kg.$$

Loss of prestress $= \Delta f_c = m\frac{P_o}{A}$

$$= \frac{6 \times 94500}{40 \times 40}\ kg./cm.^2$$

$$= 354\ kg./cm.^2$$

Design 203. *A straight post tensioned concrete member 15 metres long with a cross-section of 40 cms. × 40 cms. is prestressed with 9 sq. cms. of steel wires. This steel is made of four tendons with 2·25 cm.² per tendon. The tendons are tensioned to a stress of 10500 kg./cm.². Determine the loss of prestress in each tendon due to elastic shortening of concrete. Find also the average percentage loss of prestress.*

If it is desired that after the last tendon is tightened a stress of 10500 kg./cm.² be maintained in each tendon compute the actual stresses to which the individual tendon should be tightened. Take $m = 6$.

Solution. *Loss of prestress in the first tendon.* The total force causing the elastic shortening of concrete after the first tendon is tightened

$$= 3 \times 2{\cdot}25 \times 10{,}500\ kg.$$

$$= 70875\ kg.$$

$\therefore$ Loss of prestress in the first tendon

$$= m\frac{P_o}{A}$$

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$$= \frac{6 \times 70875}{40 \times 40} \ kg./cm.^2$$

$$= 266 \ kg./cm.^2$$

Loss of prestress in the second tendon. Total force causing the elastic shortening of concrete after the second tendon is tightened.

$$= 2 \times 2{\cdot}25 \times 10500 \ kg.$$

$$= 47250 \ kg.$$

∴ Loss of prestress in the second tendon

$$= m\frac{P_o}{A}$$

$$= \frac{6 \times 47250}{40 \times 40} \ kg./cm.^2$$

$$= 177 \ kg./cm.^2$$

Loss of prestress in the third tendon. Total force causing the elastic shortening of concrete after the third tendon is tightened

$$= 1 \times 2{\cdot}25 \times 10500 \ kg.$$

$$= 23625 \ kg.$$

∴ Loss of prestress in the third tendon

$$= m\frac{P_o}{A}$$

$$= \frac{4 \times 23625}{40 \times 40} = 89 \ kg./cm.^2$$

Loss of prestress in the fourth tendon = 0.

∴ Average loss of prestress

$$= \frac{266 + 177 + 89 + 0}{4}$$

$$= 133 \ kg./cm.^2$$

This is also equal to half the maximum loss of prestress, *i.e.*,

$$\frac{266}{2} = 133 \ kg./cm.^2$$

Average percentage loss of prestress

$$= \frac{133}{10500} \times 100\%$$

$$= 1{\cdot}27\%$$

Adjustment of initial tensions to maintain a final stress of 10500 *kg./cm.*^{2}

Initial stress required in the first tendon

$$= 10500 + 266 = 10766 \ kg./cm.^2$$

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Initial stress required in the second tendon

$$= 10500 + 177 = 10677 \text{ kg./cm.}^2$$

Initial stress required in the third tendon

$$= 10500 + 89 = 10589 \text{ kg./cm.}^2$$

and Initial stress required in the fourth tendon

$$= 10500 \text{ kg./cm.}^2$$

Design 204. *Calculate the loss of prestress due to shrinkage of concrete in a post-tensioned beam if the age of concrete at transfer is 15 days. Take* $E_s = 2{\cdot}1 \times 10^6$ *kg /cm.*^{2}

Solution.

$$\text{Residual shrinkage strain} = \frac{2 \times 10^{-4}}{\log_{10}(T+2)} = \frac{2 \times 10^{-4}}{\log_{10}(15+2)}$$

$$= \frac{2 \times 10^{-4}}{1{\cdot}2304} = 1{\cdot}6255 \times 10^{-4}$$

$\therefore$ *Loss of stress in steel* $= E_s \times$ shrinkage strain of concrete

$$= 2{\cdot}1 \times 10^6 \times 1{\cdot}6255 \times 10^{-4}$$

$$= 341{\cdot}14 \text{ kg./cm}^2.$$

Design 205 *In a prestressed concrete beam of cross-section 20 cm.* $\times$ *30 cm. and span 6 metres, an initial prestressing force of 40,000 kg. is applied at an eccentricity of 7 cm. by tendons of area 4 cm*^{2}. *Assuming* $E_s = 2 \times 10^6$ *kg /cm*^{2} *and* $E_c = 0{\cdot}333 \times 10^6$ *kg./cm*^{2}, *anchor slip* $= 0{\cdot}15$ *cm., creep coefficient in concrete* $= 2{\cdot}0$, *concrete shrinkage* $= 0{\cdot}0002$ *and creep in steel* $= 3\%$, *find the percentage loss in prestress.*

Solution.

$$A = 20 \times 30 = 600 \text{ cm.}^2, \quad I = \frac{20 \times 30^3}{12} = 45000 \text{ cm.}^4$$

$$l = 6 \text{ m.}, \quad P_o = 40{,}000 \text{ kg.},$$

$$A_t = 4 \text{ cm}^2, \quad e = 7 \text{ cm.}$$

$$m = \frac{E_s}{E_c} = \frac{2 \times 10^6}{0{\cdot}333 \times 10^6} = 6$$

$$\text{Initial stress in the tendons} = \frac{40{,}000}{4} = 10{,}000 \text{ kg./cm.}^2$$

The losses of prestress due to various causes are calculated below.

(*i*) *Loss of stress due to elastic shortening of concrete*

Initial stress in concrete at the level of the tendon

$$= f_c = \frac{P_o}{A} + \frac{P_o e^2}{I} = \frac{40{,}000}{600} + \frac{40{,}000 \times 7^2}{45{,}000} = 110{\cdot}22 \text{ kg /cm}^2$$

Loss of stress due to elastic shortening

$$= m f_c = 6 \times 110{\cdot}22 = 661{\cdot}32 \text{ kg./cm.}^2$$

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(*ii*) *Loss of stress due to anchor slip*

$$= \frac{\Delta_a}{l} E_s = \frac{0{\cdot}15}{600} \times 2 \times 10^6 = 500 \text{ kg./cm.}^2$$

(*iii*) *Loss of stress due to creep of concrete*

$$= (C_c - 1) m f_c = (2 - 1){\cdot}6 \times 110{\cdot}22 = 661{\cdot}32 \text{ kg /cm}^2$$

(*iv*) *Loss of stress due to shrinkage of concrete*

$$= \text{Shrinkage strain} \times E_s = 0{\cdot}0002 \times 2 \times 10^6 = 400 \text{ kg./cm.}^2$$

(*v*) *Loss of stress due to creep in steel*

$$= 3\% \text{ of the initial stress} = \frac{3}{100} \times 10{,}000 = 300 \text{ kg./cm.}^2$$

∴ Total loss of stress

$$= 661{\cdot}32 + 500 + 661{\cdot}32 + 400 + 300 = 2522{\cdot}64 \text{ kg./cm.}^2$$

∴ *Percentage loss of prestress*

$$\frac{2522{\cdot}64}{10{,}000} \times 100\% = 25{\cdot}23\%.$$

Design 206. *A post-tensioned prestressed concrete beam of 16 metres span is subjected to an initial prestress of 1,45,800 kg. transferred at 28 days' strength of concrete. Profile of the cable is parabolic with the maximum eccentricity of 52 cm. at the centre of the span.*

Take the following additional data :

$A = 2420$ cm.^{2}, $I = 530 \times 10^4$ cm.^{4}, $A_t = 13{\cdot}86$ cm.^{2}, $f_s = 10590$ kg /cm.^{2} at tansfer, $E_s = 2{\cdot}1 \times 10^6$ kg./cm.^{2}, $E_c = 3{\cdot}82 \times 10^5$ kg./cm.^{2}, μ = Frictional coefficient = 0·25, and Wobble correction factor $K = 0{\cdot}0015$ per metre.

Determine the following loss in prestress :

(*a*) *Loss due to elastic shortening,*

(*b*) *Shrinkage in concrete,*

(*c*) *Creep in concrete,*

(*d*) *Slip in anchorage,*

(*e*) *Frictional loss.*

Determine the effect of strain due to flexure caused by an external load of 3000 kg /metre acting on the beam on losses. Jacking is done from both ends of the beam. Estimate the net losses of prestress

Solution.

D.L. of beam $= \frac{2420}{100 \times 100} \times 2400 \text{ kg./m.} = 580{\cdot}8 \text{ kg./m.}$

D.L.B.M $= \frac{580{\cdot}8 \times 16^2}{8} \times 100 = 1858560 \text{ kg. cm.}$

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B.M. at centre due to eccentricity of the prestressing force

$$= -145800 \times 52 = -7581600 \text{ kg. cm.}$$

Net B.M. at centre $= -5723040$ kg. cm.

The various losses are calculated below.

(*a*) *Loss due to elastic shortening*

Elastic strain at end section

$$= \frac{P_o}{AE_c} = \frac{145800}{2420 \times 3{\cdot}82 \times 10^5} = 1{\cdot}57 \times 10^{-4}$$

Elastic strain at mid span

$$= \frac{P_o}{AE_c} + \frac{Me}{IE_c}$$

$$= 1{\cdot}57 \times 10^{-4} + \frac{5723040 \times 52}{530 \times 10^4 \times 3{\cdot}82 \times 10^5}$$

$$= 1{\cdot}57 \times 10^{-4} + 1{\cdot}45 \times 10^{-4}$$

$$= 3{\cdot}02 \times 10^{-4}$$

Average elastic strain $= \frac{1{\cdot}57 + 3{\cdot}02}{2} \times 10^{-4} = 2{\cdot}295 \times 10^{-4}$

$\therefore$ *Loss of prestress* $= 2{\cdot}295 \times 10^{-4} \times 2{\cdot}1 \times 10^6 = 481{\cdot}90$ *kg./cm.*^{2}

(*b*) *Loss of stress due to shrinkage in concrete.*

Shrinkage strain $= \frac{2 \times 10^{-4}}{\log_{10}(T+2)} = \frac{2 \times 10^{-4}}{\log_{10}(28+2)}$

$$= 1{\cdot}354 \times 10^{-4}$$

$\therefore$ *Loss of prestress*

$$= 1{\cdot}354 \times 10^{-4} \times 2{\cdot}1 \times 10^6 = 284{\cdot}3 \text{ kg./cm.}^2$$

(*c*) *Loss of stress due to creep in concrete and steel.*

Assuming a creep coefficient

$$C_c = 2{\cdot}5$$

Creep strain due to creep in concrete

$$= (C_c - 1) \times \text{Elastic strain}$$

$$= (2{\cdot}5 - 1) \times 2{\cdot}295 \times 10^{-4} = 3{\cdot}442 \times 10^{-4}$$

$\therefore$ *Loss of prestress* $= 3{\cdot}442 \times 10^{-4} \times 2{\cdot}1 \times 10^6 = 722{\cdot}8$ *kg./cm.*^{2}

Loss of prestress due to creep in steel

$= 2\%$ of the initial stress

$$= \frac{2}{100} \times 10590 = 211{\cdot}8 \text{ kg./cm.}^2$$

(*d*) *Loss of stress due to slip in anchorage*

Assuming a slip of 2·5 *mm.*

Loss of prestress $= \frac{0{\cdot}25}{1600} \times 2{\cdot}1 \times 10^6 = 328{\cdot}1$ *kg./cm.*^{2}

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(e) *Loss due to friction.*

Slope of cable at end $= \frac{4e}{l} = \frac{4 \times 52}{1600} = 0{\cdot}13.$

$\therefore$ From support to centre change in slope $= \alpha = 0{\cdot}13$

Loss of prestress $= (kx + \mu x) t_o$

$= [(0{\cdot}0015 \times 8) + (0{\cdot}25 \times 0{\cdot}13)] 10590$

$= 471{\cdot}2 \text{ kg./cm.}^2$

Effect of external loading of 3000 kg./m.

B.M. due to external loading

$$= M_1 = \frac{3000 \times 16^2}{8} \times 100 \text{ kg. cm.}$$

$$= 9600000 \text{ kg. cm.}$$

Strain in concrete at the level of tendon, at mid span

$$= \frac{M_1 e}{I E_c}$$

$$= \frac{9600000 \times 52}{530 \times 10^4 \times 3{\cdot}82 \times 10^5}$$

$$= 2{\cdot}466 \times 10^{-4}$$

$\therefore$ Gain in strain in steel

$$= 2{\cdot}466 \times 10^{-4}$$

$\therefore$ *Increase in prestress*

$$= 2{\cdot}466 \times 10^{-4} \times 2{\cdot}1 \times 10^{-6}$$

$$= 517{\cdot}8 \text{ kg./cm.}^2$$

§216. Design of Prestressed Concrete Beams

It is proposed to first deal in this article an approximate method of designing prestressed concrete beams.

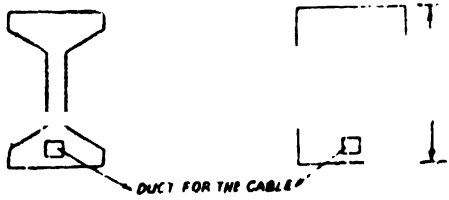

Fig. 845

Fig. 845 shows two of the usual sections of a prestressed beam. It will be taken that the section of the beam is symmetrical. Further the gross area of section itself will be considered as the useful area since the area of the tendons will be small in comparison with the gross area of the member.

Let the effective total prestress be P at an eccentricity of e. Let M_d be the maximum bending moment due to dead load alone. The

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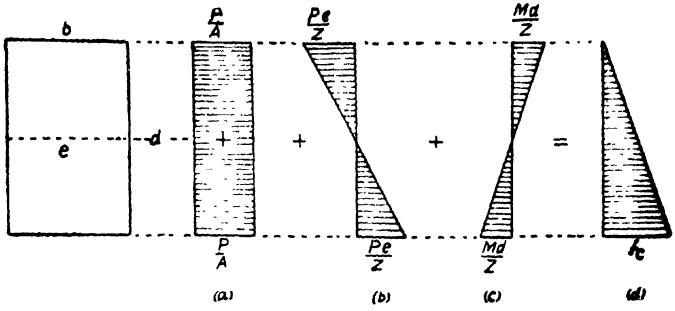

Fig. 846

stresses due to direct load, eccentricity of the prestress and due to the dead load moment are shown in Fig. 846.

The beam should be so designed that when the live load has not been applied, the resultant stress on the section due to prestress, eccentricity of the prestress and due to the dead load moment should be as shown in Fig 846 (*d*) so that the maximum compressive stress in the bottom fibre should be f_c the permissible stress in concrete. The stress in the top fibre will be zero.

To suit this condition, we have,

$$\frac{P}{A} + \frac{M_d}{Z} = \frac{Pe}{Z} \qquad \ldots(1)$$

$$\frac{P}{A} + \frac{Pe}{Z} - \frac{M_d}{Z} = f_c \qquad \ldots(2)$$

Suppose now the live load is applied.
Let the bending moment due to live load be M_l.

The beam should be so designed that after the live load is applied the bending stresses $\frac{M_l}{Z}$ caused by the live load should be such that the resultant stresses should be as in Fig. 847 (*c*) so that the compressive stress in the top fibre reaches f_c while the stress in the bottom fibre is zero.

To fulfil this condition, we have,

$$\frac{M_l}{Z} = f_c \qquad \ldots(3)$$

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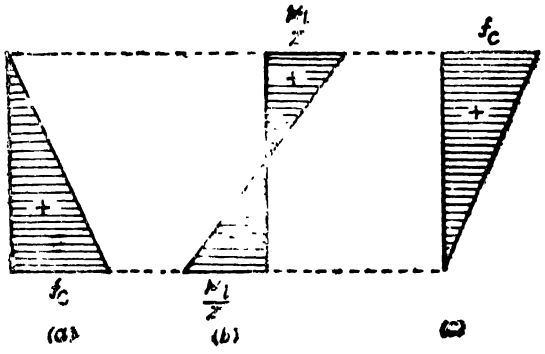

Fig. 847

(*a*) Resultant stress diagram before applying live load.

(*b*) Stresses due to live load.

(*c*) Resultant stress diagram after applying live load.

But from (2) $$f_c = \frac{P}{A} + \frac{Pe}{Z} - \frac{M_d}{Z}$$

$$\therefore \quad \frac{P}{A} + \frac{Pe}{Z} - \frac{M_d}{Z} = \frac{M_l}{Z}$$

$$\therefore \quad \boxed{\frac{Pe}{Z} = \frac{M_d + M_l}{Z} - \frac{P}{A}} \qquad \ldots(4)$$

Rewriting equations (1) and (4),

$$\frac{P}{A} + \frac{M_d}{Z} = \frac{Pe}{Z}$$

$$-\frac{P}{A} + \frac{M_d + M_l}{Z} = \frac{Pe}{Z}$$

Adding, $$\frac{2M_d + M_l}{Z} = \frac{2Pe}{Z}$$

$$\therefore \quad \boxed{e = \frac{2M_d + M_l}{2P}} \qquad \ldots(5)$$

In designing the beam, it is worth remembering the following formulae

$$Z = \frac{M_l}{}$$

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f_c (since the stress varies from 0 at one extreme fibre to f_c at the other extreme fibre)

and $$e = \frac{2M_d + M_l}{2P}$$

Hence the following steps may be adopted in designing a pre-stressed concrete beam.

Step 1. Calculate the live load moment M_l.

Step 2. Determine the section modulus required from the condition,

$$Z = \frac{M_l}{f_c}$$

Choose a convenient depth $\frac{1}{20}$ or $\frac{1}{25}$ of the span. After choosing the depth we can determine the width d from the relation

$$\frac{bd^2}{6} = Z$$

Thus the width and the depth are both known.

Step 3. Now find the prestressing force P from the relation

$$P = \frac{f_c}{2} \cdot A$$

Step 4. Find the amount of steel required from the relation

$$A_t = \frac{P}{\text{Safe stress in steel}}$$

Step 5. Determine the dead load moment M_d.

Step 6 Place the reinforcement at the eccentricity e given by

$$e = \frac{2M_d + M_l}{2P}$$

The following example shows the application of the above steps in the design of the beams.

Design 207. *A pre-stressed concrete beam of uniform rectangular cross-section and span 15 metres supports a total distributed load of 27,200 kg. excluding the weight of the beam. Determine the suitable dimensions of the beam and calculate the area of the tendons and their position. The permissible stresses are 140 kg./cm.² for concrete and 10,500 kg./cm.² for the tendons.*

Solution. B.M. due to live load

$$= M_l = \frac{Wl}{8}$$

$$= \frac{27,200 \times 15}{8} \times 100 \text{ kg. cm.}$$

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$$= 5,100,000 \text{ kg. cm.}$$

Section modulus required

$$= Z = \frac{M_1}{\quad}$$

$$= \frac{5,100,000}{140} \text{ cm.}^3$$

$$255,000 \text{ cm.}^3$$

The depth of the beam may be taken from $\frac{1}{20}$ to $\frac{1}{25}$ of the span.

In our case $\frac{1}{20}$ of the span $= \frac{15 \times 100}{20} = 75$ cm. Let the width of the beam be b cm But, the section modulus of the section

$$= Z = \frac{bd^2}{6}$$

$$b \times 75^2 \quad 255,000$$

$$b = \frac{255,000}{7} \times \frac{6}{75^2} \text{ cm.}$$

$$\therefore \quad = 39 \text{ cm.}$$

Hence the beam section will be 39 cm. × 75 cm.

∴ *Prestressing force*

$$= P = \frac{f_c}{2} \cdot A$$

$$= \frac{140}{2} \times 39 \times 75 \text{ kg.}$$

$$= 204,750 \text{ kg.}$$

∴ *Area of the tendons*

$$= A_s \cdot \text{Safe stress in tendons}$$

$$= \frac{204,750}{10,500} \text{ cm.}^2$$

$$= 19{\cdot}50 \text{ cm.}^2$$

If 6 mm. dia. bars be used,

Area of one bar $= 0{\cdot}28 \text{ cm.}^2$

∴ *Number of tendons required*

$$= \frac{19{\cdot}50}{0{\cdot}28}$$

$$= 70$$

Hence provide 70 bars of 6 mm. diameter.

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B.M. due to self-weight of the beam

$$= (0\cdot39 \times 0\cdot75 \times 2400) \times \frac{15^2}{8} \times 100 \text{ kg. cm.}$$

$$= 19,74,000 \text{ kg. cm.}$$

$\therefore$ *Eccentricity* $e = \dfrac{2M_d + M_l}{2P}$

$$= \frac{2(1,9\,4,000) + (5,1\,0,000)}{2 \times 204,750}$$

$$= 22\cdot10 \text{ cm.}$$

Design 208. *A prestressed concrete beam section is 25 cm. wide and 30 cm. deep. The initial prestressing force is 45000 kg. at an eccentricity of 6 cm. The beam has a span of 5·75 metres, and has to carry a superimposed load of 750 kg./m. Analyse the beam section for the stresses produced at mid span before and after the application of the live load. Allow a loss of prestress of 15%.*

Solution.

Area of the beam section

$= 25 \times 30$

$= 750 \text{ cm.}^2$

Section modulus

$$= Z = \frac{bd^2}{6}$$

$$= \frac{25 \times 30^2}{6} \text{ cm.}^3$$

$= 3750 \text{ cm.}^3$

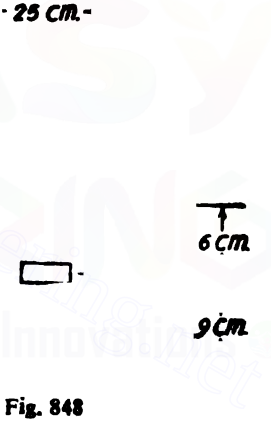

Fig. 848

Eccentricity of the prestressing force

$= e = 6 \text{ cm.}$

Stresses before the application of live load

Dead load of the beam

$= 0\cdot25 \times 0\cdot30 \times 2400 \text{ kg./m.}$

$= 180 \text{ kg./m.}$

$\therefore$ B.M. due to dead load

$$= M_o = \frac{180 \times 5\cdot75^2}{8} \times 100 \text{ kg. cm.}$$

$= 74400 \text{ kg. cm.}$

Direct stress due to prestressing force

$$= + \frac{45000}{25 \times 30} \text{ kg./cm.}^2$$

$= +60 \text{ kg./cm.}^2$

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Extreme stress due to dead load moment

$$= \frac{M_1}{Z}$$

$$= \frac{74,400}{3750} \text{ kg/cm.}^2$$

$$= \pm 19{\cdot}85 \text{ kg/cm.}^2$$

Extreme stress due to the eccentricity of the prestressing force

$$= \mp \frac{45000 \times 6}{3750} \text{ kg./cm.}^2$$

$$= \mp 72 \text{ kg./cm.}^2$$

Hence before the application of the live load the extreme stress on the section will be as follows :

Stress at the top $= +60 + 19{\cdot}85 - 72$

$= +7{\cdot}85 \text{ kg/cm.}^2$

Stress at the bottom $= 60 - 19{\cdot}85 + 72$

$= +112{\cdot}15 \text{ kg./cm.}^2$

Stresses after the application of the live load

For this analysis we will allow 15% loss of prestress

Hence the prestressing force will now be taken as

$0{\cdot}85 \times 45000 = 38,250$ kg.

∴ Direct stress due to prestressing force

$$= \frac{38250}{25 \times 30}$$

$$= +51 \text{ kg./cm.}^2$$

Extreme stress due to dead load moment

$$= \pm 19{\cdot}85 \text{ kg/cm.}^2$$

Extreme stress due to eccentricity of prestressing force

$$= \mp \frac{38250 \times 6}{3750} \text{ kg./cm.}^2$$

$$= \mp 61{\cdot}2 \text{ kg./cm.}^2$$

Live load B.M. $= M_1 = 750 \times \frac{5{\cdot}75^2}{8} \times 100$ kg. cm.

$= 309800$ kg./cm.

∴ Extreme stress due to live load moment

$$= \pm \frac{309800}{3750} \text{ kg./cm.}^2$$

$$= \pm 82{\cdot}6 \text{ kg./cm.}^2$$

Hence, the final stresses are

At top $= +51 + 19{\cdot}85 - 61{\cdot}2 + 82{\cdot}6$

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$= +92{\cdot}25$ *kg./cm.*^{2}

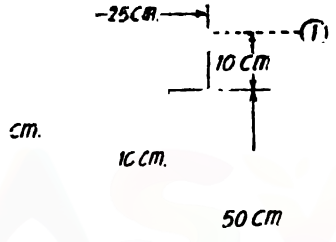
At bottom $= +51 - 19{\cdot}85 + 61{\cdot}2 - 82{\cdot}6$

$= +9{\cdot}75$ *kg./cm.*^{2}

Design 209. *Fig. 849 shows the section of a prestressed concrete beam of span 9 metres. The beam has to carry a superimposed load of 2000 kg./m. The prestressing force is transmitted by tendons in two cabl In each cable the tendons consist of 14 wires of 5 mm. diameter, subjected to an initial prestress of 10,000 kg./cm*^{2}*. Analyse the beam section at the centre, for the stresses induced before and after the application of the superimposed load. Allow 15% loss of prestress*

Solution.

It is first necessary to determine the properties of the section like area, position of centroidal axis, moment of inertia of the section about the centroidal axis. *etc.*



These are determined as shown below. (See table on page 1234).

∴ Distance of centroidal axis from 1−1.

$$= \bar{y} = \frac{\Sigma ay}{\Sigma a}$$

$$= \frac{43350}{1110}$$

$$= 39{\cdot}41 \text{ cm.}$$

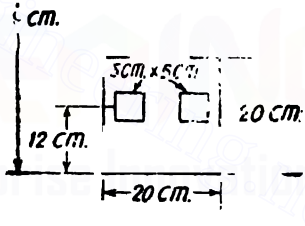


Fig. 849

Moment of inertia of the section about the axis 1−1

$$= I_{1-1} = \Sigma I_{self} + \Sigma ay^2$$

$$= 119479 + 2347550 \text{ cm.}^4$$

$$= 2467029 \text{ cm.}^4$$

Let the moment of inertia of the section about the centroidal axis be I

$$\therefore \quad I_{1-1} = I + (\Sigma a)\bar{y}^2$$

$$\therefore \quad 2467029 = I + 1100(39{\cdot}41)^2$$

$$\therefore \quad I = 758570 \text{ cm.}^4$$

$$\therefore \quad Z_t = \frac{758570}{39{\cdot}41}$$

$$= 19250 \text{ cm.}^3$$

and $$Z_b = \frac{758570}{40{\cdot}59}$$

$$18680 \text{ cm.}^3$$

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Dead load of the girder

$$\frac{1100 \times 2400}{100 \times 100}$$

$$264 \text{ kg/m.}$$

Dead load moment $= \frac{264 \times 9^2}{8} \times 100 \text{ kg. cm.}$

$$= 267300 \text{ kg. cm.}$$

Initial prestressing force

$$= 2 \times 14 \times 0{\cdot}20 \times 10{,}000$$

$$= 56{,}000 \text{ kg.}$$

Eccentricity of the prestressing force

$$= 40{\cdot}59 - 12$$

$$= 28{\cdot}59 \text{ cm.}$$

Stresses at transfer

Direct stress due to initial prestressing force

$$= \frac{56000}{1100}$$

$$= +50{\cdot}91 \text{ kg/cm.}^2$$

Component	*area a* ($cm.^2$)	*Centroidal distance y from the axis 1–1* (*cm.*)	*ay* ($cm.^3$)	ay^2 ($cm.^4$)	I_{self} ($cm.^4$)	
Top flange	250	5	1250	6250	$\frac{25 \times 10^3}{12}$ =	2083
Web	500	35	17500	612500	$\frac{10 \times 50^3}{12}$ =	104167
Bottom flange	400	70	28000	1960000	$\frac{20 \times 20^3}{12}$ =	13333
Gross Total	1150		46750	2578750		119593
Deduct for Cavities 2×5×5	50	68	3400	231200	$\frac{2 \times 5 \times 5^3}{12}$ =	104
Total	1100		43350	2347550		119479

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Bending stress at the top due to eccentricity of the prestressing force

$$= -\frac{56000 \times 28{\cdot}59}{19250}$$

$$= -83{\cdot}18 \ kg./cm.^2$$

Bending stress at the bottom due to eccentricity of the prestressing force

$$= +\frac{56000 \times 28{\cdot}59}{18680}$$

$$= +85{\cdot}72 \ kg/cm.^2$$

Bending stress at the top due to dead load moment

$$= +\frac{267300}{19250}$$

$$= +13{\cdot}89 \ kg./cm.^2$$

Bending stress at the bottom due to dead load moment

$$= -\frac{267300}{18680}$$

$$= -14{\cdot}31 \ kg./cm.^2$$

Hence, the extreme stresses before the application of the live load are as follows :

Stress at the top $= +50{\cdot}91 - 83{\cdot}18 + 13{\cdot}89$

$= -18{\cdot}38 \ kg./cm.^2$

Stress at the bottom $= +50{\cdot}91 + 85{\cdot}72 - 14{\cdot}31$

$= +122{\cdot}32 \ kg./cm.^2$

Final stresses after the application of the live load

Prestressing force is reduced by 15%

$\therefore$ Final prestressing force

$= 0{\cdot}85 \times 56000$

$= 47600 \ kg.$

Live load moment $= 2000 \times \frac{9^2}{8} \times 100 \ kg. \ cm.$

$= 2{,}025{,}000 \ kg. \ cm.$

Direct stress due to prestressing force

$$= +\frac{47600}{1100}$$

$$= +43{\cdot}27 \ kg./cm.^2$$

Bending stress at the top due to eccentricity of the prestressing force

$$= -\frac{47600 \times 28{\cdot}59}{19250}$$

$$= -70{\cdot}70 \ kg./cm.^2$$

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Bending stress at the bottom due to eccentricity of the prestressing force

$$= + \frac{47600 \times 28{\cdot}59}{18680}$$

$$= +73{\cdot}70 \ kg./cm.^2$$

Bending stress at top due to dead load moment

$$= +13{\cdot}89 \ kg./cm.^2$$

Bending stress at bottom due to dead load moment

$$= -14{\cdot}31 \ kg./cm.^2$$

Bending stress at top due to live load moment

$$= + \frac{2025000}{19250}$$

$$= +105{\cdot}2 \ kg./cm.^2$$

Bending stress at bottom due to live load moment

$$= - \frac{2025000}{18680}$$

$$= -108{\cdot}4 \ kg./cm.^2$$

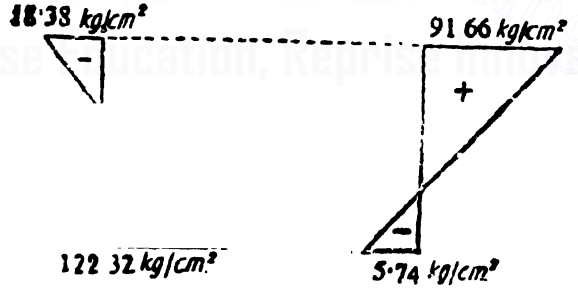
$\therefore$ The final stresses are as follows :

Stress at the top $= +43{\cdot}27 - 70{\cdot}70 + 13{\cdot}89 + 105{\cdot}2$

$= +91{\cdot}66 \ kg./cm.^2$

Stress at the bottom $= +43{\cdot}27 + 73{\cdot}70 - 14{\cdot}31 - 108{\cdot}4$

$= -5{\cdot}74 \ kg/cm.^2$



Stress distribution before application of live load

Srtess distribution after application of live load.

Fig. 850

§217. I.S. R[illegible] amendations

The Indian standard code of practice for prestressed concrete (I.S. 1343) has made the following recommendations.

(1) Quality of materials. The effective use of prestressed concrete can be obtained only when the concrete and the prestressing steel used are of high quality and strength.

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Concrete. The materials used in concrete should satisfy the following requirements.

Cement. The cement used for prestressed concrete construction shall be one of the following :

(*i*) Ordinary Portland cement or rapid hardening Portland cement complying with the requirements of IS-269.

(*ii*) Blast furnace slag cement complying with IS-455 with not more than 50 percent of slag.

Aggregates. Coarse and fine aggregates shall conform to IS-383. For adequate control of concrete, the coarse aggregate shall preferably be graded into different sizes (two grades for 20 *mm.* and three grades for 40 *mm.* aggregates) and the requisite proportions of each grade shall be taken in making the concrete mix.

Water. The water shall be clean and free from organic and inorganic matter harmful to concrete.

Admixtures. The use of admixtures is not generally recommended. However, they may be used only with the approval of the Engineer-in-charge based upon evidence, that with the passage of time, neither the compressive strength of concrete is reduced by more than 10 percent nor are other requisite qualities of concrete and/or steel impaired by the use of such admixtures. Until more information is available, the use of calcium chloride is not recommended.

Concrete mix and strength. Only controlled concrete shall be used for prestressed concrete construction. The design of the concrete mix shall conform to the requirements laid down for controlled concrete under clause 9 of I.S. 456 subject to the following further conditions :

(*a*) The minimum F_c shall not be less than 420 *kg./cm.*^{2} for pretensioned systems and 350 *kg./cm.*^{2} for post-tensioned systems. The corresponding figures for cylinder strength shall be 340 *kg./cm.*^{2} for pre-tensioning and 280 *kg./cm.*^{2} for post-tensioning .

(*b*) The mix shall contain not less than 380 *kg.* of cement per *cubic metre* of concrete for pretensioned work and 360 *kg.* of cement per *cubic metre* of concrete for post-tensioned work. The cement in the mix shall not preferably exceed 530 *kg.* per *cubic metre.*

(*c*) The mix shall contain as low a water content as consistent with adequate workability.

(*d*) The concrete shall be compacted thoroughly by vibration, pressure, shock, spinning, or other means and shall have a density of not less than 2·40 *tonnes/metre.*^{3}

(*e*) Where the design provides for a concrete of F_c higher than 530 *kg./cm.*^{2}, control measures on work shall be adequately tightened to ensure that the required high strength is fully available at all sections of the structure.

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Concrete strength at transfer. In addition to cube or cylinder strength tests required for controlled concrete at 7 and 28 days age, additional tests may be made at the age of the concrete at transfer. The concrete strengths at transfer shall not be less than half the corresponding 28 day strength required in the design.

Where strengths lower than those specified herein are required to be used, as in the case of membrane structures, special authorization shall be obtained. The provisions of this clause do not apply to components of a composite structure which are not themselves directly prestressed but are acting monolithically with prestressed concrete members.

(2) Prestressing steel. The steel used for prestressing the concrete shall be one of the following:

(*a*) Hard-drawn high tensile steel wire of diameter ranging from 1·5 *mm.* to 8 *mm.* and having tensile and other properties as specified in the following clauses. Hard-drawn steel wire may be used either as single wire or in the form of cables.

(*b*) High tensile alloy steel bars of 9·5 *mm.* to 32 *mm.* diameter having a tensile strength and other properties as specified in the following clauses.

High tensile steel wires

Ultimate tensile strength (F_s). High tensile steel wire used for prestressed concrete shall conform to the requirement shown in the table below, regarding minimum ultimate tensile strength.

Minimum Ultimate Tensile Strength of High Tensile Steel Wires

Nominal diameter of wire mm.	*Minimum ultimate tensile strength kg./cm.²*
8·00	150
7·00	150
5·00	160
4·00	175
3·00	190
2·50	205
2·00	220
1·50	235

Proof stress (F_y). The proof stress shall be not less than 80 percent of the minimum ultimate tensile strength.

Elongation at rupture. The elongation at rupture shall be not

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less than 2 per cent over a gauge length of 200 *mm.* when tested in accordance with *I.S.* 1521 *Method for tensile testing of steel wire.*

Surface condition of wire. The wire shall be free from rust, scale and other similar deleterious matters liable to affect adversely proper tensioning or its bond with concrete. Slight rust may be permitted provided it is not loose.

Coils. High tensile steel wire shall be supplied in coils having a sufficiently large diameter for the wire to layout straight

Cables. High tensile steel wire used in making the cable shall comply with the requirements mentioned in the table above. Cables used as prestressing steel shall consist of a group of single wires arranged in parallel or stranded formation. For purposes of tests, the strength of a cable shall be taken as the multiple of the strength of the single or stranded wires forming the cable.

High tensile alloy steel bars

Strength. High tensile alloy steel bars shall have a minimum ultimate tensile strength (F_s) of 95 *kg./mm.*^{2} Proof stress (F_y) of the bars shall not be less than 80 percent of the ultimate tensile strength.

Surface condition of bars. The bar as supplied shall be free from surface flows and jagged edges.

Coupling units. Coupling units and similar fixtures used in conjunction with the wires or bars shall have an ultimate tensile strength of not less than the individual strengths of the wires or bars being joined.

Mild steel bars. Mild steel bars used as non-tensioned reinforcement shall conform to IS : 432 and the method of bending, binding and placing shall conform to clause 11 of I.S. 456.

Hard drawn-steel wire. Hard drawn steel wire used as non-tensioned reinforcement shall conform to I.S. 432.

(3) **Prestressing Equipment**

Tensioning apparatus. The requirements of this clause shall apply to both the pretensioned and the post-tensioned methods of prestressing concrete except where mentioned otherwise.

Prestressing steel may be tensioned by means of levers, screw-jacks, hydraulic-jacks or similar mechanical apparatus. The method of tensioning steel covered by the I.S. code is generally by means of hydraulic or similar mechanical jack.

The type of tensioning apparatus shall be such that a controlled force can be applied. The tensioning apparatus shall not induce dangerous secondary stresses or torsional effect on the steel, concrete or on the anchorage. The anchorage provided for the temporary gripping of wires or bars on the tensioning apparatus shall be secured and such as not to damage the wire or bar.

Devices attached to the tensioning apparatus for measuring the applied force shall be such that they do not introduce errors exceeding 5 percent.

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Temporary gripping device. Prestressing tendons may be gripped by wedges, yokes, double cones or any other approved type of gripping device. The prestressing wires may be gripped single or in groups. Gripping device shall be such that in a tensile test, the wire or wires fixed by them would break before failure of the grip itself.

Releasing device. The releasing device shall be so designed that during the period between the tensioning and release, the tension in the prestressing elements is fully maintained by positive means, such as external anchorages. The device shall enable the transfer of prestress to be carried out gradually and so as to avoid large differences of tension between wires in a tendon, severe eccentricities of prestress or the sudden application of stress to the concrete.

Anchorage. The anchorage may consist of any device, patented or otherwise, which complies with the requirements laid down under the following clauses.

The anchoring device shall be capable of holding without more than nominal slip the prestressing tendon subjected to a load midway between the proposed initial prestressing load and the ultimate strength of the prestressing tendon.

The anchoring device shall be strong enough to resist in all respects a force equal to at least the breaking strength of the prestressing tendon it anchors.

The anchorage shall transfer effectively and distribute, as evenly as possible, the entire force from the prestressing tendon to the concrete without inducing undesirable secondary or local stress.

The anchorage shall be safe and secure against both dynamic and static loads as well as against impact.

The anchorage shall have provision for the introduction of a suitable protective medium such as cement grout, for the protection of the prestressing steel unless alternative arrangements are made.

(4) Design

Basic assumptions : (*a*) A plane section before bending remains a plane after bending.

(*b*) Within the range of working stresses both concrete and steel behave elastically notwithstanding the small amount of creep which occurs in both material under strained loading.

(*c*) Up to the limit of the modulus of rupture of concrete, any change in the loading produces a change of stress in the concrete only, the sole function of the prestressing tendon being to impart and maintain the prestress in the concrete notwithstanding the comparatively

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small changes in the stress of steel due to changes in the strains of concrete surrounding it.

Areas and moments of inertia of sections. In calculating the area, centroid and moment of inertia of cross-section, allowance for the prestressing tendons need not be made in the case of pretensioned members where the prestressing tendons are single wires distributed on the cross-section or strands of wires of relatively small cross-section area. Where allowance is made, it shall be on the basis of $(m-1)$ times or the area of prestressing tendons, m being the modular ratio.

In calculating the area, centroid and moment of inertia of cross-sections of post-tensioned members, deductions shall invariably be made for prestressing tendons, cable ducts or sheaths and such other openings in the concrete whether they are formed longitudinally or transversely. These deductions need not however be made for loads applied after the ducts, sheaths or openings have been grouted or filled with concrete. Where such deductions are not made a transformed area equivalent to m times the area of the prestressing element shall be taken in calculation, m being the modular ratio.

Moduli of Elasticity

Modulus of Elasticity for steel. In the absence of data the following values may be adopted :

Plain drawn wires (cold-drawn or cold worked)

$$E = 2{\cdot}1 \times 10^6 \ kg./cm.^2$$

High tensile steel bars (rolled or heat treated)

$$E = 2 \times 10^6 \ kg/cm^2$$

Modulus of Elasticity for concrete. Unless otherwise determined by tests, the modulus of elasticity of concrete E_c may be taken as $15000\sqrt{F_c}$ $kg/cm.^2$ F_c being the cube strength of concrete at 28 days where ordinary portland cement is used, and at 7 days where rapid hardening portland cement is used, on 15 *cm.* cubes expressed in $kg./cm.^2$

Permissible stresses.

Prestressing steel of any type complying with requirements of para (2) above. At the time of initial tensioning, the maximum tensile stress (f_w) immediately behind the anchorages, shall not exceed (*a*) 80 percent of the ultimate tensile strength of the wire or bar for steel without a definite yield point, and (*b*) the yield stress for steel with a guaranteed yield point.

After accounting for all losses due to creep and shrinkage of concrete, friction effects and relaxation of steel, the maximum tensile stress f_s, at the point of maximum bending moment shall not exceed (*a*) 60 percent of the ultimate tensile strength of the wire or bar for

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steel without a definite yield point, and (*b*) 80 percent of the yield stress for steel with a guaranteed yield point.

Concrete. The stresses in the concrete due to prestressing operations and working loads shall not exceed the values given in the following clauses. Unless otherwise specified, the cube strength refers to that of concrete obtained at 28 days when ordinary portland cement is used and at 7 days when rapid hardening portland cement is used on 15 *cm.* cubes.

Compressive stress due to initial prestress. The maximum permissible compressive stress due to bending and direct force at the time of transfer of prestress (f_r), shall be determined from a graph in which a straight line joins a point given by 0·5 F_r for a concrete with a F_c of 350 *kg./cm.*^{2} to a second point giving a permissible stress of 0·43 F_r for a concrete with a F_c of 530 *kg./cm.*^{2} (Fig. 851).

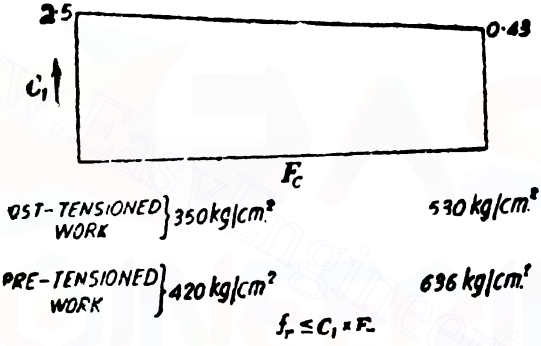

Fig. 851. Computation of compressive stress due to initial prestress.

The above F_c values apply to post-tensioned work; for pre-tensioned work they shall be increased oy 20 percent. For the purpose of the above figure, F_r shall be less than half the corresponding F_c.

The strength of concrete at the time of transfer (F_r) shall be established by tests carried out on cubes at the age of the concrete at transfer for bridges and such other major structures and other cases, if more convenient, from the straight line graph joining the ascertained cube strengths of the concrete at 28 and 7 days (7 and 3 days where rapid hardening cement is used).

The transfer of prestress shall be made only after the concrete has attained a strength of atleast half the specified F_c with due regard to the higher values required in Para (1) above for pre-tensioned work.

Compressive stress due to final prestress. The maximum permissible compressive stress (f_c) due to bending and direct force after deduction of the full losses in the specified prestress, shall be determined by a straight line relation, as in Fig. 852, but different stress limits shall apply to the concrete of the structure depending on

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whether it falls in a part of the section where the compressive stresses are not likely to increase in service (zone 1) or in a part of the section where the compressive stresses are likely to increase in service (zone 2). See Fig. 852.

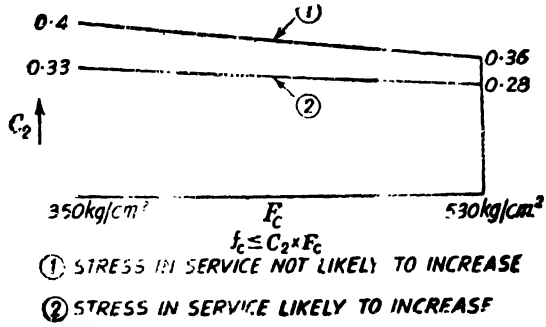

Fig. 852. Computation of compressive stress due to final prestress.

For zone 1, the straight line relation of permissible stresses (f_c) shall be determined by the straight line joining a point given by a permissible stress of 0·4 F_c for a concrete of $F_c = 350$ $kg./cm.^2$ to another point given by a permissible stress of 0·36 F_c for a concrete of $F_c = 530$ $kg/cm.^2$

For zone 2, the determining points of the graph shall be reduced to 0·33 F_c and 0·28 F_c respectively.

(*e*) *Direct compression.* Except in parts immediately behind the anchorages, the maximum stress in direct compression shall be limited to 0·8 times the permissible stress obtained from above.

(*f*) *Permissible bearing pressure behind anchorages.* On the areas immediately behind external anchorages, the permissible unit bearing pressure on the concrete, after accounting for all losses due to relaxation of steel, elastic shortening, creep of concrete, slip and/or seating of anchorages, *etc.* shall not exceed 0·48 $F_r \sqrt[3]{\frac{A_1}{A_2}}$ or 0·8 F_r, whichever is smaller, where F_r is the cube strength at transfer and A_1 and A_2 are the bearing and punching areas respectively.

During tensioning, the allowable bearing pressure specified above may be increased by 25 percent, provided that this temporary value does not exceed the average cube strength F_r at transfer. The values prescribed above for both temporary and permanent bearing pressures may be increased suitably, if adequate hoop reinforcement is provided at the anchorages.

When the anchorages are embedded in concrete, the bearing pressures shall be investigated after accounting for the surface friction between the anchorage and the concrete.

The effective punching area shall generally be the contact area

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of the anchorage devices which, if circular in shape, shall be replaced by a square of equivalent area. The bearing area shall be the maximum area of that portion of the member which is geometrically similar and concentric to the effective punching area.

Where a number of anchorages are used, the bearing area A_1 shall not overlap. Where there is already a compressive stress prevailing over the bearing area, as in the case of an anchorage placed in the body of a structure, the total pressure shall not exceed the limiting values stated above. For stage stressing of cables, the adjacent unstressed anchorages shall be neglected when determining the bearing area.

Bursting and spalling reinforcement required behind the anchorages shall be investigated and provided for, they shall preferably be placed parallel to the bearing plane. In detailing the anchorage portion of a member, recognized practice and successful past applications shall be taken into consideration.

(*g*) *Flexural stresses.* (*i*) In structures subjected to dynamic loading under the severest combination of service loads and forces applying, all direct force and bending stresses in any direction shall remain compressive. In the extreme fibre closest to the line of action of the prestressing steel the minimum compressive stress shall not be less than 5 percent of the maximum compressive stress permitted on that fibre unless otherwise approved by the appropriate authority. In the latter case, the minimum compressive stress may be reduced up to a limit of zero.

(*ii*) On the other structures under the same circumstances, a tensile stress (f_t) may be permitted, not exceeding one-tenth of the maximum permissible compressive stresses specified in para (*d*) or 14 *kg./cm.*^{2} for pretensioned and 10 *kg /cm.*^{2} for post-tensioned work whichever is less.

(*iii*) In the case of members assembled out of pre-cast blocks, no tensile stress shall be permitted at any stage of loading upto cracking.

(*iv*) The provisions of (*i*), (*ii*) and (*iii*) do not take into account stresses due to differential shrinkage, bearing friction and other time-dependent factors arising due to the sequence and type of construction adopted. These shall be investigated and allowed for without exceeding the stresses specified above.

(*h*) *Principal tensile stress.* Principal tensile stress resulting from direct forces with or without bending and shear combined shall be calculated at critical sections at the centroid of the section and at points where there is a material change in the width of the section (as at the junctions of flange and web in the compression zone) and shall not exceed $0{\cdot}4\sqrt{F_c}$, where F_c is in *kg./cm*^{2}. when the case of reinforced concrete shall be provided to take up fully all the principal stresses.

(*i*) *Transmission lengths.* Transmission by bond is governed by many variables, such as surface quality of wire, concrete strength

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at transfer, creep characteristics, available mechanical locking, wedge action. In view of these many variables, defined stipulations have not been made in the I.S. Code regarding transmission lengths ; tests are recommended to determine the values under the most unfavourable conditions in each casting yard both under normal conditions and under ultimate loads. While investigating the transmission length check shall also be made for principal tensile stresses to comply with the requirement of clause (*h*) above.

For general guidance it is recommended that one half the transmission length shall overhang the support in simply supported beam. Where there is end fixing, the whole of the transmission length shall overhang.

(*i*) *Bond stress.* Where prestress is imparted and maintained by bond, the average bond stress under working loads shall be limited to one-thirtieth of the cube strength F_c.

§218. Stages of Loading to be Considered in Design

The design shall allow for effect of all loads to which the member will be subjected during construction, handling and under service.

In order to determine all the possible extremes of stresses. the various loading conditions shall each be investigated in the same order in which they occur.

The different stages of loading defined below shall be investigated and in the cases of stages I and III, the various stresses to which the member is subjected shall be maintained within the limits specified under the clauses mentioned above. In respect of other stages, the resulting factor of safety shall be investigated and shall be brought within the permissible limits specified.

(*i*) Stage I : At transfer of prestress

(*ii*) Stage II : At handling and erection

(*iii*) Stage III : At design load

(*iv*) Stage IV : At appearance of first crack

(*v*) Stage V : At failure.

Condition at transfer of prestress. At this stage, the maximum possible prestress, that is, before any losses have occurred, shall be considered. Any relieving bending moment due to the dead weight of the prestressed member may be taken into account, provided the member is so handled and hoisted as to allow such bending moment to act uninterruptedly throughout the operation.

It shall be ensured that any tensile stress allowed to develop in the member at this stage does not exceed one-tenth of the maximum permissible compressive stress specified in para (*d*) above or 14 $kg./cm.^2$ for pretensioned and 10 $kg./cm.^2$ for post-tensioned work whichever is less. However, in the case of members assembled out of precast blocks, no tensile stress shall be permissible.

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Condition during handling and erection. At this stage, the residual prestress, at the age of the particular operation, shall be considered in conjunction with any stresses caused by the handling or erection of the member. The compressive stresses thus computed shall not exceed 50 per cent of cube strength of the concrete at the time of handling and erection. Tensile stress upto a limit of 50 per cent above those specified shall be permissible.

The design shall specify the points from which the prestressed concrete member can be safely lifted or slung for erection.

Condition under design load. At this stage, both the maximum effective prestress (that is immediately after and before any losses have taken effect) shall be considered in conjunction with the corresponding worst possible combination of dead loads, live loads and other forces, such as wind, temperature variation, *etc.*, and it shall be ensured that the stresses specified above for concrete are not exceeded.

Condition at first crack. This stage requires the calculation of the load which, in conjunction with the minimum effective prestress, produces a tensile stress in the member equal to the modulus of rupture of the concrete. The load thus computed shall not be less than 1·1 times the dead load plus 1·25 times the live load placed in such a way as to induce the maximum stresses in the member concerned. For the purpose of this clause, the modulus of rupture shall be taken as $2{\cdot}25\sqrt{F_c}$.

The cracking load of a prestressed member assembled out of precast blocks shall be computed on the assumption that the concrete is incapable of taking any tensile stresses.

Condition at failure. The ultimate loads assumed for purposes of determining the condition at failure shall not be less than the following :

(*a*) For statically determinate structures subjected to dynamic loading, such as bridges.	1·5 $DL+3LL$.
(*b*) For statically determinate structures not subjected to dynamic loading as buildings.	1·5 $DL+2{\cdot}5\ LL$ or $2[DL+LL]$ whichever is greater.

Where DL and LL are respectively the design dead load and the maximum live load inclusive of the impact.

Concrete cover and spacing of prestressing steel

(*a*) In pretensioned work, the cover of concrete measured from the outside of the prestressing tendon shall be at least 20 *mm*.

(*b*) Where cables and large sized bars are used, the minimum clear cover shall be at least 30 *mm*. or the size of the cable or bar whichever is bigger.

(*c*) Where prestressed concrete members are exposed to weather, the cover specified above shall be increased by 12 *mm*.

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(*d*) The minimum clear spacing between single wires in pre-tensioned system shall not be less than three times the diameter of the wire used or 1½ times the maximum size of aggregate whichever is more.

(*e*) The minimum clear distance between cables or large bars shall be not less than 40 *mm.* or the maximum dimension of the cable or bar, 6 5 *mm.* in excess of the largest size of aggregate used, whichever is greater. Grouping of cables is permissible but not more than two cables should be touching each other in a vertical plane with the spacing between the so grouped cables being not less than 5 *cm.* (Fig. 853).

Fig. 853. Spacing of Groups of cables.

§219. Preliminary Design

The lever arm corresponding to the moment of resistance is found to the approximately 0·65 *h* under the working load, where *h* is the total depth of the beam section.

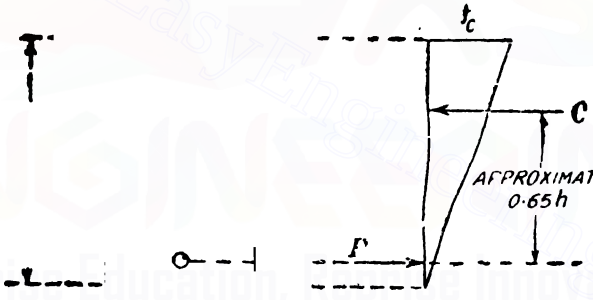

Fig. 854

Let *F* be the prestressing force in the tendons.

Let M_t = Total bending moment (after the application of the live load)

C = Total compressive resistance of the section

$$P = C = \frac{M_t}{0{\cdot}65\,h} \qquad \ldots(i)$$

Area of steel required

$$= A_t = \frac{}{f_s} = \frac{M_t}{0{\cdot}65\,hf_s} \qquad \ldots(ii)$$

where f_s = Safe stress in steel.

Let f_c = maximum compressive stress in concrete permissible. The conditions are initially assumed so that the stress distribution in concrete is as shown in Fig. 854.

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$\therefore$ Total compressive resistance

$$= C = \frac{f_c}{2} A_c$$

$$= 0{\cdot}5 f_c A_c$$

where A_c = Total area of concrete.

$$\therefore \quad A_c = \frac{C}{0{\cdot}5 f_c}$$

or $$A_c = \frac{A_t f_s}{0{\cdot}5 f_c} \qquad \ldots (iii)$$

The depth of the beam section may be estimated in a number of ways. An approximate rule is to adopt 70 per cent of the corresponding depth for conventional reinforced concrete construction. Other empirical rules are also adopted. For prestressed slabs the thickness may be taken as $\frac{1}{35}$ of the span for heavy loads to $\frac{1}{55}$ of the span for light loads.

For beams of usual proportion the following formula will be found useful.

$$h = K\sqrt{M_t}$$

where k = a coefficient between 10 and 14

and M_t = total B.M. in *tm*.

Design 210. *Design a prestressed concrete beam to the following requirements :*

(*i*) *Span*	= *15 metres*
(*ii*) *Superimposed load*	= *3400 kg./metre*
(*iii*) *Cube strength of concrete at 28 days*	= F_c = *350 kg./cm.*^{2}
(*iv*) *Safe strees in concrete at transfer of prestress*	= $f_r = 0{\cdot}5\ F_c$
(*v*) *Safe stress in concrete due to final prestress*	= $f_s = 0{\cdot}4\ F_c$
(*vi*) *Total loss of prestress*	= *20%*
(*vii*) *Allowable tensile stress in concrete*	= *10 kg./cm.*^{2}
(*viii*) *Ultimate stress in steel*	= *15000 kg./cm.*^{2}
(*ix*) *Safe stress in steel*	= *60% of ultimate stress.*

Solution.

Permissible stresses

$$f_r = 0{\cdot}5\ F_c = 0{\cdot}5 \times 350 = 175 \text{ kg./cm.}^2$$

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$$f_c = 0{\cdot}4\ F_c = 0{\cdot}4 \times 350$$
$$= 140\ kg./cm.^2$$

Permissible tensile stress in concrete
$$= 10\ kg./cm.^2$$

Permissible tensile stress in steel
$$= 0{\cdot}6 \times 15000$$
$$= 9000\ kg./cm.^2$$

Maximum B.M. due to live load
$$= M_l = \frac{wl^2}{8}$$
$$= \frac{3400 \times 15^2}{8}$$
$$= 95630\ kg.\ m.$$
$$= 95{\cdot}63\ tm.$$

Approximate dead load moment
$$= 20\%\ M_l$$
$$= 0{\cdot}2 \times 95{\cdot}63\ tm.$$
$$= 19{\cdot}13\ tm.$$

$\therefore$ Total B.M. $= M_t = 95{\cdot}63 + 19\ 13$
$$= 114{\cdot}76\ tm.$$

Overall depth $= h = k\sqrt{M_t}$
$$= 11\sqrt{M_t}$$
$$= 11\sqrt{114{\cdot}76}\ cm.$$
$$= 117{\cdot}9\ cm.$$

Let us provide an overall depth of 120 *cm.*

$\therefore$ Prestressing force
$$= P = \frac{M_t}{0\ 65\ h}$$
$$= \frac{114{\cdot}76 \times 1000 \times 100}{0{\cdot}65 \times 120}\ kg.$$
$$= 147{,}200\ kg.$$

Area of concrete required
$$= \frac{P}{0{\cdot}5\ f_c}$$
$$= \frac{147200}{0{\cdot}5 \times 140}\ cm.$$
$$= 2103\ cm.^2$$

Fig. 855 shows the proposed section for the beam.

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Actual area of the section

$$= 2 \times 50 \times 12 + 12 \times 96 \ cm.^2$$

$$= 2352 \ cm.^2$$

Moment of Inertia of the section

$$= I = \frac{50 \times 120^3}{12} - \frac{38 \times 96^3}{12} \ cm.^4$$

$$= 4,398,000 \ cm.^4$$

$\therefore$ Section modulus

$$= \frac{4,398,000}{60} \ cm.^3$$

$$= 73,300 \ cm.^3$$

Actual dead load of the beam per *metre* run

$$= 2352 \times 0{\cdot}24$$

$$= 564{\cdot}5 \ kg./m.$$

$\therefore$ Actual dead load moment

$$= 564{\cdot}5 \times \frac{15^2}{8} \times 100 \ kg.\ cm.$$

$$= 158800 \ kg.\ cm.$$

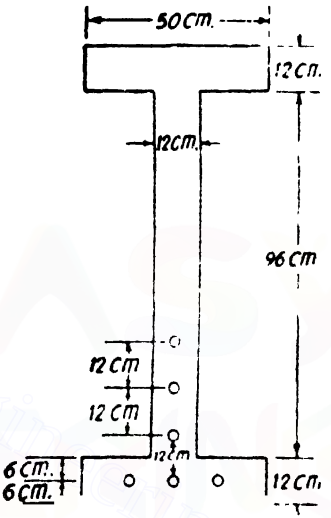

Fig. 855

Providing 6 *mm.* wires (area of each wire $= 0{\cdot}28 \ cm.^2$)

$\therefore$ Number of wires required

$$= \frac{147200}{0{\cdot}28 \times 9000}$$

$$= 59 \ wires.$$

Provide 6 cables with 10 wires per cable as shown in Fig. 855.

Actual area of steel $= 60 \times 0{\cdot}28$

$$= 16{\cdot}8 \ cm.^2$$

$\therefore$ Final prestressing force

$$= 9000 \times 16{\cdot}8 \ kg.$$

$$= 151200 \ kg.$$

Hence initial prestressing force

$$= \frac{151200}{0{\cdot}8} \ kg.$$

$$= 189,000 \ kg.$$

Check for Stresses

Distance of the centroid of cables from the centre at the bottom row of cables

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$$1 \times 12 + 1 \times 24 + 1 \times 36$$

$$= 12 \text{ cm.}$$

Actual eccentricity $e = 60 - 6 - 12$ cm

$$= 42 \text{ cm.}$$

Initial Stresses

Stress at top $= -\frac{189000}{2352} - \frac{189000 \times 42}{73300} + \frac{1588000}{73300}$ kg/cm.²

$= 80{\cdot}54 - 108{\cdot}2 + 21{\cdot}67$ kg./cm.²

$= -5{\cdot}99$ kg/cm.²

Stress at bottom $= \frac{189000}{2352} + \frac{189000 \times 42}{73300} - \frac{1588000}{73300}$ kg./cm.²

$= 80{\cdot}54 + 108{\cdot}2 - 21{\cdot}67$

$= 167{\cdot}07$ kg/cm.²

(less than 175 kg./cm.²)

Final Stresses

Stress at top

$$= \frac{151200}{2352} - \frac{151200 \times 42}{73300} + \frac{1588000}{73300} + \frac{95630{\cdot}0}{73300} \text{ kg/cm.}^2$$

$= 64{\cdot}28 - 86{\cdot}64 + 21{\cdot}67 + 130{\cdot}5$ kg./cm.²

$= 129{\cdot}81$ kg./cm.² (less than 140 kg/cm.²)

Stress at bottom

$$= \frac{151200}{2352} + \frac{151200 \times 42}{73300} - \frac{1588000}{73300} - \frac{9563000}{73300} \text{ kg./cm.}^2$$

$= 64{\cdot}28 + 86{\cdot}64 - 21{\cdot}67 - 130{\cdot}5$ kg./cm.²

$= -1{\cdot}25$ kg./cm.²

Hence the beam section is safe.

§220. The Lever Arm Conception

Fig. 856 (*a*) shows the cross-section of a prestressed concrete beam. The stress distribution in concrete due to dead load is shown in Fig. 856 (*b*). Let f_d be the extreme fibre stress due to dead load moment.

The stresses caused by the prestressing force P alone are shown in Fig 856 (*c*). Let f_{pt} and f_{pb} be the stresses in the extreme top and bottom fibres due to this condition. Let C be the resultant compressive resistance offered by the section. Obviously P and C will have the same line of action, for this condition. Of course $P = C$ and the forces act in opposite directions.

The stresses produced due to the combined action of prestressing force and dead load moment are shown in Fig. 856 (*d*). For this condition also the resultant compressive resistance equals C.

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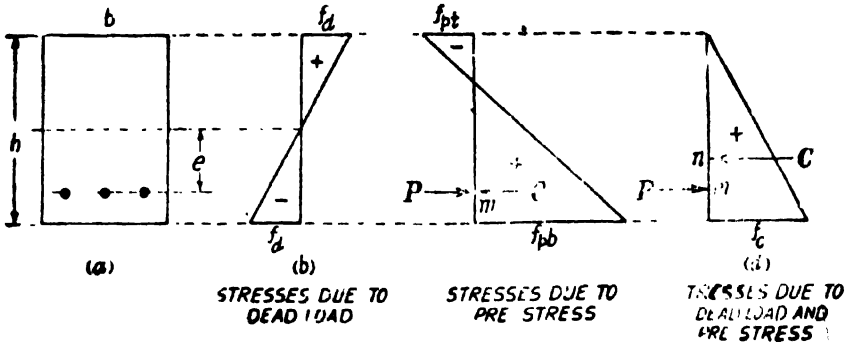

Fig. 856

The line of action of C will be shifted upwards from m to n But the line of action P has not changed. The shift mn of C shall be such that the forces P and C will form a couple or moment of resistance to balance the dead load moment, M_d. The distance mn *i.e.* the distance between the line of action of the prestressing force P and the resultant compressive resistance C is called the lever arm of the beam section.

Corresponding to the condition shown in Fig. 856 (*c*) the lever arm 0. But as the beam reaches the condition (*d*) the line of action of the compressive resistance C shifts by mn

Equating the dead load moment to the moment of resistance.

We have $$M_d = P \times mn \qquad ...(i)$$

$$mn = \frac{M_d}{P}$$

Now consider the effect of the live load. Fig. 857 (*c*) shows the stress distribution due to live load alone. Let M_l be the live load moment. Fig. 857 (*d*) shows the final stress distribution. For this

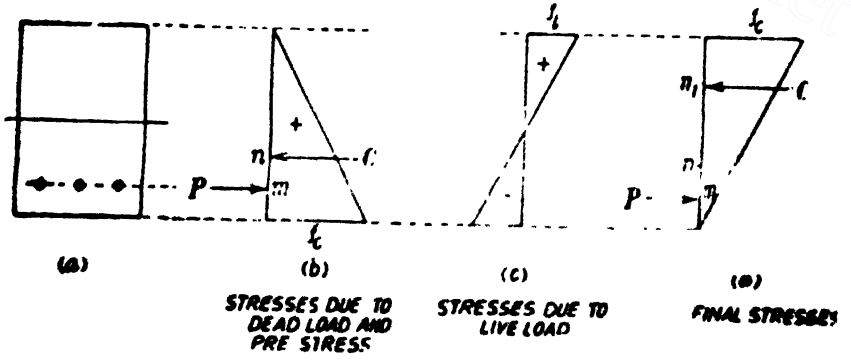

Fig. 857

condition the line of action of C has shifted to n_1. The total compressive resistance, however, is the same as in the case of condition shown in Fig. 857 (*b*).

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Now the lever arm of the beam $= mn_1$ and the moment of resistance of the beam $= P \times mn_1$. Equating the maximum bending moment to the moment of resistance, we have,

$$M_d + M_l = P \times mn_1$$

But $M_d = P \times mn$ ...eq (i)

$\therefore$ $M_l = P \times nn_1 = P \times$ shift of C ...(ii)

In general, if a moment M is applied, then C will shift such that,

$P \times$ shift of C = applied moment ...(iii)

Further, change in extreme fibre stress

$$= \frac{\text{Applied or additional moment}}{\text{section modulus}}$$

i.e. $$\partial f = \frac{\partial M}{Z}$$

The above properties are illustrated in the following example. Fig. 858 (*a*) shows the section of a prestressed concrete beam of span 10 *metres*. The dead load of the beam is 144 *kg./m.*

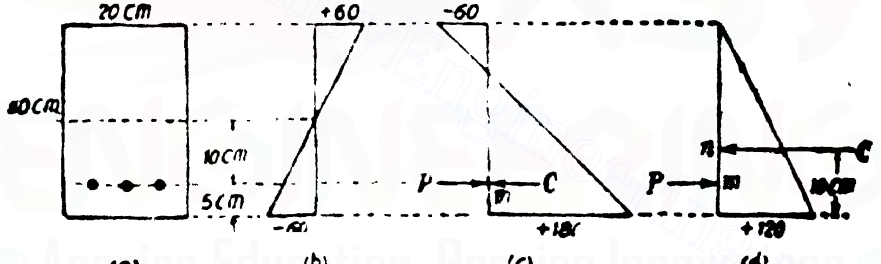

Fig. 858

The dead load moment

$$M_d = 144 \times \frac{10^2}{8} \times 100$$

$$= 180{,}000 \text{ kg. cm.}$$

Section modulus $= Z = 20 \times \frac{30^2}{6}$

$$= 3000 \text{ cm.}^3$$

Extreme stress due to dead load

$$= \frac{180{,}000}{3000}$$

$$= 60 \text{ kg./cm.}^2$$

Let the prestressing force be $P = 36000$ *kg.* at an eccentricity of 10 *cm.*

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Extreme fibre stresses due to prestressing force are given by

$$f = \frac{36000}{600} \pm \frac{36000 \times 10}{3000}$$

$$= 60 \pm 120$$

$$= -60 \text{ kg./cm.}^2 \text{ at top}$$

and $+180 \text{ kg./cm.}^2$ at bottom.

For this condition the forces P and C have the same line of action and the lever arm equals zero. The stresses due to the combined action of the prestressing force and the dead load moment are shown in Fig. 858 (d) In this condition the force C is shifted to the position n.

The internal couple formed by C and P will balance the dead load moment M_d.

$$[P \text{ or } C] \times mn.$$

$$= 36000 \times (10 - 5) \text{ kg. cm.}$$

$$= 180000 \text{ kg. cm.} = M_d$$

This satisfies the condition

$$P \times \text{Shift of } C = \text{applied moment.}$$

In case the applied moment is zero, the shift of C is zero and thus P and C are along the same line.

Also the change in stress between condition shown in Figs. 858 (c) and (d).

$$= 180 - 120 \text{ kg /cm.}^2$$

$$= 60 \text{ kg /cm.}^2$$

$\therefore$ Moment which causes this change in stress

$$= \partial M = \partial f . Z$$

$$= 60 \times 3000$$

$$= 180000 \text{ kg. cm.}$$

$$= M_d.$$

Suppose we now apply an additional load of 150 kg./m.

$$\text{Additional moment} = \partial M = 150 \times \frac{10^2}{8} \times 100 \text{ kg. cm.}$$

$$= 187500 \text{ kg. cm.}$$

Additional extreme fibre stress

$$= \pm \frac{187500}{3000} \text{ kg./cm.}^2$$

$$= \pm 62{\cdot}5 \text{ kg./cm.}^2$$

Hence actual extreme stresses will be as follows

Stress at top $= 0 + 62{\cdot}5$

$= +62{\cdot}5 \text{ kg./cm.}^2$

Stress at bottom $= +120 - 62{\cdot}5$

$= 57{\cdot}5 \text{ kg./cm.}^2$

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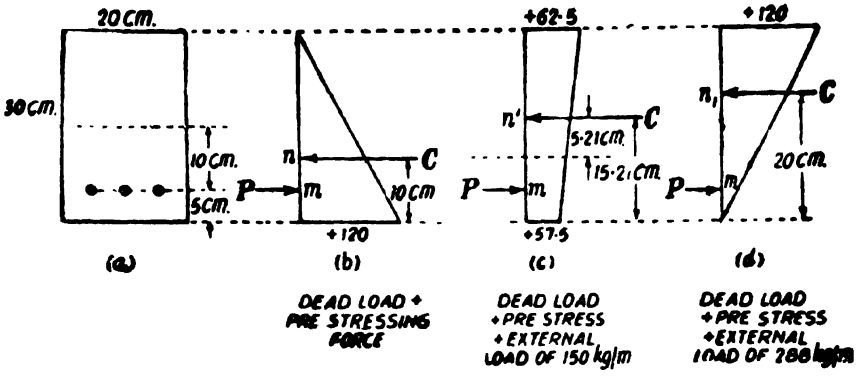

Fig. 859

$$\therefore \text{ Shift of } C = -\frac{\text{additional moment}}{P}$$

$$= \frac{187500}{36000}$$

$$= 5\,21 \text{ cm.}$$

Additional moment = Additional stress $\times Z$

$= 62{\cdot}5 \times 3000$

$= 187500$ *kg. cm.*

Hence as the beam is made to carry heavier loads C changes its position and thus the lever arm increases. The beam thus has carried the additional load of 150 *kg/m.* without exceeding the safe limits.

In fact if the safe compressive stress be 120 *kg./cm.*^{2} the beam can still further carry an additional load equal to $\frac{57{\cdot}5}{62{\cdot}5} \times 150 = 138$ *kg/m.*

Corresponding to this condition the load carried by the beam = dead load + External load of 150 + 138 = 288 *kg/m* (See Fig. 859).

§221. Kern Distance

Consider the beam section shown in Fig. 860. Let the prestressing force P be 36000 *kg.*

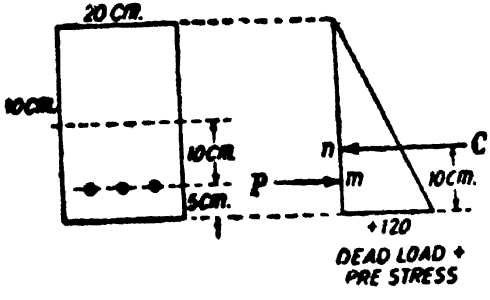

Fig. 860

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The dead load moment for the beam = 180,000 *kg. cm.* Let the safe compressive stress be 120 *kg./cm²*. Let *m* and *n* be the points of action of *P* and *C*. The distance *mn* = 10 − 5 = 5 *cm.* and the dead moment = $M_d = P \times mn$

$$= 36000 \times 5 = 180{,}000 \text{ kg. cm.}$$

Suppose the dead load moment had been less than 180,000 *kg. cm.* then the distance *mn.* would be less than 5 *cm.* and the stress distribution on the section would have exceeded the safe limits as shown in Fig. 861.

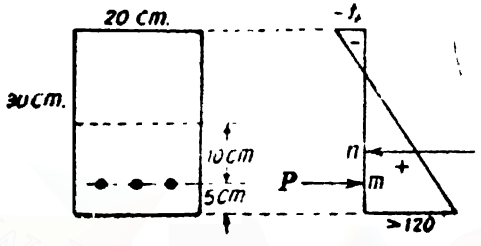

Fig. 861

Hence, in order the safe stresses are not exceeded, the point *n* which is the point of application of *C* when the beam is under the action of prestress and dead load should not be below a certain level. For the example considered *mn* should not be less than 5 *cm.*

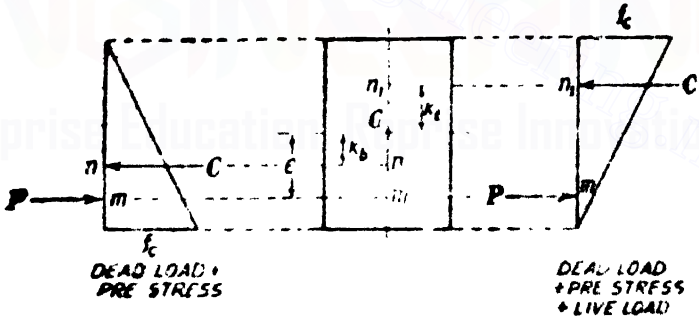

Fig 862

The point *n* below which *C* should not act so that the safe stresses are not exceeded is called *the lower kern point.* Now as the beam is subjected to live load, the bending moment increases and hence the point of application of *C* reaches a higher level. In order the safe stresses may not be exceeded the point n_1 corresponding to the line of action of *C* should not reach a higher level than a certain definite level. The point n_1 above which *C* should not act so that the safe stresses are not exceeded is called *the upper kern point.* Let *G* be the centroid of the beam section. The distance $Gn = K_b$ and $Gn_1 = K_t$ are called the *kern distances.*

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As the point of the application of C reaches the lower kern point the maximum permissible stress is induced at the bottom fibre and the minimum permissible stress is induced in the top fibre. But as the point of application of C reaches the upper kern point, the maximum permissible stress is induced in the top fibre and the minimum permissible stress is induced in the bottom fibre.

Let f_o = Average compressive stress on the section
= Stress at G.

$\therefore$ $P = f_o A$

The following relations will be found useful

$$M_d = P(e - K_b) \quad ...(i)$$

$$M_l = P(K_t + K_b) \quad ...(ii)$$

$$M_d + M_l = M_t = P(e + K_t) \quad ...(iii)$$

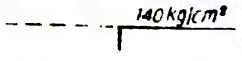
$$P = f_o A \quad ...(iv)$$

$$\partial M = \partial f Z \quad ...(v)$$

Design 211. *Design a prestressed concrete slab to the following requirements :*

Span of the slab	$= 15$ *metres*
Superimposed load	$= 2000$ *kg./metre*^{2}
Safe stress in concrete	$= 140$ *kg./cm.*^{2}
Safe stress in steel	$= 9000$ *kg./cm.*^{2}

Solution Consider a one *metre* wide strip of the slab.



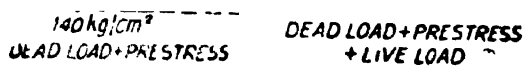


Fig 863

The stress distribution in concrete before and after application of the live load will be as shown in Fig. 863.

$$\partial M = M_l = 2000 \times \frac{15^2}{8} \times 100 \text{ kg. cm.} = 5{,}625{,}000 \text{ kg. cm.}$$

$$\partial M = \partial f \,.\, Z$$

$\therefore$ $5625000 = 140 \times Z$

$$Z = \frac{5625000}{140} \text{ cm.}^3$$

$$= 40180 \text{ cm.}^3$$

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$$Z = \frac{bd^2}{6}$$

$$= 100 \times \frac{d^2}{6} = 40180$$

$$d = 49{\cdot}1 \text{ cm.}$$

Let us provide a slab 50 *cm.* thick.

Actual area of slab section per *metre* width

$$= A = 100 \times 50 = 5000 \text{ cm.}^2$$

Prestressing force $= \frac{f_c}{2} \times A$

$$= \frac{140}{2} \times 5000 = 350{,}000 \text{ kg.}$$

$$K_t = K_b = \frac{d}{6}$$

$$= \frac{50}{6} = 8{\cdot}33 \text{ cm.}$$

Dead load $= 1 \times 0{\cdot}5 \times 2400$

$= 1200$ *kg./m.* for one *metre* wide strip.

∴ Dead load moment

$$= M_d = 1200 \times \frac{15^2}{8} \times 100 \text{ kg. cm.}$$

$$= 3{,}375{,}000 \text{ kg. cm.}$$

eccentricity $e = \frac{M_d}{F} + K_b$

$$= \frac{3375000}{350000} + 8{\cdot}33$$

$$= 18 \text{ cm.}$$

Design 212. *Design a simply supported prestressed concrete slab for the following conditions :*

Span of the slab = 12 metres.

Safe stress in concrete = 140 kg./cm.^{2}

Safe stress in steel = 9000 kg./cm.^{2}

Superimposed load = 2000 kg./metre.^{2}

Solution. Consider a one *metre* wide strip of the slab.

Live load bending moment

$$= M_l = \frac{2000 \times 12^2}{8} \times 100 = 3{,}600{,}000 \text{ kg. cm.}$$

Section modulus required

$$= Z = \frac{M_l}{f_c} = \frac{3600000}{140} = 25714{\cdot}285 \text{ cm.}^3$$

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$$Z = \frac{bd^2}{6} = 100 \frac{d^2}{6} = 25714{\cdot}285$$

$\therefore$ $d = 39{\cdot}28$ *cm.*

Provide a thickness of 40 *cm.*

D.L. per *metre* $= 1 \times 0{\cdot}4 \times 2400 = 960$ *kg./m.*

D.L. moment $= \frac{960 \times 12^2}{8} \times 100 = 1728000$ *kg. cm.*

Prestressing force

$$= P = \frac{f_c}{2} A = \frac{140}{2} (100 \times 40) = 280000 \text{ kg.}$$

Eccentricity $e = \frac{2M_d + M_l}{2P}$

$$= \frac{2(1728000) + 3600000}{2 \times 280000} = 12{\cdot}60 \text{ cm.}$$

Area of tendons $= A_t = \frac{280000}{9000} = 31{\cdot}11$ *cm.*^{2}

Using cables of 10 wires of 5 *mm.* diameter each,

Area of steel in one cable

$$= 10 \times 0{\cdot}196 = 1{\cdot}96 \text{ cm.}^2$$

$\therefore$ Spacing of cables

$$= \frac{1{\cdot}96 \times 100}{31{\cdot}11} = 6{\cdot}30 \text{ say } 6 \text{ cm. centres.}$$

Design 213. *A rectangular prestressed concrete beam has a span of 12 metres and bears a live load of 1500 kg./m, excluding self-weight. Given $f_c = 160$ kg./cm.^{2}, $f_s = 10500$ kg./cm.^{2}, design the beam using 6 mm. tendons.*

Solution Live load bending moment

$$= M_l = \frac{1500 \times 12^2}{8} \times 100 = 2{,}700{,}000 \text{ kg. cm.}$$

Section modulus required

$$= Z = \frac{2{,}700{,}000}{160} = 16875 \text{ cm.}^3$$

Let the width of the beam be 30 *cm.*

$$Z = \frac{bd^2}{6} = \frac{30d^2}{6} = 16875 \quad \therefore \quad d = 58 \text{ cm.}$$

Hence the beam section is 30 cm. × 58 cm.

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Dead load of the beam

$$= 0{\cdot}30 \times 0{\cdot}58 \times 2400 = 417{\cdot}6 \text{ kg/metre}$$

D.L. moment $= \frac{417{\cdot}6 \times 12^2}{8} \times 100 = 751680 \text{ kg. cm.}$

Prestressing force

$$P = \frac{f_c}{2} A$$

$$= \frac{160}{2} \times (30 \times 58) = 139200 \text{ kg.}$$

Area of tendons $= A_t = \frac{139200}{10500} = 13{\cdot}26 \text{ cm.}^2$

Number of 6 *mm.* diameter wires required

$$= \frac{13{\cdot}26}{0{\cdot}28} = 48 \text{ wires.}$$

These can be arranged in 6 cables with 8 wires per cable

Eccentricity $e = \frac{2M_d + M_l}{2P}$

$$= \frac{2(751680) + 2700000}{2 \times 139200} = 15{\cdot}10 \text{ cm.}$$

§222. Shear

We know that in the case of reinforced concrete beams the design for shear was made on the assumption that the shear failure is liable to be caused by diagonal tension *i.e.*, the failure was taken to be due to principal tensile stresses But, in the case of prestressed concrete members, however, due to the introduction of compressive

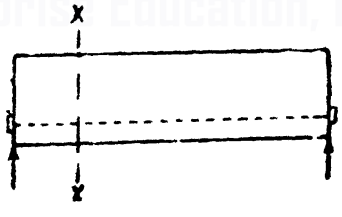

Fig. 864

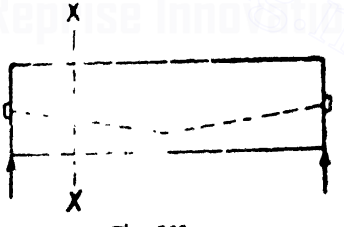

Fig. 865

stresses by prestressing, the principal tensile stresses are reduced. Further if the cables are inclined or curved, the vertical component of the tensions in the cables also will resist shear.

Consider the prestressed beam shown in Fig. 864. In this case the beam is prestressed by a straight tendon. Consider any arbitrary section *XX*. The shear force at the section has to be resisted entirely by concrete alone.

Consider the prestressed beam provided with inclined tendons. (Fig. 865).

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Consider the section XX. Let the shear force at the section XX due to dead load and live load be V_d and V_l.

Let P be the tension in the tendons.

Let θ be the inclination of the tendon with the horizontal.

The actual shear force to be resisted by concrete

$$= V_c = V_d + V_l - P \sin \theta.$$

Similarly consider the prestressed beam provided with curved tendons, Fig. 867.

In this case also at the section XX the shear to be resisted by concrete

$$= V_c = V_d + V_l - P \sin \theta$$

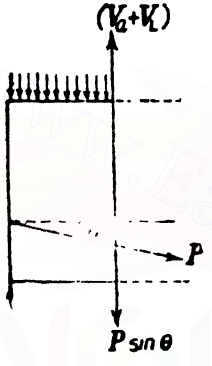

Fig. 866

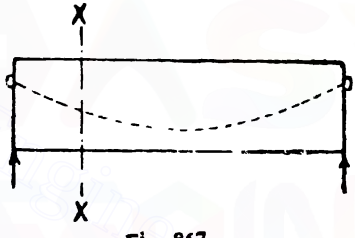

Fig. 867

where θ = Inclination of the tendon with the horizontal at the section XX.

Now the principal tensile stress in a beam section at any point can be determined as follows :

(*i*) Determine the net shear force V_c to be resisted by concrete at the section

$$V_c = V_d + V_l - P \sin \theta.$$

It may also be noted, in some exceptional cases

$$V_c = V_p + V_l + P \sin \theta.$$

This may happen when the inclination of the cable is such that the vertical component of the tension in the cable adds to the shear.

(*ii*) Now find the shear stress q at any level by the relation

$$q = \frac{V_c A\bar{y}}{I b}$$

where q = Shear stress intensity at the level

$A\bar{y}$ = Static moment of the cross-sectional area of the beam *above* or *below* that level *about* the centroidal axis.

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I = Moment of inertia of the beam section about the centroidal axis.

b = Width of the beam at the level.

(*iii*) Now find the normal stress distribution due to prestress, its eccentricity and the external bending moment. Let p be the normal stress at any point.

(*iv*) Now find the principle tensile stress p_2 from the relation,

$$p_2 = \sqrt{q^2 + \frac{p^2}{4}} - \frac{p}{2}$$

The principal tensile stress may also be determined by Mohr's circle of stress.

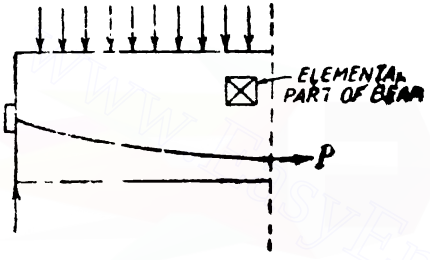

Fig. 868

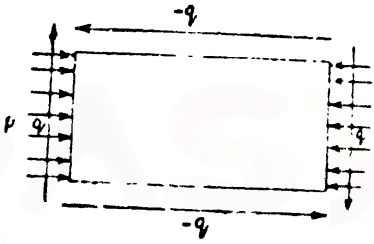

Fig. 869

The graphical solution may be carried out as follows :

Draw the rectangular axes OX and OY. Draw OE perpendicular to OX equal to q. Draw similarly FD also equal to q. Now describe a circle with DE as diameter. Measure OB. OB represents the *principal tensile stress* P_2 and OA represents the *principle compressive stress* P_1. The distance $CG = CH$ represents the *maximum or principal shear stress* q_{max}. In the figure AJ represents a plane perpendicular to AB.

The graphical method may be found convenient to adopt as it even indicates the planes of principal tensile stress. The different planes involved are the following :

Plane AE = Horizontal plane

Plane AD = Vertical plane

Plane AB = Principal plane carrying the principal tensile stress

Plane AJ = Principal plane carrying the principal compressive stress.

It can be realized that the inclination of the principal tensile plane AB with the vertical plane AD is greater than 45°. It may also be noted that the principal compressive stress is a little greater than the compressive fibre stress p; but still, is not usually considered in the design of the beam. The design is based only on the compressive fibre stress. In a likewise manner, it is also a cus-

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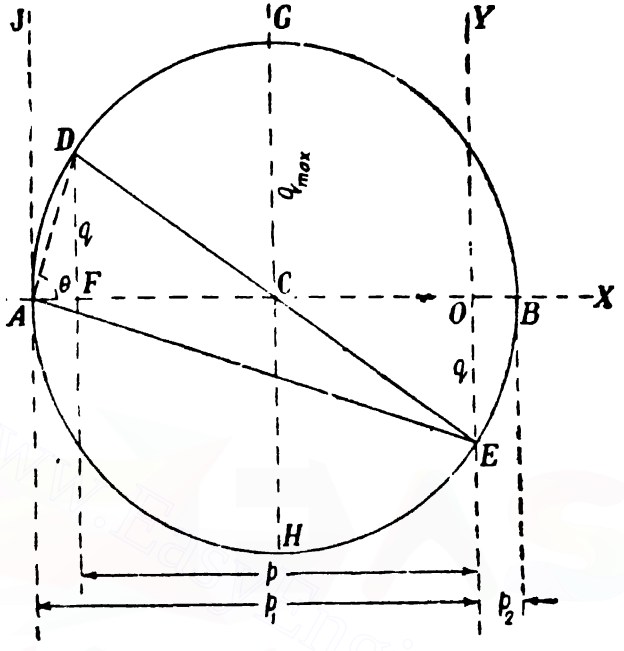

Fig. 870

tom not to take account of the maximum shearing stress q_{max} occurring along planes at 45° to the principal planes. This is due to the fact that the failure will not be due to shear itself but due to tension. It may also be observed that the maximum principal tensile stress does not necessarily occur at the centroidal axis where the maximum vertical shearing stress occurs. Studying the expression for the maximum principal tensile stress given above, it may be realized that the principal tensile stress, at some point, where p is diminished, may work out to be higher, even though the shearing stress may not be a maximum In the case of I-sections the maximum principal tensile stress occurs at a critical point corresponding to the junction of the tension flange and the web.

Design 214. *A prestressed concrete beam section, when subjected to a certain bending moment has a fibre stress distribution as shown in Fig. 871. The total vertical shear in concrete at the section is 236000 kg. Find the principal tensile stress at the centroidal axis and at the junction of the web with the lower flange.*

Solution. Let us first determine the centroidal axis and the moment of inertia of the beam section about the centroidal axis.

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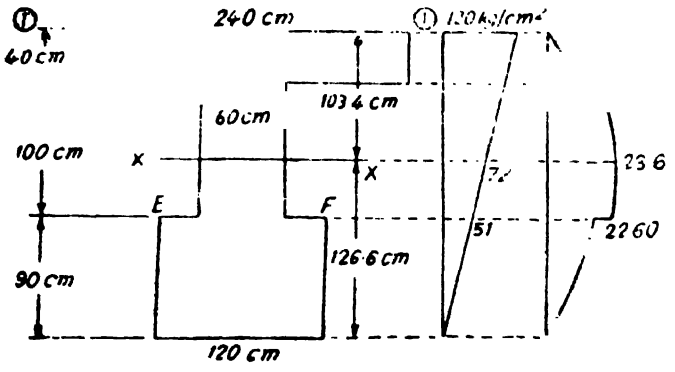

Fig. 871

These calculations are shown in the table below :

Component	area a cm^2	Centroidal distance y from the axis 1-1 cm.	ay $cm.^3$	ay^2 $cm.^4$	I_{self} cm^4
Top flange	9600	20	192000	3840000	$\frac{240\times40^3}{12}=1280000$
Web	6000	90	540000	48600000	$\frac{60\times100^3}{12}=5000000$
Bottom flange	10800	185	1998000	359630000	$\frac{120\times90^3}{12}=7290000$
Total	26400		2730000	422070000	13570000

$\therefore$ Distance of the centroidal axis from the top edge

$$\frac{\Sigma ay}{\Sigma a}$$

$$=\frac{2730000}{26400}=103{\cdot}4\ cm.$$

Moment of Inertia of section about the axis 1—1

$$=I_{1-1}=\Sigma I_{self}+\Sigma ay^2$$

$$=13570000+422070000\ cm.^4$$

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$$= 435640000 \ cm.^4$$

But $I_{1-1} = I_{xx} + Ay^2$

$\therefore$ $I_{xx} = 435640000 - 26400 \times (103{\cdot}4)^2$

$= 153390000 \ cm.^4$

Shear stresses

Shear stress in the web at the level EF

$$a\bar{y} = 120 \times 90 \times 81{\cdot}6 \ cm.^3$$

$$= 881280 \ cm.^3$$

$\therefore$ Shear stress $= q = \dfrac{S\,a\bar{y}}{I\,b}$

$$= \frac{236000 \times 881280}{153390000 \times 60} \ kg./cm.^2$$

$$= 22{\cdot}60 \ kg./cm.^2$$

Shear stress in the web at the neutral axis

$$a\bar{y} = 120 \times 90 \times 81{\cdot}6 + 60 \times 36{\cdot}6 \times 18{\cdot}3 \ cm.^3$$

$$= 881280 + 40190 = 921470 \ cm.^3$$

$\therefore$ Shear stress $= q = \dfrac{S\,a\bar{y}}{I\,b}$

$$= \frac{236000 \times 921470}{153390000 \times 60} \ kg./cm.^2$$

$$= 23{\cdot}60 \ kg./cm.^2$$

Principal tensile stress in the web at the level EF

$$p_2 = \sqrt{q^2 + \frac{p^2}{4}} - \frac{p}{2}$$

$$= \sqrt{22{\cdot}6^2 + \frac{51^2}{4}} - \frac{51}{4} \ kg./cm.^2$$

$$= 8{\cdot}57 \ kg./cm.^2$$

Principal tensile stress in the web at the neutral axis

$$p_2 = \sqrt{23{\cdot}6^2 + \frac{72^2}{4}} - \frac{72}{2} \ kg./cm.^2$$

$$= 7{\cdot}03 \ kg./cm.^2$$

The above results may also be obtained by *Mohr's circle*.

§223. Shear Reinforcement

If the maximum principal tensile stress is less than the allowable limit (10 $kg./cm.^2$), shear reinforcement is not theoretically necessary. Generally nominal stirrups are provided using 10 *mm*.

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diameter, mild steel bars, at a spacing of 30 *cm.* to 60 *cm.* centres.

But in case the maximum principle tensile stress exceeds the allowable limit it will be necessary to provide shear reinforcement. The following empirical method may be adopted :

(*i*) First determine the maximum principal tensile stress p_2 for each section of the beam and also the direction of the principal plane carrving this stress. Let θ be the inclination of the principal plane with the vertical plane.

(*ii*) Let the spacing of the vertical stirrups be p_w.

Then the load to be resisted by one stirrup will be

where b = width of the beam section at the point.

(*ii*) Let A_w be the area provided by one stirrup.

Let t_w be the stress in the stirrup.

$\therefore$ Load taken by one stirrup $= A_w t_w$.

$$\therefore \qquad A_w = \frac{p_2 b p_w}{\sin \theta}$$

$$\therefore \qquad p_w = \frac{A_w t_w \sin \theta}{p_2 b}$$

Design 215. *A prestressed concrete beam is 25 cm. wide and 150 cm. deep. The effective prestress in the tendons is 136200 kg. Under the working loads on the beam, the shear force at a certain section is 77100 kg. The effective prestress at the section may be taken to be acting at an angle of* $\sin^{-1} \frac{1}{6}$ *with the horizontal. The fibre stress under the working loads is 70 kg/cm.*^{2} *at the topmost fibres and zero at the bottommost fibres. If the permissible principal tensile stress be 7 kg./cm.*^{2} *find the spacing of 12 mm. vertical stirrups. Assume that all the tension in concrete is to be carried by the stirrups.*

Solution. Shear resistance provided by the vertical component of the tension in the tendon

$$= P \sin \alpha$$

$$= 136200 \times \tfrac{1}{6} = 22700 \text{ kg.}$$

$\therefore$ Shear force to be resisted by concrete

$$= 77100 - 22700 = 54400 \text{ kg.}$$

We will consider, as an approximation, the stress condition at the centroidal axis.

Shear stress at the centroidal axis

$$= q = \frac{3}{2} \frac{S}{A}$$

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$$= \frac{3}{2} \times \frac{54000}{25 \times 150} \text{ kg./cm.}^2$$

$$= 21{\cdot}75 \text{ kg./cm.}^2$$

Fibre stress at the neutral axis

$$= p = \frac{70}{2} = 35 \text{ kg./cm.}^2$$

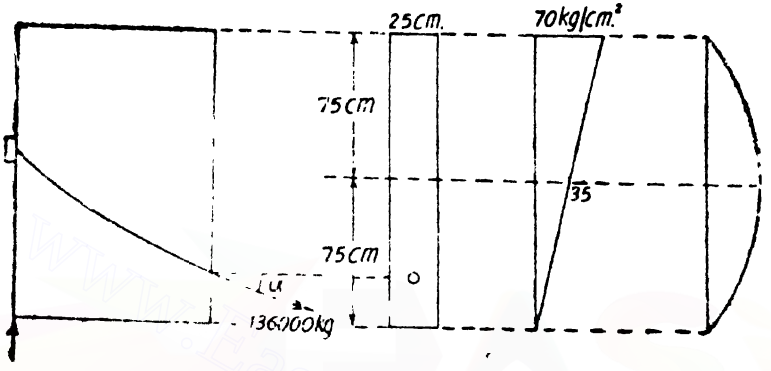

Fig. 872

∴ Principal tensile stress

$$= p_2 = \sqrt{a^2 + \frac{p^2}{4}} - \frac{p}{2}$$

$$= \sqrt{21{\cdot}75^2 + \frac{35^2}{4}} - \frac{35}{2} \text{ kg./cm.}^2$$

$$= 10{\cdot}42 \text{ kg./cm.}^2$$

(This exceeds the given safe limit of 7 kg./cm.²)

If θ be the inclination of the principal plane with the vertical plane

$$\tan 2\theta = \frac{2q}{p}$$

$$= \frac{2 \times 21{\cdot}75}{35}$$

$2\theta = 52^\circ 2'$ or $232^\circ 2'$

∴ $\theta = 26^\circ 13'$ or $116^\circ 13'$

But the plane corresponding to $\theta = 26^\circ 13'$ is the principal compressive plane.

∴ The principal tensile plane is at 116° 13′ with the vertical plane.

∴ Spacing of two-legged stirrups

$$= p_w = \frac{A_w f_w \sin \theta}{p_2 b}$$

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$$= \frac{(2 \times 1{\cdot}13) \times 1400 \times \sin 116° 13'}{10{\cdot}42 \times 25} \text{ cm.}$$

$$= 11{\cdot}15 \text{ cm.}$$

say 11 cm. centres.

§224. Vertical Prestressing

Besides the longitudinal prestressing, sometimes it may be desirable to provide vertical prestressing to reduce or eliminate the principal tensile stress. Vertical prestressing is done by providing high tension vertical steel wires of small diameter at a suitable pitch and stressed adequately.

If at a point in the section of the beam,

p = longitudinal compressive stress in concrete,

p = vertical compressive stress in concrete due to prestressing,

q = shear stress.

Then, the principal stresses will be as follows :

Major principal stress

$$= \frac{p+p'}{2} + \sqrt{\left(\frac{p-p'}{2}\right)^2 + q^2}$$

Minor principal stress

$$= \frac{p+p'}{2} - \sqrt{\left(\frac{p-p'}{2}\right)^2 + q^2}$$

or, the principal tensile stress

$$= \sqrt{\left(\frac{p-p'}{2}\right)^2 + q^2} - \frac{p+p'}{2}$$

Design 216. *The horizontal stress at the centroid of a concrete beam of rectangular cross-section 12·5 cm. × 25 cm. is 70 kg./cm.² and the maximum shearing force on the beam section is 6800 kg. Find the maximum principal tensile stress. Also find the minimum vertical prestress required to eliminate this principal tensile stress.*

Solution

$$A = 12{\cdot}5 \times 25 = 312{\cdot}5 \text{ cm.}^2$$

$$p = 70 \text{ kg./cm.}^2$$

$$q = \frac{3}{2} \times \text{Average shear stress}$$

$$= \frac{3}{2} \times \frac{6800}{312{\cdot}5} = 32{\cdot}64 \text{ kg./cm.}^2$$

$$\textbf{Principal tensile stress} = p_2 = \sqrt{\frac{p^2}{4} + q^2} - \frac{p}{2}$$

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$$= -\sqrt{\frac{70^2}{4} + 32{\cdot}64^2} - \frac{70}{2}$$

$$= 47{\cdot}86 - 35$$

$$= 12{\cdot}86 \text{ kg./cm.}^2$$

Let p' be the minimum vertical prestress required to eliminate principal tensile stress.

$$\sqrt{\left(\frac{p-p'}{2}\right)^2 + q^2} - \frac{p+p'}{2} = 0$$

$$\sqrt{\left(\frac{70-p'}{2}\right)^2 + 32\,64^2} = \frac{70+p'}{2}$$

$$\therefore \quad \left(\frac{70-p'}{2}\right)^2 + 32{\cdot}64^2 = \left(\frac{70+p'}{2}\right)^2$$

$$p' = 15{\cdot}22 \text{ kg./cm}^2.$$

§225. Tension Members

Long tension members like staybacks for retaining walls and the rods for arches may be made of prestressed concrete.

The analysis of prestressed tension members can be made by the following approaches :

(*a*) The member may be taken to be made of concrete and subjected to uniform compression by prestressing so that the member may be able to carry tension due to external loads. If no cracks are developed in the concrete the member will be able to carry a total tensile force equal to the precompression plus the tensile capacity of the concrete in the section.

(*b*) We may regard the member as mainly of high tensile steel which is pretensioned to reduce the deflection under the external loading.

(*c*) We may also consider that the member is of composite section consisting of steel and concrete.

Of the above approaches, it will be found convenient to adopt the last approach.

Fig. 873 shows a prestressed concrete member subjected to an external tension T.

Let P_o be the initial prestressing force in tendons.

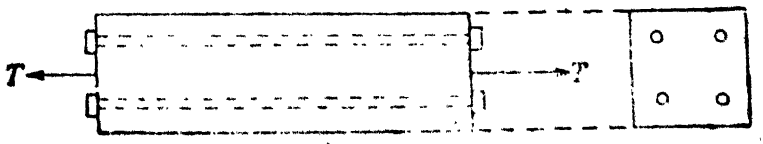

Fig. 873

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∴ Compressive stress in concrete due to prestressing force alone

$$= f_c = \frac{P_e}{A_c}$$

where A_c = actual area of concrete

Let A_s = area of steel

Equivalent concrete area

$$A_c = A_c + mA_s$$

where m = modular ratio.

Or, if A_g be the gross area of the section

Equivalent concrete area

$$= A_c = A_g + (m-1)\,A_s$$

The above expression for the equivalent concrete area will be valid only if the section is grouted well. If the section is not grouted well then it is more correct to take the equivalent concrete area as

$$A_c + mA_s$$

Tensile stress in concrete due to external tension $= f_t = \frac{T}{A_c}$

And, the tensile stress in steel

$$= f_s = m\frac{T}{A_c}$$

Hence, the resultant stresses in concrete and steel due to effective prestress and the external load will be as follows :

Resultant stress in concrete

$$= \frac{P}{A_c} - \frac{T}{A_c}$$

Resultant stress in steel

$$= f_e + m\frac{T}{A_c}$$

where f_e = effective prestress intensity in steel.

If tension in concrete should be just avoided the condition to be satisfied is

$$\frac{P}{A_c} = \frac{T}{A_c}$$

$$\therefore \quad T = \frac{A_c}{A_c} P.$$

Design 217. *A prestressed concrete member is 20 cm. × 20 cm. the tendons being subjected to a stress of 9000 kg/cm². The tendons provide an area of 5 cm². Find the external tension the member can carry if the stress in concrete under the action of external loading is*

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zero. Take m=6. Find also the tensile stress in the tendons after the application of the external tension on the member.

Solution. Equivalent concrete area

$$= A_e = A_c + (m-1)\, A_s$$
$$= 20 \times 20 + (6-1) \times 5 \ cm.^2$$
$$= 425 \ cm.^2$$

Prestressing force $= P = 9000 \times 5$

$$= 45000 \ kg.$$

Actual concrete area $= A_c = 20 \times 20 - 5$

$$= 395 \ cm.^2$$

$\therefore$ External tension that can be resisted

$$= T = \frac{A_e}{A_c} P$$

$$= \frac{425}{395} \times 45000 \ kg.$$

$$= 48420 \ kg.$$

Resultant stress is tendon

$$= f_c + m \frac{T}{A_e} = 9000 + 6 \times \frac{48420}{425} \ kg./cm.^2$$

$$= 9684 \ kg./cm.^2$$

§226. Strains in Prestressed Concrete Tension Members

It may be realized from the above discussion that with the condition of zero stress in the member, the external tension which the member can carry is somewhat greater than the effective prestressing force P. This is so since there will be an increase in the tensile stress in steel under the action of the external tension T.

It is worthy to investigate the strains in the member due to not only the prestressing force but also due to external tension. When the member is subjected to initial prestressing force P_o, the compressive stress in concrete is $\frac{P_o}{A_c}$ and the consequent strain in concrete is $\frac{P_o}{E_c A_c}$. This will obviously decrease to $\frac{P}{E_c A_c}$ after the losses have occurred. When the member is under the action of the external tensile load T, the consequent strain in concrete will be $\frac{T}{E_c A_c}$.

Let us make a comparison between the strains produced in concrete in a prestressed concrete member and an ordinary

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structural steel member. Suppose the steel member is stressed to 1400 *kg./cm.*^{2} ; then the tensile strain

$$= \frac{1400}{2{\cdot}1 \times 10^6} = 0{\cdot}00067.$$

Now for a prestressed concrete member the strain in concrete corresponding to a stress of say 70 *kg./cm.*^{2} is $\frac{70}{2{\cdot}8 \times 10^5} = 0{\cdot}00025.$ This is lesss than one half of the strain in structural steel.

In the case of long tension members, where elongation must be limited, high tensile steel alone cannot be used. Suppose the high tensile steel be stressed to a working stress of say 8750 *kg /cm.*^{2} the strian will be $\frac{8750}{2{\cdot}1 \times 10^6} = 0{\cdot}00417.$ This is more than 6 times that of structural steel and 16 times that of prestressed concrete.

Several factors influence the strains in prestressed concrete members. If the concrete is maintained in the precompressed state for a sufficient time, the shortening of the member due to creep could be appreciable. Such creep strains however would be gradually regained, though not fully, under the applications of the external tension. Since a good part of the creep may ultimately be regained, the increase in the length of the member under continued external tensile load may be greater than what is determined by calculations based on elasticity.

§227. Tension Members Designed on Cracking and Ultimate Strength Consideration

In the discussions made so far, the condition to which the prestressed concrete tension member is subjected is such that the final compressive stress in the member is zero. Such a condition may or may not be the basis of the design of the member, depending on the probable extent of overloading to which the member may be subjected. If it is desired to obtain a sufficient factor of safety, it may be necessary to design the member such that, when the member is subjected to working loads there will always exist a certain residual compressive stress in concrete. The need to satisfy such a condition will be clear after a study of the cracking and ultimate strength of the member. It is quite possible that the design of a tension member by the allowable stress method may err considerably on the unsafe side and is liable to yield inconsistent results.

In general, it may be noted that prestressed concrete tension members possess very low reserve strength above the points of zero stress. Suppose the tension member is not cast as a single piece *i.e.*, if it is made up of blocks, then cracking may occur when the condition of zero stress is attained. After this stage, any additional load if applied will be carried by the steel alone. The prestressing steel wires, in view of their small cross-sectional areas, will be

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subjected to excessive elongation immediately at the instant of cracking of concrete which may lead to the failure of other parts of the structure. Hence, for such a member it is advisable that a considerable amount of residual compression is maintained in order to ensure safety, the amount of residual compression being dependent on the magnitude of the probable overloads.

If the member is cast as a single piece and if the shrinkage and other cracks have not yet developed, the member will be able to take a certain amount of tension before developing cracks. The direct tensile strength of concrete is variable and ranges from 0·06 f_c' to 0·10 f_c'. For example, for a concrete of 300 *kg./cm.*^{2}, the tensile strength may range from 18 *kg./cm.*^{2} to 30 *kg./cm.*^{2}, which may no doubt provide a sufficient margin of safety if the strength exists and has not been lost. But the moment the concrete has developed cracks, the margin of safety is suddenly lost. In fact, the entire structure may fail as soon as the concrete has cracked since at such a stage the tensile load carried by concrete is suddenly transmitted to the steel. This will lead to a sudden elongation of steel which may have damaging effects, though the ultimate strength of steel is yet to be reached

The discussion made above should not be interpreted that such tension members are not safe. They are, in reality, as safe as any other type of tension member and may probably be even safer if properly designed. When there are possibilities of heavy loads, they should not be designed on the basis of allowable stress but rather on the basis of the cracking or ultimate strength with suitable load factors.

Design 218. *A straight concrete member 45 metres long is prestressed with high tension steel through the centroid of the section. Taking $A_c = 500$ cm.^{2}, $A_s = 5$ cm.^{2}, $m = 7{\cdot}5$, ultimate tensile strength of concrete = 24 kg./cm.^{2}. Final prestress in the steel = 8925 kg./cm.^{2}, calculate what working load can it carry using a factor of safety of 2 against the cracking of concrete. Find also the residual compressive stress in concrete under that working load.*

Solution

Final prestressing force $P = 5 \times 8925 = 44625$ *kg.*

Let T' = cracking tensile load.

Equivalent concrete area $= A_e = A_c + mA_s$

$= 500 + (7{\cdot}5 \times 5) = 537{\cdot}5$ *cm.*^{2}

$$\frac{P}{A_e} - \frac{T'}{A_e} = -24$$

$$\frac{44625}{500} - \frac{T'}{537{\cdot}5} = -24$$

$$\therefore \quad T' = 60872 \text{ kg.}$$

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With a factor of safety of 2,

$$\text{working load} = T = \frac{60872}{2} = 30436 \text{ kg.}$$

Residual compressive stress, under the above working load

$$= -\frac{P}{A_c} - \frac{T}{A_s} = -\frac{44625}{500} - \frac{30436}{537.5}$$

$$= -32.63 \text{ kg./cm.}^2$$

Design 219. *Design a pretensioned member to carry a tension of 100 tonnes. Safe compressive stress in concrete = 105 kg./cm.², m = 8, and initial prestress in steel = 12 t./cm². Assume λ = 0·8.*

Solution.

Analysis at transfer

P_o = Tension in steel = Compression in concrete

$$12000\ A_s = 105\ A_c$$

$$A_s = 0.00875\ A_c$$

Equivalent concrete area

$$= A_e = A_c + m\ A_s$$

$$= A_c + 8 \times 0.00875\ A_c$$

$$= 1.07\ A_c.$$

Analysis at working load condition

Let the final stress in concrete reach zero at this stage.

$$\therefore \quad \frac{P}{A_c} - \frac{T}{A_s} = 0$$

But

$$P = 0.8 \times 12000 \times A_s$$

$$= 0.8 \times 12000 \times 0.00875\ A_c = 84\ A_c$$

$$\therefore \quad \frac{84\ A_c}{A_c} - \frac{100{,}000}{A_s} = 0$$

$$\therefore \quad A_s = \frac{100000}{84} = 1190.5 \text{ cm.}^2$$

$$\therefore \quad A_s = 1.07\ A_c = 1190.5$$

$$\therefore \quad A_c = 1112.62 \text{ cm.}^2$$

$$A_s = 0.00875 \times 1112.62 = 9.74 \text{ cm.}^2$$

$$\therefore \quad A = 1112.62 + 9.74 = 1122.36 \text{ cm.}^2$$

Provide 33·5 cm. × 33·50 cm. section.

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Design 220. *A straight tension member 25 cm. × 25 cm. is provided with 7·50 cm.² of high tension steel wires which are subjected to a stress of 9000 kg./cm.². Allowing a factor of safety of 1·50 against cracking of concrete find the safe tension for the member, assuming a cracking stress of 30 kg./cm.². Find also the residual compressive stress under working load. Take m=6.*

Solution.

Actual concrete area $= A_c = 625 - 7\cdot50 = 617\cdot50$ *cm.²*

Equivalent concrete area $= A_e = A + (m-1)\,A_s$

$= 625 + (6-1)\,7\cdot50$

$= 662\cdot50$ *cm²*.

Prestressing force $= P = 7\cdot5 \times 9000 = 67500$ *kg.*

Let T' be the tension at which concrete will just develop cracking stress

$$\therefore \quad \frac{P}{A_c} - \frac{T'}{A_e} = -30$$

$$\frac{67500}{617\cdot50} - \frac{T'}{662\cdot50} = -30$$

$$\therefore \quad T' = 92294 \text{ kg.}$$

With a factor of safety of 1·5,

$$\text{Safe tension} = T = \frac{92294}{1\cdot5} = 61529 \text{ kg.}$$

When the member is subjected to this safe tension,

Residual compressive stress in concrete

$$= \frac{P}{A_c} - \frac{T}{A_e}$$

$$= \frac{67500}{617\cdot50} - \frac{61529}{662\cdot50}$$

$$= 109\cdot31 - 92\cdot87$$

$$= 16\cdot44 \text{ kg./cm.}^2$$

§228. Compression Members

For axially loaded columns, prestressing the member will not be found necessary. But sometimes a compression member, in addition to being subjected to axial loads, may also be subjected to bending moment caused by transverse loads. If the effect of bending moment is predominant so as to produce excessive tensile stresses in concrete, it will be advisable to reinforce the tension zones.

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Consider a column of gross area A_g. Let A_c and A_s be the area of concrete and the reinforcement respectively so that

$$A_g = A_c + A_s$$

Let P be the prestressing force at an eccentricity e.

Let M be the bending moment caused by lateral loading.

The extreme fibre stresses are given by

$$f = \frac{P}{A_c} \pm \frac{Pe}{Z} + \frac{W}{A_c} \mp \frac{M}{Z}$$

where

A_c = equivalent concrete area

$$Z = \frac{I_c}{y}$$

and I_c = moment of inertia of the equivalent concrete area.

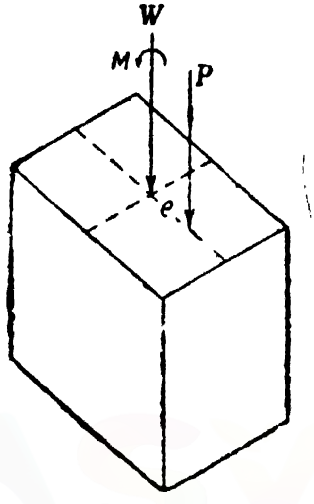

Fig. 874

Design 221. *A prestressed concrete column is 40 cm. × 40 cm. and 6 metres high and is provided with eight H T. wires of 10 mm. diameter, the effective prestress in the wires being 7000 kg./cm². The column carries an axial load of 36000 kg. and is also subjected to a transverse load of 3600 kg. acting at the mid height of the column. Determine the extreme stresses on the column section. Take $m = 6$ and assume the column to be hinged at both the ends.*

Solution. Area of concrete

$$= A_c = 40 \times 40 - 8 \times 0{\cdot}79$$

$$= 1594 \text{ cm}^2$$

Prestressing force $= 8 \times 0{\cdot}79 \times 7000$

$$= 44240 \text{ kg.}$$

Stress due to prestress

$$= \frac{44240}{1594} \text{ kg./cm}^2$$

$$= 27{\cdot}75 \text{ kg./cm.}^2 \text{ (compressive)}$$

Stress due to axial external load

$$= \frac{W}{A_g + (m-1)A_s}$$

$$= \frac{36000}{40 \times 40 + (6-1)8 \times 0{\cdot}79} \text{ kg./cm.}^2$$

$$= 22{\cdot}05 \text{ kg./cm.}^2 \text{ (compressive)}$$

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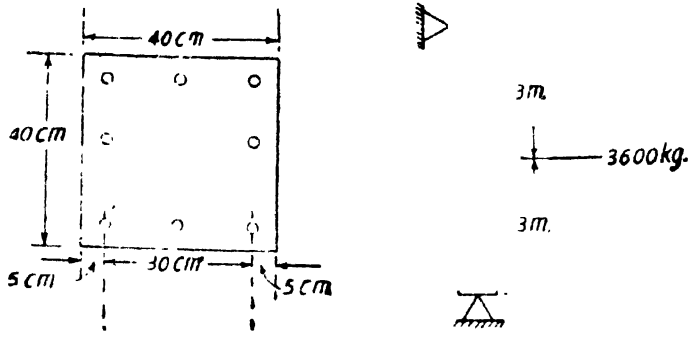

Fig. 875

Maximum bending moment due to transverse load

$$= M = \frac{3600 \times 6}{} \times 100 \ kg.\ cm.$$

$$= 540{,}000 \ kg.\ cm.$$

Moment of inertia of the equivalent concrete section

$$= I_c = \frac{40 \times 40^3}{12} + 6 \times 0{\cdot}79(6-1) \times 15^2 \ cm.^4$$

$$= 218670 \ cm.^4$$

∴ Extreme fibre stress due to bending moment

$$\frac{M}{I_c} \times y$$

$$\frac{540000}{218670} \times \left(\frac{40}{2}\right) kg./cm.^2$$

$$= 49{\cdot}38 \ kg./cm.^2$$

Hence the maximum and minimum streases on the section are,

$27{\cdot}75 + 22{\cdot}05 + 49{\cdot}38 = +99{\cdot}18 \ kg/cm.^2$ (*compressive*)

and $27{\cdot}75 + 22{\cdot}05 - 49{\cdot}38 = +0{\cdot}42 \ kg./cm.^2$ (*compressive*)

§229. End Block

End block means the portion of the prestressed concrete beam surrounding the anchorages of the tendons. Large concentrated forces are transmitted on the bearing surfaces at the ends of the beam by the anchorages. The prestress is transferred throughout the length of the end block from nearly concentrated areas. These stresses spread out into the concrete setting up complicated stress patterns. The stress distribution close to the anchorage are different from the stress distribution at sections away from the anchorage.

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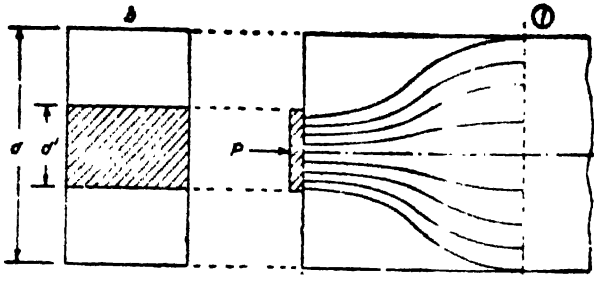

Fig. 876

Consider a beam of width b and depth d. Let the prestressing force P be applied on a bearing area bd' on the end face of the beam. At sections close to the end face, the stress on the section will be greater than $\frac{P}{bd}$. There exists a section 1—1 beyond which the stress is $\frac{P}{bd}$. The part of the beam from the end face to the section 1—1 is called the end block. Fig. 876 shows the lines of pressure transfer which spread out from the area bd' to the area bd along the end block. The length of the end block is usually taken equal to d. These lines of pressure may be taken to act as individual slender struts. It may be observed that these struts have a curvature which changes gradually from convexity towards the centroidal axis near the end face to concavity towards the centroidal axis, at sufficient distance from the end face. Transverse stresses are caused by these curvatures. Transverse compressive stresses are developed for a certain length and beyond this zone, transverse tensile stresses are developed.

The transverse stress distribution in the length of the end block is shown in Fig. 877.

Many methods are available to study the stress distribution in the end block. The following methods are due to Prof Magnel and Prof. Guyon.

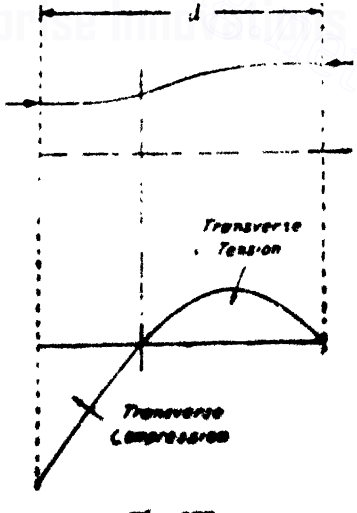

Fig. 877

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§230. Magnel's Method

In this method the stresses due to anchorage bearing are taken to be dispersed over a length of beam equal to the depth of beam. See Fig. 878. Consider any elemental unit situated within this end zone. Such an elemental unit is subjected to three stresses f_x, f_z and q.

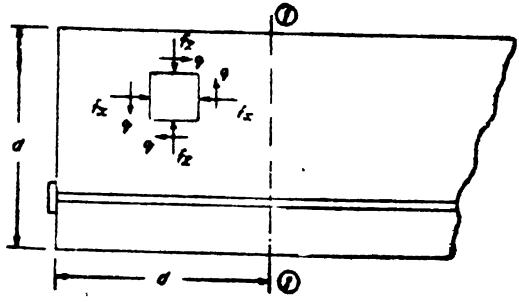

Fig. 878

f_x and f_z are horizontal and vertical stresses and q is a shear stress. We will now consider the distribution of the stresses f_x, f_z and q.

(i) *Horizontal stresses* f_x

It will be assumed that the compressive force P from the anchorage will disperse at 45°. Let the extreme lines of dispersion meet the upper and lower faces of the beam at D and A. See Fig. 879. Now it is easy to locate the centroid of the stressed zone for all sections from NM to FE. The locus of the centroid of stressed sections is given by $0_1 0_2 0_3 0_4$.

For any general vertical section XX the cable eccentricity e_y is therefore known.

The stress distribution due to the cable force P can be easily analysed for the section XX.

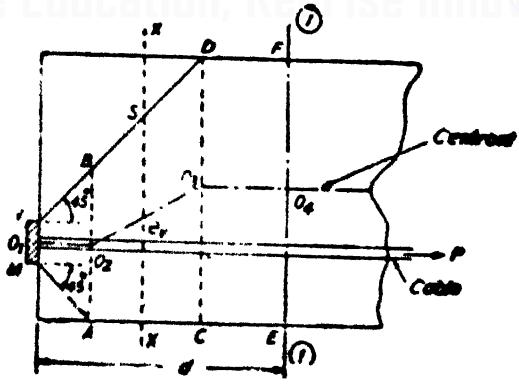

Fig. 879

For the section XX, the stress at any point distant y from the centroidal line is given by

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where d_s = depth of the stressed zone at $XX = XS$
I_s = Moment of inertia of the stressed section

(*ii*) *Shear stresses*

According to Magnel, the shear stress distribution on any horizontal plane say *H-H* (see Fig. 880) is given by

$$q = K_q \frac{S}{bd}$$

where K_q = a shear stress factor which can be determined from Fig. 881.

S = The part of the prestressing force at the section 1—1 acting above the level *H-H*. This can be determined from the horizontal stress distribution for the section 1—1. S can be calculated by summing up the stresses f_x acting above the level *H-H*.

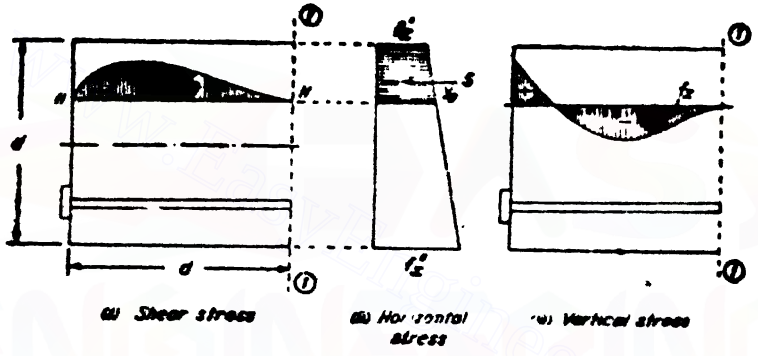

Fig. 880

(Note : If the level *H-H* is such that the cable force P also acts above the level *H-H* then this cable force P should be deducted).

It may also be realized that a certain part of the end block covers the end seating for the member. The shear stresses due to the bearing reaction acting on this part of the end block should also be taken into account. The bearing pressure distribution may be assumed to be uniform over the length of the seating.

(*ii*) *Vertical stresses*

According to Magnel, the vertical stress distribution on any horizontal plane *H-H* (Fig. 880) is given by

$$f_z = k_z \cdot \frac{M}{bd^2}$$

where K_z = vertical stress factor whose value may be determined from Fig 881.

M = Moment due to S

$= S_e y_e$

y_e = the distance of line of action of S from *H-H*.

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Note : In case the plane *H-H* is below the level of the cable, *M* has to be reduced by the quantity, cable force × distance of cable above *H-H*

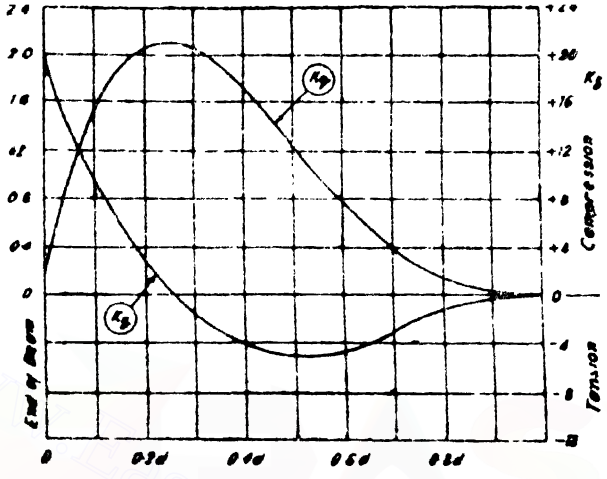

Fig. 881

Determination of principal stresses

After determining the stresses f_x, f_z and q we can determine the principal stresses easily.

$$\text{Principal stress} = \frac{f_x + f_z}{2} \pm \sqrt{\left(\frac{f_x - f_z}{2}\right)^2 + q^2}$$

The position of the principal planes is given by

$$\tan 2\theta = \frac{2q}{f_x - f_z}.$$

Design 222. *For the prestressed beam shown in Fig. 882 determine the horizontal vertical and shear stresses at the point Q in the end block. Find also the principal stresses at Q. The beam section is*

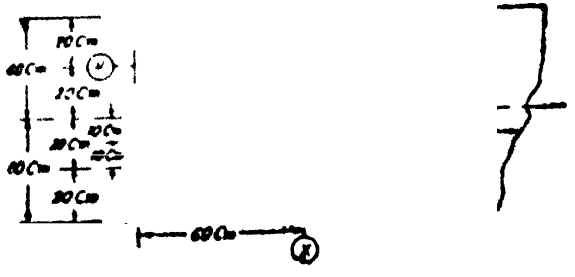

Fig. 882

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40 cm. wide and 80 cm. deep. The tendons are placed at an eccentricity of 10 cm. The anchor plate is 40 cm. wide and 20 cm. deep.

Solution. Allow a 45° stress dispersion from the top and bottom edges of the anchor plate.

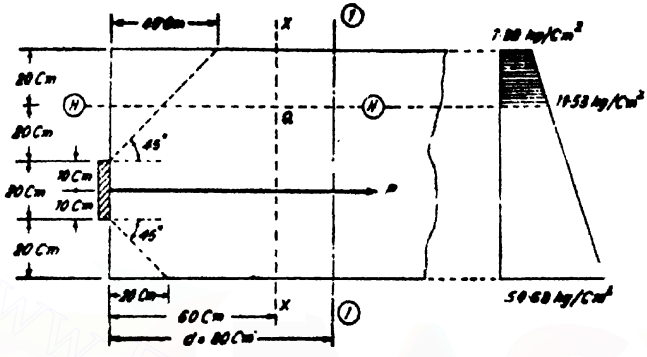

Fig. 883

Length of end block $= d = 80$ *cm.*

See Fig. 883. It is clear that the depth of the stressed vertical section equals 80 *cm.*

(*i*) **Horizontal Stress at Q**

$$= \frac{P}{bd_e} - \frac{Pey}{I_e}$$

$$= \frac{100{,}000}{40 \times 80} - \frac{100{,}000 \times 10}{40 \times \frac{80^3}{12}} \times 20 \text{ kg./cm.}^2$$

$$= 31{\cdot}25 - 11{\cdot}72 \text{ kg./cm.}^2$$

$$= 19{\cdot}53 \text{ kg./cm.}^2$$

Extreme horizontal stress at section 1 – 1

$$= \frac{P}{bd} \mp \frac{Pe}{I} \cdot \left(\frac{d}{2}\right)$$

$$= \frac{100{,}000}{40 \times 80} \mp \frac{100{,}000 \times 10}{40 \times \frac{80^3}{12}} \times \frac{80}{2} \text{ kg./cm.}^2$$

$$= 31{\cdot}25 \mp 23{\cdot}43 \text{ kg./cm.}^2$$

$$= 7{\cdot}82 \text{ kg./cm.}^2 \text{ at top}$$

and 54·68 kg./cm.² at bottom

Total horizontal force above the level *H-H*

$$= S = \left(\frac{7{\cdot}82 + 19{\cdot}53}{2}\right)(40 \times 20) = 10{,}940 \text{ kg.}$$

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This force acts at $y_o = \frac{19\cdot53+(2\times7\cdot82)}{19\cdot53+7\cdot82}\times\frac{20}{3} = 8\cdot57$ *cm.* above *H-H*

The given section is at a distance of 60 *cm.* from the end face of the beam.

Length of the end block $= d = 80$ *cm.*

∴ Distance of the given section from end face

$$= \frac{60}{80}\, d = 0\cdot75\, d \text{ from the end face.}$$

Referring to Fig. 881 at 0·75 *d* from the end face,

$$K_q = 0\cdot23 \text{ and } K_s = -2\cdot40$$

∴ Shear stress at $Q - q = K_q \frac{S}{bd}$

$$= 0\cdot23 \times \frac{10940}{40\times80} \text{ kg./cm.}^2$$

$$= 0\cdot78 \text{ kg/cm.}^2$$

Vertical stress at $Q = f_s = K_s \cdot \frac{M}{bd^2}$

where $M = S.y_o = 10940 \times 8\cdot57$ *kg. cm.*

$$f_s = -2\cdot40 \times \frac{10940\times8\cdot57}{40\times80^2} \text{ kg./cm.}^2$$

$$= -0\cdot88 \text{ kg/cm}^2 \text{ (tensile)}$$

Thus, at *Q* we have the following stresses .

$f_a = +19\cdot53$ *kg./cm.*^{2}, $f_s = -0\cdot88$ *kg./cm.*^{2} and $q = 0\cdot78$ *kg./cm.*^{2}

∴ Principal stress

$$= \frac{f_a+f_s}{2} \pm \sqrt{\left(\frac{f_a-f_s}{2}\right)^2+q^2}$$

$$= \frac{19\cdot53-0\cdot88}{2} \pm \sqrt{\left(\frac{19\cdot53+0\cdot88}{2}\right)^2+0\cdot78^2}$$

$$= 19\cdot56 \text{ kg./cm.}^2 \text{ and } -0\cdot93 \text{ kg/cm}^2.$$

§231. Guyon's Method

The end block may be analysed by the following theory.

When the prestressing force is applied through the anchor plate, the force is dispersed in a certain length of beam to the entire cross-section. The length of beam within which dispersion of the prestressing force takes place is called the *zone of transmission.*

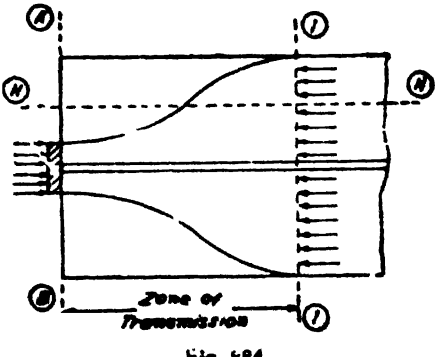

Fig. 884

Within the zone of transmission the distribution of stress can be analysed as follows.

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In a direction transverse to the axis of this concentrated force tensile forces are developed. These are called *bursting forces*. Further at the end section the surface adjacent to the anchor plate is also subjected to a tensile force called the *spalling force*.

Fig. 884 shows the end of a beam to which a prestressing force P is transmitted through a centrally placed anchor plate. Let AB be the end face of the beam. Let 1—1 be the end of the zone of transmission.

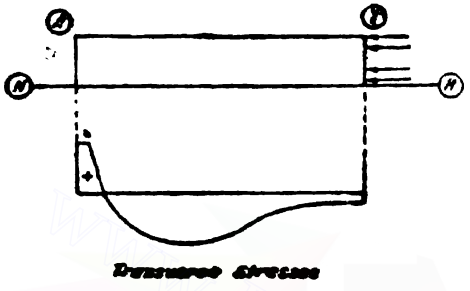

Fig. 885

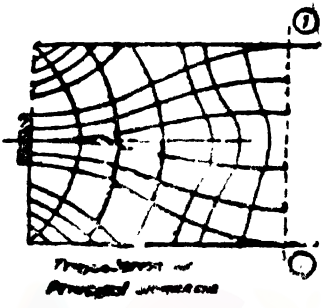

Fig. 886

Consider any horizontal section H-H. This horizontal section is subjected to transverse stress and shear stresses.

Fig. 886 shows the trajectories of principal stresses due to horizontal, vertical and shear stresses. The depth of the anchor plate influences the length of transmission.

The length of transmission and the bursting force (transverse tensile force) depend on the ratio of the depth of the anchor plate to the depth of the beam. Usually the transmission length is taken equal to the depth of the beam.

If d' be the depth of the anchor plate and d the depth of the beam, the bursting force is approximately given by

$$P_b = 0{\cdot}3P\left[1 - \frac{d'}{d}\right]$$

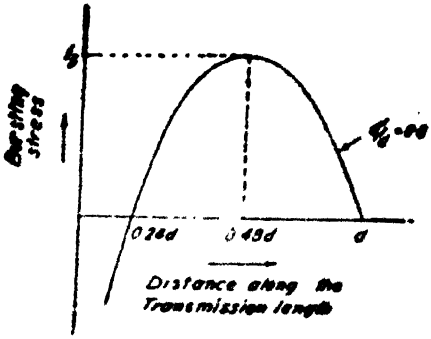

Fig. 887

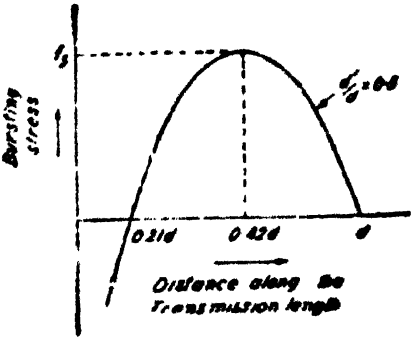

Fig. 888

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The bursting stress distribution, along the transmission length for different values of $\frac{d'}{d}$ are given by separate charts. The bursting stress distribution for $\frac{d'}{d}=0{\cdot}8$ and $\frac{d'}{d}=0{\cdot}5$ are shown in Fig. 887 and Fig. 888.

Design 223. *A prestressed concrete beam 25 cm. wide and 60 cm. deep is subjected to an axial prestressing force of 150 t. Design the end block.*

Solution.

Beam section : $b=25$ *cm.*
$d=60$ *cm.*

Size of anchor plate $b'=0{\cdot}8\ b=0{\cdot}8\times25=20$ *cm.*
$d'=0{\cdot}5\ d=0{\cdot}5\times60=30$ *cm.*

Design of the anchor plate

We will assume that the various cables provided are all passing through one duct only and are anchored to one anchor plate only, with an overhang of 2·5 *cm.* on all sides.

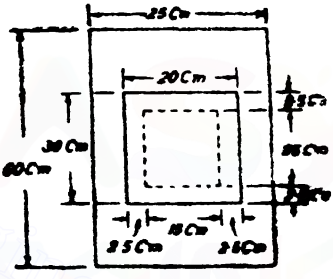

Fig. 889

Induced intensity of bearing pressure on the anchor plate

$$=\frac{150{,}000}{20\times30}=250\ kg./cm.^2$$

Consider a 1 *cm.* wide cantilevering strip of the anchor plate.

$$\therefore\ \text{Max. B.M. for the strip}=250\times\frac{2{\cdot}5^2}{2}\ kg.\ cm.$$

$$=781{\cdot}25\ kg.\ cm.\ \text{say}\ 782\ kg.\ cm.$$

Allowing a bending stress of 1500 *kg./cm.*^{2} and equating the M.R. to the max. B.M.

$$\frac{1}{2}\times1500\times1\times t^2=782$$

$$t=\sqrt{\frac{6\times782}{1500}}=1{\cdot}77\ cm.$$

Provide 18 mm. thick anchor plate.

Analysis for bursting forces

There will be bursting forces (*i*) on vertical planes and (*ii*) on horizontal planes.

(*i*) Bursting force on the horizontal plane

$$=0{\cdot}3\ P\left[1-\frac{d'}{d}\right]$$

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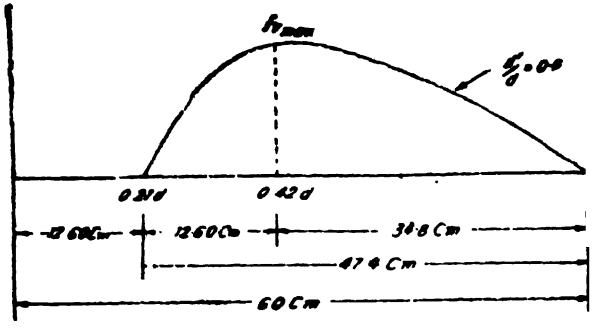

Fig. 890

$$= 0{\cdot}3 \times 150{,}000\left[1 - \frac{30}{60}\right] = 22500 \text{ kg.}$$

∴ Bursting force per *cm.* width

$$= \frac{22500}{20} = 1125 \text{ kg./cm.}$$

Equating the area of the bursting stress diagram to the bursting force per *cm.* width.

$$\frac{2}{3} \times 47{\cdot}4 \times f_{s\,max} = 1125$$

$$f_{s\,max} = \frac{1125 \times 3}{2 \times 47{\cdot}4} \text{ kg./cm.}^2$$

$$= 35{\cdot}60 \text{ kg./cm.}^2$$

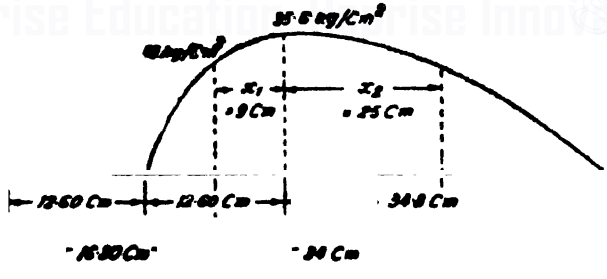

Fig. 89.

The permissible stress is 18 *kg /cm.*^{2}

Let the bursting stress be 18 *kg./cm.*^{2} at distances x_1 and x_2 from the point of maximum stress. See Fig. 891. Assuming a parabolic law,

$$\frac{35{\cdot}6 - 18}{35{\cdot}6} = \frac{x_1^2}{12{\cdot}6^2} = \frac{x_2^2}{34{\cdot}8^2}$$

$$\therefore \quad x_1 = 8{\cdot}8 \text{ cm.} \quad \text{say } 9 \text{ cm.}$$

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and $x_2 = 24{\cdot}5\ cm.$ say $25\ cm.$

$x_1 + x_2 = 9 + 25 = 34\ cm.$

Total tension for which reinforcement is necessary

$$= \frac{2}{3} \times 34(17{\cdot}6) \times 20\ kg.$$

$$= 7980\ kg.$$

$\therefore$ Steel required $= A_{st} = \dfrac{7980}{1400} = 5{\cdot}70\ cm.^2$

Provide 8 bars of 10 mm. dia.

i.e. Provide 4 stirrups of two legs each.

(*ii*) **Bursting force on the vertical plane**

$$= 0{\cdot}3\ P\left[1 - \frac{b'}{b}\right]$$

$$= 0{\cdot}3 \times 150{,}000\left[1 - \frac{20}{25}\right] kg.$$

$$= 9000\ kg.$$

Bursting force per *cm.* height

$$= \frac{9000}{30} = 300\ kg/cm.$$

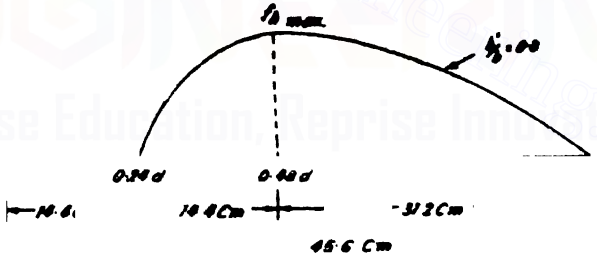

Fig. 892

Equating the area of the bursting stress diagram to the bursting force per *cm.* height,

$$\frac{2}{3} \times 45{\cdot}6\ f_{h\ max} = 300$$

$$\therefore \quad f_{h\ max} = \frac{300 \times 3}{2 \times 45{\cdot}6}\ kg./cm.^2$$

$$= 9{\cdot}87\ kg./cm.^2$$

Provide nominal horizontal steel. The horizontal legs of the vertical stirrups are sufficient for providing the necessary resistance. See Fig. 893.

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§232. Beam with Two Anchor plates symmetrically placed on the End face of the Beam.

Fig. 894 shows a beam provided with two symmetrically arranged anchor plates. The transmission length for this case will be taken equal to $d/2$. The bursting force can be determined by assuming that the beam consists of two beams each of depth $d/2$. The following problem illustrates this case

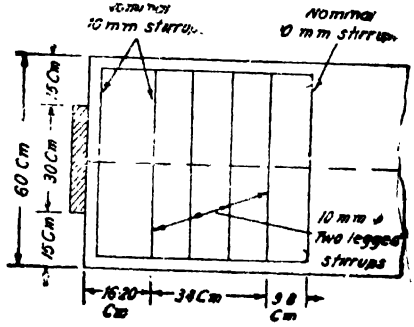

Fig 893

Design 224. *A prestressed concrete beam 25 cm. wide and 60 cm. deep is subjected to a total prestressing force of 150 t. This force is transmitted by two symmetrically arranged cables each transmitting a force of 75 t. Two anchor plates 20 cm. wide and 24 cm. deep are provided one for each cable as shown in Fig. 895. Calculate the reinforcement required in the transmission zone.*

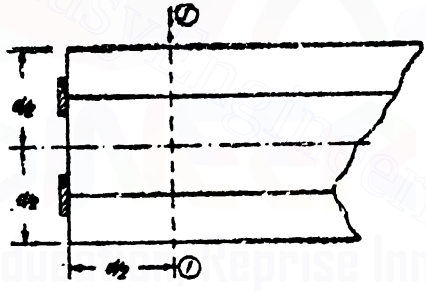

Fig. 894

Solution. Size of an anchor plate $= 20\ cm. \times 24\ cm.$

The total depth of 60 *cm.* of the beam may be taken to consist of two prism units of 30 *cm.* depth each.

Bursting force for each prism unit, on the horizontal plane

$$= 0.3\ P \left[1 - \frac{d}{d_1} \right]$$

$$= 0.3 \times 75000 \left[1 - \frac{24}{30} \right] kg.$$

$$= 4500\ kg.$$

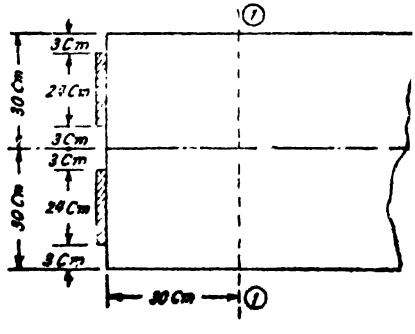

Fig. 895

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Bursting force per *cm.* width

$$\frac{4500}{20} = 225 \text{ kg./cm.}$$

The section where the bursting stress is zero and maximum are at distances $0{\cdot}24d_1$ and $0{\cdot}48d_1$ from the end of the beam.

$$0{\cdot}24\ d_1 = 0{\cdot}24 \times 30 = 7{\cdot}20 \text{ cm.}$$

$$0{\cdot}48\ d_1 = 0{\cdot}48 \times 30 = 14{\cdot}4 \text{ cm.}$$

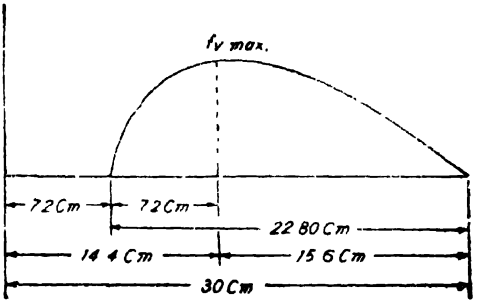

Fig. 896

Equating the area of the bursting stress diagram to the bursting force per *cm.* width,

$$\frac{2}{3} \times 22{\cdot}8 \times f_{v\ max} = 225$$

$$f_{v\ max} = \frac{225 \times 3}{2 \times 22{\cdot}8} = 14{\cdot}80 \text{ kg./cm.}^2$$

This is less than the permissible limit of 18 *kg./cm.*^{2}. Provide four nominal two-legged 10 *mm.* diameter stirrups at a spacing of 10 *cm. c/c.*

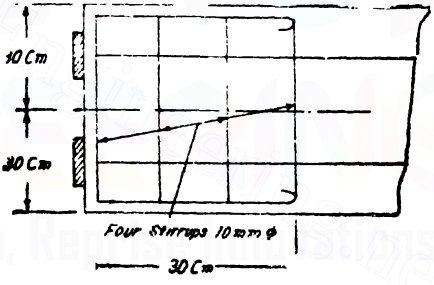

Fig. 897

Bursting force on the vertical plane also will be small and hence the horizontal legs of the stirrups will be sufficient to provide the necessary resistance.

§233. **Cable at an Eccentricity**

In this case the anchor plate is placed eccentrically on the end face of the beam. In this case an equivalent prism is considered whose axis is the same as the axis of the prestressing force.

The depth of the prism is taken equal to double the distance of the nearer edge from the axis of the prestressing force. See Fig. 898. The following example illustrates this case.

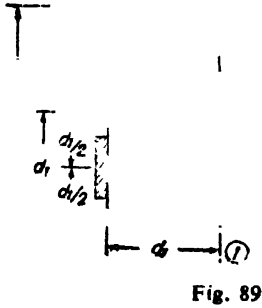

Fig. 898

Design 25. *A prestressed concrete beam is 60 cm. deep and 25 cm. wide and is*

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subjected to a prestressing force of 100 t. at an eccentricity of 10 cm. The anchoring plate is 20 cm wide and 32 cm. deep. See Fig. 899. Calculate the bursting stress and the reinforcement required in the zone of transmission

Solution. Distance between the axis of the cable and the nearer edge = 20 *cm.*

$\therefore$ Depth of the equivalent prism $= d_1 = 2 \times 20 = 40$ *cm.*

$$\frac{d'}{d_1} = \frac{32}{40} = 0{\cdot}8$$

Total bursting force on the horizontal plane

$$= 0{\cdot}3\,P\left[1 - \frac{d'}{d_1}\right] = 0{\cdot}3 \times 100{,}000\,(1 - 0{\cdot}8) = 6000\ kg.$$

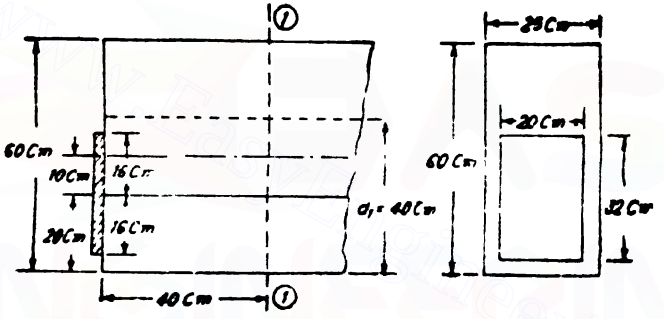

Fig. 899

$\therefore$ Bursting force per *cm.* width $= \frac{6000}{20} = 300\ kg.$

Bursting tensile stress is zero and maximum at distances $0{\cdot}24d_1$ and $0{\cdot}48d_1$ from the end face of the beam.

In our case, $0{\cdot}24d_1 = 0{\cdot}24 \times 40 = 9{\cdot}60$ *cm.*

$0{\cdot}48d_1 = 0{\cdot}48 \times 40 = 19{\cdot}20$ *cm.*

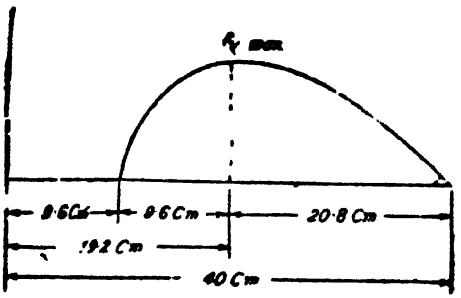

Fig. 900

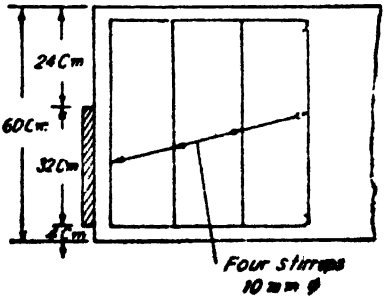

Fig. 901

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Equating the area of the bursting tensile stress diagram to the bursting force per *cm* width, we get,

$$\frac{2}{3}(9{\cdot}6+20{\cdot}8)f_{v\,max}=330$$

$$f_{v\,max}=\frac{3}{2}\times\frac{300}{30{\cdot}4}=14{\cdot}8\ kg/cm.^2$$

This is less than 18 kg/cm^2.

Similarly it will be seen that the bursting stress on the vertical plane is less than 18 $kg./cm^2$

∴ *Provide four nominal 10 mm. diameter two-legged stirrups.*

§234. Continuous Beams

Some important conceptions

The P-line and the C-line

Fig. 902 shows a beam of span l simply supported at the ends. Let the beam be provided with a tendon at an eccentricity e. If the beam is not subjected to any external loading then the bending moment at any section is Pe. If the resultant compression at any section be C then P and C act along the same line.

Beam not subjected to external loading, *P*-line coincides with the *C*-line.
Fig. 902

obviously $P=C$

The *P-line* is the line of action of the prestressing force. The *C-line* is the line of action of the resultant compressive force.

When the beam does not carry any other loading the *P-line* and the *C-line* will coincide.

If now the beam be subjected to an external bending moment then the *C*-line will be shifted from the *P*-line by the distance a equal to the lever arm

$$=\text{shift of } C\text{-line from } P\text{-line}=\frac{\text{External } B.M.}{P}$$

Now consider the simply supported beam of rectangular section, subjected to end couples M.

Hence at any section, additional B M. $=M$.

Hence the lever arm $=a=$ Shift of the *C*-line from the *P*-line

$$=\frac{M}{P}$$

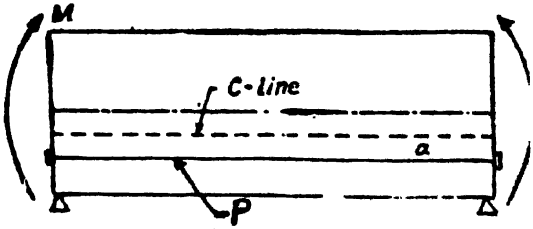

Fig. 903

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Hence for this case the *C*-line is parallel to the *P*-line and at a height *a* above it.

At any section the stresses produced are entirely due to the compressive force *C*. If the position of the *C*-line at any section is known then the extreme stresses for the case shown in Fig. 903 for the section are given by

$$\frac{C}{A} \mp \frac{C \times \text{eccentricity of } C}{Z}$$

It is very important to realize that at any section of the beam, the stress distribution can be completely determined if the position of the *C*-line at that section is known. *The stresses produced for any section are entirely due to C.*

In the discussions to follow, when we talk about external loads, we mean all the superimposed loads on the beam but not including the forces transmitted to the cable.

Primary moment. This is the product of the prestressing force and its eccentricity at the section.

If the tendons are provided at a uniform eccentricity *e* the primary moment at every section is *Pe*.

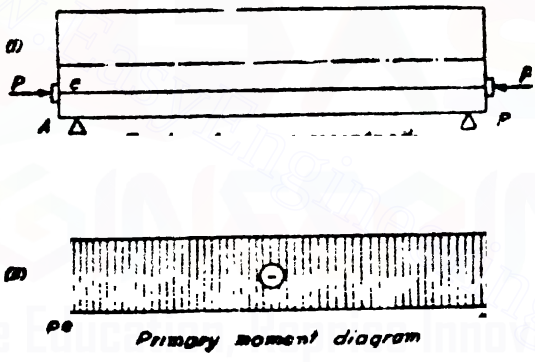

Fig. 904

Fig. 904 shows this case and the corresponding primary moment diagram.

Fig. 905 (*i*) shows a simply suptpored beam provided with inclined tendon. Fig. 905(*ii*) is the primary moment diagram. (It may be assumed that the inclination θ of the tendon with the horizontal is small so that the primary moment at any section *XX* which is equal to *P* cos θ. *y* may be taken as *Py*). It may be noted that no reactions are produced due to the

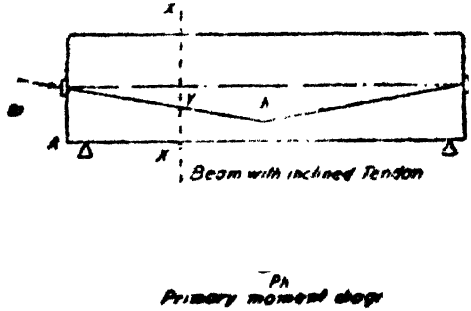

Fig. 905

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primary moments in this case and hence the resultant moment at any section is equal to the primary moment itself.

Now consider the continuous beam shown in Fig. 906 (*i*). Fig. 906 (*ii*) shows the corresponding primary moment diagram.

Secondary moment

Consider the continuous beam mentioned above. Under the action of the primary moment, the beam deflects. If due to the action of the primary moment no reactions are produced at the supports, then the resultant moment at any section would be equal to the primary moment itself. But since the supports do not allow any deflection (For example the deflection at B is resisted fully by the support at B) reactions will be induced over the supports. These reactions produce moment at every section. The bending moment at any section due to reactions alone caused by the primary moment, is called the *secondary moment*. Hence the resultant moment at any section is the algebraic sum of the primary moment and the secondary moment.

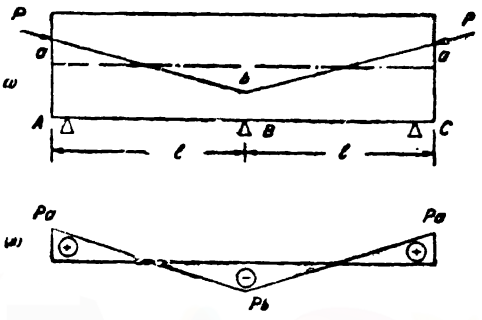

Fig 906

The resultant moments at A, B and C can be determined by either theorem of three moments or by the principle of moment distribution.

End moment at $A = +Pa$ (*clockwise*)

End moment at $C = -Pa$ (*anticlockwise*)

A	B	C
$+Pa$	$+\frac{Pa}{2}$ $-\frac{Pa}{2}$	$-Pa$

Fig. 907

Applying the moment $(+Pa)$ at A, half the applied moment *i.e.*, $\left(+\frac{Pa}{2}\right)$ is carried over to B.

Similarly applying the moment $-Pa$ at C, half the applied moment $-\frac{Pa}{2}$ is carried over to B. Thus the resultant moments are as shown in the table above.

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Fig. 908 (*i*) shows the continuous beam with the tendons.

Fig. 908 (*ii*) shows the primary moment diagram.

Fig. 908 (*iv*) shows the resultant moment diagram. By studying the primary moment diagram and the resultant moment diagram, we can easily draw the secondary moment diagram.

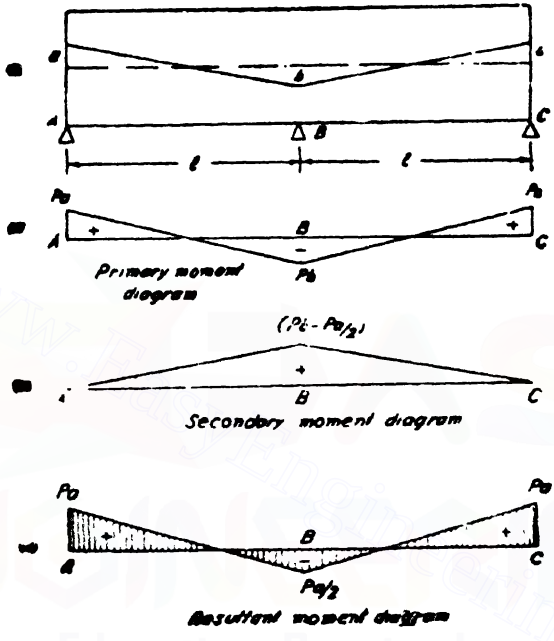

Fig. 905

Now we can establish the *C-line* for the beam easily.

The shift of the *C-line* from the *P-line* at any section

$$= \frac{\text{secondary moment at the section}}{P}$$

(*i*) Shift of *C-line* from *P-line* at $A = 0$

(*ii*) Shift of *C-line* from *P-line* at $B = \dfrac{Pb - \dfrac{Pa}{2}}{P} = \left(b - \dfrac{a}{2}\right)$

(*iii*) Shift of *C-line* from *P-line* at $C = 0$

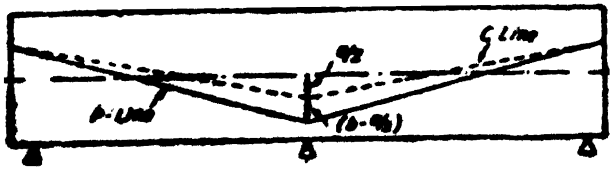

Fig. 909

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By making the above shifts from the *P-line* the *C-line* can be located.

Alternatively, we can directly calculate the eccentricity of the *C-line* from the longitudinal axis.

The eccentricity of the *C-line* at any section

$$= \frac{\text{Resultant Moment}}{P}$$

Thus,

(*i*) Actual eccentricity of the *C-line* at $A = 0$

(*ii*) Actual eccentricity of the *C-line* at $B = \frac{\frac{Pa}{2}}{P} = -\frac{a}{2}$

(*iii*) Actual eccentricity of the *C-line* at $C = 0$.

Now for any section the extreme stresses can be calculated by considering *the force C alone and its eccentricity.*

It can be realised from the above discussion that the profile of the *C-line* does not depend upon *b* the eccentricity of the cable at *B*. The profile of the *C-line* depends on the position of the end anchorages only. If the eccentricity *b* of the cable at *B* is changed, the primary and secondary moment will change but the resultant moment will not be affected.

Suppose the cable profile is such that there is no secondary moment for the beam considered above then, without changing the position of the end anchorages, the condition to be satisfied is,

$$Pb - \frac{Pa}{2} = 0.$$

$$\therefore \qquad b = \frac{a}{2}.$$

If this condition is satisfied, there will be no secondary moment and thus the resultant moment will be the same as the primary moment, and, the *P-line and C-line will coincide.*

The cable profile corresponding to this condition is called a *concordant profile.*

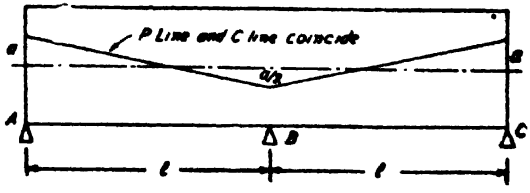

Fig. 910. Concordant Profile.

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General analysis of a continuous beam

Before proceeding further it is worth realising the assumptions introduced in the analysis of continuous beams.

(*i*) Compared with the length of the members, the eccentricities of the tendons are very small.

(*ii*) Loss of prestress due to friction is ignorable. (In any particular case where this loss is considerable it should also be taken note of).

(*iii*) The tendons are continuous throughout the whole length of the members.

Subject to the above assumptions, we have the following deductions :

(*a*) The component of the prestressing force along the longitudinal axis is equal to the prestressing force *P* itself

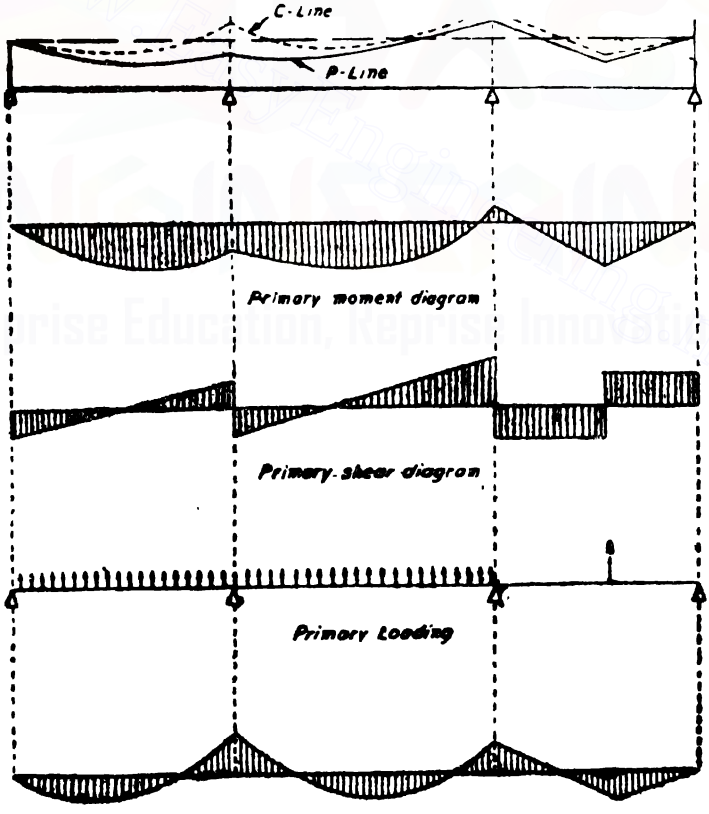

Fig. 911

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(*b*) The primary moment at any section is equal to *Pe* where *e* is the eccentricity of the tendon at that section with respect to the longitudinal axis

On the basis of the above, the continuous beam can be analysed as follows :

We will first consider as though the beam has no support and is subjected to the prestressing force alone. We can now plot the primary moment diagram. Now we can determine the loading exerted by the tendons on the beam. Applying the loading on the continuous beam with its actual supports the beam is analysed by moment distribution method or any other method. The calculations may be made as per the following scheme.

Analysis of the continuous beam

Step 1. Corresponding to the given cable profile, plot the primary moment diagram for the beam The primary moment at any section $= P \times$ eccentricity of the tendon at that section.

Step 2. Now studying the geometry of the primary moment diagram obtain the primary shear diagram.

Step 3. Studying the geometry of the primary shear diagram, obtain the primary loading on the beam. Primary loading means the load system transmitted to the beam by the cables alone.

Step 4. Now for the primary loading determined above, and considering also any *end moment* at each end of the continuous beam due to the eccentricity of the tendons, determine the *resultant moments*

Step 5 Now plot the *C-line* for the beam. The eccentricity of the *C-line* at any section

$$= \frac{\text{Resultant B M. at the section}}{P}$$

See Fig. 911.

If the external load system is also to be considered, this should be algebraically added to the primary loading determined in step 3 above.

Design 226. *A continuous prestressed concrete beam is shown in Fig. 912. The tendon has an eccentricity at A and is bent sharply at D and B and has a parabolic profile for the span BC. Locate the line of pressure (C-line) due to prestress alone. The prestressing force is 100 t.*

Solution

Let us determine the primary moments at the various sections.

Primary moment at $A = -100 \times 0{\cdot}06 = -6$ *tm.*

Primary moment at $D = -100 \times 0{\cdot}24 = -24$ *tm.*

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Primary moment at $B = +100 \times 0{\cdot}12 = +12$ *tm.*
Primary moment at $E = -100 \times 0{\cdot}27 = -27$ *tm.*
Primary moment at $C = 0$.

Primary shear analysis

From A to D, $S.F.$ = Rate of change of primary moment

$$= -\frac{24-6}{9} = -2t$$

From D to B, $S.F. = +\frac{24+12}{6} = +6t.$

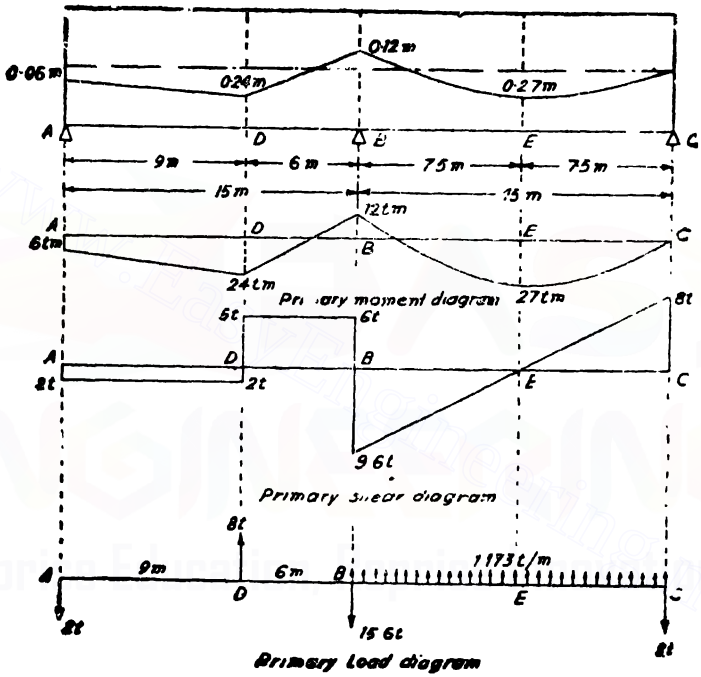

Fig. 912

From B to C, let the law for the primary moment at any section distant x from B be

$$M = ax^2 + bx + c$$

At $x = 0$, $M = +12$ $\quad \therefore \quad +12 = 0 + 0 + c \quad \therefore \; c = +12$

At $x = 7{\cdot}5$, $M = -27$ *tm.*

$$\therefore \quad -27 = 56{\cdot}25\,a + 7{\cdot}5\,b + 12$$

$$\therefore \quad 56{\cdot}25\,a + 7{\cdot}5\,b = -39 \qquad \ldots(i)$$

At $x = 15$, $M = 0$

$$\therefore \quad 0 = 225\,a + 15\,b + 12$$

$$\therefore \quad 225\,a + 15\,b = -12 \qquad \ldots(ii)$$

Solving equations (i) and (ii), we get,

$$a = \frac{44}{75} \text{ and } b = -9{\cdot}6$$

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$\therefore$ At any section in BC distant x from B,

The primary moment $= M = \frac{44}{75} x^2 - 9{\cdot}6\, x + 12$

$\therefore$ Primary shear $= S = \frac{dM}{dx} = \frac{2}{75} x - 9{\cdot}6$

At $x = 0$, $S = -9{\cdot}6\ t$

At $x = 15\ m$, $S = \frac{88}{75} \times 15 - 9{\cdot}6 = 17{\cdot}6 - 9{\cdot}6 = +8\ t.$

Hence for the span, BC the primary shear changes uniformly from $-9{\cdot}6\ t$ to $+8\ t$.

$\therefore$ Uniformly distributed load between B and C

$$= \text{Rate of change of primary shear} = \frac{9{\cdot}6 + 8}{15} = \frac{17{\cdot}6}{15} = 1{\cdot}173\ t/m.$$

Studying the primary shear diagram, we can easily realise that the following loads are transmitted to the beam by the cable,

(*i*) Downward load 2 *t* at *A*
(*ii*) Upward load $2 + 6 = 8\ t$ at *D*
(*iii*) Downward load $6 + 9{\cdot}6 = 15{\cdot}6\ t$ at *B*
(*iv*) Upward uniformly distributed load

$$= \frac{17{\cdot}6}{15} = 1{\cdot}173\ t/m \text{ on } BC$$

(*v*) Downward load 8 *t* at *C*.

We will now analyse the continuous beam for the above loading.

Fixed end moments

Using Hardy Cross conventions,

$$m_{ab} = + \frac{8 \times 9 \times 6^2}{15^2} = +11{\cdot}52\ tm.$$

$$m_{ba} = - \frac{8 \times 9^2 \times 6}{15^2} = -17{\cdot}28\ tm.$$

$$m_{bc} = + \frac{1{\cdot}173 \times 15^2}{12} = +22\ tm.$$

$$m_{cb} = - \frac{1{\cdot}173 \times 15^2}{12} = -22\ tm.$$

The analysis is now made by moment distribution as shown in the next page.

Max. free B.M. for $AB = - \frac{8 \times 9 \times 6}{15} = -28{\cdot}8\ tm.$

Max. free B.M. for $BC = - \frac{1{\cdot}173 \times 15^2}{8} = -33{\cdot}0\ tm.$

Fig. 913 shows the fixed, free and the resultant B.M. diagrams.
Now we can determine the *C-line* easily.
Eccentricity of the *C-line* at any section

$$= \frac{\text{Resultant moment at the section}}{P}.$$

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B			
$\frac{1}{2}$	$\frac{1}{2}$		
+11·52	−17·28	+22·00	−22·00
−11·52 − 6·00 →	−8·76	+11·00	←−+22·00
	−3·48	− 3·48	
− 6·00	−29·52	+29·52	0

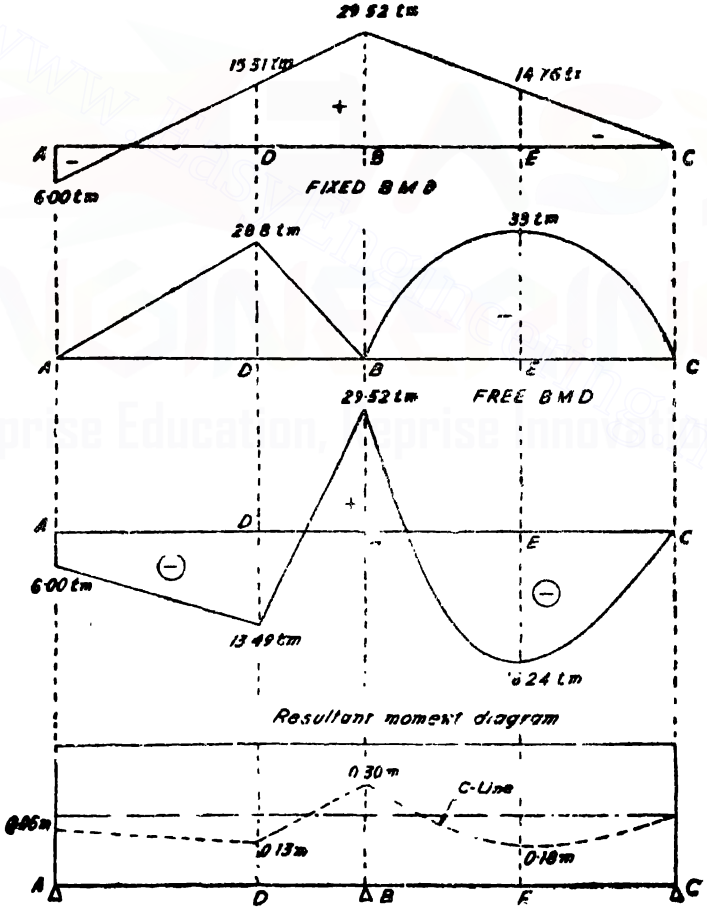

Fig. 913

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$\therefore$ Eccentricity of the *C-line* at $A = -\frac{6}{100} = -0{\cdot}06\ m.$

Eccentricity of the *C-line* at $D = -\frac{13{\cdot}49}{100} = -0{\cdot}1349\ m.$

say $-0{\cdot}13\ m.$

Eccentricity of the *C-line* at $B = +\frac{29{\cdot}52}{100} = +0{\cdot}2952\ m.$

say $+0{\cdot}30\ m.$

Eccentricity of the *C-line* at $E = -\frac{18{\cdot}24}{100} = -0{\cdot}1824\ m.$

say $-0{\cdot}18\ m.$

Eccentricity of the *C-line* at $C = 0$.

Design 227. *If the prestressed beam in the previous design is subjected to an external concentrated load of 12 t at 9 m. from the left end and a uniformly distributed load of 1·5 t per metre run on the span BC, locate the actual position of the C-line and calculate the extreme stresses in concrete at the section over the middle support. The section of the beam is as shown in Fig. 914. The dead load of the beam is 430 kg /m.*

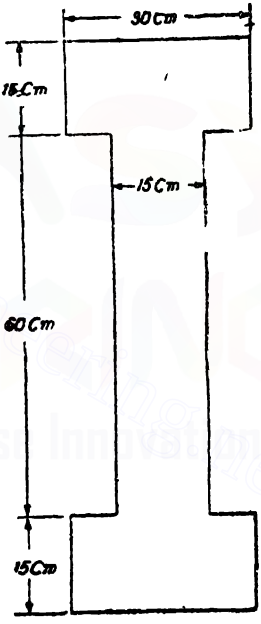

Fig. 914

Solution.

The beam has already been analysed due to prestress. Let us now analyse the beam due to external loading.

Fig. 915 shows the beam subjected to the external loading. Consider each span as a separate fixed beam the fixed end moments are as follows :

$$m_{ab} = -\frac{12 \times 9 \times 6^2}{15^2} - \frac{0{\cdot}43 \times 15^2}{12} = -25{\cdot}34\ tm.$$

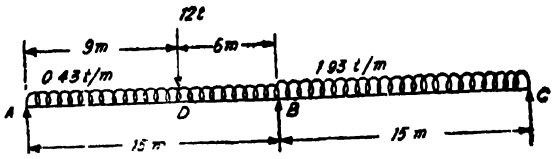

Fig. 915

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$$m_{ba} = + \frac{12 \times 9^2 \times 6}{15^2} + \frac{0{\cdot}43 \times 15^2}{12} = +33{\cdot}98 \ tm.$$

$$m_{bc} \quad \frac{1{\cdot}93 \times 15^2}{12} = -36{\cdot}19 \ tm.$$

$$m_{cb} = + \frac{1{\cdot}93 \times 15^2}{12} = +36{\cdot}19 \ tm.$$

	½	½	
−25 34	+33·98	−36·19	+36·19
+25·34	+12·67	−18·10	−36·19
	+ 3 82	+ 3·82	
0	+50·47	−50·47	0

Free B.M. at $D = + \frac{12 \times 9 \times 6}{15} + \frac{0{\cdot}43 \times 15}{2} \times 9 - \frac{0{\cdot}43 \times 9^2}{2}$

$= +54{\cdot}81 \ tm.$

Free B.M. at the middle of $BC = + \frac{1{\cdot}93 \times 15^2}{8} = +54{\cdot}28 \ tm.$

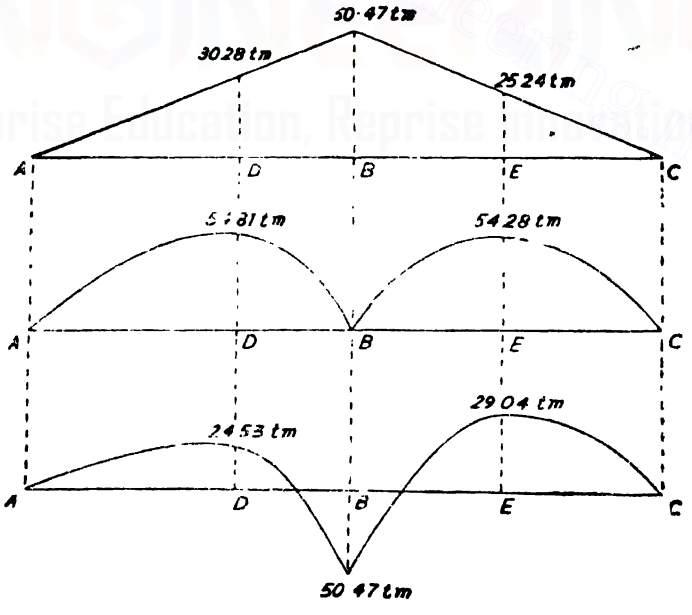

Fig. 916. B.M. due to external loads.

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Actual B.M. due to external load at $D = 54{\cdot}81 - 30{\cdot}28$
$= +24{\cdot}53$ *tm.*

" " " at $B = -50{\cdot}47$ *tm.*

" " " at $E = +54{\cdot}28 - 25{\cdot}24$
$= +29{\cdot}04$ *tm.*

Fig. 916 shows the B.M. diagram due to external loads alone. Due to the external bending moment the *C-line* will be shifted. Shift of the *C-line* at any section from the earlier position

$$= -\frac{\text{External moment}}{P}$$

Let us calculate the shift of the *C-line* at the various critical points and thus determine the actual eccentricity of the *C-line*.

(*i*) Shift of *C-line* at $A = 0$.

$\therefore$ Actual eccentricity of the *C-line* at $A = -0{\cdot}06$ *m.*

(*ii*) Shift of the *C-line* at $D = +\frac{24{\cdot}53}{100} = +0{\cdot}25$ *m.*

$\therefore$ Actual eccentricity of the *C-line* at
$D = +0{\cdot}25 - 0{\cdot}13 = +0{\cdot}12$ *m.*

(*iii*) Shift of the *C-line* at $B = \frac{-50{\cdot}47}{100} = -0{\cdot}50$ *m.*

Actual eccentricity of the *C-line* at $B = +0{\cdot}30 - 0{\cdot}50$
$= -0{\cdot}20$ *m.*

(*iv*) Shift of the *C-line* at $E = \frac{+29{\cdot}04}{100} = +0{\cdot}29$ *m.*

$\therefore$ Actual eccentricity of the *C-line* at $E = +0{\cdot}29 - 0{\cdot}18$
$= +0{\cdot}11$ *m.*

(*v*) Shift of the *C-line* at $C = 0$

$\therefore$ Actual eccentricity of the *C-line* at $C = 0$

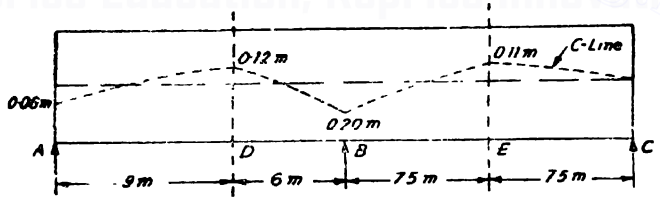

Fig. 917

Fig. 917 shows the actual position of the *C-line* after the application of the external load system.

The extreme stresses at the section *B* can now be calculated entirely due to the action of the force *C* at an eccentricity of $-0{\cdot}20$ *m.*

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Area of the beam section

$$=(2\times 30\times 15)+(15\times 60)$$
$$=900+900=1800\ cm.^2$$

Moment of inertia of the beam section

$$=I=\frac{30\times 90^3}{12}-\frac{15\times 90^3}{12}=1552500\ cm.^4$$

$C=100000\ kg.$

Eccentricity of $C=-0{\cdot}20\ m.$

∴ Extreme stresses are given by,

$$\frac{C}{A}\mp\left(\frac{C\times\text{eccentricity of }C}{I}\right)\frac{d}{2}$$

$$=\frac{100000}{1800}\mp\frac{100.000\times 20}{1552500}\times\frac{90^3}{2}\ kg./cm.^2$$
$$=55{\cdot}45\mp 57{\cdot}97\ kg/cm.^2$$
$$=-2{\cdot}52\ kg/cm.^2 \text{ and } 113{\cdot}42\ kg./cm.^2$$

Alternative method of analysis. We know the loading transmitted to the beam by the cable This loading may be algebraically added to the external loading and the beam can be analysed for the resultant loading.

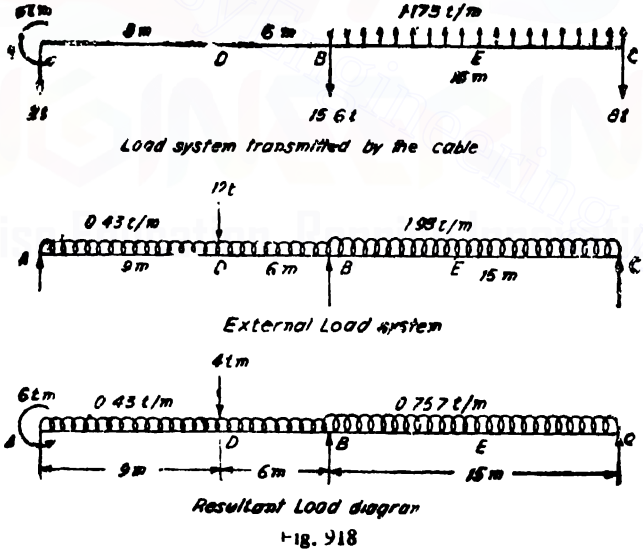

Fig. 918

Fig. 918 shows the load system transmitted by the cable, the external load system and the resultant load system.

Let us now analyse the beam for the resultant load system.

Fixed end moments

$$m_{ab}=-\frac{4\times 9\times 6^2}{15^2}-\frac{0{\cdot}43\times 15^2}{12}=-13{\cdot}82\ tm.$$

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$$m_{ba} = +\frac{4 \times 9^2 \times 6}{15^2} + \frac{0{\cdot}43 \times 15^2}{12} = +16{\cdot}10 \text{ tm.}$$

A	½	½	C
−13·82	+16·70	−14·19	+14·19
+13·82	→+3·91	−7·10 ←———	−14·19
− 6·00			
	+0·34	+0·34	
−6·08	+20·95	−20·95	0

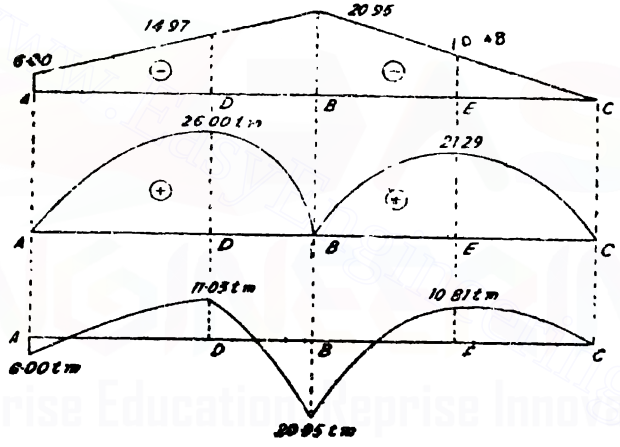

Fig. 919

$$m_{bc} = -\frac{0{\cdot}757 \times 15^2}{12} = -14{\cdot}19 \text{ tm.}$$

$$m_{cb} = +\frac{0{\cdot}757 \times 15^2}{12} \quad +14{\cdot}19 \text{ tm.}$$

Free B.M. for AB at $D = \frac{0{\cdot}43 \times 15}{2} \times 9 - \frac{0{\cdot}43 \times 9^2}{2} + \frac{4 \times 9 \times 6}{15}$

$= 26{\cdot}00$ tm.

Free B.M. at the centre of $BC = +\frac{0{\cdot}757 \times 15^2}{8} = +21{\cdot}29$ tm.

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Resultant moment at $A = -6{\cdot}00$ *tm.*

Resultant moment at $D = +26{\cdot}00 - 14{\cdot}97 = +11{\cdot}03$

Resultant moment at $B = -20{\cdot}95$ *tm.*

Resultant moment at $E = +21{\cdot}29 - 10{\cdot}48 = +10{\cdot}81$ *tm.*

Now we can calculate the actual eccentricity of the *C-line* from the resultant bending moment.

Actual eccentricity of the *C-line* at any section

$$= \frac{\text{Actual resultant B.M. at the section}}{P}$$

$\therefore$ Actual eccentricity of the *C-line* at $A = \frac{-6}{100} = -0{\cdot}06$ *m.*

,, ,, ,, ,, at $D = \frac{+11{\cdot}03}{100} = +0{\cdot}11$ *m.*

,, ,, ,, ,, at $B = \frac{-20{\cdot}95}{100} = 0{\cdot}21$ *m.*

,, ,, ,, ,, at $E = +\frac{10{\cdot}81}{100} = +0{\cdot}11$ *m.*

,, ,, ,, ,, at $C = 0$

$\therefore$ Extreme stresses at the section B

$$= \frac{C}{A} \mp \frac{M}{I} \cdot \left(\frac{d}{2}\right)$$

$$= \frac{100000}{1800} \mp \frac{(20{\cdot}95 \times 1000) \times 100}{1552500} \times \frac{90}{2}$$

$$= 55{\cdot}45 \mp 60{\cdot}59$$

$$= -5{\cdot}14 \text{ kg./cm.}^2 \text{ and } 116{\cdot}04 \text{ kg./cm.}^2$$

(The slight deviation of these results from the values obtained by the first method are due to the fact, that in the earlier method the actual eccentricity of the *C-line* at *B* was rounded off to the value (0·20 *m*).

Design of continuous beams

In the previous articles we studied about the analysis of continuous prestressed concrete beams In the case of analysis of the beam, the section of the beam, the span, and the amount and location of steel were known, and we had to determine only the stresses at any section corresponding to a given load system. But in the case of design we have to take resort to a *trial and error* process. After a set of dimensions and cable profile are selected by a trial and error process the stresses are computed and the safety of the design is checked. To plan the above process it is necessary to be familiar with the mechanics of continuous prestressed concrete beams.

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Linear transformation

Fig. 920 (*i*) shows a curve $y=f(x)$. Fig. 920 (*ii*) shows a straight line $y=(mx+c)$. Fig. 920 (*iii*) shows a curve $y=f(x)+(mx+c)$. At any point the ordinate of the curve shown in Fig. 920 (*iii*) is the sum of the ordinates at the corresponding points of the curve shown in Fig. 920(*i*) and the straight line shown in Fig. 920 (*ii*).

The curve shown in Fig. 920 (*iii*) is called a *linear transformation* of the curve shown in Fig. 920 (*i*).

Consider the two curves

$$y=f(x)$$

and $$y=f(x)+(mx+c)$$

For each of these curves

$$\frac{d^2y}{d^2x}=f''(x)$$

This means that the curvature and the bend of the curve $y=f(x)$ and the curve $y=f(x)+(mx+c)$ are the same.

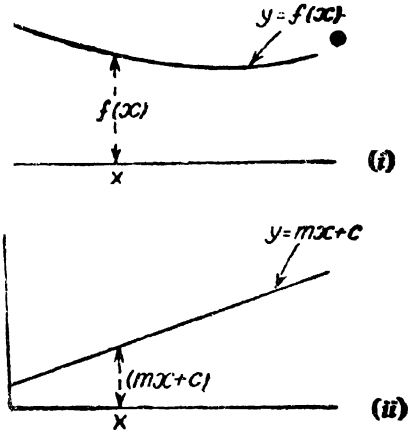

Fig. 920

Suppose in a beam the position of the *P-line* is changed over the i] ere will be no change in the *intrinsic shape*, (... and curvature of the line within individual span *then the original P-line is said to be linearly transformed to a new position.*

It should be noted in the following discussions, the position of the line —say the *P-line* will be moved *only over the interior supports if felt necessary but these will not provide any displacement to the line at the end of the beam.* Hence, in the case of a linear transformation of the *P-line* subject to the above condition, the intrinsic shape of the line within each span remains undisturbed but there will be a change in the amount of bending of the line over the interior supports.

We know, in the case of a continuous beam, the shift of the *C-line* from the *P-line* under the action of the prestressing force above is entirely due to the secondary moments. The secondary moment diagram being a linear diagram, it may be realized that when the beam is not subjected to any external loading *the C-line resulting from prestressing is a linear transformation of the P-line.*

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Another useful principle to be borne in mind is that *when an existing P-line is given a linear transformation, it will not affect the position of the C-line. Since the stresses in concrete depend on the C-line only, it follows that any linear transformation of the P-line will not affect the stresses in the concrete.*

The above principle can be easily proved.

Suppose at any section if the primary moment is M, the corresponding primary loading (*i.e.*, loading transmitted to the beam by the cable) is given by

$$\text{Primary load} = \frac{d^2M}{dx^2}$$

The primary moment $M = P \cdot y = P \times [f(x)]$ where $y = f(x)$ is the equation to the cable with respect to the end of the beam on the longitudinal axis as origin.

$$\therefore \quad \text{Primary load} = \frac{d^2M}{d^2x} = P \cdot f''(x)$$

If now the cable is linearly transformed to $y = f(x) + (mx + c)$, the corresponding primary moment would be

$$M = P[f(x) + (mx + c)]$$

$$\text{Now the primary load} = \frac{d^2M}{dx^2} = P \cdot f''(x)$$

Hence the loading transmitted to the beam will not be affected when the *P-line* is linearly transformed. This means the resultant moment will not be affected due to a linear transformation of the *P-line*, which means that the *C*-line will not be affected.

It may be noted that the primary moment and the secondary moments will be affected by a linear transformation of the *P-line*. But the resultant moment remains unchanged.

It is very important to note that at no stage we are changing the position of the *P-line* at the ends of the beam. Any change in the position of the *P-line* at the ends of the beam introduces changes in the end moments and will affect the moments at all sections of all spans and will thus disturb the *C-line*.

Design 228 *Fig. 921 shows two proposed P-lines for a prestressed concrete beam which are linear transformations of each other. Show that the C-line in the concrete is the same for both the cable positions. Assume a prestressing force of 100t.*

Solution :

Consider the beam with *P-line No. 1.* See Fig. 921 (*a*).

Fig. 921 (*b*) shows the primary moment diagram.

$$\text{Primary shear between } A \text{ and } D = \frac{-14-10}{4} = -6t$$

$$\text{Primary shear between } D \text{ and } B = \frac{-8+14}{6} = 1t$$

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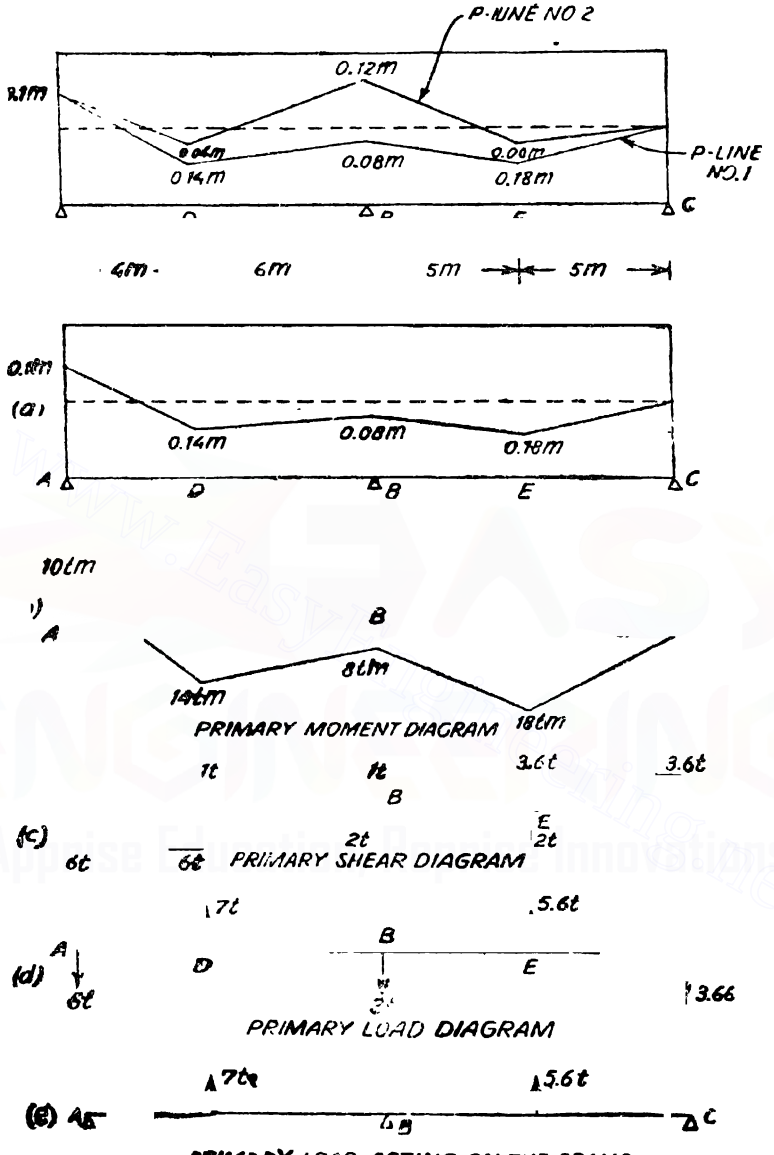

Fig. 921. Primary load acting on the spans.

Primary shear between B and $E = \frac{-18+8}{4} = -2t$

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Primary shear between E and C $\frac{0+18}{5} = +3{\cdot}6\ t$

Fig. 921 (*c*) shows the primary shear diagram.

Now it is easy to determine the primary loading.

The primary loading consists of the following :

(*i*) Downward load at $A = 6t \downarrow$
(*ii*) Upward load at $D = 6+1 = 7t \uparrow$
(*iii*) Downward load at $E = 1+2 = 3t \downarrow$
(*iv*) Upward load at $E = 2+3{\cdot}6 = 5{\cdot}6t \uparrow$
(*v*) Downward load at $C = 3{\cdot}6t \downarrow$

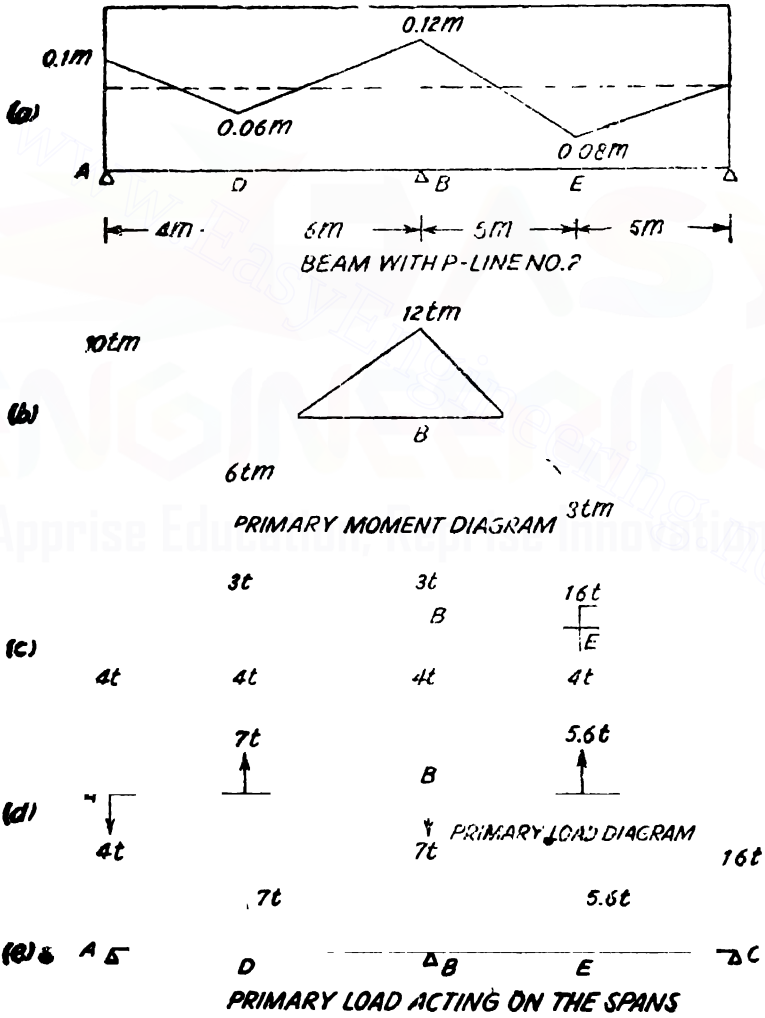

Fig. 922. Primary load acting on the spans.

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The loads directly transmitted to the supports A, B and C do not produce any moments.

Fig 921 (e) shows the primary load system acting on the two spans AB and BC.

Now consider the beam with *P-line No. 2*. See Fig. 922 (a)

Fig. 922 (b) shows the primary moment diagram.

Primary shear between A and $D = \frac{-6-10}{4} = -4t$

Primary shear between D and $B = \frac{+12+6}{6} = +3t$

Primary shear between B and $E = \frac{-8-12}{5} = -4t$

Primary shear between E and $C = \frac{0+8}{5} = +1{\cdot}6t.$

Fig. 922 (c) shows the primary shear diagram.

Now it is easy to determine the primary loading. The primary loading consists of the following :

(i) Downward load at $A = 4t \downarrow$
(ii) Upward load at $D = 4+3-7t \uparrow$
(iii) Downward load at $B = 3+4 = 7t \downarrow$
(iv) Upward load at $E = 4+1{\cdot}6 = 5{\cdot}6t \uparrow$
(v) Downward load at $C = 1{\cdot}6t \downarrow$

The loads directly transmitted to the supports A, B and C do not produce any moments.

Fig. 922 (e) shows the primary load system acting on the two spans AB and BC.

We find from the above analysis that the primary load system on the spans AB and CD are the same for the two *P-lines* given.

Since the primary load system on the spans are the same for the two *P-lines*, it follows that the resultant moment diagram will be the same for the two *P-lines*. This means for each of the *P-lines* given, the *C-line* will be the same since the *C-line* profile depends only on the resultant moments.

§235. Concordant Cable Profile

A concordant cable profile is a profile given for the cable such that the consequent *C-line* coincides with the *P-line*, when external loads are not applied. This means when a concordant profile is provided there will be no reactions at the supports due to prestressing alone

In any general profile given to the cable, reactions are produced at the supports due to prestressing alone. These reactions produce the secondary moments, and hence shift the *C-line* from the *P-line*. When a concordant profile is provided there will be no deflection of the beam at the supports due to prestressing alone.

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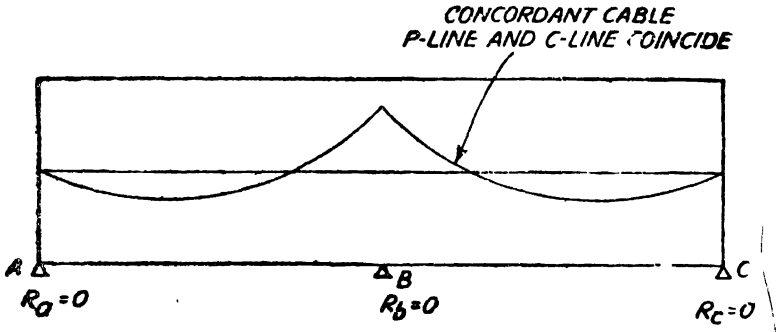
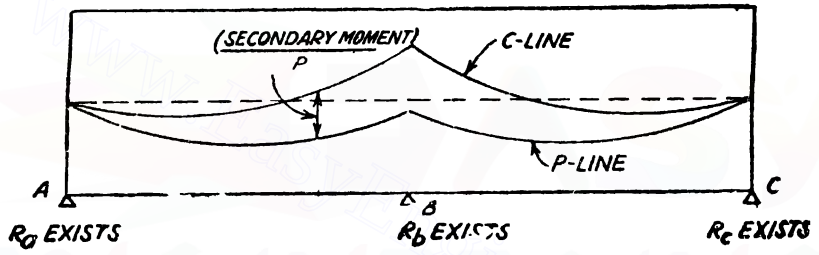


(*a*) Beam with concordant cable
Secondary moments do not exist
Primary moment = Resultant moment

Fig. 923



(*b*) Beam with non-concordant cable
Secondary moments exist
Primary moment + Secondary moment = Resultant moment

Fig. 924

The main differences between the beam with concordant profile and the beam with non-concordant profile are shown in Figs. 924 (*a*) and (*b*).

It is worthy to note the secondary moment diagram in a linear diagram and hence the resultant moment diagram is a linear transformation of the primary moment diagram. This obviously leads to the conclusion that the *C-line itself is a linear transformation of the P-line.*

This means, a C-line obtained from a non-concordant profile is one possible location for a concordant profile. This is due to the fact that all linear transformations of a given *P-line* produce the same *C-line.*

Hence a new *P-line* along the *C-line* corresponding to a non-concordant cable will not change the position of the *C-line.* This means the new *P-line* and the *C-line* coincide and thus the new *P-line* is a concordant profile.

If a concordant profile is provided there is no other benefit from the structural consideration except that the analysis is simple. The resultant moment due to prestress will be the same as the

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primary moment. Generally we find it convenient to obtain a concordant cable which gives good positions in resisting the external moments.

Though many methods are available to locate a concordant profile it may be found convenient to adopt the following method. The method is based on the following principle :

A profile following the bending moment diagram for a continuous beam due to any external loading drawn to any scale is one of the locations for a concordant profile.

The above principle can be realized easily We know the bending moments for a continuous beam are determined to satisfy the condition of no deflections at the supports. Hence a cable profile corresponding to the bending moment diagram will also produce such moments which do not result in the deflection at the supports. Hence a profile following the bending moment diagram is a concordant profile.

Two span continuous beam carrying uniformly distributed loading

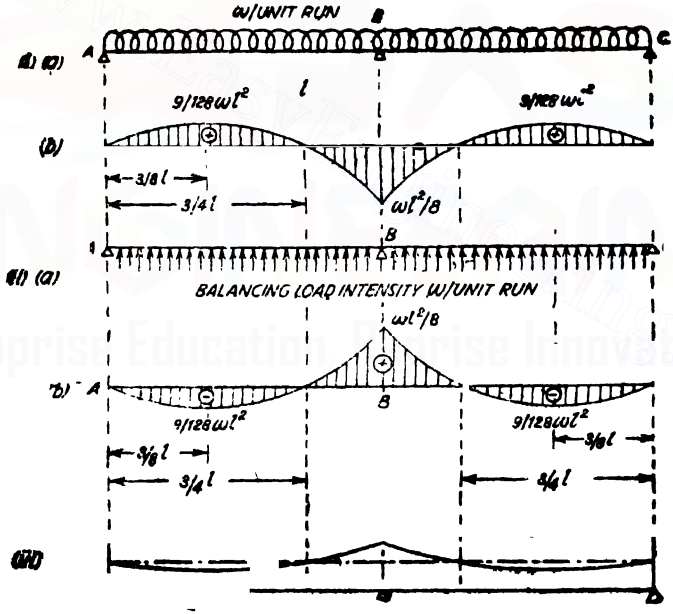

Fig. 925

Fig. 925 (*i*) (*a*) shows a continuous beam consisting of the spans *AB* and *BC* each of length *l* carrying a downward uniformly distributed load of intensity *w* per unit run.

Fig. 925 (*i*) (*b*) shows the bending moment diagram for the above loading.

The maximum negative bending moment is $\frac{wl^2}{8}$ occurring at the middle support.

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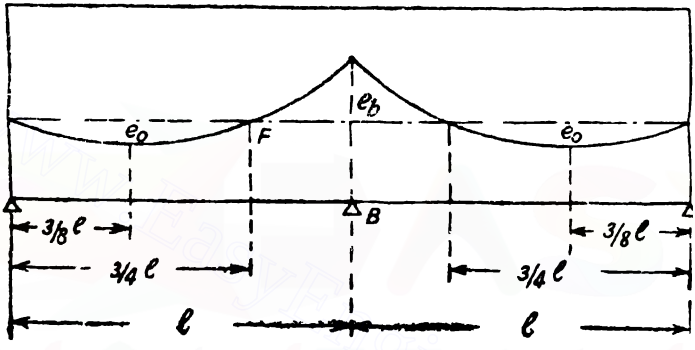
The maximum positive bending moment is $\frac{9}{128}\ wl^2$ occurring at a distance $\frac{3}{8}\ l$ from each end support.

A good cable profile should be such that it should provide an upward uniformly distributed load. Let the upward uniformly distributed load intensity provided by the cable be w' per unit run as shown in Fig. 925 (*ii*)(*a*). The bending moment diagram corresponding to this upward uniformly distributed loading is shown in Fig. 925 (*ii*)(*b*). If the cable profile corresponds to the above bending moment diagram to any scale then the profile will be a concordant profile.



ig. 926

Let e_b be the eccentricity of the cable at the middle support.

Let e_o be the maximum eccentricity of the cable below the centroidal axis See Fig. 926.

For the cable to follow a profile similar to the bending moment diagram, corresponding to the upward uniformly distributed loading,

$$e_o : e_b = \frac{9}{128}\ w'l^2 : \frac{w'l^2}{8}$$

$$e_o : e_b = 9 : 16$$

Now consider the part EF of the cable. Since w' is the upward loeding intensity, we have,

$$\frac{w'EF^2}{8e_o} = P \text{ where } P \text{ is the prestressing force}$$

$$\therefore \quad P = \frac{w'}{8e_o}\left(\frac{3}{4}\ l\right)^2 = \frac{9w'l^2}{16 \times 8e_o}$$

But $$e_o = \frac{9}{16}\ e_b$$

$$\therefore \quad P = \frac{9}{16}\ \frac{w'l^2}{8\left(\frac{9}{16}\ e_b\right)}$$

$$\therefore \quad P = \frac{w'l^2}{8e_b}$$

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The following steps may be adopted in the design of a two span continuous beam subjected to uniformly distributed loading.

Data : Span between consecutive supports $= l$

Live load : w_l per unit run

Safe stresses in concrete and tendon.

1. Choose a suitable depth of beam—*say* $\frac{1}{20}$ to $\frac{1}{25}$ of the span.

2. Choose a suitable maximum eccentricity e of the cable for the section at the middle support. This may be about $\frac{1}{3}$ to $\frac{1}{4}$ of the depth of the beam. The maximum eccentricity below the longitudinal axis $= e_0 = \frac{9}{16}\ e$ *at a distance* $\frac{3}{8}\ l$ *from the end.*

3. The parabolic cable should provide a certain amount of balancing load to counteract the downward loading Choose a certain amount of balancing load intensity. This may be nearly 0·7 to 0 9 times the live load intensity. Let w' be the balancing load intensity chosen.

4. Now determine the prestressing force necessary from the relation

$$P = \frac{w' l^2}{8 e}$$

This may be rounded off to a convenient value.

5. Allowing an average stress of 80 *kg/cm.*^{2} estimate the sectional area required for the beam.

$$\text{Sectional area} = A = \frac{\text{Prestressing force}}{\text{Average stress on the section}}$$

Now finalize the sectional dimensions of the beam and calculate the dead load intensity w_d per unit length of the beam.

6. Check for the stresses induced

(*a*) *Stresses at transfer of prestress*

Allowing *say* a possible loss of prestress of 18%,

Prestressing force at the stage of transfer of prestress $\Big\} = P_t = \frac{P}{0\ 82}$

At this stage the beam is subjected to the following forces :

(*i*) Dead load $= w_d$ per unit run

(*ii*) Upward pressure $= w' = \frac{8\ P_t\ e}{l^2}$

∴ Net upward loading per unit run $= (w' - w_d)$ per unit run.

Max. sagging moment over the middle support

$$= \frac{(w' - w_d)}{8} l^2$$

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Max. hogging moment at $\frac{3}{8}$ l from each end support

$$=\frac{9}{16}(w'-w_d)\frac{l^2}{8}$$

Stresses induced due to these moments may be compared with the safe stresses.

(*b*) *Stresses during the working load conditions.*

At this stage the prestressing force in the cable is P and the corresponding upward pressure transmitted by the cable per unit run,

$$=w'=\frac{8\,Pe}{l^2}$$

Net downward loading on the beam

$$=(w_d+w_l-w') \text{ per unit run}$$

$\therefore$ Max. hogging moment over the middle support

$$=\frac{(w_d+w_l-w')\,l^2}{8}$$

Max. sagging moment at $\frac{3}{8}$ l from each end support

$$=\frac{9}{16}(w_d+w_l-w')\frac{l^2}{8}$$

Stresses induced due to these moments may be compared with the safe stresses.

Design 229. *A post tensioned continuous beam consists of two spans each 20 metres long. The external loading other than the dead load of the beam is 2000 kg./m.*

Solution.

(*i*) Choose a depth of beam equal to $\frac{1}{20}$ of the span

$\therefore$ $$d=\frac{1}{20}\times 20\times 100=100 \text{ cm.}$$

(*ii*) Maximum eccentricity e of the cable for the section at the middle support

$$=e=\text{say } \frac{1}{3} \text{ depth}$$

$$=\frac{100}{3}=33{\cdot}3 \text{ cm.}$$

provide $e=32$ cm.

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Maximum eccentricity below the longitudinal axis

$= e_o = \frac{9}{16}\ e$ at $\frac{3}{8}\ l$ from the end support

$= \frac{9}{16} \times 32 = 18$ *cm.* at $\frac{3}{8} \times 20 = 7{\cdot}50$ *m.* from each end support

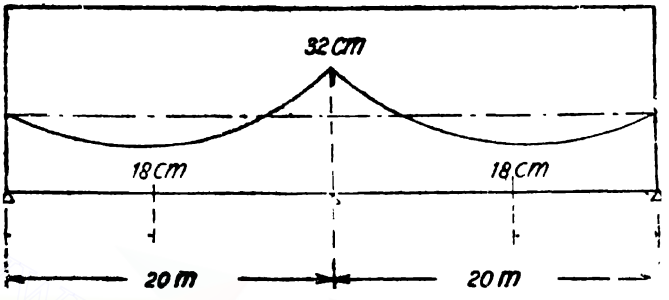

Fig. 927

(*iii*) Choose a balancing load equal to say 0·9 times the live load.

$\therefore$ Balancing load intensity $= w' = 0{\cdot}9 \times 2000 = 1800$ *kg./m.*

(*iv*) Prestressing force necessary

$$= P = \frac{w'\, l^2}{8e} = \frac{1800 \times 20^2}{8 \times 0{\cdot}32}\ kg.$$

$$= 281250\ kg.$$

Let us provide a prestressing force of 300,000 *kg.*

(*v*) Making an allowance of 80 *kg./cm.*^{2} for the average stress on the section of the beam,

Sectional area required $= \frac{300{,}000}{80} = 3750\ cm.^2$

Fig. 928 shows the proposed section for the beam.

Actual area provided

$= A = (2 \times 50 \times 25) + (25 \times 50)\ cm.^2$

$= 3750\ cm.^2$

$\therefore$ Dead load of the beam

$= \frac{3750}{100 \times 100} \times 2400$

$= 900\ kg./m.$

Moment of inertia of the section

$= I = \frac{50 \times 100^3}{12} - \frac{(50-25)50^3}{12}$ *cm*^{4}.

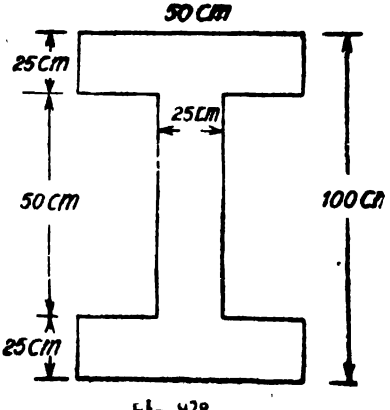

Fig. 928

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$$= 3,906,000 \ cm.^4$$

$\therefore$ Section modulus $= Z = \dfrac{3,906,000}{50} \ cm.^3$

$$= 78120 \ cm.^3$$

(vi) Check for stresses

(a) Stresses at transfer of prestress

To make allowance for a loss of prestress of say 18%, the initial prestressing force

$$= P_t = \frac{P}{0.82} = \left(\frac{300,000}{0.82}\right) kg.$$

At this stage, the beam is subjected to the following loads

1. Dead load $= 900 \ kg/m.$
2. Upward pressure $= w' = \dfrac{8 P_t e}{l^2} = 8 \times \dfrac{300,000}{0.82} \times \dfrac{0.32}{(20)^2}$

$$= 2342 \ kg/m.$$

$\therefore$ Net upward loading $= 2342 - 900 = 1442 \ kg./m.$

$\therefore$ Maximum sagging moment at mid support

$$= \frac{1442 \times 20^2}{8} \times 100 \ kg. \ cm.$$

$$= 7,210,000 \ kg. \ cm.$$

$\therefore$ Extreme stresses on the section $= \dfrac{P_t}{A} \pm \dfrac{M}{Z}$

$$= \frac{300,000}{0.82 \times 3750} \pm \frac{7,210,000}{78120} \ kg./cm.^2$$

$$= 97.57 \pm 92.30$$

$$= 189.87 \ kg./cm.^2 \text{ and } 5.27 \ kg./cm.^2$$

Safe stress at transfer $= 0.5 \ F_c = 0.5 \times 450 = 225 \ kg./cm.^2$

The stresses will be obviously within the safe limit for the max. hogging moment also.

(b) Stresses during the working load condition.

At this stage the loading on the beam consists of the following :

1. Dead load $= 900 \ kg/m.$
2. Live load $= 2000 \ kg./m.$
3. Upward pressure

$$= w' = \frac{8 Pe}{l^2} = 8 \times \frac{300,000 \times 0.32}{20 \times 20}$$

$$= 1920 \ kg./m.$$

$\therefore$ Net downward load $= 900 + 2000 - 1920 = 980 \ kg/m.$

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$\therefore$ Max. bogging bending moment at the support

$$= 980 \times \frac{20^2}{8} \times 100 \text{ kg. cm.}$$

$$= 4{,}900{,}000 \text{ kg. cm.}$$

$\therefore$ Extreme stresses on the section

$$= \frac{P}{A} \pm \frac{M}{Z}$$

$$= \frac{300{,}000}{3750} \pm \frac{4{,}900{,}000}{78120} \text{ kg./cm.}^2$$

$$= 80 \pm 62{\cdot}72 \text{ kg./cm.}^2$$

$$= 142{\cdot}72 \text{ kg./cm.}^2 \text{ and } 17{\cdot}28 \text{ kg./cm.}^2$$

Safe stress $= 0{\cdot}4\, F_c$

$$= 0{\cdot}4 \times 450 = 180 \text{ kg./cm.}^2$$

§236. Prestressed Circular Tanks and Pipes

Pipes and tanks are prestressed by providing prestressed wires round a core of concrete. The wires are covered with pneumatic mortar. Sometimes longitudinal prestress is also done by pretensioning the longitudinally placed wires In Australia a pretensioning method called the Rocla Roller compaction method is popular. In this method dense concrete is placed to form a thin-walled pipe, by a rotating rolling mould. After placing the concrete the reinforcement embedded in the concrete is tensioned by applying a high pressure inside the pipe. After the hardening of the concrete the inside pressure is released. The tensioned wires compress the concrete.

Another method often used is the pre-load method in which the concrete cylinder is wrapped by tensioned wires using the wire winding machines.

§237. Principles of Circumferential Prestressing

In a tank a circumferential prestress is provided to resist hoop tension produced by internal liquid pressure. The tank may be taken to consist of a number of circular rings, each of which is designed for hoop tension Fig 929 shows one half of such a ring. This ring is surrounded by prestressing wires carrying an initial circumferential prestressing

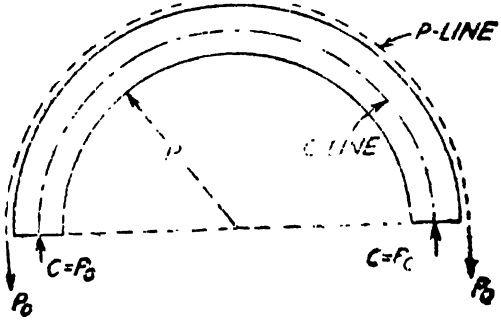

Fig. 929

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force P_o. The *C-line* or the line of pressure will coincide with the circumferential centroidal line. This can be realized by applying the theory of linear transformation. The closed ring is statically indeterminate and a cable provided along the circumferential centriodal axis is a concordant cable. Hence any cable profile parallel to this line is only a linear transformation of the circumferential centroidal axis and will not therefore alter the position of the *C-line*. The circumferential hoop compression produced in concrete is thus an axial compression irrespective of the position of the prestressing wires.

Let P_o = Initial prestressing force

Compressive stress in concrete due to initial prestressing force

$$-f_o = \frac{P_o}{A_c}$$

where A_c = actual area of concrete

After allowing for losses if the net prestressing force is P, then the compressive stress in concrete due to final prestressing force

$$= f_c = \frac{P}{A_c}$$

If now the internal liquid pressure p is applied, hoop tension due to this internal pressure

$$= pR$$

This hoop tension is resisted by concrete and steel. Using the principle of transformed section based on the elastic theory,

Equivalent concrete area $= A_e = A_c + mA_s$

where m = modular ratio

$\therefore$ tensile stress in concrete due to internal liquid pressure

$$= -\frac{pR}{A_e}$$

$\therefore$ Resultant stress in concrete under the action of the final prestressing force and internal liquid pressure,

$$= \frac{P}{A_c} - \frac{pR}{A_e}$$

The practice followed is to provide a little residual compression in the concrete under the working conditions.

§238. Methods of Design

Some of the methods of design are given below :

First method. The following procedure is adopted :

1. We will assume that the hoop tension caused by the internal liquid pressure is resisted completely by the final prestress in the steel alone.

To satisfy this condition,

$P = pR$ = area of steel × final stress intensity in steel

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$\therefore$ area of steel $= A_t = \dfrac{p \cdot R}{\text{Final stress intensity in steel}} = \dfrac{p.R}{f_s}$

where f_s = final stress intensity in steel

2. Initial prestressing force

$= P_o$ = area of steel × Initial stress in steel

$= A_t \times f_o$

where f_o = initial stress intensity in steel.

3. The compressive stress in concrete due to the initial prestressing force should be limited to the allowable compressive stress f_c.

$\therefore$ area of concrete $= A_c = \dfrac{P_o}{f_o}$

After finding A_c the thickness of the tank wall can be calculated.

4. When the tank is subjected to internal pressure the resultant compressive stress in concrete

$$= \frac{P}{A_c} - \frac{pR}{A_s}$$

The tensile stress in steel

$$= f_s + nf_c$$

Since P is a compressive force equal to the tensile force pR and also, since A_s is greater than A_c it is evident that $\dfrac{P}{A_c} > \dfrac{pR}{A_s}$ and hence there will be a little residual compression in concrete under the working conditions.

Design 230. *A prestressed concrete tank of diameter 20 metres has to resist an internal pressure head of 5 metres of water. Find the reinforcement required per metre height and the thickness of concrete required. Strength of mortar coating over the wires may be ignored.*

Take F_c = *ultimate strength of concrete* = 350 *kg./cm.*^{2}

f_c = *Safe stress in concrete* = 0·5 F_c *at transfer. Stress in concrete at servicing condition shall remain compressive.*

f_s = 10000 *kg./cm.*^{2}

Loss of prestress = 18%, m = 8·00

Solution. (1) Assuming the hoop tension to be resisted completely by the steel alone

area of steel required per *metre* height $= A_t = \dfrac{pR}{f_s}$

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$$= \frac{(1000 \times 5) \times 10}{10,000} \ cm^2$$

$$= 5 \ cm.^2$$

Spacing of 6 *mm.* dia wires

$$= \frac{0.28 \times 100}{5} = 5.6 \ cm. \ c/c$$

Provide 6 mm. wires at 5 cm. c/c

Actual steel provided $= \frac{0.28 \times 100}{5} = 5.60 \ cm.^2$

(2) Initial prestressing force

$= P_o =$ area of steel $\times$ initial stress in steel

$= A_t \times f_o$

$= 5.60 \times \frac{10000}{0.82} \ kg.$

$= 68290 \ kg.$

(3) Safe stress in concrete at transfer of stress

$= f_c = 0.5 \ F_c$

$= 0.5 \times 350 = 175 \ kg./cm.^2$

$\therefore$ area of concrete $= A_c = \frac{P_c}{f_c}$

$= \frac{68290}{175} = 390.3 \ cm.^2$

$\therefore$ thickness of wall $= \frac{390.3}{100} \ cm.$

$= 3.903 \ cm.$

Provide a thickness of 4 cm.

(4) When the tank is subjected to internal pressure the resultant compressive stress in concrete

$$= f_c = \frac{P}{A_e} - \frac{pR}{A_e}$$

Equivalent concrete area

$= A_e = A_c + mA_t$

$= (4 \times 100) \times (8 \times 5.60) \ cm.^2$

$= 444.8 \ cm.^2$

Resultant compressive stress in concrete

$$f_c = \frac{10000 \times 5.60}{100 \times 4} - \frac{1000 \times 5 \times 10}{444.8} \ kg./cm.^2$$

$= 140 - 112.4 = 27.6 \ kg./cm.^2$

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Second method. In this method the tank wall is designed to satisfy the following two conditions :

(*i*) At transfer, when the tank is empty and the initial pre-stressing force acts, the compressive stress in concrete reaches the permissible stress $f_c = 0{\cdot}5\ F_c$.

(*ii*) At service condition, when the tank is full, and the final prestressing force acts, the resultant compressive stress in concrete shall reach a desired residual value f_{c1}.

The following steps may be adopted in the design :

1 Consider unit height of wall at bottom.

At transfer, tension in steel = compression in concrete

$$\therefore \qquad P_o = f_s A_t = f_c A_c$$

$$\therefore \qquad A_t = \frac{f_c}{f_s} A_c$$

or $$A_t = K A_c \qquad \ldots(i)$$

2. Equivalent concrete area

$$= A_e = A_c + m A_t$$
$$= A_c + m K A_c$$
$$= K_1 A_c$$

Hoop tension $\quad -T = pR$

Final pre-stressing force

$$= P = \lambda P_o = \lambda f_s A_t = \lambda f_c A_c$$

Residual compressive stress in concrete

$$= \frac{P}{A_c} - \frac{T}{A_e} = f_{c1}$$

$$\lambda f_c - \frac{T}{K_1 A_c} = f_1 \qquad \ldots(ii)$$

We can find A_e from the above relation. After finding A_e we can find A_t from equation (*i*) and also the thickness of wall.

Design 231. *Design a free edge water tank of 18 metres radius to store water to a depth of 5 metres. Assume $f_c' = 450$ kg./cm.^{2} $f_s' = 15$ tonnes/cm.^{2}, stress in steel at transfer $= 0{\cdot}7^* f_s'$, compressive stress in concrete at transfer $= 0{\cdot}5\ f_c'$ concrete stress at service condition $= 0{\cdot}1\ f_c'$. $\eta = \frac{P_o}{P_t} = 0{\cdot}8$, and modular ratio $= 5{\cdot}5$.*

Solution.

Safe stresses

Safe stress in steel at transfer

$$= f_s = 0{\cdot}7\ f_s' = 0{\cdot}7 \times 15000 = 10500 \text{ kg./cm.}^2$$

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Final safe stress in steel

$$= f_s = 0{\cdot}8\, f_o = 0{\cdot}8 \times 10500 = 8400 \text{ kg./cm.}^2$$

Safe stress in concrete at transfer

$$= f_c = 0{\cdot}5\, f_c' = 0{\cdot}5 \times 450 = 225 \text{ kg./cm.}^2$$

Residual compressive stress in concrete at service condition

$$= f_{c1} = 0{\cdot}1\, f_c' = 0{\cdot}1 \times 450 = 45 \text{ kg./cm.}^2$$

Analysis at transfer

Consider the bottom 1 *metre* height of wall.

Tension in steel = compression in concrete

$$P_o = f_s A_t = f_c A_c$$

$$\therefore \quad A_t = \frac{f_c}{f_o} A_c = \frac{225}{10500} A_c$$

$$A_t = 0{\cdot}0214\, A_c \qquad \ldots(i)$$

Equivalent concrete area

$$= A_e = A_c + mA_t$$

$$= A_c + 5{\cdot}5 \times 0{\cdot}0214\, A_c$$

$$\therefore \quad A_e = 1{\cdot}1177\, A_c \qquad \ldots(ii)$$

Analysis at servicing condition

Hoop tension $= T = pR = wHR$

$$= 1000 \times 5 \times 18 = 90{,}000 \text{ kg.}$$

Final prestressing force

$$= P = 0{\cdot}8\, P_o$$

$$= 0{\cdot}8\, f_s A_t$$

$$= 0{\cdot}8\, f_c A_c$$

$$= 0{\cdot}8 \times 225\, A_c$$

$$= 180\, A_c$$

$$\frac{P}{A_c} - \frac{T}{A_e} = f_{c1}$$

$$\therefore \quad \frac{180\, A_c}{A_c} - \frac{90{,}000}{1{\cdot}1177\, A_e} = 45$$

$$\therefore \quad \frac{90{,}000}{1{\cdot}1177\, A_e} = 135$$

$$\therefore \quad A_e = \frac{90{,}000}{1{\cdot}1177 \times 135} = 596{\cdot}50 \text{ cm.}^2$$

Thickness of wall $= \frac{596{\cdot}5}{100} = 5{\cdot}965$ *cm. say 6 cm.*

$$A_t = 0{\cdot}0214\, A_e$$

$$= 0{\cdot}0214 \times 596{\cdot}50 = 12{\cdot}76 \text{ cm.}^2$$

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Spacing of 8 mm. ϕ wires

$$= \frac{0{\cdot}5 \times 100}{12{\cdot}76} = 3{\cdot}92 \text{ say } 3{\cdot}90 \text{ cm. c/c.}$$

Third method. In this method of design, a stipulated factor of safety is introduced.

Suppose a factor of safety of F against cracking is to be allowed.

Tensile strength of concrete at cracking stage

$$-f_t = 0{\cdot}08\ F_c.$$

(But, this will be zero if cracks are already formed or in case precast blocks are used)

Corresponding to the above condition

$$\frac{P}{A_c} - F\frac{pR}{A_c} = -f_t$$

The compressive stress corresponding to the initial pre-stress should not exceed the permissible value f_c.

$$\therefore \qquad A_c = \frac{P_o}{f_c}$$

From the relation

$$\begin{cases} \dfrac{P}{A_c} - \dfrac{FpR}{A_c} = -f_t \\ \\ A_c = \dfrac{P_o}{f_c} \end{cases}$$

and also

$$\begin{cases} A_c = A_c + mA_t \\ P = f_c A_t \\ P_o = f_c A_t \end{cases}$$

we have,

$$\frac{f_c A_t f_c}{f_c A_t} - \frac{FpR}{\dfrac{f_c A_t}{f_c} + mA_t} = -f_t$$

Solving for A_t, we get,

$$A_t = \frac{FpR}{\left[f_s - \dfrac{f_t f_c}{f_c}\right]\left[1 - m\dfrac{f_c}{f_c}\right]}$$

The area of steel A_t can be calculated from the above formula to provide a requisite factor of safety F.

After calculating A_t we can calculate P_o and A_c from the relations

$$P_o = A_t f_c \text{ and } A_c = \frac{P_o}{f_c}$$

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The stresses in concrete and steel may be calculated from the relation,

Stress in concrete $= \frac{P}{A_c} - \frac{pR}{A_s}$

Stress in steel $= f_s + m f_c$.

Design 232. *A prestressed concrete tank of diameter 20 metres has to resist an internal pressure head of 5 metres of water. Find the reinforcement required per metre height and the thickness of the concrete required. Strength of the mortar coating over the wires may be neglected. Provide a factor of safety of 2 at which the stress in concrete should reach zero. Safe stress in concrete at transfer = 175 kg./cm².. Stress in steel at transfer = 10500 kg./cm.², stress in steel at working condition = 8400 kg /cm². Take m = 8.*

Solution. Consider the bottom 1 *metre* height of the tank wall.

Steel area required

$$= A_t = \frac{FpR}{\left[f_s - f_t \frac{f_c}{f_c} \right] \left[1 - m \frac{f_c}{f_c} \right]}$$

$$= \frac{1{\cdot}25\,(1000 \times 5)\,10}{(8400 - 0)\left(1 - 8 \times \frac{175}{10500} \right)}$$

$$= 8{\cdot}58 \text{ cm.}^2$$

Spacing of 6 *mm.* ϕ wires

$$\frac{0{\cdot}28 \times 100}{8{\cdot}58} = 3{\cdot}26 \text{ say } 3{\cdot}25 \text{ cm. c/c.}$$

Actual steel provided $= \frac{0{\cdot}28 \times 100}{3{\cdot}25} = 8{\cdot}61 \text{ cm.}^2$

$\therefore$ $P_c = 8{\cdot}61 \times 10500 = 90405$ kg.

$\therefore$ $A_c = \frac{90405}{175} = 516{\cdot}60 \text{ cm.}^2$

$\therefore$ Thickness of wall

$$= \frac{516{\cdot}6}{100} = 5{\cdot}166 \text{ cm.}$$

Provide a thickness of 5·25 cm.

Equivalent concrete area

$$= A_e = (100 \times 5{\cdot}25) + (8 \times 8\,61) = 593{\cdot}88 \text{ cm.}^2$$

Final prestressing force $= P = 8{\cdot}61 \times 8400 = 72324$ kg.

Hoop tension $= T = pR = 1000 \times 5 \times 10 = 50{,}000$ kg.

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Residual compressive stress in concrete at service condition

$$= \frac{P}{A_c} - \frac{T}{A_e}$$

$$= \frac{72324}{100 \times 5\,25} - \frac{50,000}{593\,88}$$

$$= 137\,76 - 84\,19 = 53{\cdot}57\ kg./cm^2.$$

§239. Prestressed Concrete Piles

Instead of using the conventional reinforced concrete piles, if prestressed concrete piles are used, there is bound to be economy. Prestressed concrete piles are of smaller sectional area than reinforced concrete piles, to carry the same load. (Reduction of the consequent surface area is however not advantageous in the case of friction piles). Usually prestressed concrete piles are relatively easier to handle and pitch and are more efficiently driven. By providing sufficient amount of prestress, it is possible to avoid tension cracks completely due to handling stresses. The pile is more resilient due to the pre-compression and can resist better the stresses due to shock of driving.

§240. Prestressed Concrete Road Slabs

Road slabs may also be prestressed and used with the advantages mentioned below :

(*i*) The slab may be made of longer units between joints.

(*ii*) To provide the same strength as the reinforced concrete slab, it requires a lesser thickness if a prestressed concrete slab is provided

(*iii*) The prestressed slab has a greater resilience and flexibility than the reinforced concrete slabs

Road slabs can be individually prestressed between joints at great intervals The interval may be up to 200 *m*. Usually the main cables are provided in a longitudinal direction (*i.e*, parallel to the direction of the road). In addition, in a transverse direction secondary prestress is introduced using secondary reinforcements. In some designs, it is also a practice to provide the cables forming a diagonal grid. It is preferable to provide a smooth layer (of building paper or polythene sheet over a thin smooth finished layer of sand) over which the slab is placed in order to overcome the subgrade frictional resistance to compression of the slabs. The design is usually based on the requirement to overcome the subgrade resistance and then, over and above this condition an additional prestress of 15 $kg./cm^2$ is provided as a margin. The thickness need for the slab is mostly from the point of view of accommodating the cables with a safe cover. Generally a thickness of 15 *cm.* is found sufficient.

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§241. Prestressed Concrete Railway Sleepers

These days, prestressed concrete sleepers are being manufactured and successfully used. The ordinary reinforced concrete sleepers develop cracks and fail ultimately by crushing. But prestressed concrete sleepers are more resilient and have the property to recover on the removal of loading Bond stresses are usually greater near the ends of sleepers. These problems may be overcome by providing wires of small diameter with crimpled surfaces The cover to the reinforcement is not less than three diameters. This is found to increase the bond. Moreover, when the wires are tensioned the diameter of wires decreases by about 0·15% and when the prestress tension is transmitted to concrete by bond, the unstressed parts of the wires regain their free diameters and provide a conical wedging action. This wedging action is found to be very satisfactory when the cover of three diameters is provided.

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23

Composite Construction

Composite construction consists of providing monolithic action between (*i*) *Prefabricated units* like steel beams, precast reinforced or prestressed concrete beams and (*ii*) *Cast-in-situ* concrete. This method is found to provide a greater structural efficiency compared with the conventional methods of construction.

We know, in the conventional method of the steel beam and slab construction, there is no composite action between the two and each component carries the entire load transmitted by the slab. It can be realized that if sufficient shear connection be provided between the prefabricated beam and the cast-in-situ slab the two will act as one unit and the load will be resisted by composite action as in an R.C. tee beam. Only prefabricated or only cast-in-situ construction have their own advantages and disadvantages. But composite construction will have both the advantages of prefabricated and cast-in-situ construction. Composite construction has the added advantage, that the prefabricated units can be used also to serve as form work for the cast-in-situ work.

There are two methods of assembly adopted in composite construction. These are (*i*) the *unpropped method* and (*ii*) *the propped method.*

In the unpropped method, the prefabricated units are made strong enough to support the dead load of wet concrete and any constructional live load and load due to accidental form work.

In the propped method the prefabricated units-remain supported as the props even during the laying as well as curing of cast-in-situ concrete. Hence, when the props are removed the whole unit *i.e.*, prefabricated and the cast-in-situ unit, will act as a monolithic unit strong enough to carry the dead load and the live load.

Design principles. Consider the case of a composite construction in which the prefabricated unit is a steel beam. The construction involves the three main elements ; namely,

(*i*) The reinforced concrete slab.

(*ii*) The steel beam.

(*iii*) The shear connectors connecting the steel beam and the R.C. slab.

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These are shown in Fig. 930

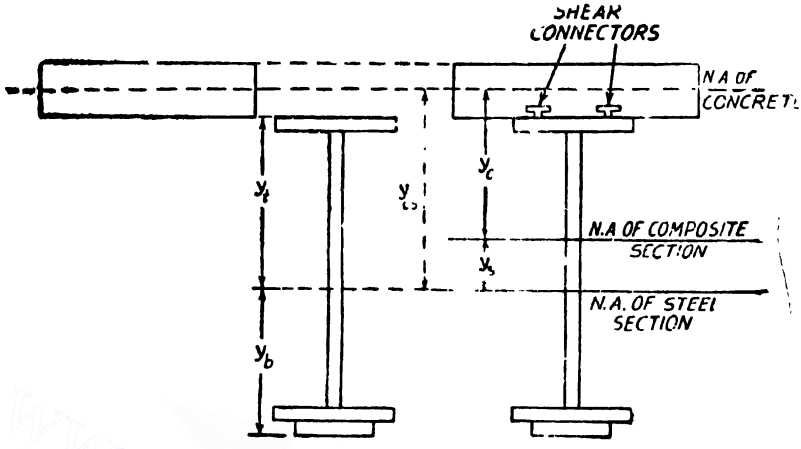

Fig. 930

Let A_s = area of the steel component.

A_c = area of the concrete component.

A_{es} = area of the equivalent steel section.

y_t = Distance of the most distant top fibre of steel from the neutral axis of the steel section.

y_b = Distance of the most distant bottom fibre of steel from the neutral axis of the steel section.

y_{cs} = Distance between the centre of gravity of the concrete section and the centre of gravity of the steel section.

y_c = Distance of the centre of gravity of the concrete section from the neurtal axis of the composite section.

y_s = Distance of the centre of gravity of the steel section from the neutral axis of the composite section.

I_c = Moment of inertia of the concrete section about its own centroidal axis

I_s = Moment of inertia of the steel section about its own neutral axis

I_{es} = Moment of inertia of the equivalent steel section about the neutral axis of the composite section.

m = Modular ratio.

$$\text{Equivalent steel area} = \text{Actual steel area} + \frac{\text{concrete area}}{m}$$

$$\therefore A_{es} = A_s + \frac{I}{m} A_c. \qquad \ldots(i)$$

Consider the composite section.

Taking moments of areas about the centre of gravity of the concrete component, we have,

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$$A_s \cdot y_{cs} = A_{cs} \cdot y_c$$

$$= \left(A_s + \frac{1}{m} A_c \right) y_c$$

$$= A_s y_c + \frac{1}{m} A_c \cdot y_c \qquad \ldots(ii)$$

Again consider the composite section and take moments about the neutral axis of the composite section.

$$A_s y_s = \frac{1}{m} A_c \cdot y_c \qquad \ldots(iii)$$

Moment of inertia of the equivalent steel section about the neutral axis of the composite section

$$= I_{cs} = I_s + A_s y_s^2 + \frac{1}{m} I_c + \frac{1}{m} A_c y_c^2$$

$$= I_s + \frac{1}{m} I_c + A_s y_s^2 + A_s y_s y_c$$

$$= I_s + \frac{1}{m} I_c + A_s y_s (y_s + y_c)$$

$$= I_s + \frac{1}{m} I_c + A_s y_s y_{cs} \qquad \ldots(iv)$$

I.S. recommendations. The Indian standard code of practice for composite construction (I.S. : 3935-1966) has made the following recommendations

(1) Composite action. For design purposes, in case of the prefabricated unit is adequately supported before placing the *in-situ* concrete, it shall be designed to sustain self load only. If the load of the form work, constructional live load and the *in-situ* concrete, is carried directly by the prefabricated unit without adequate props, this additional load shall also be taken into account in addition to self load. The composite section shall be designed for all the loads imposed on the member taking note of the fact that the composite action of the member is effective only for the loads imposed after the composite action has started to function.

In prescribing the requirements of the I.S. code, full composite action has been assumed between the prefabricatsd member and the *in-situ* concrete. For such full composite action to be considered effective, the *in-situ* concrete shall have attained at least 75 per cent of the designed 28-day strength of 15 *cm.* cubes.

The composite action should preferably be proportioned in such a manner that the neutral axis of the composite section is generally located below the *in-situ* concrete slab.

If the neutral axis is located inside the *in-situ* concrete slab, the portion of the slab below the neutral axis shall not be considered effective for computing moments of inertia or resisting moments except for deflection calculations.

(2) Equivalent section. For prefabricated units in prestressed

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concrete or reinforced concrete, consideration shall be given to the different moduli of elasticity of the concrete of the precast and of the *in-situ* portions.

For prefabricated units in steel, the effective gross area of concrete slab shall be converted into the corresponding equivalent area of steel. This shall be done by dividing the effective area of the concrete slab by the modular ratio.

(3) Differential shrinkage and creep of concrete. The effects of shrinkage and creep of the cast *in-situ* concrete on the prefabricated member shall be considered. It shall be ensured that the stresses in the prefabricated member do not exceed the safe stresses by more than 25 per cent when these effects are superimposed on the stresses caused by the worst combination of other loads

(4) Deflection

Live load deflections. Live load deflections shall be calculated on the basis of the moment of inertia of the transformed composite section using the full value of the moduli of elasticity of the concrete.

Dead load deflections. For beams shored during construction the dead load deflections shall be calculated on the basis of the moment of inertia of the transformed composite section using one-half the value of the moduli of elasticity of concretes.

For beams not shored during construction, the dead load deflection shall be calculated on the basis of the moment of inertia of the prefabricated beam alone except that deflections due to dead loads applied after the concrete slab has attained 75 per cent of the specified 28-day strength shall be calculated according to the above paragraph.

Steps such as giving a reverse camber to compensate for the full dead load plus half the live load deflections shall be taken in the design and construction in order to prevent excessive :

(*a*) dishing of the slabs and beams built with shores,
(*b*) thickening of slabs and beams built without shores, and
(*c*) deflection of beams in service.

Limiting deflections. For simply supported beams the total deflection due to dead load, live load and impact should preferably not exceed $\frac{1}{600}$ of the span, or the deflection due to live load and impact should preferably not exceed $\frac{1}{800}$ of the span. The deflection

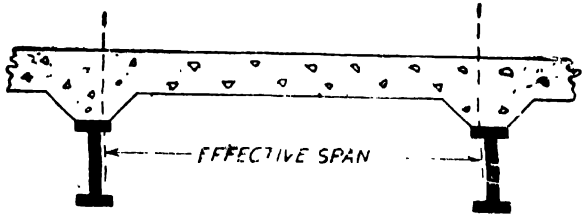

Fig. 931. Effective span of slab supported on steel prefabricated units.

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of cantilever arms due to dead load, live load and impact shall not exceed $\frac{1}{100}$ of the cantilever arms and due to live load and impact shall not exceed $\frac{1}{200}$ of the cantilever arm.

(5) Design of slabs. In continuous spans, the effective span of the slab shall be taken as follows :

(*a*) Central distance between the outstand of supporting flanges of the steel fabricated units. (See Fig. 931).

(*b*) Clear distance between the webs of precast reinforced concrete or prestressed concrete units, reduced by two-thirds of the total thickness of the slab and flange of the precast unit at the face of the web. (See Fig. 932).

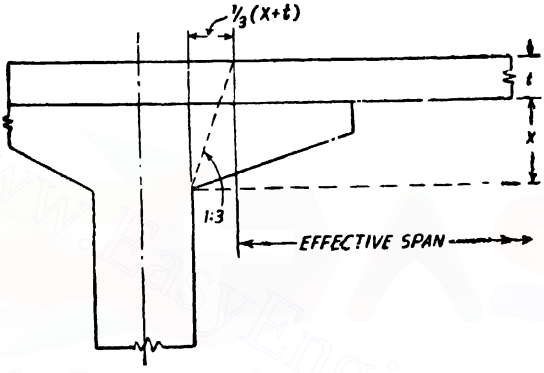

Fig. 932. Effective span of slab supported on precast concrete units.

(6) Flange width of Composite Beams

Beams having flanges on both sides. The width of flange (slab) considered effective in the design of the composite the beam except in the case of edge beam shall not exceed the least of the following :

(*a*) One-fourth the span of the beam,

(*b*) Distance, centre-to centre of the beams ; and

(*c*) The web (or rib) thickness plus twelve times the least thickness of the slab plus, in the case of slabs resting on wide flange (Fig. 932) two-thirds the total thickness of slab and the flange of the prefabricated unit at the face of the web (or rib)

For edge beams. The effective flange width for inner and outer parts (measured from the centre line of the beam) to be taken in the case of an edge beam shall not exceed the following :

(*a*) One-twelfth the span of the beam (for both the inside and the outside parts) ;

(*b*) Half the distance to the adjoining beam (for the inner part) and the actual width (for the outer part) ; and

(*c*) Six times the least thickness of the slab plus half the web-(or rib) thickness plus, in the case of slabs resting on wide flanges

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Fig. (932), one-third the total thickness of the slab and the flange, at the face of the web (or rib) (for both the inside and the outside parts).

Allowance for openings. Any permanent openings which exist shall be deducted from the calculated width of the flange (slab) at the section under consideration. The loss of section due to the openings may, however, be compensated by other suitable provisions, such as trimmer beams, in which case the full flange width shall be taken into account.

(7) Prefabricated steel and in-situ concrete composite members

Steel structural members. The steel structural members may be of rolled steel joists or any other built up sections. The structural members shall preferably be symmetrical about the vertical axis. The top flanges and web plates shall be able to absorb and transmit the forces from the connectors. The minimum thickness of the free overhang shall not be less than one-tenth of the free overhang (Fig. 933) so that heavy distortion at the junction with the connectors does not occur.

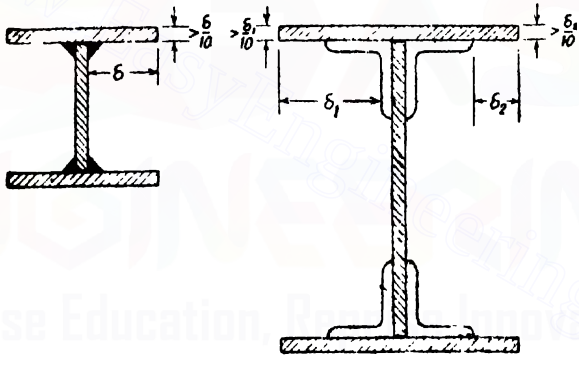

Fig. 933. Minimum thickness of flange for steel prefabricated units.

With steel prefabricated units, the depth of steel section should preferably be not less than $\frac{1}{30}$ of the span and the depth of the composite section should be not less than $\frac{1}{25}$ of the span. If depths smaller than these are used, the section should be adequate to limit deflections to the values obtained with the limiting depths specified above.

Slab and haunch. The minimum thickness of the concrete above the steel structural member shall be not less than 100 *mm.* and therefore, haunches should necessarily be provided where thinner slabs are used The slope of the haunches shall not be greater than one vertical to three horizontal for slabs thinner than 100 *mm.*

The depth of the haunch shall be chosen so that the composite structural member is not greater than one and half times the depth of the steel structural member and further the depth of the haunch

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shall not be greater than one and half times the thickness of the slab.

Where a supporting fillet is provided between the prefabricated steel section and the concrete slab, its section shall be ignored in computing the total composite section.

Shear connectors. In the case of connections between *in-situ* concrete and the prefabricated steel units, resistance to horizontal shear shall be provided by mechanical shear connectors at the junction of the concrete slab and the steel beam or girder. The connectors shall be capable of resisting the shear force between the slab and the structural steel member and at the same time prevent the vertical separation of the slab from the structural steel member at the inner face. The shear connectors shall be of the type which permit a thorough compaction of concrete in order to ensure their entire surfaces are in contact with concrete.

The shear connectors shall be of weldable steel and shall be end welded to the structural members. The capacity of the welds at permissible stress shall be not less than the shear resistance of the connectors

Note. In the case of studs, specialized fusion welding will be necessary and hence expert advice and necessary equipment should be available. To permit satisfactory welding of studs, the gap between the heads of two adjacent connectors should not be less than 15 *mm.*

Studs and channel shear connectors shall not be spaced further apart than 600 *mm.* The clear distance between the edge of a beam flange and edge of the connectors shall not be less than 25 *mm.* The concrete cover over the shear connectors in all the directions shall not be less than 25 *mm.*

In order to ensure that the concrete slab is sufficiently tied down to the steel flange, the overall height of the shear connector (that is, the length of stud, distance of the helix, height of the channel hoop *etc.*) should not be less than 50 *mm.* nor project less than 25 *mm.* into the compression zone of the concrete slab. The thickness of the compression zone shall be that at the section of maximum bending moment.

Types of shear connectors. Shear connectors shall consist of any or a combination of the following types :

(*a*) *Rigid connectors.* These consist of short lengths of bars angles or tees welded to the flange of the steel fabricated units (Fig. 934).

These connectors derive their resistance to horizontal shear from the bearing pressure of the concrete. Failure or slip is generally associated with the crushing of concrete. Some suitable means (anchors) capable of preventing the separation of the *in-situ* concrete from the prefabricated units in the direction perpendicular to the contact surface should be introduced with these connectors.

(*b*) *Flexible connectors.* Flexible connectors such as studs and channels welded to the contact surface of the prefabricated

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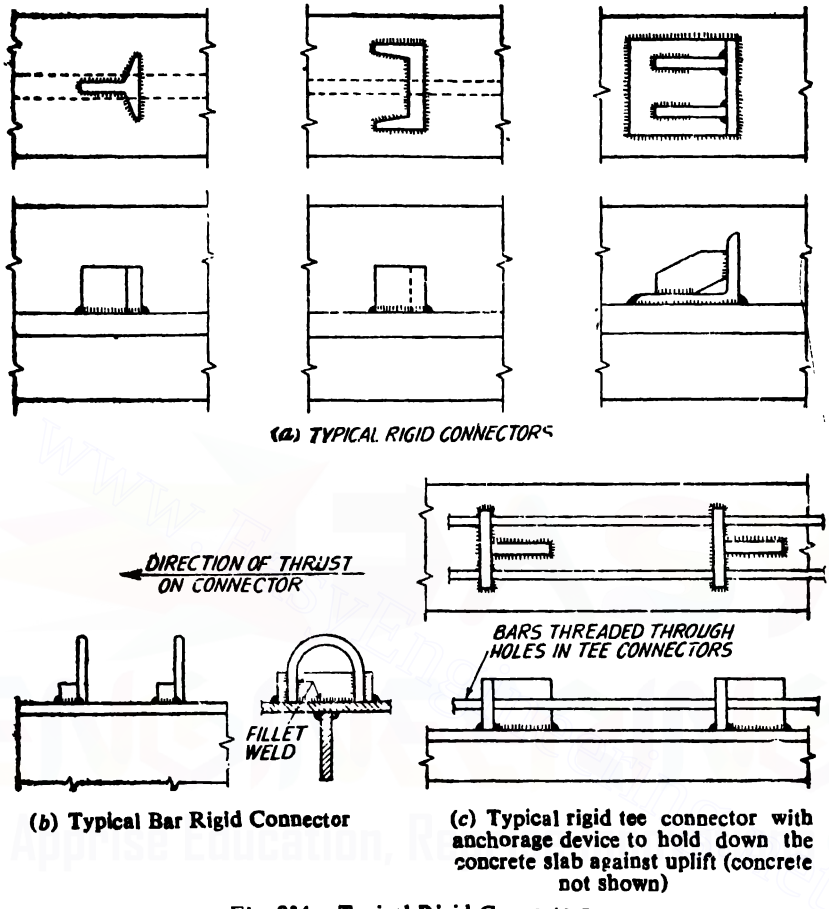

(b) Typical Bar Rigid Connector

(c) Typical rigid tee connector with anchorage device to hold down the concrete slab against uplift (concrete not shown)

Fig. 934. Typical Rigid Connectors.

units are also used. These derive resistance entirely through the bending of the connectors.

(c) *Bond or anchorage connectors* consisting of (Fig. 937) :

(*i*) Mild steel bars welded to the flange of the prefabricated unit in the form of vertical or inclined loop stirrups, or

(*ii*) inclined bars with one end welded to the flange of the steel unit and the other end suitably bent, or

(*iii*) bar stirrups welded to the flange of the steel unit at each loop. These derive their resistance through bond and/or anchorage action.

(*d*) Any other mechanical device to resist horizontal shear and to prevent vertical separation of *in-situ* concrete from prefabricated unit.

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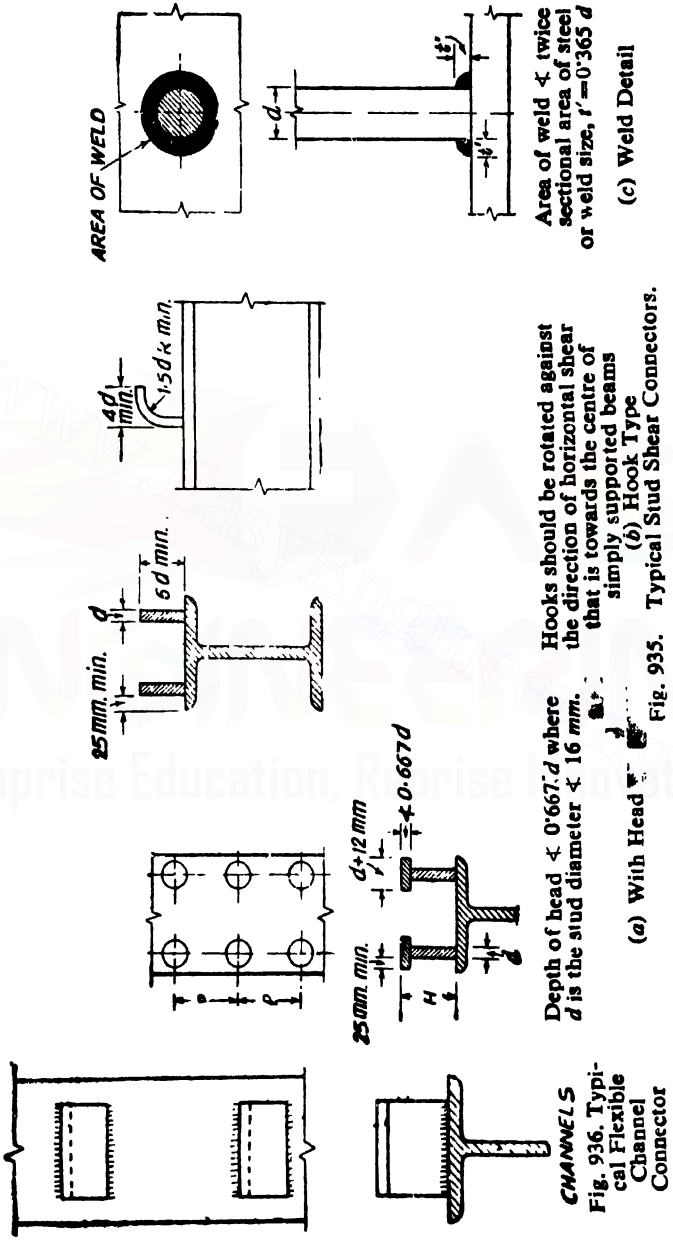

(c) Weld Detail

Area of weld ≮ (twice sectional area of steel or weld size, $t' = 0{\cdot}365\,d$

Hooks should be rotated against the direction of horizontal shear that is towards the centre of simply supported beams

(b) Hook Type

Depth of head ≮ 0·667.d where d is the stud diameter ≮ 16 mm.

(a) With Head

Fig. 935. Typical Stud Shear Connectors.

Fig. 936. Typical Flexible Channel Connector

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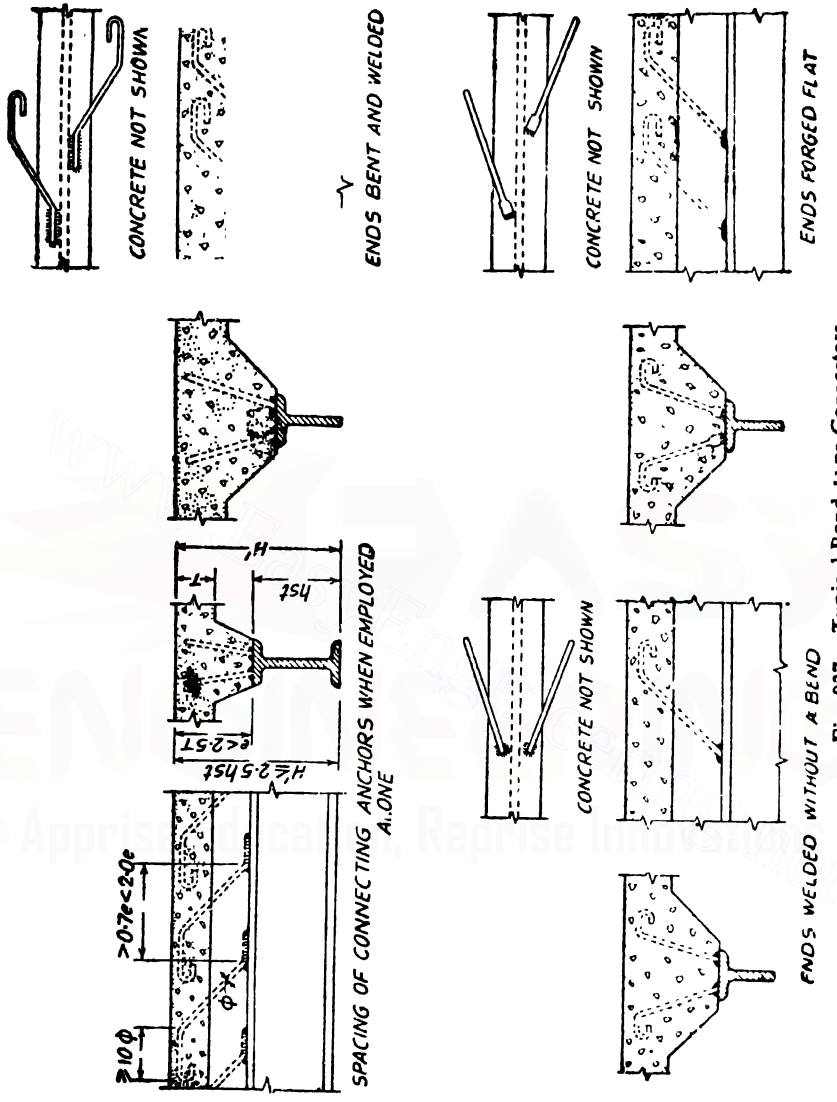

Fig. 937. Typical Bond Type Connectors.

Connectors, such as channels, tees and angles, more closely spaced with smaller faces are preferable to those with larger faces and widely placed, since the former arrangement induces a uniform distribution of shear stress in the concrete. The spacing of the connectors shall not exceed three times the thickness of the slab. Connectors should be as stiff as possible so that an even distribution of stress on the surface is achieved. Channels and tees should be preferred to angles Welding seams should be taken around the connectors as continuous welds.

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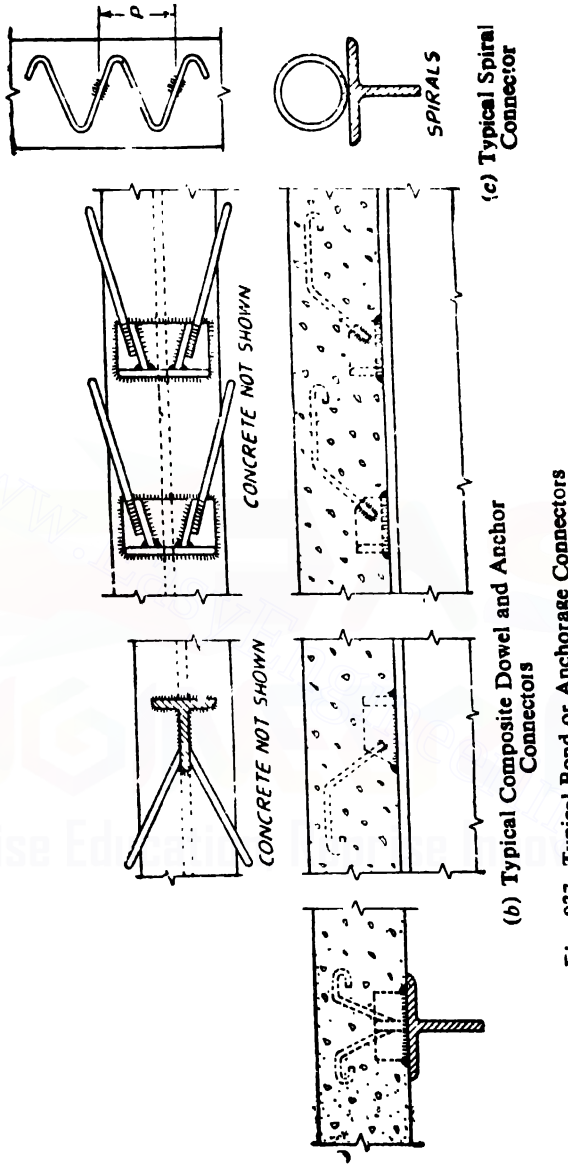

Fig. 937. Typical Bond or Anchorage Connectors

Design requirements of shear connectors. The connection between the steel prefabricated unit and the *in-situ* concrete slab shall be checked for integral action of the composite structure at all loads such that

(*a*) Shear along the contact surface is transferred without slip, and

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(*b*) Separation of the prefabricated unit and the *in-situ* slab in a direction perpendicular to the slab is prevented.

Horizontal shear force. The horizontal shear to be transferred by the shear connectors, *i.e.*, horizontal shear at the plane of contact of the prefabricated and *in-situ* unit shall be computed from the equation,

$$S_h = \frac{Vm_s}{I}$$

where

S_h = horizontal shear per linear *cm.* at the plane of contact of the *in-situ* concrete slab and the prefabricated beam at the cross-section of the composite beam under consideration.

V = The total external (vertical) shear due to the superimposed load acting on the composite section.

I = Moment of inertia of the transformed composite section.

m_s = The static moment of the transformed area on the slab side of the contact surface about the neutral axis of the composite section or the statical moment of area of reinforcement embedded in the concrete slab for negative moment.

Note 1. For beams erected without temporary props the total external shear V is the total external shear from live load and impact plus any shear from the dead load added after the concrete has attained a strength compatible to the composite action assumed in para (1) above. For beams provided with properly designed props during construction, V is the external shear from dead load, live load and prop removal loads.

Note 2. The compressive concrete area is transformed into an equivalent area of steel by dividing the effective concrete area with the modular ratio m.

When negative (hogging) moments are to be resisted by the prefabricated section alone, shear connection between the perfabricated section and the slab need not be provided in the regions of negative bending moments.

When negative moments are to be resisted by the composite section, shear connection should be provided throughout the full length of the beam, but the concrete on the tension side of the neutral axis shall not be taken as effective except as a device to develop the full stress in the reinforcing steel embedded in.

Shear resistance of connectors. Shear resistance values of any connector devices may be calculated by a rational method after duly taking into account the stress conditions, the permissible bearing pressure of concrete, the permissible bond stress in concrete, the structural properties in the steel used for the connectors and the strength of the weld. The design requirements for different types of shear connectors are given below.

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(*a*) *Rigid connectors.* The safe shear resistance capacity of a rigid connector is given by the equation,

$$Q = F_b A_b.$$

where,

F_b = The permissible bearing pressure on concrete found by the expression

$$0{\cdot}25\ \sigma_{cu} \sqrt[3]{\frac{A}{A_b}},$$ this value being limited to $0{\cdot}6\ \sigma_{cu}$.

A = The area to which the bearing force is transmitted, that is $a'\ b'\ c'\ d'$, being equal to the product of width of the flange of steel unit at surface of contact and the depth of concrete slab including the haunch (See Fig. 938).

A_b = The bearing area of the connector, that is area of the transmitting face of the connector, say $a\ b\ c\ d$ (Fig. 938).

σ_{cu} = crushing strength of 150 *mm.* concrete cube at 28 days.

The spacing and size of the inside connectors shall satisfy the following requirements :

(*i*) The bearing pressuse on the face of the connector should not exceed the permissible value Q

(*ii*) The longitudinal shear stress along the shearing surface between two successive connectors should not be greater than two and a half times the permissible shear stress for concrete. This condition shall be deemed to be satisfied, if, $2\ 5\ b_q = S_h$.

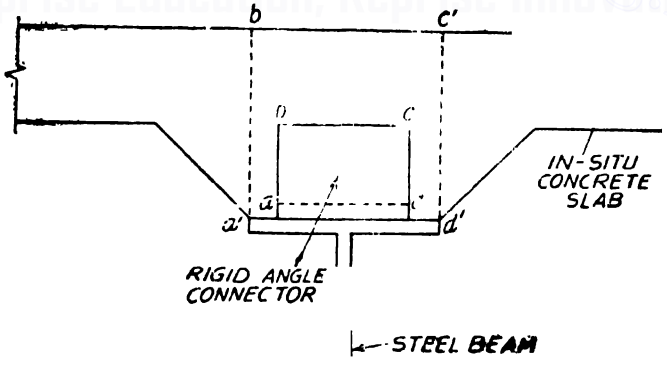

Fig. 938

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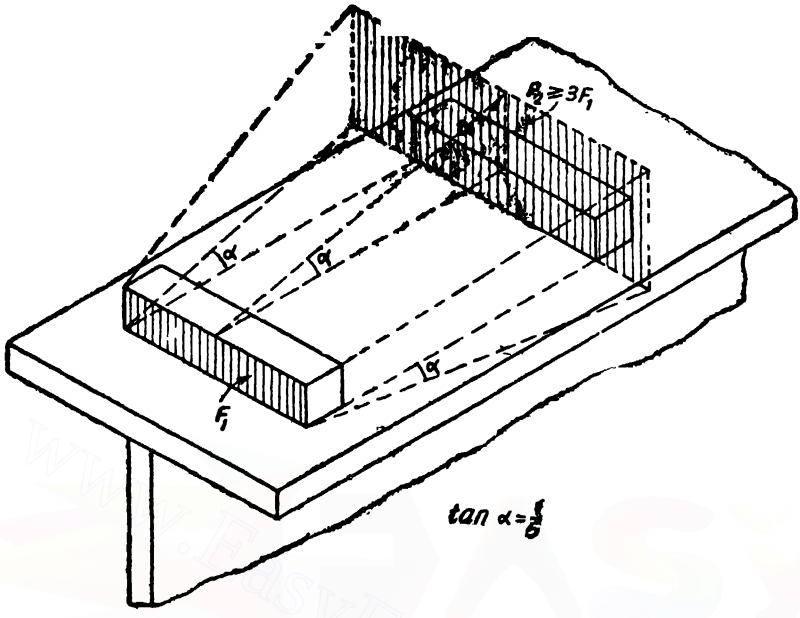

Fig. 939. Rule for determining the minimum spacing of rigid connectors.

where b = width of the steel flange of the ri id connector at surface of contact.

q = permissible shear stress measured as indirect tension in concrete.

S_h = the horizontal shear per linear *cm.* at the plane of contact of the *in-situ* concrete slab and the prefabricated beam at the cross-section of the composite beam under consideration.

(*iii*) The projected area along a slope of 1 in 5 from one rigid connector on to another should be at least three times the area of the face of the connector (Fig 939).

The following precautions are necessary with rigid connectors :

(*i*) Angular or wedge shaped placing of the connectors will tend to split the concrete slab and shall, therefore, be prohibited.

(*ii*) The area of the bearing face of the connector shall not be less than one-fifth of the area to which the bearing force is transmitted.

(*iii*) As far as possible, rigid connectors shall be associated with anchors so that shear is resisted partly by bond of the concrete and partly by the bearing pressure of the concrete against the face of the inside connectors.

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(*b*) *Anchor connector* Anchor connectors are either used alone or are used in conjunction with rigid connectors. The safe shear resistance of an anchor connector is given by the following equation.

$$Q = K\sigma_{st} A_t$$

where A_t = cross-sectional area of the anchor bar.

σ_{st} = permissible tensile stress in the anchor bar

and, K = a coefficient having the following values.

Type of anchor	
Anchors with bond length of not less than 40 times the diameter of the bar including the rock in the compression zone of the concrete.	1·0
Looped anchors with diameter of loop greater than 15 times the diameter of bar.	1·0
Looped anchor usually used in combination with rigid connectors with diameter of loop less than 15 times the diameter of the bar.	0·7
Hooked anchors usually used in combination with rigid connector where the ends of the straight anchors are hooked but the bond length in the compression zone of the concrete is less than 40 times the diameter of the bar.	0·5

The above expression for the shear resistance of the anchor bar is independent of the angle of inclination of the bar. But anchors may generally be either vertical or inclined at about 45°.

Spacing of anchors when used alone shall not be less than 0·7 times the depth of the slab and shall not be greater than two times the depth of the slab.

Spacing of anchors when associated with rigid connectors shall not be greater than two and a half times the depth of the slab. The minimum spacing of anchors associated with rigid connectors shall be governed by the design conditions of rigid connectors.

The anchors may be welded to the beam or the ends bent over and well placed completely around. The anchors shall be brought up to the top surface of the flange, then bent over and ends hooked. The bond length should be adequate to develop the full bond strength, as in accordance with the provisions of I.S. 456 with further provision that the portion from the upper bend to the hook is atleast ten times the diameter of the bar.

(*a*) ***Flexible connectors.*** (*i*) ***Welded steel connector.*** The safe shear resistance of welded connectors (Fig. 934) with minimum stud head diameter of $d+12$ *mm.* and stud head height of 12 *mm.*

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and of steel of ultimate strength of 4600 *kg./cm.*^{2} yield point of 3500 *kg./cm.*^{2} and an elongation of 20 percent is given by the following equation :

(*b*) For ratio of $\frac{H}{d}$ less than 4·2

$$Q = 4{\cdot}8\ H\ d\ \sqrt{\sigma_{cu}}\ kg.$$

For a ratio of $\frac{H}{d}$ equal to or greater than 4·2

$$Q = 19{\cdot}6\ d^2\ \sqrt{\sigma_{cu}}\ kg.$$

where d = diameter of the stud connector (*cm.*)

(*ii*) *Channel flexible connector.* In the case of channel connectors made of steel with minimum ultimate strength of 4200 to 5000 *kg./cm.*^{2}, yield point of 2300 *kg./cm.*^{2} and an elongation of 21 percent, the safe shear resistance is given by the equation (See Fig. 934).

$$Q = 10{\cdot}7\ (h + 0{\cdot}5t)\ \frac{L}{\sqrt{\sigma_{cu}}}$$

where h = Maximum thickness of the flange of the channel connector in *cm.*

t = Thickness of the web of the channel shear connector in *cm.*

L = Length of the channel shear connector

Note. Flange shall be oriented against the direction of horizontal shear, that is towards the centre in case of simply supported beams.

(*iii*) *Spiral connectors.* (Fig. 936). For spiral connectors the shear resistance shall be given by the equation

$$Q = 315 d \sqrt{\sigma_{cu}},$$

where Q = the safe shear resistance in *kg.* of one pitch of a spiral bar, and

d = the diameter of the round bar used in spiral connectors in *cm.*

The diameter of the spiral bars shall preferably be between 12 and 20 *mm.*, smaller diameter bars may be used but care shall be exercised in welding as the welding heat is likely to impair the ductility of small bars. In all composite beams the spirals shall extend at least half way in the slab. The ratio of the pitch of the spiral to the diameter shall be between 0·5 and 4·0. The developed length of one spiral per pitch shall not be less than 20 times the diameter of the bar. From fabrication consideration, the spiral pitch shall be within the limits of 100 *mm.* and 400 *mm.*

Spacing of connectors. The aggregate capacity of all connectors located at a transverse section of a beam shall be equal to the horizontal shear provided by the pitch. Thus the required pitch or

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spacing parallel to the beam axis of the connectors may be determined by the equation

$$I = \frac{NQ}{S_h}$$

where,

P = spacing or pitch of connectors in the direction of the axis, and

N = Number of connectors at one transverse section of beam.

End shear connectors. At each end of a simply supported girder in composite construction in steel and concrete, end connectors shall be provided to counteract the effects of temperature, shrinkage and creep in addition to the external shear forces. The shear resistance of such an end connector shall have the following values unless otherwise determined by rational analysis.

Shear resistance of each end connector

$$= \frac{M\, m_s}{I}$$

where,

M = maximum bending moment on the composite beam due to additional loads operating after the composite action has been effected.

m_s = the static moment of the transformed area on the slab side of the contact surface about the neutral axis of the composite section or the statical moment of area of reinforcement embedded in concrete slab for negative moment.

The end shear connector shall consist of mechanical device having adequate shear resistance as calculated above. Such mechanical device may consist of cut piece of rolled steel with the bearing face directed towards the centre of the beam or it may consist of anchor bars spreading out into the slab away from the centre of the beam (Fig. 936).

Prefabricated Prestressed or Reinforced Concrete and in-situ concrete composite members

Composite structures in which the *in-situ* concrete is assumed to act integrally with the precast beam shall be inter-connected to transfer the horizontal shear along the contact surfaces and to prevent the vertical separation of these units. Shear transfer shall be by shear bars, castellations and by bond. The units shall further be tied together by the extension of web reinforcement.

Ties. Separation of the component elements in the direction perpendicular to the contact surface shall be prevented by ties adequately embedded on each side of the contact surface. The spacing of such ties shall not exceed four times the thickness of the slab or 60 *cm.* whichever is less. The minimum cross-sectional area of the ties, in each *metre* of the span shall not be less than 0·15 per

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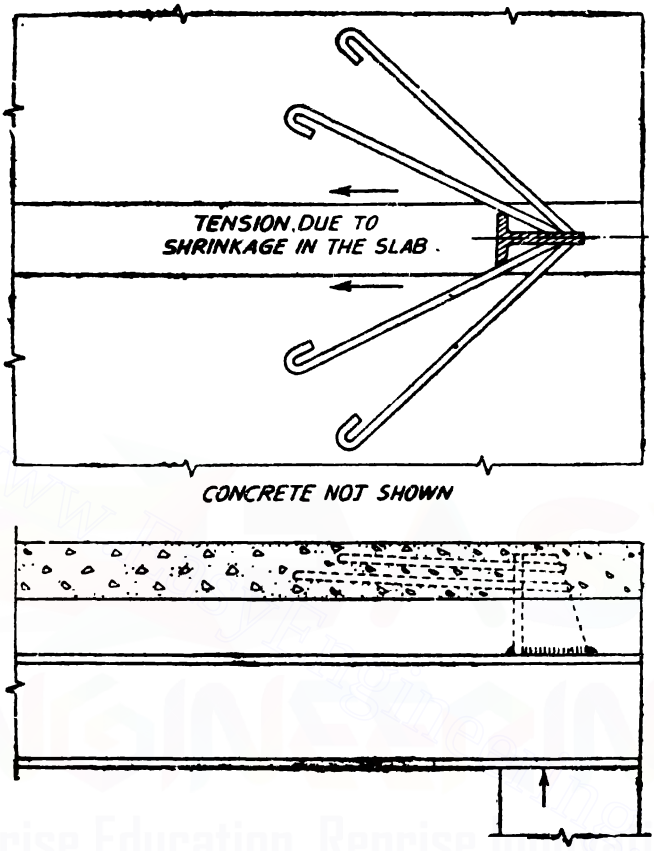

Fig. 940. Typical End Connector with Anchoring Rod.

cent of the contact area or 130 *sq. mm*. All web reinforcement of the prefabricated unit shall be extended into the cast *in-situ* concrete.

In the case of either dynamically or statically loaded structures where the horizontal shear at the inter-face at ultimate load is less than $\frac{\sigma_{cu}}{40}$ it is not necessary to provide vertical ties.

Bond strength at the inter-face. The bond strength at the inter-face shall be checked for ultimate load. The inter-face shall always be made rough for effective bonding.

The ultimate values of the horizontal shear stress at the inter-face shall be calculated by using the formula

$$S_h = \frac{V m_s}{I}$$

If the calculated shear stresses are more than the values given under no slip condition in the table below, it shall be taken that the slip has occurred. The design shear then be made taking a frictional

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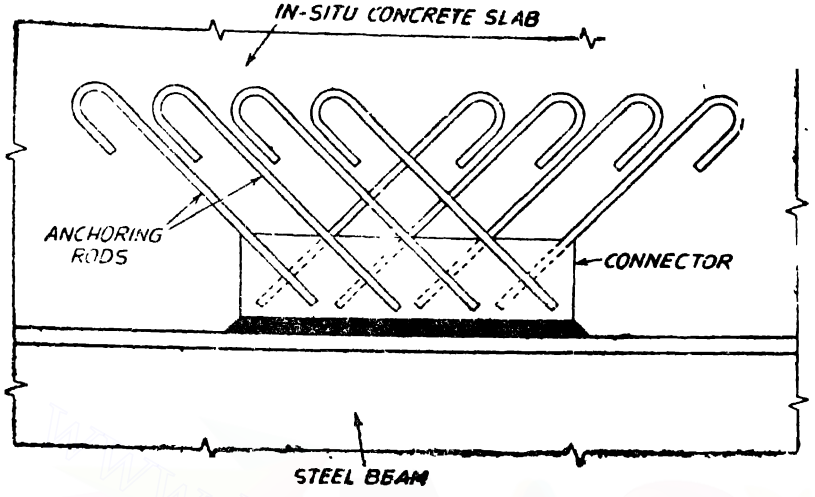

Fig. 94i. Alternative arrangement of End Connectors, showing Anchoring Rods in Opposite direction.

shear resistance of 10 *kg./cm*^{2} and the balance stress to be resisted by steel shear connectors stressed to a maximum of 1340 *kg./cm.*^{2} The inter-face shear shall not, however, exceed the value given under the maximum permissible shear stress in the table below :

Permissible shear stress in the inter-face at ultimate load

150 mm. cube strength of in situ concrete at 28 days	*Permissible shear at no slip*	*Permissible shear at slip*	*Maximum Permissible inter-face shear*
150 *kg./cm.*^{2} 200 *kg./cm.*^{2} 250 *kg./cm.*^{2}	12 *kg./cm.*^{2}	10 *kg./cm.*^{2}	14 *kg /cm.*^{2}
300 *kg./cm.*^{2} 350 *kg./cm.*^{2} 400 *kg /cm.*^{2}	14 *kg./cm.*^{2}	10 *kg./cm.*^{2}	20 *kg./cm.*^{2}

Shear bars. The shear bars at the ends of the girders to a length of one-half to three-fourth depth of the girder shall be spaced closer and designed to take full shear force under ultimate conditions.

Design 233. ***Design a beam of composite construction to the following requirements :***

(i) Flange width provided by slab — ***150 cm.***

(ii) Thickness of the slab — ***10 cm.***

(iii) Prefabricated unit shall be of a steel section.

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(iv) *Span of the beam=12 metres.*

(v) *Total load=2400 kg./m.*

Solution. Let us adopt the following working stresses adopting M 200 mix.

$$c=70 \text{ kg./cm.}^2 \text{ (bending stress in concrete)}$$

$$t=1400 \text{ kg./cm.}^2 \text{ (Tensile stress in steel)}$$

$$m=\frac{2800}{3c}=\frac{2800}{3\times 70}=13$$

Maximum bending moment

$$M=\frac{2400\times 12^2}{8}\times 100 \text{ kg. cm.}$$

$$=4,320,000 \text{ kg. cm.}$$

$\therefore$ Section modulus required

$$=\frac{M}{\frac{4,320,000}{1400}} \text{ cm.}^3$$

$$=3086 \text{ cm.}^3$$

Let us try an *ISMB* 420 joist section with a cover plate 120 *mm*×25 *mm*. cover plate. See Fig. 942.

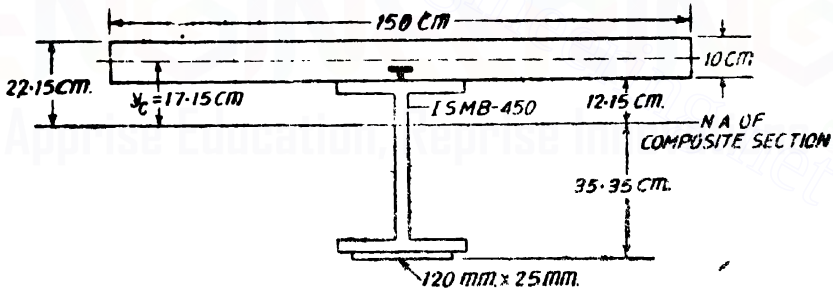

Fig. 942

Area of steel (Joist and plate)

$$=A_s=92\cdot27+12\times 2\cdot5$$

$$=122\cdot27 \text{ cm.}^2$$

Area of slab $=A_c=150\times 10=1500 \text{ cm.}^2$

$$y_t=\frac{97\cdot27\times 22\cdot5+12\times 2\cdot5\times 46\cdot25}{122\cdot27}$$

$$=28\cdot32 \text{ cm.}$$

$$y_b=47\cdot5-28\cdot32 \text{ cm.}$$

$$=19\cdot18 \text{ cm.}$$

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I_s = Moment of Inertia of the steel section about its own neutral axis.

$\therefore \quad I_s = 30390{\cdot}8 + 92{\cdot}27\,(28{\cdot}32 - 22{\cdot}50)^2$

$$+\frac{12\times 2{\cdot}5^3}{12} + 12\times 2{\cdot}5 \times (19{\cdot}18 - 1{\cdot}25)^2 \ cm.^4$$

$$= 43182 \ cm.^4$$

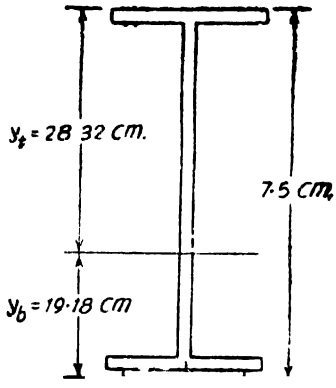

Fig. 943

I_c = Moment of inertia of the concrete section about its own neutral axis

$$= \frac{150\times 10^3}{12} \ cm.^4$$

$$= 12500 \ cm.^4$$

y_c = Distance between C.G. of the concrete section and the C.G. of the steel section

$$= 28{\cdot}32 + 5 = 33{\cdot}32 \ cm.$$

A_{es} = area of equivalent steel section

$$= A_s + \frac{1}{m} A_c$$

$$= 122{\cdot}27 + \frac{1500}{13} = 237{\cdot}7 \ cm.^2$$

y_c = distance of the C.G. of the concrete section from the neutral axis of the composite section

$$= \frac{33{\cdot}32\times 122{\cdot}27}{237{\cdot}7} \ cm.$$

$$= 17{\cdot}15 \ cm.$$

= Distance between neutral axis of the composite section and C.G. of steel section

$$= y_{cs} - y_c$$

$$= 33{\cdot}32 - 17{\cdot}15 = 16{\cdot}17 \ cm.$$

I_{es} = Moment of inertia of the equivalent section about the neutral axis of the composite section

$$= I_s + \frac{1}{m} I_c + A_s y_s y_{cs}$$

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$$=43182+\frac{12^{c}00}{13}$$

$$+122{\cdot}27\times16{\cdot}17\times33{\cdot}32\ cm.^4$$

$$=43182+962+65810=109954\ cm.^4$$

Check for bending stresses,

Maximum tensile stress in steel

$$=\frac{4,320,000}{109954}\times35{\cdot}35\ kg./cm.^2$$

$$=1390\ kg./cm.^2$$

Maximum compressive stress in steel

$$=\frac{4,230,000}{109954}\times12{\cdot}15\ kg./cm.^2$$

$$=478\ kg./cm.^2$$

Maximum compressive stress in concrete

$$=\frac{1}{13}\times\frac{4,320,000}{109954}\times22{\cdot}15\ kg./cm.^2$$

$$=67\ kg./cm.^2$$

Analysis for shear. Horizontal shear per *cm.* length at the junction of the concrete slab and steel component near the support,

$$=S_h=\frac{V\,m_s}{I}$$

where, V = Maximum shear force (at the support)

$$=2400\times\frac{12}{2}=14400\ kg.$$

m_s = static moment of the transformed area of the slab about the neutral axis of the composite section

$$=\left(\frac{150\times10}{13}\right)\times17{\cdot}15\ cm.^3$$

$$=1980\ cm.^3$$

$$I=I_{ns}=109954\ cm.^4$$

$$\therefore\quad S_h=\frac{14400\times1980}{109954}\ kg./cm.$$

$$=259{\cdot}2\ kg/cm.$$

Let 20 *mm.* diameter studs be provided.

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Let the height of the studs be 80 *mm*.

$$\therefore \quad H = 8 \text{ cm.}$$

$$d = 2 \text{ cm.}$$

$$\therefore \quad \frac{H}{d} = \frac{8}{2} = 4$$

The ratio being less than 4·2,

The safe shear resistance of one stud

$$= Q = 4{\cdot}8\ Hd\sqrt{\sigma_{cu}} \text{ kg.}$$

where σ_{cu} = crushing strength of 15 *cm.* concrete cubes at 28 days = 200 *kg/cm.*^{2}

$$\therefore \quad Q = 4{\cdot}8 \times 8 \times 2\sqrt{200} \text{ kg.}$$

$$= 1086 \text{ kg.}$$

If a single row of studs be provided, the pitch of the studs will be

$$= \frac{1086}{259{\cdot}2} = 4{\cdot}19 \text{ cm. say } 4 \text{ cm.}$$

If two rows of studs be provided, the spacing of the studs will be

$$= 2 \times 4 = 8 \text{ cm.}$$

Similarly the spacing of the studs may also be calculated at other intermediate sections.

§242. Composite construction with prefabricated prestressed concrete beam and a cast-in-situ slab

A composite section can also be made by precasting (by pre-tensioning or post-tensioning) part of the beam and then cast the remaining part of the section separately. The precast and the *cast-in-situ* components act together and constitute a composite section.

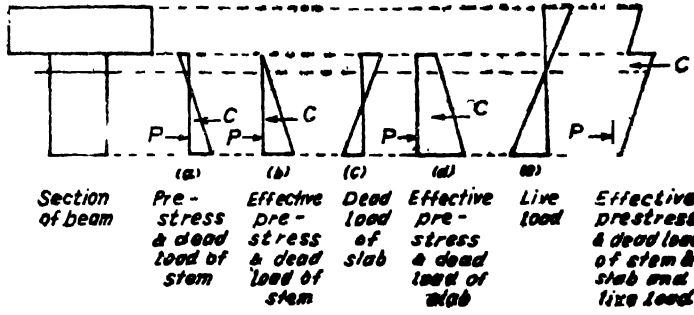

Fig. 944

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Fig. 944 shows the composite section provided for a beam, simply supported at the ends. The lower part, namely the stem, is the precast component. The top slab is cast after the stem is lifted into position. The slab rests directly on the stem. At the time of casting if temporary intermediate props are not provided, then the precast unit has to support its own load and that of the slab. After the *cast-in-situ* flange has hardened, the composite section formed will support any live or dead load on it.

Fig. 944 also shows the stress distribution for the following various load conditions :

(*a*) under the action of the initial prestressing force and the load of the stem the stress distribution will be as shown in Fig. 944 (*a*) the tension P in the cable and the resultant compressive force C in the concrete will provide a moment of resistance whose lever arm is the distance between them.

(*b*) The stress distribution will slightly alter to what is shown in Fig. 944 (*b*) consequent to the losses in prestress. The couple $C-P$ will now act with a slightly greater lever arm.

(*c*) As the slab is now cast, due to the additional load, there will be an additional moment. The stresses due to dead load of the slab are shown in Fig. 944 (*c*).

(*d*) Under the action of the effective prestress, weight of the stem and the weight of the slab the stress distribution can be determined by the addition of stress due to the conditions (*b*) and (*c*). Fig. 944 (*d*) shows the stress distribution corresponding to this stage. The stress at the bottom fibres is slightly reduced. This lever arm of the C-P couple is slightly increased.

(*e*) Now the slab becomes composite with the stem. The live load is applied at this stage. The live load bending moment is resisted by the composite section. Stresses due to live load are shown in Fig. 944 (*e*).

(*f*) The final stresses can now be calculated by adding the stresses due to the conditions (*d*) and (*e*). High compressive stress will be produced in the top fibres of the stem and the slab. The lever arm of the C-P couple is appreciably increased.

The above explanations refer to a simple case of a composite construction. There may be many variations in the actual situ conditions.

For instance it is quite possible, that the precast stem may be supported on false work when the slab is cast. In this case we need not consider the stress distribution shown in Figs. 944 (*c*) and (*d*). Sometimes the falsework may be propped up so that the stem may not carry any load. In such a case the moments due to dead load of

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stem, and dead load of slab will be resisted by the composite section itself.

Design 234. *Fig. 945 shows the cross-section of a composite beam which consists of a 30 cm. × 90 cm. precast stem and a cast-in-situ flange 90 cm. × 15 cm. The stem is a post-tensioned unit with an initial prestressing force of 250 tonnes. The effective prestress available after making deduction for losses, is 220 tonnes. The dead load moment at midspan due to the weight of the precast section is 25 tonne metre. The dead load moment due to the weight of the flange is 12·5 tonne metre. After the hardening of the flange concrete, the composite section has to carry a live load which produces a bending moment of 70 tonne metre.*

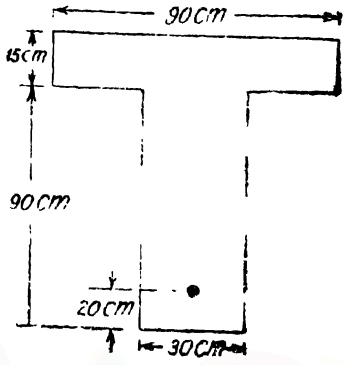

Fig 945

Determine the stresses in concrete at the various stages of loading.

Solution. The beam will be analysed for the following conditions.

(*a*) *Analysis of the rectangular stem section subjected to initial prestressing force and the dead load immediately after prestressing.*

Area of the stem section $= A = 30 \times 90 = 2700$ *cm.*^{2}

Section modulus of the stem $= Z = \dfrac{30 \times 90^2}{6} = 40500$ *cm*^{3}

Extreme stress for the stem section

$$= \frac{P_o}{A} \mp \frac{(P_o e - M_d)}{Z}$$

$$= \frac{250{,}000}{2700} \mp \frac{(250{,}000 \times 25) - 25 \times 1000 \times 100}{40500} \text{ kg./cm.}^2$$

$$= 92{\cdot}59 \mp 92{\cdot}59 \text{ kg./cm.}^2$$

$$= 0 \text{ and } 185{\cdot}18 \text{ kg./cm.}^2$$

(*b*) *Analysis of the rectangular stem section subjected to effective prestressing force and the dead load (after losses occur in prestress).*

At this stage, the extreme stresses for the stem section

$$= \frac{P}{A} \mp \frac{(Pe - M_d)}{Z}$$

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$$= \frac{220.000}{2700} \mp \frac{(220{,}000 \times 25) - 25 \times 1000 \times 100}{40500} \text{ kg./cm.}^2$$

$$= 81{\cdot}48 \mp 74{\cdot}07 \text{ kg./cm.}^2$$

$$= 7{\cdot}41 \text{ kg./cm}^2 \text{ and } 155{\cdot}55 \text{ kg./cm}^2$$

(*c*) *Analysis of the rectangular stem section subjected to effective prestressing force, dead lood of the stem and the dead load of the flange.*

At this stage the extreme stresses for the stem section

$$= \frac{P}{A} \mp \frac{[Pe - (M_a + M_f)]}{Z}$$

$$= \frac{220000}{2700} \mp \frac{(220000 \times 25) - 37{\cdot}5 \times 1000 \times 100}{40500} \text{ kg./cm.}^2$$

$$= 81{\cdot}48 \mp 43{\cdot}21 \text{ kg./cm.}^2$$

$$= 38{\cdot}27 \text{ kg./cm.}^2 \text{ and } 124{\cdot}69 \text{ kg./cm}^2$$

(*d*) *Analysis of the composite section subjected to live load.*

For this case let us determine the position of the centroidal axis

Area component	*Area a* ($cm.^2$)	*Centroidal distance y from the axis 1—1* (*cm*)	*ay* ($cm.^3$)	ay^2 ($cm.^4$)	I_{self} ($cm.^4$)
Flange	1350	7 5	10125	75937·5	$\frac{90 \times 15^3}{12} = 25312{\cdot}5$
Stem	2700	60	162 00	9720000	$\frac{30 \times 90^3}{1} = 18225{:}0$
Total	4050		172125	9795937·5	1847812·5

and the moment of inertia of the composite section about the centroidal axis. The relevant calculations are shown above.

Distance of the centroidal axis of the composite section from 1—1.

$$= \bar{y} = \frac{\Sigma ay}{(\Sigma a)} = \frac{172125}{4050} = 42{\cdot}50 \text{ cm.}$$

Moment of inertia about the axis 1—1 (*corresponding to top edge*)

$$= I_{1-1} = \Sigma I_{self} + \Sigma ay^2$$

$$= 1847812{\cdot}5 + 9795937{\cdot}5 = 11643750 \text{ cm.}^4$$

But $I_{1-1} = I_{cc} + A\bar{y}^2$

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$$\therefore \quad I_{nn} = 11643750 - 4050 \times 42{\cdot}5^2 \ cm.^4$$
$$= 4328437{\cdot}5 \ cm.^4$$

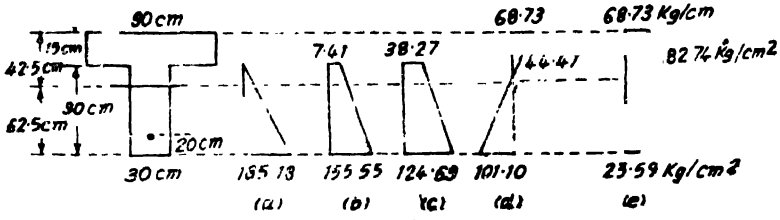

Fig. 946

Stress due to live load moment in the top fibre,

$$= + \frac{70 \times 1000 \times 100}{4328437{\cdot}5} \times 42{\cdot}5 \ kg./cm.^2$$
$$= +68{\cdot}73 \ kg./cm.^2$$

Stress due to live load moment in the bottom nore

$$= - \frac{70 \times 1000 \times 100}{4328437{\cdot}5} \times 62{\cdot}5 \ kg./cm.^2$$
$$= -101{\cdot}1 \ kg./cm.^2$$

Resultant stresses :

Stress at upper edge of flange $= +68{\cdot}73 \ kg/cm^2$

Stress at lower edge of flange $= \frac{27{\cdot}5}{42{\cdot}5} \times 68{\cdot}73 = 44{\cdot}47 \ kg./cm.^2$

Stress at the upper edge of the stem

$$= 44{\cdot}47 + 38{\cdot}27 = +82{\cdot}74 \ kg./cm.^2$$

Stress at the lower edge of the stem

$$= 124{\cdot}69 - 101{\cdot}10 = +23{\cdot}59 \ kg./cm.^2$$

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24

Ultimate Load Design

Introduction

Stress-strain curves for mild steel and concrete. Fig. 9 47 shows typical stress-strain curves for mild steel and concrete. The following characteristics must be noted.

(*a*) *Mild steel.* Stress is directly proportional to the strain up to the elastic yield point. As the yield point is reached, the steel becomes plastic and in this condition, its stress remains practically constant while the strain increases. Ultimately, due to *work-hardening* of the steel, there is another increase of stress which is not proportional to strain.

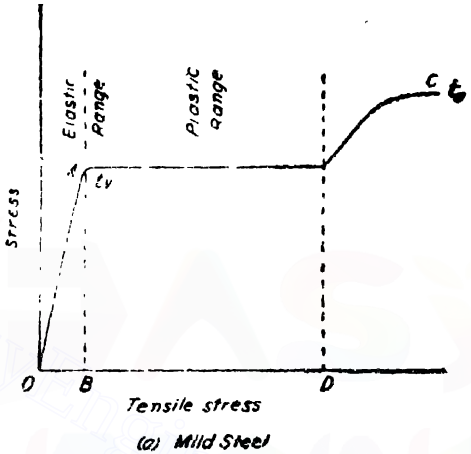

Fig. 947

(*b*) *Concrete.* In the case of concrete, the yield point is not well defined, nor is its stress even exactly proportional to strain. However, that part of the stress strain curve between *O* and *E* in Fig. 948 (*b*) is nearly a straight line. In the plastic range beyond *E*, the stress at first increases slightly but then falls to *G* as strain increases. At the stress *G* collapse takes place by crushing.

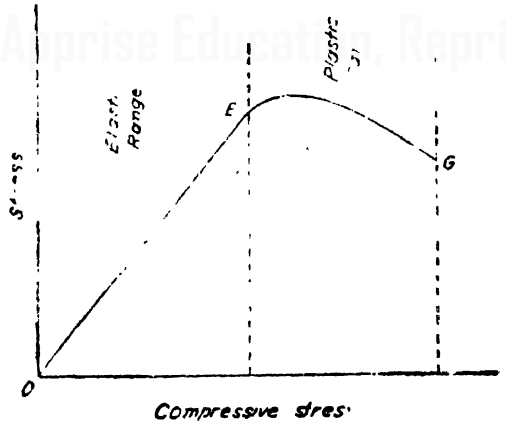

Fig. 948

In designing, following the Elastic theory, the maximum stresses are limited by two requirements *viz.*

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(*i*) All stresses, in both concrete and steel, must be proportional to strain. Hence they must not exceed the values represented in Fig. 948 by the points *A* and *E*.

(*ii*) Every structure must be safe even when overloaded to some extent. This condition requires that the permissible design stresses must be lower than the elastic yield stresses ; usually they are taken to be half the yield stresses to give a factor of safety of 2.

We know that in reinforced concrete members, the failure of the member is dependent on the *strength properties* of the two component materials, *i.e.*, concrete and steel. If a beam is over-reinforced and loaded up to destruction, the failure will occur due to crushing of concrete. In the case of ordinary concrete the failure may be preceded by cracks ; but when high-strength concrete is used, the failure may occur without warning and may occur explosively

Suppose the beam is under-reinforced. If such a beam is loaded up to destruction the failure will occur due to exhaustion of strength of steel. This failure may occur in any of the following manner depending on the quality of steel used.

If the steel has a well defined yield point the stress intensity in steel will not increase after reaching the yield point. But as further bending takes place, the strain in steel increases without increase in stress. The neutral axis will now get shifted towards the compression zone so that a failure may occur due to crushing of concrete.

In the case of lightly reinforced beam, another type of failure may take place. The stress condition may be such that the strength of the concrete in the tension zone is greater than the strength of the steel reinforcement. If now the tensile resistance of concrete is lost due to formation of the initial crack, the steel may not be able to take over the tension which was earlier resisted by the concrete. Hence, a failure will occur suddenly with the concrete in the tension zone cracking and the steel fractured almost simultaneously

It is desirable to design a beam so that when fracture takes place the steel should fail by yielding and subsequently the concrete in the compression zone should fail by crushing. The failure should not occur due to the crushing of the concrete alone as this type of failure is sudden and without notice.

Assumptions in ultimate load design. The following assumptions are made in the design of members based on ultimate load theory.

(*a*) Plane sections normal to the axis of the member remain plane after bending.

(*b*) Unless otherwise specified the tensile strength of concrete is neglected in sections subjected to bending.

(*c*) At ultimate strength, stresses and strains are not proportional and distribution of compressive stresses in a section subjected

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to bending is non-linear. The diagram of compressive concrete stress distribution may be assumed to be a rectangle, trapezoid, parabola or any other shape which results in ultimate strength in reasonable agreement with tests.

(*d*) Maximum fibre stress in concrete does not exceed 0·68 σ_{cu},

where σ_{cu} = permissible compressive strength of concrete in compression.

Ultimate load. This is the load or the load system which produces the actual failure of the member under consideration.

Working load (safe load). This is the maximum load to which the member is liable to be applied during its life time.

Factor of safety (load factor). This is the ratio of the ultimate load to the working load.

Design of beams. Fig. 949 shows the cross-section of a beam and the stress distribution corresponding to the failure condition.

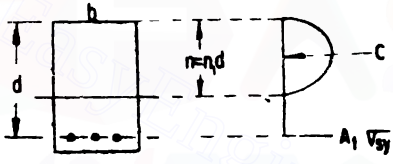

Fig. 949

Let f_c' = cylinder strength of concrete.

Let f_{av} = Average compressive stress on the cross-section of the beam.

Let $f_{av} = K_1 \times$ extreme compressive fibre stress

where K_1 = a coefficient.

The extreme compressive fibre stress may be taken as $0{\cdot}85 f_c'$

$\therefore \quad f_{av} = K_1 (0{\cdot}85 f_c')$

$\therefore$ Total compressive force

$= f_{av} \times$ area of the compression zone.

$= K_1 \times 0{\cdot}85 f_c' \times b\, n_1\, d$

$= 0{\cdot}85\, k_1\, n_1 f_c'\, b\, d$

Suppose the beam fails due to yielding of steel.

For this condition, stress in steel

$= \sigma_{sy}$ = yield stress

$\therefore \quad$ Total tension $= f_y A_t$.

Equating the total compression to total tension, we have,

$0{\cdot}85\, k_1\, n_1\, f_c'\, b\, d = f_y A_t$.

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$$\therefore \quad n_1 = \frac{\sigma_{sy} A_t}{0{\cdot}85\, k_1 f_c' b d}$$

Let the steel ratio be p

$$\therefore \quad p = \frac{A_t}{bd}$$

$$\therefore \quad n_1 = \frac{p\, \sigma_{sy}}{0{\cdot}85\, k_1 f_c'}$$

Let $$n = \frac{p\, \sigma_{sy}}{f_c'}$$

$$\therefore \quad n_1 = \frac{n}{0{\cdot}85\, k_1}$$

∴ Moment of resistance corresponding to the failure condition

$$= M_u = \text{Total compression} \times \text{lever arm}$$
$$= 0{\cdot}85\, k_1 n_1 f_c' \, bd\,(d - k_2 n_1 d)$$
$$= 0{\cdot}85\, k_1 n_1 f_c' \, b\, d^2 (1 - k_2 n_1)$$

where k_2 = a coefficient.

But M_u is equal to total tension × lever arm

$$\therefore \quad M_u = A_t\, \sigma_{sy}\,(d - k_2 n_1 d)$$
$$= A_t\, \sigma_{sy}\, d\,(1 - k_2 n_1)$$

$$\therefore \quad M_u = A_t\, \sigma_{sy}\, d \left[1 - \frac{k_2 q}{0{\cdot}85\, k_1} \right]$$

Thus the ultimate moment of resistance will be known if the coefficients k_1 and k_2 are known.

§243. I.S. Code Method — Whitneys stress block

A number of theories have been offered corresponding to an equivalent concrete stress block. Of these the rectangular concrete stress block suggested by Whitney is commonly adopted. The I.S. code also has recommended a rectangular concrete stress block. The stress distribution in concrete in the compression zone is assumed to be uniform corresponding to failure condition. The compressive stress in concrete at this condition is assumed to be $0{\cdot}55\, \sigma_{cu}$ where σ_{cu} is the 28 day ultimate cube strength of concrete in compression.

Modes of failure. A reinforced concrete beam may fail in any of the two ways mentioned below.

(*i*) *When steel provided is moderate* (*under reinforced section*). Suppose the beam is subjected to a gradually increasing load up to collapse condition. The steel will reach the yield point at a certain value of the load. As the loading is increased the steel yields and extends largely. Tension cracks appear and become widened and move upwards. The stresses in concrete in the compression zone go on increasing till the concrete is crushed. This type of failure is

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called *secondary compression failure*. In this case the failure of the beam will occur at a load slightly greater than the load at which the yielding of steel was produced.

(*ii*) *When large amount of steel is provided cover reinforced section*). In this case the amount of steel is so large that even when the concrete reaches its ultimate stress the steel will not have yielded. As the loading is gradually increased, the beam collapses by crushing of the concrete when the strains produced are large enough to cause disruption of concrete. This stage is anticipated when the strain in concrete is at the order of 0·3% to 0·4%. This type of failure takes place abruptly and explosively without giving any signs earlier. It is not desirable to design the beam to fail in this way.

Ultimate strength properties—*Singly reinforced beams* As mentioned earlier a rectangular equivalent stress block is considered for the compression zone. Fig. 950 shows the strain diagram and the stress diagram at ultimate condition.

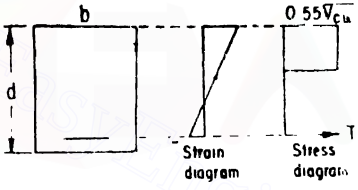

Fig. 950

The beam section may be a balanced section or an under-reinforced section. Let a be the depth of the stress block.

Balanced section. For a balanced section the depth of the stress block $= a = 0{\cdot}43\ d$.

$$\text{Lever arm} = d - \frac{a}{2} = d - \frac{0{\cdot}43\ d}{2} = 0{\cdot}785\ d$$

Ultimate moment of resistance

$$= M_u = \text{Total compression} \times \text{Lever arm}$$
$$= b \times 0{\cdot}43\ d \times 0{\cdot}55\ \sigma_{cu} \times 0{\cdot}785\ d$$

$$\therefore\ M_u = 0{\cdot}185\ \sigma_{cu}\ bd^2$$

Total compression = Total tension

$$b \times 0{\cdot}43\ d \times 0{\cdot}55\ \sigma_{cu} = A_t\ \sigma_{sy}$$

$$\therefore\ A_t = 0{\cdot}236\ \frac{\sigma_{cu}}{\sigma_{sy}}\ bd$$

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Under reinforced section. Equating the total compression to total tension

$$ba \times 0{\cdot}55\ \sigma_{cu} = A_t\ \sigma_{sy}$$

$$\therefore \qquad a = \frac{A_t\ \sigma_{sy}}{0{\cdot}55\ \sigma_{cu}\ b}$$

We can find a the height of the stress block from this relation. The section is an under reinforced section if $a < 0{\cdot}43\ d$

Lever arm $= d - \frac{a}{2}$

Ultimate moment of resistance

$= M_u = C$ or $T \times$ lever arm

Yield Strength of Steel Reinforcement

Type of steel	*Yield stress (kg./cm.²)*	*Yield stress (N/mm.²)*
(1) Medium tensile steel conforming to IS : 432 – 1960		
Bars up to and including 25 *mm.*	3600	350
Bars over 25 *mm.* up to and including 38 *mm*	3450	345
Bars over 38 *mm.* up to and including 50 *mm.*	3300	330
(2) Medium tensile steel conforming to IS : 1139 – 1959		
Bars up to and including 25 *mm.*	3600	360
Bars over 25 *mm.* up to and including 38 *mm.*	3450	345
(3) Cold twisted plain or deformed bars conforming to IS : 1786 – 1961		
Bars under 10 *mm.*	4950	495
Bars from 10 *mm.* and above	4250	425
(4) Mild steel conforming to IS : 432 – 1960		
Grade I		
Bars up to and including 20 *mm.*	2600	260
Bars over 20 *mm.*	2400	240
Grade II		
Bars up to and including 20 *mm*	2350	235
Bars over 20 *mm.*	2150	215
(5) Mild steel conforming to IS : 1139 – 1959		
Bars up to and including 20 *mm*	2600	260
Bars over 20 *mm*	2400	240

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$$= 0{\cdot}55\, \sigma_{cu}\, ba \left(d - \frac{a}{2} \right)$$

$$= A_s\, \sigma_{sy} \left(d - \frac{a}{2} \right)$$

Yield strength of steel reinforcement. The table on page 1361 shows the yield strength of various qualities of steel reinforcement.

Load factor. This is the ratio of the collapse load to the working load. The load factor has to be judiciously selected giving due consideration to the various factors contributing to failure. The load factors assist in estimating the ultimate loading

Critical load conditions. Unless otherwise specified by the engineer-in-charge or the appropriate authority, every member should be designed to carry without failure, the critical loads given below.

Let U = Ultimate load

DL = Dead load

LL = Live load

WL = Wind load

EL = Earthquake load

(*a*) For those structures in which the effect of wind and earthquake loads can be neglected :

$$U = 1{\cdot}5\, DL + 2{\cdot}2\, LL.$$

(*b*) *For those structures in which wind load should be considered* :

$$U = 1{\cdot}5\, DL + 2{\cdot}2\, LL + 0{\cdot}5\, WL$$

or $$U = 1{\cdot}5\, DL + 0{\cdot}5\, LL + 2{\cdot}2\, WL,$$ whichever gives critical conditions, provided that no member shall have a capacity less than required by the condition in (*a*) above.

(*c*) For those structures in which earthquake load should be considered, substitute EL for WL in the preceding equations.

Note :—These equations are applicable to both dynamic and static loads, provided in the case of dynamic loads, the equivalent static loads are used after allowing for impact.

Design 235. *A singly reinforced concrete beam is 25 cm. wide and 45 cm. deep to the centre of the tensile reinforcement. Find the steel area required for a balanced section and the ultimate moment of resistance. Take* $\sigma_{cu} = 150$ *kg./cm.*^{2} *and* $\sigma_{sy} = 2600$ *kg./cm.*^{2}.

Solution. $b = 25$ *cm.* ; $d = 45$ *cm.*

For a balanced section,

$$A_s = 0{\cdot}236\, \frac{\sigma_{cu}}{\sigma_{sy}}\, bd$$

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$$= 0{\cdot}236 \times \frac{150}{2600} \times 25 \times 45 \text{ cm.}^2$$

$$= 15{\cdot}32 \text{ cm.}^2$$

Ultimate moment of resistance of the balanced section

$$= M_u = 0{\cdot}185\ \sigma_{cu}\ bd^2$$

$$= 0{\cdot}185 \times 150 \times 25 \times 45^2 \text{ kg. cm.}$$

$$= 1404840 \text{ kg. cm.}$$

Design 236 (S.I.). *An R C beam 25 cm. × 45 cm. effective depth, is reinforced on the tensile side only. Find the steel area required for a balanced section and the ultimate moment of resistance. Take* $\sigma_{cu} = 15\ N/mm.^2$ *and* $\sigma_{sy} = 260\ N/mm.^2$

Solution. $b = 25$ cm. ; $d = 45$ cm.

$$\sigma_{cu} = 15\ N/mm.^2 = 1500\ N/cm.^2$$

$$\sigma_{sy} = 260\ N/mm.^2 = 26000\ N/cm.^2$$

For a balanced section

$$A_t = 0{\cdot}236\ \frac{\sigma_{cu}}{\sigma_{sy}}\ bd$$

$$= 0{\cdot}236 \times \frac{1500}{26000} \times 25 \times 45 \text{ cm.}^2$$

$$= 15{\cdot}32 \text{ cm.}^2$$

Ultimate moment of resistance of the balanced section

$$= M_u = 0{\cdot}185\ \sigma_{cu}\ bd^2$$

$$= 0{\cdot}185 \times 1500 \times 25 \times 45^2 \text{ N cm.}$$

$$= 14048440 \text{ N cm.}$$

$$= 140484{\cdot}4 \text{ N m.}$$

$$= 140{\cdot}48 \text{ k Nm.}$$

Design. 237. *A singly reinforced concrete beam is 25 cm. wide and 45 cm. deep to the centre of the tensile reinforcement. The reinforcement consists of 4 bars of 18 mm. diameter. Find the ultimate moment of resistance. Take* $\sigma_{cu} = 200\ kg/cm.^2$ *and* $\sigma_{sy} = 2600\ kg./cm.^2$

Solution.

$b = 25$ cm. ; $d = 45$ cm.

$$A_t = 4 \times 2{\cdot}54 = 10{\cdot}16 \text{ cm.}^2$$

Let the depth of the stress block be a cm.

Total compression = Total tension

$$b\ a \times 0{\cdot}55\ \sigma_{cu} = A_t\ \sigma_{sy}$$

$$a = \frac{A_t\ \sigma_{sy}}{0{\cdot}55\ \sigma_{cu}\ b}$$

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$$= \frac{10{\cdot}16 \times 2600}{0{\cdot}[illegible] \times 200 \times 25} \ cm.$$

$$= 9\,6[illegible]\ cm.$$

But $0{\cdot}43\ d = 0{\cdot}43 \times 45 = 19{\cdot}35\ cm.$

$\therefore$ $a < 0{\cdot}43\ d$

Hence, the beam section is *under reinforced*

Lever arm $= d - \frac{a}{2}$

$$= 45 - \frac{9{\cdot}61}{2} = 40\ 195\ cm$$

Ultimate moment of resistance

$= M_u = A_t \sigma_{sy} \times$ lever arm

$= 10{\cdot}16 \times 2600 \times 40\ 195\ kg.\ cm.$

$= 1061790\ kg.\ cm.$

Design 238 (S I.). *An R.C beam reinforced on the tension side is 25 cm wide with an effective depth of 45 cm It is reinforced with 4 bars of 18 mm. diameter Calculate the ultimate moment of resistance. Take* $\sigma_{cu} = 20\ N/mm^2$ *and* $\sigma_{sy} = 260\ N/mm.^2$

Solution.

$b = 25\ cm\ ,\ d = 45\ cm.$

$A_t = 4 \times 2{\cdot}54 = 10{\cdot}16\ cm.^2$

$\sigma_{cu} = 20\ N/mm^2 = 2000\ N/cm.^2$

$\sigma_{sy} = 260\ N/mm.^2 = 26000\ N/cm.^2$

Let the depth of the stress block be a *cm*

Total compression = Total tension

$b\ a \times 0{\cdot}55\ \sigma_{cu} = A_t\ \sigma_{sy}$

$\therefore$ $$a = \frac{A_t\ \sigma_{sy}}{0{\cdot}55\ \sigma_{cu}\ b}$$

$$= \frac{10{\cdot}16 \times 26000}{0{\cdot}55 \times 2000 \times 25}\ cm.$$

$$= 9{\cdot}61\ cm.$$

But $0{\cdot}43\ d = 0{\cdot}43 \times 45 = 19{\cdot}35\ cm.$

$\therefore$ $a < 0{\cdot}43\ d$

Hence, the beam section is *under reinforced*

Lever arm $= d - \frac{a}{2} = 45 - \frac{9{\cdot}61}{2}$ 40 195 cm.

Ultimate moment of resistance

$= M_u = A_t\ \sigma_{sy} \times$ lever arm

$= 10{\cdot}16 \times 26000 \times 40{\cdot}195\ N\ cm.$

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$$= 10617900 \text{ N cm.}$$
$$= 106179 \text{ N m.}$$
$$= 106{\cdot}18 \text{ kNm}$$

Design 239. *A singly reinforced concrete beam section of width 25 cm. and effective depth 45 cm. has an ultimate moment of resistance of 1,450,000 kg/cm.². Find the area of steel. Take* $\sigma_{cu} = 200$ *kg./cm.² and* $\sigma_{sy} = 2600$ *kg/cm²*

Solution.

$$b = 25 \text{ cm} ; \; d = 45 \text{ cm.}$$
$$M_u = 1{,}450{,}000 \text{ kg. cm.}$$

Ultimate M.R. of balanced section

$$= 0{\cdot}185 \, \sigma_{cu} \, bd^2$$
$$= 0{\cdot}185 \times 200 \times 25 \times 45^2 \text{ kg. cm.}$$
$$= 1{,}873{,}125 \text{ kg cm.}$$

But, the ultimate M.R. of the given beam section is only 1,450,000 *kg. cm.* Hence the beam section is *under reinforced.*

$$\therefore \quad a < 0{\cdot}43 \, d$$

Lever arm $= d - \dfrac{a}{2}$

$$= \left(45 - \frac{a}{2} \right) \text{cm.}$$

Ultimate M.R. $= M_u = 0{\cdot}55 \, \sigma_{cu} \, ba \left(d - \dfrac{a}{2} \right) = 1450000$ *kg. cm.*

$$\therefore \quad 0{\cdot}55 \times 200 \times 25 \; a \left(45 - \frac{a}{2} \right) = 1450000$$

$$\therefore \quad a \left(45 - \frac{a}{2} \right) = 527{\cdot}27$$

$$\therefore \quad a^2 - 90 \, a + 1054{\cdot}54 = 0$$

$$\therefore \quad a = 13{\cdot}85 \text{ cm.}$$

Total compression = Total tension

$$0{\cdot}55 \, \sigma_{cu} \, b \, a = A_t \, \sigma_{sy}$$

$$\therefore \quad A_t = \frac{0{\cdot}55 \, \sigma_{cu} \, ba}{\sigma_{sy}}$$

$$= \frac{0{\cdot}55 \times 200 \times 25 \times 13{\cdot}85}{2600} \text{ cm.}^2$$

$$= 14{\cdot}65 \text{ cm.}^2$$

Design 240 (S.I.). *A singly reinforced concrete beam section of width 25 cm. and effective depth 45 cm. has an ultimate moment of resistance of 145 kNm Find the area of steel Take* $\sigma_{cu} = 20$ *N/mm²* *and* $\sigma_{sy} = 260$ *N/mm²*

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Solution.

$b = 25$ *cm.*, $d = 45$ *cm.*

$M_u = 145$ *kNm.*

$= 145000$ *Nm.*

$= 14500000$ *N cm.*

$\sigma_{cu} = 20$ *N/mm.*$^2 = 2000$ *N/cm.*^{2}

$\sigma_{sy} = 260$ *N/mm.*$^2 = 26000$ *N/cm.*^{2}

Ultimate M.R. of the balanced section

$$= 0{\cdot}185\ \sigma_{cu}\ bd^2$$

$$= 0{\cdot}185 \times 2000 \times 25 \times 45^2\ N\ cm.$$

$$= 18731250\ N\ cm.$$

But ultimate M.R. of the given beam section is only

$$14500000\ N\ cm.$$

Hence, the beam section is *under-reinforced*

$$\therefore \quad a < 0{\cdot}43\ d$$

Lever arm $= d - \frac{a}{2}$

$$\left(45 - \frac{a}{2}\right) cm.$$

Ultimate M.R. $= M_u = 0{\cdot}55\ \sigma_{cu}\ ba \left(d - \frac{a}{2}\right) = 14500000$

$$\therefore \quad 0{\cdot}55 \times 2000 \times 25\ a \left(45 - \frac{a}{2}\right) = 14500000$$

$$\therefore \quad a\left(45 - \frac{a}{2}\right) = 52{\cdot}72$$

$$\therefore \quad a^2 - 90\ a + 1054{\cdot}54 = 0$$

$$\therefore \quad a = 13{\cdot}85\ cm.$$

Total compression = Total tension

$$0{\cdot}55\ \sigma_{cu}\ ba = A_s\ \sigma_{sy}$$

$$\therefore \quad A_s = \frac{0{\cdot}55\ \sigma_{cu}\ ba}{\sigma_{sy}}$$

$$= \frac{0{\cdot}55 \times 2000 \times 25 \times 13{\cdot}85}{26000}\ cm.^2$$

$$= 14{\cdot}65\ cm^2$$

Design 241. *A simply supported beam of span 6 m. is subjected to a dead load of 850 kg./m. and a live load of 1400 kg./m. Design the beam making the breadth equal to two-thirds the effective depth. Take* $\sigma_{cu} = 200$ *kg./cm*^{2}. *and* $\sigma_{sy} = 2400$ *kg./cm*^{2}.

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Solution

Span $= l = 6$ *m.*

DL : 850 *kg./m.*

LL : 1400 *kg./m*

$b = \frac{1}{2} d$

Ultimate uniformly distributed load on the beam

$= U = 1{\cdot}5\ DL + 2{\cdot}2\ LL$

$= (1{\cdot}5 \times 850) + (2{\cdot}2 \times 1400)$ *kg./m.*

$= 4355$ *kg./m.*

Ultimate moment $M_u = \frac{4355 \times 6^2}{8} \times 100$ *kg. cm.*

$= 1959750$ *kg. cm.*

For a balanced section $M_u = 0{\cdot}185\ \sigma_{cu}\ bd^2$

Equating the ultimate M.R. of the balanced section to the ultimate moment,

$0{\cdot}185 \times 200 \times \frac{1}{2} d \cdot d^2 = 1959750$

$d^3 = 79449{\cdot}32$

$d = 42{\cdot}99$ *cm.* *say 43 cm.*

$\therefore$ $b = \frac{1}{2}(43) = 28{\cdot}7$ *cm.* *say 29 cm.*

$\therefore$ *The size of the beam will be 29 cm. × 43 cm. effective*

$A_t = 0{\cdot}236 \frac{\sigma_{cu}}{\sigma_{sy}} bd$

$= 0{\cdot}236 \times \frac{200}{2400} (29 \times 43)$ *cm.*^{2}

$= 24{\cdot}52$ *cm.*^{2}

Design 242 (S.I.). *A simply supported beam of span 6 m. is subjected to a dead load 8500 N/metre and a live load of 14000 N/m. Design the beam making the breadth equal to two-thirds the effective depth.*

Take $\sigma_{cu} = 20$ *N/mm.*^{2} *and* $\sigma_{sy} = 240$ *N/mm.*^{2}

Solution

Span $= l = 6$ *m.*

$DL = 8500$ *N/m.*

$LL = 14000$ *N/m.*

$b = \frac{2}{3} d$

$\sigma_{cu} = 20$ *N/mm.*$^2 = 2000$ *N/cm.*^{2}

$\sigma_{sy} = 240$ *N/mm.*$^2 = 24000$ *N/cm.*^{2}

Ultimate uniformly distriouted load on the beam

$= U = 1{\cdot}5\ DL + 2{\cdot}2\ LL$

$= (1{\cdot}5 \times 8500) + (2{\cdot}2 \times 14000) = 43550$ *N/m.*

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Ultimate moment $M_u = \frac{43550 \times 6^2}{8}$ *Nm.*

$= 195975$ *N m.*

$= 19597500$ *N cm.*

For a balanced section,

$$M_u = 0{\cdot}185\ \sigma_{cu}\ bd^2$$

Equating the ultimate M.R. of the balanced section to the ultimate moment,

$0{\cdot}185 \times 2000 \times \frac{2}{3}\ d \cdot d^2 = 19597500$

$\therefore \quad d = 42{\cdot}99$ *cm., say, 43 cm.*

$\therefore \quad b = \frac{2}{3}(43) = 28{\cdot}7$ *cm., say 29 cm.*

$\therefore$ *The size of the beam will be 29 cm × 43 cm. effective*

$$A_t = 0{\cdot}236 \frac{\sigma_{cu}}{\sigma_{sy}}\ bd$$

$$= 0{\cdot}236 \times \frac{2000}{24000} \times 29 \times 43\ cm.^2$$

$$= 24{\cdot}52\ cm.^2$$

Design 243 *Design a simply supported slab of effective span 3·75 m. for a live load of 250 kg /m². Take* $\sigma_{cu} = 200$ *kg./cm.²* and $\sigma_{sy} = 2600$ *kg /cm.²*

Solution.

Thickness of slab $= \frac{1}{30}$ span

$= \frac{1}{30} \times 375 = 12{\cdot}5$ *cm. say 12 cm.*

Loads

DL of slab :	$12 \times 25 =$	300 *kg /m.²*
Floor finish : (2 *cm.*)	$2 \times 24 =$	48 *kg./m.²*
		348 *kg /m.²*
Live load		250 *kg./m.²*

Ultimate load $= U = 1{\cdot}5\ DI + 2{\cdot}2\ LL$

$= (1{\cdot}5 \times 348) + (2{\cdot}2 \times 250)$

$= 522 + 550 = 1072$ *kg./m.²*

Ultimate moment $M_u = \frac{1072 \times 3{\cdot}75^2}{8} \times 100$ *kg. cm.*

$= 188440$ *kg. cm.*

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Equating M.R. to B.M.

$$0{\cdot}185\ \sigma_{cu}\ bd^2 = 0{\cdot}185 \times 200 \times 100\ d^2 = 188440$$

$$\therefore \quad d = 7{\cdot}1\ cm.$$

$$\therefore \quad A_t = 0{\cdot}236\ \frac{\sigma_{cu}}{\sigma_{sy}}\ bd$$

$$= 0{\cdot}236 \times \frac{200}{2600} \times 100 \times 7{\cdot}10$$

$$= 12{\cdot}89\ cm^2$$

Spacing of 12 *mm.* ϕ bars

$$= \frac{1{\cdot}13 \times 100}{12{\cdot}89} = 8{\cdot}77\ cm.$$

say, 8·75 cm c/c

Overall depth required

$$= d + \text{half dia} + \text{clear cover}$$

$$= 7{\cdot}1 + \frac{1{\cdot}2}{2} + 1{\cdot}5 = 9{\cdot}2\ cm.$$

say 9·5 cm.

Design 244 *A reinforced concrete slab 15 cm. thick is provided with 10 mm ϕ bars at 12 cm centres. The effective cover to steel is 2 cm. Find the safe uniformly distributed load on the slab if the effective span is 3·75 metre. Take $\sigma_{cu} = 150$ kg./cm^2 and $\sigma_{sy} = 2600$ kg./cm^2.*

Solution. Consider a 1 *metre* wide strip of the slab.

$$\therefore \quad \text{Steel provided} = A_t = \frac{0{\cdot}79 \times 100}{12} = 6{\cdot}58\ cm.^2$$

Total compression = Total tension

$$C = T$$

$$ba \times 0{\cdot}55\ \sigma_{cu} = A_t\ \sigma_{sy}$$

$$a = \frac{A_t\ \sigma_{sy}}{0{\cdot}55\ \sigma_{cu}\ b}$$

$$= \frac{6{\cdot}58 \times 2600}{0{\cdot}55 \times 150 \times 100} = 2{\cdot}07\ cm.$$

But $\quad 0{\cdot}43\ d = 0{\cdot}43 \times (15 - 2) = 5{\cdot}59\ cm.$

$$\therefore \quad a < 0{\cdot}43\ d$$

$\therefore$ The beam section is under-reinforced.

$$\text{Ultimate M.R.} = M_u = A_t\ \sigma_{sy} \left(d - \frac{a}{2} \right)$$

$$= 6{\cdot}58 \times 2600 \left(13 - \frac{2{\cdot}07}{2} \right) kg.\ cm.$$

$$= 204700\ kg.\ cm.$$

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Let the ultimate load per square metre on slab $= U$ *kg./m.*^{2}

$$\therefore \quad M_u = \frac{U(3{\cdot}75)^2}{8} \times 100 \text{ kg. cm.}$$

$$\therefore \quad \frac{U(3{\cdot}75)^2}{8} \times 100 = 204700$$

$$\therefore \quad U = 1164{\cdot}5 \text{ kg./m.}^2$$

D.L. of slab $= 2500 \times 0{\cdot}15 = 375$ *kg./m.*^{2}

$$U = 1{\cdot}5 \text{ D.L.} + 2{\cdot}2 \text{ LL} = (1{\cdot}5 \times 375) + 2{\cdot}2 \text{ LL}$$

$$= 1164{\cdot}5$$

$$\therefore \quad LL = 274 \text{ kg./m.}^2$$

§244. Doubly Reinforced Section

A doubly reinforced section is provided in such situations when the ultimate moment of resistance required is greater than the ultimate moment of resistance of the balanced section and there is no scope to increase the depth of the beam. Besides the tensile steel, the compression steel will also be taken to have reached the yield stage at ultimate condition.

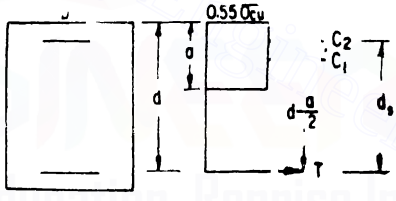

Fig. 951

Fig. 951 shows a doubly reinforced beam section.

Let b = width of the beam

d = effective depth of beam

A_c = area of compression steel

A_t = area of tensile steel

d_c = effective cover to compression reinforcement

d_0 = distance between centres of compression and tensile steel

a = depth of stress block.

Properties of under-reinforced section

Fig. 951 shows the stress diagram at ultimate condition

Total compression = Total tension

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$$b\,a\times 0{\cdot}55\,\sigma_{cu}+A_c\,\sigma_{sy}=A_t\,\sigma_{sy}$$

$$\therefore \qquad a=\frac{(A_t-A_c)\,\sigma_{sy}}{0{\cdot}55\,\sigma_{cu}\,b}$$

The section is under-reinforced if $a<0{\cdot}43\,d$.

Ultimate moment of resistance

$$M_u=b\,a\times 0{\cdot}55\,\sigma_{cu}\left(d-\frac{a}{2}\right)+A_c\,\sigma_{sy}\,d_c$$

$$=(A_t-A_c)\,\sigma_{sy}\left(d-\frac{a}{2}\right)+A_c\,\sigma_{sy}\,d_c$$

Properties of balanced section

For this case $a=0{\cdot}43\,d$

Ultimate Moment of resistance

$$M_u=b\,a\times 0{\cdot}55\,\sigma_{cu}\left(d-\frac{a}{2}\right)+A_c\,\sigma_{sy}\,d_c$$

$$=b\times 0{\cdot}43\,d\times 0{\cdot}55\,\sigma_{cu}\left(d-\frac{0{\cdot}43\,d}{2}\right)+A_c\,\sigma_{sy}\,d_c$$

$$=0{\cdot}185\,\sigma_{cu}\,bd^2+A_c\,\sigma_{sy}\,d_c$$

Design 245. *A doubly reinforced beam is 25 cm. × 45 cm. effective and is provided with 3 bars of 16 mm. diameter as compression steel at an effective cover of 5 cm. and 4 bars of 18 mm. diameter as tensile steel. Calculate the ultimate moment of resistance. Take* $\sigma_{cu}=150$ *kg./cm.*^{2} *and* $\sigma_{sy}=2600$ *kg/cm*^{2}.

Solution.

$b=25$ *cm.* ; $d=45$ *cm.* ; $d_c=5$ *cm.* ;

$A_c=3\times 2{\cdot}01=6{\cdot}03$ *cm.*^{2} ;

$A_t=4\times 2{\cdot}54=10{\cdot}16$ *cm.*^{2} ;

Let a be the depth of the stress block

Equating total compression to total tension

$$C=T$$

$$b\,a\,0{\cdot}55\,\sigma_{cu}+A_c\,\sigma_{sy}=A_t\,\sigma_{sy}$$

$$a=\frac{(A_t-A_c)\,\sigma_{sy}}{0{\cdot}55\,\sigma_{cu}\,b}=\frac{(10{\cdot}16-6{\cdot}03)\,2600}{0{\cdot}55\times 150\times 25}$$

$$=5{\cdot}21\ cm.$$

But $0{\cdot}43\,d=0{\cdot}43\times 45=19{\cdot}35$ *cm.*

$$\therefore \qquad a<0{\cdot}43\,d$$

Hence the section is *under reinforced.*

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Ultimate moment of resistance

$$= M_u = (A_t - A_c)\,\sigma_{sy}\left(d - \frac{a}{2}\right) + A_c\,\sigma_{sy}\,d_s$$

$$= (10.16 - 6.03)\,2500\left(45 - \frac{5.21}{2}\right) + 6.03 \times 2600 \times 40 \text{ kg. cm.}$$

$$= 455237.5 + 627120 \text{ kg cm.}$$

$$= 1082357.5 \text{ kg. cm.}$$

Design 246. *An R C beam 25 cm. × 45 cm. effective has an ultimate moment of resistance of 2233100 kg. cm. Find the steel required. Take $\sigma_{cu} = 200$ kg/cm.^{2} and $\sigma_{sy} = 2600$ kg/cm^2.*

Solution.

$b = 25$ cm., $d = 45$ cm., Let $d_c = 5$ cm.

$$d_s = 45 - 5 = 40 \text{ cm.}$$

$$M_u = 2233100 \text{ kg cm.}$$

Ultimate moment of a singly reinforced balanced section

$$= 0.185\,\sigma_{cu}\,bd^2$$

$$= 0.185 \times 200 \times 25 \times 45^2 \text{ kg. cm.}$$

$$= 1873125 \text{ kg cm}$$

But the Ultimate Moment of resistance required

$$= 2233100 \text{ kg cm.}$$

$\therefore$ $M_u >$ ultimate M.R. of balanced section.

Hence we must provide a doubly reinforced section.

Designing the beam as a balanced section.

$$M_u = 0.185\,\sigma_{cu}\,bd^2 + A_c\,\sigma_{sy}\,d_s$$

$$2233100 = 1873125 + A_c \times 2500 \times 40$$

$\therefore$ $A_c = 3.46$ cm.^{2}

Total compression = Total tension

$$0.55\,\sigma_{cu}\,b\,a + A_c\,\sigma_{sy} = A_t\,\sigma_{sy}$$

$$0.55 \times 200 \times 25 \times 0.43 \times 45 + 3.46 \times 2600 = A_t \times 2600$$

$\therefore$ $A_t = 23.92$ cm.^{2}

§245. T Beams

Ultimate load theory can be applied to *T*-beams on lines similar to those for rectangular beams. The following two cases arise.

Case 1. *When the depth of the stress block is less than the thickness of the flange.*

Let B = width of flange

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d = effective depth

b_r = width of rib

A_t = area of steel

t = thickness of flange

a = depth of stress block.

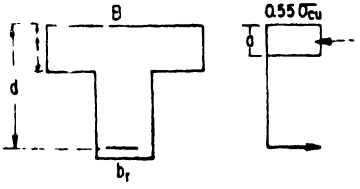

Fig. 952

Fig. 952 shows the stress diagram at ultimate condition. Equating total compression to total tension,

$$0{\cdot}55\ \sigma_{cu}\ Ba = A_t\ \sigma_{sy}$$

$$\therefore \qquad a = \frac{A_t\ \sigma_{sy}}{0{\cdot}55\ \sigma_{cu}\ B}$$

If $a < 0{\cdot}43\ d$ the section is under-reinforced.

Lever arm $\left(d - \frac{a}{2}\right)$

Ultimate moment of resistance

$$M_u = A_t\ \sigma_{sy}\left(d - \frac{a}{2}\right)$$

If $a > 0{\cdot}43\ d$, assume $a = 0{\cdot}43\ d$

and lever arm $= 0{\cdot}785\ d$

and $$M_u = 0{\cdot}55\ \sigma_{cu}\ (B \times 0{\cdot}43\ d)\ 0{\cdot}785\ d.$$

Case 2. *When the depth of the stress block is greater than the thickness of the flange.*

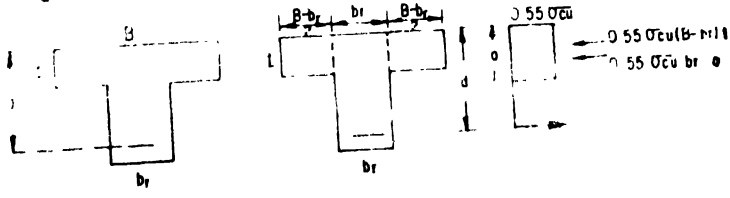

Fig. 953

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Fig. 953 shows the stress diagram at ultimate condition.

Equating to total compression to total tension

$[b_r\, a+(B-b_r)t]\sigma_{cu}=A_s\, \sigma_{sy}$

$\therefore$ Find a from the above relation.

If $a<0{\cdot}43d$ the section is under-reinforced.

Ultimate Moment of resistance

$$M_u=0{\cdot}55\ \sigma_{cu}\ b_r\ a\left(d-\frac{a}{2}\right)+0{\cdot}55\ \sigma_{cu}\ (B-b_r)\ t\left(d-\frac{t}{2}\right)$$

But if $a>0{\cdot}43\ d$, assume $a=0{\cdot}43\ d$ and

$$M_u=0{\cdot}55\sigma_{cu}\ b_r\ a\left(d-\frac{a}{2}\right)+0\ 55\ \sigma_{cu}\ (B-b_r)\ t\left(d-\frac{t}{2}\right)$$

taking $a=0{\cdot}43\ d$.

Design 247. *A reinforced concrete T-beam section has a flange width of 125 cm., a flange thickness of 10 cm., an effective depth of 45 cm. and rib width of 25 cm. It is reinforced with tensile steel of 4 bars of 25 mm. diameter. Find the ultimate moment of resistance. Take* $\sigma_{cu}=150$ *kg/cm.*^{2} *and* $\sigma_{sy}=2600$ *kg./cm*^{2}.

Solution. $A_s=4\times4{\cdot}91=19{\cdot}64$ *cm.*^{2} Assuming $a<t$ and equating the total compression to total tension at ultimate condition.

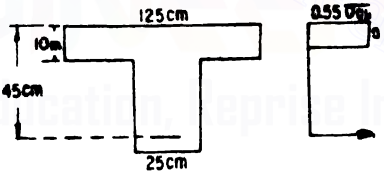

Fig. 954

$0{\cdot}55\ \sigma_{cu}\ Ba=A_s\ \sigma_{sy}$

$$a=\frac{A_s\ \sigma_{sy}}{0{\cdot}55\ \sigma_{cu}\ B}=\frac{19{\cdot}64\times2600}{0{\cdot}55\times150\times125}=4{\cdot}95\ cm.$$

This is less than t.

$0{\cdot}43\ d=0{\cdot}43\times45=19{\cdot}35\ cm.$

$\therefore$ $a<0{\cdot}43\ d$

Ultimate Moment of resistance

$$M_u=A_s\ \sigma_{sy}\left(d-\frac{a}{2}\right)$$

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$$= 19{\cdot}64 \times 200 \left(45 - \frac{4{\cdot}95}{2}\right) \text{ kg. cm}$$

$$= 2171240 \text{ kg. cm.}$$

Design 248. *A reinforced concrete T-beam section has a flange width of 125 cm., flange thickness of 10 cm., effective depth of 45 cm. and rib width of 25 cm. It is provided with tensile steel of 10 bars of 25 mm. diameter. Find the ultimate moment of resistance. Take* $\sigma_{cu} = 150$ *kg./cm.*^{2} *and* $\sigma_{sy} = 2600$ *kg./cm.*^{2}

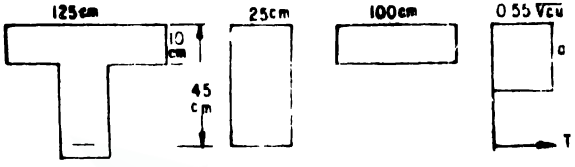

Fig. 955

Solution. Assuming $a < t$ and equating total compression to total tension

$$0{\cdot}55\ \sigma_{cu}\ B\ a = A_t\ \sigma_{sy}$$

$$a = \frac{A_t\ \sigma_{sy}}{0{\cdot}55\ \sigma_{cu}\ B} = \frac{49{\cdot}1 \times 2400}{0{\cdot}55 \times 150 \times 125} = 11{\cdot}43 \text{ cm.}$$

But $t = 10$ cm.

$a > t$

This means the depth of the stress block is greater than the thickness of the flange. Taking $a > t$ and,

Equating total compression to total tension,

$$0{\cdot}55\ \sigma_{cu}[b_r\ a + (B - b_r)\ t] = A_t\ \sigma_{sy}$$

$$0{\cdot}55 \times 150[25a + 100 \times 10] = 49{\cdot}1 \times 2400$$

$$\therefore \quad a = 17{\cdot}13 \text{ cm.}$$

But $0{\cdot}43\ d = 0{\cdot}43 \times 45 = 19{\cdot}35$ cm.

$$\therefore \quad a < 0{\cdot}43\ d$$

Ultimate Moment of resistance

$$= M_u = 100 \times 10 \times 0{\cdot}55 \times 150\ (45 - 5)$$

$$+ 25 \times 17{\cdot}13 \times 0{\cdot}55 \times 150 \left(45 - \frac{17{\cdot}13}{2}\right)$$

$$= 3300000 + 1287000 = 4587000 \text{ kg. cm.}$$

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§246. Ultimate Load Theory Applied to Prestressed Concrete Members

Assumptions in Ultimate Load Theory. The following assumptions are made in the design of prestressed concrete members based on ultimate load theory :

(*i*) Plane sections normal to the axis of the member remain plane after bending.

(*ii*) The tensile strength of concrete is neglected.

(*iii*) At ultimate strength, stresses and strains are not proportional, and the distribution of compressive stresses in a section subjected to bending is non-linear. The stress distribution diagram for concrete may be assumed to be a rectangle, trapezoid, parabola or any other shape which results in ultimate strength in reasonable agreement with tests.

(*iv*) Maximum fibre stress in concrete does not exceed 0·68 F_c where F_c = ultimate compressive strength of concrete.

§247. Modes of Failure of a Prestressed Concrete Beam Section

The following are the ways in which a prestressed concrete beam section can fail at ultimate load :

(*a*) Fracture of steel in tension.

(*b*) Excessive elongation of steel followed by crushing of concrete.

(*c*) A small elongation of steel followed by crushing of concrete.

(*d*) Failure of concrete in shear.

(*e*) Failure of bond in pretensioned systems or of the anchorage in post-tensioned systems.

Failure of concrete in shear occurs when the principal tensile stress exceeds the value which concrete can take with or without the help of reinforcement as the case may be.

Failure of bond in pretensioned systems occurs due to inadequate transmission lengths at the ends of the members for the transference of prestress through bond. Failure of an anchorage occurs in post-tensioned systems where the wires or cables are not properly grouted with cement grout.

(*a*) Fracture of steel in tension

This is a case when the percentage of steel provided is so low, that when the concrete in the precompressed tensile zone cracks, the steel is not able to take up the additional tensile stress transferred to it by the cracking of adjacent fibres of concrete. In this case the failure occurs suddenly without a warning. If this type of failure at ultimate load should be avoided the steel ratio $\frac{A_t}{bD}$ should be greater

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where F_s = Ultimate tensile strength of steel in $kg./mm.^2$

A_t = Area of steel in the tension zone.

Failure at collapse load, through premature fracture of steel, or by loss of bond or by shear or by collapse at anchorages shall be avoided.

We will now consider the failure of the member at ultimate load due to the cases (*b*) and (*c*) mentioned above.

(*b*) Analysis of rectangular beam section when the amount of steel is small, but not small enough to cause premature fracture of steel.

These sections come under the category of under reinforced sections. In this case

$$\frac{A_t}{bD} > \frac{25}{F_s}$$

and $$\frac{A_t}{bd}\frac{F_s}{F_c} < 0{\cdot}24$$

Generally beam sections should be designed in this condition.

In this case, as the loading is increased the steel reaches the yield point and continues to yield at higher loads without fracture. The area of the compression zone becomes lesser and lesser and ultimately the section fails due to crushing of concrete.

Failure of this type is preceded by very large cracks in the

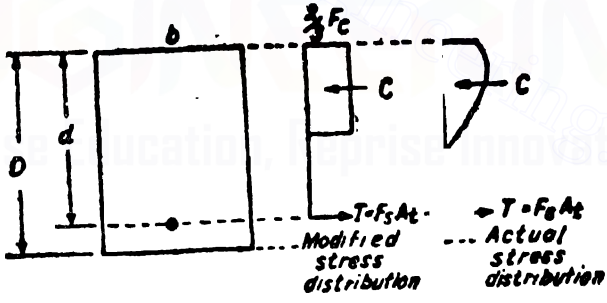

Fig. 956

tension zone and considerable deflection of the member Properties of the section for this case are determined below :

Let b and D be the width and overall depth of the beam section.

Let d be the depth from the compression edge to the tensile steel

The actual stress distribution in concrete is somewhat parabolic and for the purpose of analysis is modified to a rectangular shape (called the stress block) The average stress taken for the stress block is $\frac{2}{3} F_c$ Let h be the depth of the stress block.

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Equating total compression to total tension,

$$bh\left(\frac{2}{3}F_c\right)=A_tF_s$$

$$h=\frac{3}{2}\frac{A_t}{b}\cdot\frac{F_s}{F_c}$$

Lever arm $$a=d-\frac{h}{2}=d-0{\cdot}75\frac{A_t}{b}\frac{F_s}{F_c}$$

$$\therefore \quad a=d\left[1-0{\cdot}75\frac{A_t}{bd}\frac{F_s}{F_c}\right]$$

Ultimate moment of resistance

$$=M_{us}=A_tF_s\times\text{lever arm}$$

$$=A_tF_s\,d\left[1-0{\cdot}75\frac{A_t}{bd}\cdot\frac{F_s}{F_c}\right]$$

As per the I.S. code,

$$M_{us}=K_{us}A_tF_s\,d\left[1-0{\cdot}75\frac{A_t}{bd}\cdot\frac{F_s}{F_c}\right]$$

where, K_{us} = a constant equal to unity when the steel is effectively bonded to concrete and equal to 0·7 when the steel is unbonded.

(c) Analysis of rectangular beam sections where the amount of steel is so large, that the failure at collapse load occurs due to a small elongation of steel followed by crushing of concrete.

These sections come under the category of over-reinforced beams In this case, with gradual overloading, the tensile stress in steel increases, but is yet within the elastic range, and therefore, the neutral axis remains closer to the centre of gravity of the prestressing steel. Consequently the top compressive fibres undergo excessive plastic deformation under heavy stresses. With further increase in loads, the failure takes place by crushing of concrete. In this case the failure occurs without a warning. Designing the beam to fail in this way at collapse condition may be preferably avoided.

In this case, $\frac{A_t}{bd}\frac{F_s}{F_c}>0{\cdot}24$

For this case, as per the I.S. code the ultimate moment of resistance is given by,

$$M_{uc}=0{\cdot}25\,bd^2\times 0{\cdot}75\,F_c$$

§248. Ultimate Load Conditions

The ultimate loads assumed for purposes of determining the condition of failure shall not be less than the following :

(*a*) For statically determinate structures subjected to dynamic loading, such as bridges : $1{\cdot}5\,DL+3\,LL$

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(*b*) For statically determinate structures not subjected to dynamic loading, such as buildings ; 1·5 *DL*+2·5 *LL* or 2 (*DL*+*LL*) whichever is greater.

DL and *LL* are respectively the design dead load and the maximum live load inclusive of impact.

Design 249. *A pretensioned concrete rectangular beam 25 cm. ×60 cm. is prestressed with 6 50 cm.² of steel wires. The centre of gravity of the steel is located 12 cm. above the bottom edge. Taking F_s = 17000 kg /cm.² and F_c = 400 kg./cm.² find the ultimate moment of resistance of the beam section.*

Solution $b = 25$ *cm.* $D = 60$ *cm.*

$d = 48$ *cm.* $F_s = 17000$ *kg./cm.²*

$A_s = 6{\cdot}50$ *cm.²* $F_c = 400$ *kg./cm.²*

$$\frac{A_s}{bd} \cdot \frac{F_s}{F_c} = \frac{6{\cdot}50}{25 \times 48} \cdot \frac{17000}{400} = 0{\cdot}23 \text{ (less than } 0{\cdot}24)$$

Hence the section is under-reinforced.

∴ *Ultimate moment of resistance*

$$= M_{us} = K_{us} A_s F_s d \left[1 - 0{\cdot}75 \frac{A_s}{bd} \frac{F_s}{F_c} \right]$$

$$= 1 \times 6{\cdot}5 \times 17000 \times 48 [1 - 0{\cdot}75 \times 0{\cdot}23] \text{ kg. cm.}$$

$$= 4{,}389{,}060 \text{ kg cm.}$$

Design 250. *A pretensioned concrete rectangular beam 25 cm. × 50 cm. is prestressed with 8 cm.² of steel wires. The centre of gravity of the steel is located 10 cm. above the bottom edge. Taking F_s = 17000 kg./cm.² and F_c = 400 kg./cm.², find the ultimate moment of resistance of the beam section.*

Solution.

$b = 25$ *cm.*, $D = 50$ *cm.*,

$d = 40$ *cm.*, $F_s = 17000$ *kg./cm.²*,

$A_s = 8$ *cm.²*, $F_c = 400$ *kg./cm.²*

$$\frac{A_s}{bd} \cdot \frac{F_s}{F_c} = \frac{8}{25 \times 40} \cdot \frac{17000}{400} \quad 0{\cdot}34 \text{ (greater than } 0{\cdot}24)$$

Hence the section is over-reinforced.

Ultimate moment of resistance.

$$M_{us} = 0{\cdot}25 bd^2 \times 0{\cdot}75 F_c$$

$$= 0{\cdot}25 \times 25 \times 40^2 \times 0{\cdot}75 \times 400 \text{ kg. cm}$$

$$= 3{,}000{,}000 \text{ kg. cm.}$$

Design 251. *A pretensioned concrete rectangular beam 30 cm. ×60 cm. has a span of 10 metres. It is prestressed with 8 cm.² of steel wires. The centre of gravity of the steel is located 10 cm. above the*

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bottom edge. Taking $F_s = 17000$ kg./cm.^{2} and $F_c = 400$ kg./cm.^{2}, find the ultimate moment of resistance and safe external uniformly distributed load the beam can carry.

Solution.

$b = 30$ cm., $D = 60$ cm. $l = 10$ metre,

$d = 50$ cm., $F_s = 17000$ kg./cm.^{2},

$A_t = 8$ cm.^{2}, $F_c = 400$ kg./cm.^{2}.

$$\frac{A_t}{bd} \cdot \frac{F_s}{F_c} = \frac{8}{30 \times 50} \times \frac{17000}{400}$$

$$= 0{\cdot}227 \text{ (less than } 0{\cdot}24)$$

Hence, the section is under-reinforced

$\therefore$ *Ultimate moment of resistance*

$$M_u = K_u A_t F_s d \left[1 - 0{\cdot}75 \frac{A_t}{bd} \frac{F_s}{F_c} \right]$$

$$= 1 \times 8 \times 17000 \times 50 \, [1 - 0{\cdot}75 \times 0{\cdot}227] \text{ kg. cm.}$$

$$= 5{,}642{,}300 \text{ kg. cm.}$$

Let the ultimate uniformly distributed load be U kg./m.

$\therefore$ Ultimate bending moment

$$= U \times \frac{10^2}{8} \times 100 = 5{,}642{,}300$$

$$U = 4513{\cdot}84 \text{ kg./metre}$$

DL of the beam $= 0{\cdot}30 \times 0{\cdot}60 \times 2400 = 432$ kg./m.

$$1{\cdot}5\,DL + 2{\cdot}5\,LL = 4513{\cdot}84 \text{ kg./m.}$$

$$[1{\cdot}5 \times 432] + 2{\cdot}5\,LL = 4513{\cdot}84$$

$$\therefore \quad LL = 1546 \text{ kg./m.}$$

§249. Analysis of I Sections

Let b = width of flange of the section

D = overall depth of the section

d = Depth from the top edge to the centre of gravity of steel

b' = width of the web

t = thickness of the flange.

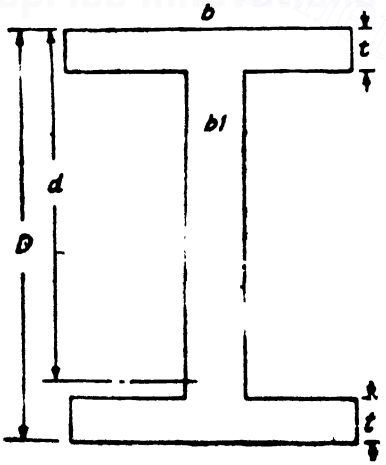

Fig. 957

The I S. code has made the following recommendations.

Condition for under reinforced ction

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$$\frac{A_{tw}}{b'd} \quad \frac{F_s}{F_c} \leq 0{\cdot}24$$

where, $A_{tw} \quad A_t - A_{tf}$

A_{tf} = Steel area required to develop the ultimate compressive strength of the overhanging portions of the flange

$$= \frac{0{\cdot}68\, F_c\, (b-b')t}{F_s}$$

(a) *Ultimate moment of resistance of under-reinforced section*

When $t \geq \frac{d}{4}$, $M_{us} = K_{us}\, A_t\, F_s\, d\left[1 - 0{\cdot}75\, \frac{A_t}{bd} \cdot \frac{F_s}{F_c}\right]$

When $t < \frac{d}{4}$, $M_{us} = K_{us}\, A_{tw}\, F_s\, d\left[1 - 0{\cdot}75\, \frac{A_{tw}}{b'd} \cdot \frac{F_s}{F_c}\right]$

$$+ 0{\cdot}70\, F_c\, t(b-b')\,(d - 0{\cdot}5t)$$

where K_{us} = a constant equal to unity when tensioned steel is effectively bonded to concrete, and equal to 0·70 when the steel unbonded.

(b) *Ultimate moment of resistance of the over-reinforced section*

$$M_{us} = 0{\cdot}25b'd^2 \times 0{\cdot}75\, F_c + 0{\cdot}68\, t \times 0{\cdot}75\, F_c\, (b-b')(d - 0{\cdot}5t)$$

Design 252. *Fig. 958 shows section of a pretensioned concrete beam provided with 80 cm.² of steel whose centre of gravity is 25 cm. from the bottom edge. Find the ultimate moment of resistance. Take $F_s = 17000$ kg./cm.² and $F_c = 400$ kg./cm.²*

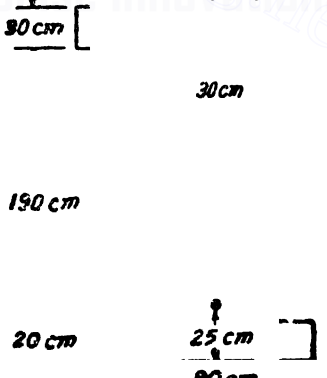

Fig. 958

Solution.

$b = 125$ cm., $b' = 30$ cm²., $d = 30 + 190 + 20 - 25 = 215$ cm., $t = 30$ cm., $A_t = 80$ cm.²

$$\frac{d}{4} = \frac{215}{4} = 53{\cdot}75 \text{ cm.}$$

$$A_{tf} = \frac{0{\cdot}68\, F_c (b-b')t}{F_s} = \frac{0{\cdot}68 \times 400(125-30)30}{17000} = 45{\cdot}60 \text{ cm.}^2$$

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$\therefore$ $A_{tw} = A_t - A_{tf}$

$= 80 - 45.60 = 34.40 \text{ cm.}^2$

$$\frac{A_{tw}}{b'd} \cdot \frac{F_s}{F_c}$$

$$= \frac{34.4}{30 \times 215} \times \frac{17000}{400} = 0.227 \text{ (less than 0.24)}$$

$\therefore$ The section is under-reinforced.

$\therefore$ *Ultimate moment of resistance*

$$= M_{us} = K_{us} \; A_{tw} \; F_s \; d \left[1 - 0.75 \frac{A_{tw} \; F_s}{b'd \; F_c} \right] + 0.70 \; F_c \; t \; (b - b')(d - 0.5t)$$

$= 1 \times 34.4 \times 17000 \times 215[1 - 0.75 \times 0.227]$

$+ 0.70 \times 400 \times 30 \; (125 - 30) \; (215 - 0.5 \times 30)$

$= 1.0432 \times 10^8 + 1.5960 \times 10^8 \text{ kg. cm.}$

$= 2.6392 \times 10^8 \text{ kg. cm.}$

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25
Limit State Method

Limit state method of design is a further improvement over the method of *Ultimate Load* design. We know that the ultimate load theory is based on the assumption that a structure reaches a collapse condition forming a mechanism, when a certain load is applied. The ratio of the ultimate load to the working load is called the load factor. This method appeared to be satisfactory and acceptable and was recommended by the codes of practice of many countries After more research and study, it has now been felt that this method of design has to be modified so that a structure can be designed to function satisfactorily at service conditions The ultimate load method appears to emphasize on the adequate strength of a member Now it is realized that the adequate strength could not be considered as the only criterion for the satisfactory performance of the member at service conditions In most of the cases a member designed by ultimate load theory is too slender to be adopted, and if adopted may exhibit new problems like excessive deflection, excessive crack widths *etc.* at service conditions.

In the limit state method a structure is designed to withstand all loads likely to act on it in the duration of its life span and also to satisfy the serviceability requirements like deflection limitations and limitations of cracks Limit state means an acceptable limit for the safety and serviceability requirements before failure can occur. The design provides a condition that the structure will not become unfit for use which it is meant or in other words, the structure will not reach a limit state.

All limit states which are relevant are considered in the design to ensure an adequate degree of safety and serviceability The structure in general shall be designed on the basis of the most critical limit state and shall also be checked for other limit states.

The most important limit states to be considered are (*a*) formation of a mechanism leading to collapse and (*b*) excessive deflection and development of excessive cracks.

The limit state corresponding to the formation depends on the ultimate strength. But, the limit state corresponding to excessive deflection and excessive crack width is determined by elastic theory. Thus, the limit state method combines the principles of ultimate load theory and elastic theory.

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§250. Flexural Members

Basic assumptions (Sec I.S. 456—1978 Clause 37·1). Design for the limit state of collapse in flexure shall be based on the assumptions given below :

(*a*) Plane sections normal to the axis of the member remain plane after bending. In other words, the strain at any point on the cross-section is directly proportional to its distance from the neutral axis.

(*b*) The maximum strain in concrete at the outermost compression fibre is 0·0035.

(*c*) The design stress-strain relationship for concrete is taken as shown in Fig. 959.

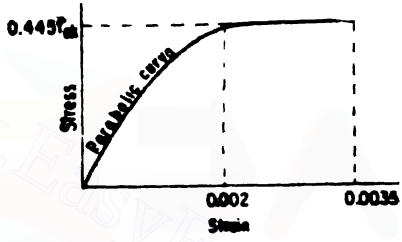

Fig. 959. Design stress-strain curve for concrete.

(*d*) The tensile strength of concrete is ignored.

(*e*) The design stresses in the reinforcement are derived from the strains using the stress-strain relationships given in Figs. 960 and 961.

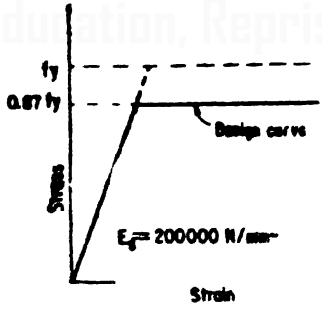

Fig. 960. Stress-strain curve for mild steel.

(*f*) The strain in the tension reinforcement is to be not less than

$$\frac{0{\cdot}87 f_y}{E_s} + 0{\cdot}002$$

where f_y = characteristic strength of steel, and
E_s = modulus of elasticity of steel.

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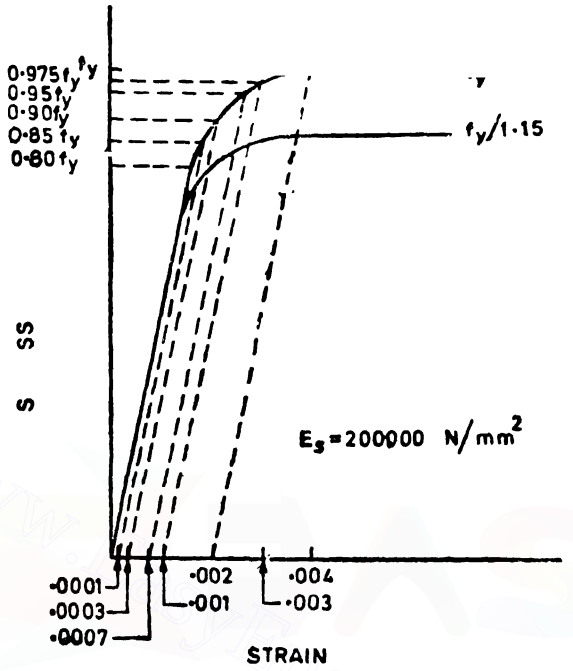

Fig. 961.

This assumption is introduced to ensure ductile failure *i.e.* the tensile reinforcement has to undergo a certain degree of inelastic deformation before the concrete fails in compression.

Maximum depth of neutral axis.

Assumptions (*b*) and (*f*) given above govern the maximum depth of neutral axis. The strain distribution across a member corresponding to those limiting conditions is shown in Fig. 962.

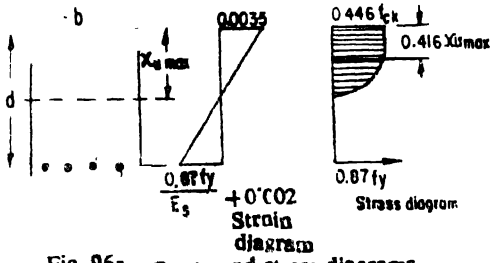

Fig. 962. Strain and stress diagrams.

The ratio of the maximum depth of neutral axis to the effective depth follows the relation,

$$\frac{x_{u\,max}}{d} = \frac{0.0035}{0.0055 + 0.87\, f_y/E_s}$$

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The table below shows the values of $\frac{x_{u\,max}}{d}$ for three graders of reinforcing steel.

f_y (N/mm.^{2})	250	415	500
$\frac{x_{u\,max}}{d}$	0·531	0·479	0·456

§251. Analysis of Rectangular Sections

Fig. 962 shows the stress block for concrete The centroid of the compressive force in the rectangular section is taken at a distance of 0·416 x_u from the extreme compression edge. The total compressive force is 0·36 $f_{ck}\ bx_u$ The lever arm (*i.e.* the distance between the centroid of the compressive force and the centroid of the tensile force) is equal to $(d - 0·416\ x_u)$.

Hence the maximum moment of resistance of the rectangular section is given by,

$$M_{u,\,lim} = 0·36\ f_{ck}\ bx_{u\,max}\ (d - 0·416\ x_{u\,max})$$

Putting the value of $x_{u\,max}$ from the table given above we can determine the limiting moment of resistance and thus determine the limiting moment of resistance factors for a singly reinforced section.

For instance consider the case when $f_y = 250\ N/mm^2$

For this case $\frac{x_{u\,max}}{d} = 0·531$

$$\therefore \quad M_{u\,lim} = 0·36\ f_{ck}\ b \times 0·531\ d\ (d - 0·416 \times 0·531\ d)$$

$$= 0·149\ f_{ck}\ bd^2$$

$$\therefore \quad \frac{M_{u\,lim}}{f_{ck}\ bd^2} = 0·149 \quad \text{...(limiting moment of resistance factor)}$$

Equating total compression to total tension

$$0·36\ f_{ck} \times b \times 0·531\ d = A_t \times 0·87\ f_y$$

$$\frac{A_t}{bd} = \frac{0·36 \times 0·531}{0·87} \cdot \frac{f_{ck}}{f_y} = 0·2197 \frac{f_{ck}}{f_y}$$

$$\therefore \text{ Percentage steel} = \frac{A_t}{bd} \times 100 = 21·97 \frac{f_{ck}}{f_y}$$

The following table shows the limiting moment of resistance $M_{u,\,lim}$ for different grades of steel and the corresponding percentage of steel.

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Limiting Moment of Resistance and Percentage of Reinforcement

f_y ($N/mm.^2$)	M_u, lim	Percentage of steel
250	$0{\cdot}149\, f_{ck}\, bd^2$	$21{\cdot}97\,\frac{f_{ck}}{f_y}$
415	$0{\cdot}138\, f_{ck}\, bd^2$	$19{\cdot}82\,\frac{f_{ck}}{f_y}$
500	$0\,133\, f_{ck}\, bd^2$	$18{\cdot}87\,\frac{f_{ck}}{f_y}$

The maximum percentage of steel permissible for singly reinforced section corresponding to the limiting moment of resistance are given below :

Maximum Percentage of Tensile Steel for Singly Reinforced Sections

f_{ck} $N/mm.^2$	f_y ($N/mm.^2$)		
	250	415	500
15	1·32	0·72	0·57
20	1·76	0·96	0·76
25	2·20	1·19	0·94
30	2·64	1·43	1·13

Under-reinforced section. An under-reinforced section means a singly reinforced beam section in which the amount of steel provided is less than the appropriate value given above. For such a section the depth of neutral axis x_u will be less than $x_{u\,max}$. At the limit state of collapse, the strain in steel will be greater than $\frac{0{\cdot}87\, f_y}{E_s} + 0{\cdot}002$ and the design stress in steel will be equal to $0{\cdot}87\, f_y$.

For such a beam section we can determine the depth of neutral axis by equating the total compression and total tension on the section.

$$\therefore \quad 0{\cdot}36\, f_{ck}\, b\, x_u = A_t \times 0{\cdot}87\, f_y$$

$$\therefore \quad x_u = \frac{A_t \times 0{\cdot}87\, f_y}{0{\cdot}36\, f_{ck}\, b}$$

$\therefore$ **Design moment for limit state**

$$= M_u = A_t \times 0{\cdot}87 f_y\,(d - 0{\cdot}416\, x_u).$$

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Design 253. *A singly reinforced rectangular beam is 30 cm. wide and 60 cm. deep and is provided with 4 bars of 18 mm. diameter. Determine the design moment for limit state (factored moment). Take* $f_{ck} = 15\ N/mm.^2$ *and* $f_y = 415\ N/mm^2$.

Solution. $b = 30\ cm.$

$$d = 60 - (2{\cdot}5 + 0{\cdot}9) = 56{\cdot}60\ cm.$$

$$f_{ck} = 15\ N/mm.^2 = 1500\ N/cm.^2$$

$$f_y = 415\ N/mm.^2 = 41500\ N/cm.^2$$

$$A_t = 4 \times 2{\cdot}54 = 10{\cdot}16\ cm.^2$$

Let x_u be the depth of neutral axis

Equating total compression to total tension,

$$0{\cdot}36\, f_{ck}\, b\, x_u = A_t \times 0{\cdot}87\, f_y$$

$$\therefore \quad x_u = \frac{A_t \times 0{\cdot}87\, f_y}{0{\cdot}36\, f_{ck}\, b}$$

$$= \frac{10{\cdot}16 \times 0{\cdot}87 \times 41500}{0{\cdot}36 \times 1500 \times 30}\ cm. = 22{\cdot}64\ cm.$$

$$x_{u\,max} = 0{\cdot}479\, d = 0{\cdot}479 \times 56{\cdot}60 = 27{\cdot}11\ cm.$$

$$x_u < x_{u\,max}$$

$\therefore$ Section is under-reinforced

$\therefore$ Design moment for limit state

$= M_u =$ (Total compression or total tension) $(d - 0{\cdot}416\, x_u)$

$= A_t \times 0{\cdot}87\, f_y\, (d - 0{\cdot}416\, x_u)$

$= 10{\cdot}16 \times 0{\cdot}87 \times 41500\ (56{\cdot}6 - 0{\cdot}416 \times 22{\cdot}64)\ N\ cm.$

$= 173068{\cdot}88\ N\ cm$

$= 173068{\cdot}88\ Nm.$

$= 173{\cdot}07\ k\ Nm.$

Design 254. *A singly reinforced rectangular beam section is 30 cm. wide and 60 cm. deep. Find the steel reinforcement required to suit the following data :*

Mix of concrete : M 15

Characteristic strength of reinforcement $= 415\ N/mm.^2$

Factored moment : 180 k Nm.

Solution. Providing 25 *mm.* diameter bars at a clear cover of 25 *mm.*, effective cover to reinforcement

$$= 25 + 12{\cdot}5 = 37{\cdot}5\ mm$$

$$= 3{\cdot}75\ cm.$$

Effective depth $= d = 60 - 3{\cdot}75 = 56{\cdot}25\ cm.$

$$f_{ck} = 15\ N/mm.^2 = 1500\ N/cm.^2$$

$$f_y = 415\ N/mm.^2 = 41500\ N/cm.^2$$

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Limiting moment of resistance

$$M_{u\,lim} = 0{\cdot}138 f_{ck} bd^2$$
$$= 0{\cdot}138 \times 1500 \times 30 \times 56{\cdot}25^2 \; N\,cm.$$
$$= 19650000 \; N\,cm.$$
$$= 196{\cdot}5 \; k\,Nm.$$

But actual factored moment

$$= 180 \; kNm.$$

This means the beam section must be designed as a singly reinforced under-reinforced section.

Let x_u = depth of neutral axis

$$0{\cdot}36 f_{ck} b x_u (d - 0{\cdot}416 x_u) = 18000000$$
$$0{\cdot}36 \times 1500 \times 30 \, x_u (56{\cdot}25 - 0{\cdot}416 x_u) = 18000000$$
$$\therefore \; x_u (56{\cdot}25 - 0{\cdot}416 x_u) = 1111$$

Solving we get $x_u = 24{\cdot}05$ *cm.*

Total compression = Total tension

$$0{\cdot}36 f_{ck} b x_u = A_t \times 0{\cdot}87 f_y$$
$$A_t = \frac{0{\cdot}36 f_{ck} b x_u}{0{\cdot}87 f_y}$$
$$= \frac{0{\cdot}36 \times 1500 \times 30 \times 24{\cdot}05}{0{\cdot}87 \times 41500} \; cm.^2$$
$$= 10{\cdot}79 \; cm.^2$$

§252. Doubly Reinforced Section

We know that a doubly reinforced section is adopted when the design moment exceeds the moment of resistance of a singly reinforced section.

M.R. of a doubly reinforced section in the limit state

= Limiting M.R. of the singly reinforced section + additional moment of resistance

i.e., $M_u = M_{u\,lim} + M_{u2}$

where M_{u2} = additional moment of resistance beyond that of the singly reinforced section.

For analysis purposes the lever arm corresponding to the additional moment of resistance is taken equal to the distance between the centroids of tension and compression reinforcement.

The reinforcement for the doubly reinforced section consists of :

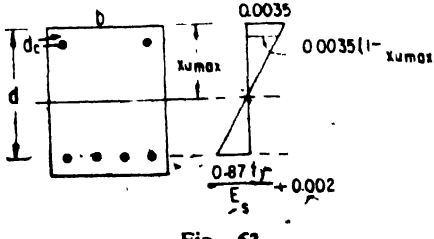

Fig. 63

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(*i*) Tensile steel component corresponding $M_{u\,lim}$ of singly reinforced section

and (*ii*) Additional tensile and the compressive reinforcement to provide the additional moment of resistance M_{u2}.

Let d_c = effective cover to compressive reinforcement.

Let A_{t2} = additional tensile steel area

A_c = area of compressive steel area

f_{sc} = stress in the compressive steel

f_{cc} = stress in concrete at the level of the compressive steel.

Additional tensile force = additional compressive force

$$A_c\,(f_{sc}-f_{cc}) = A_{t2}\,(0{\cdot}87 f_y)$$

Additional moment of resistance

$$M_{u2} = A_c(f_{sc}-f_{cc})\,(d-d_c) = A_{t2}\,(0{\cdot}87 f_y)(d-d_c)$$

Total tensile steel required

= tensile steel required for limiting balanced singly reinforced section + A_{t2}.

Design 255. *A reinforced concrete beam 30 cm. × 60 cm. is to be designed for a factored moment of resistance of 325 k Nm. Find the reinforcement needed. Take* $f_y = 415$ *N/mm.*^{2} *and* $f_{ck} = 15$ *N/mm.*^{2} *and* $d_c = 3{\cdot}75$ *cm.*

Solution. Providing 25 *mm.* ϕ bars at a clear cover of 25 *mm.* effective depth $= d = 60 - 2{\cdot}5 - 1{\cdot}25 = 56{\cdot}25$ *cm.*

$M_{u\,lim}$ for a singly reinforced section

$$= 0{\cdot}138\, f_{ck}\, bd^2$$
$$= 0{\cdot}138 \times 1500 \times 30 \times 56{\cdot}25^2$$
$$= 19650000\ N\ cm.$$

Percentage of tensile steel for balanced singly reinforced section

$$= \frac{A_{t1}}{bd} \times 100 = 19{\cdot}82\,\frac{f_{ck}}{f_y}$$
$$= 19{\cdot}82 \times \frac{1500}{41500} = 0{\cdot}716\%$$

$$\therefore \quad A_{t1} = \frac{0{\cdot}716}{100} \times 30 \times 56{\cdot}25 = 12{\cdot}05\ cm.^2$$

Additional M.R.

$$= M_{u2} = 32500000 - 19650000$$
$$= 12850000\ N\ cm$$

Additional tensile steel

$$A_{t2} = \frac{M_{u2}}{0{\cdot}87 f_y\,(d-d_c)}$$

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$$= \frac{12850000}{0.87 \times 41500(56.25 - 3.75)}$$

$$= 6.78 \ cm^2$$

Total tensile steel

$$= A_t = A_{t1} + A_{t2} = 12.08 + 6.78 = 18.86 \ cm.^2$$

Taking $f_{sc} = f_y = 41500 \ N/cm.^2$

and $f_{cc} = 0.446 \ f_{ck}$

$$= 0.446 \times 1500 = 669 \ N/cm.^2$$

$$A_c = \frac{M_{u2}}{(f_{sc} - f_{cc})(d - d_c)}$$

$$= \frac{12850000}{(41500 - 669)(56.25 - 3.75)} \ cm.^2$$

$$= 5.99 \ cm.^2$$

253. T-beams

Fig. 964 shows the strain diagram and the stress diagram for a T-beam section at utlimate limiting condition.

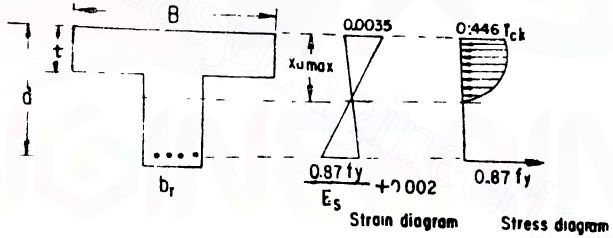

Fig. 964

Moment of resistance of the T-beam section is equal to the sum of the moment of resistance of the concrete in the web of width b_f and the moment of resistance of the flanges.

The moment of resistance is maximum when the depth of neutral axis is equal to $x_{u\,max}$. Studying the stress diagram for the beam section, we find that when the thickness t of the flange is small (less than $0.2d$), the stress is practically uniform in the flange. The resultant compressive force of the flange can be taken to act at $\frac{t}{2}$ below the extreme compression fibre On the basis of the above, the limiting moment of resistance of a T-beam section, for small values of $\frac{t}{d}$ is given by,

$$M_{u,lim} = M_{u,lim} \text{ of web } + 0.446 \ f_{ck} \ (B - b_f) \ t \left(d - \frac{t}{2} \right)$$

where $M_{u,lim}$ of web $= 0.36 \ f_{ck} \ b_f \ x_{u\,max} \ (d - 0.416 \ x_{u\,max})$

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The expression for the moment of resistance given above is applicable when the ratio $\frac{t}{d} < 0.2$.

When the ratio $\frac{t}{d}$ exceeds 0·2 the moment of resistance should be calculated by the following expression,

$$M_{u,lim} = M_{u,lim} \text{ of web} + 0.446 f_{ck} (B - b_r) y_f \left(d - \frac{y_f}{2} \right)$$

where $y_f = 0.15 x_u + 0.65 t$ but not greater than t.

Design 256. *An R C. T-beam section reinforced for tension has the following dimensions.*

(i) *Flange width* = *125 cm.*
(ii) *Thickness of flange* = *10 cm.*
(iii) *Effective depth* = *55 cm.*
(iv) *Breadth of rib* = *30 cm.*

Determine the limiting moment of resistance of the section.
Take $f_{ck} = 15$ *N/mm.*^{2} and $f_y = 415$ *N/mm.*^{2}

Solution.

$$\frac{t}{d} = \frac{10}{55} = 0.18 < 0.20$$

$M_{u,lim}$ of web $= 0.36 f_{ck} b_r x_{u\,max} (d - 0.416 x_{u\,max})$

For $f_y = 415$ *N/mm* ^{2}, $x_{u\,max} = 0.479 d$
$= 0.479 \times 55 = 26.34$ *cm.*

$\therefore$ $M_{u,lim}$ of web
$= 0.36 \times 1500 \times 30 \times 26.34 \ (55 - 0.416 \times 26.34)$ *N cm.*
$= 18793500$ *N cm.*

M R. of flange $= 0.446 f_{ck} (B - b_r) t \left(d - \frac{t}{2} \right)$
$= 0.446 \times 1500 \ (125 - 30) \ 10 \ (55 - 5)$
$= 31777500$ *N cm.*

$\therefore$ Total moment of resistance
$M_{u,lim} = 18793500 + 3177750$
$= 50571000$ *N cm.*
$= 505710$ *Nm.*
$= 505.71$ *k Nm.*

§254. Limit State in Shear—I.S. Recommendations

The nominal shear stress q_v in beams of uniform depth as per the code, is given by

$$q_v = \frac{S_u}{bd}$$

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where, S_u = Shear force due to design loads.

b = breadth of the beam for rectangular sections and breadth of the web for T sections

d = effective depth.

Design Shear Strength of Concrete

(*i*) *Members without shear reinforcement.* The design shear strength of concrete in beams without shear reinforcement is given in the table below.

Design Shearing Strength of Concrete q_c ($N/mm.^2$)

$100\frac{A_t}{bd}$	*Grades of Concrete*					
	M 15	*M 20*	*M 25*	*M 30*	*M 35*	*M 40*
0·25	0·35	0·36	0·36	0·37	0·37	0·38
0·50	0·46	0·48	0·49	0·50	0·50	0·51
0·60	0·50	0·51	0·53	0·54	0·54	0·55
0·70	0·53	0·55	0·56	0·57	0·58	0·59
0·80	0·55	0·57	0·59	0·60	0·61	0·62
0·90	0·57	0·60	0·62	0·63	0·64	0·65
1·00	0·60	0·62	0·64	0·66	0·67	0·68
1·10	0·62	0·64	0·66	0·68	0·69	0·70
1·20	0·63	0·66	0·69	0·70	0·72	0·73
1·30	0·65	0·68	0·71	0·72	0·74	0·75
1·40	0·67	0·70	0·72	0·74	0·76	0·77
1·50	0·68	0·72	0·74	0·76	0·78	0·79
1·60	0·69	0·73	0·76	0·78	0·80	0·81
1·70	0·71	0·75	0·77	0·80	0·81	0·83
1·80	0·71	0·76	0·79	0·81	0·83	0·85
1·90	0·71	0·77	0·80	0·83	0·85	0·86
2·00	0·71	0·79	0·82	0·84	0·86	0·88
2·10	0·71	0·80	0·83	0·86	0·88	0·90
2·20	0·71	0·81	0·84	0·87	0·89	0·91
2·30	0·71	0·82	0·86	0·88	0·91	0·93
2·40	0·71	3·82	0·87	0·90	0·92	0·94
2·50	0·71	0·82	0·88	0·91	0·93	0·95
2·60	0·71	0·82	0·89	0·92	0·94	0·97
2·70	0·71	0·82	0·90	0·93	0·96	0·98
2·80	0·71	0·82	0·91	0·94	0·97	0·99
2·90	0·71	0·82	0·92	0·95	0·98	1·00
3·00	0·71	0·82	0·92	0·96	0·99	1·01

For solid slabs, the design shear strength for concrete shall be $k\,q_c$ where k has the value given below. :

Overall depth of slab (mm)	*300 or more*	*275*	*250*	*225*	*200*	*175*	*150 or less*
k	1·00	1·05	1·10	1·15	1·20	1·25	1·30

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(*ii*) *Members with shear reinforcement.* Under no circumstances, even with shear reinforcement shall the nominal shear stress in beams q_v exceed $q_{c\,max}$ given in the table below :

Maximum Shear Stress $q_{c\,max}$ ($N/mm.^2$)

Concrete grade	*M 15*	*M 20*	*M 25*	*M 30*	*M 35*	*M 40*
$q_{c\,max}$ $N/mm.^2$	2·5	2·8	3·1	3·5	3·7	4·0

Shear reinforcement shall be provided to carry a shear equal to $V_{us} = V_u - q_c\, bd$. The shear reinforcement may be provided in the form of stirrups or bent up bars. When stirrups are provided

$$\text{Spacing of stirrups} = 0{\cdot}87\,\frac{A_w f_y d}{V_{us}}$$

For bent up bars bent at θ with the horizontal the shear resistance $= 0{\cdot}87\, A_w f_y \sin\theta$.

Design 257. *Determine the shear reinforcement for a reinforced concrete beam 30 cm. wide and 60 cm. deep to the following data :*

Grade of concrete = *M 15*

$f_y = 250\ N/mm.^2$

Percentage of tensile steel $= 0{\cdot}8\%$

Factored shear force $V_u = 175\ kN$

Solution. Assuming that 25 *mm.* ϕ bars are provided at a clear cover of 25 *mm.*

$$\text{Effective depth} = d = 60 - 2{\cdot}5 - \frac{2{\cdot}5}{2} = 56{\cdot}25\ cm.$$

Nominal shear stress

$$q_v = \frac{S_u}{bd} = \frac{175000}{30 \times 56{\cdot}25} = 103{\cdot}7\ N/cm.^2$$

$$= 1{\cdot}037\ N/mm.^2$$

For *M* 15 concrete $q_{c\,max} = 2{\cdot}5\ N/mm.^2$

$\therefore$ $q_v < q_{c\,max}$

For 0·8% steel, $q_c = 0{\cdot}55\ N/mm.^2 = 55\ N/cm.^2$

$\therefore$ Shear resistance of concrete

$$= q_c\, bd$$

$$= 55 \times 30 \times 56{\cdot}25 = 92813\ N$$

$\therefore$ Shear to be resisted by stirrups

$$= V_{us} = S_u - q_c\, bd$$

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$$= 175000 - 92813 = 82187\ N$$

Spacing of twoelgged 8 *mm.* ϕ stirrups

$$= 0{\cdot}87\ \frac{A_w f_y d}{V_{us}}$$

$$= \frac{0{\cdot}87 \times (2 \times 0{\cdot}5) 25000 \times 56{\cdot}25}{82187} = 14{\cdot}9\ cm$$

Provide 2-legged 8 mm. ϕ *stirrups @ 14 cm. c/c.*

§255. Limit State in Compression—Columns

Axially loaded compression members. As per code, all columns shall be designed for minimum eccentricity equal to the unsupported length/500 plus lateral dimension/30 subject to a minimum of 20 *mm*

After calculating the eccentricity the column section should be designed for the combined effect of the axial load and bending. But, however, the code recommends that, in cases where the minimum eccentricity determined as above is less than 0·05 times the lateral dimension of the column in the direction under consideration, the ultimate load in axial compression may be determined from the following equation :

$$P_u = 0{\cdot}4 f_{ck} A_c + 0{\cdot}67 f_y A_{sc}$$

where P_u = axial ultimate load
A_c = area of concrete
A_{sc} = area of reinforcement.

Design. 258. *A reinforced column of unsupported length 2·75 metres is to be designed for a factored axial load of 3200 kN. Find the cross-sectional dimensions of the column and the reinforcement required. Use M 20 concrete and take* $f_y = 415\ N/mm^2$.

Solution. Let area of the column $= A\ cm.^2$

Let the area of steel $= A_{sc} = 1\%\ A = 0{\cdot}01\ A\ cm.^2$

Area of concrete $A_c = 0{\cdot}99\ A\ cm.^2$

$$P_u = 0{\cdot}4 f_{ck} A_c + 0{\cdot}67 f_y A_{sc} = 3200 \times 1000\ N$$

$$0{\cdot}4 \times 2000 \times 0{\cdot}99\ A + 0{\cdot}67 \times 41500 \times 0{\cdot}01\ A = 3200 \times 1000$$

$$A = 2990{\cdot}5\ cm.^2$$

Provide 60 cm. $\times$ *50 cm.*

Area of steel $= \frac{1}{100} \times 2990{\cdot}5 = 29{\cdot}91\ cm.^2$

Provide 8 bars of 22 mm. ϕ *(30·41 cm.*^{2}*)*

Check for minimum eccentricity

$$e_{min} = \frac{l}{500} + \frac{D}{30} = \frac{275}{500} + \frac{60}{30} = 0{\cdot}55 + 2$$

$$= 2{\cdot}55\ cm.$$

$$\therefore \quad \frac{e_{min}}{D} = \frac{2{\cdot}55}{60} = 0{\cdot}0425$$

This is less than 0·05.

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26
Form Work

For all concrete constructions some sort of temporary staging, either of timber or steel, is required so as to form moulds or forms to hold the concrete till it sets. Such a temporary staging is called *centering, form work, shuttering* or *false work*. Approximately the cost of the formwork works out to 15 per cent of the total cost of the structure. Hence great care should be taken to effect economy in the design of form work.

Good form work should satisfy the following requirements :

(*i*) The form work should have sufficient strength to carry the dead weight of the concrete and the liveloads which are likely to be present during the construction stage.

(*ii*) All joints in form work should be stiff so that the deformations caused by the dead and live loads shall be a minimum. All joints shall be watertight.

(*iii*) Surfaces of the form work shall be plane or exactly following the requirements, so that after removal of form work, it should not be a costly item to finish the surface of concrete.

(*iv*) The form work shall be as light as possible.

It is necessary to see that the concrete poured into the form work will not stick to it. Otherwise, it may become difficult to remove the shuttering and during this process some amount of surface concrete may be liable to be damaged. To remedy this, it is usual that the interior faces of form work should be given a coating of a thin layer of oil. Sometimes these faces are given a coating of soft soap or they are whitewashed. Canvas or oil paper may also be used for this purpose.

Brief details of form work provided for various structures are given below :

(*i*) *Form work for foundations*

The type of form work depends upon whether the sides of the foundation are taken up vertically or on slope.

If the sides of the foundation are to be taken up vertically to the top, it is usual to construct a hollow box with sides only without any top or bottom. It should be rigid when filled with concrete and also when the concrete is being tamped.

If sloping sides are required for the foundation the box must extend to 15 *cms*. height after which the trapezoidal forms should be

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fixed to shutter the slopes. Figs. 965 and 966 show the form work for foundations.

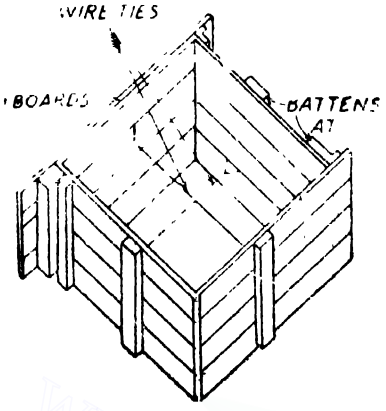

Fig. 965.
Form work for square footing.

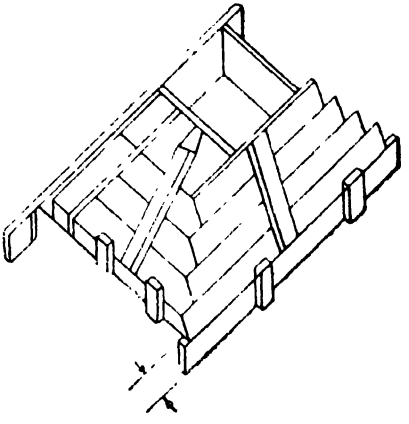

Fig. 966.
Form work for sloping footing.

(*ii*) *Form work for columns*

Fig. 967 shows a typical mould for a column. It is made of two sides of 30 *mm* boards on end of width equal to the width of the column. The two other sides are built similarly but their widths will be greater by twice the thickness of the boards. 75 *mm.* × 100 *mm.* yokes and bolts are used to secure the boards at intervals of 60 *cm.* near the top of the column and reducing to 30 *cm.* near the bottom.

It is found necessary that good stout timber should be used for form work of columns. This is because the form work has to remain rigid when the concrete is subjected to hard ramming.

(*iii*) *Form work for beams and slabs*

The form work for the beams will usually be made along with that of the slab. The soffits of the beams must be strutted by means of vertical members which should rest on folding wedges. This is done to avoid any settlement during the construction. To lower the support the wedges are slowly knocked out.

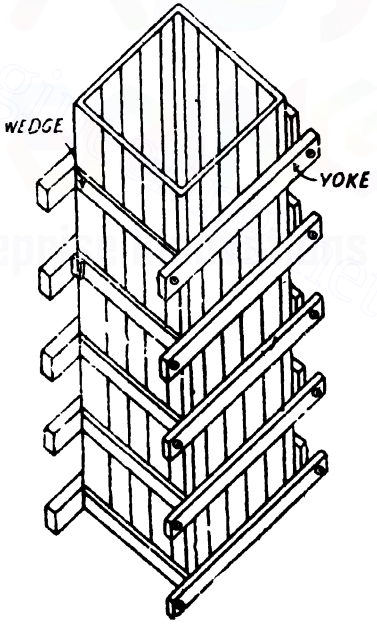

Fig. 967

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Fig. 968 shows a typical arrangement of form work for beam and slab construction.

(*iv*) *Form work for precast lintels*

Lintels may be precast on the ground in a suitable mould as shown in Fig. 969. The wood used should not be of less than 40 *mm.* thickness.

Fig. 969 shows details of form work often adopted for precast lintels.

(*v*) *Form work for stair case*

The form work for the stair case depends upon the methods of designing the stairs. Fig. 970 shows a typical form work for stair cases.

(*iv*) *Form work for circular water tanks*

The form work will consist of outer and inner forms The space between the outer and the inner forms should be equal to the thickness of the tank wall.

The forms usually consist of a number of circular wooden frames acting as stiffeners holding vertical wooden strips.

Fig. 971 shows details of form work for a circular tank.

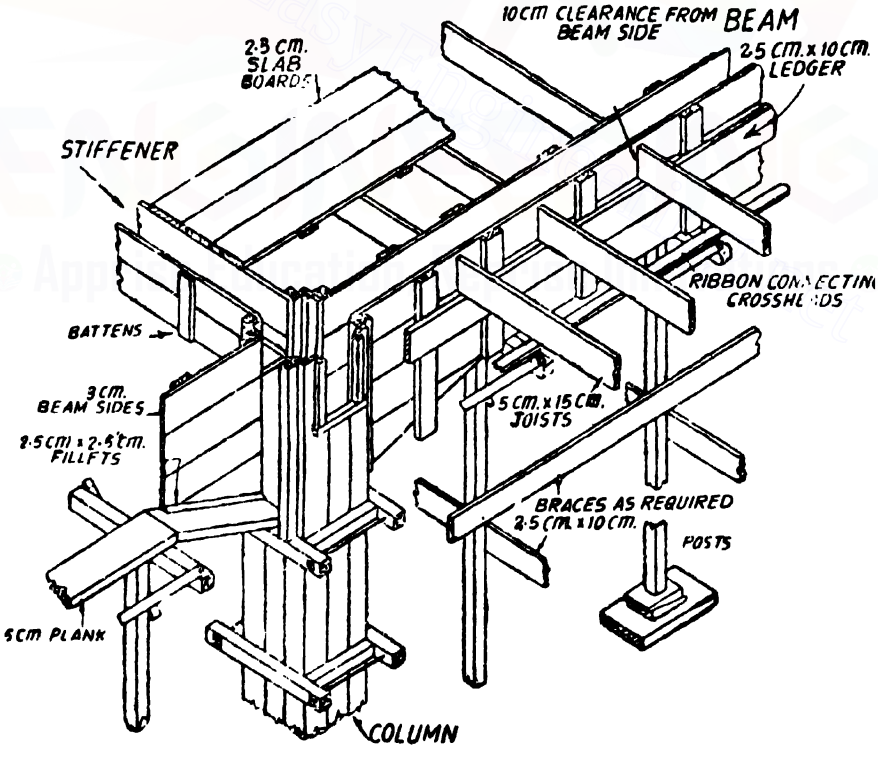

Fig. 968. Form work for col

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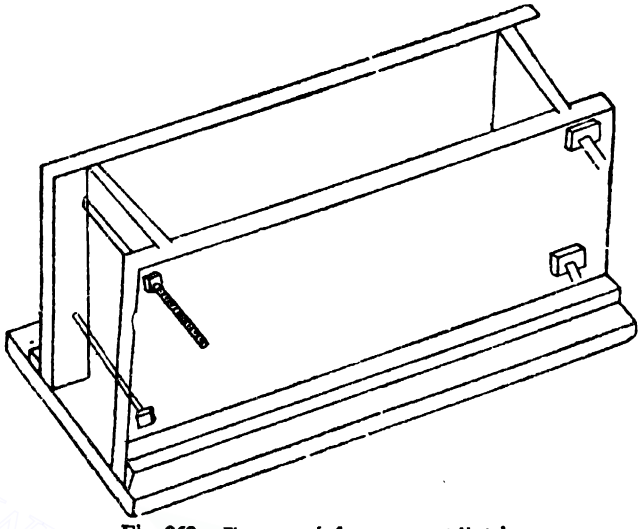

Fig. 969. Form work for a precast lintel.

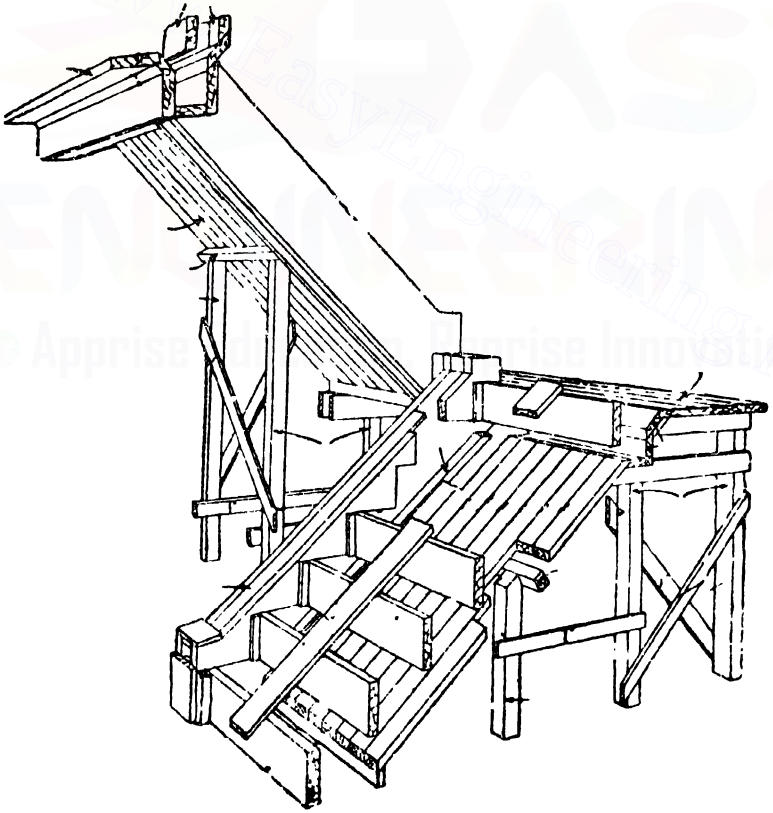

Fig. 970. Form work for staircase.

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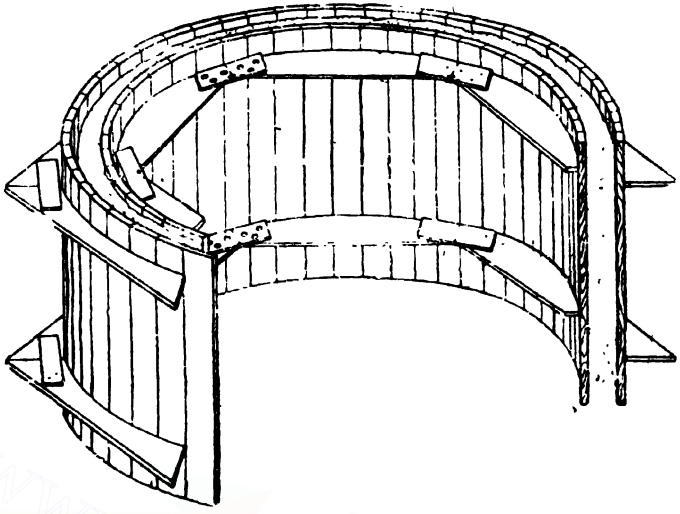

Fig. 971 Form work for a circular tank.

(*ii*) *Form work for square or rectangular tanks*

Fig 972 shows a usual type of form work provided for square and rectangular tanks.

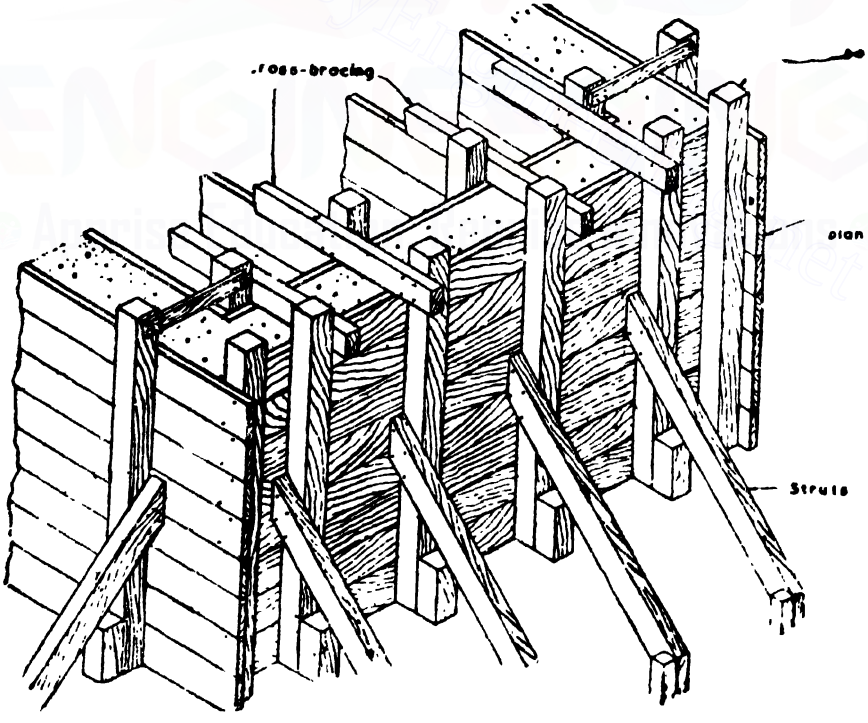

Fig. 972. Form work for the walls of square and rectangular tanks.

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I.S. Recommendation (*IS : 456*)

General. The form work shall conform to the shape, lines and dimensions as shown on the plans, and be so constructed as to remain sufficiently rigid during the placing and compacting of the concrete, aud shall be sufficiently tight to prevent loss of liquid from the concrete.

Cleaning and Treatment of Forms. All rubbish, particularly chippings, and saw dust, shall be removed from the interior of the forms before the concrete is placed and the form work in contact with the concrete shall be cleaned and thoroughly wetted or treated with an approved composition Care shall be taken that such a proved composition is kept out of contact with the reinforcement.

Stripping time. In no circumstances shall forms be struck until the concrete reaches a strength of at least twice the stress to which the concrete may be subjected at the time of striking.

The strength referred to shall be that of concrete using the same cement and aggregates, with the same proportions, and cured, under conditions of temperature and moisture similar to those existing on the work. Where possible, the formwork should be left longer, as it would assist the curing.

In normal circumstances (generally temperatures are above 20°C), and where ordinary cement is used forms may be struck after expiry of following periods :

(*a*) Walls, columns and vertical side of beams	...24 to 48 hours as may be decided by the engineer-in-charge
(*b*) Slabs (Props left under)	... 3 days
(*c*) Beams soffits (Props left under)	... 7 days
(*d*) Removal of props to slabs :	
(1) Spanning up to 4·5 *metres*	... 7 days
(2) Spanning over 4·5 *metres*	...14 days
(*e*) Removal of props to beams and arches :	
(1) Spanning up to 4·5 *metres*	...14 days
(2) Spanning over 6 *metres*	...21 days

For rapid hardening cement $\frac{3}{7}$ of the above period will be sufficient in all cases except vertical sides of slabs, beams and columns which should be retained for 24 hours.

Note. The number of props, their sizes and disposition shall be such as to be able to safely carry the full dead of the slab, beam or arch as the case may be.

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Procedure when removing the form work. All form work shall be removed without such shock or vibration as would damage the reinforced concrete. Before the soffits and struts are removed, the concrete surface shall be exposed, where necessary, in order to ascertain that the concrete has sufficiently hardened.

Proper precautions shall be taken to allow for the decrease in the rate of hardening that occurs with all cements in the cold weather.

Camber. It is generally desirable to give forms an upward camber to ensure that the beams do not have a sag when they have taken up their deflection, but this should not be done unless allowed for in the design calculations of the beams.

Tolerances. Formwork shall be so constructed that the internal dimensions are within the permissible tolerances specified by the designer.

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APPENDIX I

GENERAL INFORMATION ABOUT CONCRETE

Hand-mixing of concrete

For small and important works mixing of concrete is done by hand. For large-scale work, concrete mixers are used.

Hand-mixing requires about 10% more of cement than what is required in machine mixing. It is very important that mixing should be done continuously instead of intermittently. It is very important to note that the component materials of concrete should be uniformly spread in the concrete mass. The surfaces of coarse as well as fine aggregates should be completely covered with the cement-water paste.

Mixing of concrete should not be done on ground. It should be done on a hard clean surface like a concrete floor or a metal tray

Method of Mixing

The measured quantity of sand should be first spread on the mixing surface evenly. The cement is now dumped on the sand and then evenly distributed. The two materials should be mixed well till the mixture appears to be of uniform colour. The mixture obtained is again evenly spread on the mixing surface. Now the measured quantity of coarse aggregate is spread. Coarse aggregate should not be dumped at one place, because, if this is done the aggregates of larger sizes are liable to roll out. Now the whole mass is mixed at least three times by shovelling from the centre to the side and from the side to the centre. After the mixture is evenly done a hollow is made at the middle. Now about half to three-fourths of the total quantity of water required is poured. The materials are now turned in towards the hollow. The remaining quantity of water is now poured and the mixture is continuously turned until the whole mass appears to have the same colour and consistency. The concrete made is now ready to be placed where required.

After the day's work it is necessary that the mixing platform should be thoroughly cleaned to remove all concrete left over. If this is not done, this concrete will set and the platform cannot be used for the next day.

Mixing concrete by machine. The quality of machine mixed concrete is better than that of hand mixed concrete. Mixing machines are easily available since they are manufactured in this country. Mixers are usually of two types, *viz.*, *rotary* and *tilting* mixers. In the case of rotary mixers, the drum in which the concrete is mixed is cylindrical. The drum rotates about a horizontal axis. It is fed from a charging hopper and the concrete, after it is

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mixed, is collected in a discharge chute. In the tilting type of mixers the drum is usually conical or bowl-shaped. The drum rotates about an inclined axis. The mixed concrete can be discharged by tilting the drum.

Mixing Interval. The interval of time for which the concrete should be mixed depends upon the quantity of the mixture. A period of one and half minutes may be found satisfactory for mixers up to 800 *litres* capacity. This interval should be reckoned from the instant that *all* the materials have been deposited into the drum. For each additional 800 *litres* the interval of mixing may be increased by 15 seconds. The speed of the mixer should be as advised by the makers of the mixer.

After the concrete has been well mixed in the mixer, the whole mixture should be discharge from the mixer. The drum should be *completely* emptied before it is recharged. If this is not done the quality of each batch of mix will not be uniform.

After the day's work, or at any time if the mixer is not liable to be used for an hour, it is very important that the drum as well as the blades should be cleaned, otherwise any concrete left adhering to the drum or blades will harden. Cleaning the mixer may be done as follows. Two to three baskets full of coarse aggregates should be put into the drum and two three buckets of water should be poured into it. The drum should be rotated for one and half minutes. After this, the contents in the mixture should be discharged. Very often it is seen that the mixers are not cleaned and every time a certain quantity of concrete is left hardened in the drum. Not only the capacity of the mixer is reduced, its efficiency also goes down. Hence it is advisable that cleaning the mixers should be insisted.

Curing of concrete. The process of hardening requires moisture. Even though strictly, the quantity of water used for mixing the concrete is sufficient for this purpose, actually the water used gets evaporated. Surface cracks may occur and the strength of concrete comes down, if the concrete should be allowed to dry too quickly by exposure to wind and sun. It is therefore necessary to maintain the newly placed concrete moist for several days. This process is called *curing* For road slabs or other large flat areas, the surface should be covered with burlap as soon as it is found that the concrete is hard enough to permit this. This burlap must be kept moist for a duration of 24 hours After this process, moist earth or sand may be deposited over the concrete and the whole thing is kept wet for a period not less than 10 days. In winter, the period may be extended to 14 days.

I.S. recommendations (*IS 456*)

Measuring. The quantity of cement shall be determined by weight. Ordinary portland cement has a weight of 1440 *kg./metres*^{3}

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and rapid hardening portland cement has a weight of 1210 *kg./metres*^{3}. The quantities of fine and coarse aggregates shall be determined either by volume or preferably by weight. The following table gives the suitable proportions based on the assumption that the aggregates are dry.

Concrete Mix Proportions

Ordinary Concrete

Grade of concrete	Total quantity of dry Aggregates by volume per 50 *kg.* of cement, to be taken as the sum of the individual volumes of fine and coarse aggregates (*max*) (*litres*)	Proportion of Fine Aggregate to coarse Aggregate	Quantity of water per 50 *kg.* of cement (*Max*) (*litres*)
M 100	300	Generally 1 : 2 for fine aggregate to coarse aggregate by volume but subject to an upper limit of 1 : 1½ and a lower limit of 1 : 3	34
M 150	220		32
M 200	160		30
M 250	100		27

Mixing. Concrete shall normally be mixed in a mechanical mixer.

Mixing shall be continued until there is a uniform distribution of the materials and the mass is uniform in colour and consistency, but in no case mixing be done for less than two minutes.

When hand-mixing is permitted by the engineer in charge, it shall be carried out on a water-tight platform and care shall be taken to ensure that mixing is continued until the mass is uniform in colour and consistency.

Transporting. Concrete shall be handled from the place of mixing to the place of final deposit as rapidly as practicable in such a manner that will prevent the segregation of loss of any ingredients.

During hot or cold weather, concrete shall be tranported in deep containers.

Placing and Compacting. The concrete shall be placed and compacted before setting commences and should not be subsequently disturbed Concrete shall be thoroughly compacted during the operation of placing and thoroughly worked around the reinforcement around embedded fixtures and into corners of the form work.

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APPENDIX II
TABLE 1
Area of Groups of round bars (*Area in cm.*²)

Diam. mm.	*Numbers of bars*									
	1	2	3	4	5	6	7	8	9	10
5	0·20	0·39	0·59	0·79	0·98	1·18	1·37	1·57	1·77	1·96
6	0·28	0·56	0·85	1·13	1·41	1·70	1·98	2·26	2·54	2·83
7	0·38	0·77	1·15	1·54	1·92	2·31	2·69	3·08	3·46	3·85
8	0·50	1·00	1·51	9·01	2·51	3·01	3·52	4·02	4·52	5·03
10	0·79	1·57	2·36	3·14	3·93	4·71	5·50	6·28	7·07	7·85
12	1·13	2·26	3·39	4·52	5·65	6·79	7·92	9·05	10·18	11·31
14	1·54	3·08	4·62	6·16	7·70	9·24	10·78	12·32	13·85	15·39
16	2·01	4·02	6·03	8·04	10·05	12·06	14·07	16·08	18·10	20·11
18	2·54	5·09	7·63	10·18	12·72	15·26	17·81	20·36	22·90	25·45
20	3·14	6·28	9·42	12·57	15·71	18·84	21·99	25·14	28·28	31·42
22	3·80	7·60	11·40	15·21	19·01	22·81	26·61	30·41	34·21	38·01
24	4·52	9·05	13·57	18·10	22·62	27·14	31·67	36·19	40·72	45·24
26	5·31	10·62	15·93	21·24	26·55	31·86	37·17	42·47	47·78	53·09
28	6·16	12·31	18·47	24·63	30·79	36·94	43·10	49·26	55·42	61·58
30	7·07	14·14	21·21	28·27	35·34	42·41	49·48	56·55	63·62	70·69
32	8·04	16·08	24·13	32·17	40·21	48·26	56·30	64·34	72·38	80·42
34	9·08	18·16	27·24	36·32	45·40	54·48	63·56	72·63	81·71	90·79
36	10·18	20·36	30·54	40·72	50·90	61·07	71·25	81·43	91·61	101·79
38	11·34	22·68	34·02	45·36	56·70	68·04	79·38	90·73	102·07	113·41
40	12·57	25·13	37·70	50·26	62·83	75·40	87·96	100·53	113·10	125·66
45	15·90	31·38	47·71	63·62	79·52	95·43	111·33	127·23	143·14	159·04
50	19·64	39·27	58·91	78·54	98·15	117·81	137·45	157·08	176·72	196·35

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TABLE 2

Perimeter of round bars

Dia. of bars (mm.)	*Perimeter in cms*	*Dia. of bars (mm.)*	*Perimeter in cms*
5	1·57	24	7·54
6	1·89	26	8·17
7	2·20	28	8·80
8	2·51	30	9·42
10	3·14	32	10·05
12	3·77	34	10·68
14	4·40	36	11·31
16	5·03	38	11·94
18	5·65	40	12·57
20	6·28	45	14·14
22	6·91	50	15·71

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TABLE 3

Areas of round bars in slabs per metre width for different spacings

Diam. of bars (mm.)	Spacing of bars (cm)												
	6	8	10	12	14	16	18	20	22	24	26	28	30
5	3·27	2·45	1·96	1·64	1·40	1·23	1·09	0·98	0·89	0·82	0·75	0·70	0·65
6	4·71	3·53	2·83	2·63	2·02	1·76	1·57	1·41	1·28	1·18	1·09	1·01	0·94
7	6·41	4·81	3·85	3·20	2·75	2·40	2·14	1·92	1·75	1·60	1·48	1·37	1·28
8	8·37	6·28	5·02	4·19	3·59	3·14	2·79	2·51	2·28	2·09	1·93	1·79	1·67
10	13·09	9·82	7·85	6·54	5·61	4·91	4·36	3·93	3·57	3·27	3·02	2·80	2·62
11	15·84	11·88	9·50	7·92	6·79	5·94	5·28	4·75	4·32	3·96	3·65	3·39	3·17
12	18·85	14·14	11·31	9·42	8·08	7·07	6·28	5·65	5·14	4·71	4·35	4·04	3·77
14	25·65	19·24	15·39	12·83	11·00	9·62	8·55	7·70	7·00	6·41	5·92	5·50	5·13
16	33·51	25·13	20·10	16·76	14·36	12·56	11·18	10·05	9·14	8·38	7·73	7·18	7·70
18	42·41	31·80	25·44	21·20	18·17	15·90	14·14	12·72	11·56	10·60	9·78	9·08	8·48
20	52·36	39·27	31·41	26·18	22·44	19·64	17·45	15·71	14·29	13·09	12·08	11·22	10·47
22		47·51	38·01	31·67	27·15	23·75	21·12	19·00	17·28	15·84	14·62	13·57	12·67
25		61·36	49·08	40·90	35·06	30·68	27·27	24·54	22·31	20·45	18·88	17·53	16·36

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TABLE 4

Shear value of bent up bars

Shear value of one bar bent at 45°

$= 0{\cdot}707\ A_w\ t_w\ kg.$

where, A_w = Area of the bar in $cm.^2$

t_w = Allowable tensile stress in $kg./cm.^2$

$= 1400\ kg./cm.^2$

The following table gives the shear value of bent up bars.

Diameter of bar (mm.)	Shear value (kg.)			
	one bar	two bars	three bars	four bars
10	778	1556	2334	3112
12	1120	2240	3360	4480
14	1524	3048	4572	6096
16	1990	3980	5970	7960
18	2517	5034	7551	10068
20	3111	6222	9333	12444
22	3761	7522	11283	15044
24	4774	8948	13422	17896
25	4860	9720	14580	19440
26	5257	10514	15771	21028
28	6098	12196	18294	24392
30	6998	13996	20994	27992
32	7960	15920	23880	31840
34	8989	17978	26967	35956
36	10080	20160	30240	40320
40	12450	24900	37350	49800

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TABLE 5

Shear value of bent up bars

Shear value of one bar bent at 45°
$= 0{\cdot}707\ A_w\ t_w$ *Newton*

A_w = Area of the bar in $cm.^2$

t_w = Allowable tensile stress in $N/cm.^2$
$= 14000\ N/cm.^2$

The following table gives the shear value of the bent up bars.

Diameter of bar (mm.)	*Shear value*			
	one bar	*two bars*	*three bars*	*four bars*
10	7774	14548	23322	31096
12	11194	22389	33583	44778
14	15237	30474	45710	60947
16	19901	39802	59703	79605
18	25187	50375	75562	100750
20	31095	65191	93287	124382
22	37626	76251	112877	150502
24	44778	89555	134333	179110
25	48528	97174	145760	194347
26	52551	105103	157654	210206
28	60947	121895	182843	243789
30	69965	139930	209895	279860
32	79605	159209	238814	318418
34	89866	179742	269598	359465
36	100750	201499	302249	402998
40	124382	248764	373147	497529

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TABLE 6

Minimum length of embedment of bars in concrete

This table gives the length of embedment required for bars of various diameters for different stresses in the bars. The average bond stress has been taken at 7 kg./cm.² (70 N/cm.²).

Stress in steel N/cm.²	Stress in steel in kg./cm.²	Minimum length of embedment (cms.)								
		10 mm. dia.	12 mm. dia	14 mm. dia	16 mm. dia.	18 mm. dia.	20 mm. dia	22 mm. dia.	24 mm. dia	25 mm. dia.
4000	400	15	18	20	23	2	29	32	35	36
5000	500	19	22	26	29	33	37	40	44	46
6000	600	22	26	30	35	39	43	48	52	54
7000	700	25	30	35	40	45	50	55	60	63
8000	800	29	35	40	46	52	57	63	69	72
9000	900	32	39	45	52	58	65	71	7	81
10,000	1000	36	43	50	57	65	72	79	86	90
11,000	1100	40	47	55	63	71	79	87	94	98
12,000	1200	43	52	60	69	78	86	95	103	107
12,600	1260	45	54	63	72	81	90	99	108	112
13,000	1300	47	56	65	74	84	93	102	112	116
14,000	1400	50	60	70	80	90	100	110	120	125

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TABLE 6

Minimum length of embedment of bars in concrete. This table gives the length of embedment required for bars of various diameters for different stresses in the bars. The average bond stress has been taken at 6 *kg /cm.* (*60 N/cm²*).

Stress in steel in N/cm²	*Stress in steel kg /cm.²*	*Minimum length of embedment (cms.)*								
		10 mm. diam	*12 mm. diam*	*14 mm diam.*	*16 mm. diam.*	*18 mm. diam.*	*20 mm. diam.*	*22 mm. diam.*	*24 mm. diam.*	*25 mm. diam.*
4000	400	17	20	24	27	30	34	37	40	42
5000	500	21	25	30	34	38	42	46	50	53
6000	600	25	30	35	40	45	50	55	60	63
7000	700	30	35	41	47	53	59	65	70	73
8000	800	34	40	47	54	60	67	74	80	84
9000	900	38	45	53	60	68	75	83	90	94
10000	1000	42	50	59	67	75	84	92	100	105
11000	1100	46	55	65	74	83	92	101	110	115
12000	1200	50	60	70	80	90	100	110	120	125
13000	1300	55	65	76	87	98	109	120	130	136
14000	1400	59	70	82	94	105	117	129	140	146

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TABLE 7

Properties of column sections

Dia. or side cms	Round columns			Square columns		
	Area cm.^{2}	Volume per metre cu. metre	Moment of inertia (cm.^{4})	Area (cm.^{2})	Volume per metre (cu. metre)	Moment of inertia (cm.^{4})
30	706·86	0·0707	39770	900	0·0900	67480
32	804·25	0·0804	51480	1024	0·1024	87380
34	907·92	0·0908	65580	1156	0·1156	111400
37	1017·9	0·1018	82450	1296	0·1296	140000
38	1134·1	0·1134	102300	1444	0·1444	173800
40	1256·5	0·1257	125600	1600	0·1600	213600
42	1385·4	0·1385	152700	1764	0·1764	259300
44	1520·5	0·1520	184000	1936	0·1936	312300
46	1661·9	0·1662	219800	2116	0·2116	373100
48	1809·6	0·1810	260600	2304	0·2304	442200
50	1963·5	0·1964	306800	2500	0·2500	520700
52	2123·7	0·2124	358800	2704	0·2704	609200
54	2290·2	0·2290	417400	2916	0·2916	708600
56	2463·0	0·2463	482600	3136	0·3136	819300
58	2642·1	0·2642	555400	3364	0·3364	942800
60	2827·4	0·2827	636000	3600	0·3600	1080000
62	3019·1	0·3019	725400	3844	0·3844	1231000
64	3217·0	0·3217	823300	4096	0·4096	1397000
66	3421·2	0·3421	931300	4356	0·4356	1581000
68	3631·7	0·3632	1050000	4624	0·4624	1781000
70	3848·5	0·3849	1179000	4900	0·4900	2001000
72	4071·5	0·4071	1319000	5184	0·5184	2239000
74	4300·8	0·4301	1472000	5476	0·5476	2500000
76	4536·5	0·4537	1639000	5776	0·5776	2381000
78	4778·4	0·4778	1817000	6084	0·6084	3084000
80	5026·5	0·5027	2010000	6400	0·6400	3414000

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TABLE 8

Properties of short R. C. columns

Size cm. × cm.	Vertical steel			Load on bars in tonnes at 1400 kg/cm²	Safe load on column (tonnes)		
	diam mm.	Number	Area cm²		with $c=60$ kg/cm²	with $c=50$ kg/cm²	with $c=40$ kg/cm²
15 × 15	12	4	4·52	6·33	19·56	17·35	15·15
16 × 16	12	4	4·52	6·33	21·42	18·90	16·39
17 × 17	12	4	4·52	6·33	23·40	20·55	17·71
18 × 18	12	4	4·52	6·33	25·50	22·30	19·11
19 × 19	12	4	4·52	6·33	27·72	24·15	20·59
20 × 20	12	4	4·52	6·33	30·06	26·10	22·15
21 × 21	12	4	4·52	6·33	32·59	28·15	23·79
22 × 22	12	4	4·52	6·33	35·10	30·30	25·51
22 × 22	20	4	12·57	17·60	45·88	41·17	36·46
23 × 23	12	4	4·52	6·33	37·80	32·55	27·31
23 × 23	24	4	18·10	25·34	55·99	50·88	45·78
23 × 23	24	8	36·19	50·67	80·24	75·31	70·38
24 × 24	12	4	4·52	6·33	40·60	34·90	29·19
24 × 24	24	4	18·10	25·34	58·81	53·24	47·66
24 × 24	24	8	36·19	50·67	83·06	77·66	72·26
25 × 25	16	4	8·04	11·26	48·28	42·11	35·94
25 × 25	28	4	24·63	34·48	70·50	64·50	58·49
25 × 25	28	8	49·26	68·96	103·50	97·75	91·99
26 × 26	18	4	10·18	14·25	54·20	47·54	40·83
26 × 26	30	4	28·27	39·58	78·44	71·97	65·49
26 × 26	30	8	56·55	79·17	116·34	110·14	103·94
27 × 26	18	4	10·18	14·25	57·38	50·19	43·00
27 × 27	30	4	28·27	39·58	81·59	74·79	67·59
27 × 27	30	8	56·55	79·17	119·52	112·79	106·07
28 × 28	18	4	10·18	14·25	60·68	52·94	45·20
28 × 28	30	4	28·27	39·58	84·92	77·37	69·81
28 × 28	30	8	56·55	79·17	122·82	115·54	108·27
29 × 29	20	4	12·57	17·60	67·31	59·08	50·74
29 × 29	30	4	28·27	39·58	84·92	77·37	69·81
29 × 29	30	8	56·55	79·17	126·24	118·39	110·55
30 × 30	20	4	12·57	17·60	70·85	61·97	53·10
30 × 30	36	4	40·72	57·01	108·57	99·97	91·38
30 × 30	36	8	81·43	114·00	163·11	154·93	146·74

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TABLE 8 (Contd.)

Properties of Short R C. Columns

Size cm. × cm.	Vertical steel			Load on bars in tonnes at 1400 kg./cm.²	Safe load on column (tonnes)		
	Diameter (mm.)	Number	Area cm.²		with c=60 kg./cm.²	with c=50 kg /cm²	with c=40 kg /cm.²
32×32	20	4	12·57	17 60	78 2ɜ	68 17	58·06
32×32	30	4	40·72	57·01	116·01	106 17	96·54
32×32	36	8	81·43	114·00	170·55	161·13	151·70
34×34	22	4	15·21	21·29	89 74	78 33	66·92
34×34	30	8	56·55	79·17	145·14	134·14	123·15
34×34	40	8	100·53	140·47	202 47	192 18	181·89
36×36	22	4	15·21	21·29	18·14	85·33	72·52
36×36	32	8	64·34	90·38	164·20	151·96	139·65
36×36	40	8	100·53	140 74	212 74	200·51	188·55
38×38	22	4	15·21	21·29	107·02	92·73	78·44
38×38	34	8	72·63	101·68	183·96	170·25	156·33
38×38	45	8	127·23	278·12	257·13	243·96	230·79
40×40	22	4	15·21	21 29	116·38	100·53	84·08
40×40	34	8	72·63	101·68	193·32	178·05	162·77
40×40	45	1	127·23	178·12	266·49	251·76	237 03
45×45	24	4	18·10	25·34	145·35	125·69	105·62
45×45	36	1	81·43	114 00	230·01	211·18	191·74
45×45	45	8	127·23	178·12	291 99	273·01	254 03
50×50	20	8	25·14	35·20	183 69	158·94	135·19
50×50	34	12	108·95	152·54	295·99	272·08	248·17
50×50	45	12	190·85	276·19	405·74	382·65	359·50
55×55	22	8	30·41	42·57	222·25	192·30	162 89
55×55	34	12	108·95	152 53	327·49	298·33	296·17
55×55	45	12	190·85	267·19	437 24	408·89	380·56
60×60	24	8	36·19	50·67	264·50	223·86	193 22
60×60	36	12	122·15	171·01	379·68	344·90	310·15
60×60	45	12	190·85	267·19	471·74	437·65	403·56
65×65	24	8	36·19	50·67	302 00	260·11	218·22
65×65	36	12	122·15	171·01	417·18	376·15	335·12
65×65	45	12	190·85	267·19	509·24	368·90	428·57
70×70	26	8	40·37	59·46	350 93	308·34	253·76
70×70	38	12	136·09	190·53	476·36	428·73	381·09
70×70	45	12	190·85	267·19	549·74	502·65	455·56

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TABLE 9

Moment Coefficients for Continuous Beams

(Equal Spans) (Dead Loads)

M = Coefficient × total load on span × span

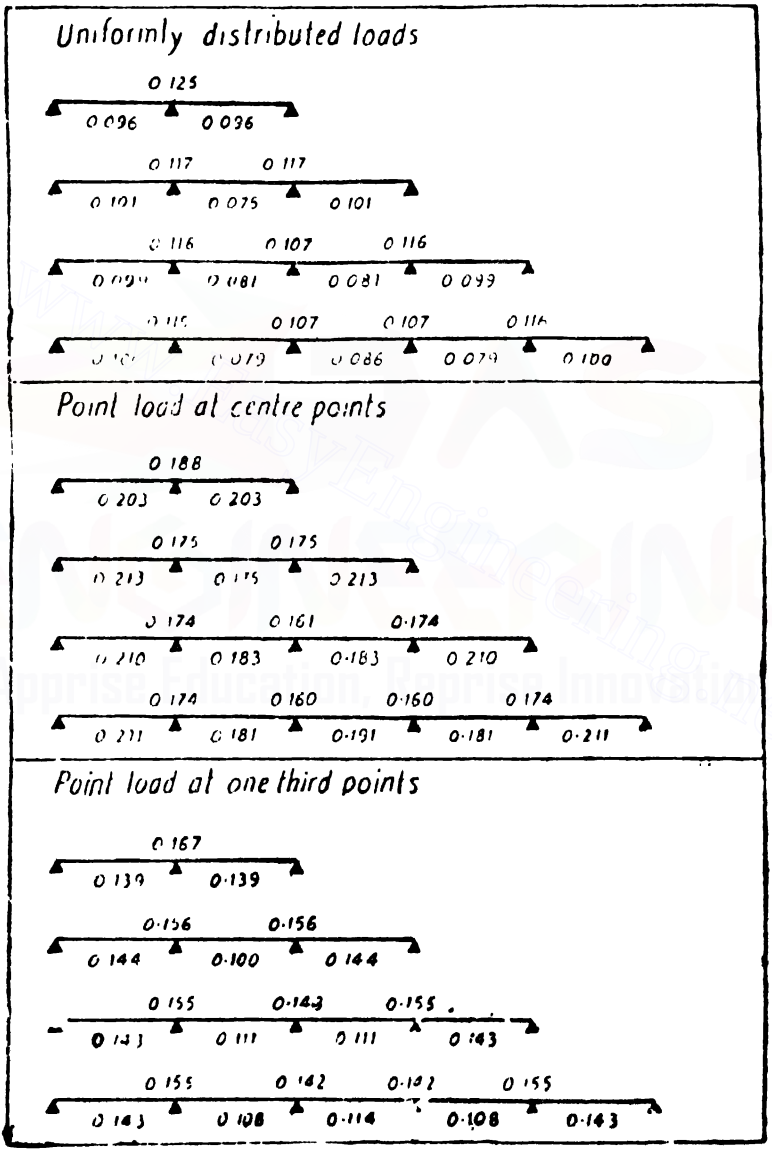

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TABLE 10

Moment Coefficients for Continuous Beams

(Equal Spans) (Live Load)

M = Coefficient × total load on span × span

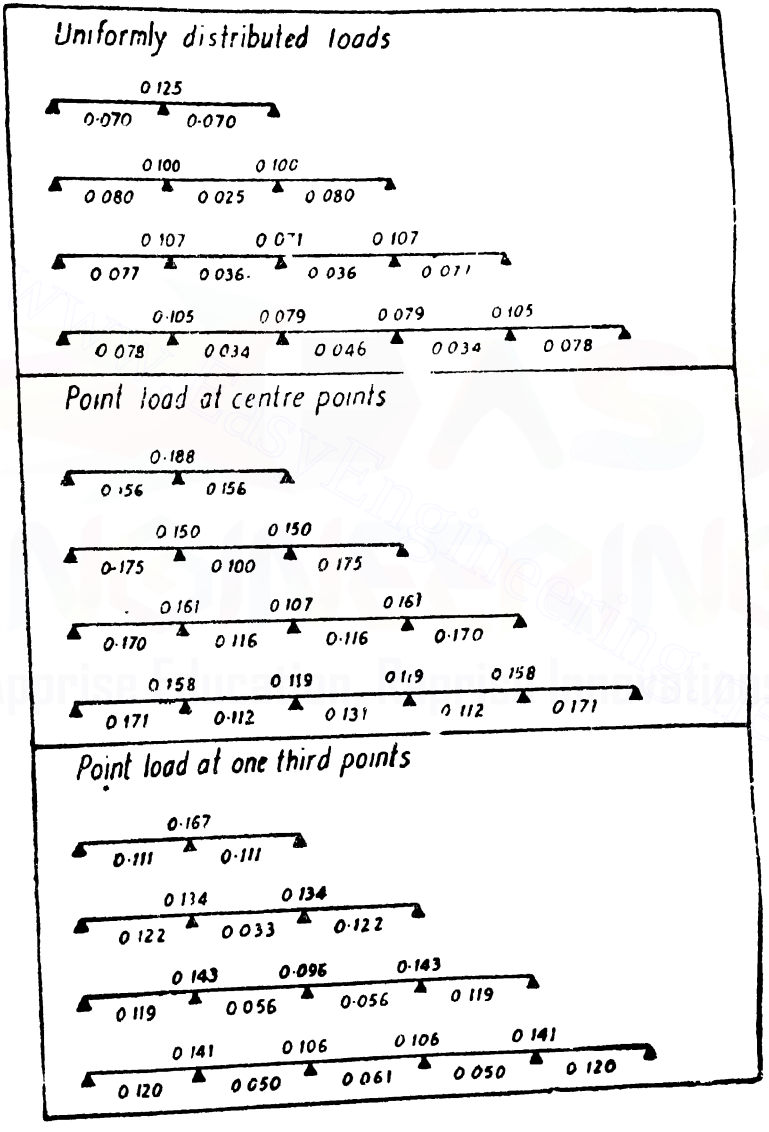

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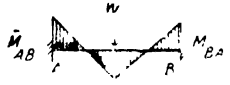
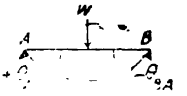
TABLE 11

Fixed end moment $\bar{M}_{AB}$ and $\bar{M}_{BA}$ and end slope angles θ_{AB} and θ_{BA}

Concentrated Loads





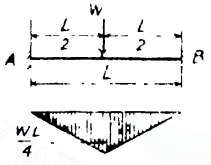
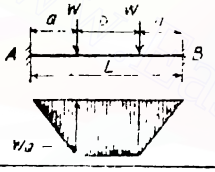
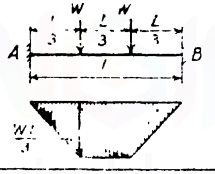
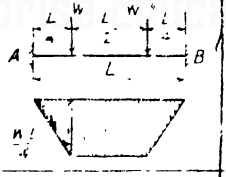
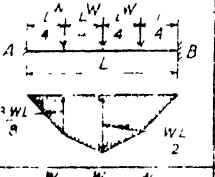
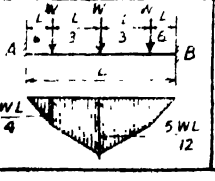
No	LOADING SYSTEMS Free B M DIAGRAMS	$\bar{M}_{AB}$ AND $\bar{M}_{BA}$	θ_{AB} AND θ_{BA}
1		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{WL}{8}$	$\theta_{AB} = -\theta_{BA} = \frac{WL^2}{16EI}$
2		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{Wa(L-a)}{L}$	$\theta_{AB} = -\theta_{BA} = \frac{Wa(L-a)}{2EI}$
3		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{2WL}{9}$	$\theta_{AB} = -\theta_{BA} = \frac{WL^2}{9EI}$
4		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{3WL}{16}$	$\theta_{AB} = -\theta_{BA} = \frac{3WL^2}{32EI}$
5		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{5WL}{16}$	$\theta_{AB} = -\theta_{BA} = \frac{5WL^2}{32EI}$
6		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{19WL}{12}$	$\theta_{AB} = -\theta_{BA} = \frac{19WL^2}{144EI}$

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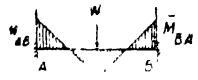
TABLE 11 (Contd.)

Fixed end moments M_{AB} and M_{BA} and end slope angles θ_{AB} and θ_{BA}

Concentrated Loads



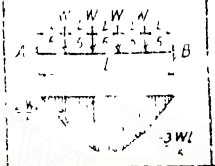
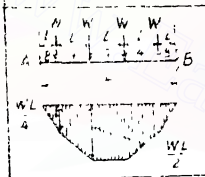
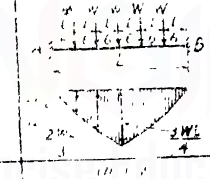
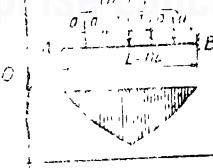
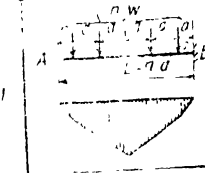
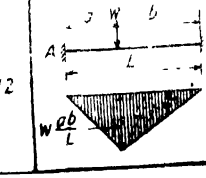
No.	LOADING SYSTEMS Free B.M. DIAGRAMS	$\bar{M}_{AB}$ AND $\bar{M}_{BA}$	θ_{AB} AND θ_{BA}
7		$\bar{M}_{AB} = -\bar{M}_{BA} = \frac{2WL}{5}$	$\theta_{AB} = -\theta_{BA} = \frac{WL^2}{5EI}$
8		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{11WL}{32}$	$\theta_{AB} = -\theta_{BA} = \frac{11WL^2}{64EI}$
9		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{35WL}{72}$	$\theta_{AB} = -\theta_{BA} = \frac{35WL^2}{144EI}$
10		$\bar{M}_{AB} = -\bar{M}_{BA} = \frac{WL}{12}\cdot\frac{n^2-1}{n}$	$\theta_{AB} = -\theta_{BA} = \frac{WL^2}{24EI}\cdot\frac{n^2-1}{n}$
11		$\bar{M}_{AB} = -\bar{M}_{BA} = \frac{WL}{24}\cdot\frac{2n^2+1}{n}$	$\theta_{AB} = -\theta_{BA} = \frac{WL^2}{48EI}\cdot\frac{2n^2+1}{n}$
12		$\bar{M}_{AB} = -\frac{Wab^2}{L^2}$ $\bar{M}_{BA} = +\frac{Wa^2b}{L^2}$	$\theta_{AB} = \frac{Wab}{6LEI}(b+L)$ $-\theta_{BA} = \frac{Wab}{6LEI}(a+L)$

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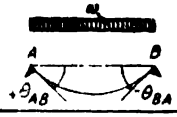
TABLE 12

Fixed end moments $\bar{M}_{AB}$ and $\bar{M}_{BA}$ and end slope angles θ_{AB} and θ_{BA}

Uniformly Distributed Loads



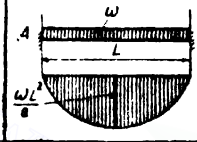
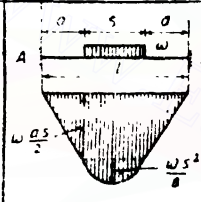
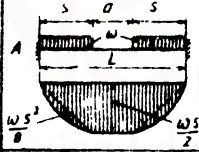
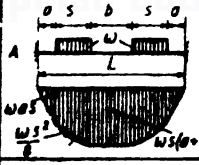
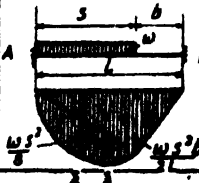
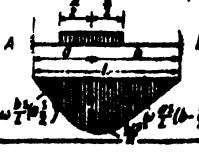
No	LOADING SYSTEMS Free B M DIAGRAM	$\bar{M}_{AB}$ AND $\bar{M}_{BA}$	θ_{AB} AND θ_{BA}
1		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{\omega L^2}{12}$	$\theta_{AB} = -\theta_{BA} = \frac{\omega L^3}{24EI}$
2		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{\omega S}{24L}(3L^2 - S^2)$ For $S = \frac{L}{2}$, $\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{11\omega L^2}{192}$ For $S = \frac{L}{3}$, $\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{13\omega L^2}{324}$ For $S = \frac{L}{4}$, $\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{47\omega L^2}{1536}$	$\theta_{AB} = -\theta_{BA} = \frac{\omega S}{48EI}(3L^2 - S^2)$ For $S = \frac{L}{2}$, $\theta_{AB} = -\theta_{BA} = \frac{11\omega L^3}{384EI}$ For $S = \frac{L}{3}$, $\theta_{AB} = -\theta_{BA} = \frac{13\omega L^2}{648EI}$ For $S = \frac{L}{4}$, $\theta_{AB} = -\theta_{BA} = \frac{47\omega L^3}{3072EI}$
3		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{\omega S^2}{6l}(2L + a)$ For $a = S = \frac{L}{3}$, $\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{7\omega L^2}{162}$	$\theta_{AB} = -\theta_{BA} = \frac{\omega S^2}{12EI}(2L + a)$ For $a = S = \frac{L}{3}$, $\theta_{AB} = -\theta_{BA} = \frac{7\omega L^3}{324EI}$
4		$\bar{M}_{AB} = -\bar{M}_{BA} =$ $= -\frac{\omega S}{12L}\left[3L^2 - 3(b+S)^2 - S^2\right]$ For $a = S = b = \frac{L}{3}$, $\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{31\omega L^2}{750}$	$\theta_{AB} = -\theta_{BA} =$ $= \frac{\omega S}{24EI}\left[3L^2 - 3(b+S)^2 - S^2\right]$ For $a = S = b = \frac{L}{3}$, $\theta_{AB} = -\theta_{BA} = \frac{31\omega L^3}{1500EI}$
5		$\bar{M}_{AB} = -\frac{\omega S^2}{12L^2}\left[2L(3L - 4S) + 3S^2\right]$ $\bar{M}_{BA} = +\frac{\omega S^2}{12L^2}(4L - 3S)$ For $S = b = \frac{L}{2}$, $\bar{M}_{AB} = -\frac{11\omega L^2}{192}$ $\bar{M}_{BA} = \frac{5\omega L^2}{192}$	$\theta_{AB} = \frac{\omega S^2}{24EIL}(2L - S)^2$ $-\theta_{BA} = \frac{\omega S^2}{24EIL}(2L^2 - S^2)$ For $S = b = \frac{L}{2}$, $\theta_{AB} = \frac{9\omega L^3}{384EI}$ $-\theta_{BA} = \frac{7\omega L^3}{384EI}$
6		$\bar{M}_{AB} = -\frac{\omega S}{12L^2}\left[12ab^2 + S^2(L - 3b)\right]$ $\bar{M}_{BA} = +\frac{\omega S}{12L^2}\left[12a^2b + S^2(L - 3a)\right]$	$\theta_{AB} = \frac{\omega bS}{24L}\left[4a(b+L) - S^2\right]\frac{1}{EI}$ $-\theta_{BA} = \frac{\omega aS}{24L}\left[4b(a+L) - S^2\right]\frac{1}{EI}$

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TABLE 13

Fixed end moments M_{AB} and M_{BA} and end slope angles θ_{AB} and θ_{BA}

Triangular Loads, Applied Loads
Parabolic Loads

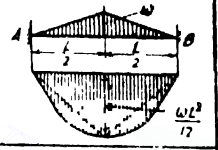
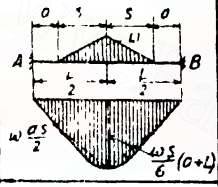
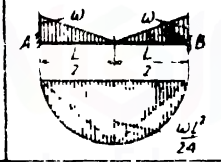
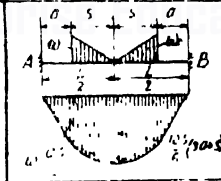
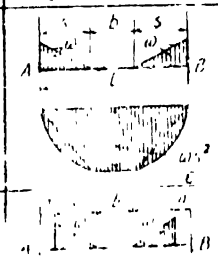
No	LOADING SYSTEMS Free B M DIAGRAM	$\bar{M}_{AB}$ AND $\bar{M}_{BA}$	θ_{AB} AND θ_{BA}
1		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{5wL^2}{96}$	$\theta_{AB} = -\theta_{BA} = \frac{5wL^3}{192EI}$
2		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{ws}{24L}(3L^2 - 2s^2)$ For $a = s = \frac{L}{4}$, $\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{23wL^2}{768}$	$\theta_{AB} = -\theta_{BA} = \frac{ws}{48EI}(3L^2 - 2s^2)$ For $a = s = \frac{L}{4}$, $\theta_{AB} = -\theta_{BA} = \frac{23wL^3}{1536EI}$
3		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{wL^2}{32}$	$\theta_{AB} = -\theta_{BA} = \frac{wL^3}{64EI}$
4		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{ws}{8L}(L^2 - 2s^2)$ For $a = s = \frac{L}{4}$, $\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{7wL^2}{256}$	$\theta_{AB} = -\theta_{BA} = \frac{ws}{16EI}(L^2 - 2s^2)$ For $a = s = \frac{L}{4}$, $\theta_{AB} = -\theta_{BA} = \frac{7wL^3}{512EI}$
5		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{ws^2}{12L}(2L - s)$ For $s = b = \frac{L}{3}$, $\bar{M}_{AB} = -\bar{M}_{BA} = \frac{5wL^2}{324}$ $\bar{M}_{AB} = \frac{ws}{12L}[a(L-a) + s(2b+3s)]$ For $a = b = s = \frac{L}{4}$, $\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{29wL^2}{1500}$	$\theta_{AB} = -\theta_{BA} = \frac{ws^2}{24EI}(2L - s)$ For $s = b = \frac{L}{3}$, $\theta_{AB} = -\theta_{BA} = \frac{wL^3}{648EI}$ $\theta_{AB} = \theta_{BA} = \frac{ws}{24EI}[a(L-a) + s(2b+3s)]$ For $a = b = s = \frac{L}{4}$, $\theta_{AB} = -\theta_{BA} = \frac{29wL^3}{3000EI}$

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TABLE 13 (Contd.)

Fixed end moments $\bar{M}_{AB}$ and $\bar{M}_{BA}$ and end slope angles θ_{AB} and θ_{BA}

Angular Loads
Applied Moments
Parabolic Loads

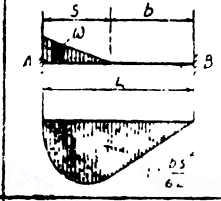
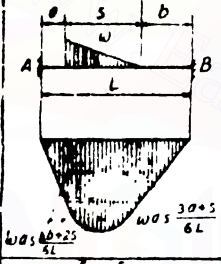
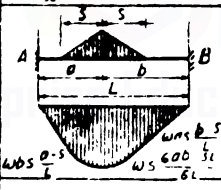
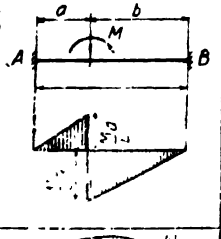
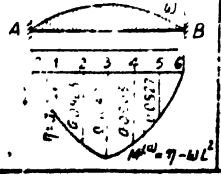
No.	LOADING SYSTEMS Free B.M. DIAGRAM	$\bar{M}_{AB}$ AND $\bar{M}_{BA}$	θ_{AB} AND θ_{BA}
7		$\bar{M}_{AB} = -\frac{wS^2}{60L^2}(10bL + 3S^2)$ $\bar{M}_{BA} = +\frac{wS^3}{60L^2}(5b + 2S)$ For $S = L$, $\frac{L}{2}$, $M_{AB} = -\frac{3wL^2}{960}$ $\bar{M}_{BA} = +\frac{7wL^2}{960}$	$\theta_{AB} = \frac{wS^2}{360L}\left[5b(4L+S) - 8S^2\right]\frac{1}{EI}$ $-\theta_{BA} = \frac{wS^2}{360L}\left[10b(L+S) - 7S^2\right]\frac{1}{EI}$ For $S = b = \frac{L}{2}$, $\theta_{AB} = \frac{53wL^3}{5760EI}$ $-\theta_{BA} = \frac{37wL^3}{5760EI}$
8		$\bar{M}_{AB} = -\frac{wS}{60L^2}\left[10b^2(3a+S) + S^2(15a+10b+3S) + 40abS\right]$ $\bar{M}_{AB} = +\frac{wS}{60L^2}\left[10a^2(3b+2S) + S^2(10a+5b+2S) + 20abS\right]$ For $a = S$, $b = \frac{L}{3}$, $\bar{M}_{AB} = -\frac{wL^2}{45}$, $\bar{M}_{BA} = +\frac{29wL^2}{1620}$	$\theta_{AB} = \frac{wS}{360L}\left[10b^2(3b+2S) + 20b^2(3a+S) + S^2(40a + 25b + 8S) + 100abS\right]\frac{1}{EI}$ $-\theta_{BA} = \frac{wS}{360L}\left[20a^2(3b+2S) + 10b^2(3a+S) + S^2(35a + 20b + 7S) + 80abS\right]\frac{1}{EI}$ For $a = S = b = \frac{L}{3}$ $\theta_{AB} = \frac{101wL^3}{9720EI}$, $-\theta_{BA} = \frac{47wL^3}{4860EI}$
9		$M_{AB} = -\frac{wS}{6L^2}\left[6ab^2 + S^2(a-2b)\right]$ $M_{BA} = +\frac{wS}{6L^2}\left[6a^2b + S^2(b-2a)\right]$	$\theta_{AB} = \frac{wbS}{12L}\left[2a(b+L) - S^2\right]\frac{1}{EI}$ $-\theta_{BA} = \frac{waS}{12L}\left[2b(a+L) - S^2\right]\frac{1}{EI}$
10		$M_{AB} = M\frac{b}{L}\left(2 - \frac{3b}{L}\right)$ $\bar{M}_{BA} = +M\frac{a}{L}\left(2 - \frac{3a}{L}\right)$ For $a = 0$, $M_{AB} = -M$, $M_{BA} = 0$ For $a = \frac{L}{2}$, $M_{AB} = M_{BA} = +\frac{M}{4}$ For $a = L$, $M_{AB} = 0$, $\bar{M}_{BA} = +M$	$\theta_{AB} = M\frac{L}{6}\left[1 - \frac{3b}{L^2}\right]\frac{1}{EI}$ $-\theta_{BA} = M\frac{L}{6}\left[1 - \frac{3a}{L^2}\right]\frac{1}{EI}$ For $a = 0$, $\theta_{AB} = M\frac{L}{3EI}$, $\theta_{BA} = M\frac{L}{6EI}$ For $a = \frac{L}{2}$, $\theta_{AB} = \theta_{BA} = \frac{ML}{24EI}$ For $a = L$, $\theta_{AB} = -M\frac{L}{6EI}$ $\theta_{BA} = -M\frac{L}{3EI}$
11		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{wL^2}{15}$	$\theta_{AB} = -\theta_{BA} = \frac{wL^3}{30EI}$

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TABLE 13 (Contd.)

Fixed end moments $\bar{M}_{AB}$ and $\bar{M}_{BA}$ and end slope angles θ_{AB} and θ_{BA}

Triangular Loads
Applied Moments
Parabolic Loads

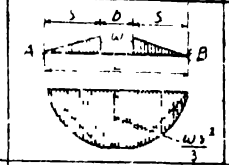
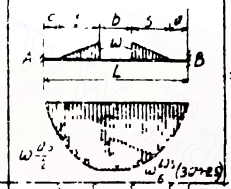
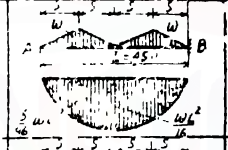
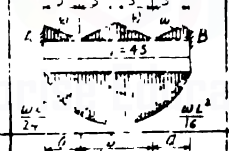
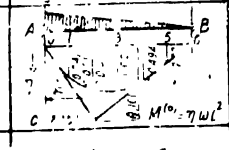
No	LOADING SYSTEMS Free B M DIAGRAM	$\bar{M}_{AB}$ AND $\bar{M}_{BA}$	θ_{AB} AND θ_{BA}
12		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{ws^2}{12L}(4L-3s)$ For $s = b = \frac{L}{3}$, $\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{wL^2}{36}$	$\theta_{AB} = -\theta_{BA} = \frac{ws^2}{24EI}(4L-3s)$ For $s = b = \frac{L}{3}$, $\theta_{AB} = -\theta_{BA} = \frac{wL^3}{12EI}$
13		$\bar{M}_{AB} = -\bar{M}_{BA} =$ $= +\frac{ws}{12L}\left[6a(L-a) - s(4b+5s)\right]$ For $a = s = b = \frac{L}{5}$, $\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{11wL^2}{500}$	$\theta_{AB} = -\theta_{BA} =$ $= \frac{ws}{24EI}\left[6a(L-a) + s(4b+5s)\right]$ For $a = b = \frac{L}{5}$, $\theta_{AB} = -\theta_{BA} = \frac{11wL^3}{9000EI}$
14		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{17wL^2}{384}$	$\theta_{AB} = -\theta_{BA} = \frac{17wL^3}{768EI}$
15		$\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{5wL^2}{128}$	$\theta_{AB} = -\theta_{BA} = \frac{5wL^3}{256EI}$
16		$\bar{M}_{AB} = -\bar{M}_{BA} =$ $= -\frac{w}{12L}\left[L^3 - a^2(2L-a)\right]$ For $a = b = \frac{L}{3}$, $\bar{M}_{AB} = -\bar{M}_{BA} = -\frac{11wL^2}{}$	$\theta_{AB} = \theta_{BA} = \frac{w}{24EI}\left[L^3 - a^2(2L-a)\right]$ For $a = b = \frac{L}{3}$, $\theta_{AB} = -\theta_{BA} = \frac{11wL^3}{324EI}$
17		$\bar{M}_{AB} = \frac{wL^2}{20}$ $\bar{M}_{BA} = +\frac{wL^2}{30}$	$\theta_{AB} = \frac{wL^3}{45EI}$ $-\theta_{BA} = \frac{7wL^3}{360EI}$
18		$\bar{M}_{AB} = \frac{ws^2}{30L^2}\left[10a^2 + 5(5a+s)\right]$ $\bar{M}_{BA} = \frac{ws^3}{20L^2}(5a+s)$ For $s = a = \frac{L}{2}$, $\bar{M}_{AB} = -\frac{wL^2}{30}$ $\bar{M}_{BA} = +\frac{wL^2}{160}$	$\theta_{AB} = \frac{ws^2}{360L}\left[40a^2 - 7s(5a+s)\right]$ $\theta_{BA} = \frac{ws^2}{180L}\left[10a^2 + 45(5a+s)\right]$ For $s = a = \frac{L}{2}$, $\theta_{AB} = \frac{41wL^3}{2880EI}$ $\theta_{BA} = \frac{17wL^3}{1440EI}$

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TABLE 14

Fixed end moments $\bar{M}^{\circ}_{AB}$ for hinged members

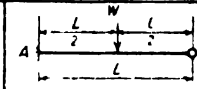
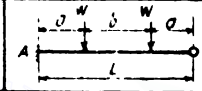
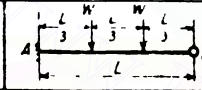
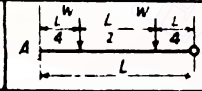
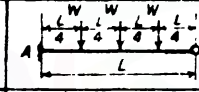
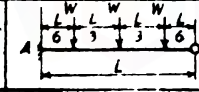
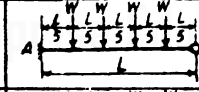
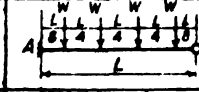
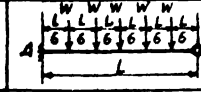
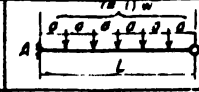
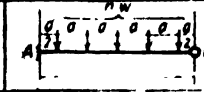
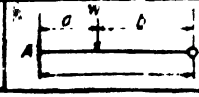
No	LOADING SYSTEMS Free B M DIAGRAM	$\bar{M}^{\circ}_{AB}$
1		$\bar{M}^{\circ}_{AB} = -\frac{3WL}{16}$
2		$\bar{M}^{\circ}_{AB} = -\frac{3Wa(L-a)}{2L}$
3		$\bar{M}^{\circ}_{AB} = -\frac{WL}{3}$
4		$\bar{M}^{\circ}_{AB} = -\frac{9WL}{32}$
5		$\bar{M}^{\circ}_{AB} = -\frac{15WL}{32}$
6		$\bar{M}^{\circ}_{AB} = -\frac{19WL}{48}$
7		$\bar{M}^{\circ}_{AB} = -\frac{3WL}{5}$
8		$\bar{M}^{\circ}_{AB} = -\frac{33WL}{64}$
9		$\bar{M}^{\circ}_{AB} = -\frac{35WL}{48}$
10		$\bar{M}^{\circ}_{AB}$ $\frac{WL}{8}$ $\frac{n^2-1}{n}$
11		$\bar{M}^{\circ}_{AB} = \frac{WL}{16}$ $\frac{2n^2+1}{n}$
12		$\bar{M}^{\circ}_{AB} = \frac{Wab}{2L^2}(b+L)$

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TABLE 15

Fixed end moments M°_{AB} for hinged members

$\overline{M}^{\circ}_{AB}$ Uniformly Distributed Loads

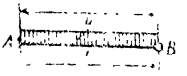
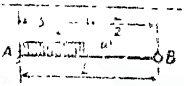
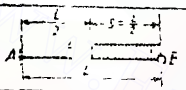
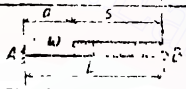
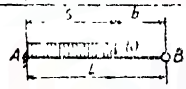
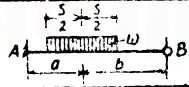
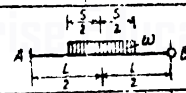
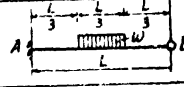
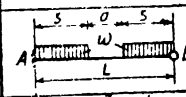
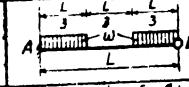
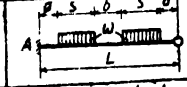
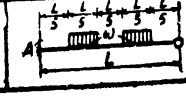
No	LOADING SYSTEMS Free Body DIAGRAM	$\overline{M}^{\circ}_{AB}$
1		$\overline{M}^{\circ}_{AB} = -\frac{w L^2}{12}$
2		$\overline{M}^{\circ}_{AB} = -\frac{9wL^2}{128}$
3		$\overline{M}^{\circ}_{AB} = -\frac{7wL^2}{128}$
4		$\overline{M}^{\circ}_{AB} = -\frac{w s^2}{2L^2}(2L^2 - s^2)$
5		$\overline{M}^{\circ}_{AB} = -\frac{w s^3}{8L^2}(4b + s^2)$
6		$\overline{M}^{\circ}_{AB} = -\frac{wbs}{8L^2}(2a^2 + 8ab - s^2)$
7		$\overline{M}^{\circ}_{AB} = -\frac{ws}{16L}(3L^2 - s^2)$
8		$\overline{M}^{\circ}_{AB} = -\frac{13wL^2}{216}$
9		$\overline{M}^{\circ}_{AB} = -\frac{ws^2}{4L}(2L + a)$
10		$\overline{M}^{\circ}_{AB} = -\frac{7wL^2}{108}$
11		$\overline{M}^{\circ}_{AB} = -\frac{ws}{8L}\left[3L^2 - 3(b+s)^2 - s^2\right]$
12		$\overline{M}^{\circ}_{AB} = -\frac{91wL^2}{500}$

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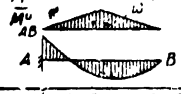
TABLE 16

Fixed end moments $\bar{M}^{o}_{AB}$ for hinged members

Triangular Loads
Applied Moments, Parabolic Loads



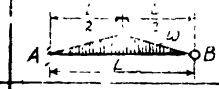
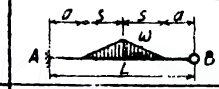
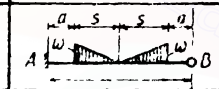
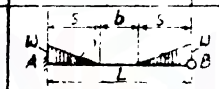
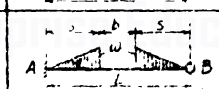
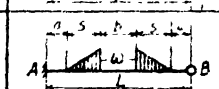
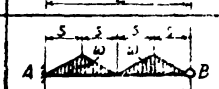
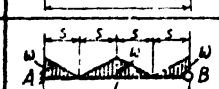
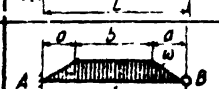
No	LOADING SYSTEMS Free B M DIAGRAM	$\bar{M}^{o}_{AB}$
1		$\bar{M}^{o}_{AB} = -\frac{5\omega L^2}{64}$
2		$\bar{M}^{o}_{AB} = -\frac{\omega s}{16L}(3L^2 - 2s^2)$, For $a = s = \frac{L}{4}$, $\bar{M}^{o}_{AB} = -\frac{23\omega L^2}{256}$
3		$\bar{M}^{o}_{AB} = -\frac{3\omega L^2}{64}$
4		$\bar{M}^{o}_{AB} = -\frac{3\omega s}{16L}(L^2 - 2s^2)$, For $a = s = \frac{L}{4}$, $\bar{M}^{o}_{AB} = -\frac{41\omega L^2}{512}$
5		For $s = b = \frac{L}{4}$, For $c = \frac{L}{4}$, $\bar{M}^{o}_{AB} = \frac{\omega s^2}{8L}(2L - s)$, $\bar{M}^{o}_{AB} = -\frac{5\omega L^2}{216}$, $\bar{M}^{o}_{AB} = -\frac{73\omega L^2}{512}$
6		For $a = b = s = \frac{L}{5}$, $\bar{M}^{o}_{AB} = -\frac{\omega s}{8L}\left[6a(L-a) + s(2b+3s)\right]$, $\bar{M}^{o}_{AB} = -\frac{53\omega L^2}{1000}$
7		$\bar{M}^{o}_{AB} = -\frac{\omega s^2}{8L}(4L - 3s)$, For $s = b =$, $\bar{M}^{o}_{AB} = -\frac{\omega L^2}{24}$
8		For $a = b = s = \frac{L}{5}$, $\bar{M}^{o}_{AB} = -\frac{\omega s^2}{8L}\left[5a(L-a) + s(4b + 5s)\right]$, $\bar{M}^{o}_{AB} = -\frac{33\omega L^2}{1000}$
9		$\bar{M}^{o}_{AB} = -\frac{17\omega L^2}{256}$
10		$\bar{M}^{o}_{AB} = -\frac{15\omega L^2}{256}$
11		For $a = b = \frac{L}{3}$, $\bar{M}^{o}_{AB} = -\frac{\omega}{8L}\left[L^3 - a^2(2L - a)\right]$, $\bar{M}^{o}_{AB} = \frac{33\omega L^2}{324}$
12		$\bar{M}^{o}_{AB} = -\frac{\omega L^2}{15}$

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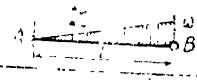
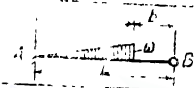
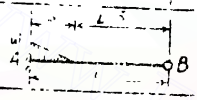
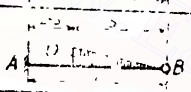
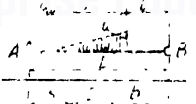
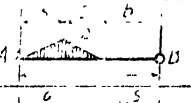
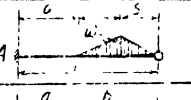
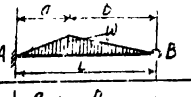
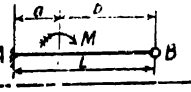
TABLE 16 (Contd.)

Fixed end moments $\overline{M}^{o}_{AB}$ for hinged members

Triangular Loads
Applied Moments
Parabolic Loads

No	LOADING SYSTEMS [illegible] B M DIAGRAM	$\overline{M}^{o}_{AB}$
13		$\overline{M}^{o}_{AB} = -\frac{7wL^2}{120}$
14		For $s = b = \frac{L}{2}$, $\overline{M}^{o}_{AB} = -\frac{41wL^2}{960}$
15		For $s = \frac{L}{2}$, $\overline{M}^{o}_{AB} = -\frac{53wL^2}{1920}$
16		$\overline{M}^{o}_{AB} = -\frac{ws^2}{30L^2}(3L^2 - 3s^2)$ For $s = \frac{L}{2}$, $\overline{M}^{o}_{AB} = -\frac{17wL^2}{480}$
17		$\overline{M}^{o}_{AB} = -\frac{ws^2}{120L^2}(10L^2 - 3s^2)$. For $s = \frac{L}{2}$, $\overline{M}^{o}_{AB} = -\frac{37wL^2}{1920}$
18		$\overline{M}^{o}_{AB} = -\frac{ws}{120L^2}\left[10s^2(3b+2s) + 20a(3s^2+2s^2) + 5bs(4b+5s) + 4s(25ab+2s^2)\right]$ For $a = s = b = \frac{L}{3}$, $\overline{M}^{o}_{AB} = -\frac{101wL^2}{3240}$
19		$\overline{M}^{o}_{AB} = -\frac{ws}{120L^2}\left[10s^2(3b+s) - 20a(3b^2+s^2) + 5bs(8b+7s) + s(85ab+7s^2)\right]$ For $a = s = b = \frac{L}{3}$, $\overline{M}^{o}_{AB} = -\frac{47wL^2}{1620}$
20		For $b = \frac{L}{2}$, $\overline{M}^{o}_{AB} = -\frac{ws^2}{4L^2}(4L^2 - 7Ls + 3s^2)$, $\overline{M}^{o}_{AB} = -\frac{39wL^2}{1024}$
21		$\overline{M}^{o}_{AB} = -\frac{ws^2}{4L^2}(2L^2 - 3s^2)$. For $a = \frac{L}{2}$, $\overline{M}^{o}_{AB} = -\frac{25wL^2}{1024}$
22		$\overline{M}^{o}_{AB} = -\frac{w}{120L}(b+L)(7L^2 - 3b^2)$
23		$\overline{M}^{o}_{AB} = +\frac{M}{2}\left[1 - \frac{3b^2}{L^2}\right]$ For $b = \frac{L}{2}$, $\overline{M}^{o}_{AB} = +\frac{M}{8}$ For $b = 0$, $\overline{M}^{o}_{AB} = +\frac{M}{2}$
24		$\overline{M}^{o}_{AB} = -\frac{wL^2}{10}$

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LOGARITHMS AND ANTILOGARITHMS

TABLE 17

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170						5 9 13	17 21 25	30 34 38
						0212	0253	0294	0334	0374	4 8 12	16 20 24	28 32 36
11	0414	0453	0492	0531	0569						4 8 12	16 20 23	27 31 35
						0607	0645	0682	0719	0755	4 7 11	15 18 22	26 29 33
12	0792	0828	0864	0899	0934						3 7 11	14 18 21	25 28 32
						0969	1004	1038	1072	1106	3 7 10	14 17 20	24 27 31
13	1139	1173	1206	1239	1271						3 6 10	13 16 19	23 26 29
						1303	1335	1367	1399	1430	3 7 10	13 16 19	22 25 29
14	1461	1492	1523	1553	1584						3 6 9	12 15 19	22 25 28
						1614	1644	1673	1703	1732	3 6 9	12 14 17	20 23 26
15	1761	1790	1818	1847	1875						3 6 9	11 14 17	20 23 26
						1903	1931	1959	1987	2014	3 6 8	11 14 17	19 22 25
16	2041	2068	2095	2122	2148						3 6 8	11 14 16	19 22 24
						2175	2201	2227	2253	2279	3 5 8	10 13 16	18 21 23
17	2304	2330	2355	2380	2405						3 5 8	10 13 15	18 20 23
						2430	2455	2480	2504	2529	3 5 8	10 12 15	17 20 22
18	2553	2577	2601	2625	2648						2 5 7	9 12 14	17 19 21
						2672	2695	2718	2742	2765	2 4 7	9 11 14	16 18 21
	2788	2810	2833	2856	2878						2 4 7	9 11 13	16 18 20
						2900	2923	2945	2967	2989	2 4 6	8 11 13	15 17 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8

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TABLE 17 (Contd.)

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	123	456	789
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 4 4

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TABLE 18

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	123	456	789
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1	1 1 1	2 2 2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 2	2 2 2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1	1 2 2	2 3 3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1	1 2 2	2 3 3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1	1 2 2	2 3 3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1	1 2 2	3 3 3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1	2 2 2	3 3 3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1	2 2 2	3 3 3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0 1 1	2 2 3	3 4 4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1 1 2	2 3 3	4 4 5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1 1 2	2 3 3	4 4 5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 2 2	3 3 4	4 5 6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 2 2	3 3 4	4 5 6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

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TABLE 18 (Contd.)

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
·60	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1 1 2	3 4 4	5 6 7
·61	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1 2 2	3 4 5	5 6 7
·62	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1 2 2	3 4 5	5 6 7
·63	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1 2 2	3 4 5	5 6 7
·64	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1 2 2	3 4 5	6 6 7
·65	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1 2 2	3 4 5	6 6 7
·66	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1 2 3	3 4 5	6 7 7
·67	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1 2 3	3 4 5	6 7 8
·68	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1 2 3	4 4 5	6 7 8
·69	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1 2 3	4 5 5	6 7 8
·70	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1 2 3	4 5 6	6 7 8
·71	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1 2 3	4 5 6	7 8 9
·72	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1 2 3	4 5 6	7 8 9
·73	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1 2 3	4 5 6	7 8 9
·74	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1 2 3	4 5 6	7 8 9
·75	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1 2 3	4 5 6	7 8 9
·76	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1 2 3	4 5 6	7 8 9
·77	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1 2 3	4 5 7	8 9 10
·78	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1 2 3	4 6 7	8 9 10
·79	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1 2 3	5 6 7	8 9 10
·80	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1 2 4	5 6 7	8 9 11
·81	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1 2 4	5 6 7	8 10 11
·82	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1 2 4	5 6 7	9 10 11
·83	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1 3 4	5 6 8	9 10 11
·84	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1 3 4	5 6 8	9 10 12
·85	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1 3 4	5 7 8	9 10 12
·86	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1 3 4	5 7 8	9 11 12
·87	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1 3 4	5 7 8	10 11 12
·88	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1 3 4	6 7 8	10 11 13
·89	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1 3 4	6 7 9	10 11 13
·90	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1 3 4	6 7 9	10 12 13
·91	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2 3 5	6 8 9	11 12 14
·92	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2 3 5	6 8 9	11 12 14
·93	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2 3 5	6 8 9	11 13 14
·94	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2 3 5	6 8 10	11 13 15
·95	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2 3 5	7 8 10	12 13 15
·96	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2 3 5	7 8 10	12 13 15
·97	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2 3 5	7 9 10	12 14 16
·98	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2 4 5	7 9 11	12 14 16
·99	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2 4 5	7 9 11	13 14 16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2 4 6	7 9 11	13 15 17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2 4 6	8 9 11	13 15 17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2 4 6	8 10 12	14 15 17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2 4 6	8 10 12	14 16 18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2 4 6	8 10 12	14 16 18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2 4 6	8 10 12	15 17 19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2 4 6	8 11 13	15 17 19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2 4 7	9 11 13	15 17 20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2 4 7	9 11 13	16 18 20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2 5 7	9 11 14	16 18 20

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TABLE 19

NATURAL SINES

Deg.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differs. 1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0193	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	·2588	2605	2622	2639	2655	2672	2689	2706	2723	2740	3	6	8	11	14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	·2923	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	·3250	3272	[illegible]	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

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TABLE 19 (Contd.)

NATURAL SINES

Deg	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
45	·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	·7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	8
54	·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	4	6	8
59	·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	5	7
60	·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	5	7
61	·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	2	3
78	·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	·9962	9963	9965	9966	9965	9969	9971	9972	9973	9974	0	0	1	1	1
86	·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	·9998	9999	9999	9999	1·000	1·000	1·000	1·000	1·000	1·000	0	0	0	0	0
90	1·000														

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TABLE 20

NATURAL COSINES

(Numbers in difference columns to be subtracted, not added.)

Degrees	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1	2	3	4	5
0	1·000	1·000	1·000	1·000	1·000	1·000	·9999	·9999	·9999	·9998	0	0	0	0	0
1	·9998	·9998	·9998	·9997	·9997	·9997	·9996	·9996	·9995	·9995	0	0	0	0	0
2	·9994	·9993	·9992	·9992	·9991	·9990	·9990	·9989	·9988	·9987	0	0	0	1	1
3	·9986	·9985	·9984	·9983	·9982	·9981	·9980	·9979	·9978	·9977	0	0	1	1	1
4	·9976	·9974	·9973	·9972	·9971	·9969	·9968	·9966	·9965	·9963	0	0	1	1	1
5	·9962	·9960	·9959	·9957	·9955	·9954	·9952	·9951	·9949	·9947	0	1	1	1	2
6	·9945	·9943	·9942	·9940	·9938	·9936	·9934	·9932	·9930	·9928	0	1	1	1	2
7	·9925	·9923	·9921	·9919	·9917	·9914	·9912	·9910	·9907	·9905	0	1	1	2	2
8	·9903	·9900	·9898	·9895	·9893	·9890	·9888	·9885	·9882	·9880	0	1	1	2	2
9	·9877	·9874	·9871	·9869	·9866	·9863	·9860	·9857	·9854	·9851	0	1	1	2	2
10	·9848	·9845	·9842	·9839	·9836	·9833	·9829	·9826	·9823	·9820	1	1	2	2	3
11	·9816	·9813	·9810	·9806	·9803	·9799	·9796	·9792	·9789	·9785	1	1	2	2	3
12	·9781	·9778	·9774	·9771	·9767	·9763	·9759	·9755	·9751	·9748	1	1	2	3	3
13	·9744	·9740	·9736	·9732	·9728	·9724	·9720	·9716	·9712	·9707	1	1	2	3	3
14	·9703	·9699	·9694	·9690	·9686	·9681	·9677	·9673	·9668	·9664	1	1	2	3	4
15	·9659	·9655	·9650	·9646	·9641	·9636	·9632	·9627	·9622	·9617	1	2	2	3	4
16	·9613	·9608	·9603	·9598	·9593	·9588	·9583	·9578	·9573	·9568	1	2	2	3	4
17	·9563	·9558	·9553	·9548	·9542	·9537	·9532	·9527	·9521	·9516	1	2	3	3	4
18	·9511	·9505	·9500	·9494	·9489	·9483	·9478	·9472	·9466	·9461	1	2	3	4	5
19	·9455	·9449	·9444	·9438	·9432	·9426	·9421	·9415	·9409	·9403	1	2	3	4	5
20	·9397	·9391	·9385	·9379	·9373	·9367	·9361	·9354	·9348	·9342	1	2	3	4	5
21	·9336	·9330	·9323	·9317	·9311	·9304	·9298	·9291	·9285	·9278	1	2	3	4	5
22	·9272	·9265	·9259	·9252	·9245	·9239	·9232	·9225	·9219	·9212	1	2	3	4	6
23	·9205	·9198	·9191	·9184	·9178	·9171	·9164	·9157	·9150	·9143	1	2	3	5	6
24	·9135	·9128	·9121	·9114	·9107	·9100	·9092	·9085	·9078	·9070	1	2	4	5	6
25	·9063	·9056	·9048	·9041	·9033	·9026	·9018	·9011	·9003	·8996	1	3	4	5	6
26	·8988	·8980	·8973	·8965	·8957	·8949	·8942	·8934	·8926	·8918	1	3	4	5	7
27	·8910	·8902	·8894	·8886	·8878	·8870	·8862	·8854	·8846	·8838	1	3	4	5	7
28	·8829	·8821	·8813	·8805	·8796	·8788	·8780	·8771	·8763	·8755	1	3	4	6	7
29	·8746	·8738	·8729	·8721	·8712	·8704	·8695	·8686	·8678	·8669	1	3	4	6	7
30	·8660	·8652	·8643	·8634	·8625	·8616	·8607	·8599	·8590	·8581	1	3	4	6	7
31	·8572	·8563	·8554	·8545	·8536	·8526	·8517	·8508	·8499	·8490	2	3	5	6	8
32	·8480	·8471	·8462	·8453	·8443	·8434	·8425	·8415	·8406	·8396	2	3	5	6	8
33	·8387	·8377	·8368	·8358	·8348	·8339	·8329	·8320	·8310	·8300	2	3	5	6	8
34	·8290	·8281	·8271	·8261	·8251	·8241	·8231	·8221	·8211	·8202	2	3	5	7	8
35	·8192	·8181	·8171	·8161	·8151	·8141	·8131	·8121	·8111	·8100	2	3	5	7	8
36	·8090	·8080	·8070	·8059	·8049	·8039	·8028	·8018	·8007	·7997	2	3	5	7	9
37	·7986	·7976	·7965	·7955	·7944	·7934	·7923	·7912	·7902	·7891	2	4	5	7	9
38	·7880	·7869	·7859	·7848	·7837	·7826	·7815	·7804	·7793	·7782	2	4	5	7	9
39	·7771	·7760	·7749	·7738	·7727	·7716	·7705	·7694	·7683	·7672	2	4	6	7	9
40	·7660	·7649	·7638	·7627	·7615	·7604	·7593	·7581	·7570	·7559	2	4	6	8	9
41	·7547	·7536	·7524	·7513	·7501	·7490	·7478	·7466	·7455	·7443	2	4	6	8	10
42	·7431	·7420	·7408	·7396	·7385	·7373	·7361	·7349	·7337	·7325	2	4	6	8	10
43	·7314	·7302	·7290	·7278	·7266	·7254	·7242	·7230	·7218	·7206	2	4	6	8	10
44	·7193	·7181	·7169	·7157	·7145	·7133	·7120	·7108	·7096	·7083	2	4	6	8	10

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TABLE 20 (Contd.)

NATURAL COSINES

[Numbers in different columns to be subtracted, added.]

Deg	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	M. Diff. 1	2	3
45	·7071	7059	7046	7034	[illegible]	[illegible]	6987	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
46	·6947	6934	6921	6909	[illegible]	[illegible]	6871	6858	6845	[illegible]	[illegible]	[illegible]	3
47	·6820	6807	6794	6782	6769	[illegible]	6743	6730	6717	[illegible]	[illegible]	[illegible]	[illegible]
48	·6691	6678	6665	6652	[illegible]	[illegible]	[illegible]	[illegible]	6587	[illegible]	[illegible]	[illegible]	[illegible]
49	·6561	6547	6536	[illegible]	[illegible]	[illegible]	[illegible]	6468	6455	6441	2	4	7
50	·6428	6414	[illegible]	[illegible]	[illegible]	[illegible]	6347	6333	6320	6307	2	4	7
51	·6293	6280	6266	6252	6239	[illegible]	6211	6198	6184	[illegible]	2	5	7
52	·6157	6143	6129	6115	6101	[illegible]	6074	6060	6046	6032	2	5	7
53	·6018	6004	[illegible]	5976	5962	[illegible]	5934	5920	5906	[illegible]	2	5	7
54	·5878	5864	[illegible]	[illegible]	[illegible]	[illegible]	5793	5779	5764	5750	2	5	[illegible]
55	·5736	5721	5707	[illegible]	[illegible]	[illegible]	5650	[illegible]	[illegible]	5606	2	5	7
56	·5592	5577	5563	5548	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5461	2	5	[illegible]
57	·5446	5432	5417	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	5	7
58	·5299	5284	5270	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5150	5135	[illegible]	[illegible]	[illegible]
59	·5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	[illegible]	[illegible]	[illegible]
60	·5000	4985	4970	4955	4939	4924	[illegible]	[illegible]	[illegible]	4863	3	5	8
61	·4848	4833	4818	4802	4787	4772	4756	[illegible]	4726	4710	3	5	[illegible]
62	·4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	[illegible]
63	·4540	4524	4509	4493	4478	4462	[illegible]	4431	[illegible]	4399	3	[illegible]	[illegible]
64	·4384	4368	[illegible]	4337	[illegible]	[illegible]	[illegible]	4274	[illegible]	4242	3	5	8
65	·4226	4210	4193	4179	4163	4147	4131	4115	4099	4083	3	5	8
66	·4067	4051	4035	4019	4003	[illegible]	[illegible]	3955	3939	3923	3	5	8
67	·3907	3891	3875	3859	3843	[illegible]	3811	3795	3779	3762	3	5	[illegible]
68	·3746	3730	3714	3697	[illegible]	[illegible]	3649	3633	3616	3600	3	5	8
69	·3584	3567	3551	3535	[illegible]	3502	3486	3469	3453	3437	3	5	8
70	·3420	3404	[illegible]	3371	[illegible]	3339	3322	3305	3289	3272	3	6	8
71	·3256	3239	[illegible]	3206	[illegible]	3173	3156	3140	3123	3107	[illegible]	6	8
72	·3090	3074	3057	3040	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6	8
73	·2924	2907	2890	2874	2857	[illegible]	2823	2807	2790	2773	3	6	8
74	·2756	2740	2723	2706	2689	[illegible]	2656	2639	2622	2605	3	6	8
75	·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	9
76	·2419	2402	2385	2368	2351	2334	[illegible]	2300	2284	2267	3	6	8
77	·2250	2233	2215	2198	2181	2164	[illegible]	2130	2113	2096	3	6	9
78	·2079	2062	2045	2028	[illegible]	1994	1977	[illegible]	1942	1925	3	6	9
79	·1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9
80	·1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9
81	·1564	1547	1530	1513	1495	1478	1461	1444	1425	1408	3	6	9
82	·1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9
83	·1219	1201	1184	1167	1149	1132	1115	1097	[illegible]	[illegible]	3	6	9
84	1045	1028	1011	0993	0976	0958	[illegible]	0924	0906	[illegible]	3	6	9
85	·0872	0854	0837	0819	0802	0785	[illegible]	0750	0732	0715	3	6	9
86	·0698	0680	0663	0645	[illegible]	0610	0593	0576	0558	0541	3	6	9
87	·0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9
88	·0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9
89	0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9
	·0000												

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TABLE 21

NATURAL TANGENTS

′	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

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TABLE 21 (Contd.)

NATURAL TANGENTS

Degree	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
45	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	2·0057	2·0145	2·0233	2·0323	2·0413	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5783	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2·9042	9208	9375	9544	9714	9887	3·0061	3·0237	3·0415	3·0595	29	58	87	116	145
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	127	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	185	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4·7046	7453	7867	8288	8716	9152	9594	5·0045	5·0504	5·0970					
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5·6713	7297	7894	8502	9124	9758	6·0405	6·1066	6·1742	6·2432					
81	6·3138	3859	4596	5350	6122	0912	7720	8548	9395	7·0264					
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	8·0285					
83	8·1443	2636	3863	5126	6427	7769	9152	9·0579	9·2052	9·3572					
84	9·5144	9·6771	9·8451	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0					
90															

Mean differences cease to be sufficiently accurate

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TABLE 22

LOGARITHMS OF SINES

′	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences				
°	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1	2	3	4	5
0	∞	[illegible]	[illegible]	7108	8439	9408	1·0200	[illegible]	[illegible]	[illegible]					
1	$\bar{2}$·2419	[illegible]	[illegible]	3558	3880	4179	4459	4723	4971	5208					
2	$\bar{2}$·5428	[illegible]	[illegible]	6035	6220	6397	6567	6731	6889	7041					
3	$\bar{2}$·7188	[illegible]	[illegible]	7602	7731	7857	7979	8098	8213	8326					
[illegible]	[illegible]	[illegible]	[illegible]	8749	8849	8946	9042	9135	9226	9315	16	32	48	64	80
[illegible]	[illegible]	9489	9573	9655	9736	9816	9894	9970	1·0046	1·0120	13	26	39	52	65
[illegible]	$\bar{1}$·0192	0264	0334	0403	0472	0539	0605	0670	0734	0797	11	22	33	44	55
7	$\bar{1}$·0859	0920	0981	1040	1099	1157	1214	1271	1326	1381	10	19	29	38	48
8	$\bar{1}$·1430	1489	1542	1594	1646	1697	1747	1797	1847	1895	8	17	25	34	42
9	$\bar{1}$·1943	1991	2038	2085	2131	2176	2221	2266	2310	2353	8	15	23	30	38
10	$\bar{1}$·2397	2439	2482	2524	2565	2606	2647	2687	2727	2767	7	14	20	27	34
11	$\bar{1}$·2806	2845	2883	2921	2959	2997	3034	3070	3107	3143	6	12	19	25	31
12	$\bar{1}$·3179	3214	3250	3284	3319	3353	3387	3421	3455	3488	6	11	17	23	28
13	$\bar{1}$·3521	3554	3586	3618	3650	3682	3713	3745	3775	3806	5	11	16	21	26
14	$\bar{1}$·3837	3867	3897	3927	3957	3986	4015	4044	4073	4102	5	10	15	20	24
15	$\bar{1}$·4130	4158	4186	4214	4242	4269	4296	4323	4350	4377	5	9	14	18	23
16	$\bar{1}$·4403	4430	4456	4482	4508	4533	4559	4584	4609	4634	4	9	13	17	21
17	$\bar{1}$·4659	4684	4709	4733	4757	4781	4805	4829	4853	4876	4	8	12	16	20
18	$\bar{1}$·4900	4923	4946	4969	4992	5015	5037	5060	5082	5104	4	8	11	15	19
19	$\bar{1}$·5126	5148	5170	5192	5213	5235	5256	5278	5299	5320	4	7	11	14	18
20	$\bar{1}$·5341	5361	5382	5402	5423	5443	5463	5484	5504	5523	3	7	10	14	17
21	$\bar{1}$·5543	5563	5583	5602	5621	5641	5660	5679	5698	5717	3	6	10	13	16
22	$\bar{1}$·5736	5754	5773	5792	5810	5828	5847	5865	5883	5901	3	6	9	12	15
23	$\bar{1}$·5919	5937	5954	5972	5990	6007	6024	6042	6059	6076	3	6	9	12	15
24	$\bar{1}$·6093	6110	6127	6144	6161	6177	6194	6210	6227	6243	3	6	8	11	14
25	$\bar{1}$·6259	6276	6292	6308	6324	6340	6356	6371	6387	6403	3	5	8	11	13
26	$\bar{1}$·6418	6434	6449	6465	6480	6495	6510	6526	6541	6556	3	5	8	10	13
27	$\bar{1}$·6570	6585	6600	6615	6629	6644	6659	6673	6687	6702	2	5	7	10	12
28	$\bar{1}$·6716	6730	6744	6759	6773	6787	6801	6814	6828	6842	2	5	7	9	12
29	$\bar{1}$·6856	6869	6883	6896	6910	6923	6937	6950	6963	6977	2	4	7	9	11
30	[illegible]	7003	7016	7029	7042	7055	7068	7080	7093	7106	2	4	6	9	11
31	$\bar{1}$·7118	7131	7144	7156	7168	7181	7193	7205	7218	7230	2	4	6	8	10
32	[illegible]	7254	7266	7278	7290	7302	7314	7326	7338	7349	2	4	6	8	10
33	$\bar{1}$·7361	7373	7384	7396	7407	7419	7430	7442	7453	7464	2	4	6	8	10
34	$\bar{1}$·7476	7487	7498	7509	7520	7531	7542	7553	7564	7575	2	4	6	7	9
[illegible]	$\bar{1}$·7586	7597	7587	7518	7629	7640	7650	7661	7671	7682	2	4	5	7	9
[illegible]	[illegible]	7703	7713	7723	7734	7744	7754	7764	7774	7785	2	3	5	7	9
[illegible]	[illegible]	[illegible]	7815	7825	7835	7844	7854	7864	7874	7884	2	3	5	7	8
[illegible]	[illegible]	[illegible]	7913	7922	7932	7941	7951	7960	7970	7979	2	3	5	6	8
[illegible]	[illegible]	[illegible]	[illegible]	8017	[illegible]	8035	8044	8053	8063	8072	2	3	5	6	8
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	8108	8117	8125	[illegible]	8143	8152	8161	1	3	4	6	7
41	[illegible]	8178	8187	8195	8204	8213	8221	8230	8238	8247	1	3	4	6	7
42	[illegible]	8259	8264	8272	8280	8289	8297	8305	8313	8322	8330	1	3	4	6	7
43	[illegible]	8338	8346	8354	8362	8370	8378	8386	8394	8402	8410	1	3	4	5	7
44	$\bar{1}$·8418	8426	8433	8441	8449	8457	8464	8472	8480	8487	1	3	4	5	6

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TABLE 22 (Contd.)

LOGARITHMS OF SINES

Deg	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1 2 3	4	5
45	$\bar{1}$·8495	8502	8510	8517	8525	8532	8540	8547	8555	8562	1 2 4	5	6
46	$\bar{1}$·8569	8577	8584	8591	8598	8606	8613	8620	8627	8634	1 2 4	5	6
47	$\bar{1}$·8641	8648	8655	8662	8669	8676	8683	8690	8697	8704	1 2 3	5	6
48	$\bar{1}$·8711	8718	8724	8731	8738	8745	8751	8758	8765	8771	1 2 3	4	6
49	$\bar{1}$·8778	8784	8791	8797	8804	8810	8817	8823	8830	8836	1 2 3	4	5
50	$\bar{1}$·8843	8849	8855	8862	8868	8874	8880	8887	8893	8899	1 2 3	4	5
51	$\bar{1}$·8905	8911	8917	8923	8929	8935	8941	8947	8953	8959	1 2 3	4	5
52	$\bar{1}$·8965	8971	8977	8983	8989	8995	9000	9006	9012	9018	1 2 3	4	5
53	$\bar{1}$·9023	9029	9035	9041	9046	9052	9057	9063	9069	9074	1 2 3	4	5
54	$\bar{1}$·9080	9085	9091	9096	9101	9107	9112	9118	9123	9128	1 2 3	4	5
55	$\bar{1}$·9134	9139	9144	9149	9155	[illegible]	9165	9170	9175	9181	1 2 3	[illegible]	4
56	$\bar{1}$·9186	9191	9196	9201	[illegible]	[illegible]	[illegible]	9221	9226	9231	1 2 3	3	4
57	$\bar{1}$·9236	9241	9246	9251	9255	9260	9265	9270	9275	9279	1 2 2	3	4
58	$\bar{1}$·9284	9289	9294	9298	[illegible]	[illegible]	9312	9317	[illegible]	9326	1 2 2	3	4
59	$\bar{1}$·9331	9335	9340	9344	9349	9353	9358	9362	9367	9371	1 1 2	3	4
60	$\bar{1}$·9375	9380	9384	9388	9393	9397	9401	[illegible]	9410	9414	1 1 2	3	4
61	$\bar{1}$·9418	9422	9427	9431	9435	9439	9443	9447	9451	9455	1 1 2	3	3
62	$\bar{1}$·9459	9463	9467	9471	[illegible]	9479	9483	9487	[illegible]	9495	1 1 2	3	3
63	$\bar{1}$·9499	9503	9506	[illegible]	[illegible]	9518	9522	[illegible]	[illegible]	9534	1 1 2	3	3
64	$\bar{1}$·9537	9540	9544	[illegible]	9551	9555	[illegible]	[illegible]	[illegible]	[illegible]	1 1 2	2	3
65	$\bar{1}$·9573	9576	9580	9583	9587	[illegible]	9594	9597	[illegible]	[illegible]	1 1 2	2	3
66	$\bar{1}$·9607	9611	9614	9617	9621	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1 1 2	2	3
67	$\bar{1}$·9640	9643	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1 1 2	2	3
68	$\bar{1}$·9672	9675	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0 1 1	2	2
69	$\bar{1}$·9702	9704	9707	[illegible]	9713	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0 1 1	2	[illegible]
70	$\bar{1}$·[illegible]	[illegible]	9735	9738	9741	9743	[illegible]	9749	9751	9754	0 1 1	2	2
71	$\bar{1}$·9757	[illegible]	9764	9767	9770	[illegible]	[illegible]	[illegible]	9777	9780	0 1 1	2	2
72	$\bar{1}$·9782	[illegible]	9787	[illegible]	9792	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0 1 1	2	2
73	$\bar{1}$·9806	9808	9811	[illegible]	[illegible]	9817	9820	9822	9824	9826	0 1 1	2	2
74	$\bar{1}$·9828	9831	9833	9835	[illegible]	[illegible]	[illegible]	9843	9845	9847	0 1 1	1	[illegible]
75	$\bar{1}$·9849	9851	9853	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9865	9867	0 1 1	1	2
76	$\bar{1}$·[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0 1 1	1	2
77	$\bar{1}$·9887	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0 1 1	1	1
78	$\bar{1}$·9904	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0 1 1	1	1
79	$\bar{1}$·[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0 0 1	1	
80	$\bar{1}$·9934	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0 0 1	1	1
81	$\bar{1}$·9946	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0 0 1	1	1
82	$\bar{1}$·[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0 0 1	1	1
83	$\bar{1}$·9968	9968	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0 0 0	1	1
84	$\bar{1}$·9976	9977	9978	9978	[illegible]	[illegible]	9981	9981	9982	9983	0 0 0	0	1
85	$\bar{1}$·9983	9984	9985	9985	9986	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0 0 0	0	0
86	$\bar{1}$·9989	9990	9990	9991	9991	[illegible]	9992	9993	9993	9994	0 0 0	0	0
87	$\bar{1}$·9994	9994	9995	9995	[illegible]	[illegible]	[illegible]	[illegible]	9997	9997	0 0 0	0	0
88	$\bar{1}$·9997	9998	9998	9998	9998	9999	9999	9999	9999	9999	0 0 0	0	0
89	$\bar{1}$·9999	9999	0000	0000	0000	0000	0000	0000	0000	0000			
90	0·0000												

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TABLE 23

LOGARITHMS OF COSINES

Degree	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1′ 2′ 3′	Mean Differences 4′ 5′
0	0·0000	0000	0000	0000	0000	0000	0000	0000	0000	T9999	0 0 0	0 0
1	T·9999	9999	9999	9999	9999	9999	9998	9998	9998	9998	0 0 0	0 0
2	T·9997	9997	9997	9996	9996	9996	9996	9995	9995	9994	0 0 0	0 0
3	T·9994	9994	9993	9993	9992	9992	9991	9991	9990	9990	0 0 0	0 0
4	T·9989	9989	9988	9988	9987	9987	9986	9985	9985	8984	0 0 0	0 0
5	T·9983	9983	9982	9981	9981	9980	9979	9978	9978	9977	0 0 0	0 1
6	T·9976	9975	9975	9974	9973	9972	9971	9970	9969	9968	0 0 0	1 1
7	T·9968	9967	9966	9965	9964	9963	9962	9961	9960	9959	0 0 1	1 1
8	T·9958	9956	9955	9954	9953	9952	9951	9950	9949	9947	0 0 1	1 1
9	T·9946	9945	9944	9943	9941	9940	9939	9937	9936	9935	0 0 1	1 1
10	T·9934	9932	9931	9929	9928	9927	9925	9924	9922	9921	0 0 1	1 1
11	T·9919	9918	9916	9915	9913	9912	9910	9909	9907	9906	0 1 1	1 1
12	T·9904	9902	9901	9899	9897	9896	9894	9892	9891	9889	0 1 1	1 1
13	T·9887	9885	9884	9882	9880	9879	9876	9875	9873	9871	0 1 1	1 2
14	T·9869	9867	9865	9863	9861	9859	9857	9855	9853	9851	0 1 1	1 2
15	T·9849	9847	9845	9843	9841	9839	9837	9835	9833	9831	0 1 1	1 2
16	T·9828	9826	9824	9822	9820	9817	9815	9813	9811	9808	0 1 1	2 2
17	T·9806	9804	9801	9799	9797	9794	9792	9789	9787	9785	0 1 1	2 2
18	T·9782	9780	9777	9775	9772	9770	9767	9764	9762	9759	0 1 1	2 2
19	T·9757	9754	9751	9749	9746	9743	9741	9738	9735	9733	0 1 1	2 2
20	T·9730	9727	9724	9722	9719	9716	9713	9710	9707	9704	0 1 1	2 2
21	T·9702	9699	9696	9693	9690	9687	9684	9681	9678	9675	0 1 1	2 2
22	T·9672	9669	9666	9662	9659	9656	9653	9650	9647	9643	1 1 2	2 3
23	T·9640	9637	9634	9631	9627	9624	9621	9617	9614	9611	1 1 2	2 3
24	T·9607	9604	9601	9597	9594	9590	9587	9583	9580	9576	1 1 2	2 3
25	T·9573	9569	9566	9562	9558	9555	9551	9548	9544	9540	1 1 2	2 3
26	T·9537	9533	9529	9525	9522	9518	9514	9510	9507	9503	1 1 2	3 3
27	T·9499	9495	9491	9487	9488	9479	9475	9471	9467	9463	1 1 2	3 3
28	T·9459	9455	9451	9447	9443	9439	9435	9431	9427	9422	1 1 2	3 3
29	T·9418	9414	9410	9406	9401	9397	9393	9388	9384	9380	1 1 2	3 4
30	T·9375	9371	9367	9362	9358	9353	9349	9344	9340	9335	1 1 2	3 4
31	T·9331	9326	9322	9317	9312	9308	9303	9298	9294	9289	1 2 2	3 4
32	T·9284	9279	9275	9270	9265	9260	9255	9251	9246	9241	1 2 2	3 4
33	T·9236	9231	9226	9221	9216	9211	9206	9201	9196	9191	1 2 3	3 4
34	T·9186	9181	9175	9170	9165	9160	9155	9149	9144	9139	1 2 3	3 4
35	T·9134	9128	9123	9118	9112	9107	9101	9096	9091	9085	1 2 3	4 5
36	T·9080	9074	9069	9063	9057	9052	9046	9041	9035	9029	1 2 3	4 5
37	T·9023	9018	9012	9006	9000	8995	8989	8983	8977	8971	1 2 3	4 5
38	T·8965	8959	8953	8947	8941	8935	8929	8923	8917	8911	1 2 3	4 5
39	T·8905	8899	8893	8887	8880	8874	8868	8862	8855	8849	1 2 3	4 5
40	T·8843	8836	8830	8823	8817	8810	8804	8797	8791	8784	1 2 3	4 5
41	T·8778	8771	8765	8758	8751	8745	8738	8731	8724	8718	1 2 3	5 6
42	T·8711	8704	8697	8690	8683	8676	8669	8662	8655	8648	1 2 3	5 6
43	T·8641	8634	8627	8620	8613	8606	8598	8591	8584	8577	1 2 4	5 6
44	T·8569	8562	8555	8547	8540	8532	8525	8517	8510	8502	1 2 4	5 6

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TABLE 23 (Contd.)

LOGARITHMS OF COSINES

[Numbers in difference columns to be subtracted, not added.]

Deg	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5	6
45	$\bar{1}$·8495	8487	8480	8472	8464	8457	8449	8441	8433	8426	1	3	4		5	6
46	$\bar{1}$·8418	8410	8402	8394	8386	8378	8370	8362	8354	8346	1	3	4		5	7
47	$\bar{1}$·8338	8330	8322	8313	8305	8297	8289	8280	8272	8264	1	3	4		6	7
48	$\bar{1}$·8255	8247	8238	8230	8221	8213	8204	8195	8187	8178	1	3	4		6	7
49	$\bar{1}$·8169	8161	8152	8143	8134	8125	8117	8108	8099	8090	1	3	4		6	7
50	$\bar{1}$·8081	8072	8063	8053	8044	8035	8026	8017	8007	7998	2	3	5		6	8
51	$\bar{1}$·7989	7979	7970	7960	7951	7941	7932	7923	7913	7903	2	3	5		6	8
52	$\bar{1}$·7893	7884	7874	7864	7854	7844	7835	7825	7815	7805	2	3	5		7	8
53	$\bar{1}$·7795	7785	7774	7764	7754	7744	7734	7723	7713	7703	2	3	5		7	9
54	$\bar{1}$·7692	7682	7671	7661	7650	7640	7629	7618	7607	7597	2	4	5		7	9
55	$\bar{1}$·7586	7575	7564	7553	7542	7531	7520	7509	7498	7487	2	4	6		7	9
56	$\bar{1}$·7476	7464	7453	7442	7430	7419	7407	7396	7384	7373	2	4	6		8	10
57	$\bar{1}$·7361	7349	7338	7326	7314	7302	7290	7278	7266	7254	2	4	6		8	10
58	$\bar{1}$·7242	7230	7218	7205	7193	7181	7168	7156	7144	7131	2	4	6		8	10
59	$\bar{1}$·7118	7106	7093	7080	7068	7055	7042	7029	7016	7003	2	4	6		9	11
60	$\bar{1}$·6990	6977	6963	6950	6937	6923	6910	6896	6883	6869	2	4	7		9	11
61	$\bar{1}$·6856	6842	6828	6814	6801	6787	6773	6759	6744	6730	2	5	7		9	12
62	$\bar{1}$·6716	6702	6687	6673	6659	6644	6629	6615	6600	6585	2	5	7		10	12
63	$\bar{1}$·6570	6556	6541	6526	6510	6495	6480	6465	6449	6434	3	5	8		10	13
64	$\bar{1}$·6418	6403	6387	6371	6356	6340	6324	6308	6292	6276	3	5	8		11	13
65	$\bar{1}$·6259	6243	6227	6210	6194	6177	6161	6144	6127	6110	3	6	8		11	14
66	$\bar{1}$·6093	6076	6059	6042	6024	6007	5990	5972	5954	5937	3	6	9		12	15
67	$\bar{1}$·5919	5901	5883	5865	5847	5828	5810	5792	5773	5754	3	6	9		12	15
68	$\bar{1}$·5736	5717	5698	5679	5660	5641	5622	5602	5583	5563	3	6	10		13	16
69	$\bar{1}$·5543	5523	5504	5484	5463	5443	5423	5402	5382	5361	3	7	10		14	17
70	$\bar{1}$·5341	5320	5299	5278	5256	5235	5213	5192	5170	5148	4	7	11		14	18
71	$\bar{1}$·5126	5104	5082	5060	5037	5015	4992	4969	4946	4923	4	8	11		15	19
72	$\bar{1}$·4900	4876	4853	4829	4805	4781	4757	4733	4709	4684	4	8	12		16	20
73	$\bar{1}$·4659	4634	4609	4584	4559	4533	4508	4482	4456	4430	4	9	13		17	21
74	$\bar{1}$·4403	4377	4350	4323	4296	4269	4242	4214	4186	4158	5	9	14		18	23
75	$\bar{1}$·4130	4102	4073	4044	4015	3986	3957	3927	3897	3867	5	10	15		20	24
76	$\bar{1}$·3837	3806	3775	3745	3713	3682	3650	3618	3586	3554	6	11	16		21	26
77	$\bar{1}$·3521	3488	3455	3421	3387	3353	3319	3284	3250	3214	6	11	17		23	28
78	$\bar{1}$·3179	3143	3107	3070	3034	2997	2959	2921	2883	2845	6	12	19		25	31
79	$\bar{1}$·2806	2767	2727	2687	2647	2606	2565	2524	2482	2439	7	14	20		27	34
80	$\bar{1}$·2397	2353	2310	2266	2221	2176	2131	2085	2038	1991	8	15	23		30	38
81	$\bar{1}$·1943	1895	1847	1797	1747	1697	1646	1594	1542	1489	8	17	25		34	42
82	$\bar{1}$·1436	1381	1326	1271	1214	1157	1099	1040	0981	0920	10	19	29		38	48
83	$\bar{1}$·0859	0797	0734	0670	0605	0539	0472	0403	0334	0264	11	22	33		44	55
84	$\bar{1}$·0192	0120	0046	$\bar{2}$·9970	$\bar{2}$·9894	$\bar{2}$·9816	$\bar{2}$·9736	$\bar{2}$·9655	$\bar{2}$·9573	$\bar{2}$·9489	13	26	39		52	65
85	$\bar{2}$·9403	9315	9226	9135	9042	8946	8849	8749	8647	8543	16	32	48		64	80
86	$\bar{2}$·8436	8326	8213	8098	7979	7857	7731	7602	7468	7330						
87	$\bar{2}$·7188	7041	6889	6731	6567	6397	6220	6035	5842	5640						
88	$\bar{2}$·5428	5200	4971	4732	4459	4179	3880	3558	3210	2832						
89	$\bar{2}$·2410	1961	1450	0870	0200	$\bar{3}$·9408	$\bar{3}$·8439	$\bar{3}$·7100	$\bar{3}$·5429	$\bar{3}$·2419						
90																

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TABLE 24

LOGARITHMS OF TANGENTS

Degrees	0′ 0°	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Difference 1 2 3	Mean Difference 4 5
0	−∞	$\bar{3}$·2419	$\bar{3}$·5429	$\bar{3}$·7190	$\bar{3}$·8439	$\bar{3}$·9409	$\bar{2}$·0200	$\bar{2}$·0870	$\bar{2}$·1450	$\bar{2}$·1962		
1	$\bar{2}$·2419	2833	3211	3559	3881	4181	4401	4725	4973	5208		
2	$\bar{2}$·5431	5643	5845	6038	6223	6401	6571	6736	6894	7046		
3	$\bar{2}$·7194	7337	7475	7609	7739	7865	7988	8107	8223	8336		
4	$\bar{2}$·8446	8554	8659	8762	8862	8960	9056	9150	9241	9331	16 32 48	64 81
5	$\bar{2}$·9420	9506	9591	9674	9756	9836	9915	9992	$\bar{1}$·0068	$\bar{1}$·0143	13 26 40	53 66
6	$\bar{1}$·0216	0289	0360	0430	0499	0567	0633	0699	0764	0828	11 22 34	45 56
7	$\bar{1}$·0891	0954	1015	1076	1135	1194	1252	1310	1367	1423	10 20 29	39 49
8	$\bar{1}$·1478	1533	1587	1640	1693	1745	1797	1848	1898	1948	9 17 26	35 43
9	$\bar{1}$·1997	2046	2094	2142	2189	2236	2282	2328	2374	2419	8 16 23	31 39
10	$\bar{1}$·2463	2507	2551	2594	2637	2680	2722	2764	2805	2846	7 14 21	28 35
11	$\bar{1}$·2887	2927	2967	3006	3046	3085	3123	3162	3200	3237	6 13 19	26 32
12	$\bar{1}$·3275	3312	3349	3385	3422	3458	3493	3529	3564	3599	6 12 18	24 30
13	$\bar{1}$·3634	3668	3702	3736	3770	3804	3837	3870	3903	3935	6 11 17	22 28
14	$\bar{1}$·3968	4000	4032	4064	4095	4127	4158	4189	4220	4250	5 10 16	21 26
15	$\bar{1}$·4281	4311	4341	4371	4400	4430	4459	4488	4517	4546	5 10 15	20 25
16	$\bar{1}$·4575	4603	4632	4660	4688	4716	4744	4771	4799	4826	5 9 14	19 23
17	$\bar{1}$·4853	4880	4907	4934	4961	4987	5014	5040	5066	5092	4 9 13	18 22
18	$\bar{1}$·5118	5143	5169	5195	5220	5245	5270	5295	5320	5345	4 8 13	17 21
19	$\bar{1}$·5370	5394	5419	5443	5467	5491	5516	5539	5563	5587	4 8 12	16 20
20	$\bar{1}$·5611	5634	5658	5681	5704	5727	5750	5773	5796	5819	4 8 12	15 19
21	$\bar{1}$·5842	5864	5887	5909	5932	5954	5976	5998	6020	6042	4 7 11	15 19
22	$\bar{1}$·6064	6086	6108	6129	6151	6172	6194	6215	6236	6257	4 7 11	14 18
23	$\bar{1}$·6279	6300	6321	6341	6362	6383	6404	6424	6445	6465	3 7 10	14 17
24	$\bar{1}$·6486	6506	6527	6547	6567	6587	6607	6627	6647	6667	3 7 10	13 17
25	$\bar{1}$·6687	6706	6726	6746	6765	6785	6804	6824	6843	6863	3 7 10	13 16
26	$\bar{1}$·6882	6901	6920	6939	6958	6977	6996	7015	7034	7053	3 6 9	13 16
27	$\bar{1}$·7072	7090	7109	7128	7146	7165	7183	7202	7220	7238	3 6 9	12 15
28	$\bar{1}$·7257	7275	7293	7311	7330	7348	7366	7384	7402	7420	3 6 9	12 15
29	$\bar{1}$·7438	7455	7473	7491	7509	7526	7544	7562	7579	7597	3 6 9	12 15
30	$\bar{1}$·7614	7632	7649	7667	7684	7701	7719	7736	7753	7771	3 6 9	12 14
31	$\bar{1}$·7788	7805	7822	7839	7856	7873	7890	7907	7924	7941	3 6 9	11 14
32	$\bar{1}$·7958	7975	7992	8008	8025	8042	8059	8075	8092	8109	3 6 8	11 14
33	$\bar{1}$·8125	8142	8158	8175	8191	8208	8224	8241	8257	8274	3 5 8	11 14
34	$\bar{1}$·8290	8306	8323	8339	8355	8371	8388	8404	8420	8436	3 5 8	11 14
35	$\bar{1}$·8452	8468	8484	8501	8517	8533	8549	8565	8581	8597	3 5 8	11 13
36	$\bar{1}$·8613	8629	8644	8660	8676	8692	8708	8724	8740	8755	3 5 8	11 13
37	$\bar{1}$·8771	8787	8803	8818	8834	8850	8865	8881	8897	8912	3 5 8	10 13
38	$\bar{1}$·8928	8944	8959	8975	8990	9006	9022	9037	9053	9068	3 5 8	10 13
39	$\bar{1}$·9084	9099	9115	9130	9146	9161	9176	9192	9207	9223	3 5 8	10 13
40	$\bar{1}$·9238	9254	9269	9284	9300	9315	9330	9346	9361	9376	3 5 8	10 13
41	$\bar{1}$·9392	9407	9422	9438	9453	9468	9483	9499	9514	9529	3 5 8	10 13
42	$\bar{1}$·9544	9560	9575	9590	9605	9621	9636	9651	9666	9681	3 5 8	10 13
43	$\bar{1}$·9697	9712	9727	9742	9757	9772	9788	9803	9818	9833	3 5 8	10 13
44	$\bar{1}$·9848	9864	9879	9894	9909	9924	9939	9955	9970	9985	3 5 8	10 13

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TABLE 24 (Contd.)

LOGARITHMS OF TANGENTS

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
45	·0000	0015	0030	0045	0061	0076	0091	0106	0121	0136	3	5	8	10	13
46	·0152	0167	0182	0197	0212	0228	0243	0258	0273	0288	3	5	8	10	13
47	·0303	0319	0334	0349	0364	0379	0395	0410	0425	0440	3	5	8	10	13
48	·0456	0471	0486	0501	0517	0532	0547	0562	0578	0593	3	5	8	10	13
49	·0608	0624	0639	0654	0670	0685	0700	0716	0731	0746	3	5	8	10	13
50	·0762	0777	0793	0808	0824	0839	0854	0870	0885	0901	3	5	8	10	13
51	·0916	0932	0947	0963	0978	0994	1010	1025	1041	1056	3	5	8	10	13
52	·1072	1088	1103	1119	1135	1150	1166	1182	1197	1213	3	5	8	10	13
53	·1229	1245	1260	1276	1292	1308	1324	1340	1356	1371	3	5	8	11	13
54	·1387	1403	1419	1435	1451	1467	1483	1499	1516	1532	3	5	8	11	13
55	·1548	1564	1580	1596	1612	1629	1645	1661	1677	1694	3	5	8	11	14
56	·1710	1726	1743	1759	1776	1792	1809	1825	1842	1858	3	5	8	11	14
57	·1875	1891	1908	1925	1941	1958	1975	1992	2008	2025	3	6	8	11	14
58	·2042	2059	2076	2093	2110	2127	2144	2161	2178	2195	3	6	9	11	14
59	·2212	2229	2247	2264	2281	2299	2316	2333	2351	2368	3	6	9	12	14
60	·2386	2403	2421	2438	2456	2474	2491	2509	2527	2545	3	6	9	12	15
61	·2562	2580	2598	2616	2634	2652	2670	2689	2707	2725	3	6	9	12	15
62	·2743	2762	2780	2798	2817	2835	2854	2872	2891	2910	3	6	9	12	15
63	·2928	2947	2966	2985	3004	3023	3042	3061	3080	3099	3	6	9	13	16
64	·3118	3137	3157	3176	3196	3215	3235	3254	3274	3294	3	6	10	13	16
65	·3313	3333	3353	3373	3393	3413	3433	3453	3473	3494	3	7	10	13	17
66	·3514	3535	3555	3576	3596	3617	3638	3659	3679	3700	3	7	10	14	17
67	·3721	3743	3764	3785	3806	3828	3849	3871	3892	3914	4	7	11	14	18
68	·3936	3958	3980	4002	4024	4046	4068	4091	4113	4136	4	7	11	15	19
69	4158	4181	4204	4227	4250	4273	4296	4319	4342	4366	4	8	12	15	19
70	·4389	4413	4437	4461	4484	4509	4533	4557	4581	4606	4	8	12	16	20
71	·4630	4655	4680	4705	4730	4755	4780	4805	4831	4857	4	8	13	17	21
72	·4882	4908	4934	4960	4986	5013	5039	5066	5093	5120	4	9	13	18	22
73	·5147	5174	5201	5229	5256	5284	5312	5340	5368	5397	5	9	14	19	23
74	·5425	5454	5483	5512	5541	5570	5600	5629	5659	5689	5	10	15	20	25
75	·5719	5750	5780	5811	5842	5873	5905	5936	5968	6000	5	10	16	21	26
76	·6032	6065	6097	6130	6163	6196	6230	6264	6298	6332	6	11	17	22	28
77	·6366	6401	6436	6471	6507	6542	6578	6615	6651	6688	6	12	18	24	30
78	·6725	6763	6800	6838	6877	6915	6954	6994	7033	7073	6	13	19	26	32
79	·7113	7154	7195	7236	7278	7320	7363	7406	7449	7493	7	14	21	28	35
80	·7537	7581	7626	7672	7718	7764	7811	7858	7906	7954	8	16	23	31	39
81	·8003	8052	8102	8152	8203	8255	8307	8360	8413	8467	9	17	26	35	43
82	·8522	8577	8633	8690	8748	8806	8865	8924	8985	9046	10	20	29	39	49
83	·9109	9172	9236	9301	9367	9433	9501	9570	9640	9711	11	22	34	45	56
84	·9784	9857	9932	1·0008	1·0085	1·0164	1·0244	1·0326	1·0409	1·0494	13	26	40	53	66
85	1·0580	0669	0759	0850	0944	1040	1138	1238	1341	1446	16	32	48	64	81
86	1·1554	1664	1777	1893	2012	2135	2261	2391	2525	2663					
87	1·2806	2954	3106	3264	3429	3599	3777	3962	4155	4357					
88	1·4569	4792	5027	5275	5539	5819	6119	6441	6789	7167					
89	1·7581	8038	8550	9130	9800	2·0591	2·1561	2·2810	2·4571	2·7581					

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TABLE 25

POWERS, ROOTS AND RĒCIPROCALS

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\sqrt{10n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$\frac{1}{n}$
1	1	1	1	1	3·162	2·154	4·642	1
2	4	8	1·414	1·260	4·472	2·714	5·848	·5000
3	9	27	1·732	1·442	5·477	3·107	6·694	·3333
4	16	64	2	1·587	6·325	3·420	7·308	·2500
5	25	125	2·236	1·710	7·071	3·684	7·937	·2000
6	36	216	2·449	1·817	7·746	3·915	8·434	·1667
7	49	343	2·646	1·913	8·367	4·121	8·879	·1429
8	64	512	2·828	2·000	8·944	4·309	9·283	·1250
9	81	729	3·000	2·080	9·487	4·481	9·655	·1111
10	100	1000	3·162	2·154	10·0	4·642	10·000	·1000
11	121	1331	3·317	2·224	10·488	4·791	10·323	·09091
12	144	1728	3·464	2·289	10·954	4·932	10·627	·08333
13	169	2197	3·606	2·351	11·402	5·066	10·914	·07692
14	196	2744	3·742	2·410	11·832	5·192	11·187	·07143
15	225	3375	3·873	2·466	12·247	5·313	11·447	·06667
16	256	4096	4·000	2·520	12·649	5·429	11·696	·06250
17	289	4913	4·123	2·571	13·038	5·540	11·935	·05882
18	324	5832	4·243	2·621	13·416	5·646	12·164	·05556
19	361	6859	4·359	2·668	13·784	5·749	12·386	·05263
20	400	8000	4·472	2·714	14·142	5·848	12·599	·0500
21	441	9261	4·583	2·759	14·491	5·944	12·806	·04762
22	484	10648	4·690	2·802	14·832	6·037	13·006	·04545
23	529	12167	4·796	2·844	15·166	6·127	13·200	·04348
24	576	13824	4·899	2·884	15·492	6·214	13·389	·04167
25	625	15625	5·000	2·924	15·811	6·300	13·572	·0400
26	676	17576	5·099	2·962	16·125	6·383	13·751	·03846
27	729	19683	5·196	3·000	16·432	6·463	13·925	·03704
28	784	21952	5·292	3·037	16·733	6·542	14·095	·03571
29	841	24389	5·385	3·072	17·029	6·619	14·260	·03448
30	900	27000	5·477	3·107	17·321	6·694	14·422	·03333
31	961	29791	5·568	3·148	17·607	6·768	14·581	·03226
32	1024	32768	5·657	3·175	17·889	6·840	14·736	·03125
33	1089	35937	5·745	3·208	18·166	6·910	14·888	·03030
34	1156	39304	5·831	3·240	18·439	6·980	15·037	·02941
35	1225	42875	5·916	3·271	18·708	7·047	15·183	·02857
36	1296	46656	6·000	3·302	18·974	7·114	15·326	·02778
37	1369	50653	6·083	3·332	19·235	7·179	15·467	·02703
38	1444	54872	6·164	3·362	19·494	7·243	15·605	·02632
39	1521	59319	6·245	3·391	19·748	7·306	15·741	·02564
40	1600	64000	6·325	3·420	20·00	7·368	15·874	·0250
41	1681	68921	6·403	3·448	20·248	7·429	16·005	·02439
42	1764	74088	6·481	3·476	20·494	7·489	16·134	·02381
43	1849	79507	6·557	3·503	20·736	7·548	16·261	·02326
44	1936	85184	6·633	3·530	20·976	7·606	16·386	·02273
45	2025	91125	6·708	3·557	21·213	7·663	16·510	·02222
46	2116	97336	6·782	3·583	21·448	7·719	16·631	·02174
47	2209	103823	6·856	3·609	21·679	7·775	16·751	·02128
48	2304	110592	6·928	3·634	21·909	7·830	16·869	·02083
49	2401	117649	7·000	3·659	22·136	7·884	16·985	·02041
50	2500	125000	7·071	3·684	22·361	7·937	17·100	·020

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TABLE 25 (Contd.)
POWERS, ROOTS AND RECIPROCALS

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\sqrt{10n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$\frac{1}{n}$
51	2601	132651	7·141		22·583	7·990	17·213	·01961
52	2704	140608	7·211		22·804	8·041	17·325	·01923
53	2809	148877	7·280		23·022	8·093	17·435	·01887
54	2916	157464	7·348		23·238	8·143	17·544	·01852
55	3025	166375	7·416		23·452	8·193	17·652	·01818
56	3136	175616	7·483		23·664	8·243	17·758	·01786
57	3249	185193	7·550		23·875	8·291	17·863	·01754
58	3364	195112	7·616		24·083	8·340	17·967	·01724
59	3481	205379	7·681		24·290	8·387	18·070	·01695
60	3600	216000	7·746		24·495	8·434	18·171	·01667
61	3721	226981	7·810		24·698	8·481	18·272	·01639
62	3844	238328	7·874		24·900	8·527	18·371	·01613
63	3969	250047	7·937		25·100	8·573	18·469	·01587
64	4096	262144	8·000		25·298	8·618	18·566	·01562
65	4225	274625	8·062		25·495	8·662	18·663	·01538
66	4356	287496	8·124		25·690	8·707	18·758	·01515
67	4489	300763	8·185		25·884	8·750	18·852	·01493
68	4624	314432	8·246		26·077	8·794	18·945	·01471
69	4761	328509	8·307		26·268	8·837	19·038	·01449
70	4900	343000	8·367		26·458	8·879	19·129	·01429
71	5041	357911	8·426		26·646	8·921	19·220	·01408
72	5184	373248	8·485		26·833	8·963	19·310	·01389
73	5329	389017	8·544		27·019	9·004	19·399	·01370
74	5476	405224	8·602		27·203	9·045	19·487	·01351
75	5625	421875	8·660		27·386	9·086	19·574	·01333
76	5776	438976	8·718		27·568	9·126	19·661	·01316
77	5929	456533	8·775		27·749	9·166	19·747	·01299
78	6084	474552	8·832		27·928	9·205	19·832	·01282
79	6241	493039	8·888		28·107	9·244	19·916	·01266
80	6400	512000	8·944		28·284	9·283	20·000	·01250
81	6561	531441	9·000		28·460	9·322	20·083	·01235
82	6724	551368	9·055		28·636	9·360	20·165	·01220
83	6889	571787	9·110		28·810	9·398	20·247	·01205
84	7056	592704	9·165		28·983	9·435	20·328	·01190
85	7225	614125	9·220		29·155	9·473	20·408	·01176
86	7396	636056	9·274		29·326	9·510	20·488	·01163
87	7569	658503	9·327		29·496	9·546	20·567	·01149
88	7744	681472	9·381		29·665	9·583	20·646	·01136
89	7921	704969	9·434		29·833	9·619	20·724	·01124
90	8100	729000	9·487		30·000	9·655	20·801	·01111
91	8281	753571	9·539		30·166	9·691	20·878	·01099
92	8464	778688	9·592		30·332	9·726	20·954	·01087
93	8649	804357	9·644		30·496	9·761	21·029	·01075
94	8836	830584	9·695		30·659	9·796	21·105	·01064
95	9025	857375	9·747		30·822	9·830	21·179	·01053
96	9216	884736	9·798		30·984	9·865	21·253	·01042
97	9409	912673	9·849		31·145	9·899	21·327	·01031
98	9604	941192	9·899		31·305	9·933	21·400	·01020
99	9801	970299	9·950		31·464		21·472	·01010
100	10000	1000000	10·000		31·623	10·000	21·544	·0100

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TABLE 26

SQUARE ROOTS. FROM 1 TO 10

	0	1	2	3	4	5	6	7	8	9	Mean Differences		
											1 2 3	4 5 6	7 8 9
1·0	1·000	1·005	1·010	1·015	1·020	1·025	1·030	1·034	1·039	1·044	0 1 1	2 2 3	3 4 4
1·1	1·049	1·054	1·058	1·063	1·068	1·072	1·077	1·082	1·086	1·091	0 1 1	2 2 3	3 4 4
1·2	1·095	1·100	1·105	1·109	1·114	1·118	1·123	1·127	1·131	1·136	0 1 1	2 2 3	3 4 4
1·3	1·140	1·145	1·149	1·153	1·158	1·162	1·166	1·170	1·175	1·179	0 1 1	2 2 3	3 3 4
1·4	1·183	1·187	1·192	1·196	1·200	1·204	1·208	1·212	1·217	1·221	0 1 1	2 2 2	3 3 4
1·5	1·225	1·229	1·233	1·237	1·241	1·245	1·249	1·253	1·257	1·261	0 1 1	2 2 2	3 3 4
1·6	1·265	1·269	1·273	1·277	1·281	1·285	1·288	1·292	1·296	1·300	0 1 1	2 2 2	3 3 3
1·7	1·304	1·308	1·311	1·315	1·319	1·323	1·327	1·330	1·334	1·338	0 1 1	2 2 2	3 3 3
1·8	1·342	1·345	1·349	1·353	1·356	1·360	1·364	1·367	1·371	1·375	0 1 1	1 2 2	3 3 3
1·9	1·378	1·382	1·386	1·389	1·393	1·396	1·400	1·404	1·407	1·411	0 1 1	1 2 2	3 3 3
2·0	1·414	1·418	1·421	1·425	1·428	1·432	1·435	1·439	1·442	1·446	0 1 1	1 2 2	2 3 3
2·1	1·449	1·453	1·456	1·459	1·463	1·466	1·470	1·473	1·476	1·480	0 1 1	1 2 2	2 3 3
2·2	1·483	1·487	1·490	1·493	1·497	1·500	1·503	1·507	1·510	1·513	0 1 1	1 2 2	2 3 3
2·3	1·517	1·520	1·523	1·526	1·530	1·533	1·536	1·539	1·543	1·546	0 1 1	1 2 2	2 3 3
2·4	1·549	1·552	1·556	1·559	1·562	1·565	1·568	1·572	1·575	1·578	0 1 1	1 2 2	2 3 3
2·5	1·581	1·584	1·587	1·591	1·594	1·597	1·600	1·603	1·606	1·609	0 1 1	1 2 2	2 3 3
2·6	1·612	1·616	1·619	1·622	1·625	1·628	1·631	1·634	1·637	1·640	0 1 1	1 2 2	2 2 3
2·7	1·643	1·646	1·649	1·652	1·655	1·658	1·661	1·664	1·667	1·670	0 1 1	1 2 2	2 2 3
2·8	1·673	1·676	1·679	1·682	1·685	1·688	1·691	1·694	1·697	1·700	0 1 1	1 1 2	2 2 3
2·9	1·703	1·706	1·709	1·712	1·715	1·718	1·720	1·723	1·726	1·729	0 1 1	1 1 2	2 2 3
3·0	1·732	1·735	1·738	1·741	1·744	1·746	1·749	1·752	1·755	1·758	0 1 1	1 1 2	2 2 3
3·1	1·761	1·764	1·766	1·769	1·772	1·775	1·778	1·780	1·783	1·786	0 1 1	1 1 2	2 2 3
3·2	1·789	1·792	1·794	1·797	1·800	1·803	1·806	1·808	1·811	1·814	0 1 1	1 1 2	2 2 2
3·3	1·817	1·819	1·822	1·825	1·828	1·830	1·833	1·836	1·838	1·841	0 1 1	1 1 2	2 2 2
3·4	1·844	1·847	1·849	1·852	1·855	1·857	1·860	1·863	1·865	1·868	0 1 1	1 1 2	2 2 2
3·5	1·871	1·873	1·876	1·879	1·881	1·884	1·887	1·889	1·892	1·895	0 1 1	1 1 2	2 2 2
3·6	1·897	1·900	1·903	1·905	1·908	1·910	1·913	1·916	1·918	1·921	0 1 1	1 1 2	2 2 2
3·7	1·924	1·926	1·929	1·931	1·934	1·936	1·939	1·942	1·944	1·947	0 1 1	1 1 2	2 2 2
3·8	1·949	1·952	1·954	1·957	1·960	1·962	1·965	1·967	1·970	1·972	0 1 1	1 1 2	2 2 2
3·9	1·975	1·977	1·980	1·982	1·985	1·987	1·990	1·992	1·995	1·997	0 1 1	1 1 2	2 2 2
4·0	2·000	2·002	2·005	2·007	2·010	2·012	2·015	2·017	2·020	2·022	0 0 1	1 1 1	2 2 2
4·1	2·025	2·027	2·030	2·032	2·035	2·037	2·040	2·042	2·045	2·047	0 0 1	1 1 1	2 2 2
4·2	2·049	2·052	2·054	2·057	2·059	2·062	2·064	2·066	2·069	2·071	0 0 1	1 1 1	2 2 2
4·3	2·074	2·076	2·078	2·081	2·083	2·086	2·088	2·090	2·093	2·095	0 0 1	1 1 1	2 2 2
4·4	2·098	2·100	2·102	2·105	2·107	2·110	2·112	2·114	2·117	2·119	0 0 1	1 1 1	2 2 2
4·5	2·121	2·124	2·126	2·128	2·131	2·133	2·135	2·138	2·140	2·142	0 0 1	1 1 1	2 2 2
4·6	2·145	2·147	2·149	2·152	2·154	2·156	2·159	2·161	2·163	2·166	0 0 1	1 1 1	2 2 2
4·7	2·168	2·170	2·173	2·175	2·177	2·179	2·182	2·184	2·186	2·189	0 0 1	1 1 1	2 2 2
4·8	2·191	2·193	2·195	2·198	2·200	2·202	2·205	2·207	2·209	2·211	0 0 1	1 1 1	2 2 2
4·9	2·214	2·216	2·218	2·220	2·223	2·225	2·227	2·229	2·232	2·234	0 0 1	1 1 1	2 2 2
5·0	2·236	2·238	2·241	2·243	2·245	2·247	2·249	2·252	2·254	2·256	0 0 1	1 1 1	2 2 2
5·1	2·258	2·261	2·263	2·265	2·267	2·269	2·272	2·274	2·276	2·278	0 0 1	1 1 1	2 2 2
5·2	2·280	2·283	2·285	2·287	2·289	2·291	2·293	2·296	2·298	2·300	0 0 1	1 1 1	2 2 2
5·3	2·302	2·304	2·307	2·309	2·311	2·313	2·315	2·317	2·319	2·322	0 0 1	1 1 1	2 2 2
5·4	2·324	2·326	2·328	2·330	2·332	2·335	2·337	2·339	2·341	2·343	0 0 1	1 1 1	2 2 2

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TABLE 26 (Contd.)

SQUARE ROOTS. FROM 1 TO 10

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1 2 3	4 5 6	7 8 9
5·5	2·345	2·347	2·349	2·352	2·354	2·356	2·358	2·360	2·362	2·364	0 0 1	1 1 1	1 2 2
5·6	2·366	2·369	2·371	2·373	2·375	2·377	2·379	2·381	2·383	2·385	0 0 1	1 1 1	1 2 2
5·7	2·387	2·390	2·392	2·394	2·396	2·398	2·400	2·402	2·404	2·406	0 0 1	1 1 1	1 2 2
5·8	2·408	2·410	2·412	2·415	2·417	2·419	2·421	2·423	2·425	2·427	0 0 1	1 1 1	1 2 2
5·9	2·429	2·431	2·433	2·435	2·437	2·439	2·441	2·443	2·445	2·447	0 0 1	1 1 1	1 2 2
6·0	2·449	2·452	2·454	2·456	2·458	2·460	2·462	2·464	2·466	2·468	0 0 1	1 1 1	1 2 2
6·1	2·470	2·472	2·474	2·476	2·478	2·480	2·482	2·484	2·486	2·488	0 0 1	1 1 1	1 2 2
6·2	2·490	2·492	2·494	2·496	2·498	2·500	2·502	2·504	2·506	2·508	0 0 1	1 1 1	1 2 2
6·3	2·510	2·512	2·514	2·516	2·518	2·520	2·522	2·524	2·526	2·528	0 0 1	1 1 1	1 2 2
6·4	2·530	2·532	2·534	2·536	2·538	2·540	2·542	2·544	2·546	2·548	0 0 1	1 1 1	1 2 2
6·5	2·550	2·551	2·553	2·555	2·557	2·559	2·561	2·563	2·565	2·567	0 0 1	1 1 1	1 2 2
6·6	2·569	2·571	2·573	2·575	2·577	2·579	2·581	2·583	2·585	2·587	0 0 1	1 1 1	1 2 2
6·7	2·588	2·590	2·592	2·594	2·596	2·598	2·600	2·602	2·604	2·606	0 0 1	1 1 1	1 2 2
6·8	2·608	2·610	2·612	2·613	2·615	2·617	2·619	2·621	2·623	2·625	0 0 1	1 1 1	1 2 2
6·9	2·627	2·629	2·631	2·632	2·634	2·636	2·638	2·640	2·642	2·644	0 0 1	1 1 1	1 2 2
7·0	2·645	2·648	2·650	2·651	2·653	2·655	2·657	2·659	2·661	2·663	0 0 1	1 1 1	1 2 2
7·1	2·666	2·666	2·665	2·670	2·672	2·674	2·676	2·678	2·680	2·681	0 0 1	1 1 1	1 1 2
7·2	2·683	2·685	2·687	2·689	2·691	2·693	2·694	2·696	2·698	2·700	0 0 1	1 1 1	1 1 2
7·3	2·702	2·704	2·706	2·707	2·709	2·711	2·713	2·715	2·717	2·718	0 0 1	1 1 1	1 1 2
7·4	2·720	2·722	2·724	2·726	2·728	2·729	2·731	2·733	2·735	2·737	0 0 1	1 1 1	1 1 2
7·5	2·739	2·740	2·742	2·744	2·746	2·748	2·750	2·751	2·753	2·755	0 0 1	1 1 1	1 1 2
7·6	2·757	2·759	2·760	2·762	2·764	2·766	2·768	2·769	2·771	2·773	0 0 1	1 1 1	1 1 2
7·7	2·775	2·777	2·778	2·780	2·782	2·784	2·786	2·787	2·789	2·791	0 0 1	1 1 1	1 1 2
7·8	2·793	2·795	2·796	2·798	2·800	2·802	2·804	2·805	2·807	2·809	0 0 1	1 1 1	1 1 2
7·9	2·811	2·812	2·814	2·816	2·818	2·820	2·821	2·823	2·825	2·827	0 0 1	1 1 1	1 1 2
8·0	2·828	2·830	2·832	2·834	2·835	2·837	2·839	2·841	2·843	2·844	0 0 1	1 1 1	1 1 2
8·1	2·846	2·848	2·850	2·851	2·853	2·855	2·857	2·858	2·860	2·862	0 0 1	1 1 1	1 1 2
8·2	2·864	2·865	2·867	2·869	2·871	2·872	2·874	2·876	2·877	2·879	0 0 1	1 1 1	1 1 2
8·3	2·881	2·883	2·884	2·886	2·888	2·890	2·891	2·893	2·895	2·897	0 0 1	1 1 1	1 1 2
8·4	2·898	2·900	2·902	2·903	2·905	2·907	2·909	2·910	2·912	2·914	0 0 1	1 1 1	1 1 2
8·5	2·915	2·917	2·919	2·921	2·922	2·924	2·926	2·927	2·929	2·931	0 0 1	1 1 1	1 1 2
8·6	2·933	2·934	2·936	2·938	2·939	2·941	2·943	2·944	2·946	2·948	0 0 1	1 1 1	1 1 2
8·7	2·950	2·951	2·953	2·955	2·956	2·958	2·960	2·961	2·963	2·965	0 0 1	1 1 1	1 1 2
8·8	2·966	2·968	2·970	2·972	2·973	2·975	2·977	2·978	2·980	2·982	0 0 1	1 1 1	1 1 2
8·9	2·983	2·985	2·987	2·988	2·990	2·992	2·993	2·995	2·997	2·998	0 0 1	1 1 1	1 1 2
9·0	3·000	3·002	3·003	3·005	3·007	3·008	3·010	3·012	3·013	3·015	0 0 0	1 1 1	1 1 1
9·1	3·017	3·018	3·020	3·022	3·023	3·025	3·027	3·028	3·030	3·032	0 0 0	1 1 1	1 1 1
9·2	3·033	3·035	3·036	3·038	3·040	3·041	3·043	3·045	3·046	3·048	0 0 0	1 1 1	1 1 1
9·3	3·050	3·051	3·053	3·055	3·056	3·058	3·059	3·061	3·063	3·064	0 0 0	1 1 1	1 1 1
9·4	3·066	3·068	3·069	3·071	3·072	3·074	3·076	3·077	3·079	3·081	0 0 0	1 1 1	1 1 1
9·5	3·082	3·084	3·085	3·087	3·089	3·090	3·092	3·094	3·095	3·097	0 0 0	1 1 1	1 1 1
9·6	3·098	3·100	3·102	3·103	3·105	3·106	3·108	3·110	3·111	3·113	0 0 0	1 1 1	1 1 1
9·7	3·114	3·116	3·118	3·119	3·121	3·122	3·124	3·126	3·127	3·129	0 0 0	1 1 1	1 1 1
9·8	3·130	3·132	3·134	3·135	3·137	3·138	3·140	3·142	3·143	3·145	0 0 0	1 1 1	1 1 1
9·9	3·146	3·148	3·150	3·151	3·153	3·15	156	3·158	3·159	3·161	0 0 0	1 1 1	1 1 1

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TABLE 27

SQUARE ROOTS. FROM 10 TO 100

N	0	1	2	3	4	5	6	7	8	9	Mean Differences 1 2 3	4 5 6	7 8 9
10	3·162	3·178	3·194	3·209	3·225	3·240	3·256	3·271	3·286	3·302	2 3 5	6 8 9	11 12 14
11	3·317	3·332	3·347	3·362	3·376	3·391	3·406	3·421	3·435	3·450	1 3 4	6 7 9	10 12 13
12	3·464	3·479	3·493	3·507	3·521	3·536	3·550	3·564	3·578	3·592	1 3 4	6 7 8	10 11 13
13	3·606	3·619	3·633	3·647	3·661	3·674	3·688	3·701	3·715	3·728	1 3 4	5 7 8	10 11 12
14	3·742	3·755	3·768	3·782	3·795	3·808	3·821	3·834	3·847	3·860	1 3 4	5 7 8	9 11 12
15	3·873	3·886	3·899	3·912	3·924	3·937	3·950	3·962	3·975	3·987	1 3 4	5 6 8	9 10 11
16	4·000	4·012	4·025	4·037	4·050	4·062	4·074	4·087	4·099	4·111	1 2 4	5 6 7	9 10 11
17	4·123	4·135	4·147	4·159	4·171	4·183	4·195	4·207	4·219	4·231	1 2 4	5 6 7	8 10 11
18	4·243	4·254	4·266	4·278	4·290	4·301	4·313	4·324	4·336	4·347	1 2 3	5 6 7	8 9 10
19	4·359	4·370	4·382	4·393	4·405	4·416	4·427	4·438	4·450	4·461	1 2 3	5 6 7	8 9 10
20	4·472	4·483	4·494	4·506	4·517	4·528	4·539	4·550	4·561	4·572	1 2 3	4 6 7	8 9 10
21	4·583	4·593	4·604	4·615	4·626	4·637	4·648	4·658	4·669	4·680	1 2 3	4 5 6	8 9 10
22	4·690	4·701	4·712	4·722	4·733	4·743	4·754	4·764	4·775	4·785	1 2 3	4 5 6	7 8 9
23	4·796	4·806	4·817	4·827	4·837	4·848	4·858	4·868	4·879	4·889	1 2 3	4 5 6	7 8 9
24	4·899	4·909	4·919	4·930	4·940	4·950	4·960	4·970	4·980	4·990	1 2 3	4 5 6	7 8 9
25	5·000	5·010	5·020	5·030	5·040	5·050	5·060	5·070	5·079	5·089	1 2 3	4 5 6	7 8 9
26	5·099	5·109	5·119	5·128	5·138	5·148	5·158	5·167	5·177	5·187	1 2 3	4 5 6	7 8 9
27	5·196	5·206	5·215	5·225	5·235	5·244	5·254	5·263	5·273	5·282	1 2 3	4 5 6	7 8 9
28	5·292	5·301	5·310	5·320	5·329	5·339	5·348	5·357	5·367	5·376	1 2 3	4 5 6	7 7 8
29	5·385	5·394	5·404	5·413	5·422	5·431	5·441	5·450	5·459	5·468	1 2 3	4 5 5	6 7 8
30	5·477	5·486	5·495	5·505	5·514	5·523	5·532	5·541	5·550	5·559	1 2 3	4 4 5	6 7 8
31	5·568	5·577	5·586	5·595	5·604	5·612	5·621	5·630	5·639	5·648	1 2 3	3 4 5	6 7 8
32	5·657	5·666	5·675	5·683	5·692	5·701	5·710	5·718	5·727	5·736	1 2 3	3 4 5	6 7 8
33	5·745	5·753	5·762	5·771	5·779	5·788	5·797	5·805	5·814	5·822	1 2 3	3 4 5	6 7 8
34	5·831	5·840	5·848	5·857	5·865	5·874	5·882	5·891	5·899	5·908	1 2 3	3 4 5	6 7 8
35	5·916	5·925	5·933	5·941	5·950	5·958	5·967	5·975	5·983	5·992	1 2 2	3 4 5	6 7 8
36	6·000	6·008	6·017	6·025	6·033	6·042	6·050	6·058	6·066	6·075	1 2 2	3 4 5	6 7 7
37	6·083	6·091	6·099	6·107	6·116	6·124	6·132	6·140	6·148	6·156	1 2 2	3 4 5	6 7 7
38	6·164	6·173	6·181	6·189	6·197	6·205	6·213	6·221	6·229	6·237	1 2 2	3 4 5	6 6 7
39	6·245	6·253	6·261	6·269	6·277	6·285	6·293	6·301	6·309	6·317	1 2 2	3 4 5	6 6 7
40	6·325	6·332	6·340	6·348	6·356	6·364	6·372	6·380	6·387	6·395	1 2 2	3 4 5	6 6 7
41	6·403	6·411	6·419	6·427	6·434	6·442	6·450	6·458	6·465	6·473	1 2 2	3 4 5	5 6 7
42	6·481	6·488	6·496	6·504	6·512	6·519	6·527	6·535	6·542	6·550	1 2 2	3 4 5	5 6 7
43	6·557	6·565	6·573	6·580	6·588	6·595	6·603	6·611	6·618	6·626	1 2 2	3 4 5	5 6 7
44	6·633	6·641	6·648	6·656	6·663	6·671	6·678	6·686	6·693	6·701	1 2 2	3 4 5	5 6 7
45	6·708	6·716	6·723	6·731	6·738	6·745	6·753	6·760	6·768	6·775	1 1 2	3 4 4	5 6 7
46	6·782	6·790	6·797	6·804	6·812	6·819	6·826	6·834	6·841	6·848	1 1 2	3 4 4	5 6 7
47	6·856	6·863	6·870	6·877	6·885	6·892	6·899	6·907	6·914	6·921	1 1 2	3 4 4	5 6 7
48	6·928	6·935	6·943	6·950	6·957	6·964	6·971	6·979	6·986	6·993	1 1 2	3 4 4	5 6 6
49	7·000	7·007	7·014	7·021	7·029	7·036	7·043	7·050	7·057	7·064	1 1 2	3 4 4	5 6 6
50	7·071	7·078	7·085	7·092	7·099	7·106	7·113	7·120	7·127	7·134	1 1 2	3 4 4	5 6 6
51	7·141	7·148	7·155	7·162	7·169	7·176	7·183	7·190	7·197	7·204	1 1 2	3 4 4	5 6 6
52	7·211	7·218	7·225	7·232	7·239	7·246	7·253	7·259	7·266	7·273	1 1 2	3 3 4	5 6 6
53	7·280	7·287	7·294	7·301	7·308	7·314	7·321	7·328	7·335	7·342	1 1 2	3 3 4	5 5 6
54	7·348	7·355	7·362	7·369	7·376	7·382	7·389	7·396	7·403	7·409	1 1 2	3 3 4	5 5 6

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TABLE 27 (Contd.)

SQUARE ROOTS FROM 10 TO 100

N	0	1	2	3	4	5	6	7	8	9	Mean Differences 1 2 3	4 5 6	7 8 9
55	7·416	7·423	7·430	7·436	7·443	7·450	7·457	7·463	7·470	7·477	1 1 2	3 3 4	5 5 6
56	7·483	7·490	7·497	7·503	7·510	7·517	7·523	7·530	7·537	7·543	1 1 2	3 3 4	5 5 6
57	7·550	7·556	7·563	7·570	7·576	7·583	7·589	7·596	7·603	7·609	1 1 2	3 3 4	5 5 6
58	7·616	7·622	7·629	7·635	7·642	7·649	7·655	7·662	7·668	7·675	1 1 2	3 3 4	5 5 6
59	7·681	7·688	7·694	7·701	7·707	7·714	7·720	7·727	7·733	7·740	1 1 2	3 3 4	4 5 6
60	7·746	7·752	7·759	7·765	7·772	7·778	7·785	7·791	7·797	7·804	1 1 2	3 3 4	4 5 6
61	7·810	7·817	7·823	7·829	7·836	7·842	7·849	7·855	7·861	7·868	1 1 2	3 3 4	4 5 6
62	7·874	7·880	7·887	7·893	7·899	7·906	7·912	7·918	7·925	7·931	1 1 2	3 3 4	4 5 6
63	7·937	7·944	7·950	7·956	7·962	7·969	7·975	7·981	7·987	7·994	1 1 2	3 3 4	4 5 6
64	8·000	8·006	8·012	8·019	8·025	8·031	8·037	8·044	8·050	8·056	1 1 2	2 3 4	4 5 6
65	8·062	8·068	8·075	8·081	8·087	8·093	8·099	8·106	8·112	8·118	1 1 2	2 3 4	4 5 6
66	8·124	8·130	8·136	8·142	8·149	8·155	8·161	8·167	8·173	8·179	1 1 2	2 3 4	4 5 5
67	8·185	8·191	8·198	8·204	8·210	8·216	8·222	8·228	8·234	8·240	1 1 2	2 3 4	4 5 5
68	8·246	8·252	8·258	8·264	8·270	8·276	8·283	8·289	8·295	8·301	1 1 2	2 3 4	4 5 5
69	8·307	8·313	8·319	8·325	8·331	8·337	8·343	8·349	8·355	8·361	1 1 2	2 3 4	4 5 5
70	8·367	8·373	8·379	8·385	8·390	8·396	8·402	8·408	8·414	8·420	1 1 2	2 3 4	4 5 5
71	8·426	8·432	8·438	8·444	8·450	8·456	8·462	8·468	8·473	8·479	1 1 2	2 3 4	4 5 5
72	8·485	8·491	8·497	8·503	8·509	8·515	8·521	8·526	8·532	8·538	1 1 2	2 3 3	4 5 5
73	8·544	8·550	8·556	8·562	8·567	8·573	8·579	8·585	8·591	8·597	1 1 2	2 3 3	4 5 5
74	8·602	8·608	8·614	8·620	8·626	8·631	8·637	8·643	8·649	8·654	1 1 2	2 3 3	4 5 5
75	8·660	8·666	8·672	8·678	8·683	8·689	8·695	8·701	8·706	8·712	1 1 2	2 3 3	4 5 5
76	8·718	8·724	8·729	8·735	8·741	8·746	8·752	8·758	8·764	8·769	1 1 2	2 3 3	4 5 5
77	8·775	8·781	8·786	8·792	8·798	8·803	8·809	8·815	8·820	8·826	1 1 2	2 3 3	4 4 5
78	8·832	8·837	8·843	8·849	8·854	8·860	8·866	8·871	8·877	8·883	1 1 2	2 3 3	4 4 5
79	8·888	8·894	8·899	8·905	8·911	8·916	8·922	8·927	8·933	8·939	1 1 2	2 3 3	4 4 5
80	8·944	8·950	8·955	8·961	8·967	8·972	8·978	8·983	8·989	8·994	1 1 2	2 3 3	4 4 5
81	9·000	9·006	9·011	9·017	9·022	9·028	9·033	9·039	9·044	9·050	1 1 2	2 3 3	4 4 5
82	9·055	9·061	9·066	9·072	9·077	9·083	9·088	9·094	9·099	9·105	1 1 2	2 3 3	4 4 5
83	9·110	9·116	9·121	9·127	9·132	9·138	9·143	9·149	9·154	9·160	1 1 2	2 3 3	4 4 5
84	9·165	9·171	9·176	9·182	9·187	9·192	9·198	9·203	9·209	9·214	1 1 2	2 3 3	4 4 5
85	9·220	9·225	9·230	9·236	9·241	9·247	9·252	9·257	9·263	9·268	1 1 2	2 3 3	4 4 5
86	9·274	9·279	9·284	9·290	9·295	9·301	9·306	9·311	9·317	9·322	1 1 2	2 3 3	4 4 5
87	9·327	9·333	9·338	9·343	9·349	9·354	9·359	9·365	9·370	9·375	1 1 2	2 3 3	4 4 5
88	9·381	9·386	9·391	9·397	9·402	9·407	9·413	9·418	9·423	9·429	1 1 2	2 3 3	4 4 5
89	9·434	9·439	9·445	9·450	9·455	9·460	9·466	9·471	9·476	9·481	1 1 2	2 3 3	4 4 5
90	9·487	9·492	9·497	9·503	9·508	9·513	9·518	9·524	9·529	9·534	1 1 2	2 3 3	4 4 5
91	9·539	9·545	9·550	9·555	9·560	9·566	9·571	9·576	9·581	9·586	1 1 2	2 3 3	4 4 5
92	9·592	9·597	9·602	9·607	9·612	9·618	9·623	9·628	9·633	9·638	1 1 2	2 3 3	4 4 5
93	9·644	9·649	9·654	9·659	9·664	9·670	9·675	9·680	9·685	9·690	1 1 2	2 3 3	4 4 5
94	9·695	9·701	9·706	9·711	9·716	9·721	9·726	9·731	9·737	9·742	1 1 2	2 3 3	4 4 5
95	9·747	9·752	9·757	9·762	9·767	9·772	9·778	9·783	9·788	9·793	1 1 2	2 3 3	4 4 5
96	9·798	9·803	9·808	9·813	9·818	9·823	9·829	9·834	9·839	9·844	1 1 2	2 3 3	4 4 5
97	9·849	9·854	9·859	9·864	9·869	9·874	9·879	9·884	9·889	9·894	1 1 1	2 3 3	4 4 5
98	9·899	9·905	9·910	9·915	9·920	9·925	9·930	9·935	9·940	9·945	0 1 1	2 2 3	3 4 4
99	9·950	9·955	9·960	9·965	9·970	9·975	9·980	9·985	9·990	9·995	0 1 1	2 2 3	3 4 4

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TABLE 28

RECIPROCALS OF NUMBERS. FROM 1 TO 10

(Numbers in difference columns to be subtracted, not added.)

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1 2 3	4 5 6	7 8 9
1·0	1·0000	9901	9804	9709	9615	9524	9434	9346	9259	9174			
1·1	·9091	9009	8929	8850	8772	8696	8621	8547	8475	8403			
1·2	8333	8264	8197	8130	8065	8000	7937	7874	7813	7752			
1·3	7692	7634	7576	7519	7463	7407	7353	7299	7246	7194			
1·4	7143	7092	7042	6993	6944	6897	6849	6803	6757	6711	5 10 14	19 24 29	33 38 43
1·5	6667	6623	6579	6536	6494	6452	6410	6369	6329	6289	4 8 13	17 21 25	29 33 38
1·6	6250	6211	6173	6135	6098	6061	6024	5988	5952	5917	4 7 11	15 18 22	26 29 33
1·7	5882	5848	5814	5780	5747	5714	5682	5650	5618	5587	3 6 10	13 16 20	23 26 29
1·8	5556	5525	5495	5464	5435	5405	5376	5348	5319	5291	3 6 9	12 15 17	20 23 26
1·9	5263	5236	5208	5181	5155	5128	5102	5076	5051	5025	3 5 8	11 13 16	18 21 24
2·0	·5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2 5 7	10 12 14	17 19 21
2·1	·4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2 4 7	9 11 13	15 17 20
2·2	·4545	4525	4505	4484	4464	4444	4425	4405	4386	4367	2 4 6	8 10 12	14 16 18
2·3	·4348	4329	4310	4292	4274	4255	4237	4219	4202	4184	2 4 5	7 9 11	13 14 16
2·4	·4167	4149	4132	4115	4098	4082	4065	4049	4032	4016	2 3 5	7 8 10	12 13 15
2·5	·4000	3984	3968	3953	3937	3922	3906	3891	3876	3861	2 3 5	6 8 9	11 12 14
2·6	·3846	3831	3817	3802	3788	3774	3759	3745	3731	3717	1 3 4	6 7 8	10 11 13
2·7	3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1 3 4	5 7 8	9 11 12
2·8	·3571	3559	3546	3534	3521	3509	3497	3484	3472	3460	1 2 4	5 6 7	9 10 11
2·9	·3448	3436	3425	3413	3401	3390	3378	3367	3356	3344	1 2 3	5 6 7	8 9 10
3·0	·3333	3322	3311	3300	3289	3279	3268	3257	3247	3236	1 2 3	4 5 6	7 8 10
3·1	·3226	3215	3205	3195	3185	3175	3165	3155	3145	3135	1 2 3	4 5 6	7 8 9
3·2	·3125	3115	3106	3096	3086	3077	3067	3058	3049	3040	1 2 3	4 5 6	7 8 9
3·3	·3030	3021	3012	3003	2994	2985	2976	2967	2959	2950	1 2 3	4 4 5	6 7 8
3·4	2941	2933	2924	2915	2907	2899	2890	2882	2874	2865	1 2 3	3 4 5	6 7 8
3·5	·2857	2849	2841	2833	2825	2817	2809	2801	2793	2786	1 2 2	3 4 5	6 6 7
3·6	·2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1 2 2	3 4 5	5 6 7
3·7	·2703	2695	2688	2681	2674	2667	2660	2653	2646	2639	1 1 2	3 4 4	5 6 6
3·8	2632	2625	2618	2611	2604	2597	2591	2584	2577	2571	1 1 2	3 3 4	5 5 6
3·9	·2564	2558	2551	2545	2538	2532	2525	2519	2513	2506	1 1 2	3 3 4	4 5 6
4·0	·2500	2494	2488	2481	2475	2469	2463	2457	2451	2445	1 1 2	2 3 4	4 5 5
4·1	·2439	2433	2427	2421	2415	2410	2404	2398	2392	2387	1 1 2	2 3 3	4 5 5
4·2	2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1 1 2	2 3 3	4 4 5
4·3	2326	2320	2315	2309	2304	2299	2294	2288	2283	2278	1 1 2	2 3 3	4 4 5
4·4	2273	2268	2262	2257	2252	2247	2242	2237	2232	2227	1 1 2	2 3 3	4 4 5
4·5	2222	2217	2212	2208	2203	2198	2193	2188	2183	2179	0 1 1	2 2 3	3 4 4
4·6	2174	2169	2165	2160	2155	2151	2146	2141	2137	2132	0 1 1	2 2 3	3 4 4
4·7	2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0 1 1	2 2 3	3 4 4
4·8	2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	0 1 1	2 2 3	3 4 4
4·9	2041	2037	2033	2028	2024	2020	2016	2012	2008	2004	0 1 1	2 2 2	3 4 4
5·0	·2000	1996	1992	1988	1984	1980	1976	1972	1969	1965	0 1 1	2 2 2	3 3 4
5·1	·1961	1957	1953	1949	1946	1942	1938	1934	1931	1927	0 1 1	2 2 2	3 3 3
5·2	·1923	1919	1916	1912	1908	1905	1901	1898	1894	1890	0 1 1	1 2 2	3 3 3
5·3	·1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0 1 1	1 2 2	2 3 3
5·4	·1852	1848	1845	1842	1838	1835	1832	1828	1825	1821	0 1 1	1 2 2	2 3 3

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TABLE 28 (Contd.)

RECIPROCALS OF NUMBERS. FROM 1 TO 10

[Numbers in difference columns to be subtracted, not added]

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1 2 3	4 5 6	7 8 9
5·5	·1818	1815	1812	1808	1805	1802	1799	1795	1792	1789	0 1 1	1 2 2	2 3 3
5·6	1786	1783	1779	1776	1773	1770	1767	1764	1761	1757	0 1 1	1 2 2	2 3 3
5·7	·1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0 1 1	1 2 2	2 3 3
5·8	1724	1721	1718	1715	1712	1709	1706	1703	1700	1697	0 1 1	1 2 2	2 3 3
5·9	1695	1692	1689	1686	1684	1681	1678	1675	1672	1669	0 1 1	1 2 2	2 3 3
6·0	1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0 1 1	1 2 2	2 3 3
6·1	1639	1637	1634	1631	1629	1626	1623	1621	1618	1616	0 1 1	1 2 2	2 3 3
6·2	1613	1610	1608	1605	1603	1600	1597	1595	1592	1590	0 1 1	1 2 2	2 3 3
6·3	1587	1585	1582	1580	1577	1575	1572	1570	1567	1565	0 0 1	1 1 2	2 2 2
6·4	·1562	1560	1558	1555	1553	1550	1548	1546	1543	1541	0 0 1	1 1 2	2 2 2
6·5	·1538	1536	1534	1531	1529	1527	1524	1522	1520	1517	0 0 1	1 1 2	2 2 2
6·6	·1515	1513	1511	1508	1506	1504	1502	1499	1497	1495	0 0 1	1 1 2	2 2 2
6·7	·1493	1490	1488	1486	1484	1481	1479	1477	1475	1473	0 0 1	1 1 2	2 2 2
6·8	·1471	1468	1466	1464	1462	1460	1458	1456	1453	1451	0 0 1	1 1 2	2 2 2
6·9	·1449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0 0 1	1 1 2	2 2 2
7·0	·1429	1427	1425	1422	1420	1418	1416	1414	1412	1410	0 0 1	1 1 2	2 2 2
7·1	·1408	1406	1404	1403	1401	1399	1397	1395	1393	1391	0 0 1	1 1 2	2 2 2
7·2	·1389	1387	1385	1383	1381	1379	1377	1376	1374	1372	0 0 1	1 1 2	2 2 2
7·3	1370	1368	1366	1364	1362	1361	1359	1357	1355	1353	0 0 1	1 1 2	2 2 2
7·4	1351	1350	1348	1346	1344	1342	1340	1339	1337	1335	0 0 1	1 1 2	2 2 2
7·5	1333	1332	1330	1328	1326	1325	1323	1321	1319	1318	0 0 1	1 1 2	2 2 2
7·6	1316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0 0 1	1 1 2	2 2 2
7·7	1299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0 0 0	1 1 2	2 2 2
7·8	1282	1280	1279	1277	1276	1274	1272	1271	1269	1267	0 0 0	1 1 2	2 2 2
7·9	·1266	1264	1263	1261	1259	1258	1256	1255	1253	1252	0 0 0	1 1 2	2 2 2
8·0	·1250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0 0 0	1 1 2	2 2 2
8·1	1235	1233	1232	1230	1229	1227	1225	1224	1222	1221	0 0 0	1 1 2	2 2 2
8·2	1220	1218	1217	1215	1214	1212	1211	1209	1208	1206	0 0 0	1 1 2	2 2 2
8·3	·1205	1203	1202	1200	1199	1198	1196	1195	1193	1192	0 0 0	1 1 2	2 2 2
8·4	·1190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0 0 0	1 1 2	2 2 2
8·5	·1176	1175	1174	1172	1171	1170	1168	1167	1166	1164	0 0 0	1 1 2	2 2 2
8·6	·1163	1161	1160	1159	1157	1156	1155	1153	1152	1151	0 0 0	1 1 2	2 2 2
8·7	·1149	1148	1147	1145	1144	1143	1142	1140	1139	1138	0 0 0	1 1 2	2 2 2
8·8	·1136	1135	1134	1133	1131	1130	1129	1127	1126	1125	0 0 0	1 1 2	2 2 2
8·9	·1124	1122	1121	1120	1119	1117	1116	1115	1114	1112	0 0 0	1 1 2	2 2 2
9·0	·1111	1110	1109	1107	1106	1105	1104	1103	1101	1100	0 0 0	1 1 2	2 2 2
9·1	·1099	1098	1096	1095	1094	1093	1092	1090	1089	1088	0 0 0	0 1 1	1 1 2
9·2	·1087	1086	1085	1083	1082	1081	1080	1079	1078	1076	0 0 0	0 1 1	1 1 2
9·3	·1075	1074	1073	1072	1071	1070	1068	1067	1066	1065	0 0 0	0 1 1	1 1 2
9·4	·1064	1063	1062	1060	1059	1058	1057	1056	1055	1054	0 0 0	0 1 1	1 1 2
9·5	·1053	1052	1050	1049	1048	1047	1046	1045	1044	1043	0 0 0	0 1 1	1 1 2
9·6	1042	1041	1039	1038	1037	1036	1035	1034	1033	1032	0 0 0	0 1 1	1 1 2
9·7	·1031	1030	1029	1028	1027	1026	1025	1024	1022	1021	0 0 0	0 1 1	1 1 2
9·8	·1020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0 0 0	0 1 1	1 1 2
9·9	·1010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0 0 0	0 0 1	1 1 1

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