CURVES

Curves are usually employed in lines of communication in order that the change of direction at the intersection of the straight line shall be gradual.

The lines connected by the curves are tangential to it and are called as tangents or straights.

NECESSITY OF CURVES:-

Straight route of road or track is always desirable, since it provides economy in construction, transportation and maintenance.

But when there is change in alignment or gradient of road or track, then it becomes a need to provide curves under following circumstances

1. Excessive cutting and filling can be prevented by providing the change in alignment by curves.
2. The obstruction which came in the way of straight alignment can be made easier by providing by pass with the help of curves.
3. In the straight route gradient are made more comfortable and easy providing diversions with help of curves.
4. In the straight route costly land comes in the way then it can avoided by providing diversions with the help of curves.

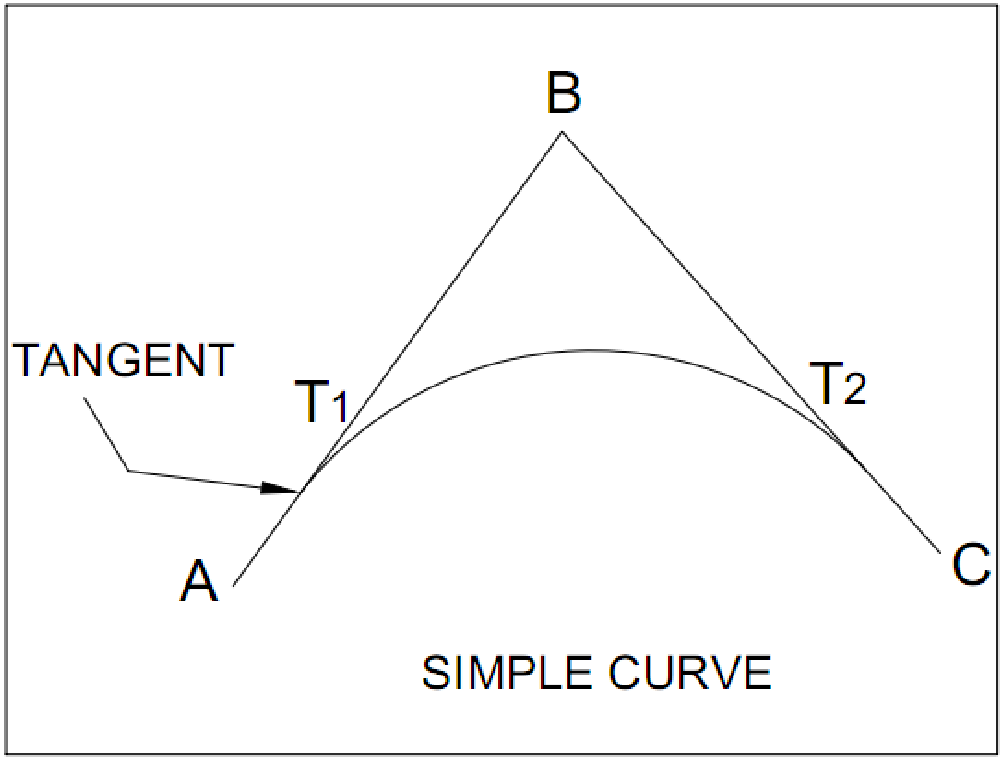
TYPES OF CURVES:-

- HORIZONTAL CURVES
  - SIMPLE CURVE
  - COMPOUND CURVE
  - REVERSE CURVE
  - TRANSITION CURVE
  - COMBINED CURVE
- VERTIAL CURVES
  - SUMMIT CURVE
  - VALLEY CURVE

1. SIMPLE CURVE;-

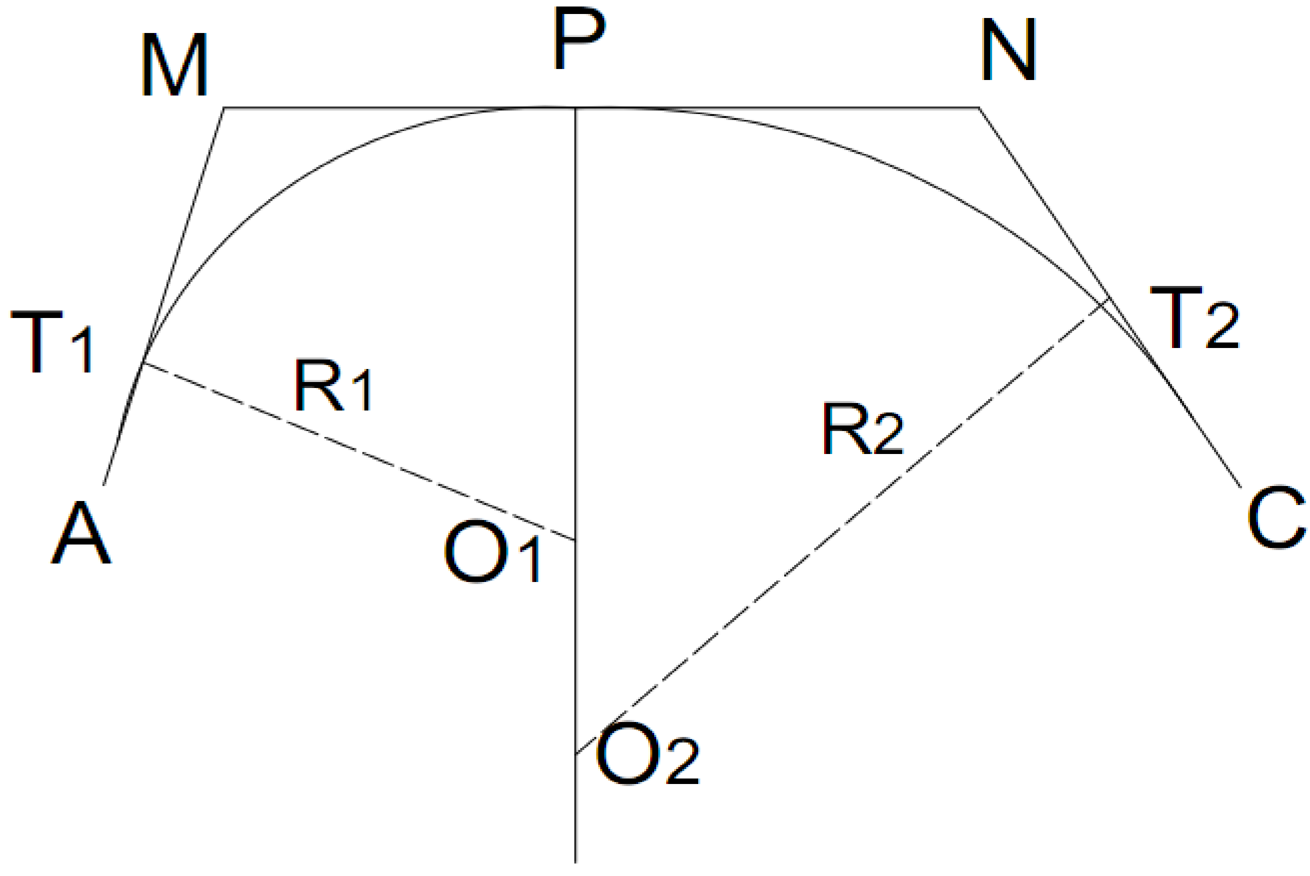
A simple curve consist of a single arc connecting two straights or tangents.

simple curve is normally represented by the length of its radius or by the degree of curve.



2. COMPOUND CURVE;-

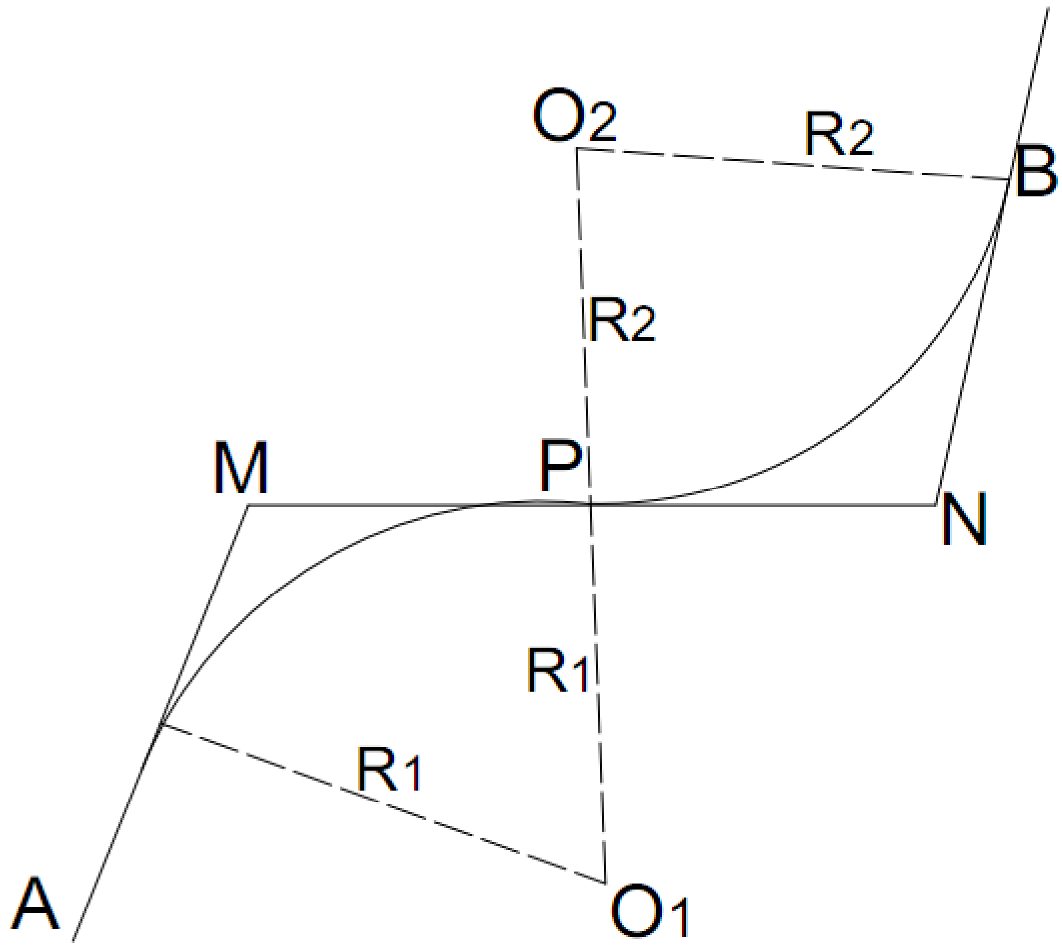
A compound curve consist of two arcs of different radii curving in the same direction and lying on the same side of their common tangent , their centers being on the same side of the curve



COMPOUND CURVE

3. REVERSE CURVE;-

A reverse curve is composed of two arcs of equal or different radii bending or curving in opposite direction with common tangent at their junction, their centers being in opposite sides of the curve.

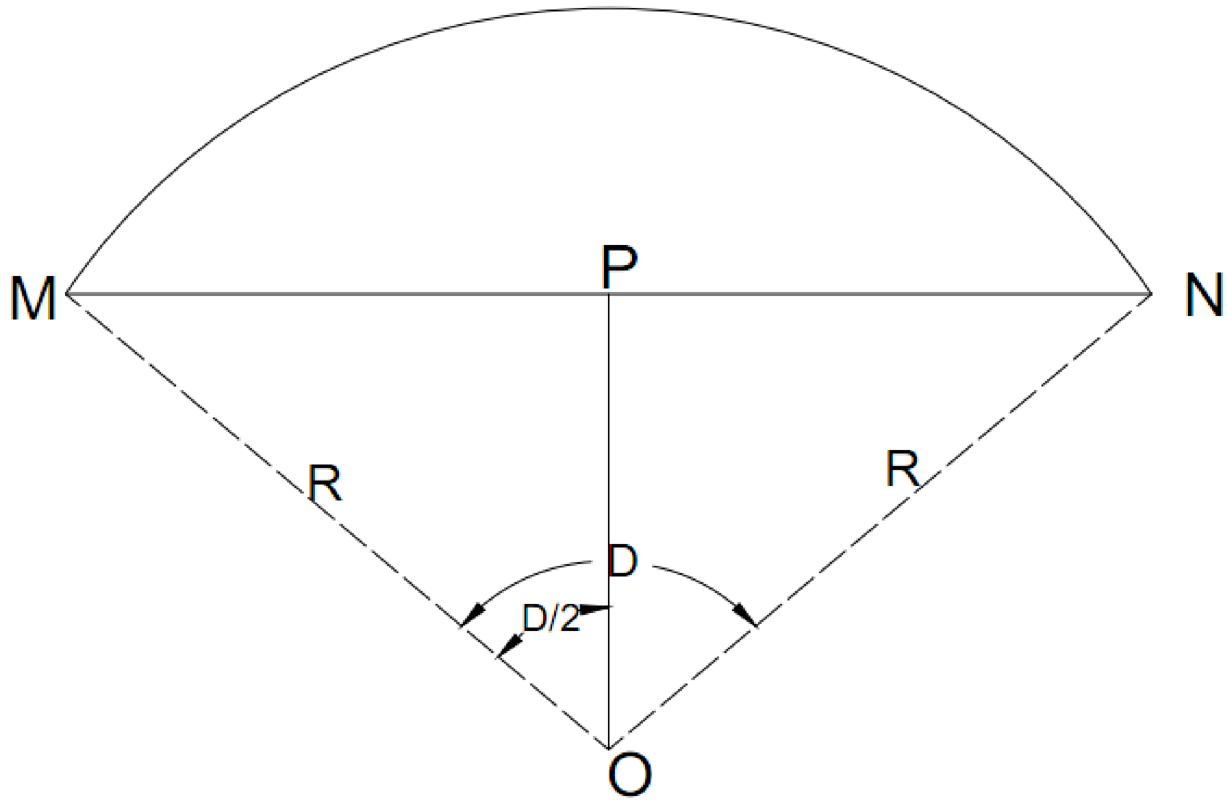


NOMENCLATURE OF CURVE

A curve is designated by the angle subtended by a chord of a specified length or by the radius.

In England the usual method of designating the curve is by its radius e.g. 400 m radius curve. Sometimes it is designated by a number of degrees subtended at the centre by a chord of 100 links e.g. 2^{o} curve.

In India so far the standard chord was 100ft. With metric conversion this may be changed to 30 m. However the length of 20 m is more convenient.



Let ,

R = the radius of a curve in m

D = the degrees of the curves

MN = the chord 30 m long

P = its mid point

IN the Δ OMP, OM=R; MP=$^1/_2$MN= 15 m. MOP= D/2

Then,

$$\sin\frac{D}{2} = \frac{MP}{OM} = \frac{15}{R} \quad or \quad R = \frac{15}{\sin\frac{D}{2}} \qquad \text{..........(1)}$$

When D is small, sin (D/2) may be taken approximately equal to D/2 in radians.

$$R = \frac{15}{\frac{D}{2} \times \frac{\pi}{180}} = \frac{15 \times 360}{\pi D} = \frac{1718.89}{D}$$

$$R = \frac{1719}{D}$$

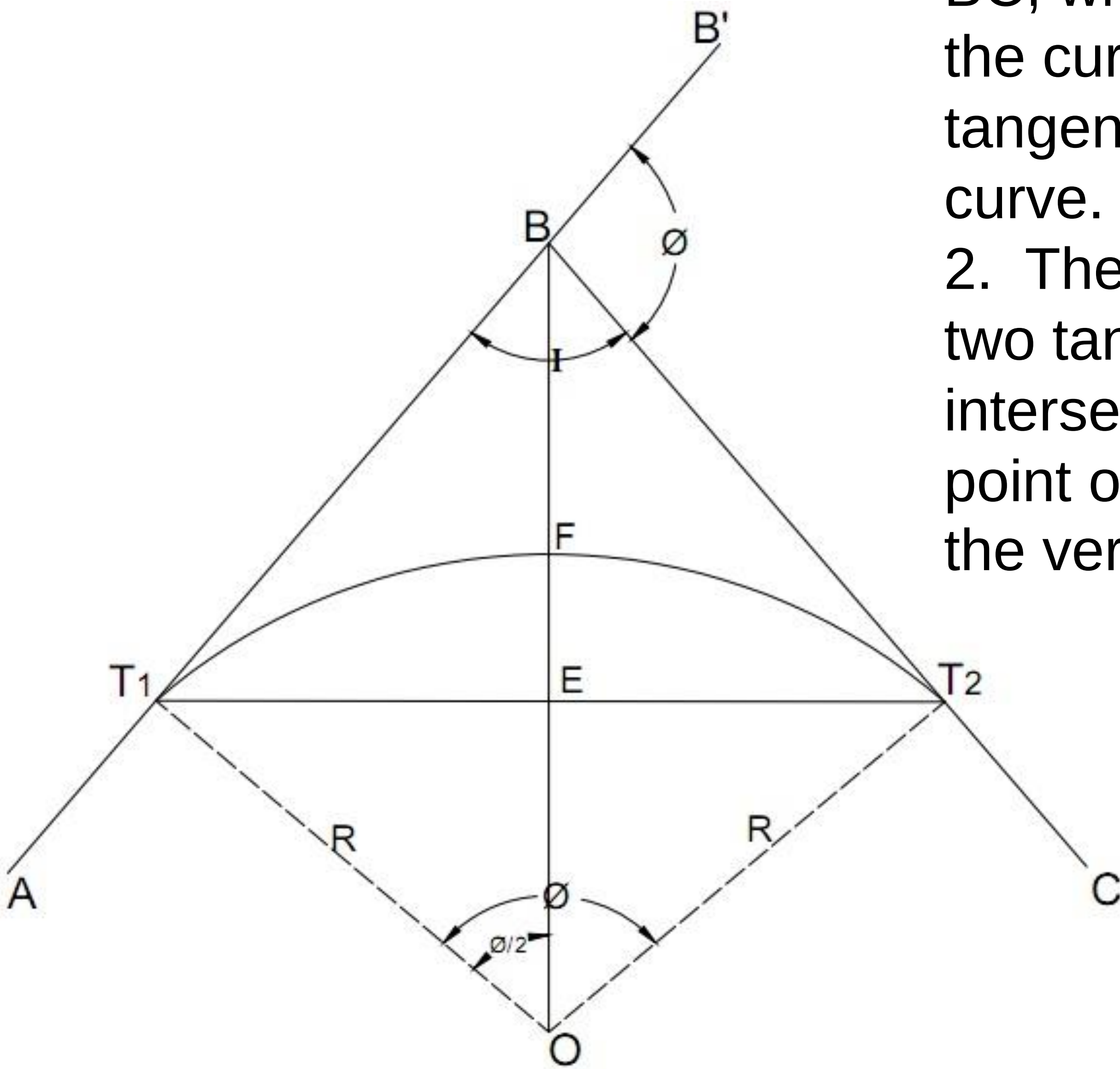
If MN =100 links (20m) and R is expressed in links degree of the curve can be shown as $D = \frac{5729.6}{R}$

With R= 600m = 3000 links

$$D = \frac{5729.6}{3000} = 1.909^0 = 1.91^0$$

A curve of 600 m radius is equivalent to 1.91^0 curve.

Notation for circular curve:-



1. The straight lines AB and BC, which are connected by the curve are called the tangents or straights to the curve.
2. The point B at which the two tangent lines AB and BC intersect is known as the point of intersection (P.I.) or the vertex (V).

3. If the curve deflects to the right of direction of progress of survey (AB), it is called as right hand curve, if to the left , it is called as left hand curve.
4. The tangent line AB is called the first tangent or rear tangent (also called the back tangent)
5. The tangent line BC is called as the second tangent or forward tangent.
6. The points (T_1 and T_2) at which the curve touches the straights are called tangent point(T.P.). The beginning of the curve (T_1) Is called the point of curve.(P.C.) or the tangent curve (T.C.). The end of the curve (T_2) is known as the point of tangency(P.T.) or the curve tangent(C.T.).
7. The ∠ABC between the tangent lines AB and BC is called the angle of intersection (**I**). The ∠B'BC (i.e. the angle by which the forward tangent deflects from the rear tangent) is known as the deflection angle (ø) of the curve.

8. The distance from the point of intersection to the tangent point is called the tangent distance or tangent length. (BT_1 and BT_2).
9. The line T_1T_2 joining the two point (T_1 and T_2) is known as the long chord.(L).
10. The arc T_1FT_2 is called the length of the curve.(ℓ).
11. The mid point F of the arc T_1FT_2 is known as the apex or the summit of the curve and lies on the bisector of the angle of the intersection.
12. The distance BF from the point of the intersection to the apex of the curve is called the apex distance of external distance.
13. The angle T_1OT_2 subtended at the centre of curve by the arc T_1FT_2 is known as the central angle, and is equal to the deflection angle.(ø)
14. The intercept EF on the line OB between the apex (F) of the curve and the midpoint (E) of the long chord is called the versed sine of the curve.

ELEMENTS OF THE CURVE;-

$T_1BT_2 + B'BT_2 = 180^0$ or $I + \varnothing = 180^0$ ………….(2)

The angle $T_1OT_2 = 180^0 - I = \varnothing$ ……………(3)

Tangent length = $BT_1 = BT_2 = OT_1 \tan(\varnothing/2) = R \tan(\varnothing/2)$ … ..(4)

Length of the chord (L) = $2T_1E = 2OT_1 \sin(\varnothing/2) = 2R \sin(\varnothing/2)$…(5)

Length of the curve(ℓ) = length of the arc T_1FT_2

$= R \times \varnothing$ (in radians) $= (\pi R\varnothing) / 180^0$ …………….(6)

If the curve be designated by the degrees of the curve(D),

Length of the curve =$(30\ \varnothing)/D$ (30 m chord) ………….(6a)

$= (20\ \varnothing)/D$ (20 m chord) …………..(6b)

Apex distance = BF = BO – OF = $OT_1 \sec(\varnothing/2)$ – OF

$$= R\left(\sec\frac{\varnothing}{2} - 1\right) = R\ exsec\frac{\varnothing}{2} \quad \text{……….(7)}$$

Versed sine of the curve = EF = OF – OE=OF – OT1 COS(Ø/2)

$$= R\left(1 - cos\frac{Ø}{2}\right) = R\ versine\ \frac{Ø}{2} \quad \ldots\ldots(8)$$

<u>Methods of curve ranging:-</u>

The methods of setting out curves may be divided in two classes according to the instrument employed.

1. **Linear or chain and tape method:-**

   Linear methods are those in which the curve is set out with chain and tape only.

2. **Angular or instrumental method:-**

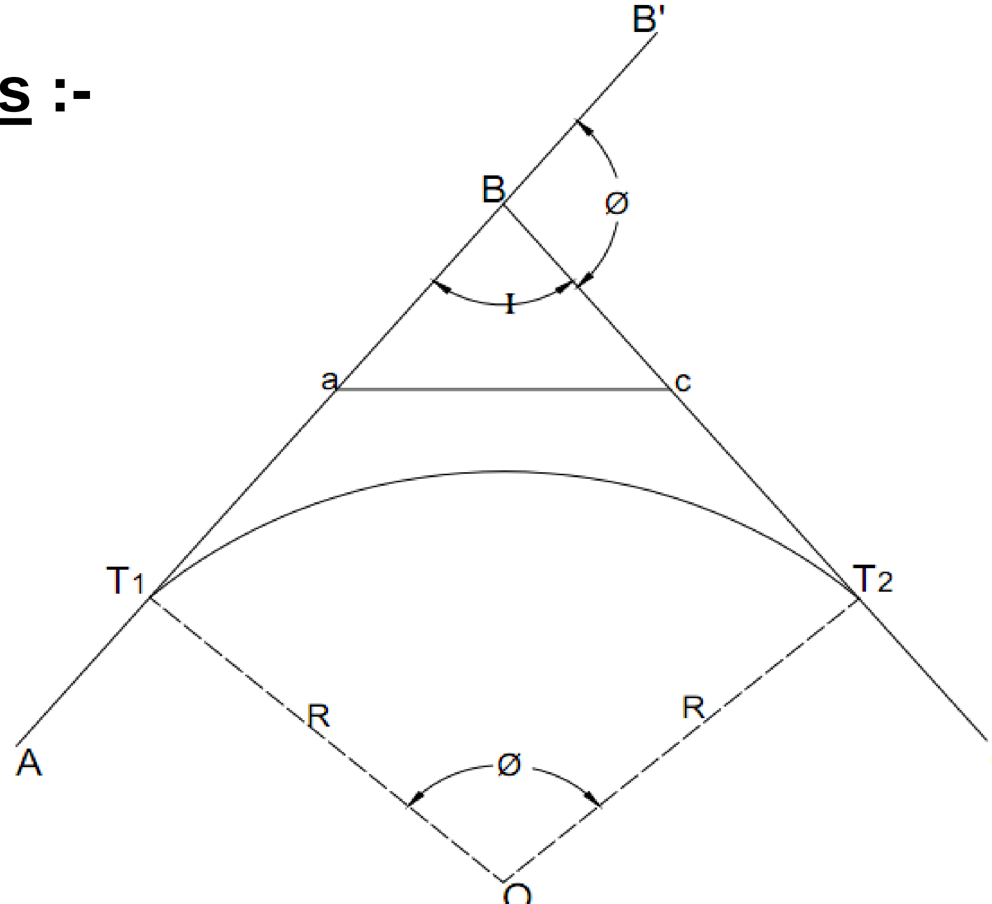
   instrumental methods are those in which theodolite with or without a chain is employed to set out curve.

Peg interval :-

It is the usual practice to fix pegs at equal interval on the curve as along the straight. The interval between the peg is usually 20 to 30 m. strictly speaking this interval must be measured as the arch intercept between them. However as it is necessarily measured along the chord, the curve consist of series of a chord rather than of arcs. In other words, the length of the chord is assume to be equal to be that of the arc. In order that the difference in length between the arc and chord may be negligible, the length of the chord should not be more than 1/20th of the radius of the curve.

The length of unit chord (peg interval) is, therefore, 30m for flat curve, 20m for sharp curve, and 10 m or less for very sharp curve. When the curve is of a small radius, the peg interval are considered to be along the arc and the length of the corresponding chords are calculated to locate the pegs.

<u>**Location of tangent points**</u> **:-**



To locate the tangent point T_1 and T_2 proceed as fallows:-

1. Having fixed the direction of the tangents, produce them so as to meet at the point B.
2. Set up the theodolite at the intersection point B and measure the angle of intersection T_1BT_2. Find the deflection angle Ø from the relation I + Ø $=180^0$

3. Calculate the tangent length from the formula (4)

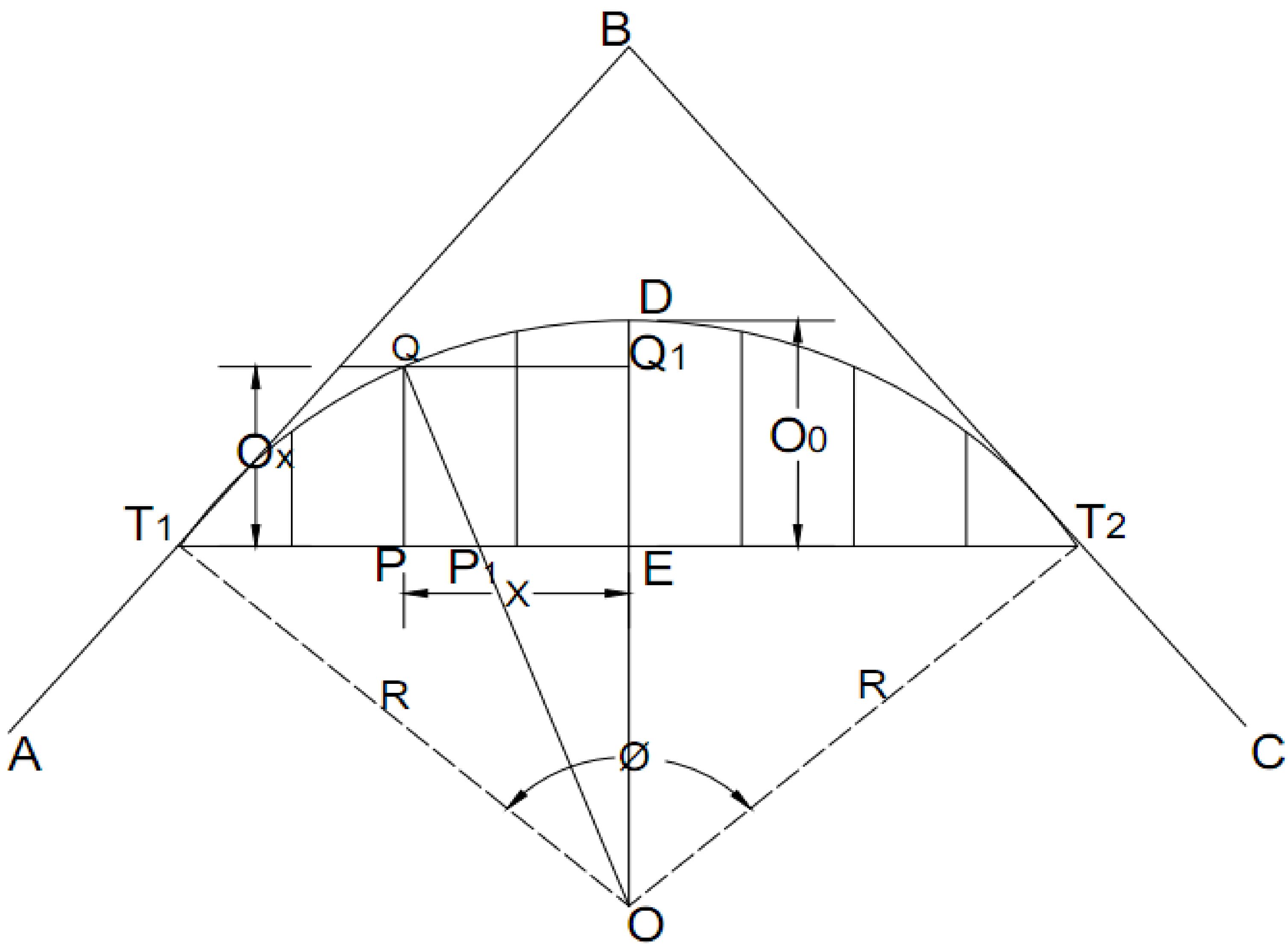
i.e. Tangent length = $BT_1 = BT_2 = OT_1 \tan(Ø/2) = R \tan(Ø/2)$

4. Locate T_1 by measuring the tangent length backward along the rear tangent AB from the intersection point B.

5. Similarly, locate the T_2 by measuring the same distance along the forward tangent BC from B.

Chain and tape (or linear) Methods of setting out curves

1. By offsets or ordinates from the long chord



Let AB and BC = the tangents to the curve T1DT2

T_1 and T_2 = tangent points

T_1T_2 = the long chord of length L.

ED = O_0 = the offsets at the midpoint of T_1T_2 (the versed sine)

PQ = Ox = the offsets at a distance x from E so that EP = x

$OT_1 = OT_2 = OD = R$ = The radius of the curve.

The exact formula for the offset at any point on the long chord may be deduced as follows:-

Draw QQ_1 parallel to T_1T_2, meeting ED at Q_1 join OQ cutting TT_1 in P_1.

Now in the ΔOT1E, $OT_1 = R$; $T_1E = (L/2)$;

$OE = OD - ED = R - O_0$,

$OT_1^2 = T_1E^2 + OE^2$ or $R^2 = (L/2)^2 + (R - O_0)^2$

$$O_0 = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2} \qquad \text{.......................(9)}$$

Of the three quantities O_0, L and R, two quantities L and R, or L and O_0 are usually known. The remaining unknown may be calculated from the above formula.

From the ΔOQQ_1, $OQ^2 = OQ_1^2 + OQ_1^2$.

But $QQ_1 = OE + EQ_1 = OE + Ox = (R-O_0) + Ox$

$QQ_1 = x$; $OQ = R$.

$$R^2 = x^2 + \{(R - O_0) + Ox\}^2$$

Or $Ox + (R – O0) = \sqrt{R^2 – Ox^2}$

Hence $Ox = \sqrt{R^2 – x^2} - (R – O0)$ (exact) ………..(10)

When the radius of curve is large as compared with the length of the long chord, the offsets may be calculated from the approximate formula, which may be deduced as fallows,

In this case PQ is very nearly equal to the radial ordinate QP_1

Then $QP_1 \times 2R = T_1P \times PT_2$

Now, $T_1P = x$; $T_1T_2 = L$ hence $PT_2 = L – x$; $QP_1 = Ox$.

$$O_x = \frac{x(L - x)}{2R}$$ (Approximate) ………..(11)

Procedure :-

To set out the curve ,

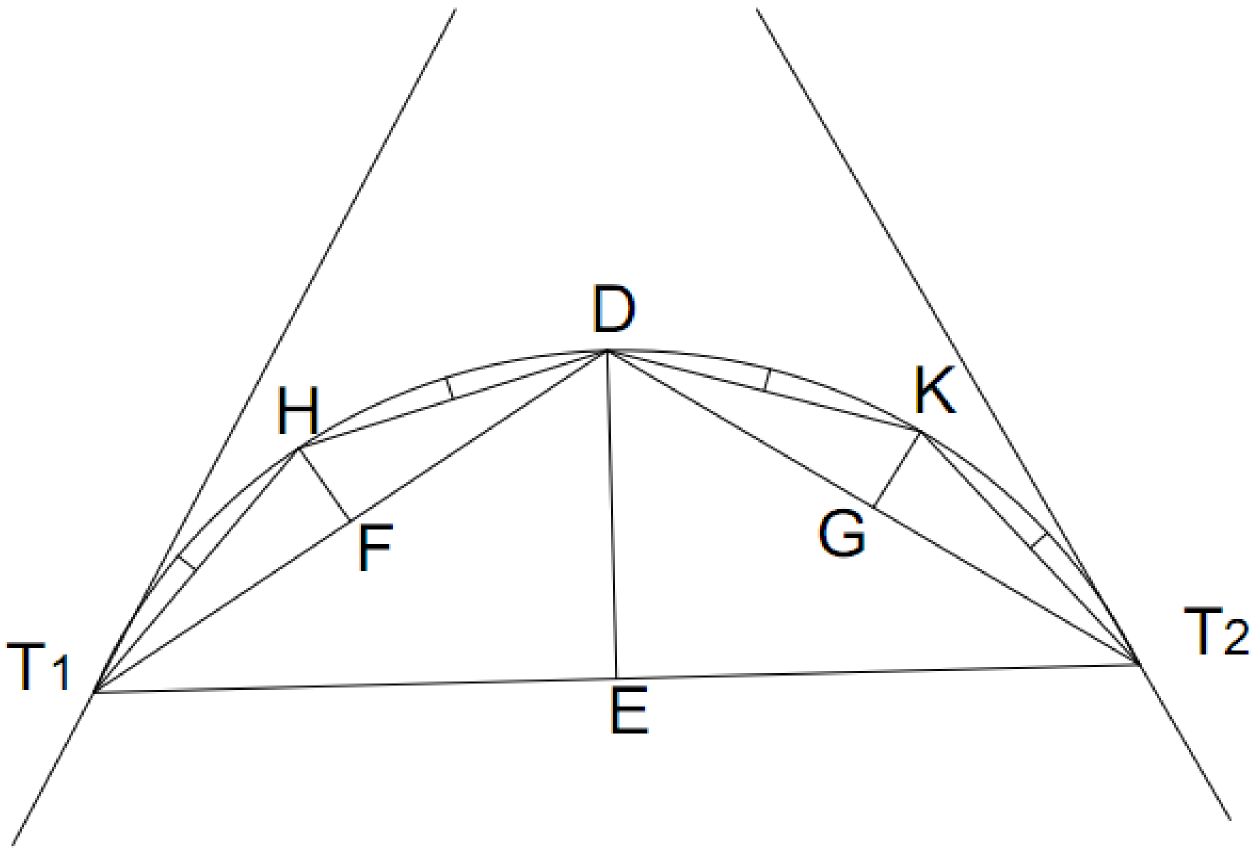
i. Divide the long chord into an even number of equal parts.
ii. Set out the offsets as calculated from formula no.10 at each of the points of division, thus obtaining the required points on the curve. Since the curve is symmetrical along ED, the offsets for the right half of the curve will be same as those for the left half.

If the offsets are calculated from formula (11), the long chord should be divided into a convenient number of equal parts and the calculated offsets set out at each points of division.

This method is usually adopted for setting out short curves.

e.g. curves for street kerbs.

2. **By successive bisections of arcs:-**



Let, T_1 and T_2 be the tangent points. Join T1and T2 and bisect at E.

Set out the offset ED $= R\left(1 - cos\frac{\emptyset}{2}\right) = R\ versine\ \frac{\emptyset}{2}$, Thus determining the point D on the curve. Join T_1D and DT_2 and bisect them at F and G respectively.

At F and G set out the offsets FH and GK each equal to R [1 – cos(ø/4)], thus obtaining two more points H and K on the curve, By repeating the process, set out as many points as required.

3. <u>By offsets from the tangents</u>:-

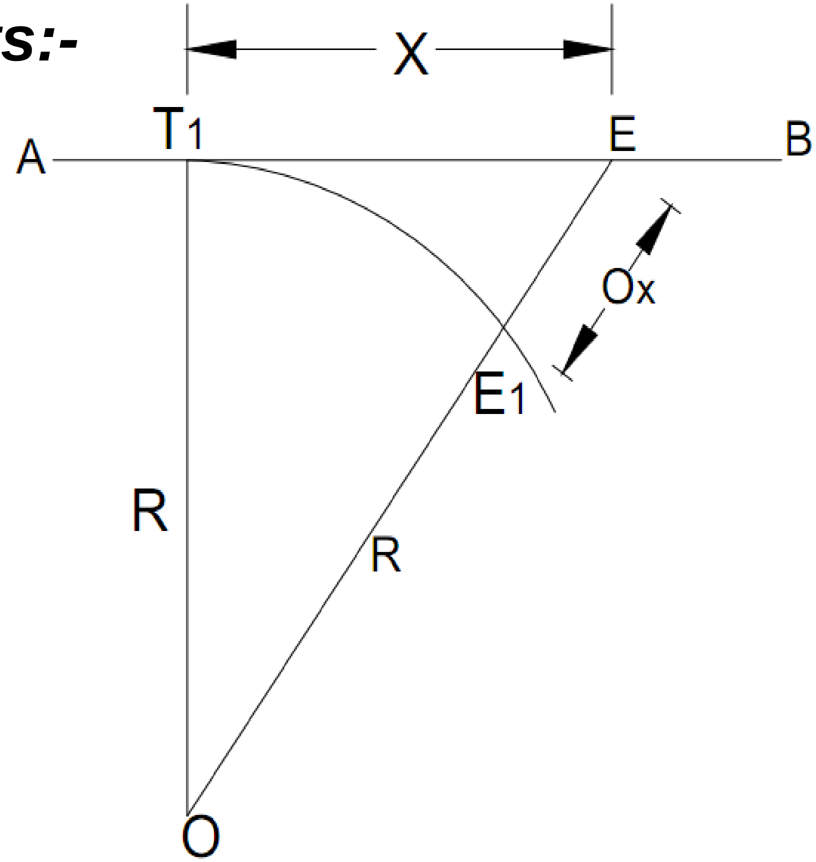
In this method the offsets are set out either radially or perpendicular to the tangents BA and BC according as the centre (O) of the curve is accessible or inaccessible.

a) By Radial offsets:-

Let T_1 be the first tangent point

EE1 = Ox= the radial offset at E at a distance of x from T1 along the tangent AB.

a) By Radial offsets:-



Let, T_1 be the first tangent point

$EE_1 = O_x$ = the radial offset at E at a distance of x from T_1 along the tangent AB.

Now in the ΔOT_1E, $OT_1 = R$; $T_1E = x$; $OE = OE_1 + EE_1 = R + O_x$

Now, $OE^2 = OT_1^2 + T_1E^2$

$(R + O_x)^2 = R^2 + x^2$. i.e. $O_x = \sqrt{R^2 - x^2} - R$ ……………….(12)

When the radius is large, the offsets may be calculated by the approximate formula, which may be deduced as follows :

$$ET_1^2 = EE_1 \times (2R + EE_1) \quad \text{i.e. } x^2 = O_x \times (2R - O_x)$$

Since O_x^2 is very small as compared with 2R, it may be neglected.

$$\therefore\ O_x = (x^2 / 2R) \text{ ………………….(13)}$$

The formula may be obtained from the exact ones thus :

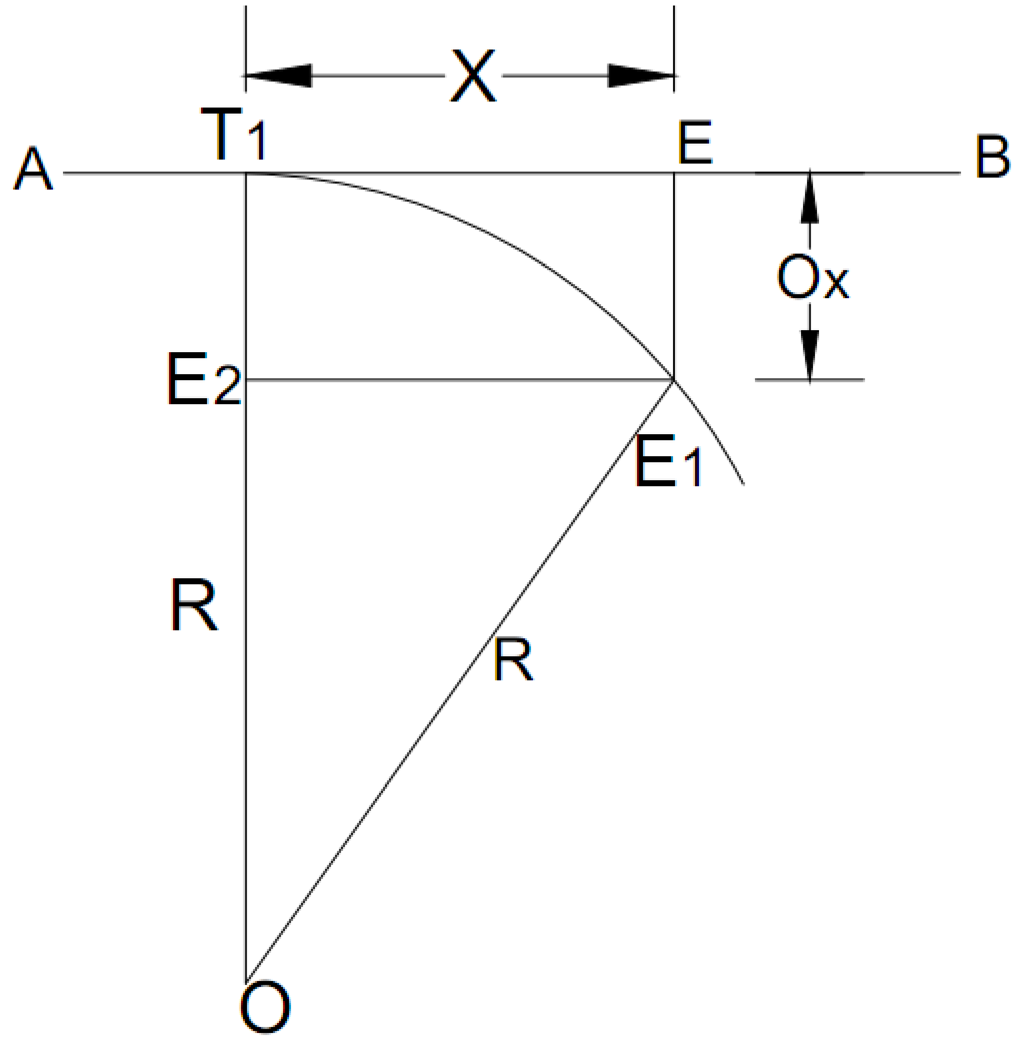
Expanding the factor = $\sqrt{R^2 - x^2}$, we have

$$O_x = R\left(1 + \frac{x^2}{2R^2} + \frac{x^4}{8R^4} + \cdots\cdots\cdots\right) - R$$

Neglecting the other terms except the first two, we get,

$$O_x = R\left(1 + \frac{x^2}{2R^2}\right) - R = \frac{x^2}{2R}$$

b) By offsets perpendicular to tangents:-



Let, EE_1 be the perpendicular offset at a distance x measured along the tangent AB from the tangent point T_1 so that $T_1E = x$.

Through E_1 draw E_1E_2 parallel to BT_1, meeting OT at E_2.

Then $E_1E_2 = T1E = x$; $T_1E_2 = EE_1 = Ox$;

$OE_2 = OT_1 - T_1E_2 = (R - Ox)$

Now, from ΔOE_1E_2, $OE_1^2 = E_1E_2^2 + OE_2^2$

i.e. $R^2 = x^2 (R - Ox)$ or $Ox = R - \sqrt{R^2 - x^2}$ exact ……(14)

From which, the corresponding approximate formula may be

obtained by expanding the factor $\sqrt{R^2 - x^2}$. Thus, we have

$$O_x = R - R\left(1 - \frac{x^2}{2R^2} - \frac{x^4}{8R^4} - \dots\dots\dots\right)$$

Neglecting higher powers of R^2 we get

$$O_x = R - R\left(1 - \frac{x^2}{2R^2}\right) = \frac{x^2}{2R} \qquad \text{(app)……..(14)}$$

Procedure:-

To set out the curve

i. Locate the tangent points T1 and T2, by measuring a distance equal to the tangent length R tan(ø/2), backward along the tangent BA from the point of intersection(B) and the same distance forward along the tangent BC.

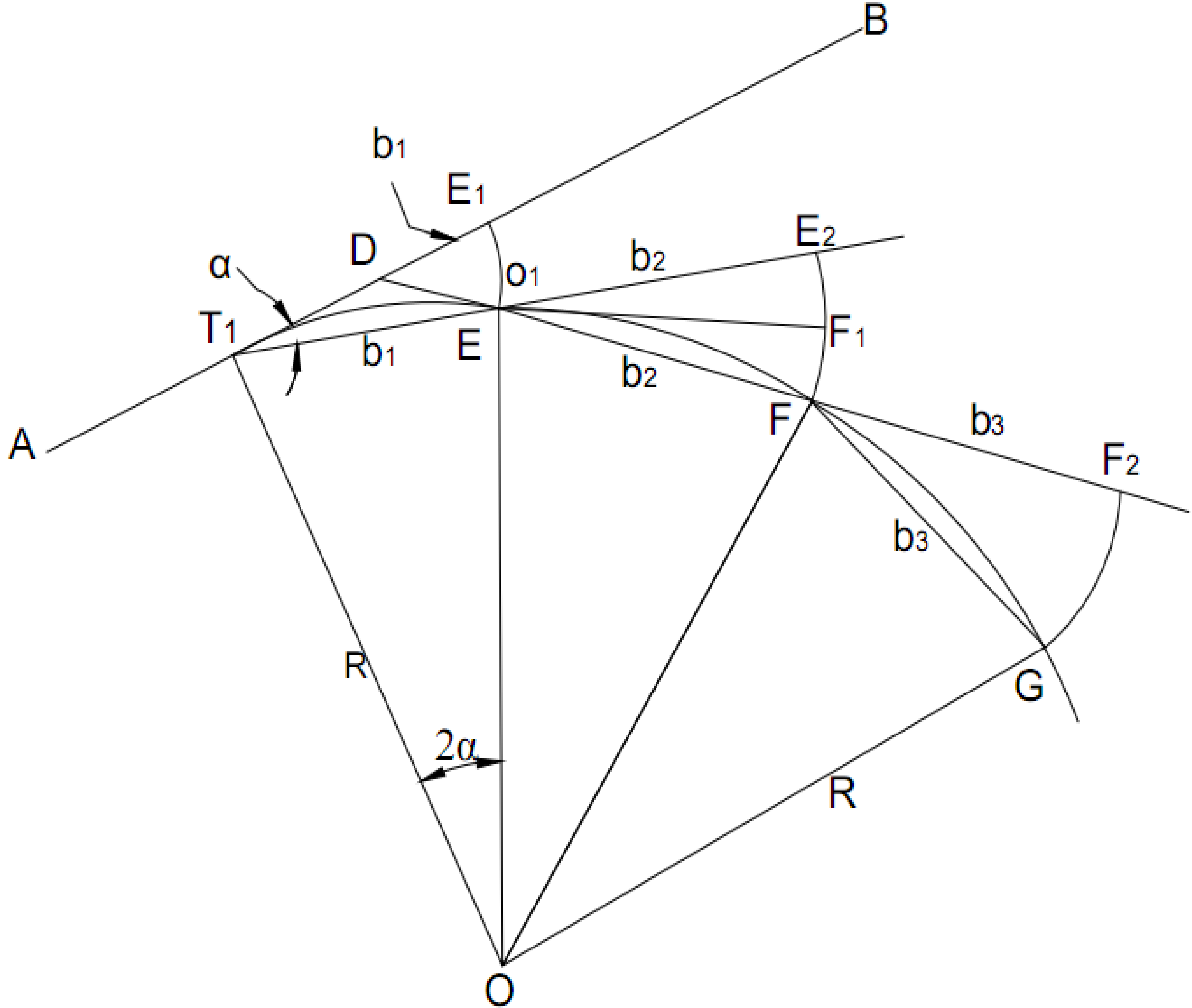
ii. Measure equal distances, say, 20 or 30m along the tangent T_1B from T_1.

iii. Set out the offsets calculated from formula(14) or (13) perpendicular to T_1B at each distance, thus obtaining the required points in the curve.

iv. Continue the process until the apex of the curve is reached.

v. Set out the remaining half of the curve from the second tangent.

4) <u>By offsets from chord produced</u> :-



Let,

AB = the rear tangent ; T_1 = the first tangent point.

E,F,G etc. = the successive points on the curve.

$T_1E_1 = T_1E$ = the first chord of length b_1.

EF,FG, etc = the successive chords of length b_2 ,b_3 etc each being equal to the length of unit chord.

$BT_1E = \alpha$ in radian = the angle between the tangent BT_1 and the first chord TE_1.

$E_1E = O_1$ = the offset from the tangent BT_1.

$E_2F = O_2$ = the offset from the preceding chord T_1E produced.

Draw the tangent DEF_1 at E_1 meeting the rear tangent at D. Produce T_1E to E_2 so that $EE_2 = b_2$.

Let, F_1 be the point of intersection of DEF_1 and E_2F. The formula for the offsets may be deduced as follows:

The angle subtended at the centre (O) of the curve by the arc T_1E is obviously equal to 2α.

But the chord T_1E = arc T_1E very nearly

$$= R \times 2\alpha \quad \text{or} \quad \alpha = (T_1E / 2R)$$

Similarly, the chord E_1E = arc E_1E nearly.

∴ The first offset (O_1) = E_1E = $T_1E \times \alpha$ = $\frac{T_1E^2}{2R} = \frac{b_1^{\,2}}{2R}$ ……(15)

Now ∠E_2EF_1 = ∠DET_1 (vertically opposite);

∠DET_1 = ∠DT_1E, since DT_1 = DE, both being tangent to the circle

∴ ∠E_2EF_1 = ∠DT_1E = ∠E_1T_1E

The Δs E_1T_1E and E_2EF_1 being nearly isosceles, may be considered approximately similar.

$$\frac{E_2F_1}{EE_2} = \frac{E_1E}{T_1E} \quad \text{i.e.} \quad \frac{E_2F_1}{b_2} = \frac{O_1}{b_1}$$

$$\text{or} \quad E_2F_1 = \frac{b_2 \times O_1}{b_1} = \frac{b_2}{b_1} \times \frac{b_1^{\,2}}{2R} = \frac{b_2\, b_1}{2R}$$

F1F being the offset from the tangent at E, is equal to

$$\frac{EF^2}{2R} = \frac{b_2^{\,2}}{2R}$$

Now the second offset (O_2) $E_2F = E_2F_1 + F_1F$

$$= \frac{b_2 \times b_1}{2R} + \frac{{b_2}^2}{2R} = \frac{b_2\,(b_1 + b_2\,)}{2R} \qquad \text{…………………..(17)}$$

Similarly, the third offset (O_3) $= \frac{b_3\,(b_2+b_3\,)}{2R} = \frac{{b_2}^2}{R}$ …………..(18)

Since $b_2 = b_3 = b_4 =$…………

Each of the successive offsets O_4, O_5, etc. except the last one (O_n) is equal to O_3. Since the length of the last chord is usually less than the length of unit chord (b_2),

The last offset $O_n = \frac{b_n\,(b_{n-1} + b_n)}{2R}$ …………..(19)

Mode of Procedure:-

i. Having the fixed directions of the tangents AB and BC, locate the first tangent point T_1 by measuring backward distance equal to the length (R tan(Ø/2)), along the tangent from the point of intersection (B).
   Similarly, mark the other tangent point T_2 by measuring forward the same distance along the tangent BC.

ii. Measure the distance equal to the length of the first chord (T_1E) along T_1B. Thus marking the point E_1.

iii. With the zero end of the chain (or tape) pinned down at T_1, swing the portion of the chain ($=T_1E_1$) around the point T_1 through the calculated offset O_1, thus fixing the first point E on the curve.

iv. Pull the chain forward in the direction of T_1E produced, until EE_2 equals the length (1 chain or ½ chain) of the second chord (b_2).

v. Hold fast the zero end of the chain at E and swing the chain around E through the second calculated offset O_2, thus locating the second point F on the curve.

vi. Repeat the process until the end of the curve is reached. The last point thus fixed should coincide with the previously located point T2. If not, find the closing error. If it is large, the whole curve must set out again . But if it is small, all the points are moved sideways by an amount proportional to the square of their distances from the beginning of the curve (T1) thus distributing the closing error among all the points.

This method is largely used for road curves. It gives better result than those obtained by the preceding method. It can be used in confined situations, since all the work is done in the immediate proximity of the curve. The most serious objection to this method is that if any point is inaccurately fixed, its error is carried forward through all the subsequent points.

INSTRUMENTAL OR ANGULAR METHODS

1. Rankine's method of tangential angle:-

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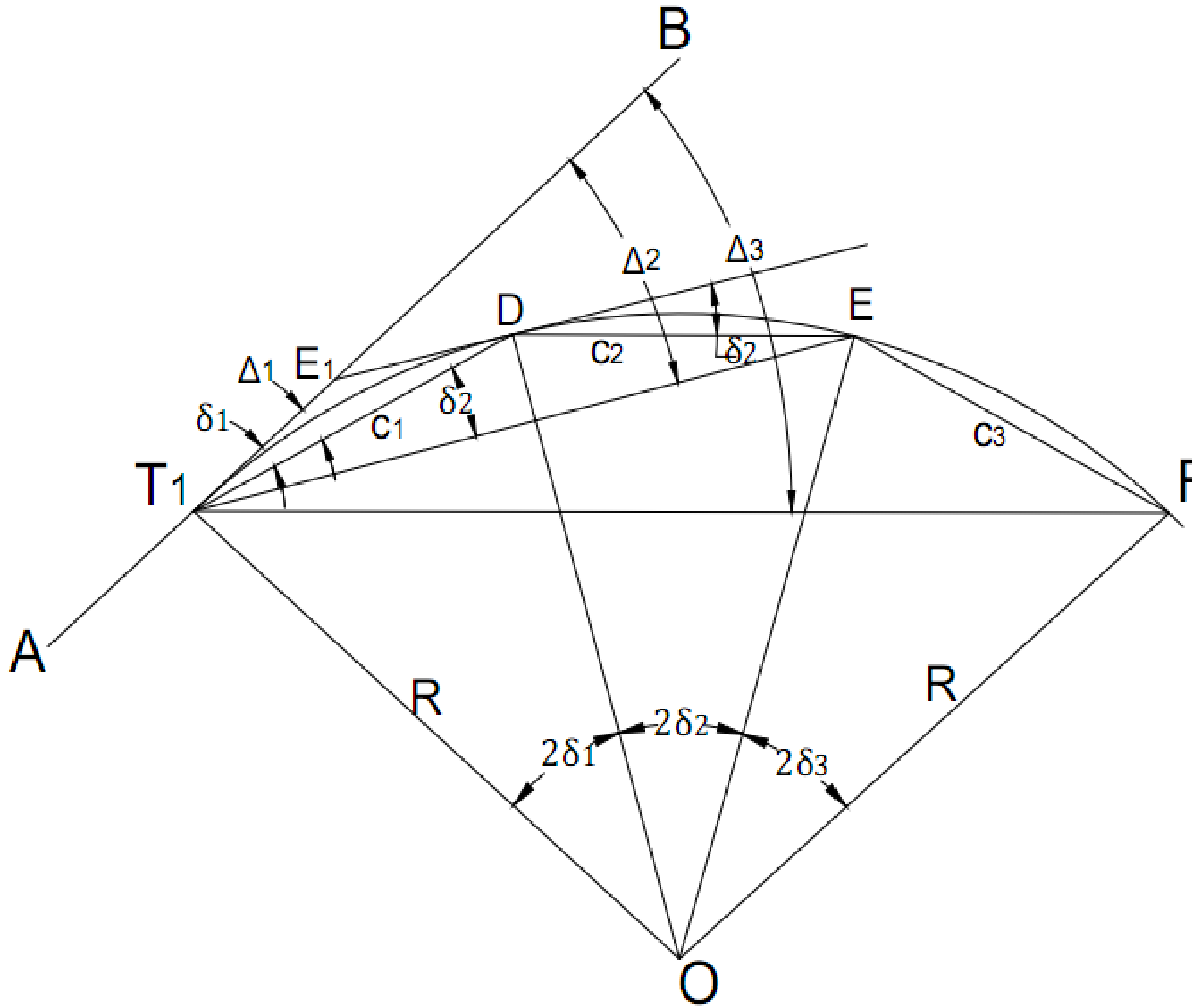


In this method the curve is set out by the tangential angles(often called the deflection angles) with a theodolite and a chain or tape.

The derivation of the formula for calculating the deflection angle it as fallows :-

Let,

AB = the rear tangent to the curve.

T_1 and T_2 = the tangent points

D,E,F, etc. = the successive points on the curve.

$\delta_1, \delta_2, \delta_3$, etc = the tangential angles which each of the successive chord T_1D, DE. EF etc makes with the respective tangents at T_1, D, E, etc.

$\Delta_1, \Delta_2, \Delta_3$, etc = the total tangential or deflection angles (between the rear tangent AB and each of the lines T_1D, DE, EF, etc.

c_1, c_2, c_3 etc = the lengths of the chord T_1D, DE, EF, etc.

R = radius of the curve

Chord T_1D = arc T_1D (very small) = c_1.

$BT_1D = \delta_1 = \frac{1}{2} T_1OD$ i.e. $T_1D = 2\delta_1$

NOW, $\frac{T_1OD}{c_1} = \frac{180°}{\pi R}$ i.e. $T_1OD = \frac{180° c_1}{\pi R} \therefore 2\delta_1 = \frac{180° c_1}{\pi R}$

Hence, $\delta_1 = \frac{90° c_1}{\pi R} Degrees = \frac{90 \times 60\ c_1}{\pi R}\ minutes$

$$\delta_1 = 1718.9 \frac{c_1}{R}\ minutes \qquad \ldots\ldots\ldots\ldots\ldots(20)$$

Similarly, $\delta_2 = 1718.9 \frac{c_2}{R}$; $\delta_3 = 1718.9 \frac{c_3}{R}$; and so on.

$$\delta_n = 1718.9 \frac{c_n}{R} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20\ a)$$

Since the chord lengths c_2, c_3, …….c_{n-1} is equal to the length of the unit chord (peg interval), $\delta_2 = \delta_3 = \delta_4 = \delta_{n-1}$.

Now, the total tangential (deflection) angle ($\Delta 1$) for the first chord (T_1D) = BT_1D. $\therefore \Delta_1 = \delta_1$.

The total tangential angle (Δ_2) for the second chord (DE) $= BT_1E$.

But $BT_1E = BT_1D + DT_1E$.

Now the angle DT1E is the angle subtended by the chord DE in the opposite segment and therefore, equals the tangential angle ($\delta 2$) between the tangent at D and the chord DE.

Therefore, $\Delta_2 = \delta_1 + \delta_2 = \Delta_1 + \delta_2$

Similarly, $\Delta_3 = \delta_1 + \delta_2 + \delta_3 = \Delta_2 + \delta_3$

$\Delta_n = \delta_1 + \delta_2 + \delta_3 + \ldots\ldots\ldots\ldots + \delta_n$

$\Delta_n = \Delta_{n-1} + \delta_n$ ……………………………..(21)

Check;- The total deflection angle (BT_1T_2) $= \Delta n = (\text{ø}/2)$

where ø is the deflection angle of the curve.

From the above, it will be seen that the deflection angle (Δ) for any chord is equal to the deflection angle for preceding chord plus the tangential angle for the chord/

If the degree of the curve (D) be given, the deflection angle for 30m chord is equal to ½ D degrees, and that for the sub chord is equal to $(c_1 \times D)/60$ degrees, where c_1 is the length of the first chord , If the degree of the curve (D) be given, the deflection angle for 30m chord is equal to ½ D degrees, and that for the sub chord is equal to $(c_1 \times D)/60$ degrees,

where c_1 is the length of the first chord,

Hence, $$\delta_1 = \frac{c_1 \times D}{60};\ \delta_2 = \delta_3 \ldots\ldots\ldots = \delta_{n-1} = \frac{D}{2}$$

$$\delta_n = \frac{c_n \times D}{60} \qquad \ldots\ldots\ldots\ldots\ldots (22)$$

In the case of left hand curve each of the values $\Delta 1, \Delta 2, \Delta 3$, etc should be subtracted from 360^0 to obtain required value to which the vernier of the instrument is to be set.

Procedure:-

To set out a curve

i. Set up the theodolite over first tangent point (T_1) and level it.

ii. With both plates clamped at zero, direct the telescope to the ranging rod at the point of intersection B and bisect it.

iii. Release the vernier plate and set the vernier A to first deflection angle (Δ_1), the telescope being thus directed along T1D.

iv. Pin down the zero end of the chain or tape at T_1, and holding the arrow at the distance on the chain equal to the length of the first chord, swing the chain around T_1 until the arrow is bisected by the cross-hairs, thus fixing the first point D on the curve.

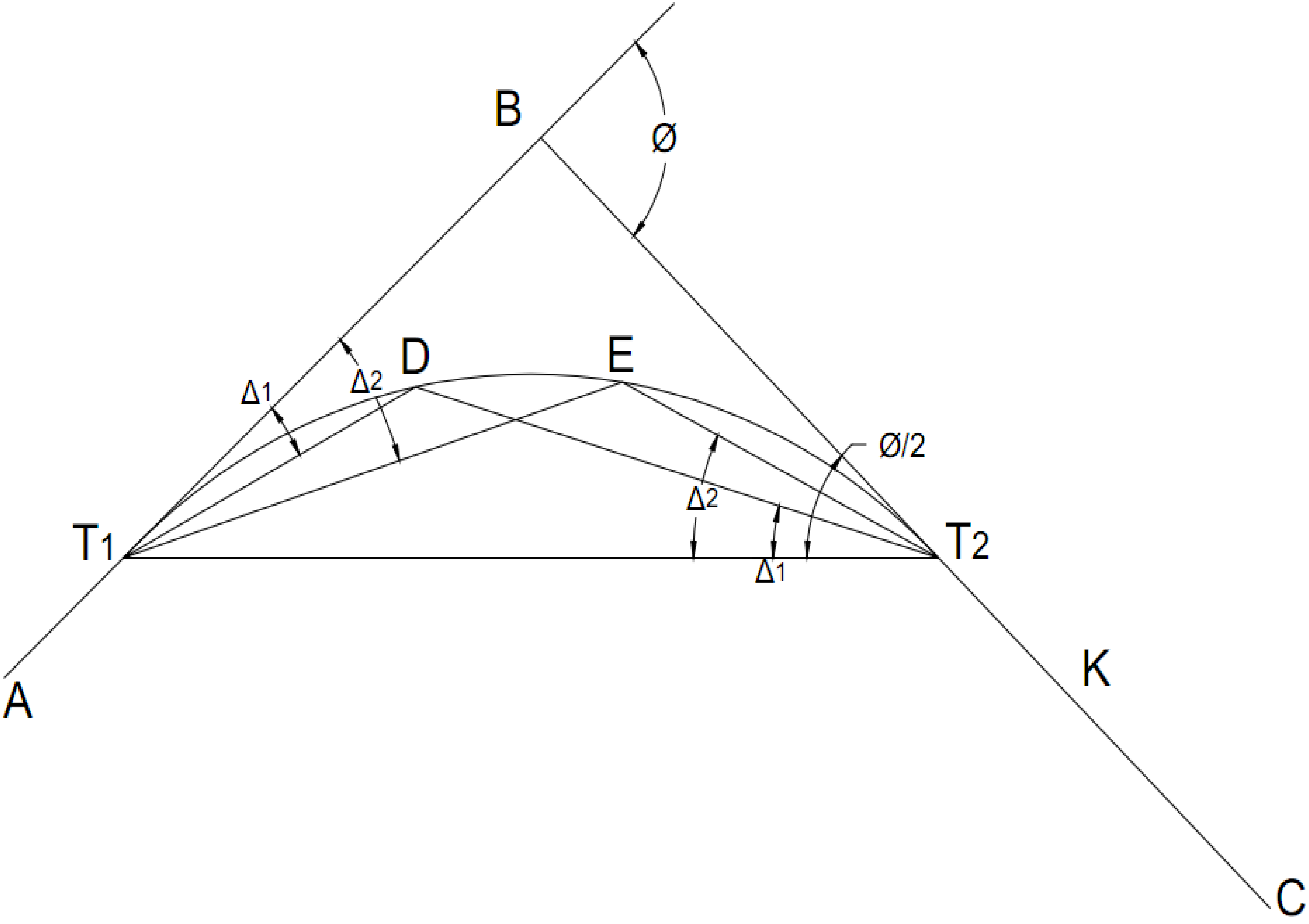
v. Unclamp the upper plate and set the vernier to the second deflection angle Δ_2, the line of sight bring now directed along T_1E.

vi. Hold the zero end of the chain at D and swing the other end around D until the arrow held at other end is bisected by the line of sight , thus locating the second point on the curve.
vii. Repeat the process until the end of the curve is reached.

Check:-

The last point thus located must coincide with the previously located tangent point T2. If not, find the distance between them which is actual error. If it is within the permissible limit, the last few pegs may be adjusted, if it is exceeds the limit, the entire work must be checked.

2. Two theodolite method :-



This method is used when the ground is not favourable for accurate chaining e.g. rough ground. It is based on the fact that angle between the tangent and the chord is equal to the angle which that chord subtends in the opposite segment.

Let, D, E, etc. be the points on the curve . The angle (Δ_1) between the tangent T_1B and the chord T_1D = BT_1D = T_1T_2D. Similarly, the angle BT_1E = Δ_2 = T_1T_2E, the total tangential or deflection angles Δ_1, Δ_2, Δ_3 being calculated as in the first method.

Procedure:-

To set out the curve

i. Set up theodolite over T1 and another over T2.
ii. Set the vernier of each instrument to zero.
iii. Direct the instrument at T1 to the ranging rod at the point of intersection B and bisect it.
iv. Direct the instrument at T2 to the first tangent point T1 and bisect it.

v. Set the vernier of each of the instrument to read the deflection angle Δ_1. Thus the line of sight of the instrument at T_1 is directed along the T_1D and that of the other instrument at T_2 along T_2D. Their point of intersection gives the required point on the curve.

vi. Move the ranging rod until it is bisected by the crosshairs of both instruments, thus locating the point D on the curve.

vii. To obtain the second point on the curve, set the vernier of each of the instruments to the second deflection angle Δ_2 and proceed as before.

If the first tangent point T_1 cannot be sighted from the instrument at T_2 the ranging rod at the point of intersection B may be sighted. The procedure will be then be as follows:-

i. With both plates of the second instrument clamped at zero, bisect the signal at B

ii. Release the vernier plate and swing the telescope (clockwise) through (360^0 – ø/2), thus directing the line of sight along T_2T_1.

iii. To obtain the first point on the curve, set the vernier to the first deflection angle Δ_1. the vernier reading will then be $(360^0 – ø/2) + \Delta_1$ instead of $\Delta 1$ as in the first case.
iv. The rest of the procedure is exactly the same as before. Instead of sighting the intersection point B, any point K in the forward direction of the tangent line T_2C may be used.

In this case, however, the angle through which the telescope has to be turned, after having the signal at K with both plates clamped at zero, is equal to $(180^0 – ø/2)$. The line of collimation is thus directed along the line T_2T_1. To obtain the first point on the curve, the vernier reading must be $(180^0 – ø/2) + \Delta_1$.

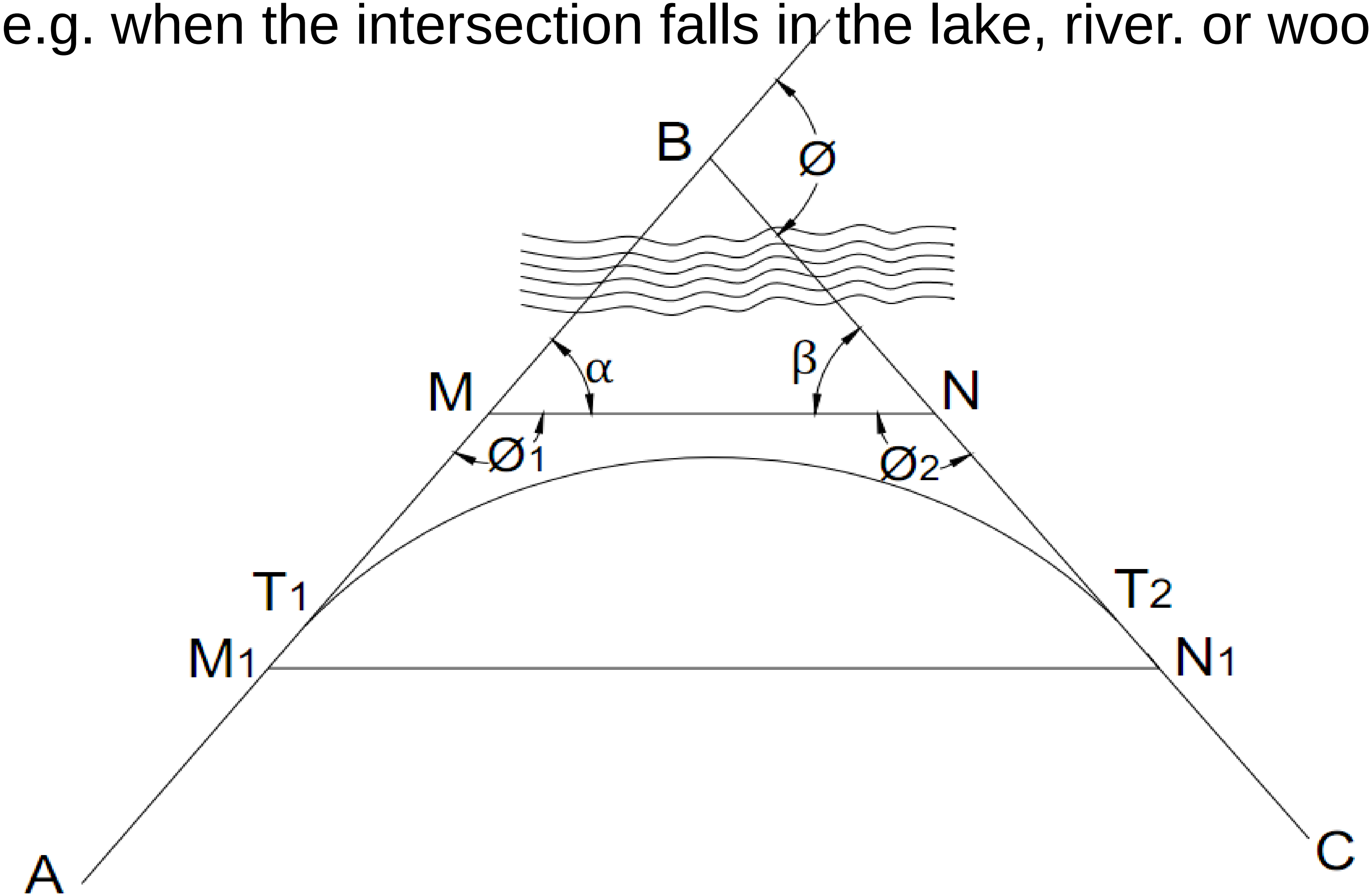
It will seen that in this method no chain or tape is used to fix the points on the curve, but they are located by the intersection of the lines of the sight of the two instruments. The method is simple and accurate, but it is expensive, since two surveyors and two instruments are required to use this method.

Obstacles in setting out curves:-

The following obstacles occurring in common practice will now be considered:-

1) **When the point of the intersection of the tangent lines is inaccessible:-**

e.g. when the intersection falls in the lake, river. or wood.



Let, AB and BC be the two tangent lines intersecting at the point B, and T_1 and T_2 the tangent points. It is required to determine the value of the deflection angle (ø) between the tangents and to locate the tangent point T_1 and T_2.

Procedure:-

a) Fix points M and N suitably on the tangents AB and BC resp. so that M and N are intervisible, and the line MN runs on moderately level ground in order that accurate chaining may be possible. If the ground beyond the curve is not suitable, the points may be fixed inside the curve as at M and N. Measure MN accurately.

b) Set up the instrument at M and measure the angle AMN(θ_1) between AB and MN.

   Transfer the instrument to N and measure the angle CNM (θ_2) between BC and MN.

Now in the Δ $\angle$BMN, $\angle BMN = \alpha = 180^0 - \angle AMN = 180^0 - \theta_1$,

$\angle BNM = \beta = 180^0 - \angle CNM = 180^0 - \theta_2$.

The deflection angle (ø) = ∠BMN + ∠BNM = α + β

or = 360^0 − sum of measured angles.

= 360^0 − (Ɵ1 + Ɵ2).

c) Solve the triangle BMN to obtain the distance BM and BN.

$$BM = \frac{MN \sin \beta}{\sin [180^\circ - (\alpha + \beta)]}; \quad BN = \frac{MN \sin \alpha}{\sin [180^\circ - (\alpha + \beta)]}$$

d) Calculate the tangent length BT1 and BT2from the formula

T= R (tan ø/2).

e) Obtain the distances MT1 and NT2.

MT1 = BT1 – BM and NT2 = BT2 – BN.

f) Measure the distance MT1 from M along the tangent line BA. Thus locating the first tangent point T1.

similarly, locate the second tangent point T2 by measuring the distance NT2 from N along the tangent BC.

If the point fixed inside the curve, the procedure is same as above, except for the distances to be measured from the points M1 and N1 to locate the tangent point T1 and T.

If the point fixed inside the curve, the procedure is same as above, except for the distances to be measured from the points M1 and N1 to locate the tangent point T1 and T2. MT1 and NT2 being respectively equal to (BM1 – BT1) and (BN1 = BT2).

When it is found impossible to obtain a clear line MN, a traverse is run between M and N to find the length and bearing of the line MN. From the known bearing of the tangent lines and the calculated bearing of the line MN, the angles α and β may easily be obtained. The distances BM and BN are then calculated as before.

2. **When the curve can not be set out from the tangent points, vision being obstructed**:-

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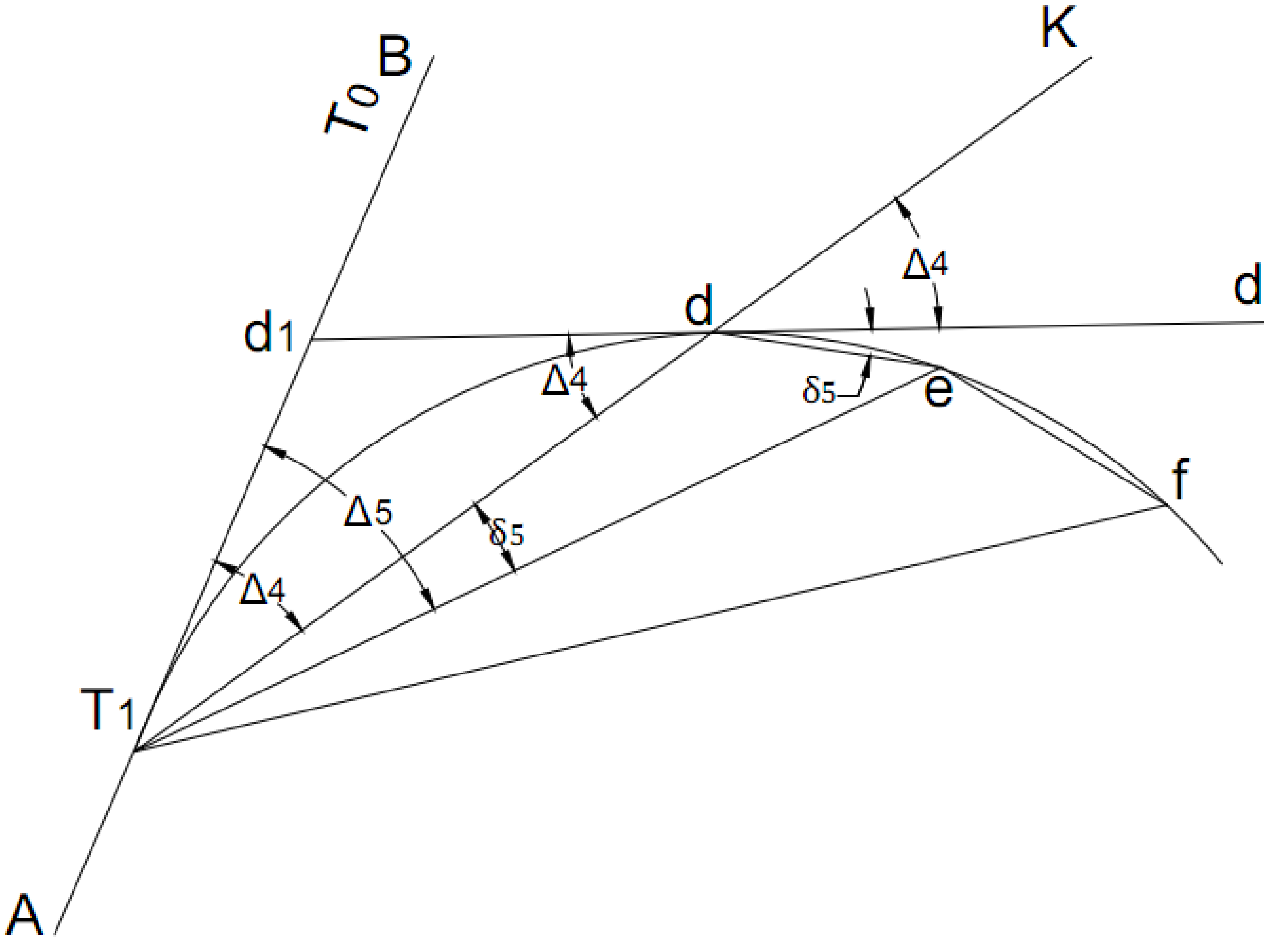


2. When the curve can not be set out from the tangent points, vision being obstructed:-

As a rule, the whole curve is set out from the first tangent point T_1. But this is possible only when the curve is short and ground is moderately level and free from all the obstructions. However, it is often found that this cannot be done on account of great length of the curve, or obstructions intervening the line of sight such as buildings, cluster of trees, plantations, etc. In such a case, the instrument requires to be set up at one or more points along the curve.

Procedure (first method):-

Suppose the first four points have been located by the deflection angles from the instrument at T1, when it is found necessary to shift the instrument.

Let, d be the last point located from T_1 and its deflection angle Δ.

i. Shift the instrument and set it up at d.

ii. With both plates clamped at zero, take a backsight on T_1 and transit the telescope.

iii. To locate the next point e, set the vernier to read the deflection angle Δ_5, thus directing the line of sight along de.

iv. Using the same tabulated deflection angles, continue the setting out of the curve from d as already explained.

Proof:-

Draw the tangent d_1dd_2 at d, meeting the first tangent BT_1 at d_1. Produce T_1d to K.

$Kdd_2 = d_1dT_1$ (vertically opposite); $d_1dT_1 = d_1T_1d$, since $d_1T_1 = d_1d$. But $d_1T_1d = \Delta_4$, so, $Kdd_2 = d_1T_1d = \Delta_4$.

The tangential angle for the chord de $= d_2de = \delta_5 = dT_1e$.

$\therefore$ $Kde = \Delta_4 + \delta_5 = \Delta_5$.

The total tangential (or deflection) angle for the chord de $= d_1T_1e = d_1T_1d + dT_1e = \Delta_4 + \delta_5 = \Delta$

Thus is well be seen that when the telescope is transited after a backsight was taken with the vernier reading zero, the line of sight directed along dK and when the telescope is turned through the angle $\Delta 4$, it is in the direction of tangent at d.

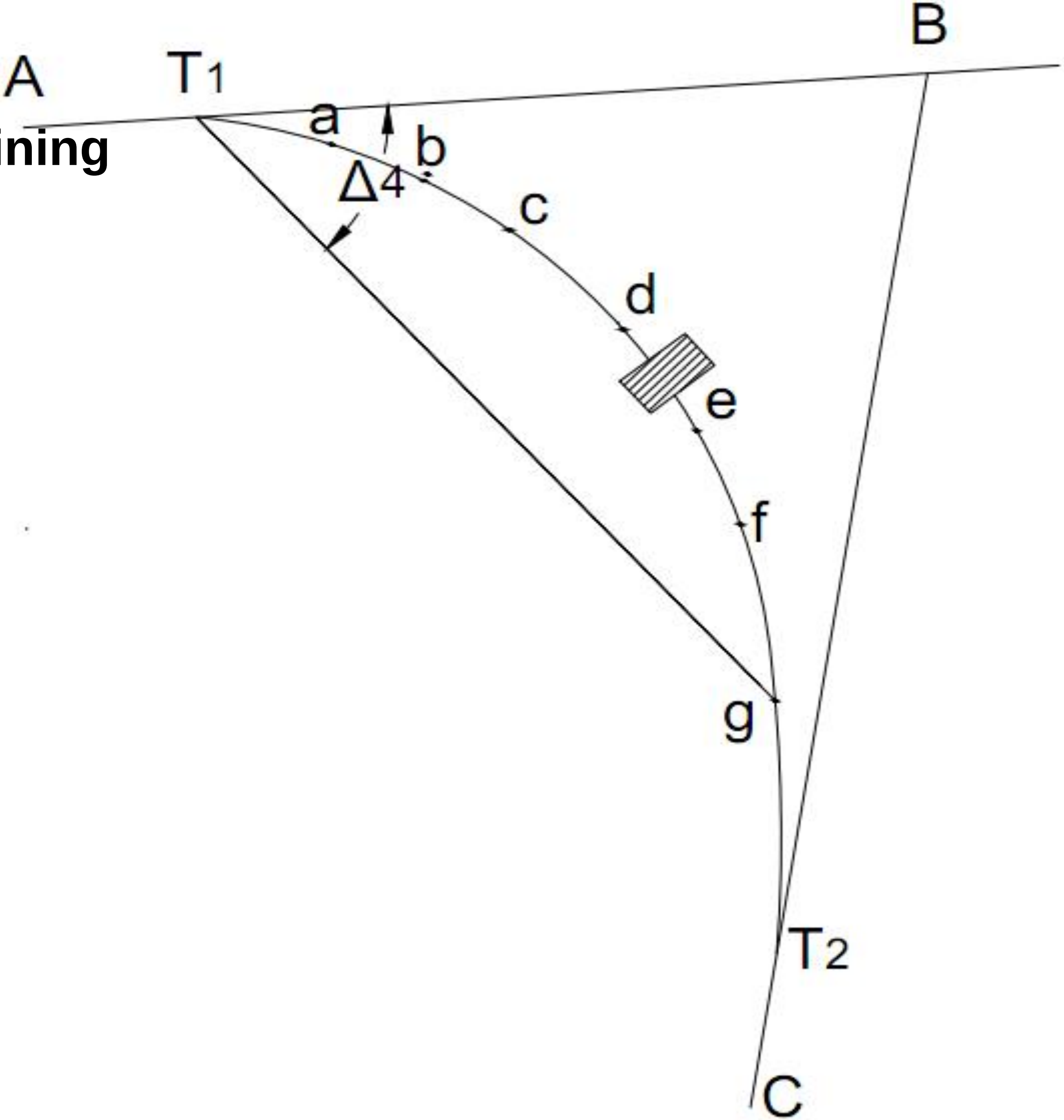
To use this method instrument must be in perfect adjustment

Second method:-

Suppose d is the last point located from the instrument at T1.

i. Suppose the instrument is at T1, fix any point K in the line T1d produced.
ii. Move the instrument to d and with the vernier set to zero bisect K.
iii. Release the vernier plate and set the vernier to $\Delta 5$, to locate the next point e.

When the obstacle to chaining occurs:-



Third method:-

Suppose that the first four points have been located from T_1, d being the last point located and that the portion of the curve between d and f cannot be chained across.

In such a case, the following procedure may be adopted:-

Let g be the next station on the curve which can be seen from T_1.

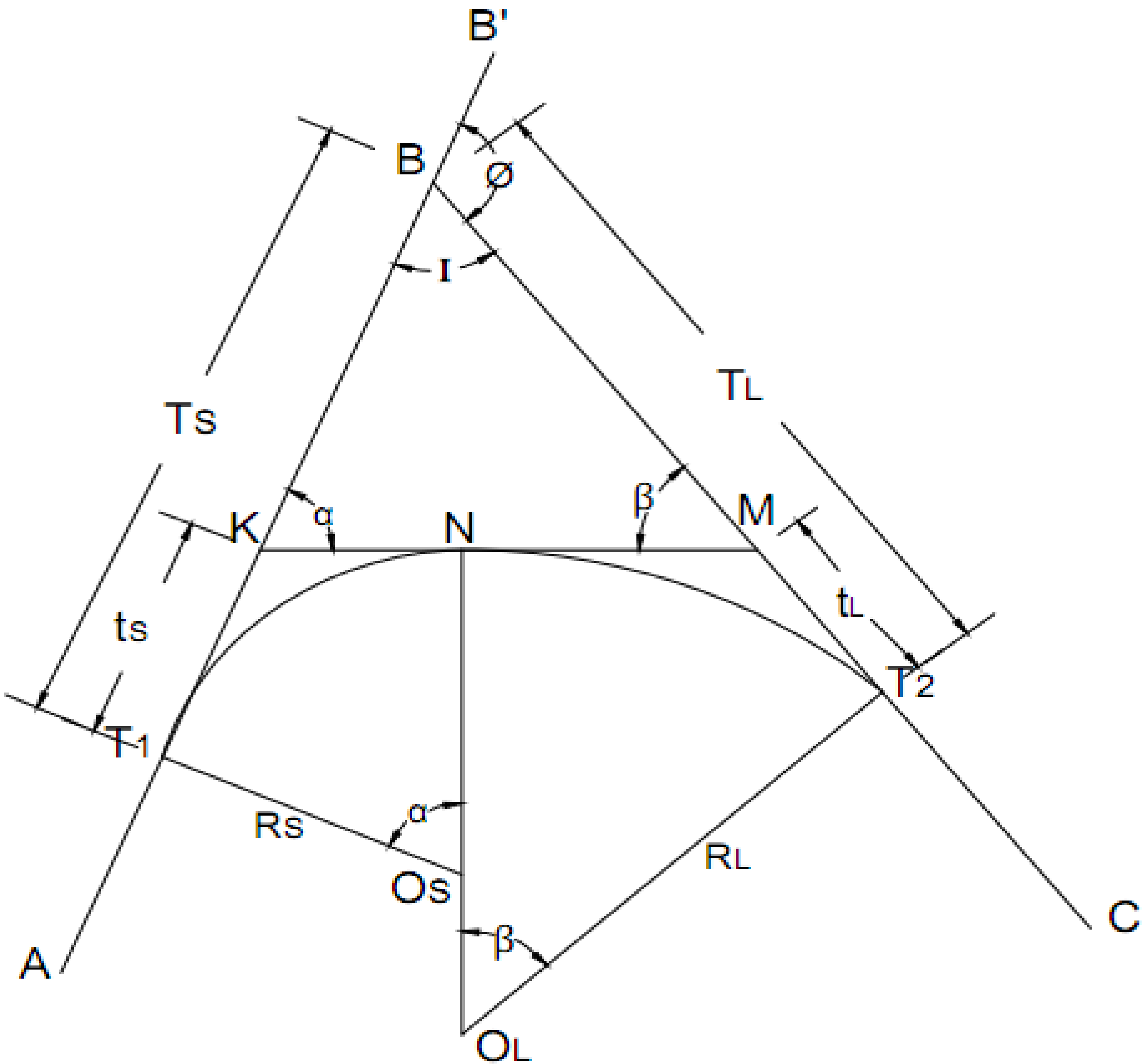
i. Calculate the length of long chord T_1g from the formula,

   $T_1g = 2R \sin \Delta_4$.

ii. Set the vernier A to the deflection angle Δ_4, thus directing the line of sight along T_1g. Measure the distance T_1g along this direction, and locate the point g on the curve. Locate the remaining points in the usual way. Alternatively, set out the curve T_2g in the reverse direction from the second tangent point T_2.

iii. The point e and f, which were left out, will be located sfter the obstruction is removed.

COMPOUND CURVE:-



COMPOUND CURVE

In the figure it is shown that a compound curve which is tangential in three straights AB, BC and KM at T1,T2 and N respectively. The two circular arcs T1N and NT2 having centres at the point N, OS and OL being in straight line. The arc having a smaller radius may be first or second.

Let, the tangents AB and BC intersect at the point B, AB and KM and BC at M.

Notation:-

Let , R_s = the smaller radius O_sT1.

R_L = the larger radius O_LT2.

T_s = the smaller tangent length (BT1)

T_L = the larger tangent length (BT2)

ø = the deflection angle between the rear tangent (AB) and forward tangent (BC)

α = the deflection angle between the rear tangent (AB) and common tangent (KM) = ∠BKM.

β = the deflection angle between the forward tangent (AB) and common tangent (KM) = ∠BMK.

t_s = the length of the tangent to the arc (NT1) having a smaller radius

*** t_L = the length of the tangent to the arc (NT2) having a larger radius.

Elements of the compound curve:-

$$\varnothing = \alpha + \beta$$

KN = KT1 = $\mathbf{t_s = R_s \tan(\alpha/2)}$

MN = MT2 = $\mathbf{t_L = R_L \tan(\beta/2)}$

Length of common tangent (KM) =

KM = KN + MN = $\mathbf{R_s \tan(\alpha/2) + R_L \tan(\beta/2)}$

In the ΔBKM, $BK = \dfrac{KM \sin\beta}{\sin(\alpha+\beta)}$; $BM = \dfrac{KM \sin\alpha}{\sin(\alpha+\beta)}$

$$BK = \frac{(t_S + \mathrm{t_L})\sin\beta}{\sin(\varnothing)}; \quad BM = \frac{(t_S + \mathrm{t_L})\sin\alpha}{\sin(\varnothing)}$$

Now $Ts = BT_1 = KT_1 + BK = t_S + \dfrac{(t_S + \mathrm{t_L})\sin\beta}{\sin(\varnothing)}$; ………….(A)

$T_L = BT_2 = MT_2 + BM = t_L + \dfrac{(t_S + \mathrm{t_L})\sin\alpha}{\sin(\varnothing)}$ …………..(B)

Substituting the values of ts and t_L in the equation A and B we get,

$$T_S = R_S\, tan\frac{\alpha}{2} + \left(R_S\, tan\frac{\alpha}{2} + R_L\, tan\frac{\beta}{2}\right)\frac{\sin\beta}{\sin\text{Ø}}$$

$$T_L = R_L\, tan\frac{\beta}{2} + \left(R_S\, tan\frac{\alpha}{2} + R_L\, tan\frac{\beta}{2}\right)\frac{\sin\alpha}{\sin\text{Ø}}$$

Of the seven quantities Rs, RL, Ts, TL, Ø, α and β four must be known. Then remaining three may be calculated from the above equations,

The following equation gives the relationship between the seven elements involved in compact form

$$T_S \sin\text{Ø} = (R_L - R_s) versin\, \beta + R_s\, versine\, \text{Ø}$$

$$T_L \sin\text{Ø} = (R_L - R_s) versin\, \alpha + R_L\, versine\, \text{Ø}$$

Ø = α + β

Procedure :-

The curve may be setout by the method of deflection angles from the two points T1 and N, the first branch from T1 and second from N.

1. The four quantities of the curve being known, calculate the other three.
2. Locate B, T_1 and T_2 as already explained, obtain the chainage of T_1 from the known chainage of B.
3. Calculate the length of the first arc and add it to the chainage of T_1 to obtain the chainage of N. Similarly, compute the length of the second arc which added to the chainage of chainage of N gives the chainage of T_2.
4. Calculate the deflection angles for both the arcs.
5. With the theodolite set up over T_1 set out the first branch already explain in Rankine's method.
6. Shift the instrument and set up at N, with the vernier set to $(\alpha/2)$ behind zero i.e. $(360 - \alpha/2)$, take a backsight on T_1 and plunge the telescope which is thus directed along T1N

produced. (if the telescope is now swing through an angle $\alpha/2$ the line of sight will be directed along the common tangent NM and the vernier will read 360)

6. Set the vernier to the first deflection angle $\Delta 1$ as calculated for the second arc.
7. Repeat the process until the end of the second arc is reached i.e. T2.

Check :- $\angle T1NT2 = \left(180^\circ - \frac{\alpha + \beta}{2}\right) \, or \left(180^\circ - \frac{\emptyset}{2}\right)$

TRANSITION CURVE

A curve of varying radius is known as ‘transition curve’. The radius of such curve varies from infinity to certain fixed value. A transition curve is provided on both ends of the circular curve. The transition curve is also called as *spiral* or *easement curve*.

OBJECTS OF PROVIDING TRANSITION CURVES:-

1. To accomplish gradually the transition from the tangent to the circular curve, and from the circular curve to the tangent.

2. To obtain a gradual increase of curvature from zero at the tangent point to the specified quantity at the junction of the transition curve with the circular curve.

3. To provide the superelevation gradually from zero at the tangent point to the specified amount on the circular curve.

4. To avoid the overturning of the vehicle.

REQUIREMENT OF IDEALTRANSITION CURVES:-

1. It should meet the original straight tangentially.
2. It should meet the circular curve tangentially.
3. Its radius at the junction with the circular curve should be the same as that of the circular curve.
4. The rate of increase of curvature along the transition curve should be same as that of in increase of superelevation.
5. The length should be such that the full superelevation is attained at the junction with the circular curve.

The types of transition curve which are in common use are

1. **A Cubic parabola**
2. **A clothoid or spiral**
3. **A lemniscate**

the first being used on railways and third one on highways.

SUPERELEVATION:-

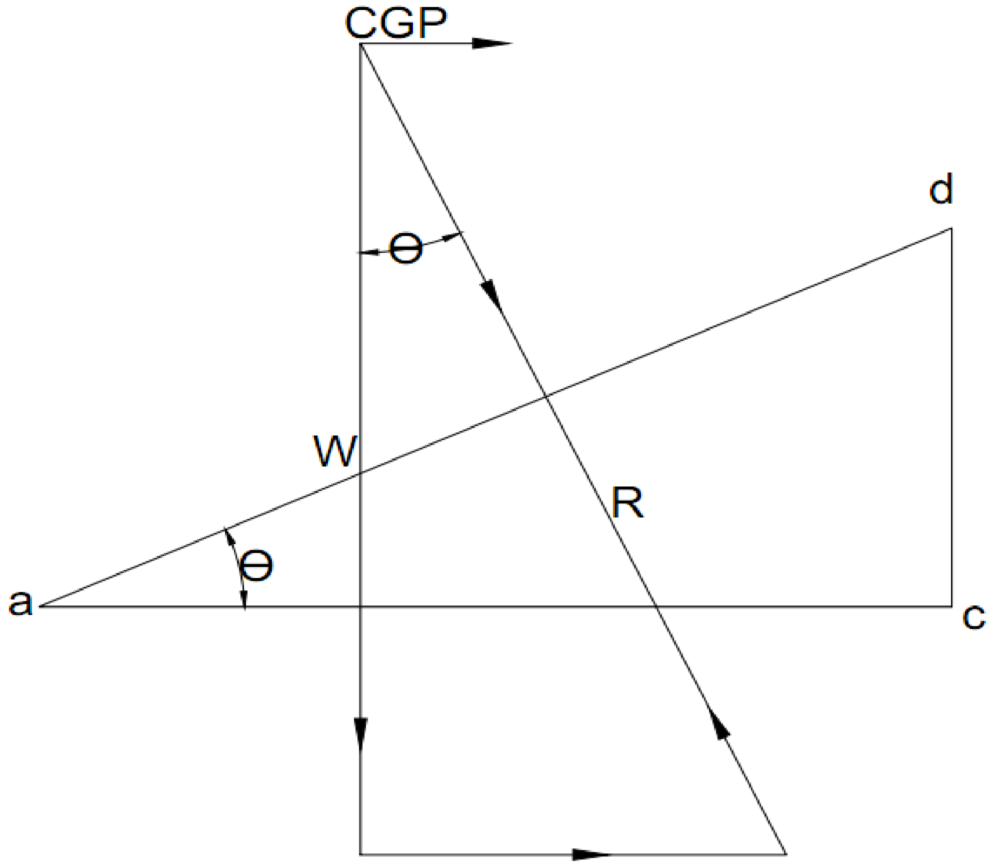
When a vehicle passes from a straight path to curved one, the forces acting on it are

i. The weight of the vehicle.
ii. The centrifugal force both acting through the CG of vehicle.

Since the centrifugal force is always acts in the direction perpendicular to the axis of rotation which is vertical, its direction is always horizontal. The effect of the centrifugal force is to push the vehicle off the track or rail. In order to counteract the action, the plane of rails or the road surface is made perpendicular to the resultant of centrifugal force and weight of the vehicle.

In other word, the outer rail is superelevated or raised above the inner one. Similarly the road should be "banked", i.e. the outer edge of the road should be raised above the inner one, the raising of the outer rail or outer edge above the inner one, being called as superelevation or cant.

The amount of cant is depend on vehicle and radius of curve.



Let,

W = weight of the vehicle

P = the centrifugal force.

v = the speed of the vehicle in m/s.

g =the acceleration due to gravity = 9.81 m/s^2

R = the radius of the curve in m

h = the superelevation in m

b = the width of the road in m

G = the distance between centers of the rails in m.

Then the equilibrium, the resultant of the weight and the centrifugal force must be equal and opposite to the reaction perpendicular to the road or rail surface.

$$P = \frac{Wv^2}{gR} \qquad \therefore \frac{P}{W} = \frac{v^2}{gR}$$

If Ɵ be the inclination of the road or rail surface, the inclination of the resultant to the vertical is also Ɵ, therefore we have

$$\tan\theta = \frac{dc}{ac} = \frac{P}{W} = \frac{bv^2}{gR}$$

Hence the amount of superelevation h

$$h = b\tan\theta = \frac{v^2}{gR} \quad \text{on roads}$$

$$h = \frac{Gv^2}{gR} \quad \text{on railways}$$

The amount of superelevation is limited about 1/12th of the gauge, 1/10th being permitted under special circumstances.

The maximum superelevation recommended for broad gauge (1676 mm) , meter gauge (1000 mm) and narrow gauge (762 mm) are 140mm (165 mm) , 90 mm (102 mm) and 65 mm (75 mm) respectively

LENGTH OF TRANSITION CURVE:-

The length of the transition curve may be determined in the following ways

1. **By an arbitrary gradient :-**

   The length may be such that the superelevation is applied at a uniform rate of 1in n, the value of n varying from 300 to 1200.

Therefore, $L = nh$ …………..(1)

where, L = the length of transition curve in m

h = the superelevation in m

1 in n = the rate of canting

2. By the time rate:- The transition curve may be such a length that the cant is applied at an arbitrary time rate of **a** cm per second, **a** varying from 2.5 to 5 cm.

Let, L = the length of transition curve in m

h’ = the amount of superelevation in cm.

v = the speed in m/sec

a = the time rate (cm/sec)

Time taken by vehicle in passing over the transition curve $= \dfrac{h'v}{\mathbf{a}}$

Superelevation attained in this time $= \dfrac{L\mathbf{a}}{v} = h'$

$$L = \frac{h'v}{\mathbf{a}} \qquad \text{................(2)}$$

3. By the rate of change of radial acceleration:-

This rate should be such that the passenger should not experience any sensation of discomfort when the train travelling over the curve. It is taken as 30 cm per sec^3 .

Now the radial acceleration on the circular curve $= \dfrac{v^2}{R}$ (m/sec^2)

Time taken by the vehicle to pass over the transition curve

$$= \frac{L}{v} \quad \text{seconds.}$$

Radial acceleration attained in L/v seconds at the rate of

0.3m/sec³ $= \frac{L}{v} \times 0.3 \quad m/sec^3$

$$\frac{v^2}{R} = \frac{L}{v} \times 0.3 \quad or \quad L = \frac{v^3}{0.3\,R} \qquad \text{...............(3)}$$

$$L = \frac{v^3}{14\,R}$$ If v = speed in km/hr.

The ratio of centrifugal force and the weight is called the centrifugal ratio.

Centrifugal ratio $= \frac{P}{W} = \frac{Wv^2}{gRW} = \frac{v^2}{gR}$ ....(4)

The maximum value of the centrifugal ratio on roads is taken as 1/4 and for railways as 1/8.

On roads $= \frac{v^2}{gR} = \frac{1}{4}$

$$v^2 = \frac{gR}{4} = 2.452\,R \quad or \quad v = \sqrt{2.452\,R}$$

Now from the formula $L = \dfrac{v^3}{0.3R}$

$$L = \frac{2.452^{3/2}\, R^{3/2}}{0.3R} = 12.80\sqrt{R} \quad \text{..........(5)}$$

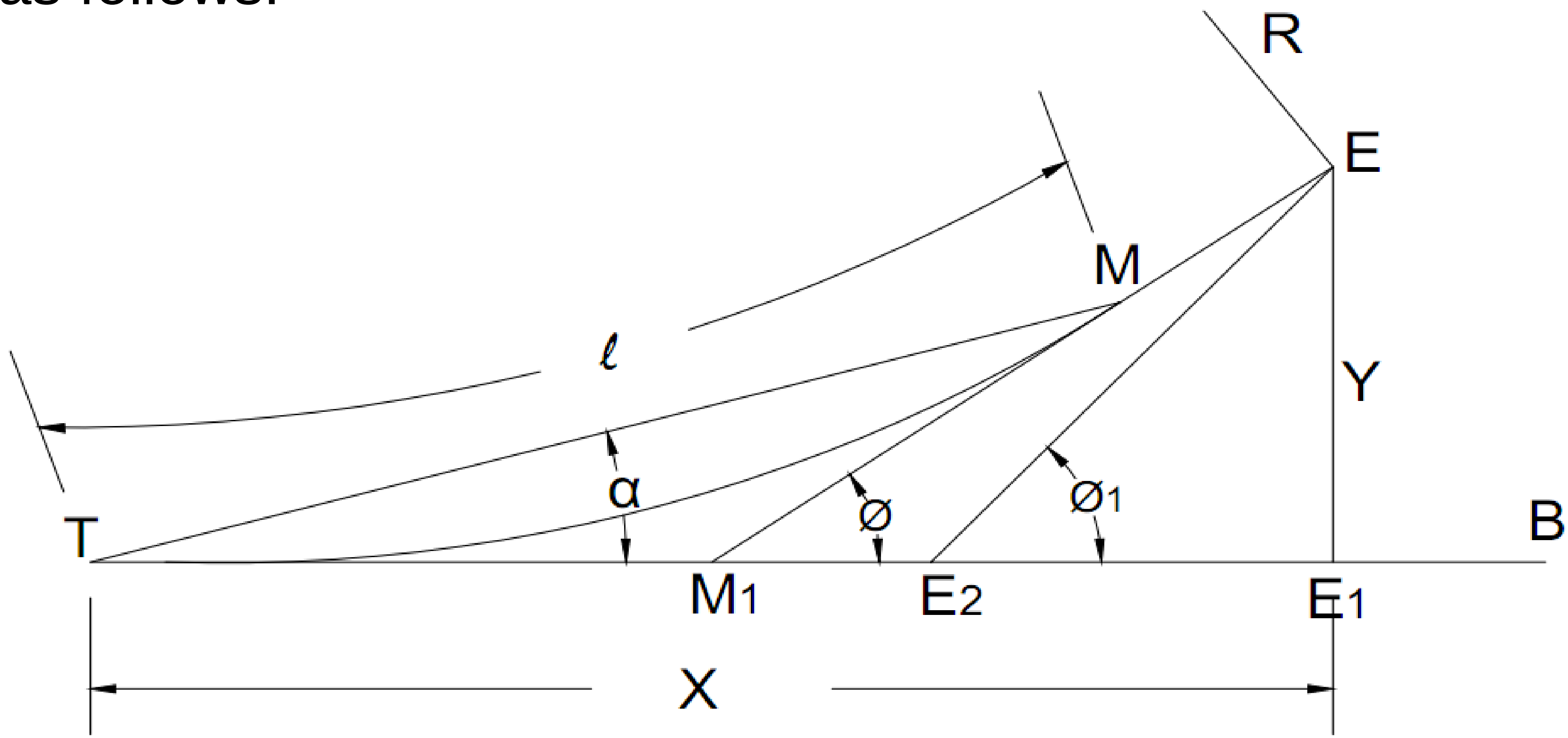
On railways $\dfrac{v^2}{gR} = \dfrac{1}{8}$ $\quad v^2 = 1.226\, R \quad or \quad v = \sqrt{1.226\, R}$

$$L = \frac{v^3}{0.3R} = \frac{1.226^{3/2}\, R^{3/2}}{0.3R} = 4.526\sqrt{R} \quad \text{......(6)}$$

The equation 5 or 6 is to be used only when the full centrifugal force is developed and when the rate of gain of radial acceleration is 0.3 m /sec.

Ideal transition curve:-

The equation of the ideal transition curve (clothoid spiral) may be as follows:-



Let, TB = the initial tangent

T = the beginning of the transition curve

E = the point of junction of the transition curve with the circular curve.

M = any point on the transition curve, ℓ m along it from T.

r = the radius of the transition curve at M.

R = the radius of the circular curve.

Ø = the inclination of the tangent to the transition curve at M to the initial tangent TB.

$Ø_1$ = the angle between the tangent TB and the tangent to the transition curve at the junction point E.

(this angle is known as spiral angle)

L = the length of the transition curve.

α is the deflection angle i.e. angle MTB between the tangent at T and line from T to any point (M) on the curve

The fundamental requirement of the spiral curve is that its radius vary inversely as the distance(ℓ) from the beginning of the curve.

Therefore, $r \propto \frac{1}{l} = \frac{1}{r} = ml$

Now for all curve $\frac{dØ}{dl} = curvature = \frac{1}{r}$

$$d\emptyset = \frac{1}{r}dl = ml \times dl$$

Integrating we get, $\emptyset = \dfrac{ml^2}{2}$ …………(7)

The constant of integration being zero, since ø = 0, when ℓ=0.
At the junction point E, ℓ = L, r = R, and ø = ø1.

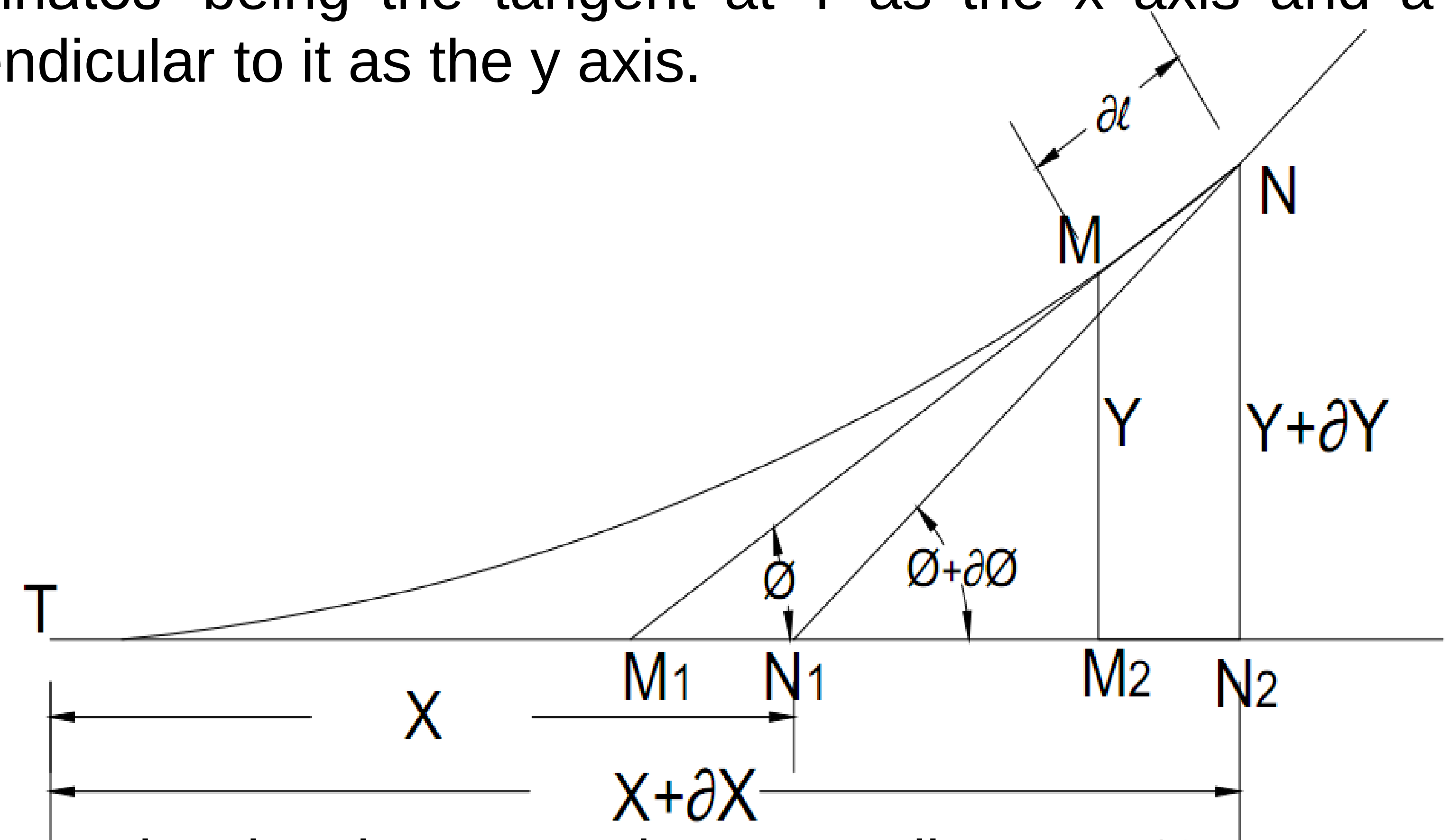
$$m = \frac{1}{RL} \; and \; \emptyset_1 = \frac{L}{2R}$$

Substituting the value of m in equation 7, we get

$$\emptyset = \frac{l^2}{2RL} \; or \; l = k\sqrt{\emptyset}$$ …………(8)

where $k = \sqrt{2RL}$

If the curve is to set out by offsets from the tangent at the commencement of the curve (T), it is necessary to calculate the rectangular (cartesian) coordinates, the ‘the axes of the coordinates’ being the tangent at T as the x axis and a line perpendicular to it as the y axis.



Let, M and N be the two points at a distance δl apart on the curve. Let the coordinates of M and N be (x,y) and $(x+\delta x, y+\delta y)$

And the respective inclinations of the tangents at M and N to the initial tangent (TB) at T Ø and Ø + δØ.

$$x = l\left(1 - \frac{\text{Ø}^2}{10} + \frac{\text{Ø}^4}{216} - etc\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

$$writing\ \frac{l^2}{k} for\ \text{Ø}, x = l\left(1 - \frac{l^4}{10k^4} + \frac{l^8}{216k^8} - etc\right)$$

$$y = \frac{l^3}{6RL}\left(1 - \frac{\text{Ø}^2}{14} + \frac{\text{Ø}^4}{440} - etc\right) \quad \ldots\ldots\ldots\ldots\ldots(10)$$

$$y = \frac{l^3}{6RL}\left(1 - \frac{l^4}{14(2RL)^2} + \frac{l^8}{440(2RL)^4} - etc\right)$$

Rejecting all the terms of the expressions 9 and 10 except the first we have,

$$x = l\ and\ y = \frac{l^3}{6RL} = \frac{x^3}{6RL} \quad \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Which is the equation of curve parabola, the length of the curve being measured along x axis the cubic parabola is known as Froude’s transition or easement curve.

Taking the first two terms of the expressions 9 and 10

We get,

$$x = l\left(1 - \frac{\emptyset^2}{10}\right) = l\left(1 - \frac{l^4}{10(2RL)^2}\right) \quad \text{.................(12)}$$

$$y = \frac{l^3}{6RL}\left(1 - \frac{\emptyset^2}{14}\right) = \frac{l^3}{6RL}\left(1 - \frac{l^4}{14(2RL)^2}\right) \quad \text{.............(13)}$$

From which the coordinates of any point on the true or clothoid spiral can be obtained.

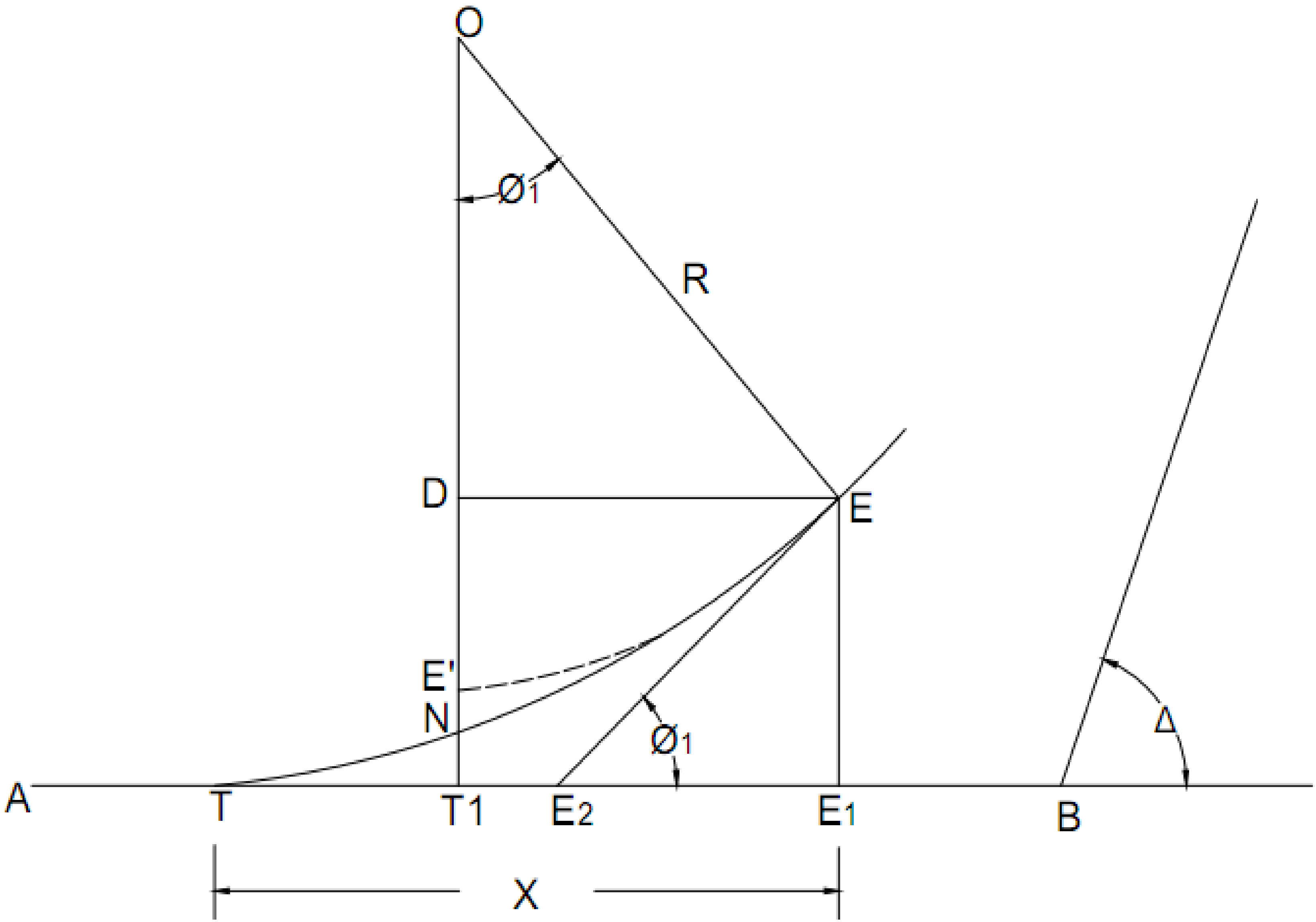
If we take the first term only of expansion 10, we get the equation for cubic parabola.

$$y = \frac{l^3}{6RL} \quad \text{......................(14)}$$

$$\alpha = \frac{\emptyset}{3} \quad \textit{very nearly} \quad \alpha = \frac{1}{3}\frac{l^2}{k^2} = \frac{l^3}{6RL} \quad \textit{radians}$$

$$\alpha = \frac{1800\, l^3}{\pi RL} \quad \textit{minutes} \quad \text{.............(15)}$$

Characteristics of a transition curve:-



Let, TB = the original tangent

T = the commencement of the transition curve.

E = the end of the transition curve

EE_2 = the tangent to both the transition and the circular curve at E.

$Y = EE_1$ = the offset to the junction point (E) of both the curves.

$X = TE_1$ = the x coordinate of E.

EE' = the redundant circular curve.

T_1 = the point of intersection of the line (OE') perpendicular to the tangent to the circular curve at E' and the origin at tangent TB.

$S = E'T_1$ = the shift of the circular curve.

N = the point in which OE' cuts the transition curve.

$Ø_1$ = the spiral angle (EE_2B) between the common tangent EE_2 and the original tangent TB.

R = the radius of the circular curve (OE)

L = the length of the transition curve.

a) Now $EE' = R\,Ø1$; but $Ø1 = L/2R$ $\quad EE' = L/2$

EE' nearly equal to EN, $EN = L/2$ ………… (16) i.e. The shift (E'T1) bisect the transition curve at N.

Hence TN = L/2 ....(17)

b) Draw ED perpendicular to OE'

Now $S = E'T_1 = Y - R\,(1 - \cos Ø_1)$ ....(18)

or $S = Y - 2R\sin^2(Ø_1/2)$

$$S = Y - 2R\frac{{Ø_1}^2}{4} = \frac{L^2}{6R} - \frac{L^2}{8R} = \frac{L^2}{24R}$$ ....(19)

Also, $$NT1 = \frac{TN^3}{6RL} = \frac{L^2}{48R} = \frac{S}{2} = \frac{1}{2}E'T_1$$ ....(20)

i.e. the transition curve bisect the shift.

c) The total tangent length BT :-

a) True spiral (clothoid) $BT = BT_1 + T_1T$

$$BT = (R + S)\tan\frac{\Delta}{2} + \frac{L}{2}\left(1 - \frac{S}{5R}\right)$$ ......(21)

b) cubic parabola:- in this case of cubic parabola the length of the curve is measured along the x axis (TB)

Therefore, TE = L = TE1 = X

$$BT = (R + S)\tan\frac{\Delta}{2} + \frac{L}{2} \qquad \text{............(22)}$$

The amount S tan (Δ/2) is called the shift increment , and (x - R sinØ1) is the spiral extension.

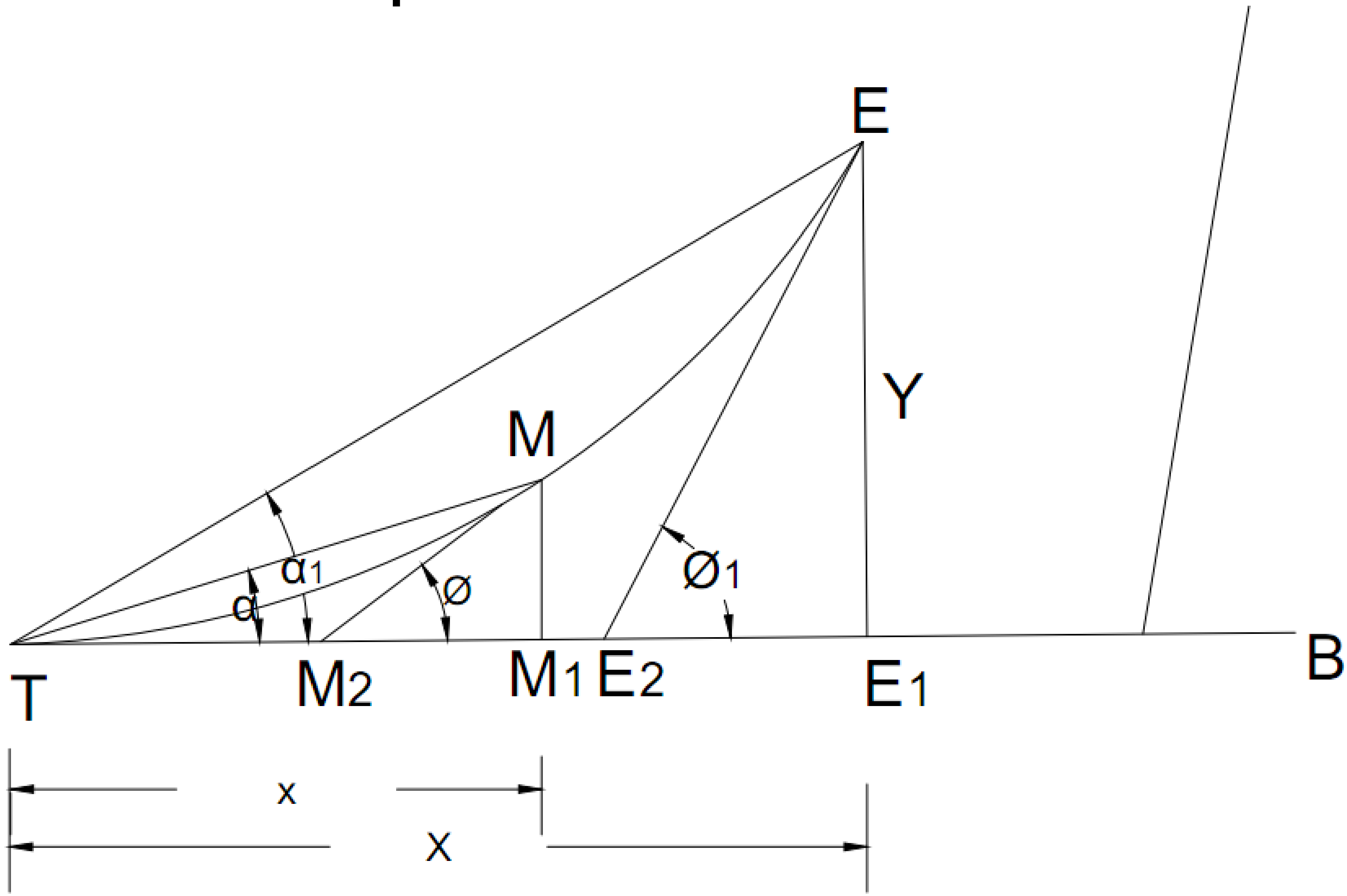
Thus it will be noticed that when the transition curve is inserted in between the tangent and the circular curve, the length of the combined curve is greater than that of simple curve (= R tan Δ/2) by an amount depending upon the transition curve used. In the case of cubic parabola this increase is equal to

$$S\ \tan\frac{\Delta}{2} + \frac{L}{2}$$

while in the case of true spiral or clothoid, it equals

$$\left\{S\ \tan\frac{\Delta}{2} + (X - R\sin\text{Ø}_1\ )\right\}\ or \left\{S\ \tan\frac{\Delta}{2} + \frac{L}{2}\left(1 - \frac{S}{5R}\right)\right\}$$

Elements of cubic parabola:-



Elements of cubic parabola:-

Let, $x = TM1$ = the distance of any point m on the curve measured along the tangent TB from the commencement T on the curve.

$y = M1M$ = the perpendicular offset to point M

ℓ = the lengthy of transition curve TM.

$X = TE1$ = the distance of the junction point E of transition curve with the circular curve from T measured along TB

$Y = E1E$ = the perpendicular offset to the junction point E.

L = the length of the transition curve.

$Ø = \angle MM2B$ = the angle between tangent line AB and tangent to the transition curve at any point M.

$Ø1 = \angle EE2B$ = the angle between TB and tangent at E.

R = the radius of the circular curve.

$\alpha = \angle MTB$ = the deflection angle to any point M.

$\alpha 1 = \angle ETB$ = the deflection angle to the junction E.

$$x = l \; and \; y = \frac{l^3}{6RL} = \frac{x^3}{6RL} \quad \text{......................(23)}$$

$$\alpha = \frac{l^2}{6RL} Radian = \frac{1800 l^2}{\pi RL} \; minutes$$

$$\alpha = = \frac{573 l^2}{RL} \; minutes \quad \text{............................(24)}$$

$$Since \; l = L, \quad \alpha_n = \frac{573 L}{R} \; Minutes$$

$$\emptyset_1 = \frac{L}{2R} Radians = \frac{3 \times 573 L}{R} \; Minutes \quad \text{.......... (25)}$$

If the degree (D) of the curve given instead of radius, the corresponding values α, α_n, $\emptyset_1$, may be found by substituting the value of R in terms of D, i.e. R=1719/D in equation 24 and 25

$$\alpha = \frac{D l^2}{3L} \; minutes \quad \text{....(26)} \qquad \alpha_n = \frac{DL}{3} \; minutes \quad \text{.........(27)}$$

$$\emptyset_1 = DL \; minutes = \frac{DL}{60} degrees \quad \text{...........(28)}$$

Elements of True spiral:-

Using the same notation the elements are:-

The coordinates of any point:

$$x = l\left(1 - \frac{\emptyset^2}{10}\right) = l\left(1 - \frac{l^4}{40R^2l^2}\right) \quad \text{............. (29)}$$

$$y = \frac{l^3}{6RL}\left(1 - \frac{\emptyset^2}{14}\right) = \frac{l^3}{6RL}\left(1 - \frac{l^4}{14(2RL)^2}\right) \quad \text{.........(30)}$$

The coordinate of the end (E) of the curve:

$$X = L\left(1 - \frac{{\emptyset_1}^2}{10}\right) = L\left(1 - \frac{L^2}{40R^2}\right) = L\left(1 - \frac{3S}{5R}\right) \quad \text{.....(31)}$$

$$Y = \frac{L^2}{6R}\left(1 - \frac{{\emptyset_1}^2}{14}\right) = \frac{L^2}{6R}\left(1 - \frac{L^2}{56R^2}\right) \quad \text{..........(32)}$$

The expression for deflection are same as cubic parabola.

The total tangent length

$$BT = (R + S)\tan\frac{\Delta}{2} + \frac{L}{2}\left(1 - \frac{S}{5R}\right)$$

Elements of cubic spiral:-

Using the same notation the elements are:-

$$\alpha = \frac{l^2}{6RL} Radian \ = \frac{1800 l^2}{\pi RL} \ minutes \quad \alpha == \frac{573 l^2}{RL} \ minutes$$

$$\alpha = \frac{Dl^2}{3L} \ minutes$$

$$Since \ l = L, \ \ \alpha_n = \frac{573L}{R} \ Minutes \qquad \alpha_n = \frac{DL}{3} \ minutes$$

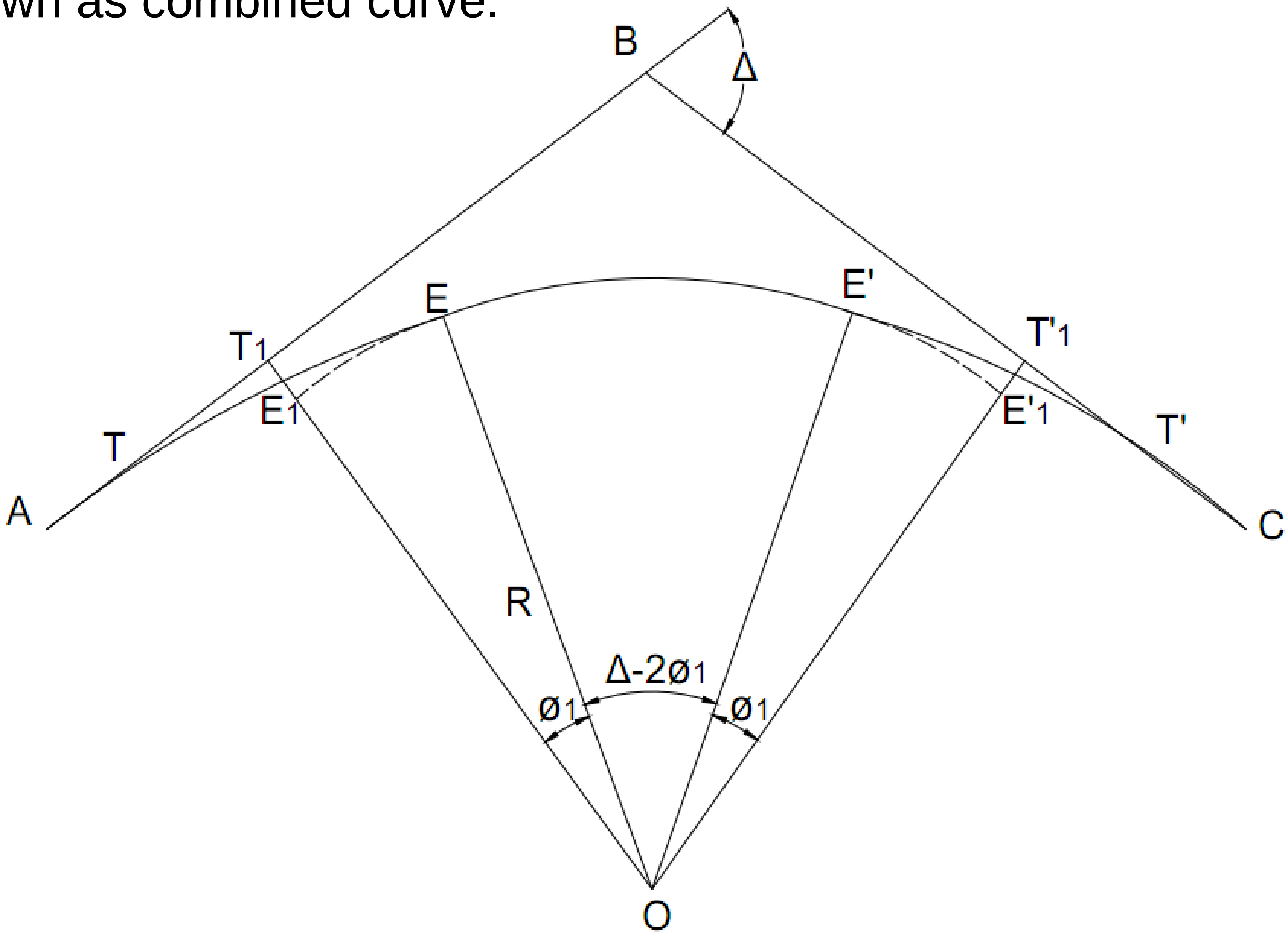
$$\emptyset_1 = \frac{L}{2R} Radians \ = \frac{3 \times 573L}{R} \ Minutes$$

$$\emptyset_1 = DL \, minutes = \frac{DL}{60} degrees$$

Total tangent length

$$BT = (R + S)\tan\frac{\Delta}{2} + \frac{L}{2}$$

Combined Curve:- when the transition curves are inserted at each end of the main circular curve, the resulting curve is known as combined curve.



Length of combined curve:-

The angle subtended at the centre by the circular arc (EE’) = (Δ - $\emptyset_1$) degrees.

length of circular arc = EE’ $= \dfrac{\pi R(\Delta - 2\emptyset_1)}{180°}$ ……….(1A)

the length of the combined curve $= \dfrac{\pi R(\Delta - 2\emptyset_1)}{180°} + 2L$ …(1B)

The length of the curve may also be found out by another way

The central angle subtended by the circular arc

E1E’1 = Δ degrees

Length of circular curve arc E’E’$_1$ $= \dfrac{\pi R\Delta}{180°}$ …………(2A)

Length of combined curve $= \dfrac{\pi R\Delta}{180°} + L$ …………(2B)

The calculation required for setting out the combined curve may be made in the following steps:-

Data:- a) the deflection angle (Δ) between the straights,

b) the radius (R) of the circular curve

c) the length of the transition curve (L)

d) the chainage of the point of intersection of the two straights.

1. Find the shift (S) of the circular curve from $S = \dfrac{L^2}{24R}$

2. Compute the total tangent length from the formula according as a cubic parabola or spiral is used.
3. Calculate the spiral angle Ø1 from $\emptyset_1 = \dfrac{L}{2R}$ radians

4. Calculate the length of the circular curve from formula 1A and length of the combined curve from formula 1B also check result from formula 2B.

5. Find the chainage of the beginning (T) of the combined curve by subtracting the total tangent length from the given chainage of point of intersection B.
6. Obtain the chainage of the junction point E of the transition curve with the circular curve by adding the length of the curve to the chainage of T.
7. Determine the chainage of the other junction point (E') of the circular arc with the other transition curve by adding the length of the circular arc to the chainage of E.
8. Obtain the chainage of the end point (T') of the combined curve by adding the length of the combined curve to the chainage of T.
9. Calculate the deflection angle for the transition curve from

$\alpha = \dfrac{573 l^2}{LR}$ minutes or $\alpha = \dfrac{D l^2}{3L}$ $minutes,$

And also for circular curve from $\delta = \dfrac{1718.9\, c}{R}$ $min,$ bearing in the mind that the points stacked on the compound curve with through chainage so that there will be sub-chords at each end of the transition curve and of the circular curve.

10. Find the total tangential angles for the circular curve from $\Delta n = \sum \delta$ and check the result s by observing if the Δn equals 0.5(Δ - 2Ø1).

11. Calculate the offsets for the transition curve from $y = \frac{l^3}{6RL}$ in case of cubic parabola and from $y = \frac{l^3}{6RL}\left(1 - \frac{\emptyset^2}{14}\right) = \frac{l^3}{6RL}\left(1 - \frac{l^4}{14(2RL)^2}\right)$ in case of of true spiral.

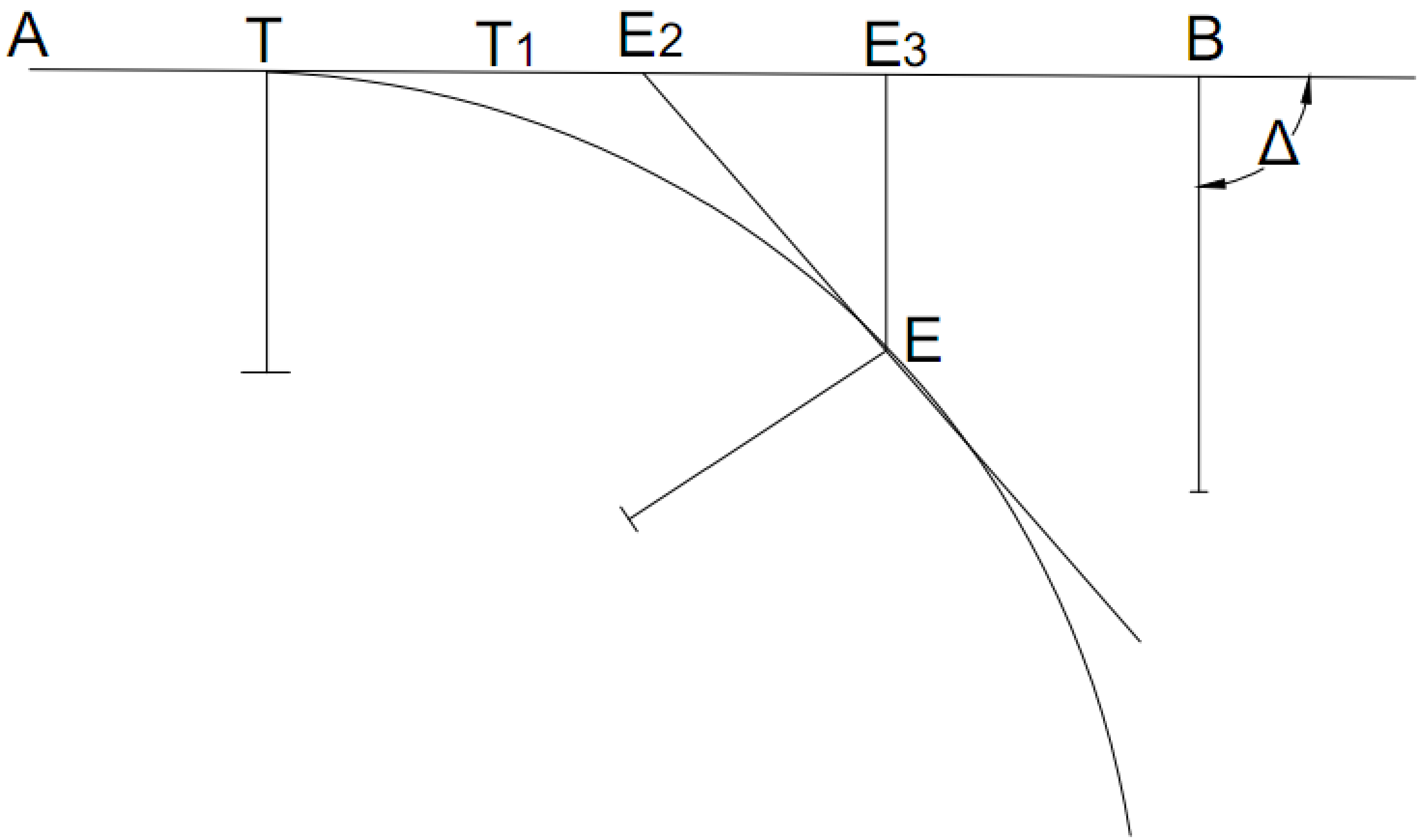
12. Finally compute the offsets from chords produced from $\mathrm{O_n} = \frac{\mathrm{b_n}\,(\mathrm{b_{n-1}} + \mathrm{b_n})}{2\mathrm{R}}$ for circular curve.

Tabulate the result as under

Station no.	**Chainage**	**Length of chord**	**Deflection angle α or δ in minutes**	**Total tangential angle Δ**	**Δ in** **° ' ″**	**Actual Instrument Reading** **° ' ″**	**Remarks**

Setting out the combined curve by deflection angle:-

The first transition curve may be set out from T (i) by the deflection angles or (ii) by the tangent offsets; and the circular curve from the junction point E. the second transition curve may be setout from T', checking on the junction point E' previously located.



1. Having the fixed the tangent AB and BC, locate the tangent point T by measuring the backward the total tangent length from B and other tangent point T’ by measuring the forward the same distance from B along the forward tangent BC.
2. From T measure along TB, the distances equal to the ½ L , 2/3L, and L, the peg these points, which are lettered T1, E2, and E3 resp.
3. Set up theodolite over T and with both plates clamped at zero, bisect B.
4. Release the vernier plate and set the vernier to the first deflection angle as obtained , thus directing the line of sight to the first point on the transition curve.
5. Pin down the zero end of the tape at T, and holding the arrow at the distance on the chain equal to the length of the first chord, swing the chain around T until the arrow is bisected by the cross-hairs, thus fixing the first point on the transition curve.

6. Repeat the procedure until the end of the curve (E) is reached. Check the location of E by measuring the distance EE2 which should be 4S.
7. To set out circular curve, shift the instrument and set it up to E.
8. With the vernier set to 2/3 Ø1, behind zero, for right hand curve, take backsight on T and plunge the telescope which is thus directed along forward direction (if the telescope is now swing through an angle 2/3 Ø1 the line of sight will be directed along the common tangent and the vernier will read 360)
9. Transit the telescope and set the vernier to first deflection angle for the circular curve and hence the first point on the curve is obtained.
10. Continue the setting out the circular curve upto E in the usual way.
11. Set out the other transition curve from T’ as before.

Setting out the Transition curve by tangent offset:-

a) Cubic parabola:-

i) From T measure the x coordinate of points along TB.

ii) Locate the points on the curve by setting out the respective offsets perpendicular to TB at each distance.

b) Cubic spiral:-

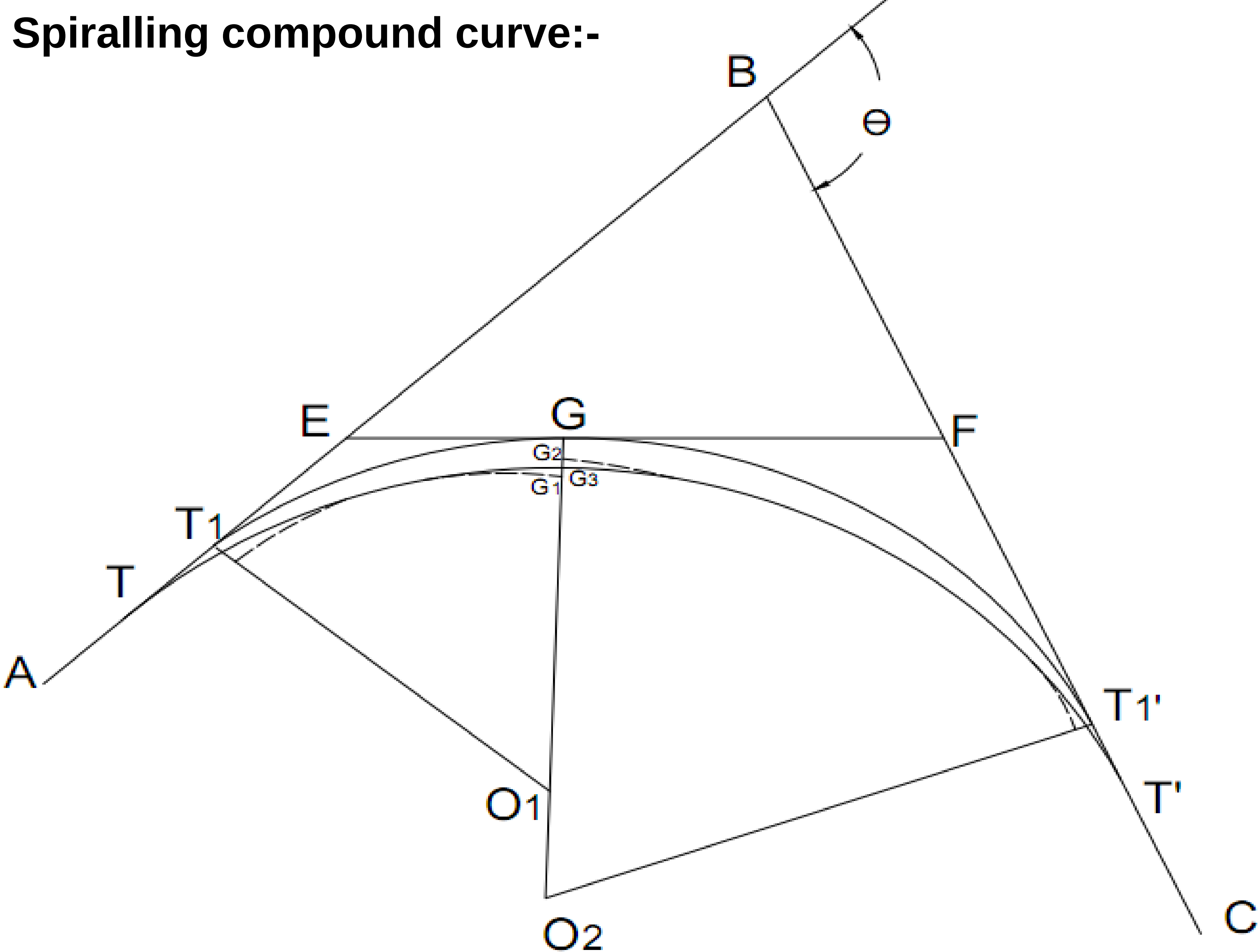
i) Each point is located by swinging the chord length from the preceding point through the calculated offset.

Setting out the transition curve by offsets from tangent (TT1) and from the circular arc (E1E):-

This method is based upon the fact that the often from the circular arc (E1E) to the transition curve at a distance x from E is equal to the offset from the tangent (TT1) to the transition curve at a distance x from T, the transition curve at a distance x from T, the tangent offsets being calculated from $y = \frac{x^3}{6RL}$.

In this method, therefore, half the transition curve is set out by means of offsets from the tangent (TT_1) and the remaining half by means of offsets from the circular arc E_1E.

Spiralling compound curve:-



When it is required to insert transition curve in between two arcs of a compound curve, the following procedure may be adopted:-

1. With the given radii of two circular arcs, the maximum speed and distance between the rail heads, calculate the amount of superelevation for each arc by the relation

VERTICAL CURVES

When two different or contrary gradients meet, they are connected by a curve in vertical plane is called a vertical curve.

It is advisable to introduce a vertical curve in road and in railway work in order to round off the angle and to obtain a gradual change in grade so that abrupt change in grade is avoided at the apex.

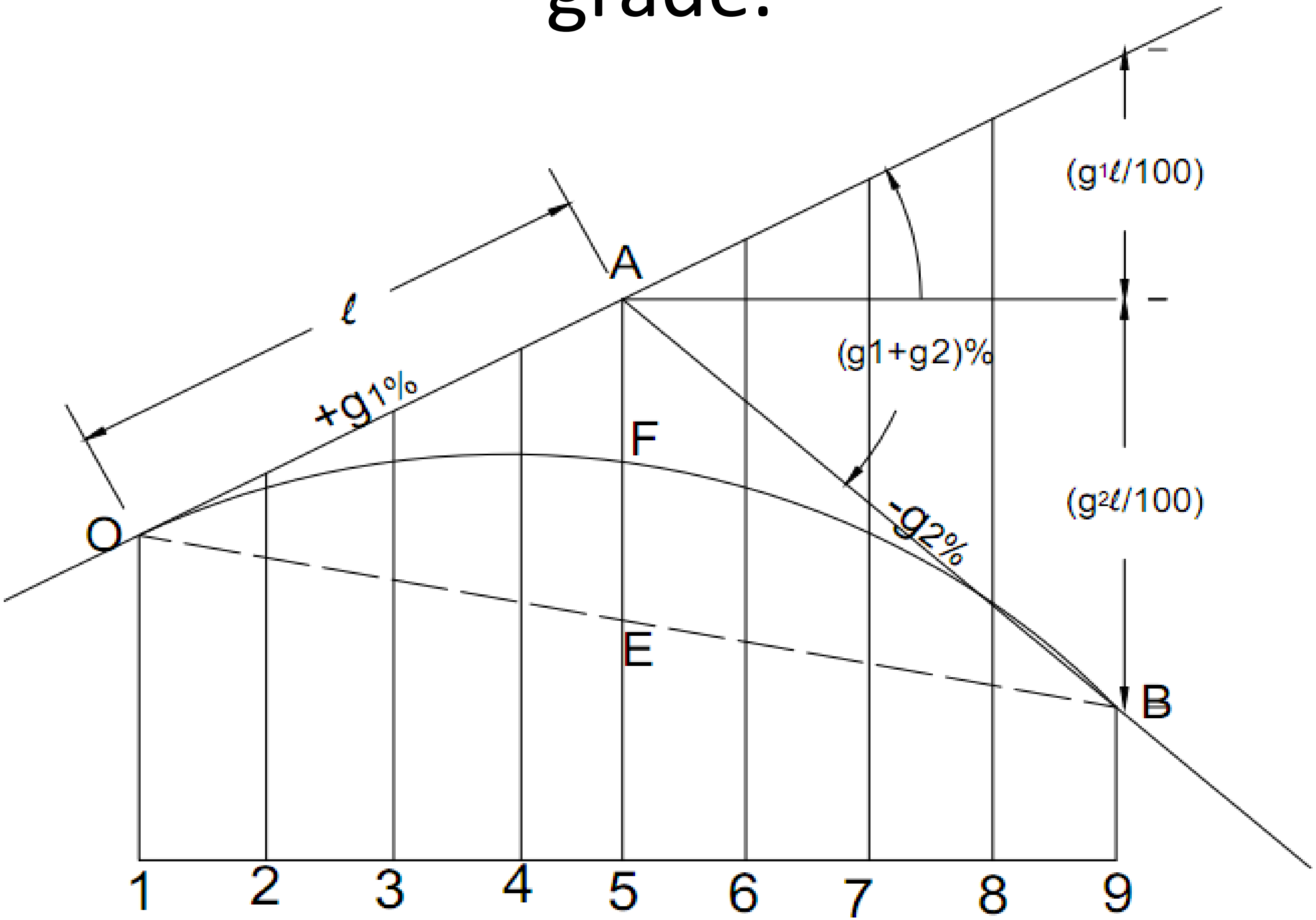
REQUIREMENTS

1. It gives adequate visibility and safety to the traffic.
2. It gives gradual change in grade or slope.
3. It gives adequate comfort to the passengers.

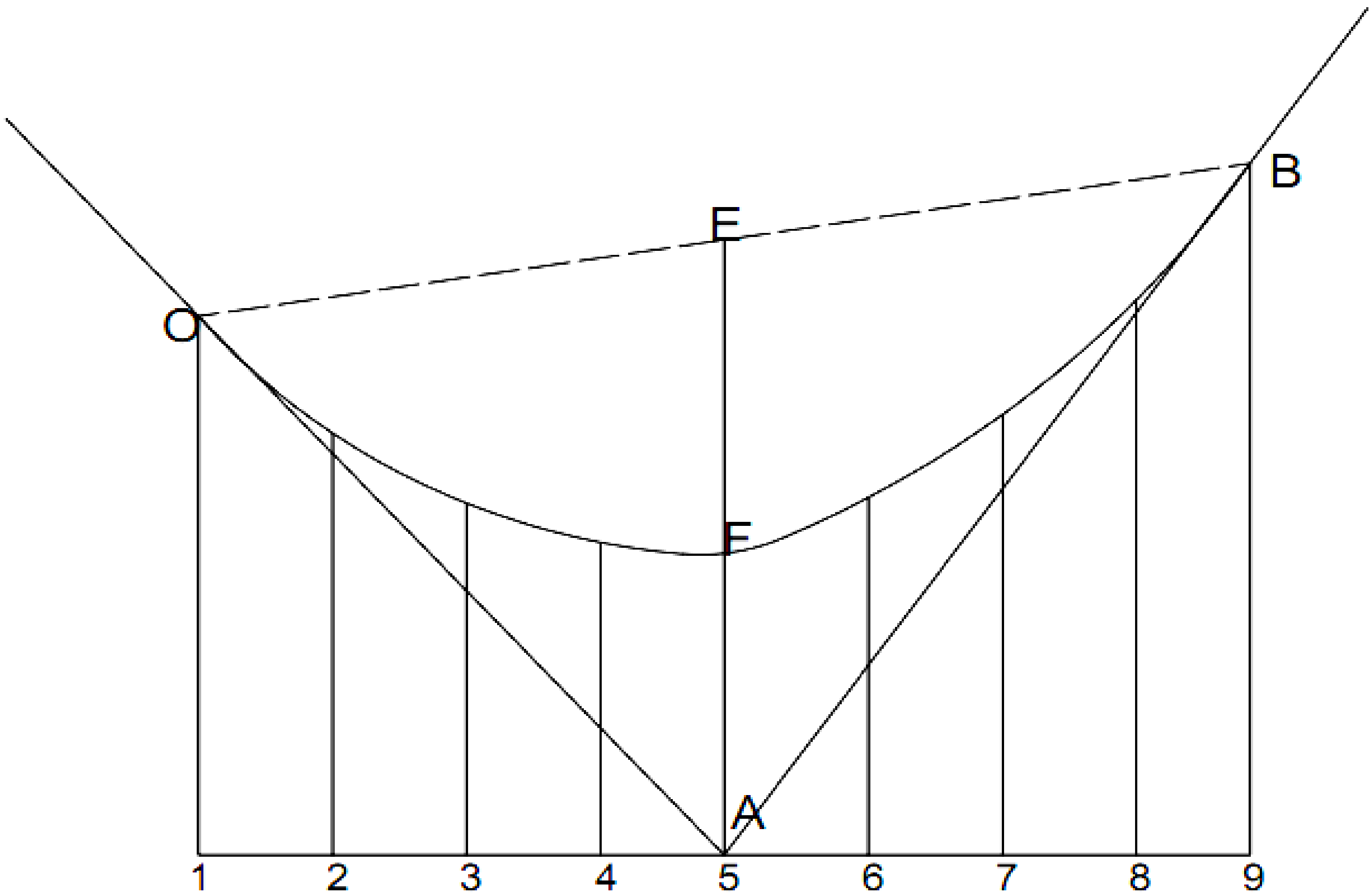
Types of Vertical Curves

1. An up grade followed by a down grade.
2. A down grade followed by an up grade.
3. An up grade followed by another up grade.
4. A down grade followed by another down grade.

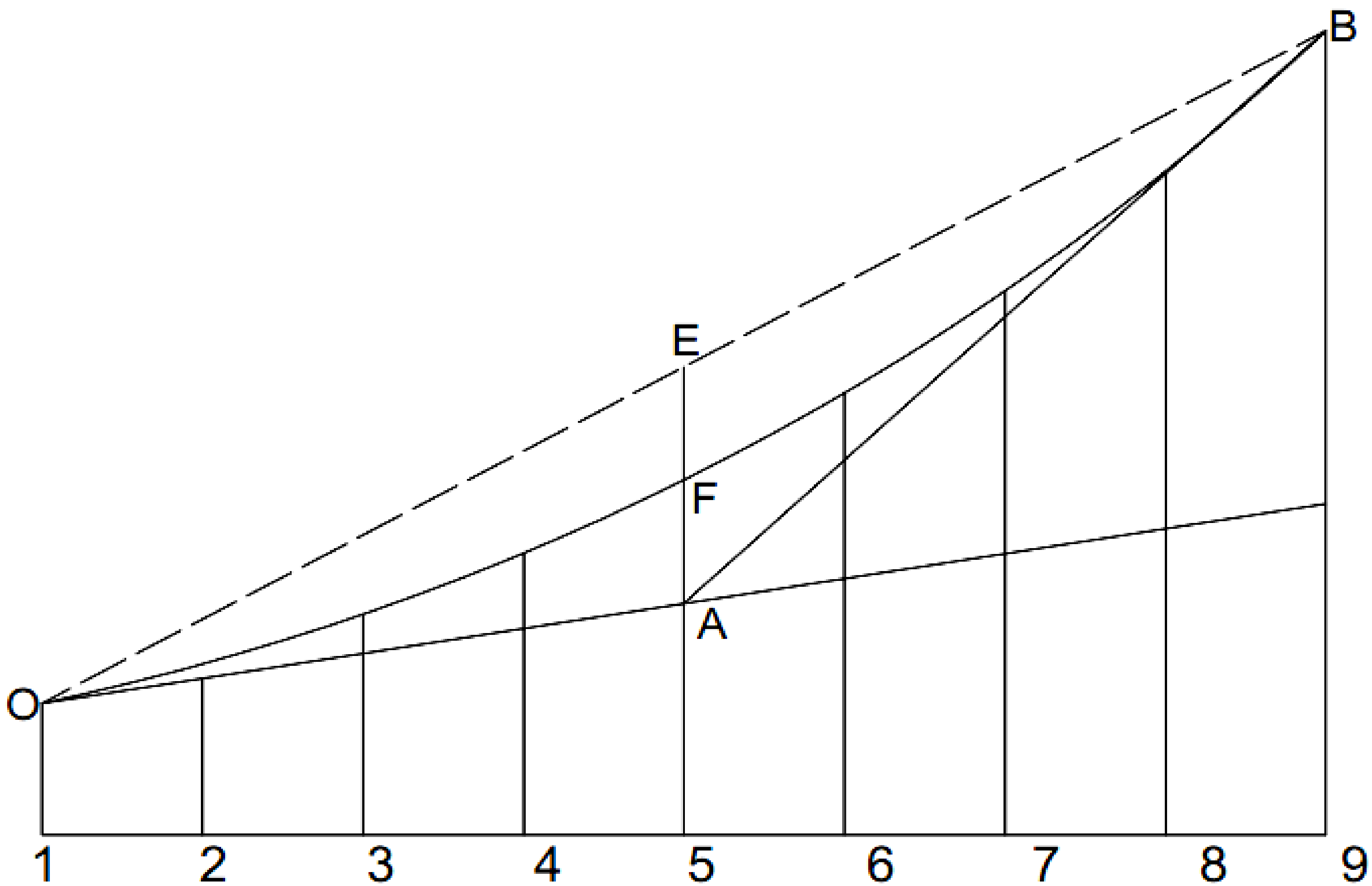
1. An up grade followed by a down grade.



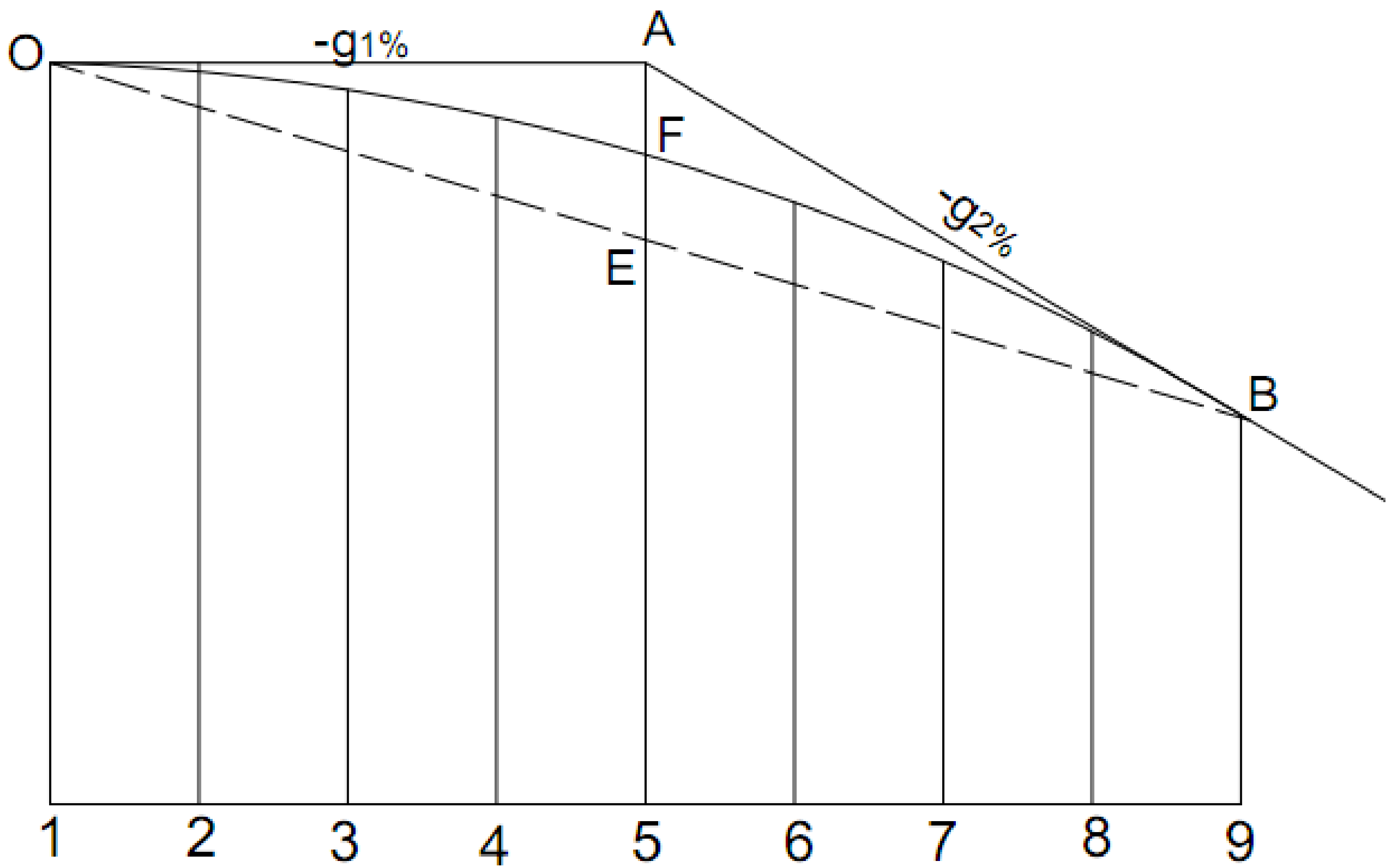
2. A down grade followed by an up grade



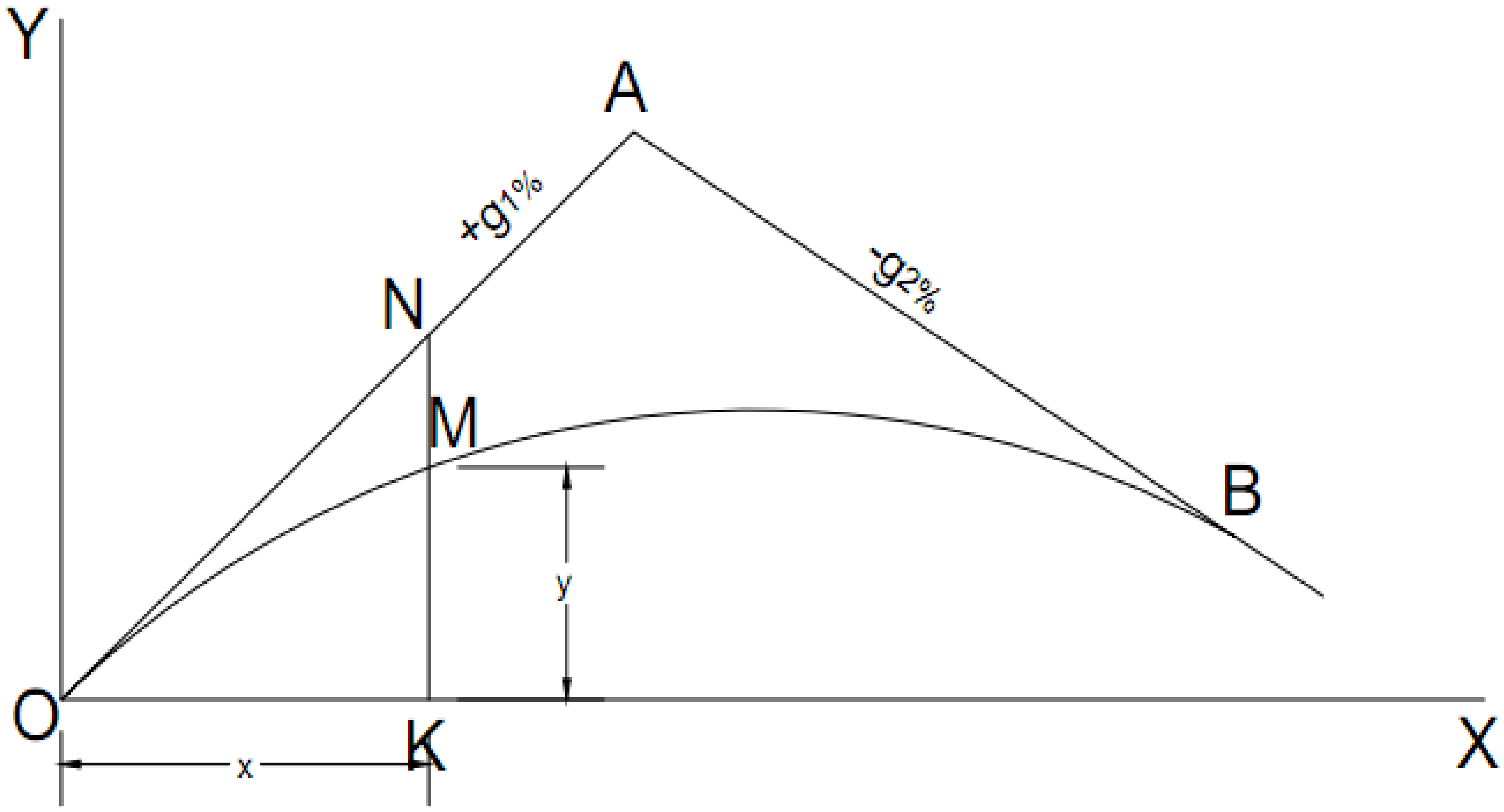
3.An up grade followed by another up grade.



4. A down grade followed by another down grade.



PROPERTIES OF PARABOLA:-



In fig,

OX and OY = the axes of the rectangular co-ordinates passes through the point (O) of the vertical curve.

O = origin of the co-ordinates.

OA and OB = the tangent to the vertical curve

+g1% = the grade of the tangent OB

-g2% = the grade of the tangent AB

M = Any point on the curve whose co-ordinates are x and y

Now, it may be shown that the equation of the parabola with respect to OX and OY is $y = cx^2 + g1x$

Now, OK = x; KM = y; KN = g1x; and NM = KN – KM

$$NM = g1x - y = -cx^2$$

From which it fallows that vertical distance from tangent to any point on the curve varies as the square of its horizontal distance from the point of commencement of the curve (the point of the tangency). This vertical distance is called as tangent correction.

$$y = -cx^2$$

When x=2l=L

$$y = \frac{(g1 - g2)}{400l} x^2$$

When x=ℓ, $y = AF = \frac{(g1 - g2)l}{400} = FE$

By knowing c, the tangent corrections may be computed in the following way:-

By tangent correction:-

Let the chainage and elevation of the apex is given

1) The length of the curve on either side of the A being ℓ m, determine the chainages of points of tangency (O and B)

chainage of O = chainage of A – ℓ.

chainage of B = chainage of A + ℓ.

2) by knowing the grades of the tangents OA and AB, and the elevation of A, compute the elevation of the tangent points O and B.

elevation of O = elevation of A – ($\ell g_1/100$)

elevation of B = elevation of A – ($\ell g_2/100$)

3) Compute the tangent correction from $y=cx^2$ for the stations on the curve $y_{x'} = cx'^2$

4) Determine the elevations of the corresponding stations on the tangent OAG

   Elevation of the tangent at any station = elevation of the point of tangency (O) + x'g1.

5) Find the elevations of the stations on the curve by adding algebraically the tangent corrections to the elevations of the corresponding stations on the tangent OA .

   Elevation of the station at a distance x' on the curve = elevation of the station on the tangent ± tangent correction $y_{x'}$

The result may be tabulated as under.

Station	Chainage	Tangent or grade elevation	Tangent correction	Elevation of the curve	Remarks

Location of Highest or Lowest Point

The position and elevation of the highest point (Summit) or the lowest point at the sag may be calculated as follows :

In fig. 1 & 2, let P be the required point at a distance x from the beginning A of the curve. The tangent to the curve at this point P being a horizontal line, its slope is zero.

The general equation of the parabola is $y = cx^2 + g_1x$.

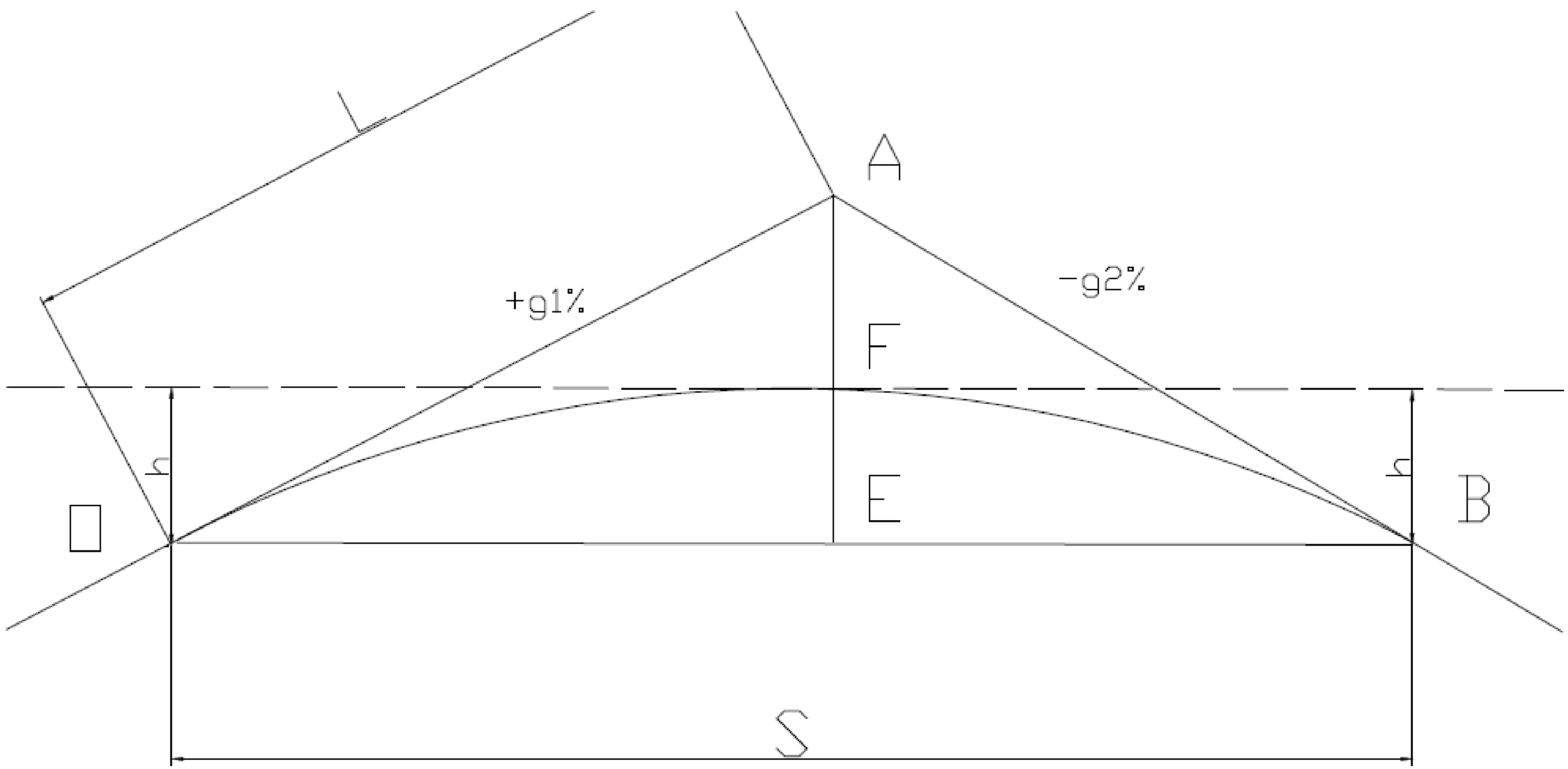
The slope of the tangent at any point on the parabola $= dy/dx$.

Now, $dy/dx = 2cx + g_1$

At summits where speeds of 100 kmph are contemplated the requirement of visibility i.e. the sight line will lead to longer curves than one obtained by the above formula.

Sight distances:-

Let two points on the curve at a height ‘h’ metres from the ground be intervisible and let the distance between be S. A value of 1.1 m is usually taken as the eye level height above the road surface for an observer sitting in a car. Sight distances are laid down in the interest of road safety and the choice for any distance depends on the nature of the road and the speed of the traffic using it.


L
A
+g1%
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F
h
E
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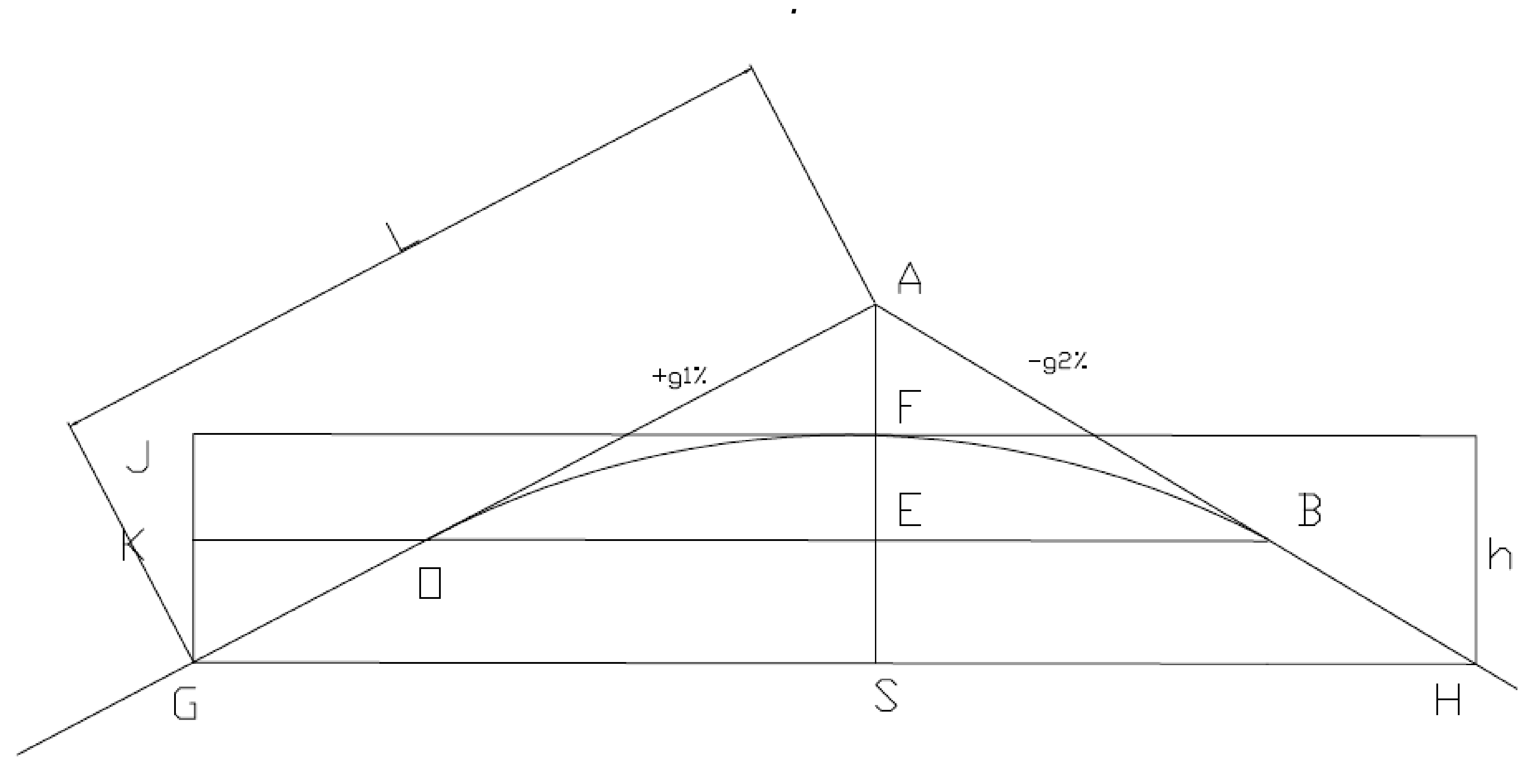
S


L
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S
H

the angle between OB and horizontal

$= L/100 \times (g_{1-}g_2)/2L$

$= (g_{1+}g_2)/ 200$ radians.

Angle KOG = $g_1/ 100 - (g_{1+}g_2)/ 200$

$= (g_1 - g_2)/ 200$ radians.

But OK = S/2 –L.

Therefore

$KG = (S/2 – L) ((g_1 - g_2)/ 200$

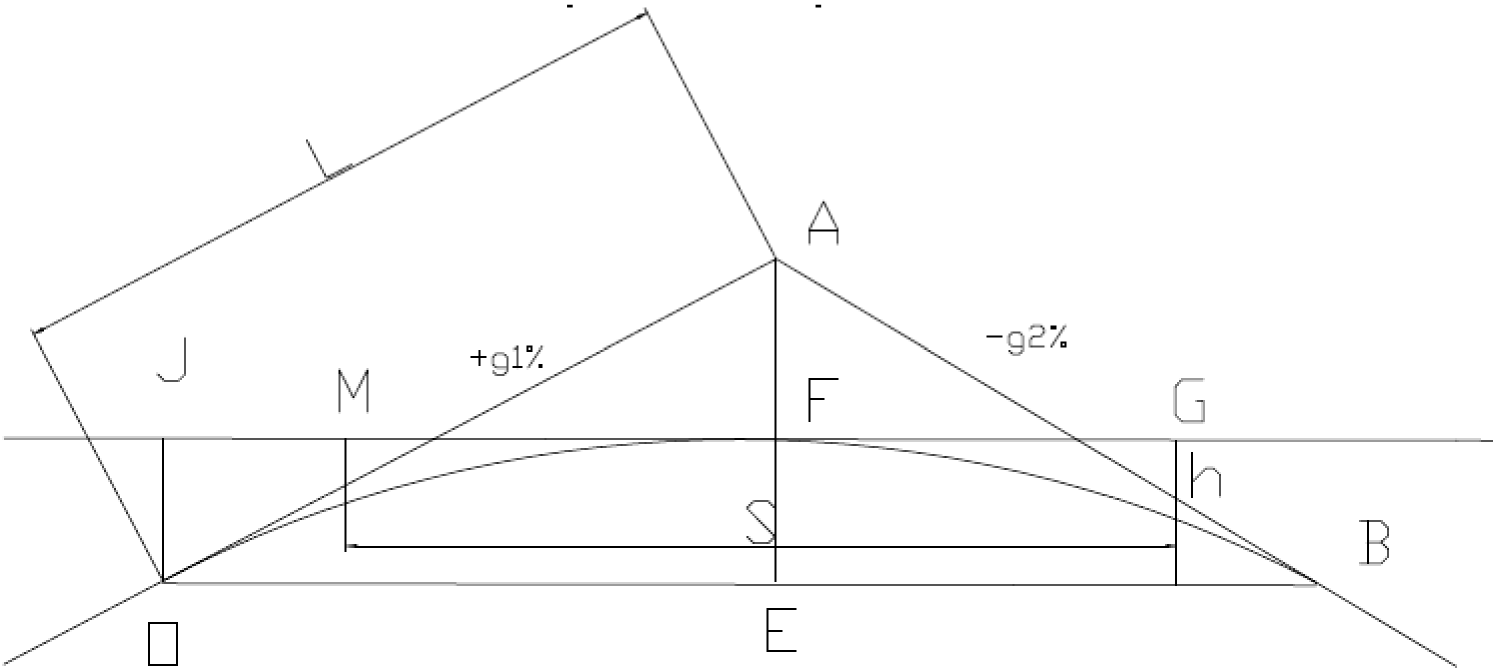
$h = KG + AF$

$= (S/2 – L) (g_1 - g_2 / 200) + = L/100 \times (g_{1-}g_2)$

$= ((S-L)/400) (g_{1-}g_2)$

(C) Sight distance less than length of the curve.

The offsets from the tangent MFG are given by



At point J, x =L .

$$JO = 4h /S^2 \ x \ L^2$$

But $JO = EF = AF = L/400 \ x \ (g_1 - g_2)$

Therefore,

$$4h /S^2 \ x \ L^2 = L/400 \ x \ (g_1 - g_2).$$

Therefore,

$$L = (g_1 - g_2) \ x \ S^2 /1600 \ x \ h \ .... (3)$$